Modelling the eruption processes of a large number of similar but not identical volcanoes

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OxWaSP Seminar, Warwick 30 Jan 2015
The return period curve

Shows the expected time in years between events of at least the specified magnitude: magnitude = $\log_{10}(\text{mass in kg}) - 7$. 

Caveat: Return periods are subtle; they are today’s assessment, but look like they are a prediction about the future. This leads to lots of confusion.
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The model for a single volcano

Magnitude, $x$

Time, $t$

$A \sim \text{Poisson}(\lambda_A)$

$\lambda_A = \int \int A \sigma (1 + \xi x - \mu \sigma)^{-\frac{1}{\xi} - 1} \, dx \, dt$
The model for a single volcano

Magnitude, $x$

$N(A) \sim \text{Poisson}(\lambda_A)$

with the parametric form

$$\lambda_A = \int_A \int \frac{1}{\sigma} \left( 1 + \xi \frac{x - \mu}{\sigma} \right)^{-(1/\xi)-1} dt \ dx$$
The model for a single volcano (cont)

We make a change of parameters:

\[(\mu, \sigma, \xi) \mapsto (\kappa, \lambda, \xi)\]

where

\[\kappa\] Maximum possible eruption magnitude (finite),
\[\lambda\] Expected time between eruptions of mag \(\geq 4\).

This makes it more appropriate for us to treat the parameters as \textit{a priori} mutually independent, and we can use informative priors for each margin.

- It also opens up the possibility of incorporating lower-quality data from other volcanoes.
The model for multiple volcanoes

Two different views of *exchangeability*:
The model for multiple volcanoes

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1. Creates a data landscape in which each volcano ‘sees’ its own data more clearly than the data from the other volcanoes.
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1. Creates a data landscape in which each volcano ‘sees’ its own data more clearly than the data from the other volcanoes.

2. Reduces the effective number of parameters below the actual number of parameters and so creates ‘degrees of freedom’.
The model for multiple volcanoes

Japanese SVs, 1600–2010

\[ D_i := \{(T_{ij}, X_{ij})\}_{j=1}^{M_i} \]
The model for multiple volcanoes

Japanese SVs, 1600–2010

\[ D_i := \{(T_{ij}, X_{ij})\}_{j=1}^{M_i} \]

All other SVs, 1800–2010

\[ M_i \]

\[ \lambda_i \]

\[ \pi_\lambda \]

\[ \xi \]

\[ \kappa_i \]

\[ \pi_{\kappa} \]

\[ \pi_\lambda \]
The model for multiple volcanoes

Japanese SVs, 1600–2010

All other SVs, 1800–2010
Computation

Japanese SVs, 1600–2010

All other SVs, 1800–2010
Computation

It’s all done with napkins:
Japanese active stratovolcano dataset

- Toya
- Hokkaido–Komagatake
- Oshima–Oshima
- Pagan
- Agrigan
- Miyakejima
- Izu–Oshima
- Fujisan
- Sakura–jima
- Kuchinoerabujima
- Suwanosejima

Year: 1600, 1700, 1800, 1900, 2000
Posterior margins

Posterior marginal for $\xi$

![Graph showing posterior marginal for $\xi$]
Posterior margins

Posterior marginal for $\kappa_i$ for a volcano not in the dataset.
Posterior margins

Prior/posterior marginal for $\lambda_i$ for a volcano not in the dataset.
Sakurajima

http://earthobservatory.nasa.gov/IOTD/view.php?id=80274
Sakurajima

Marginal distribution for \((\kappa_i, \lambda_i)\).

The coloured region shows an approximate 95% high probability region. The dashed lines show the magnitudes of the three eruptions.
Sakurajima

Some further observations on this volcano:

▶ Sakurajima had a huge eruption in 1914 (mag 5.7). Since 1955 it has been erupting almost continually.

▶ Volcanoes can operate in one of two regimes: episodic (plugged conduit) and persistent (open). Many volcanoes are dominated by one regime, but it looks as though Sakurajima switched from plugged to open in 1955.

▶ If so, Sakurajima should drop out of our analysis at 1955, because it would no longer be exchangeable with the other (plugged) volcanoes.

This illustrates how a screening procedure works. A database plus some simple rules provides a first-cut. For any particular volcano, the results can be challenged by experts, allowing us to refine the rules, and to identify important missing fields in the database.