Dependent Double Branching Annihilating Random Walk

Joint with
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Márton Balázs

University of Bristol

Oberseminar Stochastics
Bonn, 9th July, 2015.

Attractive and non-attractive models

- Totally asymmetric simple exclusion process
- A $\oplus$ $\ominus$ 0 model
- Totally asymmetric zero range process

On large scales

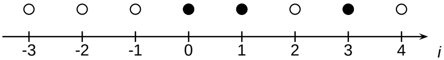
- Shocks
- Rarefaction waves

A mean field version

Positive recurrence

Two words on the proof

The totally asymmetric simple exclusion process



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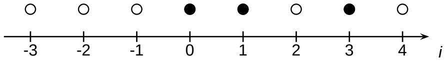

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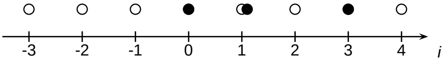

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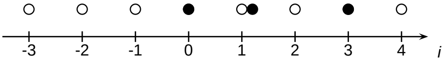

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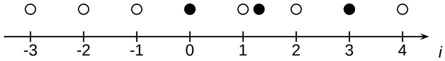

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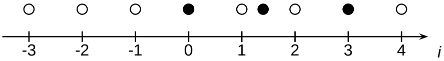

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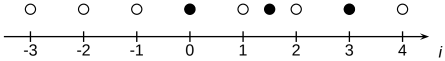

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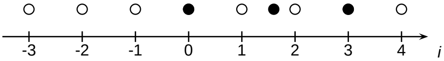

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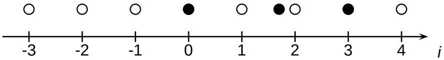

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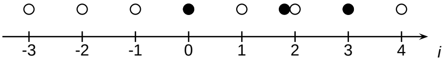

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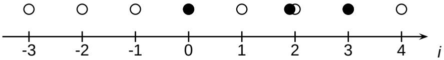

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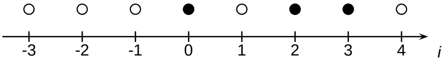

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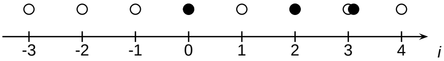

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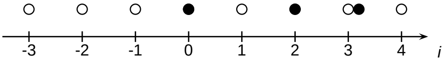

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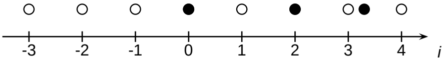

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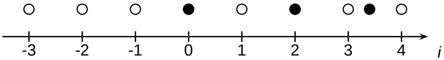

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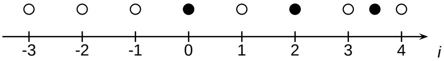

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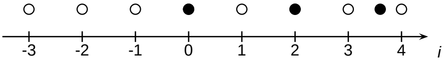

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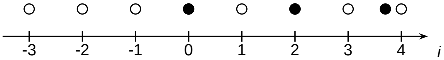

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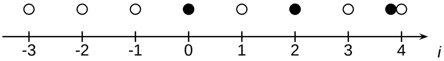

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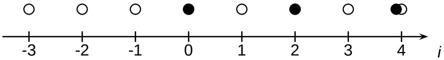

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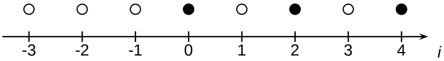

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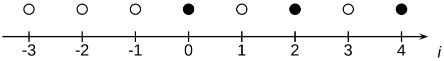

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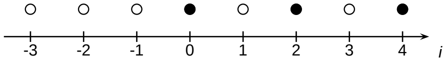

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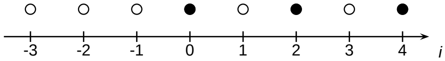

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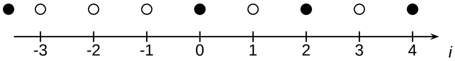

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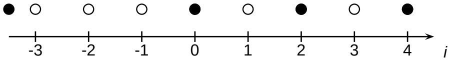

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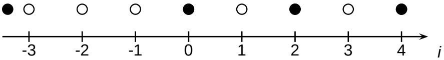

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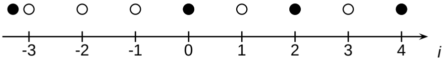

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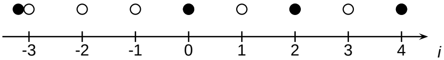

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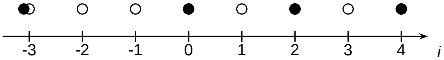

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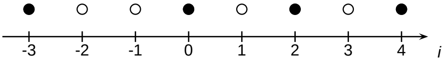

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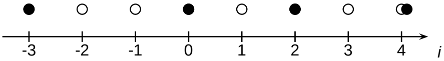

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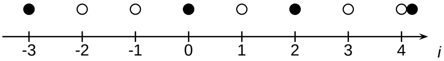

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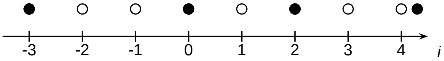

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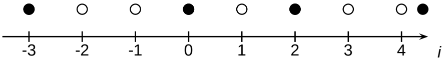

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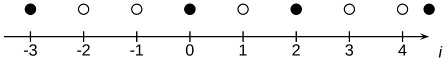

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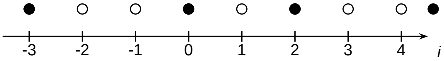

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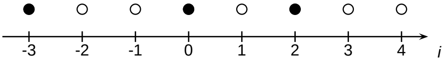

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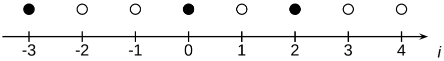

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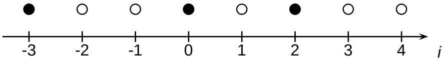

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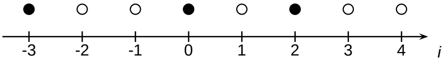

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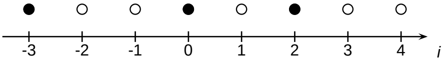

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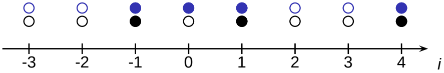
The Bernoulli(ϱ) product distribution is stationary (and non-reversible) for all $0 \le \varrho \le 1$: $\omega_i(t) \sim$ Bernoulli(ϱ).

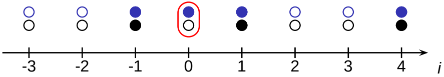
These are the important (= ergodic) stationary distributions.

The second class particle



Stochastic coupling: evolution as close as possible

The second class particle



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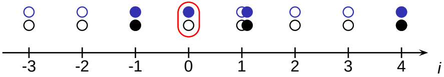

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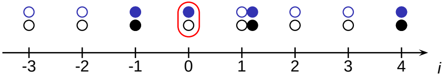

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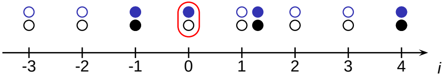

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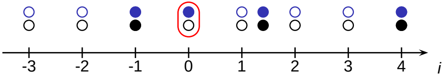

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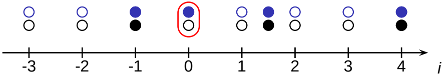

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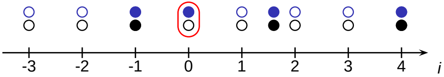

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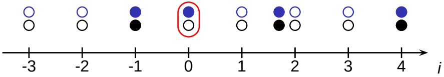

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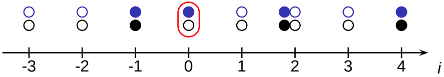

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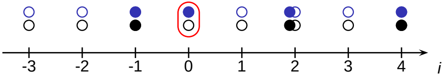

Stochastic coupling: evolution as close as possible

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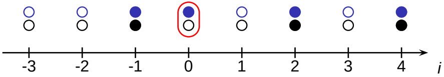

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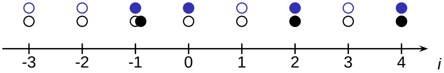

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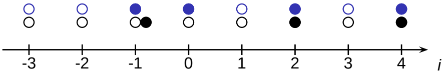

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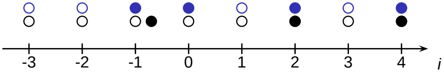

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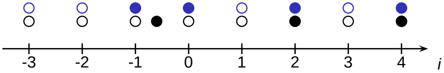

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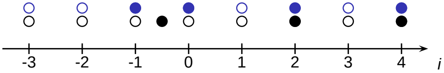

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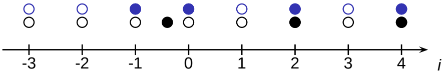

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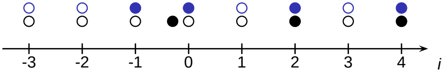

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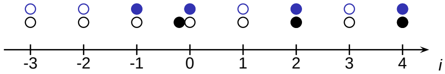

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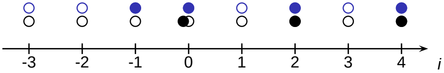

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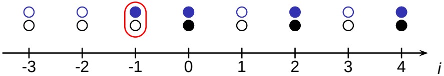

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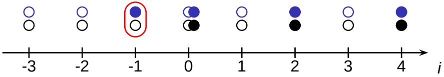

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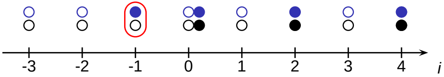

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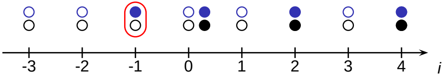

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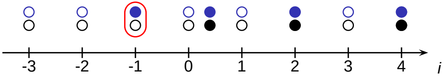

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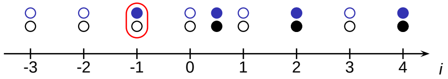

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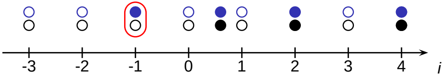

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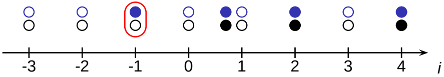

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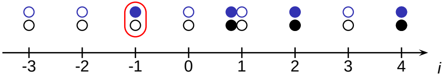

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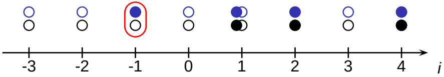

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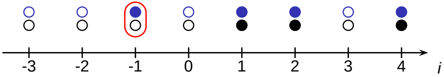

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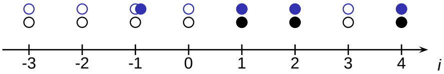

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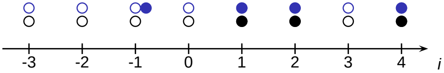

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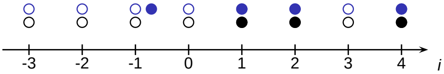

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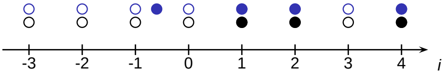

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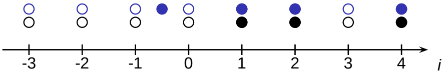

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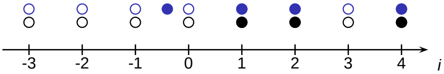

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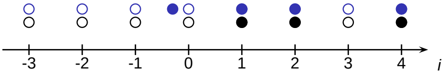

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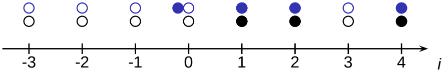

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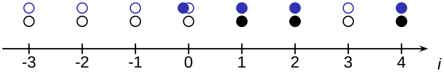

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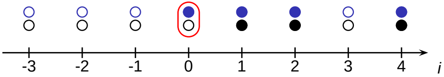
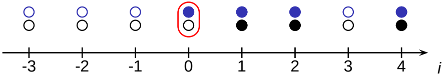


Stochastic coupling: evolution as close as possible

Second class particle. Its position at time t: $Q(t)$.

The second class particle

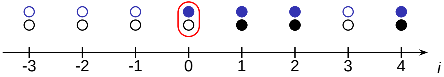


Stochastic coupling: evolution as close as possible

Second class particle. Its position at time t: $Q(t)$.

$$\begin{aligned}\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) &= \mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=0]\cdot(1-\varrho)\\ &\quad+\mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=1]\cdot\varrho-\varrho^2\\ &= \mathbf{E}[\omega_i(t)]\cdot\varrho-\varrho^2.\end{aligned}$$

The second class particle



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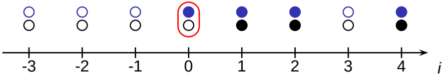
Second class particle. Its position at time t: $Q(t)$.

$$\begin{aligned}\mathbf{Cov}(\omega_i(t), \omega_0(0)) &= \mathbf{E}[\omega_i(t)\omega_0(0) \,|\, \omega_0(0) = 0] \cdot (1 - \varrho) \\ &\quad + \mathbf{E}[\omega_i(t)\omega_0(0) \,|\, \omega_0(0) = 1] \cdot \varrho - \varrho^2 \\ &= \mathbf{E}[\omega_i(t)] \cdot \varrho - \varrho^2.\end{aligned}$$

$$\mathbf{P}\{Q(t) = i\} = \mathbf{E}[\omega_i(t) - \omega_i(t)] = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)].$$

The second class particle



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$$\mathbf{P}\{Q(t)=i\}=\mathbf{E}[\omega_i(t)-\omega_i(t)]=\mathbf{E}[\omega_i(t)]-\mathbf{E}[\omega_i(t)].$$

$$\begin{aligned}\varrho=\mathbf{E}[\omega_i(t)] &= \mathbf{E}[\omega_i(t)\,|\,\omega_0(0)=0]\cdot(1-\varrho)\\ &\quad+\mathbf{E}[\omega_i(t)\,|\,\omega_0(0)=1]\cdot\varrho\\ &= \mathbf{E}[\omega_i(t)]\cdot(1-\varrho)+\mathbf{E}[\omega_i(t)]\cdot\varrho\end{aligned}$$

The second class particle

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) = \mathbf{E}[\omega_i(t)]\cdot\varrho - \varrho^2. \tag{1}$$

$$\mathbf{P}\{Q(t)=i\} = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]. \tag{2}$$

$$\varrho = \mathbf{E}[\omega_i(t)]\cdot(1-\varrho) + \mathbf{E}[\omega_i(t)]\cdot\varrho. \tag{3}$$

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So,

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) \overset{(1)}{=} \varrho\cdot\big(\mathbf{E}[\omega_i(t)]-\varrho\big)$$

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The second class particle

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) = \mathbf{E}[\omega_i(t)]\cdot\varrho - \varrho^2. \tag{1}$$

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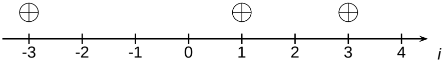
The second class particle traces information propagation.

