Model limitations: Sequential data assimilation with uncertain static parameters

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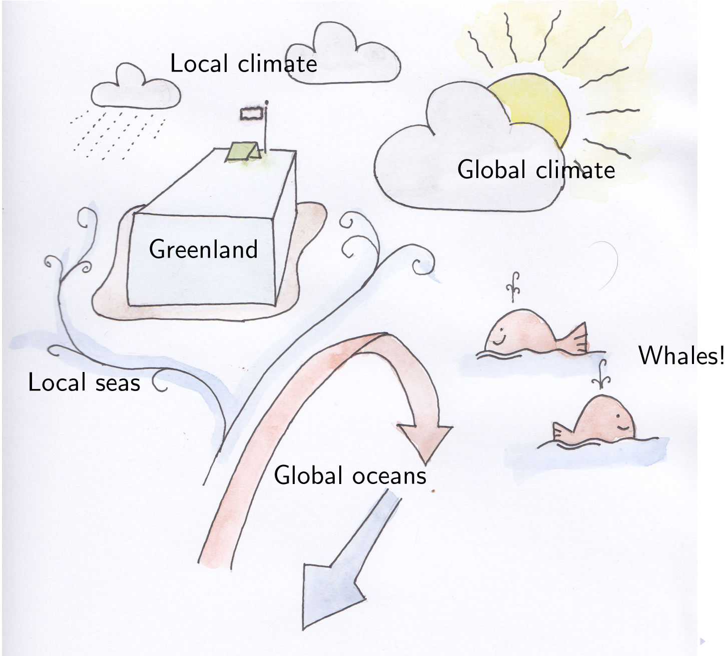
Illustration: the Greenland ice-sheet



Simplest interesting example

Conditional on θ:

$$\begin{aligned} x_0 &\sim \pi_{x_0}(\theta) && \text{(init. cond. unc.)} \\ x_t &= g(x_{t-1};\theta) + Q(x_{t-1};\theta)\,\omega_t && \text{(state eqn.)} \\ y_t &= f(x_t;\theta) + \nu_t && \text{(obs. eqn.)} \end{aligned}$$

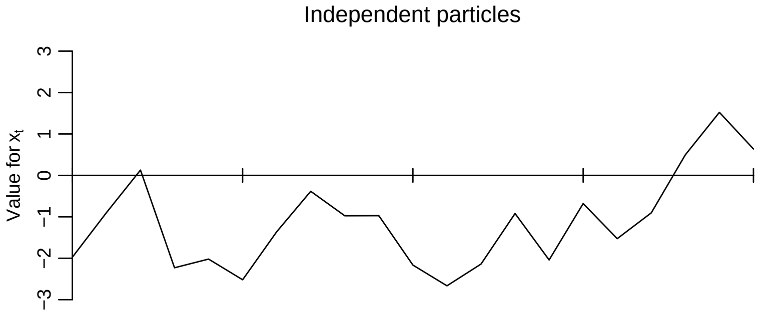
where

$$\begin{aligned} \omega_t &\overset{\text{iid}}{\sim} \mathrm{N}(0, I) && \text{(structural uncertainty)} \\ \nu_t &\overset{\text{iid}}{\sim} \mathrm{N}(0, v^2) && \text{(measurement unc.)} \end{aligned}$$

and then let $\theta \sim \pi_\theta$, to account for parametric uncertainty. The functions f, g, and Q are given, likewise the measurement uncertainty standard deviation, v.

Sampling from $\{x_{0:T}, \theta\} \mid y_{1:T}$ "intractable and unsolved" (C. Andrieu)

Particle filters, for given θ



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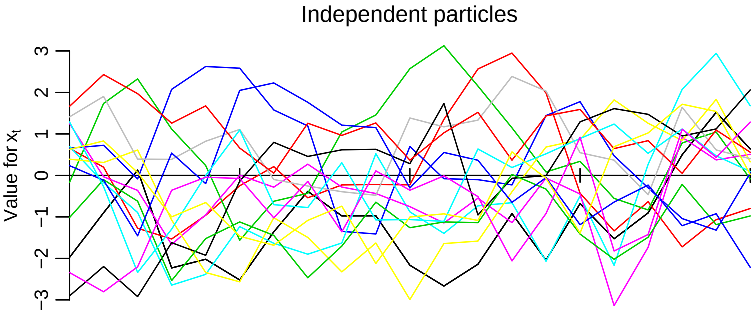

