

# Simple models for glacial cycles

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# A little background

## Reductionist models

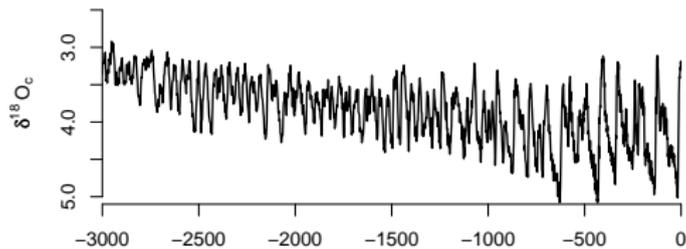
Implement physical principles at the micro-scale (e.g. symmetry or conservation, continuity). The correct macro-scale behaviour of the state vector is expected to emerge as a consequence of the correct physical principles being implemented.

## Phenomenological models

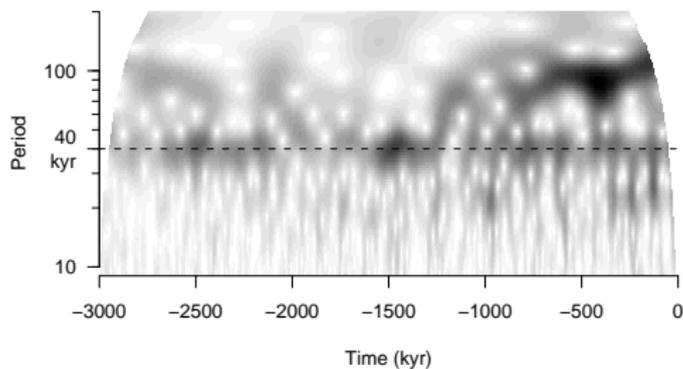
Implement observed macro-scale regularities directly into the evolution equations of a highly aggregated state vector. The choice of equation structure is more subjective, since many different structures can be consistent with the same regularities.

# Visualising the glacial cycle

Benthic Foraminifera stack (LR04)



Continuous Morlet Wavelet Transform (Amplitude)



# Saltzman & Maasch's 1991 model (SM91)

Climate Dynamics 1991, 1, 101-110

Climate  
Dynamics  
1991, 1, 101-110

## A first-order global model of late Cenozoic climatic change II. Further analysis based on a simplification of CO<sub>2</sub> dynamics\*

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**Abstract.** A model developed recently for the long-term evolution of global climate, which includes and extends a model developed through the use of a two-box ocean-atmosphere-ice system, is further analyzed. The model includes a first-order approximation of the CO<sub>2</sub> cycle, which includes the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics. The model is used to analyze the evolution of the CO<sub>2</sub> cycle and the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics. The model is used to analyze the evolution of the CO<sub>2</sub> cycle and the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics.

Major, based on the assumption that the largest and the smallest of the model would occur for the case of a constant CO<sub>2</sub> concentration. The model is used to analyze the evolution of the CO<sub>2</sub> cycle and the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics. The model is used to analyze the evolution of the CO<sub>2</sub> cycle and the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics.

**The authors declare no conflict of interest.**

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### Introduction

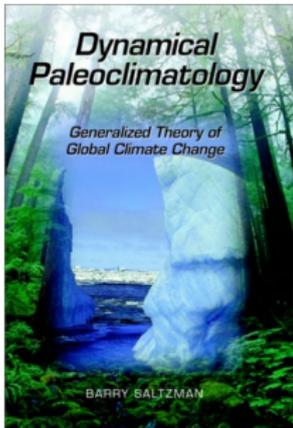
In a recent paper (Saltzman and Maasch, 1991, SM91) we developed a global climate model which includes the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics. The model is used to analyze the evolution of the CO<sub>2</sub> cycle and the effects of the CO<sub>2</sub> cycle on the climate system, including the effects of the CO<sub>2</sub> cycle on the ocean circulation and the effects of the CO<sub>2</sub> cycle on the ice sheet dynamics.

