A new statistical framework for analysing multi-model ensembles

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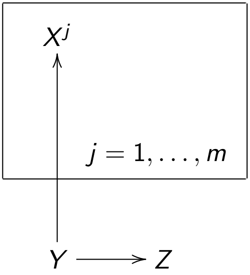
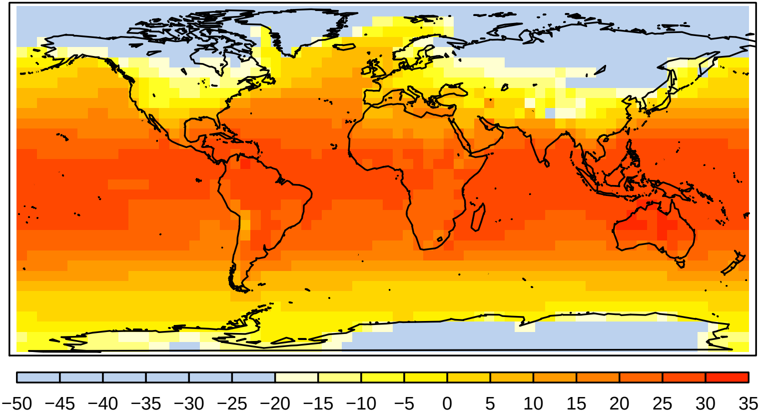
Two factorisations



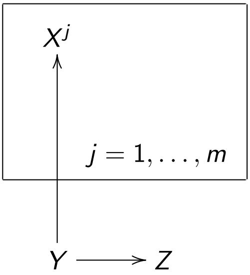
Standard approach (e.g. Smith et al, 2009, JASA)

X^j = model j; Y = actual climate; Z = climate observations.

HadCM3, DJF atmospheric temperature, 1980−1999



Two factorisations



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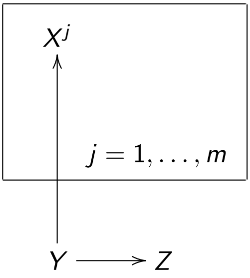

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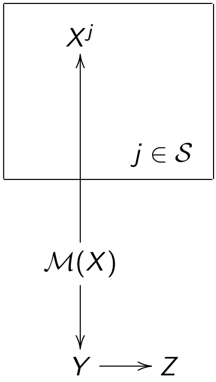

Cool new approach, $\mathcal{S} \subseteq \{1, \ldots, m\}$.

X^j = model j; Y = actual climate; Z = climate observations.

Structural statistical principle

There is a subset $\mathcal{S}$ of the simulators which we are prepared to treat as (i) second-order exchangeable, and actual climate (ii) respects exchangeability with these simulators.

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Consequences

$$\text{(i)} \quad X^j = \mathcal{M}(X) + \mathcal{R}^j(X) \qquad j \in \mathcal{S}$$

$$\text{(ii)} \quad Y = A\mathcal{M}(X) + U$$

where $\mathcal{M}(X)$ is the 'representative' simulator and $\mathcal{R}^j(X)$ and U are 'residuals'; A is specified (i.e. not uncertain).

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Objects

We take $A = I$ and then need to specify:

1. The mean and variance for $\mathcal{M}(X)$;
2. $\mathrm{Var}\{\mathcal{R}(X)\}$, same for all $j \in \mathcal{S}$, the *residual variance*;
3. $\mathrm{Var}(U)$, the *discrepancy covariance*.

Implications of our statistical model

1. Define the discrepancy as

$$D^j := Y - X^j = U - \mathcal{R}^j(X).$$

Then

$$\mathrm{Cov}(D^i, D^j) = \mathrm{Var}(U) \qquad i \neq j.$$

The discrepancies for different simulators are correlated iff the representative simulator and actual climate are not equal.

Implications of our statistical model (cont)

2. Define the ensemble mean as

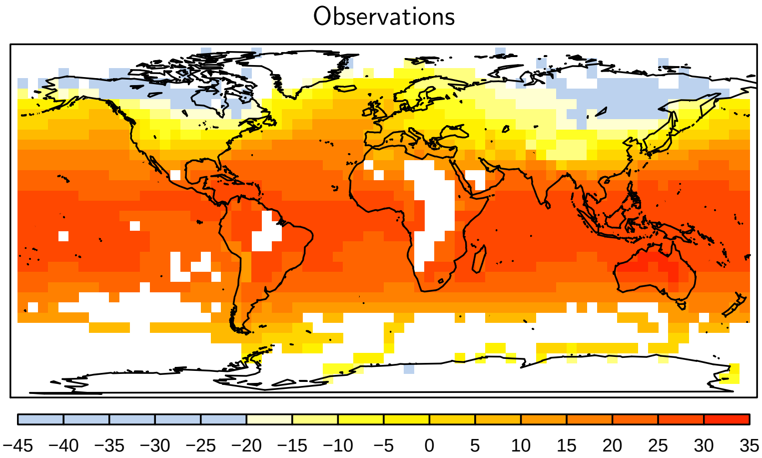
$$\bar{X} := m_S^{-1} \sum_{j \in \mathcal{S}} X^j = \mathcal{M}(X) + m_S^{-1} \sum_{j \in \mathcal{S}} \mathcal{R}^j(X).$$

Then

$$E(Y - \bar{X}) = \mathbf{0}$$
$$\mathrm{Var}(Y - \bar{X}) = \mathrm{Var}(U) + m_S^{-1} \mathrm{Var}(\mathcal{R}(X))$$

The ensemble mean is 'unbiased', and performs better than any ensemble member, but in general its error does not go to zero as $m_S \to \infty$, but to $\mathrm{Var}(U)$.

