Probability 1, Autumn 2015, Assessed HW sheet 2

Deadline: To be handed in by 10am on Friday the 4th December.
Hand in: You should hand your work along with a completed cover sheet in to the marked cabinet on the ground floor of the School of mathematics. Please staple each sheet of your work together with the cover page at the front. Cover sheets are available on the ground floor of the School of Mathematics.
Assessment: This homework will count for 5% of your total mark for Probability 1. To obtain full marks you will need to explain clearly how you obtained your answer, using appropriate notation.
Collaboration: The work you hand in should be your own work. You are welcome to discuss the problems with each other but the solutions you hand in should be written solely by you.
Solutions will be available on Blackboard on the 12th Dec.

HW A2.1 (**7 marks**) The *Pessimistic* Geometric(p) random variable counts the number of failures before the first success in a sequence of independent trials each with success probability p. (Notice the difference from the version we covered in class.) State the mass function, and find the expectation and variance of this random variable. (*No need for long computations if you look at this carefully, but explain.*)

HW A2.2 Define the function

$$F(x) = \begin{cases} 0, & \text{if } x \le -1, \\ \dfrac{1}{2} - \dfrac{x^2}{2}, & \text{if } -1 \le x \le 0, \\ \dfrac{1}{2} + \dfrac{x^2}{2}, & \text{if } 0 \le x \le 1, \\ 1, & \text{if } x \ge 1. \end{cases}$$

(a) (**4 marks**) Show that this function is a cumulative distribution function.

(b) (**3 marks**) Show that it describes a continuous random variable, and compute the density function.

(c) (**3 marks**) Determine the expectation and variance of a random variable with this distribution.

HW A2.3 (**8 marks**) The *ceil*, or *upper integer part* $\lceil x \rceil$ of a number x is the smallest integer at least as large as x. E.g., $\lceil 2 \rceil = 2$, and $\lceil 2.63 \rceil = 3$. Let $X \sim \text{Exp}(\lambda)$, and $Y = \lceil X \rceil$. Identify the distribution of Y and hence state its mean and variance in terms of λ. (No marks for this, but interesting: how do these compare to the mean and variance of X?)