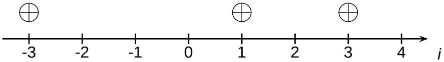
A $\oplus$ $\ominus$ 0 model



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A ⊕ ⊖ 0 model

⊕ to the right: rate 1



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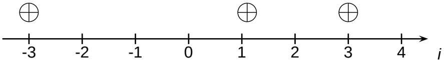

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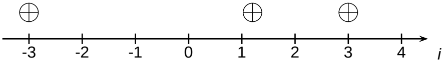

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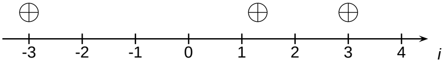

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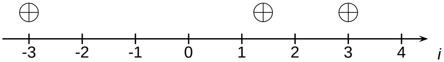

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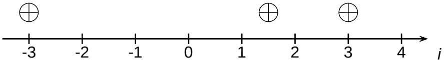

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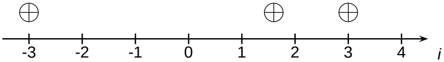

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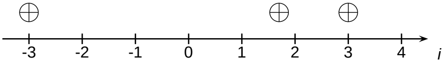
A ⊕ ⊖ 0 model

⊕ to the right: rate 1



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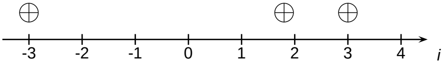

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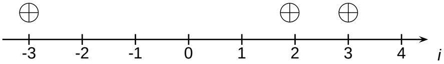

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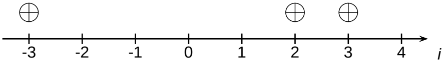

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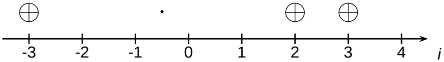
A $\oplus$ $\ominus$ 0 model

pair creation from vacuum: rate c



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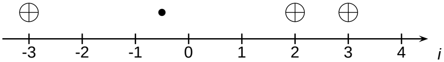
A ⊕ ⊖ 0 model

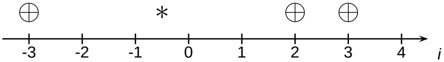
pair creation from vacuum: rate c



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A ⊕ ⊖ 0 model

pair creation from vacuum: rate c



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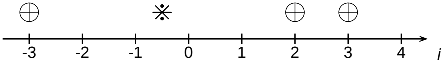
A ⊕ ⊖ 0 model

pair creation from vacuum: rate c



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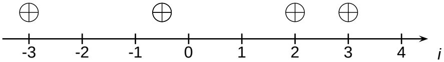
A $\oplus$ $\ominus$ 0 model

pair creation from vacuum: rate c



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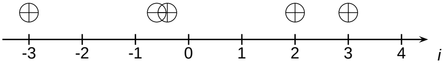
A $\oplus$ $\ominus$ 0 model

pair creation from vacuum: rate c



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A ⊕ ⊖ 0 model

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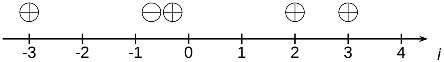

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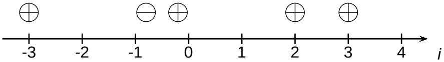

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A ⊕ ⊖ 0 model

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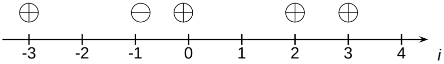

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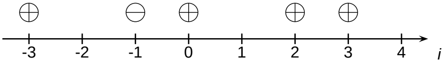
A $\oplus$ $\ominus$ 0 model

pair creation from vacuum: rate c



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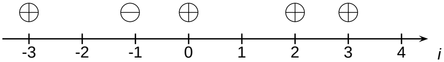
A $\oplus$ $\ominus$ 0 model

$\ominus$ to the left: rate 1



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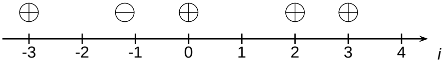
A $\oplus$ $\ominus$ 0 model

$\ominus$ to the left: rate 1



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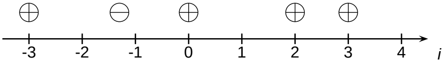
A $\oplus$ $\ominus$ 0 model

$\ominus$ to the left: rate 1



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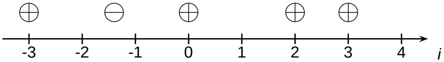
A ⊕ ⊖ 0 model

⊖ to the left: rate 1



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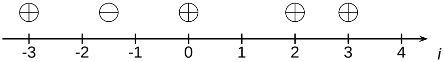
A $\oplus$ $\ominus$ 0 model

$\ominus$ to the left: rate 1



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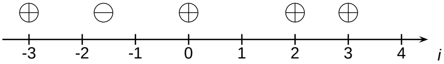
A ⊕ ⊖ 0 model

⊖ to the left: rate 1



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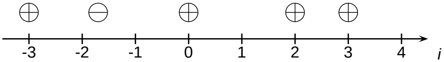
A ⊕ ⊖ 0 model

⊖ to the left: rate 1



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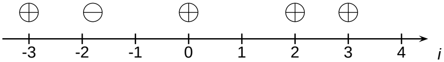

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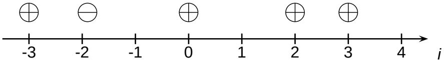
$\ominus$ to the left: rate 1



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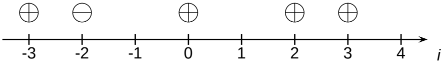
A $\oplus$ $\ominus$ 0 model

$\ominus$ to the left: rate 1



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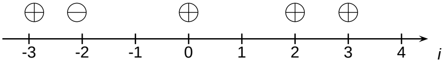
A ⊕ ⊖ 0 model

annihilation: rate 2



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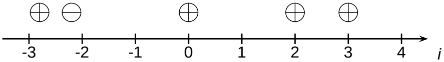
A $\oplus$ $\ominus$ 0 model

annihilation: rate 2



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A $\oplus$ $\ominus$ 0 model

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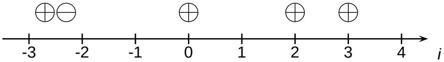

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A ⊕ ⊖ 0 model

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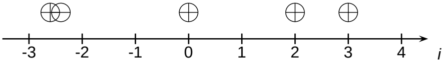

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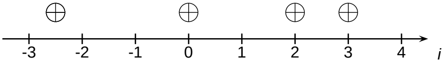

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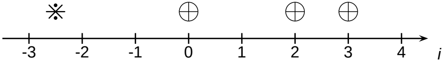

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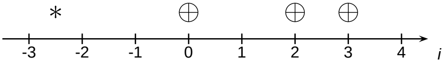

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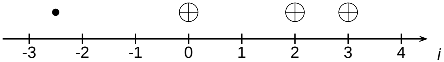

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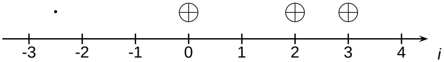

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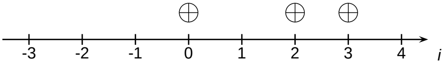

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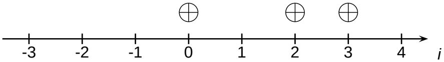

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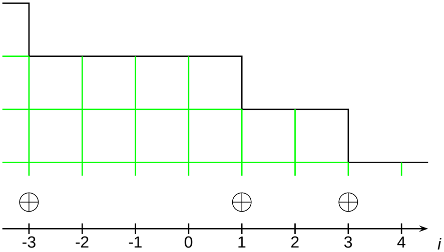
Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model



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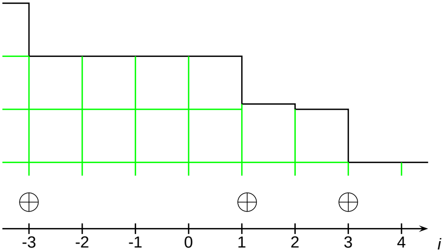

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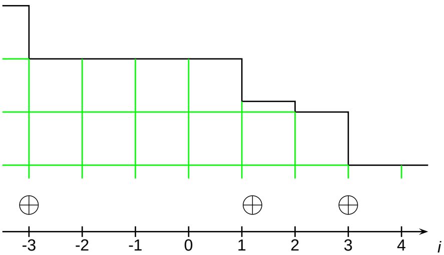

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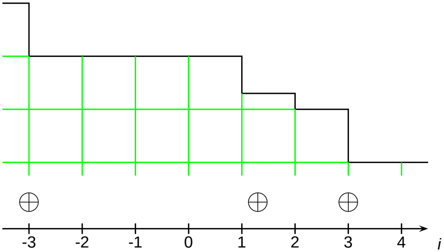

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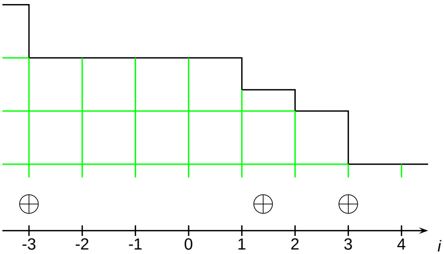

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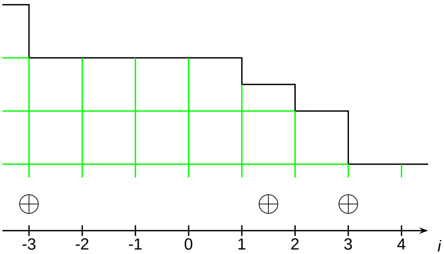

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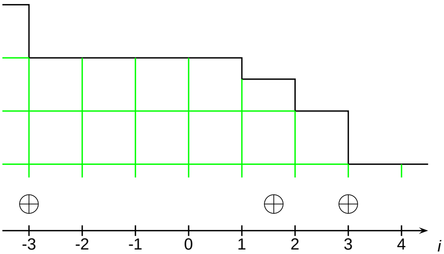

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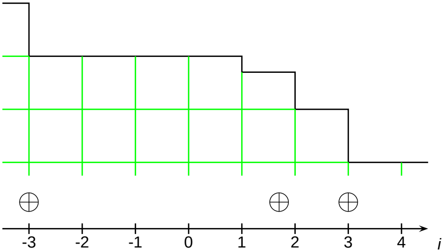

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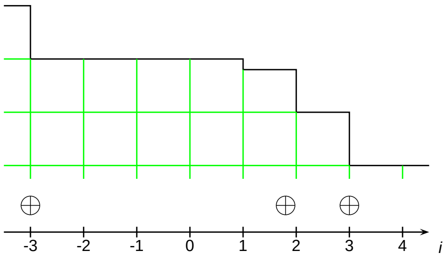

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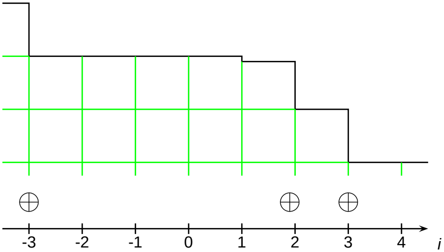

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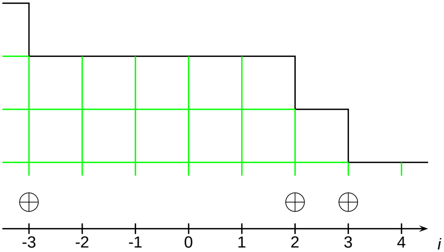

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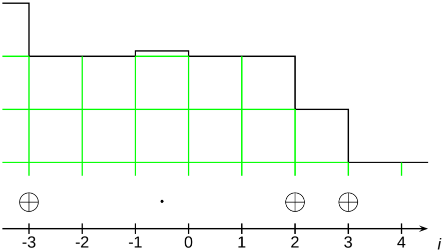

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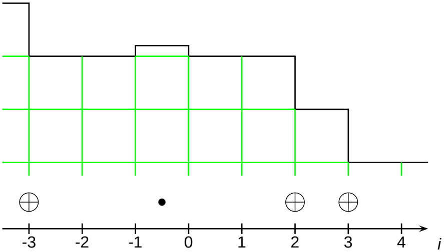

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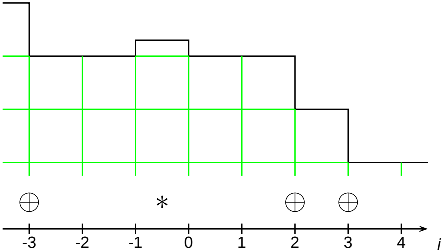

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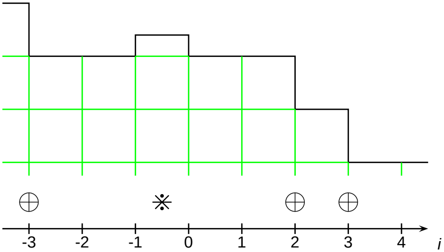

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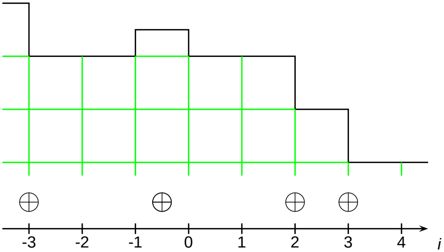

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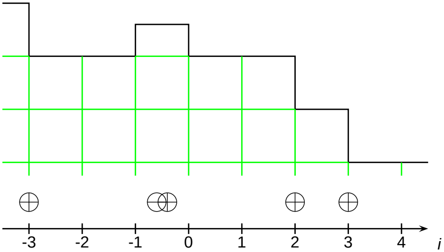

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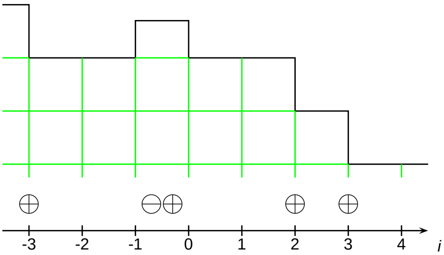