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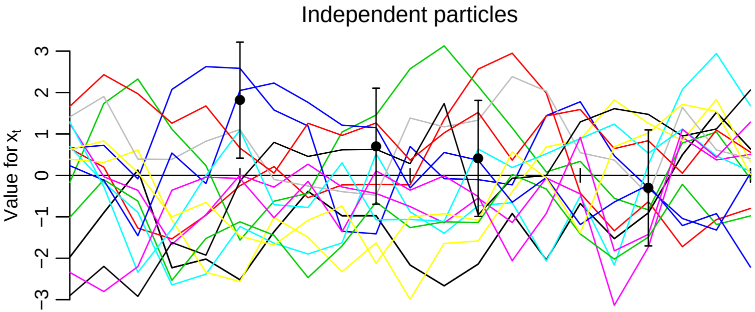

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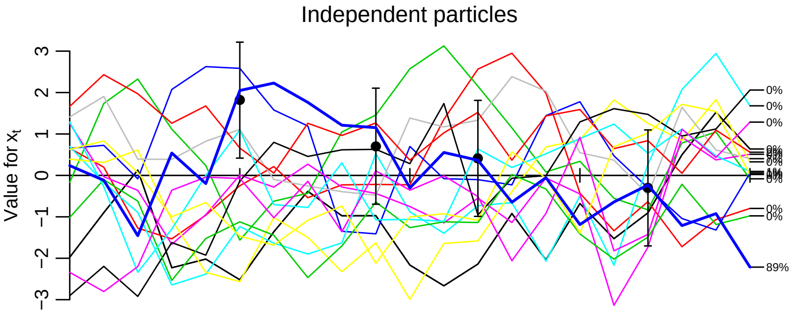

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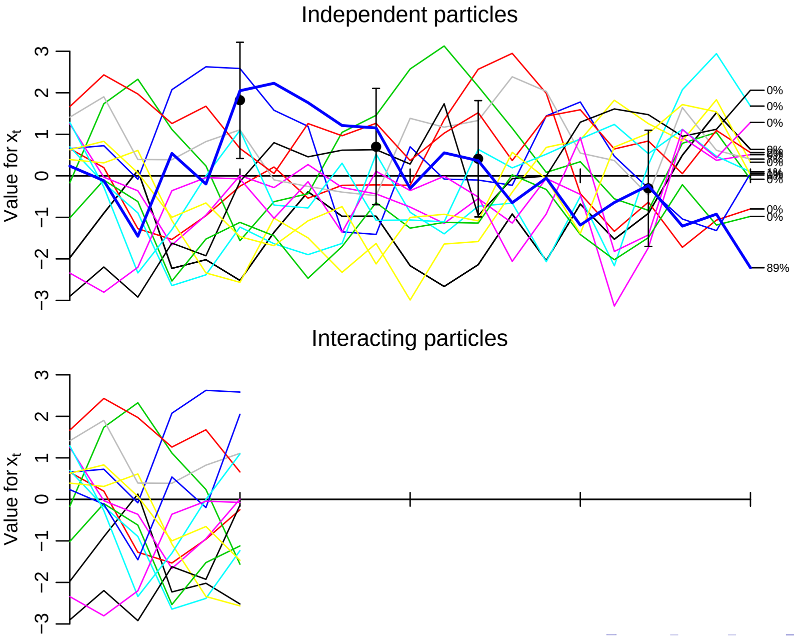

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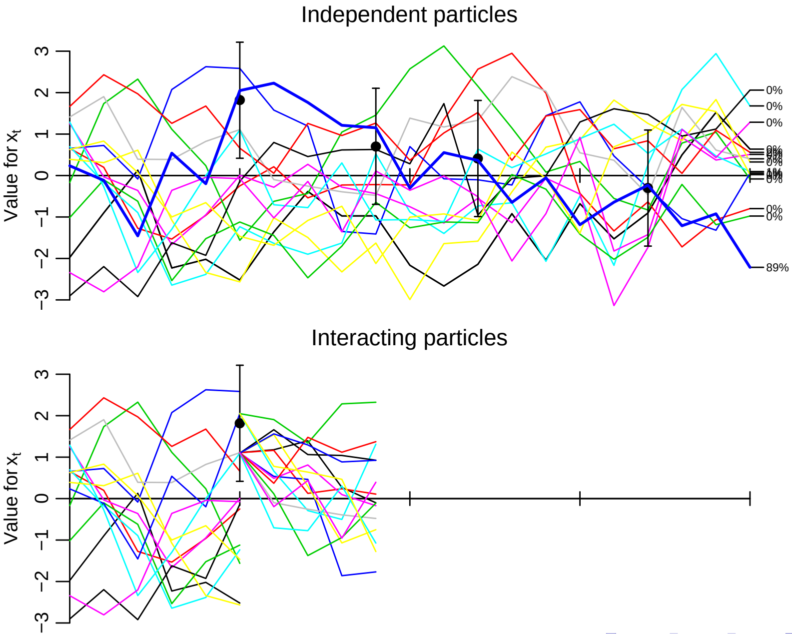