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$$\frac{dI'}{dt} = -a_1 [k_{\mu}\mu' + k_{\theta}\theta' + k_R R'(t)] - K_I I'$$

$$\frac{d\mu'}{dt} = b_1\mu' - b_2(\mu')^2 - b_3(\mu')^3 - b_{\theta}\theta'$$

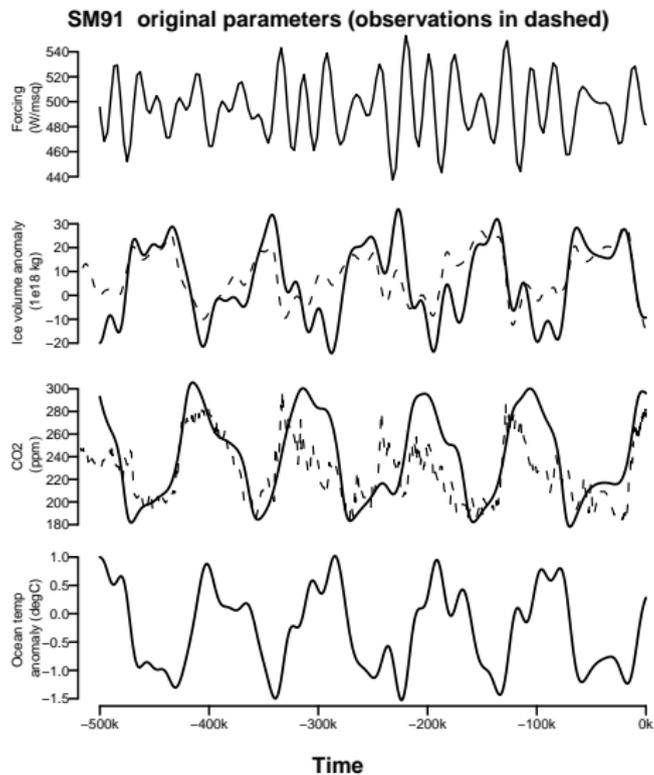
$$\frac{d\theta'}{dt} = -c_I I' - K_{\theta}\theta'$$



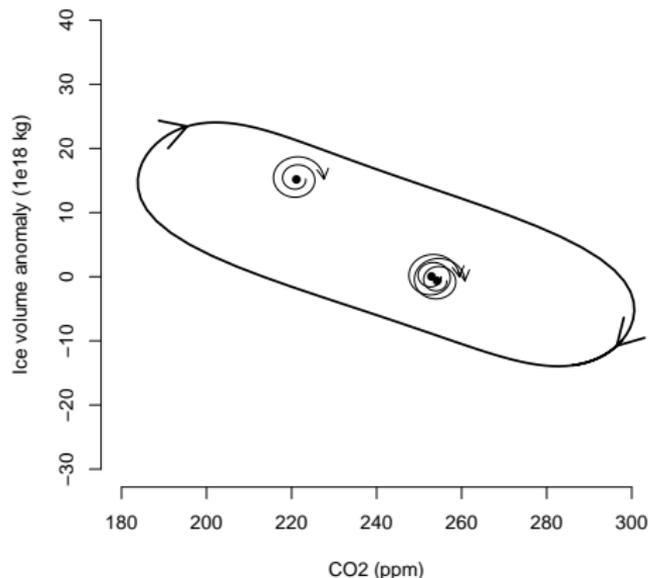
where  $I$  Ice volume,  $10^{18}$  kg  
 $\mu$  CO<sub>2</sub>, ppmv  
 $\theta$  Deep ocean temp., °C  
 $R(t)$  Solar forcing, W/m<sup>2</sup>.

Primes indicate variables centered around the tectonic mean. The coefficients  $a_1$ ,  $b_1$ ,  $b_2$ ,  $b_3$ ,  $b_{\theta}$ , and  $c_I$  are 'tunable'.