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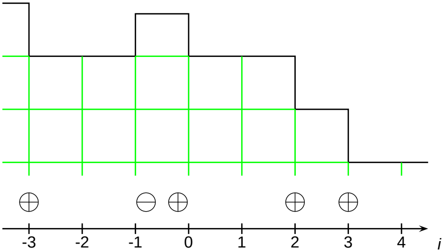

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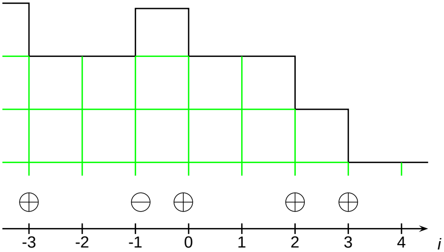

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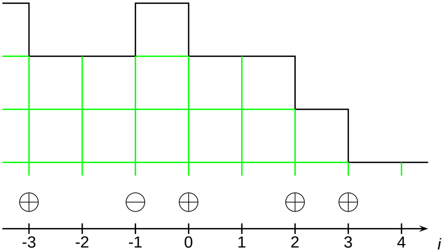

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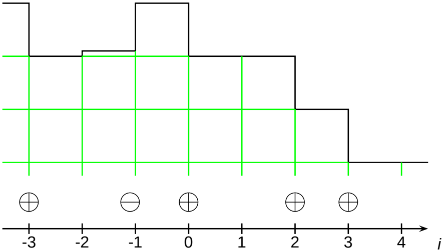

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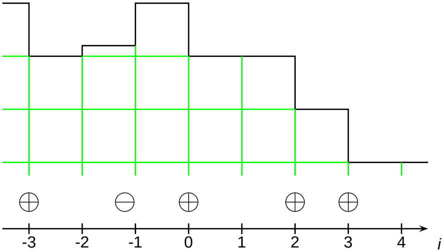

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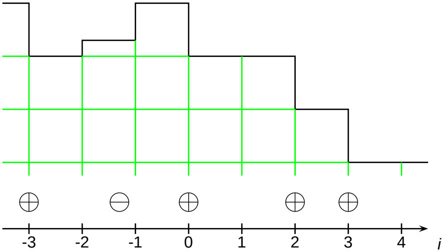

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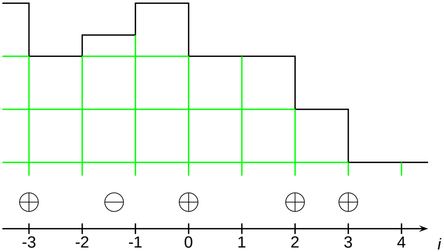

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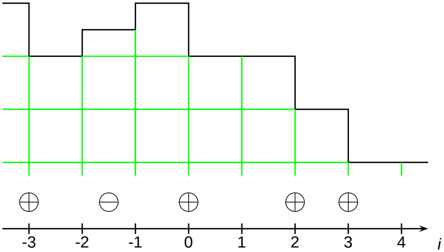

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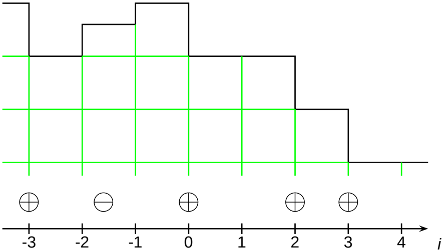

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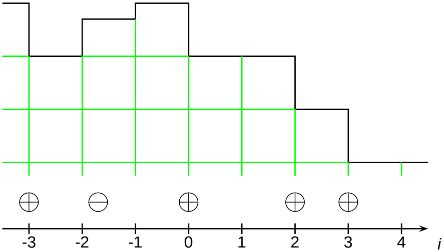

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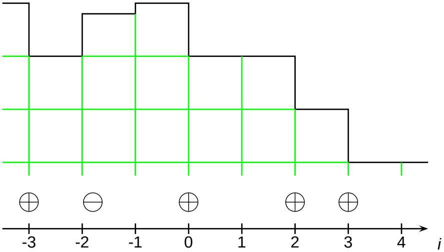

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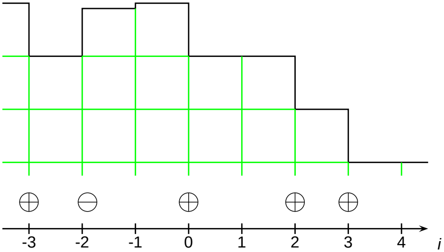

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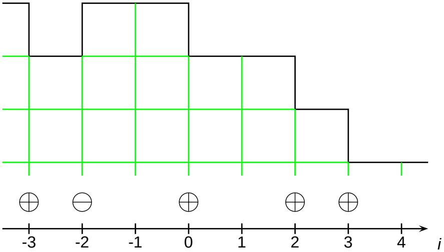

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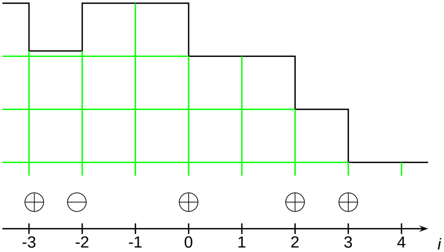

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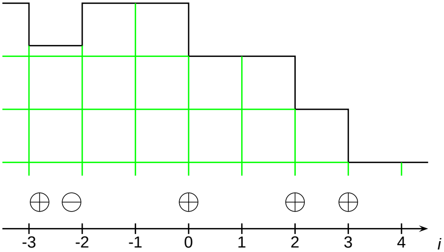

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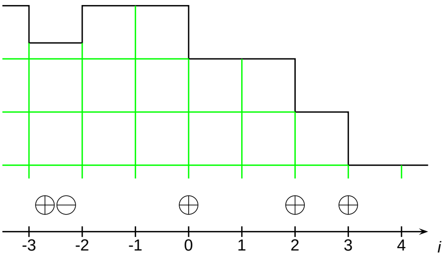

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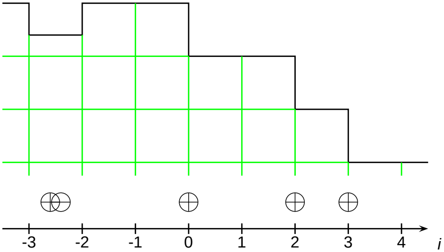

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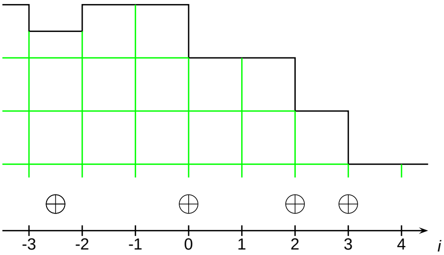

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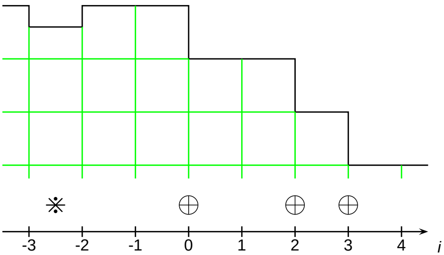

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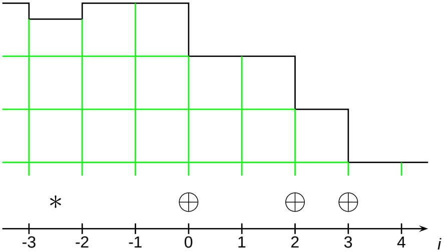

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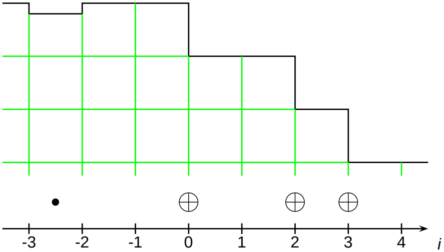

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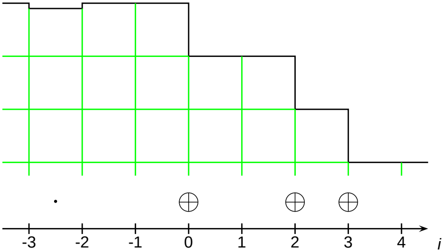

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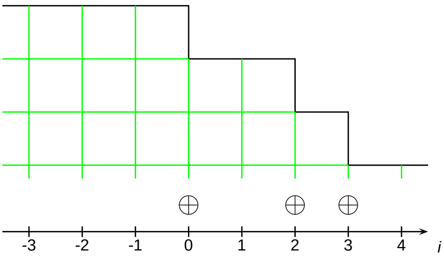

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The second class particle

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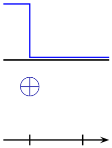

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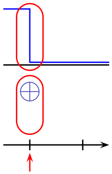

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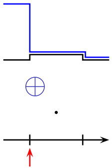

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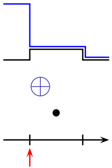

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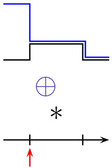

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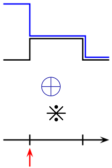

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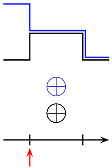

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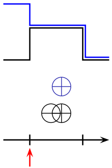

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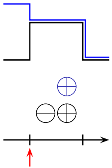

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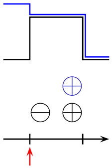

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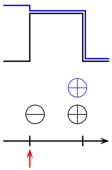

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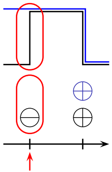

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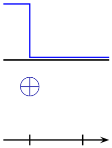

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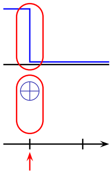

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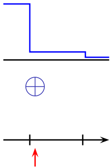

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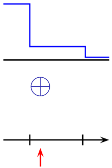

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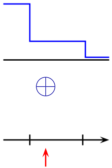

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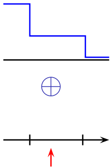

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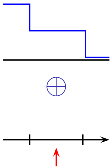

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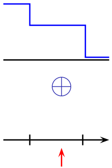

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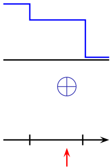

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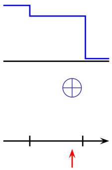

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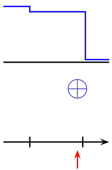

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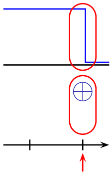

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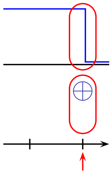
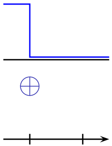


With rate $1 - c$

Attractivity

The second class particle

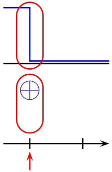
But, for $c > 1$:



With rate 1

The second class particle

But, for $c > 1$:



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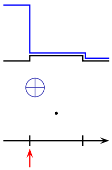

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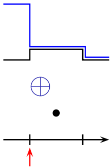

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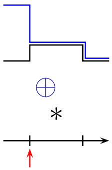

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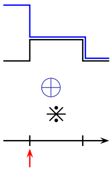

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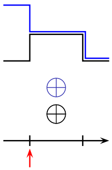

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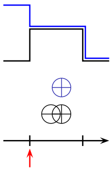

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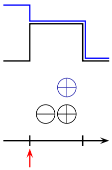

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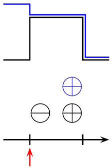

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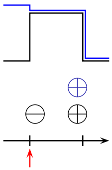

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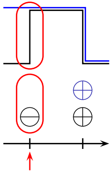

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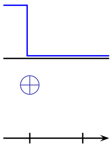

With rate $c - 1$

The second class particle

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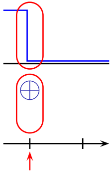
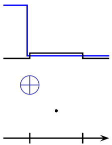


With rate $c - 1$

The second class particle

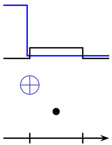
But, for $c > 1$:



With rate $c - 1$

The second class particle

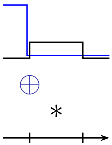
But, for $c > 1$:



With rate $c - 1$

The second class particle

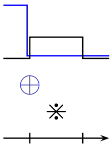
But, for $c > 1$:



With rate $c - 1$

The second class particle

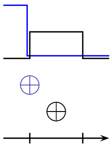
But, for $c > 1$:



With rate $c - 1$

The second class particle

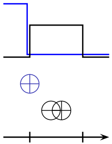
But, for $c > 1$:



With rate $c - 1$

The second class particle

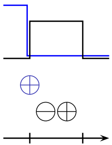
But, for $c > 1$:



With rate $c - 1$

The second class particle

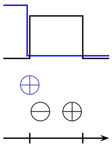
But, for $c > 1$:



With rate $c - 1$

The second class particle

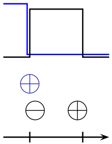
But, for $c > 1$:



With rate $c - 1$

The second class particle

But, for $c > 1$:



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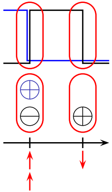
The second class particle

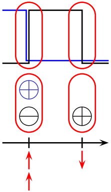
But, for $c > 1$:



With rate $c - 1$

The second class particle

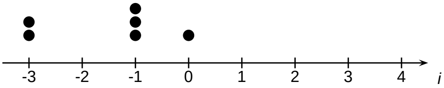
But, for $c > 1$:



