Nonexistence of bi-infinite geodesics in exponential last passage percolation - a probabilistic way

Joint with
Ofer Busani and Timo Seppäläinen

Márton Balázs

University of Bristol

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Integrable probability session
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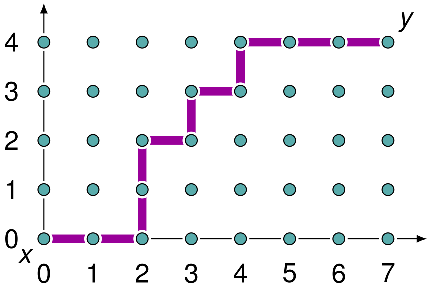
Last passage percolation

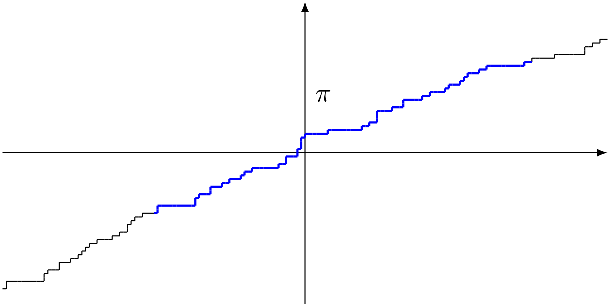
- Place ω_z i.i.d. Exp(1) for $z \in \mathbb{Z}^2$.
- The *geodesic* $\pi_{x,y}$ from x to y is the a.s. unique heaviest up-right from x to y.
- $G_{x,y} = \sum_{z \in \pi_{x,y}} \omega_z$ is its weight.



Surface growth, TASEP, queuing…

Bi-infinite geodesics

A bi-infinite up-right path is a *bi-infinite geodesic*, if any of its segments is itself a geodesic between the two endpoints.



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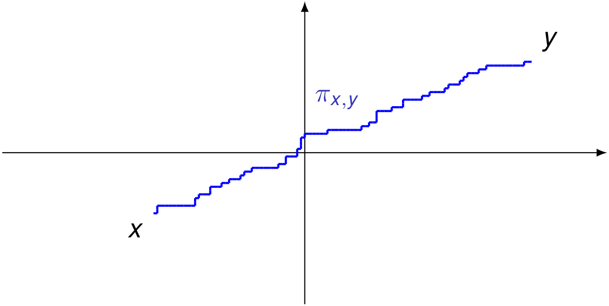

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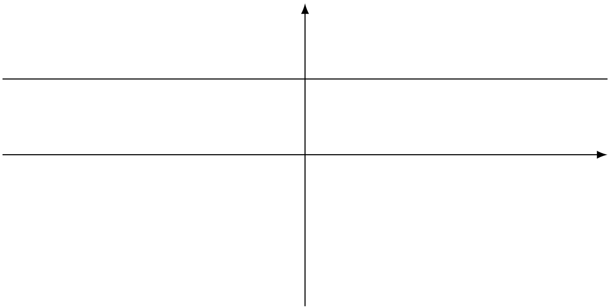

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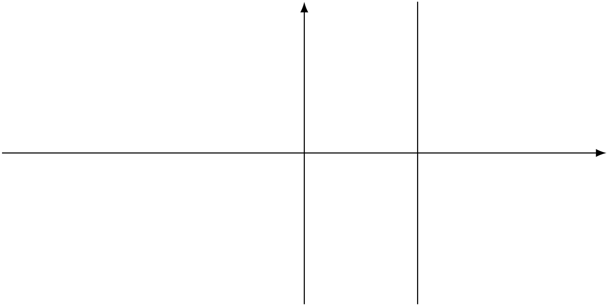

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- Full result by Basu, Hoffman, Sly '18, using estimates from integrable probability.
- We only need a bit of random walks, queuing, couplings.