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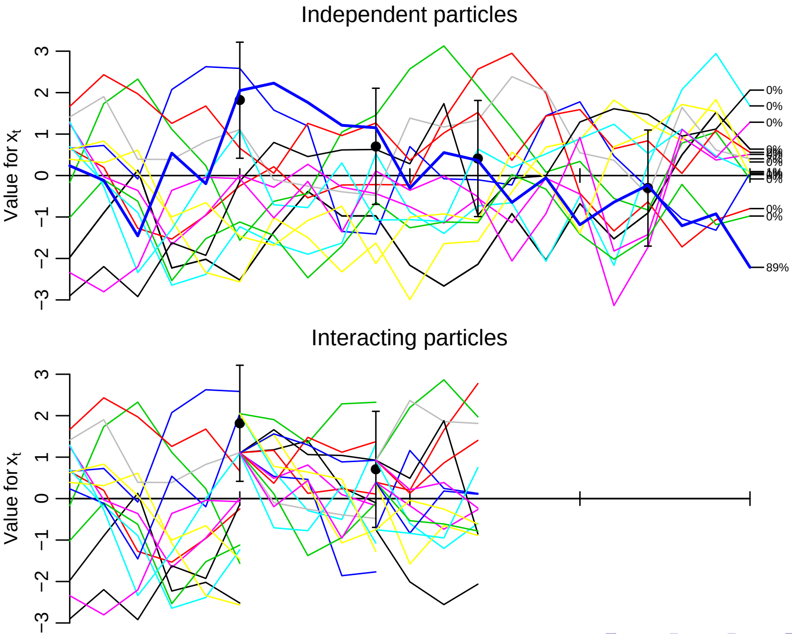

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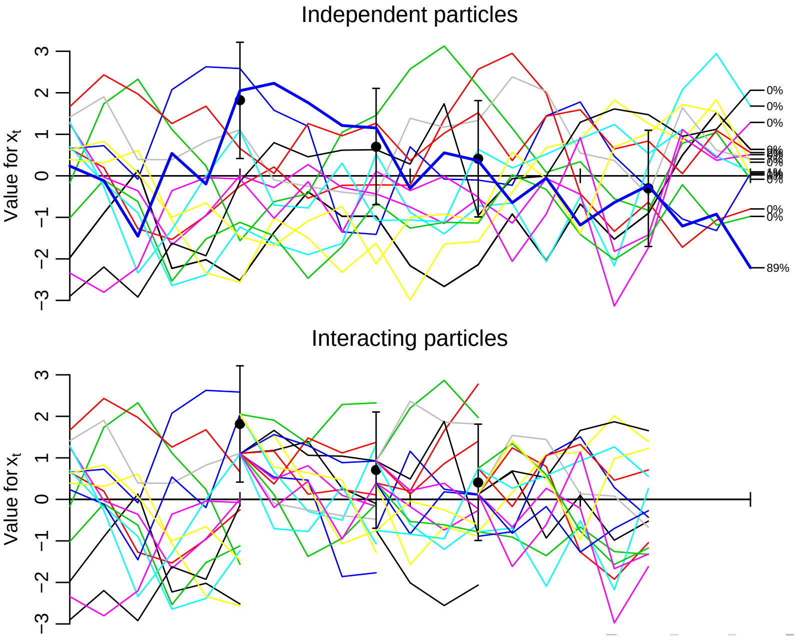