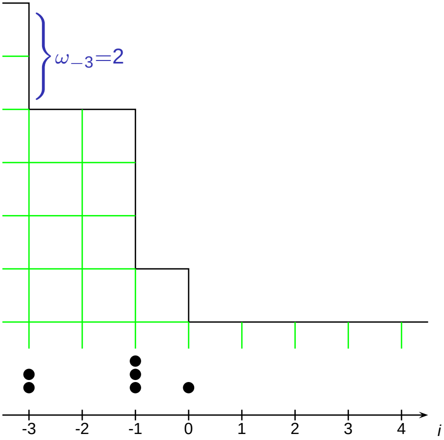
With rate $c - 1$

Non-attractivity

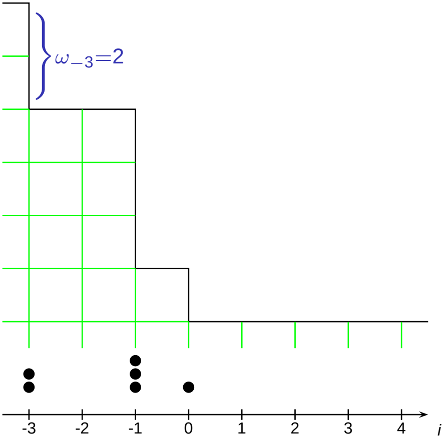
Totally asymmetric zero range process



Totally asymmetric zero range process

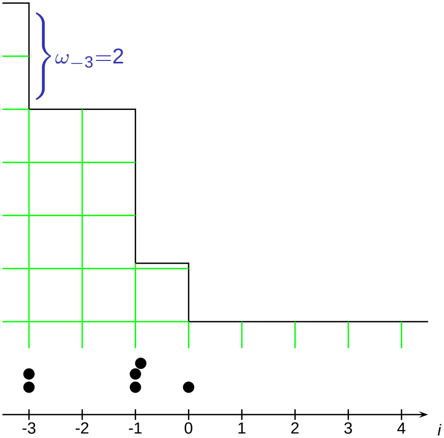


Totally asymmetric zero range process



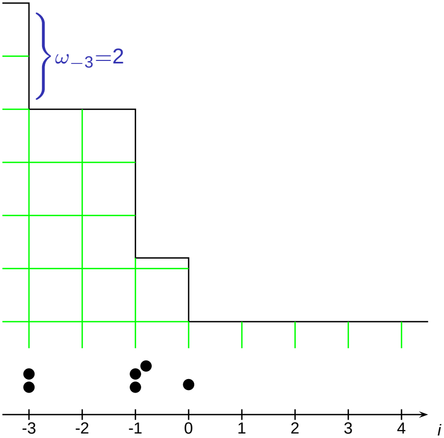
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



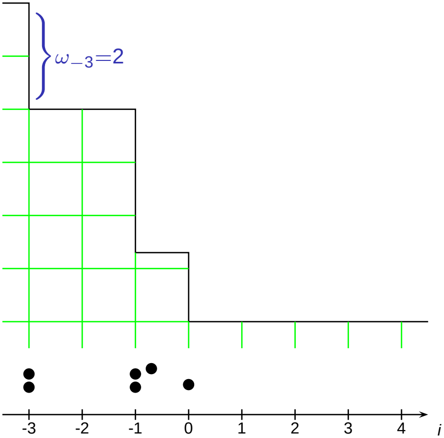
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



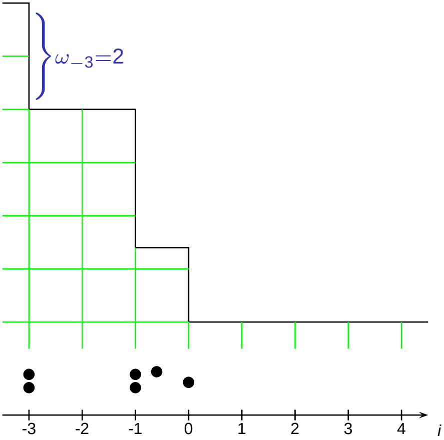
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



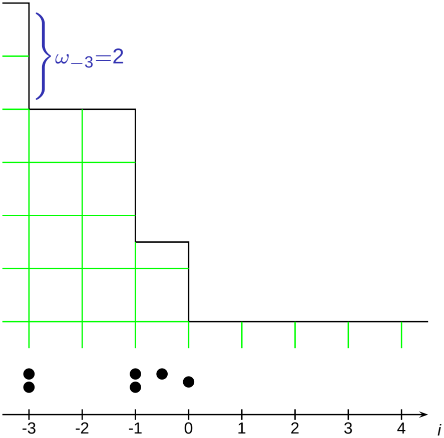
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



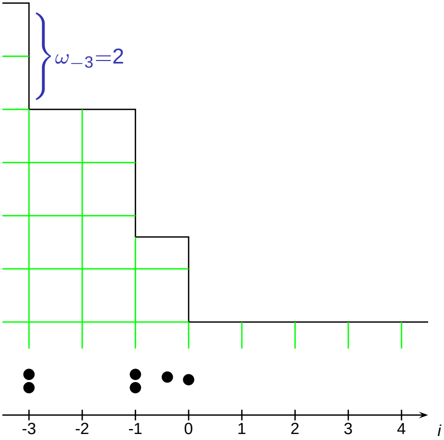
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



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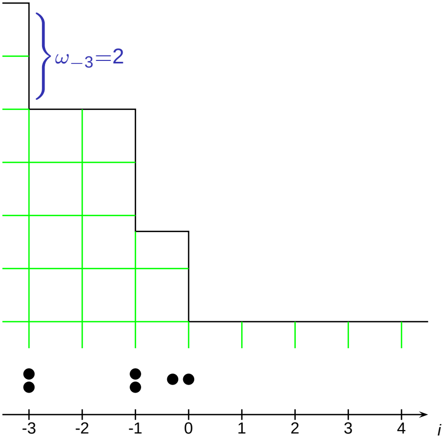

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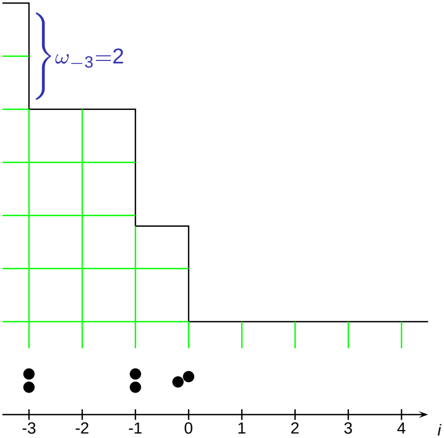

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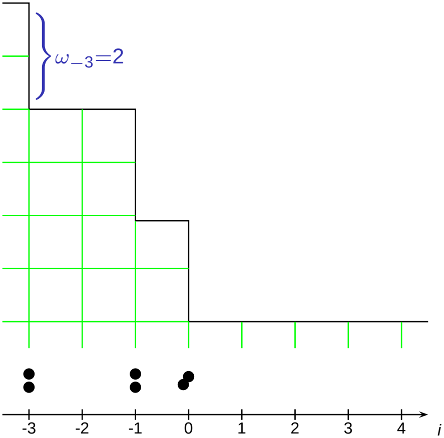

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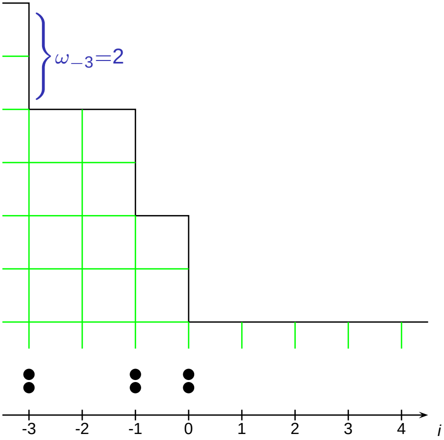

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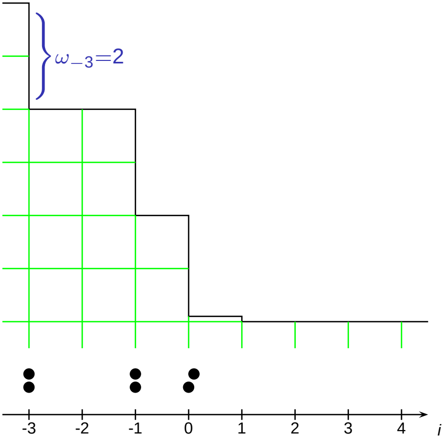

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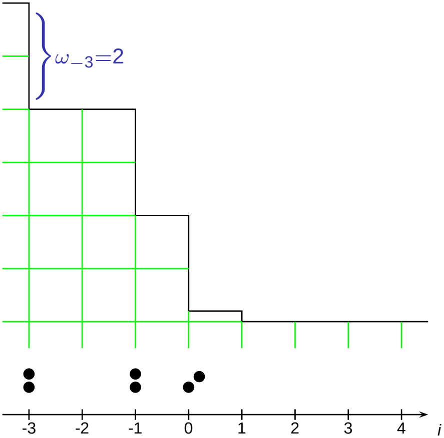

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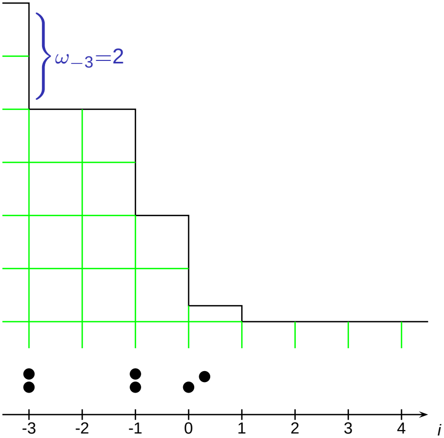

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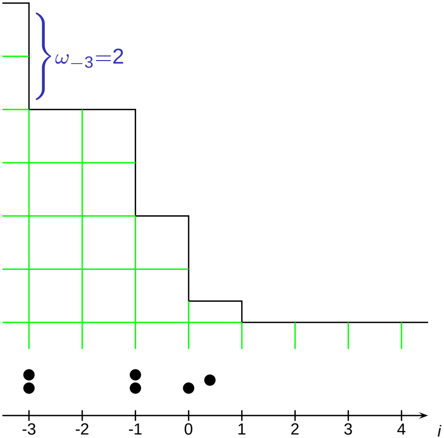

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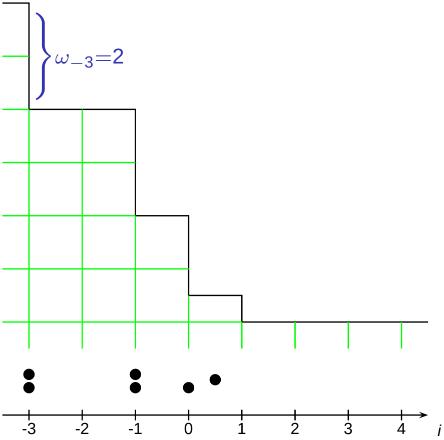

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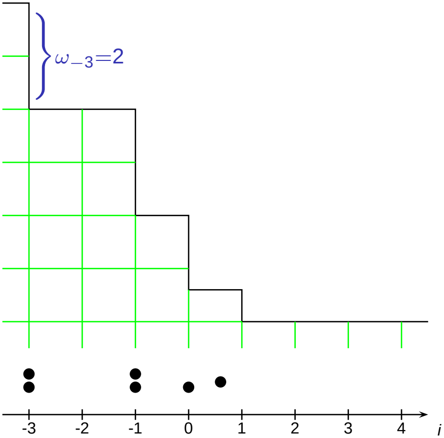

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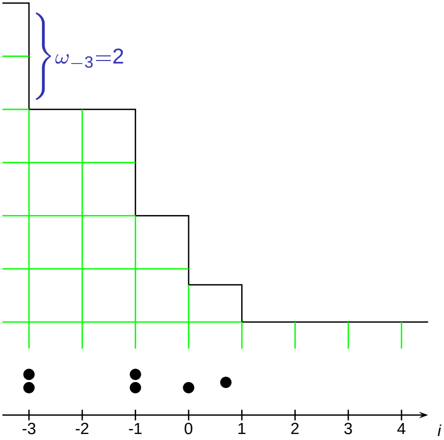

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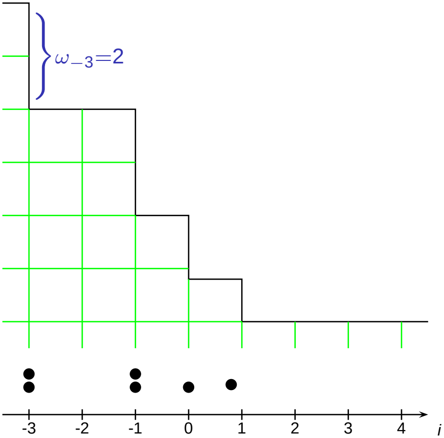

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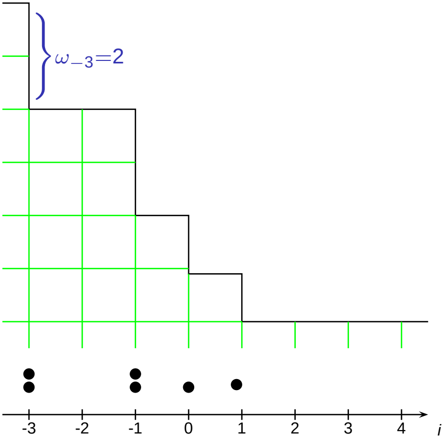

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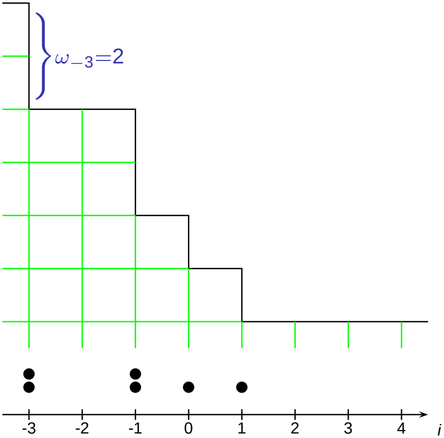
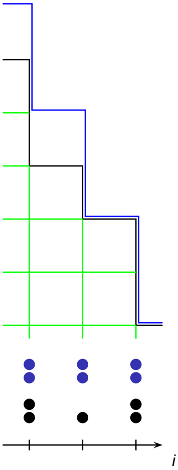


Particles jump to the right with rate $r(\omega_i)$.

The second class particle: attractive case

States ω and ω only differ at one site.



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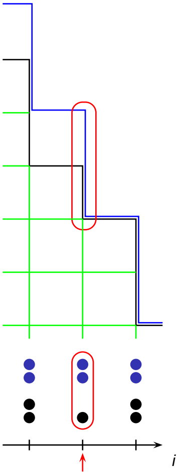

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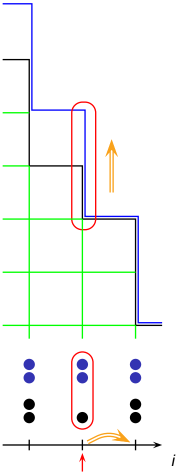
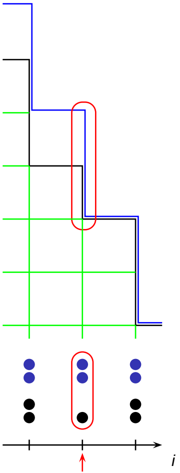


Growth on the right:

rate≤rate

The second class particle: attractive case

States ω and ω only differ at one site.