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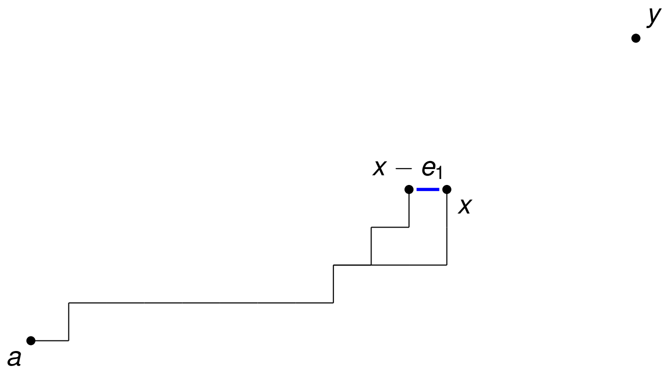

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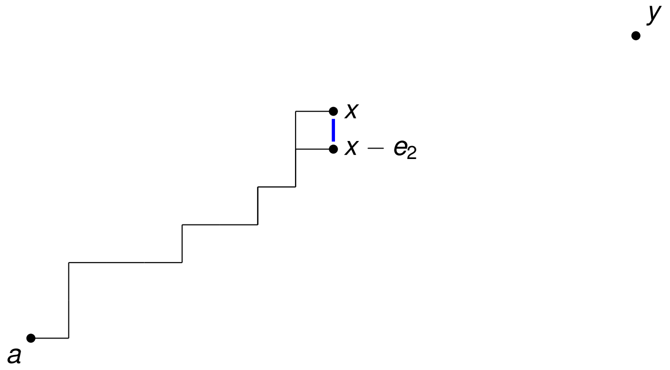

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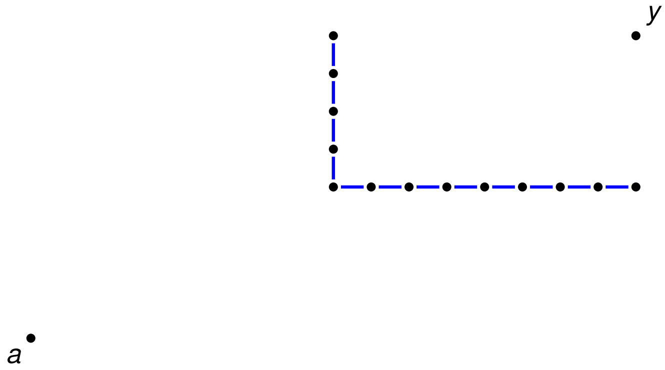

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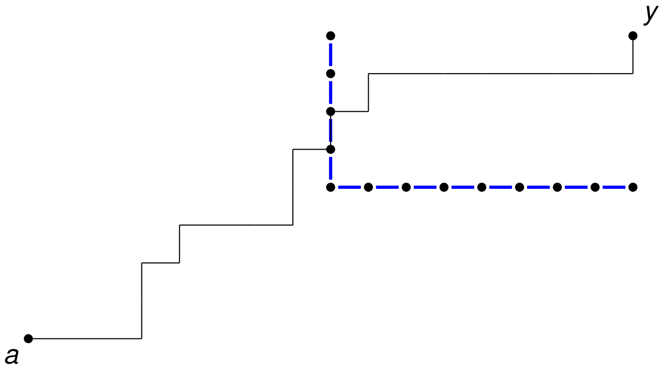

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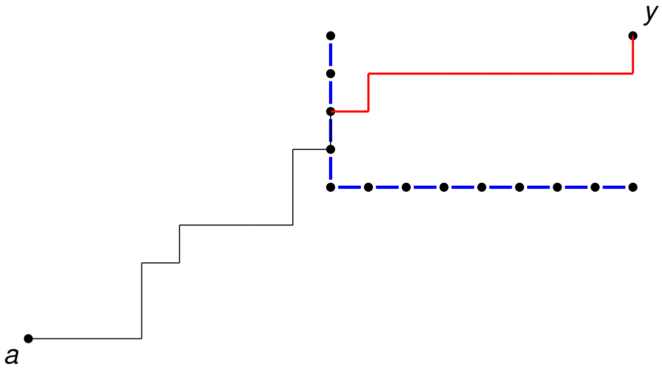