# SM91's coefficient choices



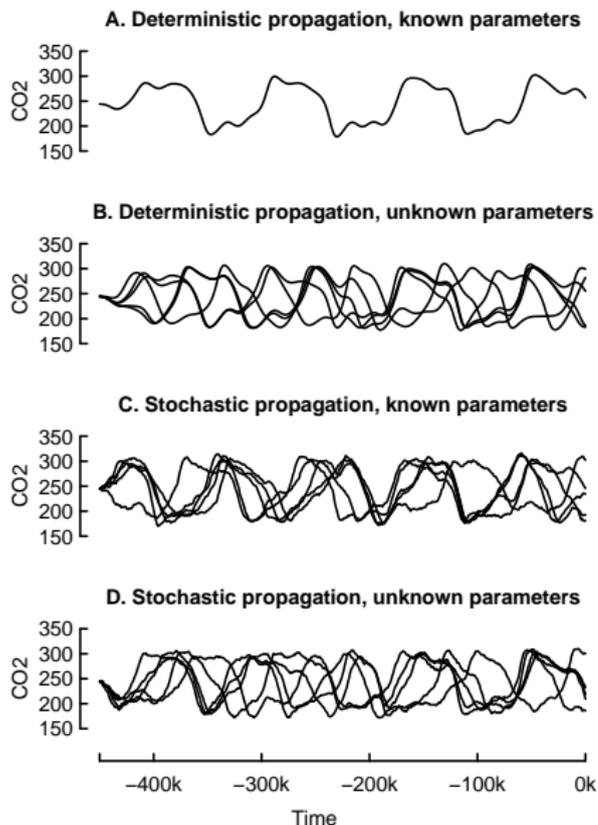
# A little dynamical systems theory



To start with, set  $R'(t) = 0$ . Write  $x' = (I', \mu', \theta')$ . Solving  $dx'/dt = \mathbf{0}$  gives three fixed points. At the SM91 parameters, these are all *unstable*. This means that the state vector is periodic.

Adding orbital forcing perturbs these fixed points through time and makes the state vector follow a much more interesting trajectory.

# 'Noising up' the SM91 model



We treat the SM91 model as the drift term in a stochastic differential equation with Itô form (conditional on  $\Psi$ )

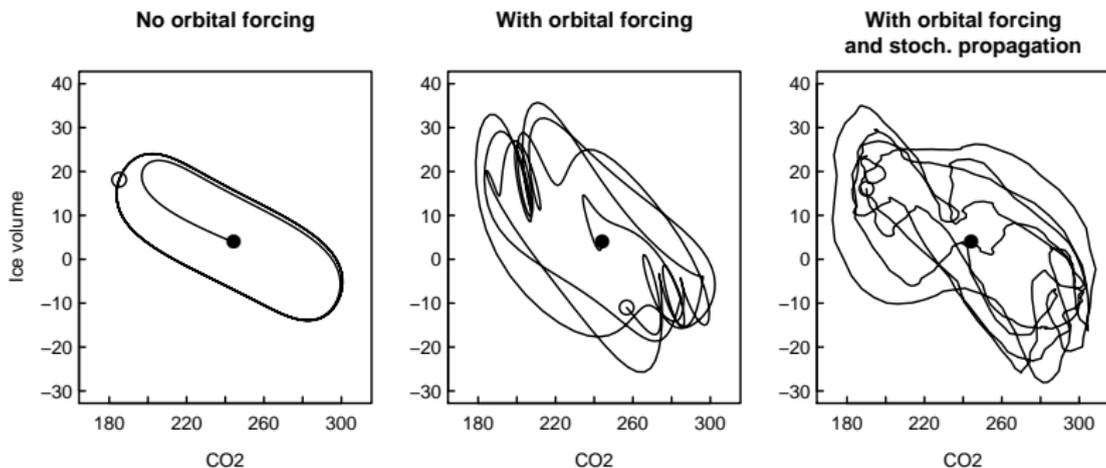
$$dX(t) = f(X(t), t, \Psi) dt + \Sigma^{1/2} dW(t)$$

where  $\Psi = (a_1, \dots, c_l)$ , and  $W(t)$  is a vector of independent Brownian motions.