Growth on the right:

rate≤rate

with rate:

The second class particle: attractive case

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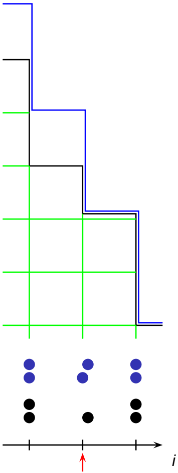

Growth on the right:

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with rate:

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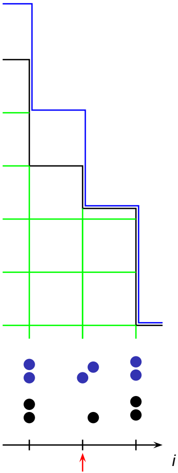

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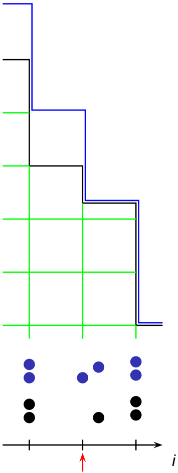

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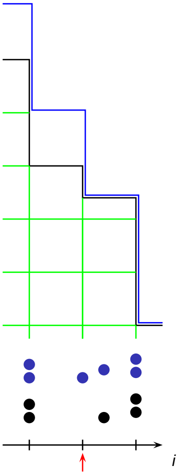

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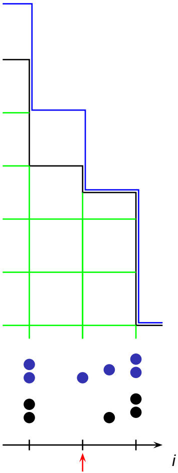

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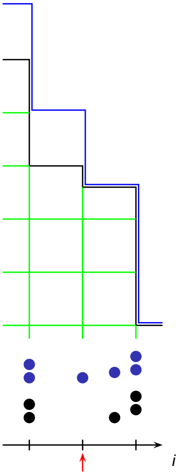

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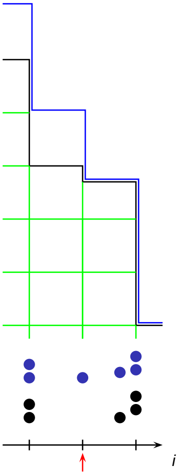

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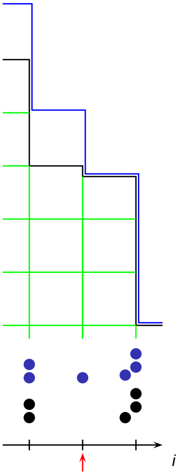

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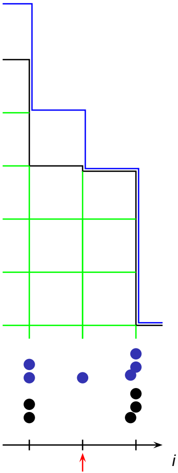

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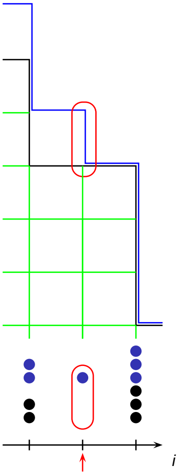

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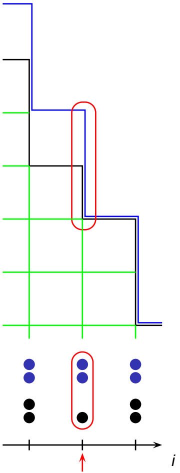

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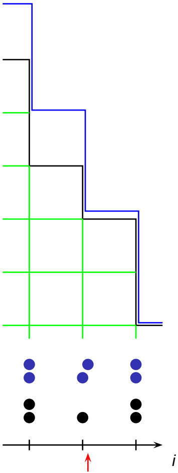

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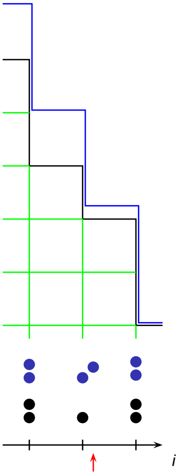

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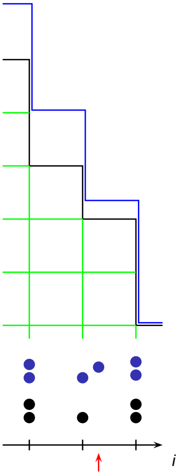

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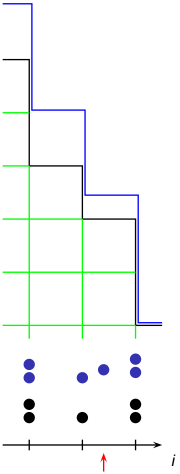

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with rate-rate:

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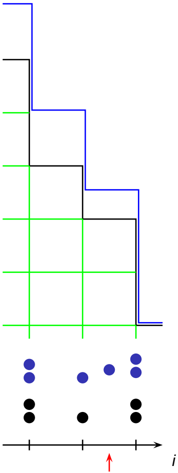


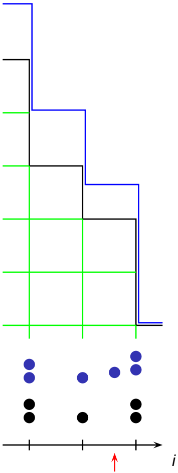
Growth on the right:

rate≤rate

with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



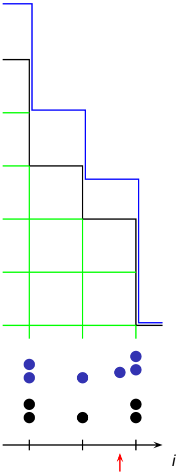
Growth on the right:

rate≤rate

with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



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with rate-rate:

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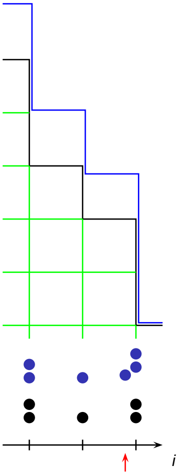

Growth on the right:
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with rate-rate:

The second class particle: attractive case

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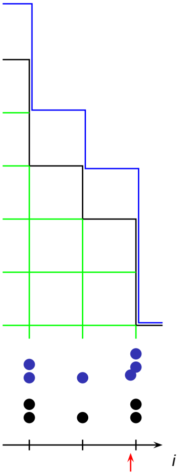

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with rate-rate:

The second class particle: attractive case

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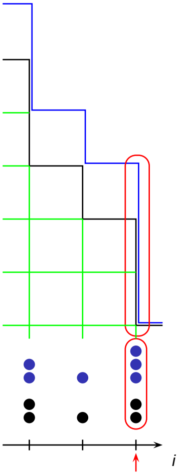


Growth on the right:

rate≤rate

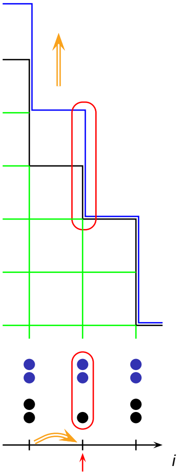
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

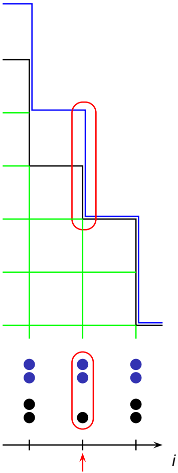
rate$\geq$rate



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate
with rate:



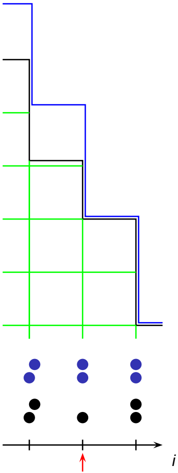
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate≥rate

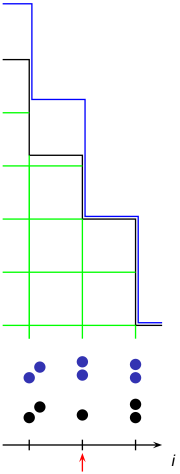
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

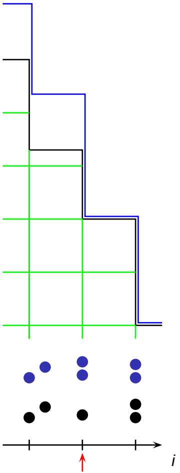
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

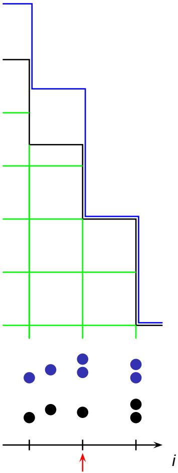
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate≥rate
with rate:



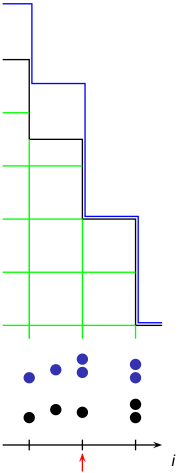
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate≥rate

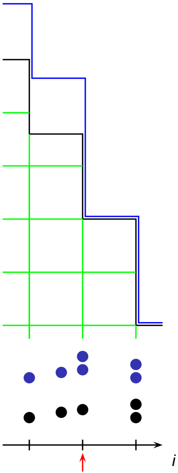
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

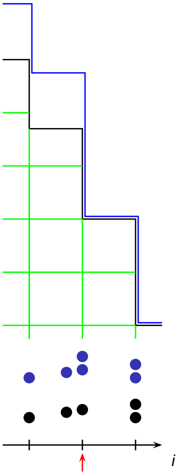
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

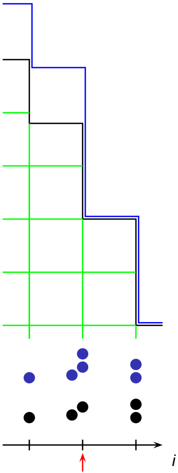
Growth on the left:
rate≥rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

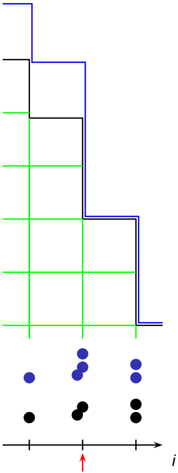
Growth on the left:
rate≥rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

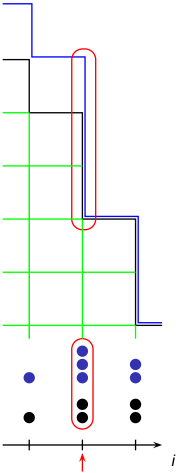
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

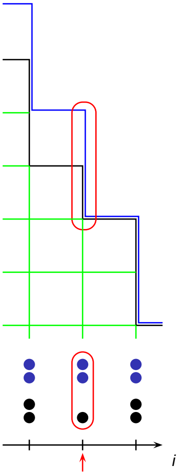
Growth on the left:
rate≥rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

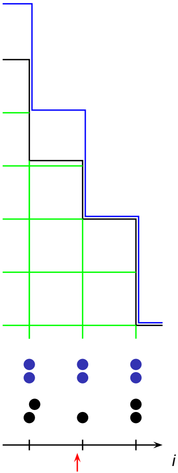
Growth on the left:
rate≥rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

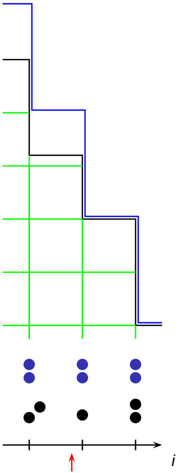
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate≥rate
with rate-rate:



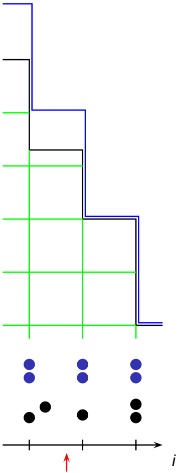
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

with rate-rate:



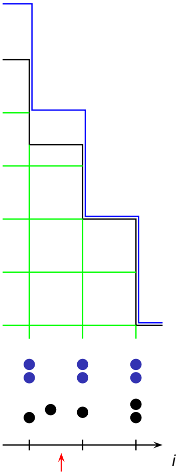
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

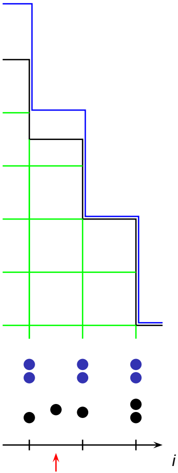
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate≥rate
with rate-rate:



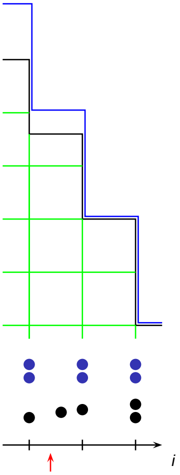
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate≥rate

with rate-rate:



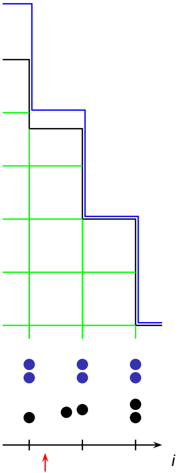
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate≥rate

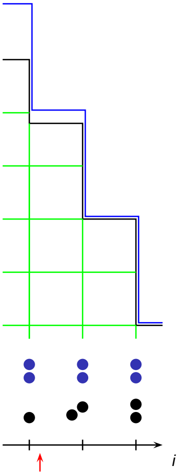
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

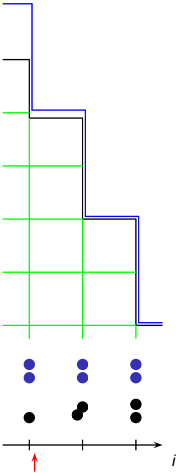
Growth on the left:
rate≥rate
with rate-rate:



The second class particle: attractive case

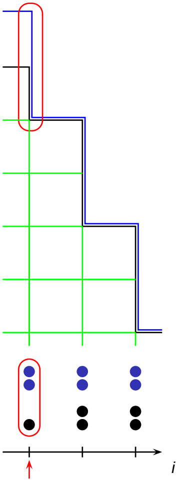
States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.