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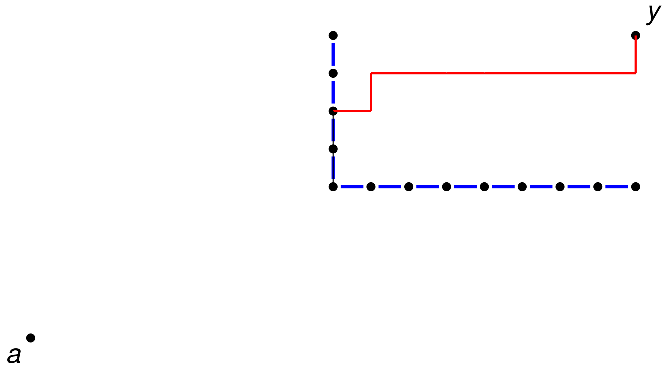

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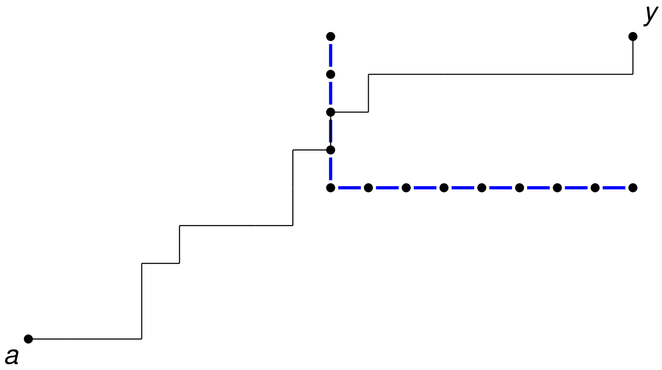
$\rightsquigarrow$ Act as boundary weights for a smaller, embedded model.

2. Stationary LPP



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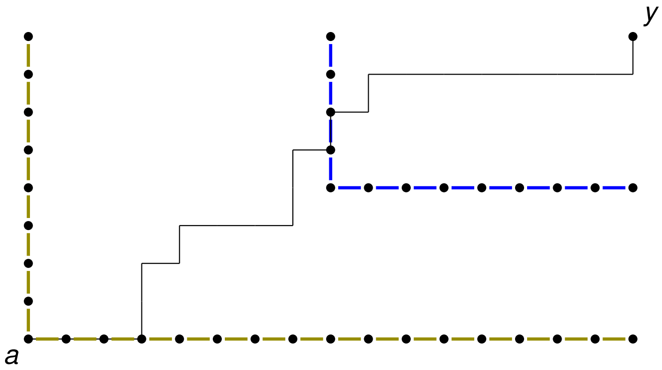
Replace the boundary to | $\sim \text{Exp}(\varrho)$, — $\sim \text{Exp}(1-\varrho)$ independent.



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Then $J_x \sim \text{Exp}(\varrho)$, $I_x \sim \text{Exp}(1-\varrho)$, independent.

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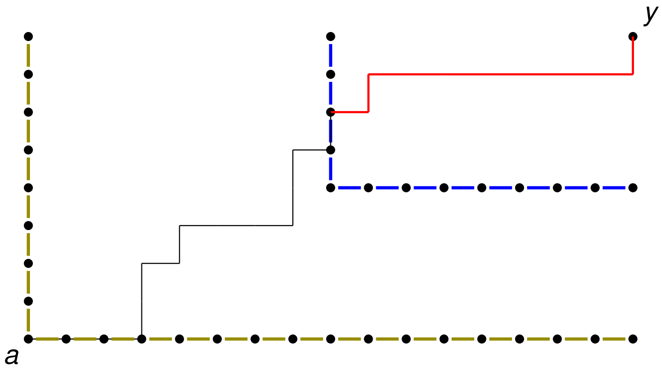
Replace the boundary to | $\sim$ Exp(ϱ), — $\sim$ Exp($1-\varrho$) independent.



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The embedded model has the same structure.