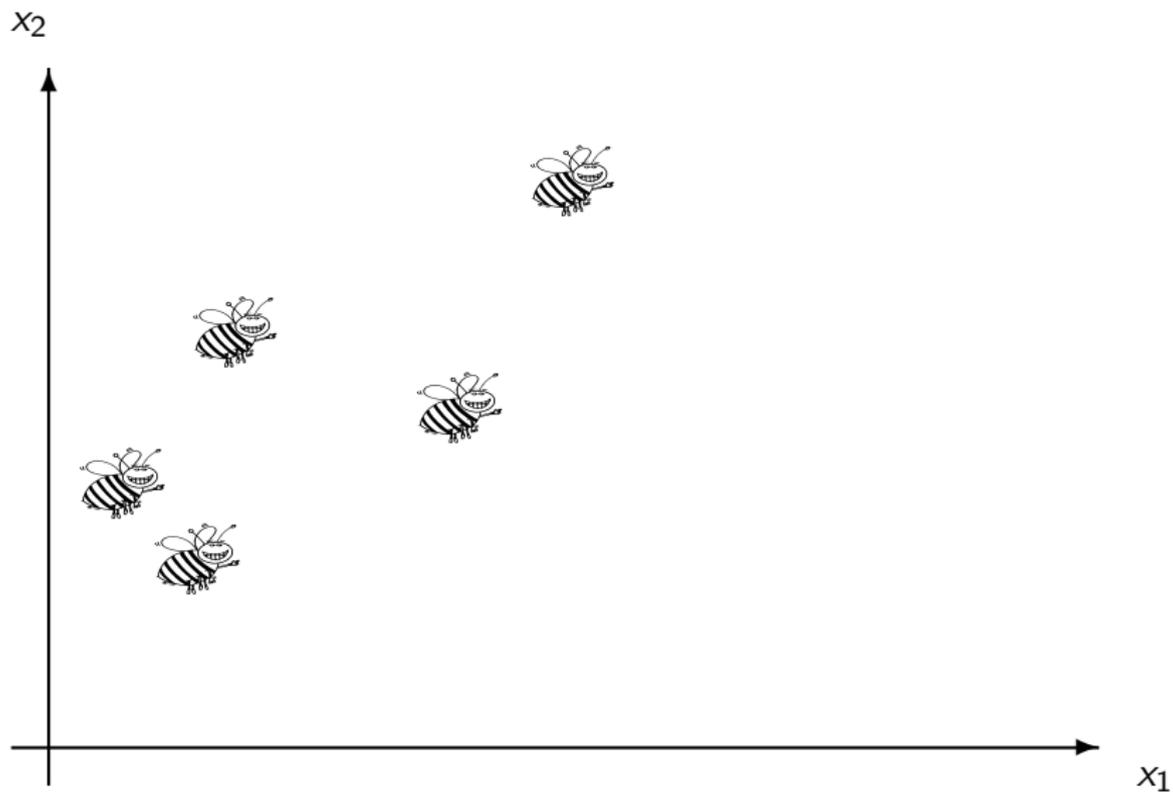
We choose  $\pi(\Psi)$  and  $\Sigma$  so that the sample paths of  $X(t)$  look 'about right'.