Growth on the left:
rate≥rate
with rate-rate:

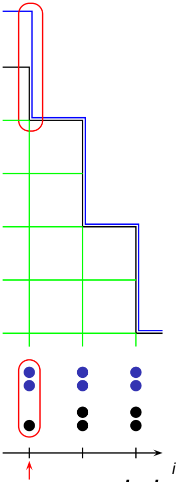
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

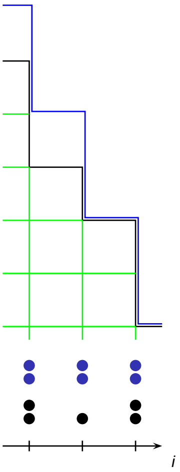
rate≥rate

with rate-rate:

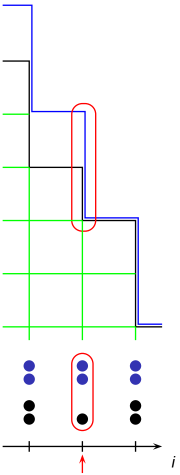


A single discrepancy↑, the *second class particle*, is conserved.

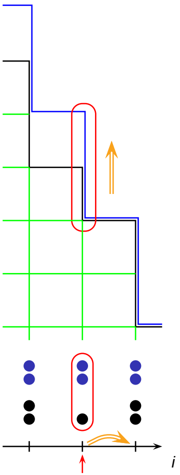
The second class particle: non-attractive case



The second class particle: non-attractive case

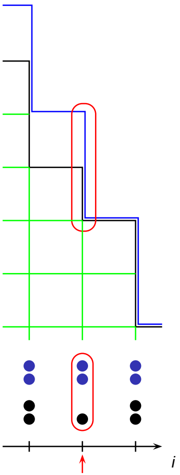


The second class particle: non-attractive case



Growth on the right:
rate>rate

The second class particle: non-attractive case

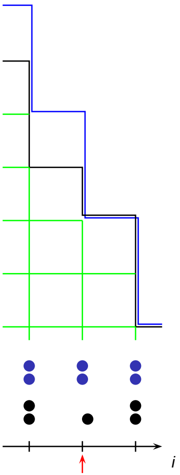


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

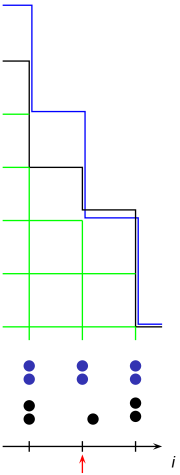


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

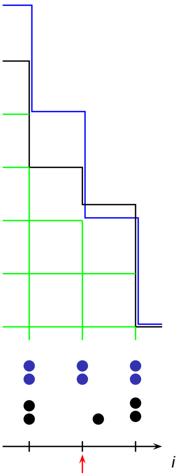


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

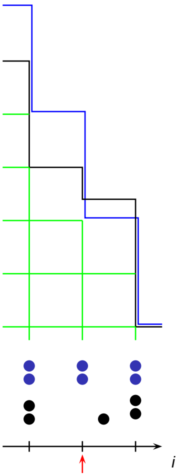


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

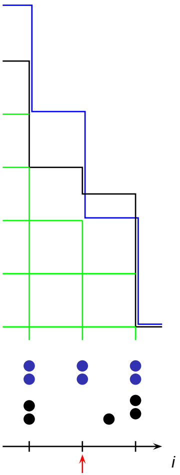


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

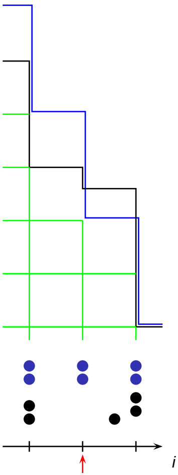


Growth on the right:

rate$>$rate

rate-rate:

The second class particle: non-attractive case

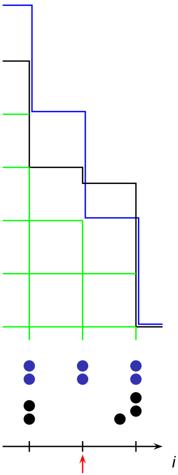


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

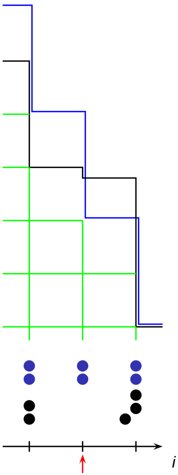


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

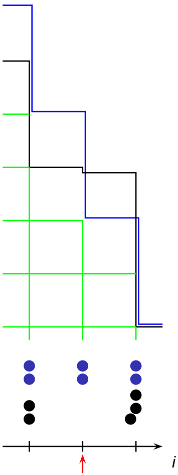


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

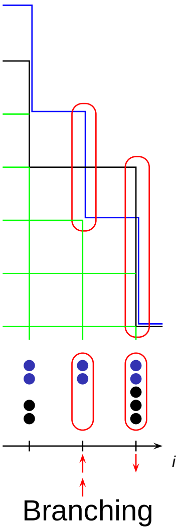


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

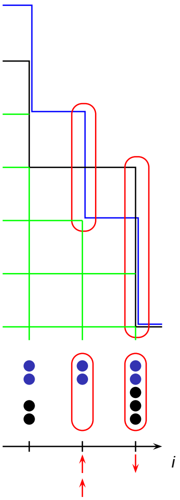


Growth on the right:

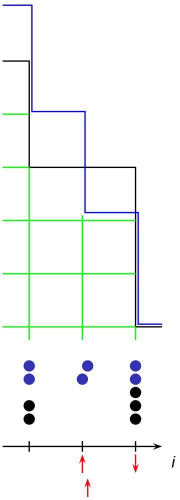
rate$>$rate

rate-rate:

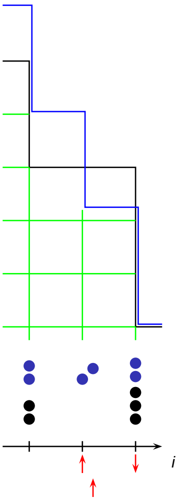
The second class particle: non-attractive case



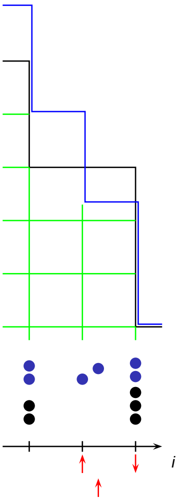
The second class particle: non-attractive case



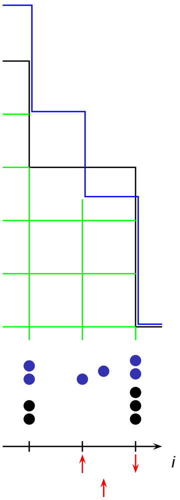
The second class particle: non-attractive case



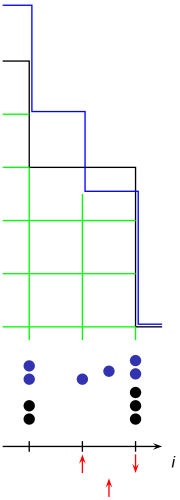
The second class particle: non-attractive case



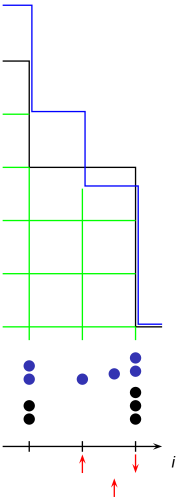
The second class particle: non-attractive case



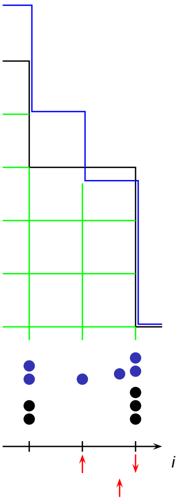
The second class particle: non-attractive case



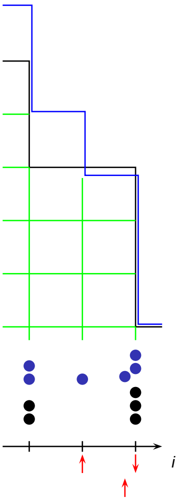
The second class particle: non-attractive case



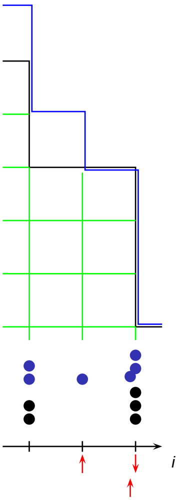
The second class particle: non-attractive case



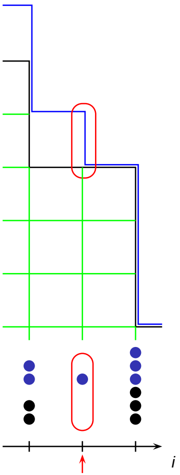
The second class particle: non-attractive case



The second class particle: non-attractive case



The second class particle: non-attractive case

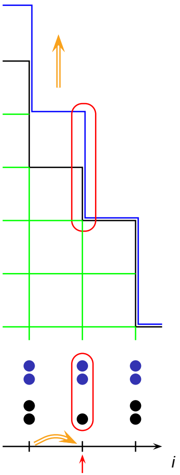


Annihilating

The second class particle: non-attractive case

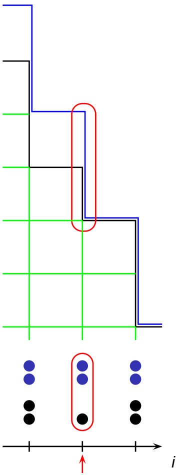
Growth on the left:

rate<rate



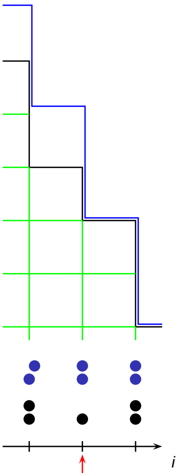
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

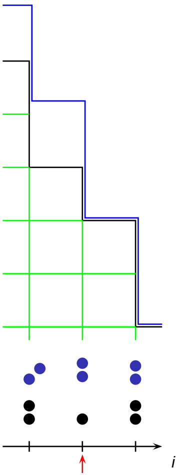


The second class particle: non-attractive case

Growth on the left:

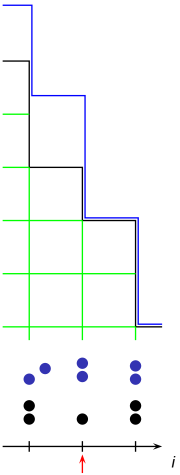
rate<rate

with rate-rate:



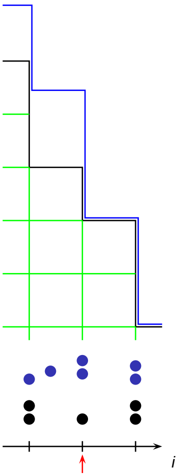
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

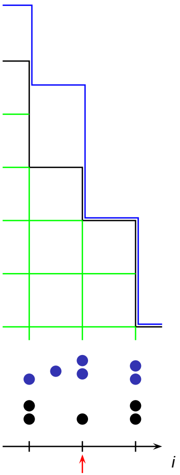


The second class particle: non-attractive case

Growth on the left:

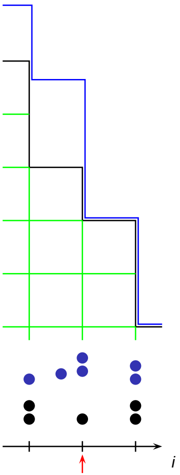
rate<rate

with rate-rate:



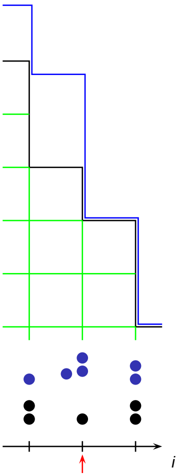
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



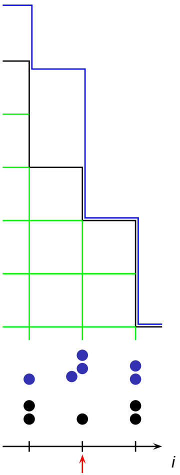
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



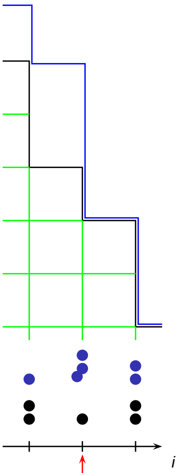
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

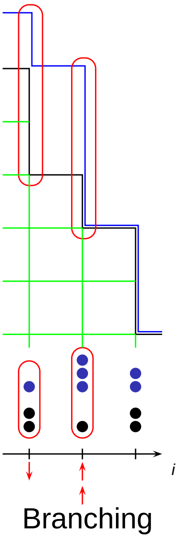


The second class particle: non-attractive case

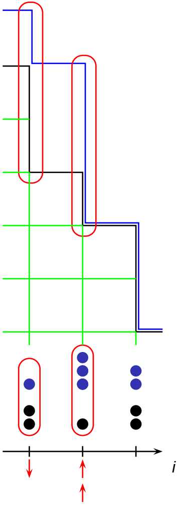
Growth on the left:

rate<rate

with rate-rate:



The second class particle: non-attractive case



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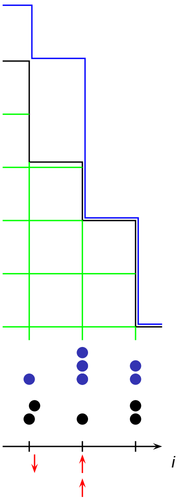

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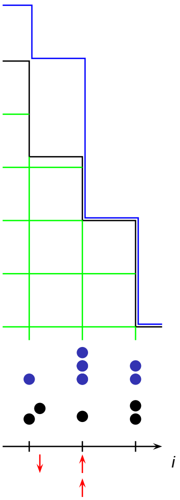

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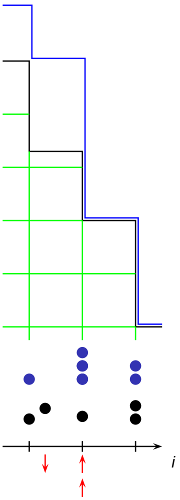