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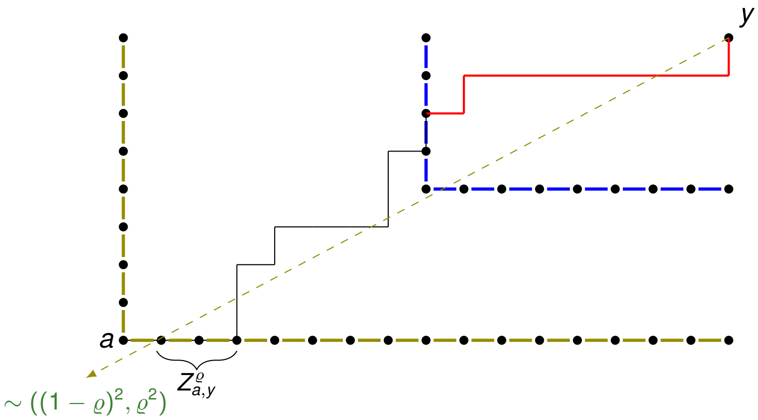
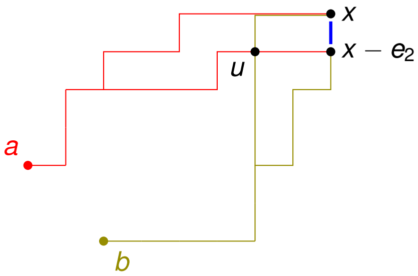


B., Cator, Seppäläinen '06: $\mathbb{P}\{|Z^{\varrho}_{a,y}| \geq \ell\} \leq \text{box}^2/\ell^3$, *good directional control*.

3. Crossing lemma

Let a be North-West of b.



Then $J_x^{(a)} \geq J_x^{(b)}$.

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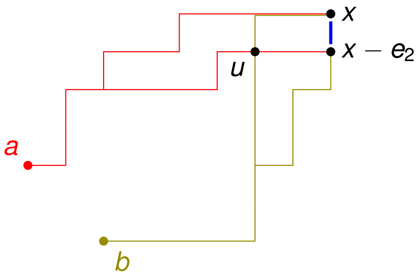
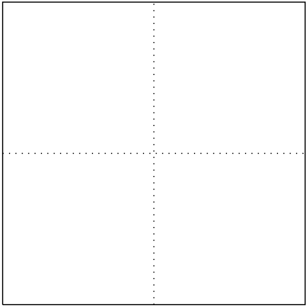


Then $\quad J_x^{(a)} \geq J_x^{(b)}.$

Similarly, $\quad I_x^{(a)} \leq I_x^{(b)}.$

Proof

Take larger and larger boxes and show that geodesics avoid more and more the origin when crossing from one side to the other (Newman '95).



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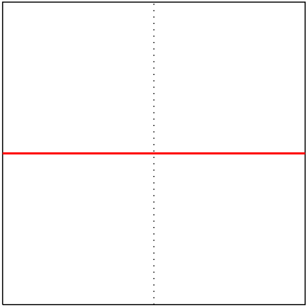
1. Close to vertical and horizontal all semi-infinite geodesics become trivial.



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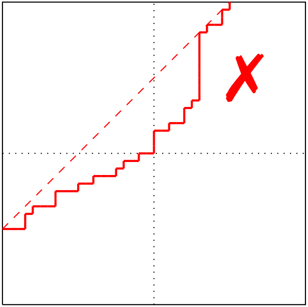
2. Otherwise, geodesics don't like to turn too much.



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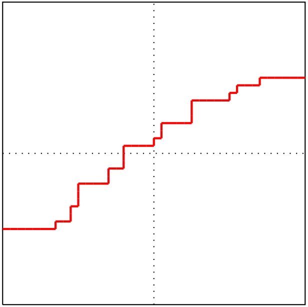
1. Close to vertical and horizontal all semi-infinite geodesics become trivial.
2. Otherwise, geodesics don't like to turn too much.
3. We are left with roughly diagonal ones, show that they fluctuate too much.