## 'Noising up' the SM91 model (cont)

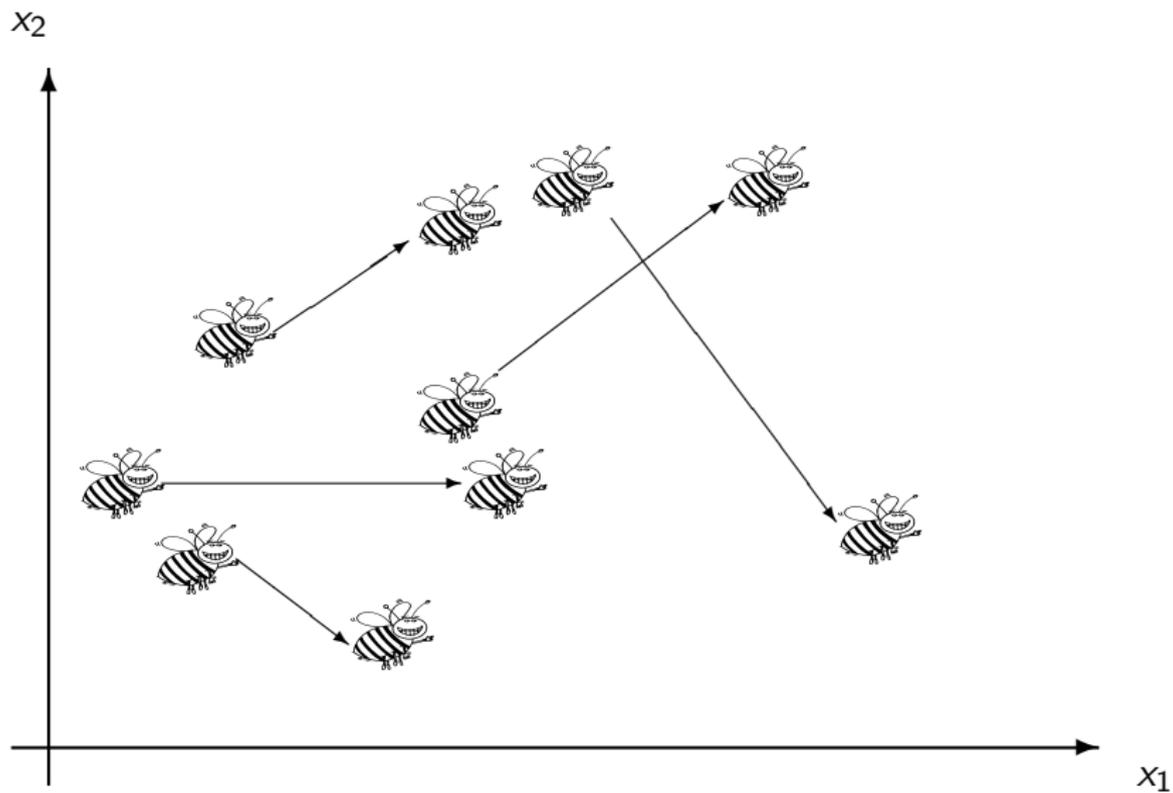
Fixing the parameters  $\Psi$  at the SM91 values, here we can see the effect of orbital forcing on the model dynamics, and the additional effect of stochastic propagation, in  $\text{CO}_2$  / Ice volume space.