Application: reconstructing mean DJF atmospheric temperature, 1980-1999



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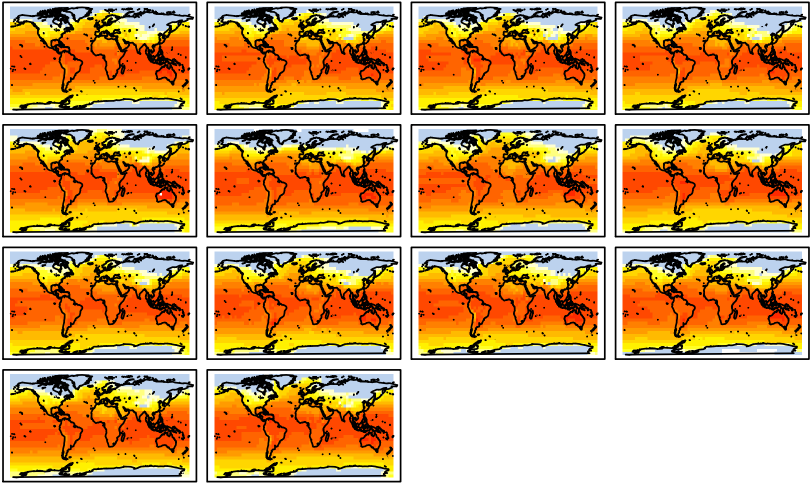
Exchangeable ensemble



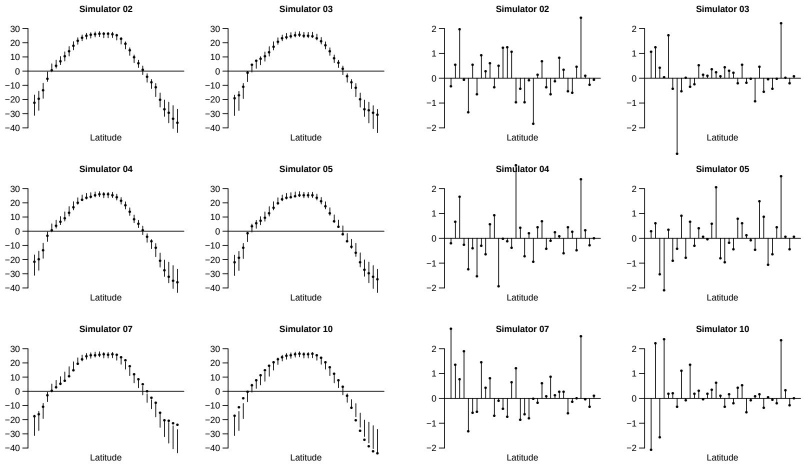
Specification

`REM: all variances must be coherent on the 2-sphere!`

1. $E\{\mathcal{M}(X)\}$: specified for zonal means using an EBM
2. $\mathrm{Var}\{\mathcal{M}(X)\}$: $\pm 10°$C for the zonal means
3. $\mathrm{Var}\{\mathcal{R}(X)\}$: mainly the sample variance of the ensemble
4. $\mathrm{Var}(U)$: judgements of simulator quality for zonal means

Diagnostics

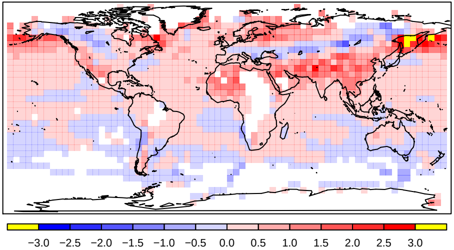
Leave-one-out assessment of the MME (first six simulators)



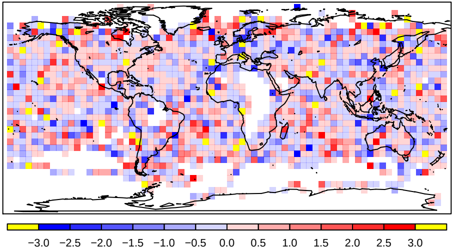
Diagnostics

Mean and variance of the observations, adjusted by the MME.

Marginal standardised prediction errors



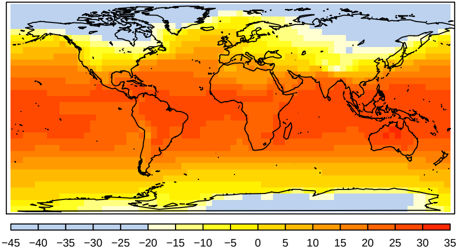
Joint standardised prediction errors



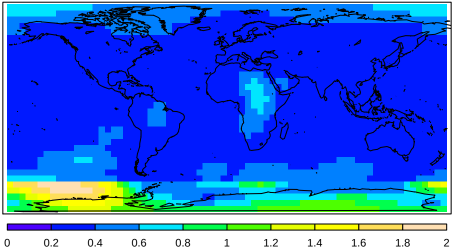
Reanalysis results

Mean and std dev. of climate adjusted by the MME and the observations.

Mean surface temperature field (degrees Celcius)



Std dev. surface temperature field (degrees Celcius)



Summary

Our approach has:

1. A different factorisation of the joint distribution to reflect how climate scientists actually use climate simulators.

2. A simple and intuitive statistical model, requiring informative judgements that are made in the domain of climate scientists.

3. A second-order inferential framework that is quick and deterministic, making detailed predictive diagnostics possible.

Further reading: J.C. Rougier, M. Goldstein, and L. House, Assessing climate uncertainty using evaluations of several different climate models, available at `http://www.maths.bris.ac.uk/~mazjcr/mme2.pdf`.