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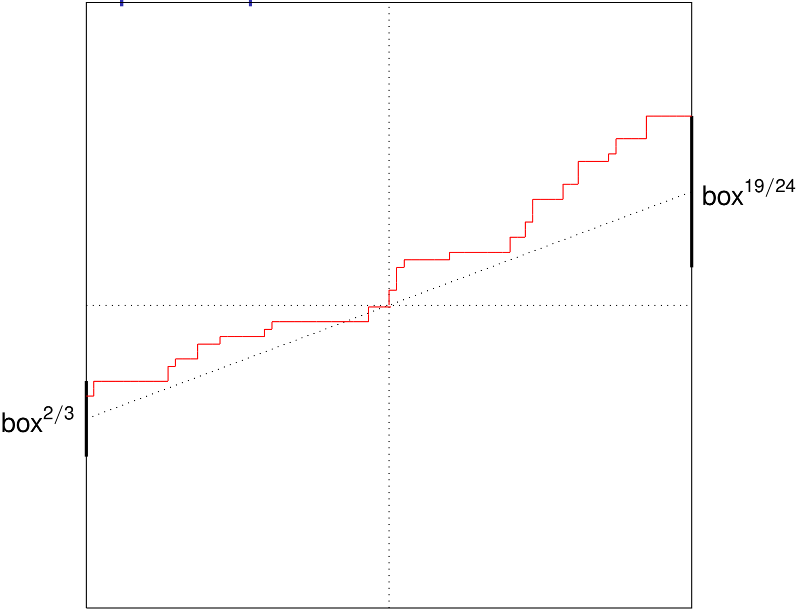

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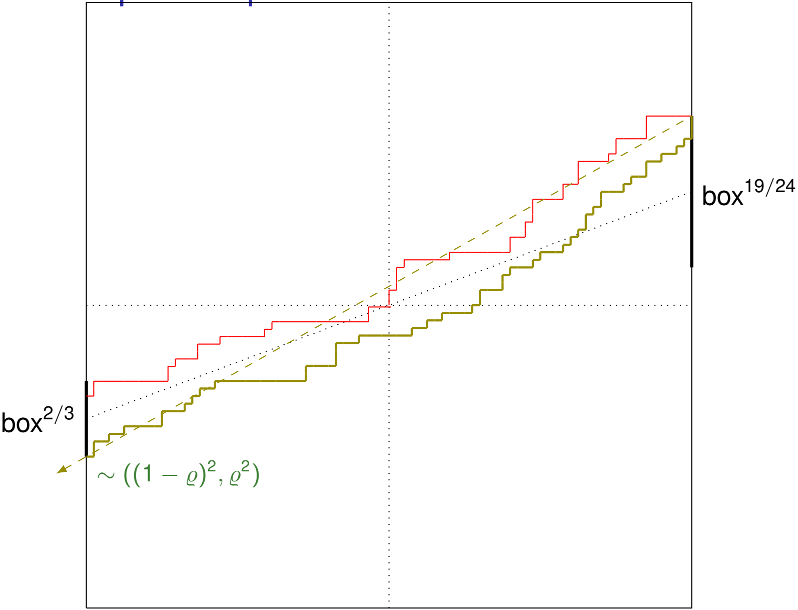