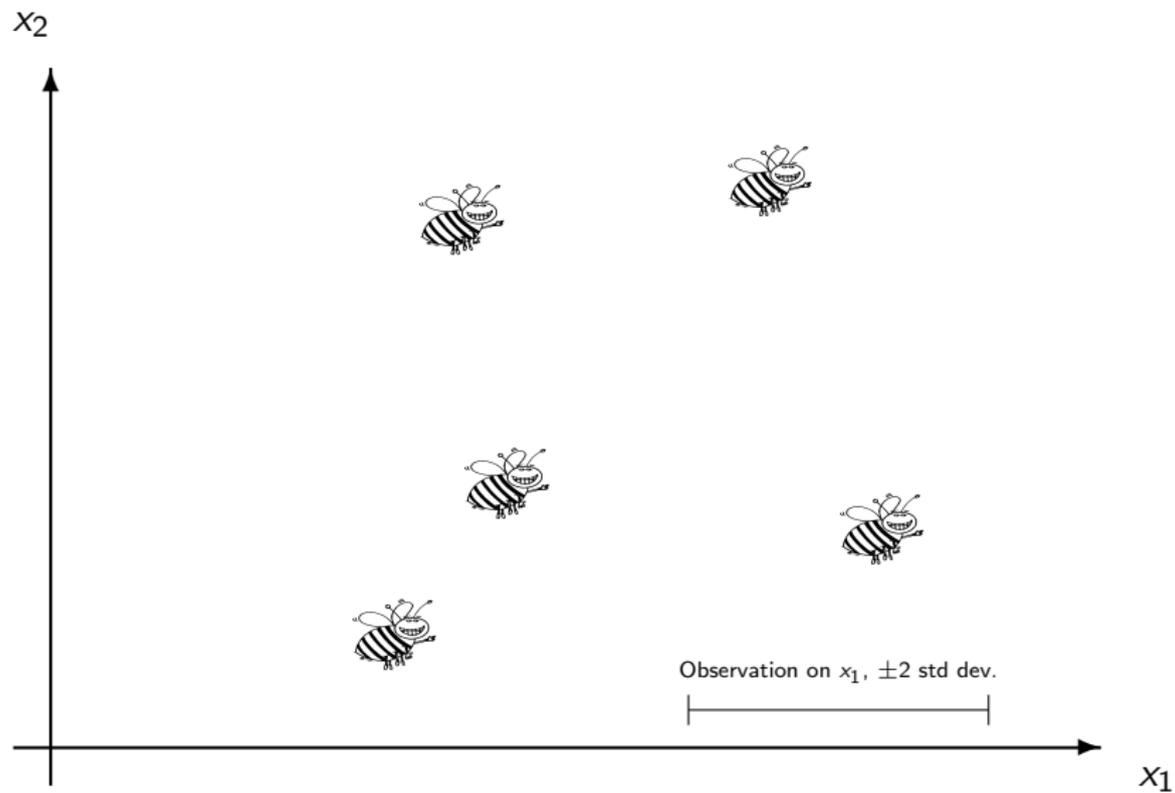
# Particle filters in one slide



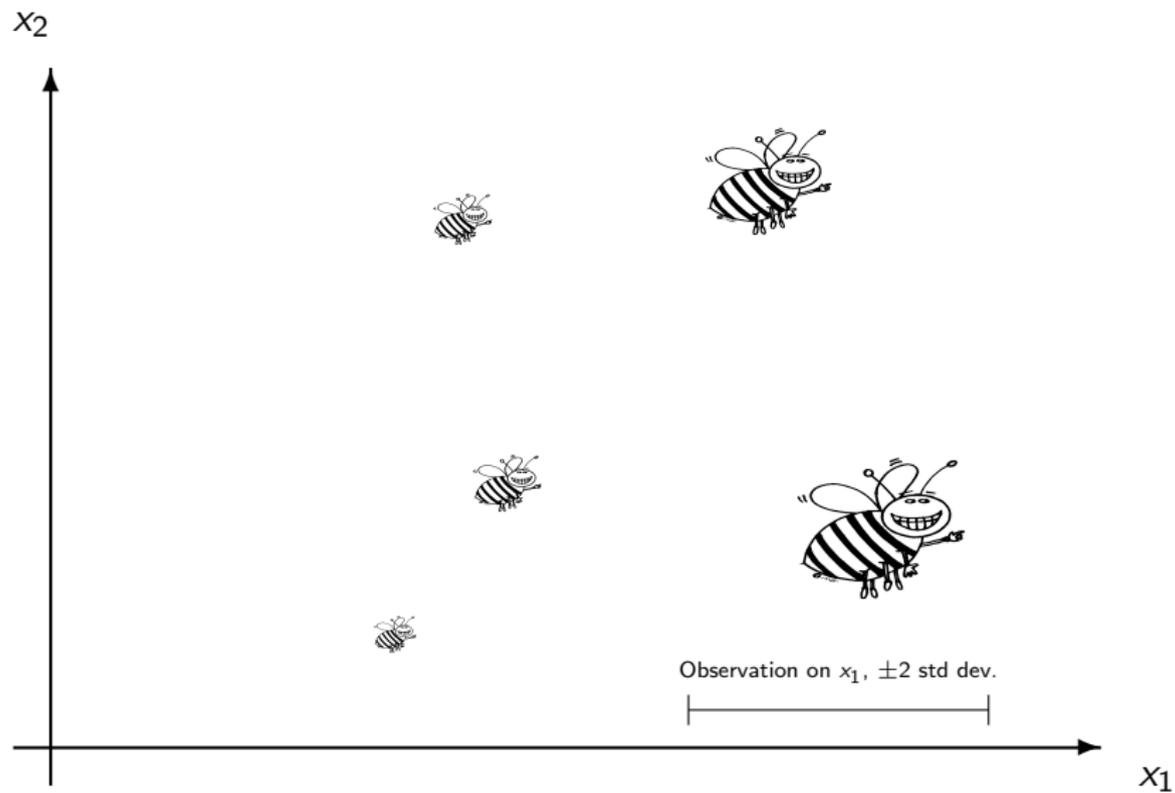
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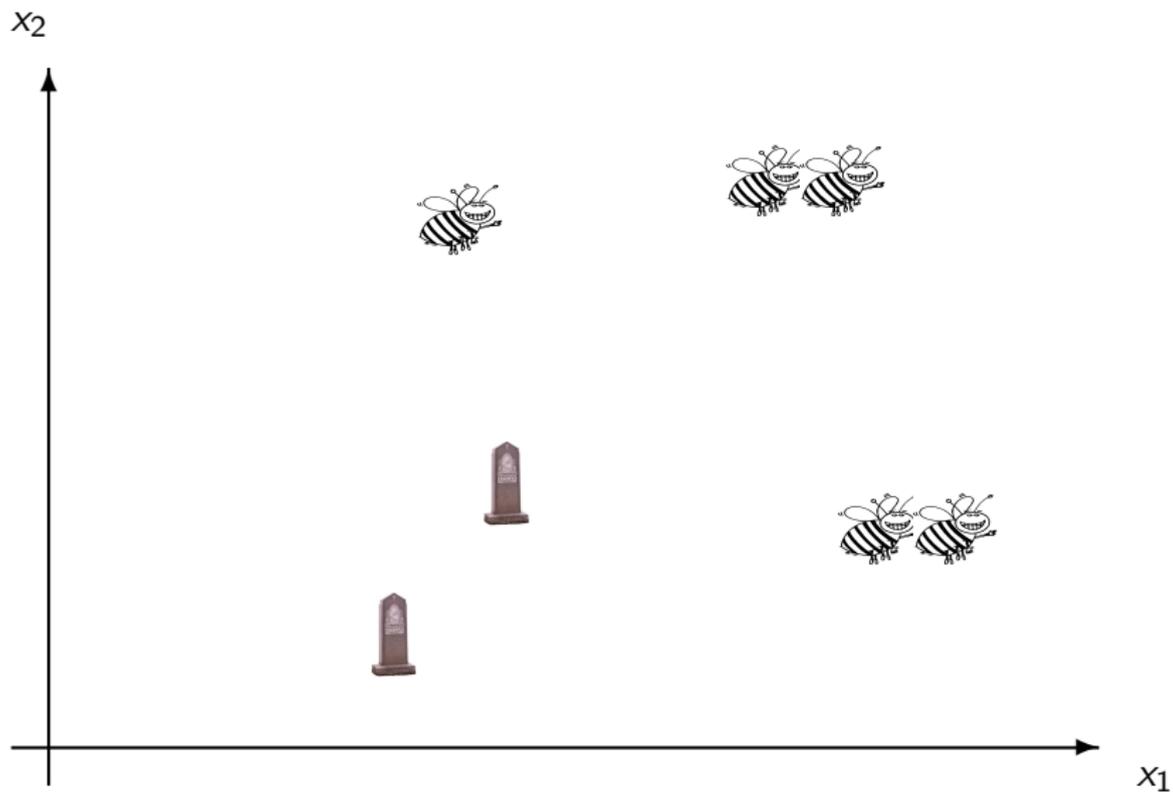