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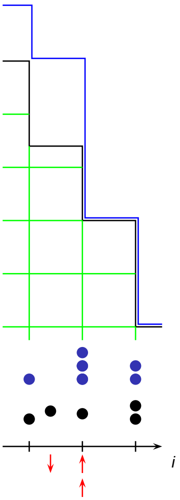

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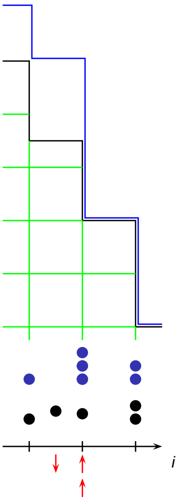

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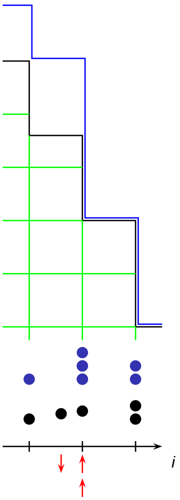

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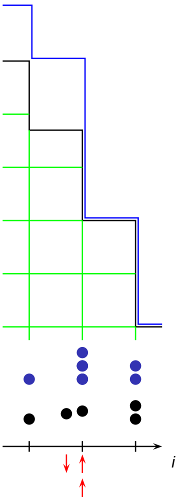

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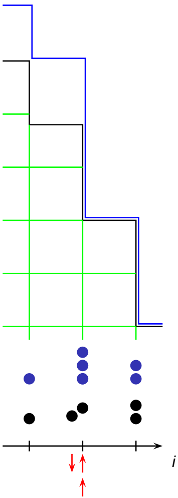

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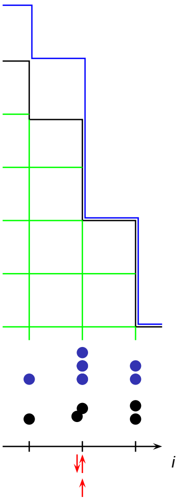

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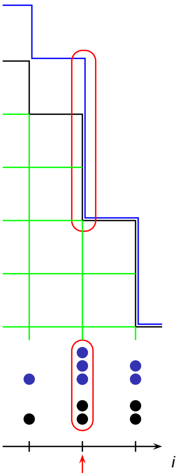

Annihilating

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

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$$= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t - \mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t + \mathfrak{o}(t)}{t}$$

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$$\varepsilon\frac{\partial}{\partial T}\hat{\varrho} = \hat{\varrho}(T, X-\varepsilon)\big[1-\hat{\varrho}(T,X)\big] - \hat{\varrho}(T,X)\big[1-\hat{\varrho}(T,X+\varepsilon)\big]$$

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$$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})] = 0 \qquad (\textit{Burgers eq.}).$$

Burgers eq.: characteristics

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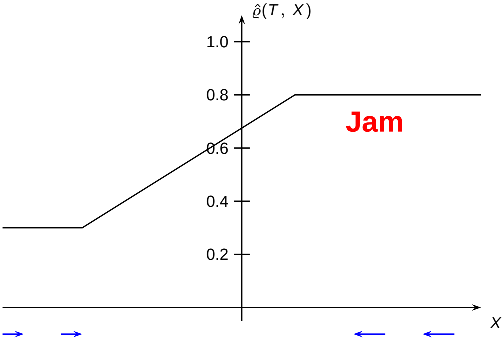
Second class particles are known to follow the characteristics.

On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

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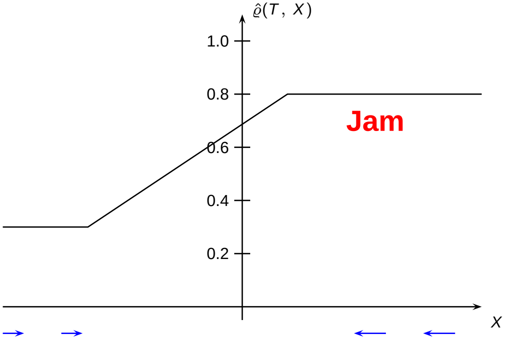

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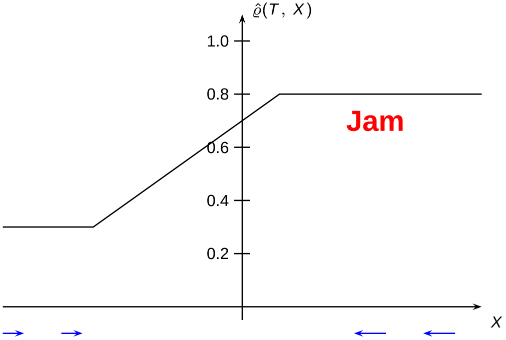

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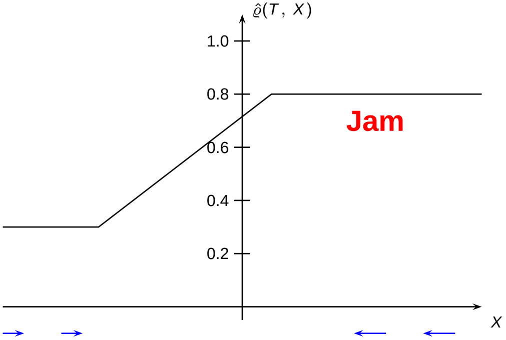

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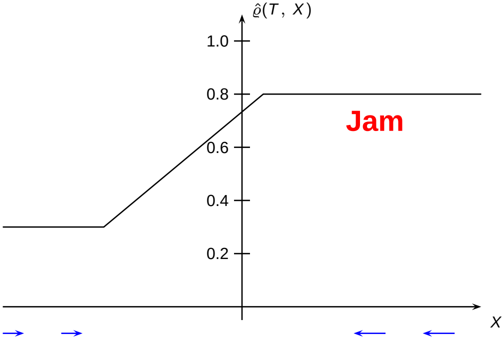

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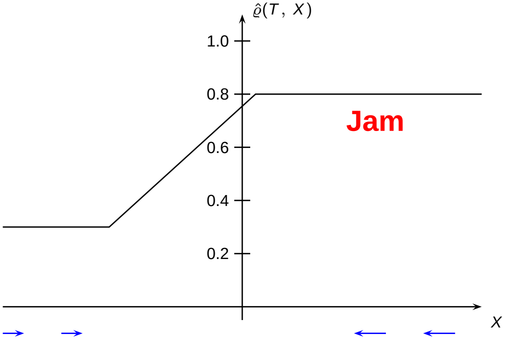

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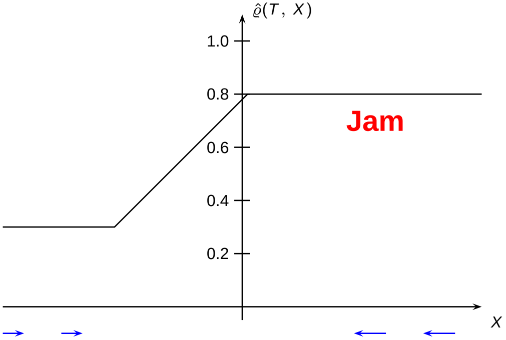

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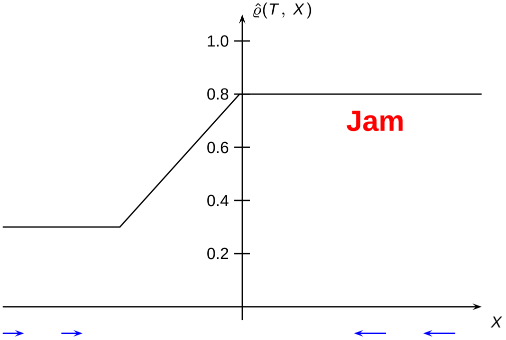

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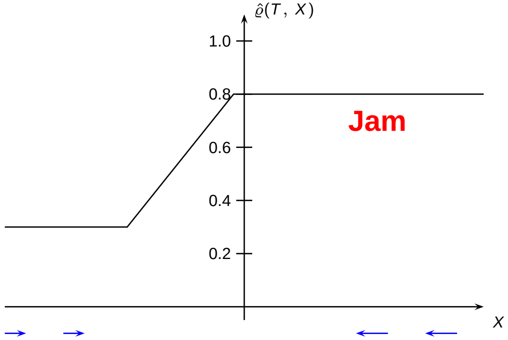

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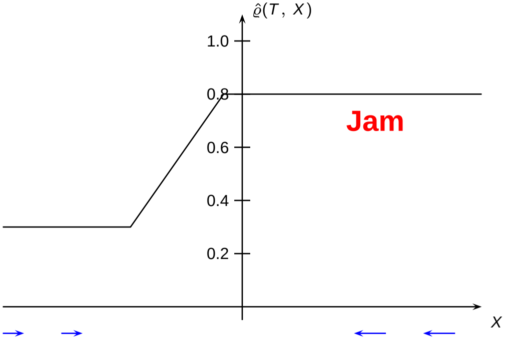

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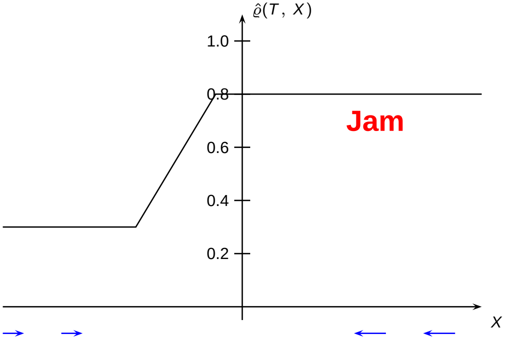

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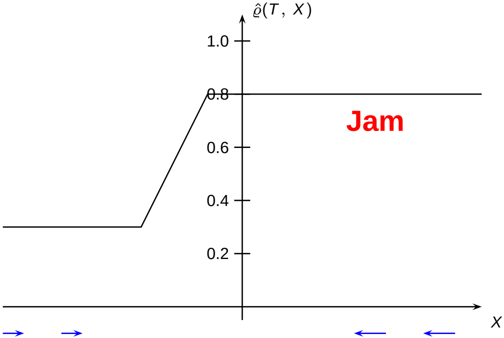

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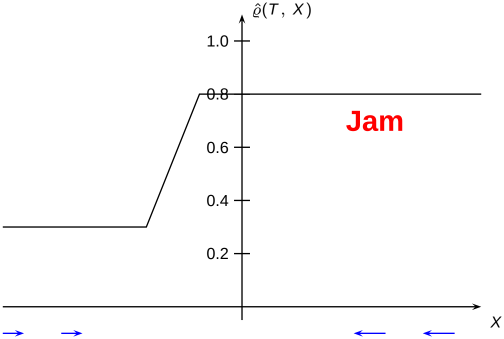

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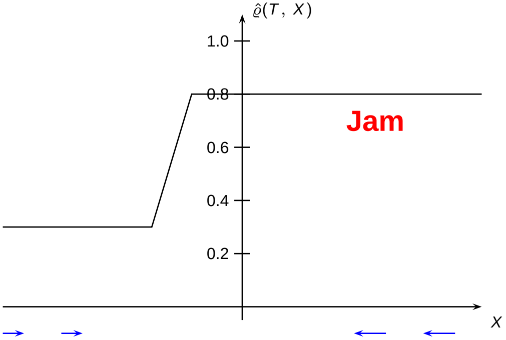

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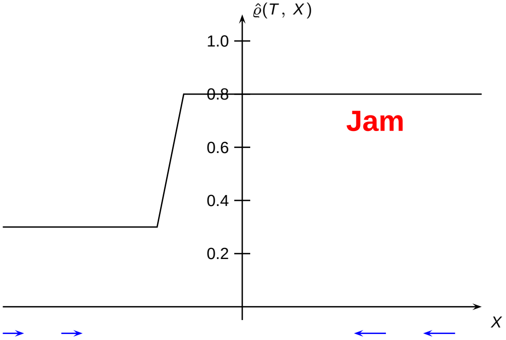

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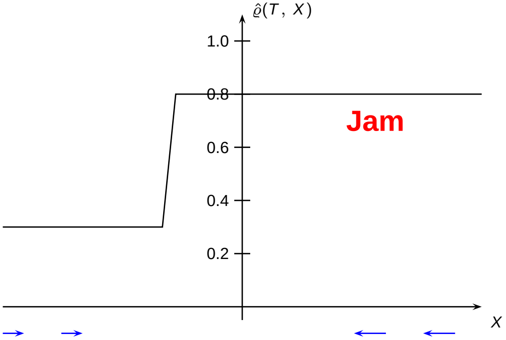

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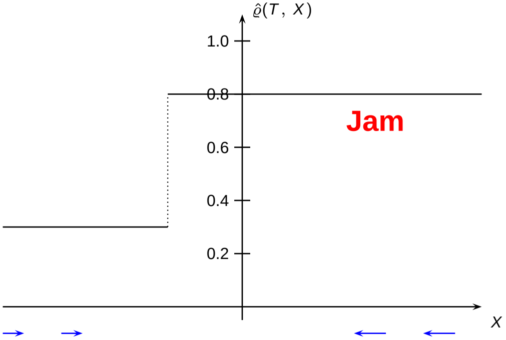

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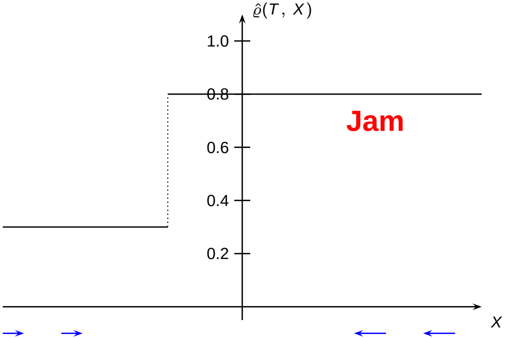

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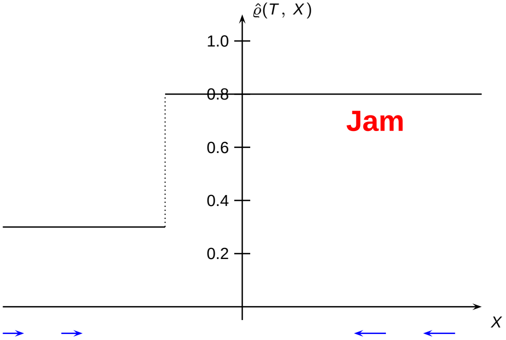