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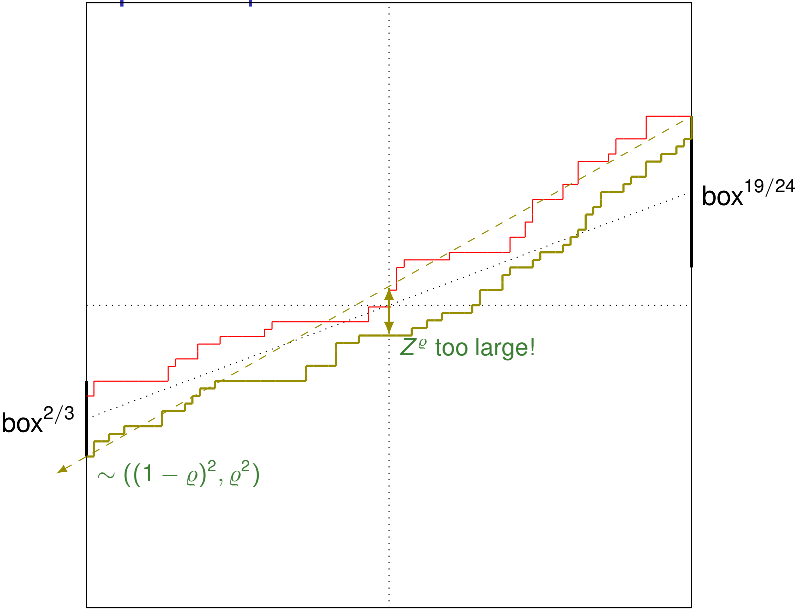
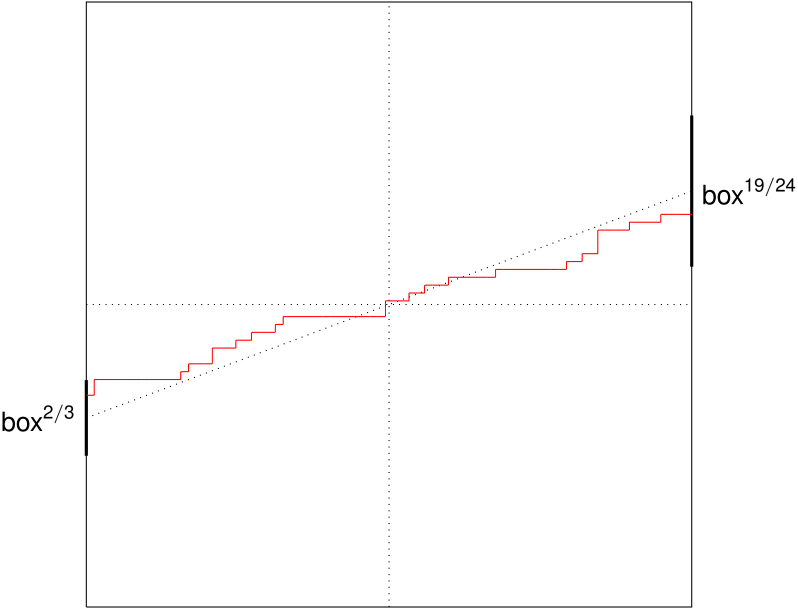


3. The diagonal case: the attack of the geodesics



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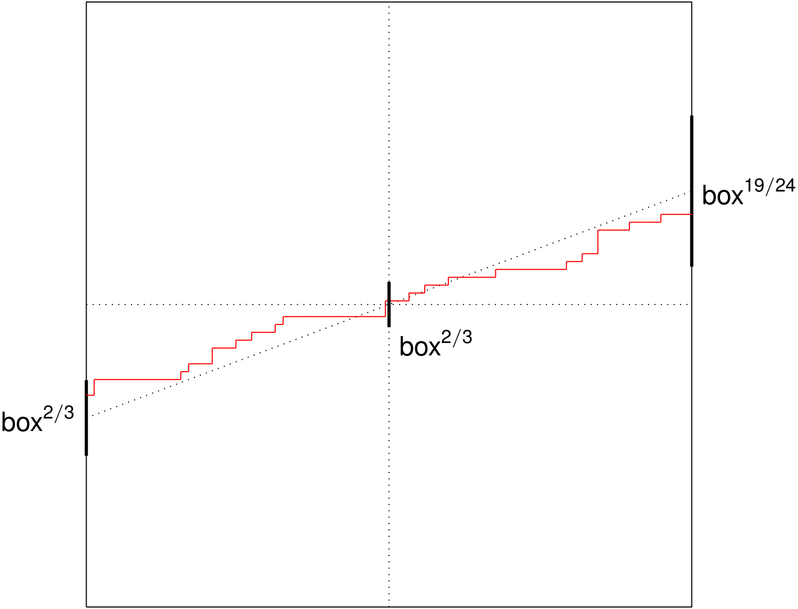