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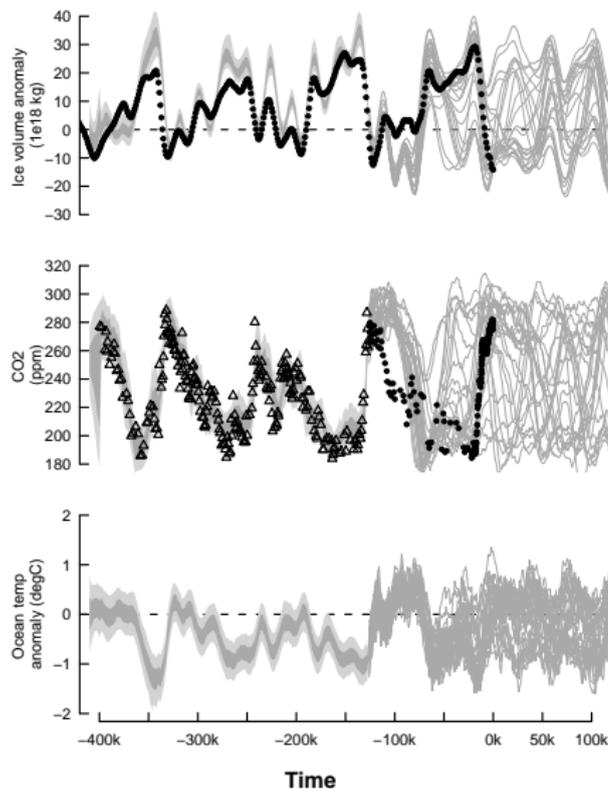
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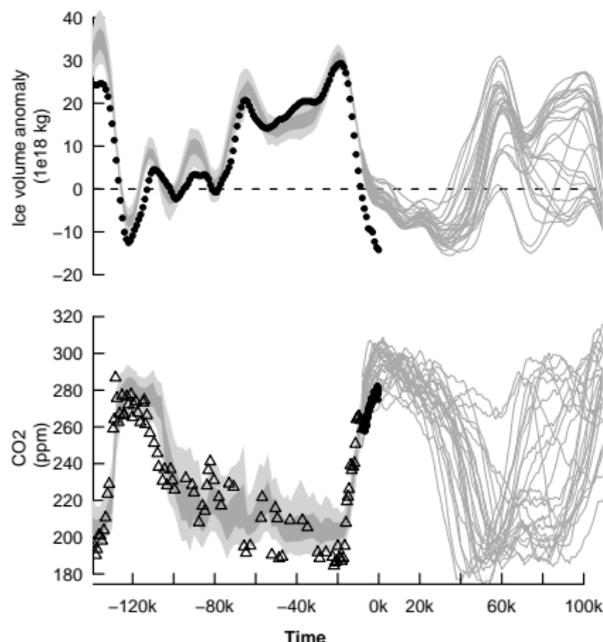