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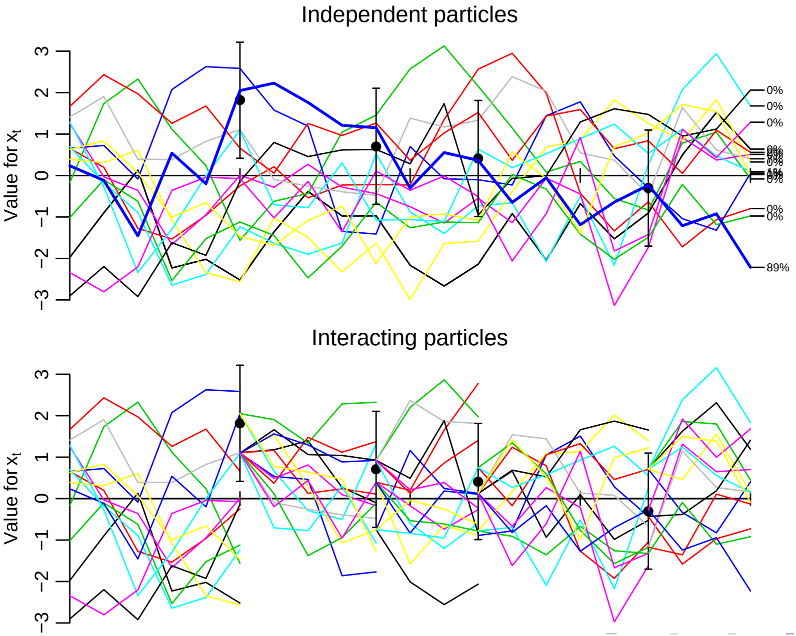

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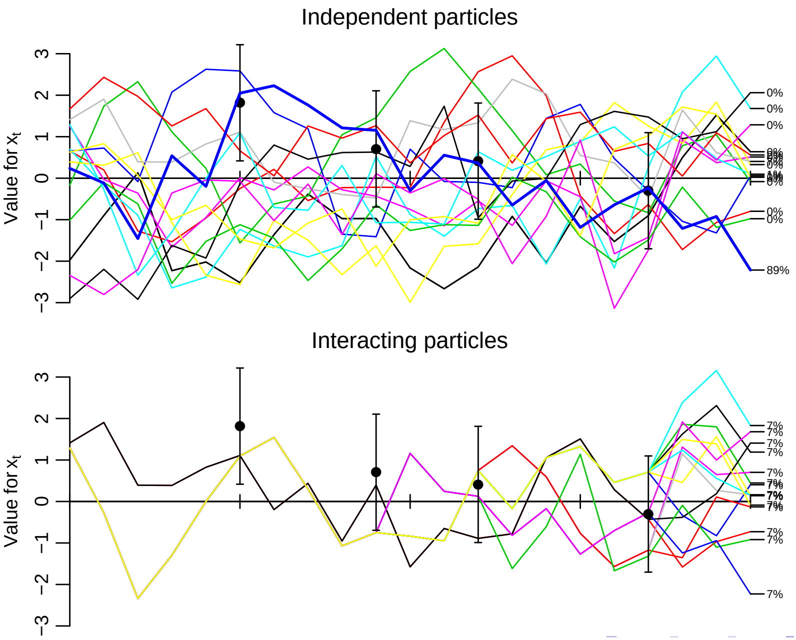

The difficulties with uncertainty θ

- One simple idea is to attach a realisation from π_θ to each particle, in order to sample jointly from $\{x_{0:T}, \theta\} \mid y_{1:T}$.

  However, static parameters do not evolve in time, so every interaction reduces the resolution of the θ distribution. Too many observations, and the θ distribution becomes degenerate, unless we have `<DrEvil>`one million`</DrEvil>` particles.

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- The solution is to 'integrate out' the state vector in some form. The two approaches are

  1. Gaussian (Laplace) approximation for $x_{1:T} \mid \{\theta, y_{1:T}\}$ turning a high-dimensional integration into a high-dimensional optimisation;
  2. Particle Markov chain Monte Carlo (P-MCMC), which uses a Gibbs sampler to swap between sampling $x_{1:T} \mid \{\theta, y_{1:T}\}$ and $\theta \mid \{x_{1:T}, y_{1:T}\}$.

Summary

In inference for environmental systems, 'high dimensional data' is ~ 1000 observations. This is at the limit of what we can compute.

- Particle filters, the most general tool for sampling from $x_{1:T} \mid \{\theta, y_{1:T}\}$, adapt naturally to parallel implementation;
- MCMC for $\theta \mid y_{1:T}$ or $\theta \mid \{x_{1:T}, y_{1:T}\}$, can also be implemented in parallel, using the approach of Cui *et al*.

Small state vectors, though!

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Two useful references

C. Andrieu *et al*, 2009, Particle Markov chain Monte Carlo methods, forthcoming in the *Journal of the Royal Statistical Society, Ser. B*, available at
`http://www.rss.org.uk/pdf/Andrieu_et_al._14.10.09.pdf`

T. Cui *et al*, 2009, Using MCMC Sampling to Calibrate a Computer Model of a Geothermal Field, available at `http://www.stats.ox.ac.uk/~nicholls/linkfiles/papers/cui09.pdf`