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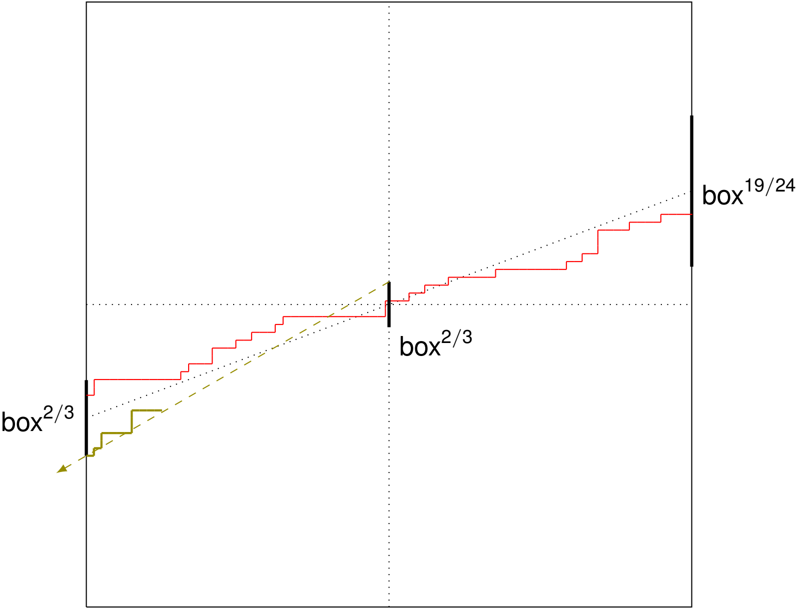

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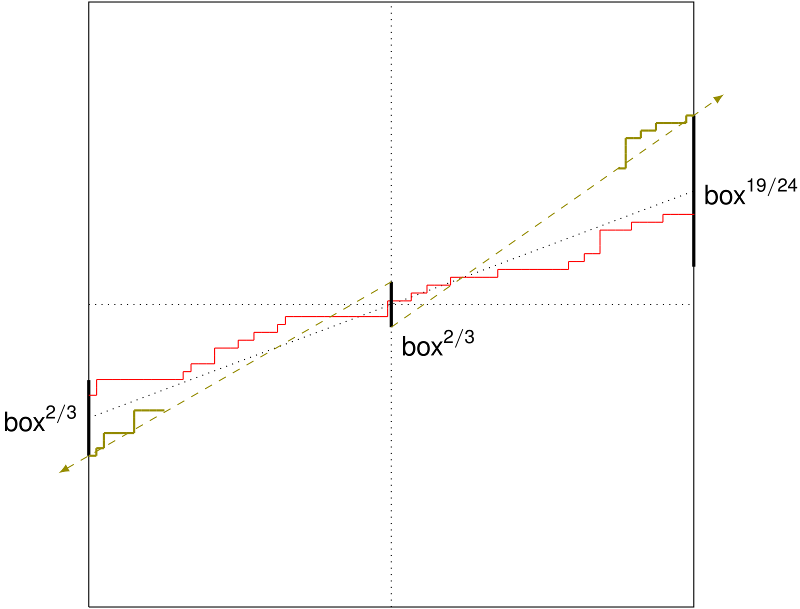

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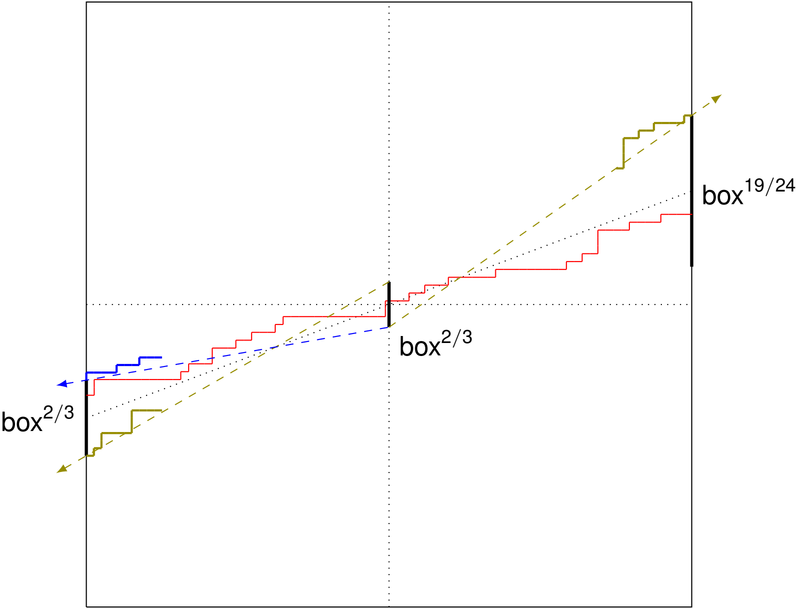