## Validation: predicting beyond –126 kyr



We initialise the model at –450 kyr, and then run forward until –126 kyr, simultaneously learning about  $\Psi$  and the state vector, using a *particle filter*. Our training data is CO<sub>2</sub> from the Vostok ice-core. Then we predict forwards, presented as samples selected from the posterior distribution. The fit to the Benthic stack data (ice volume proxy) is remarkable. But CO<sub>2</sub> is slow in responding to the deglaciation. The predictability horizon is about 50 kyr.

## Ruddiman's hypothesis, predicting beyond $-8$ kyr



Training on  $\text{CO}_2$  up to  $-8$  kyr. Ruddiman claims that after this time human activity (mainly rice cultivation in Asia) perturbed the climate and delayed the inception of the next glaciation. But according to our analysis, it seems as though this is not due for another 50 kyr.

## Beyond 'proof of concept'

This work is preliminary. The main issues we need to address more carefully are (thinking more widely about phenomenological models represented as dynamical systems):

- ▶ A generalisation to endogenous (i.e. state-dependent) deposition rates or their equivalent, to account for dating/timing uncertainty;
- ▶ More transparent approaches for choosing  $\pi(\Psi)$  and  $\Sigma$ , which are sensitive to the dynamical aspects of the model;
- ▶ A far more sophisticated learning framework (almost certainly a hybrid particle filter / MCMC);
- ▶ Techniques for parallelising the code, notably load-balancing on clusters.

All of these bits are 'sort of' in place, but the challenge is to combine them together.