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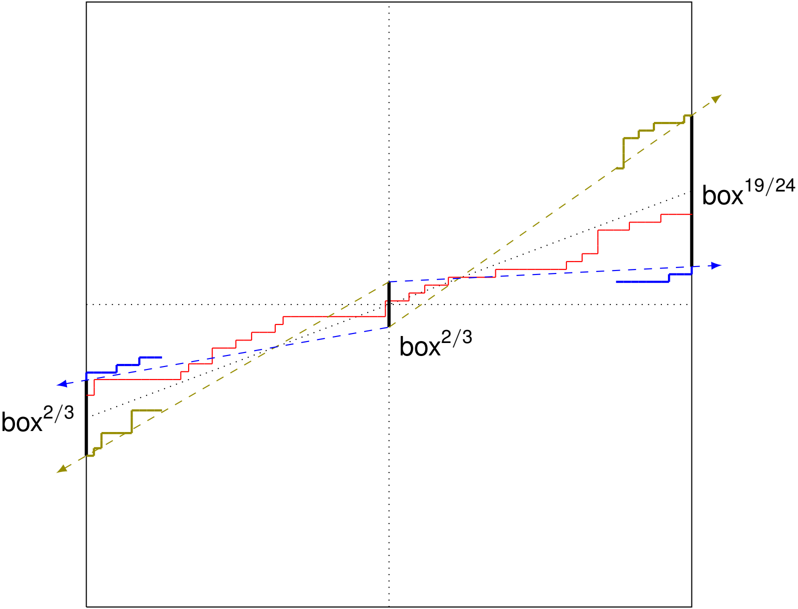

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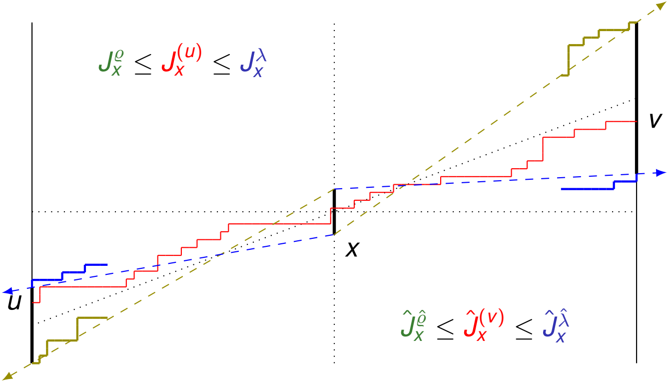
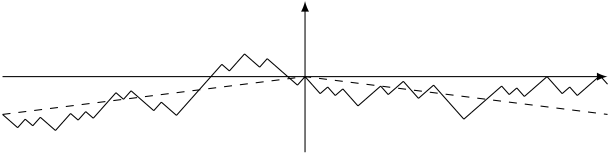
With high probability, $\forall u, x, v$:



$\rightsquigarrow$ Compare our increments $J^{(u)}$ and $J^{(v)}$ to the stationary J^{ϱ}, J^{λ}, $\hat{J}^{\hat{\varrho}}$ and $\hat{J}^{\hat{\lambda}}$ which are independent and nicely distributed.

3. The diagonal case: the attack of the geodesics

The problem boils down to whether a simple random walk minus drift reaches its maximum at 0. The answer is an asymptotic *no*, the drift is beaten by the fluctuations.



Thank you.

`https://arxiv.org/abs/1909.06883`