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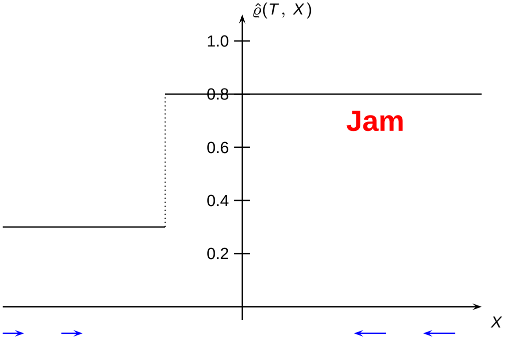

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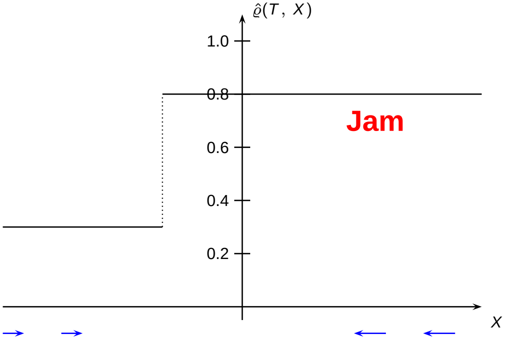

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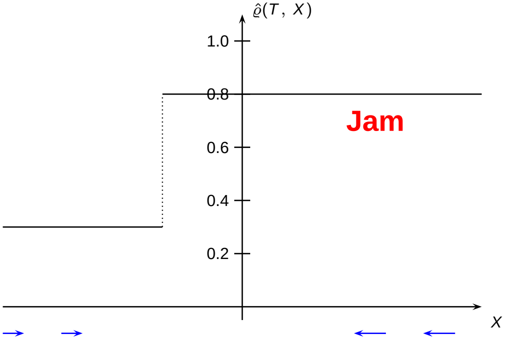

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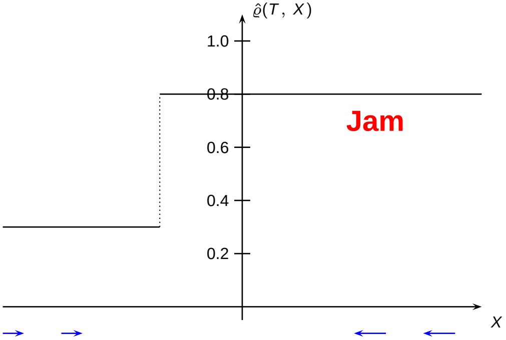

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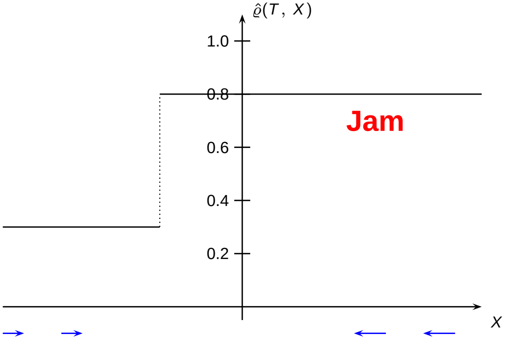

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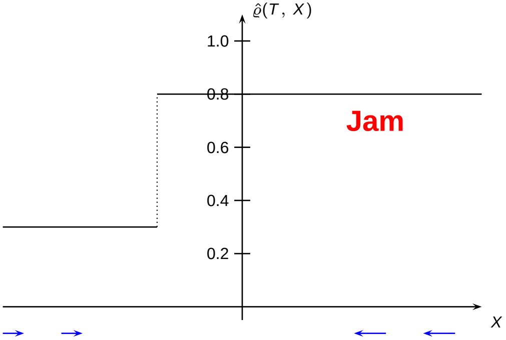

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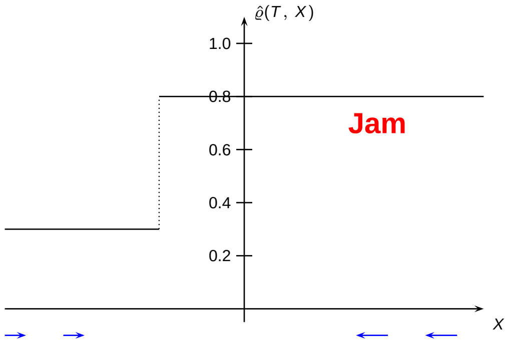

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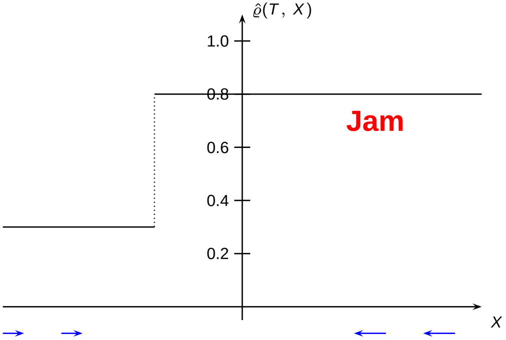

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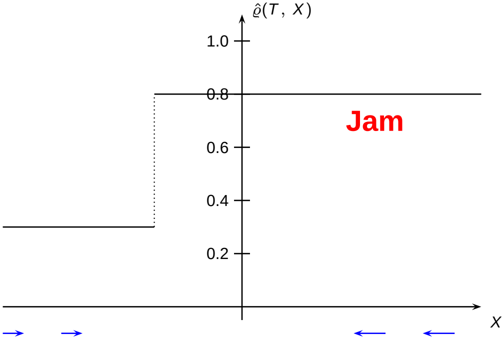

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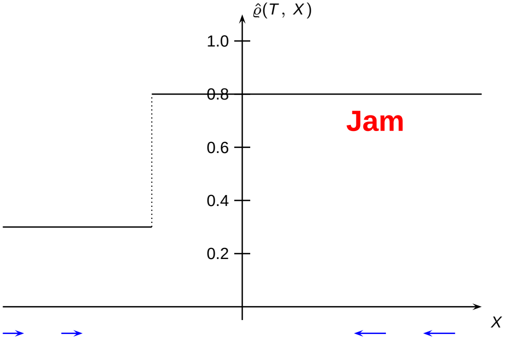

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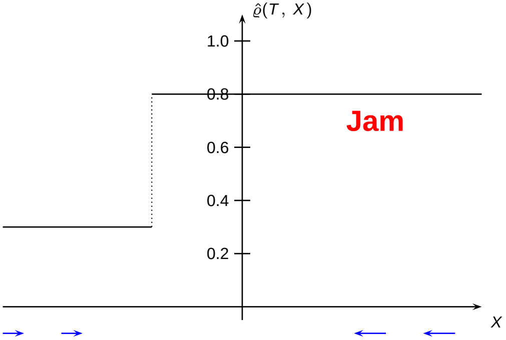

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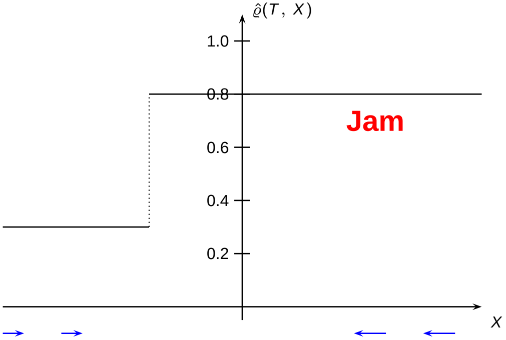

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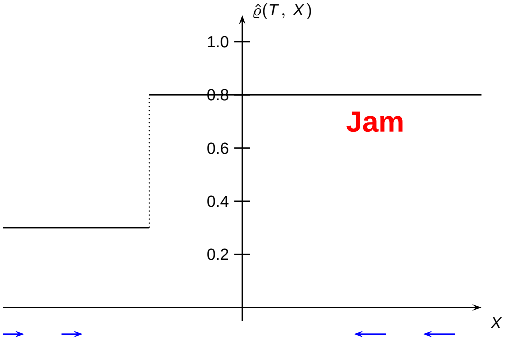
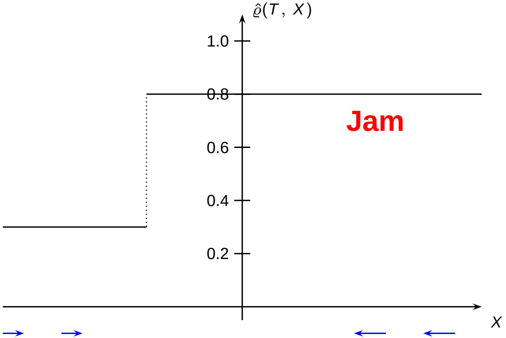


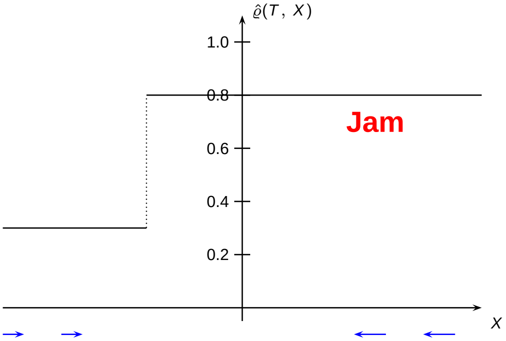
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



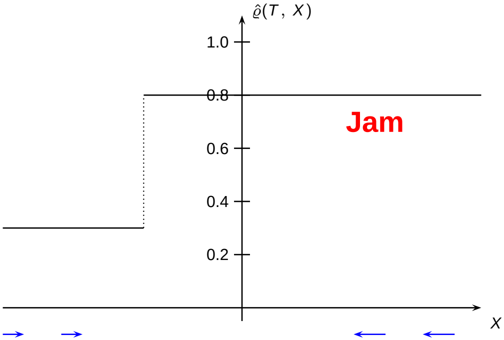
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



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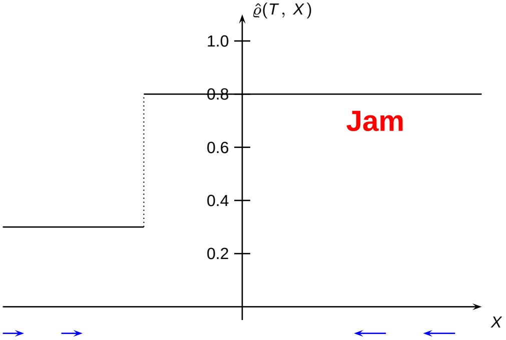
On large scales



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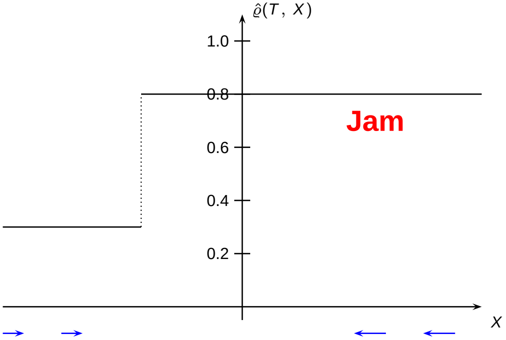
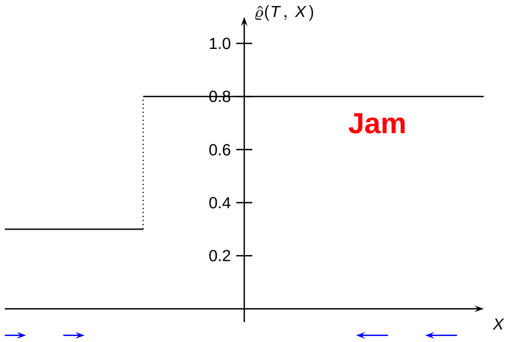
On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



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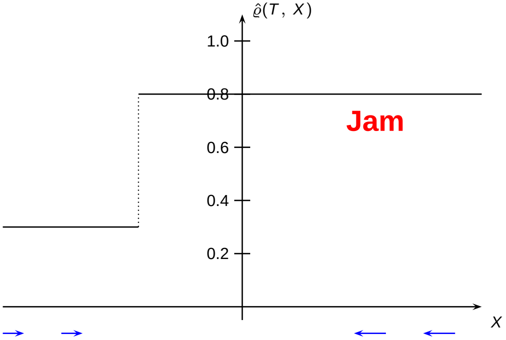
On large scales



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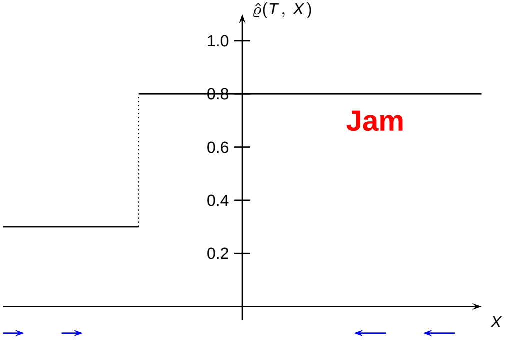

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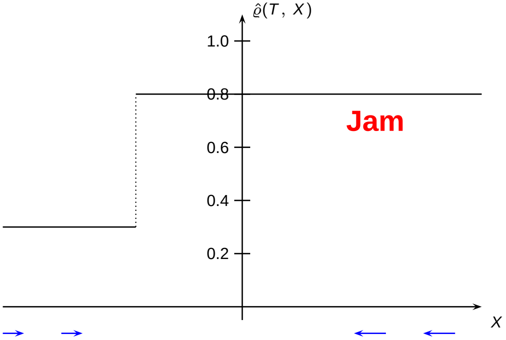

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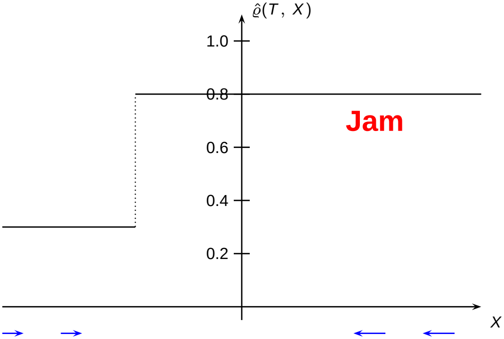

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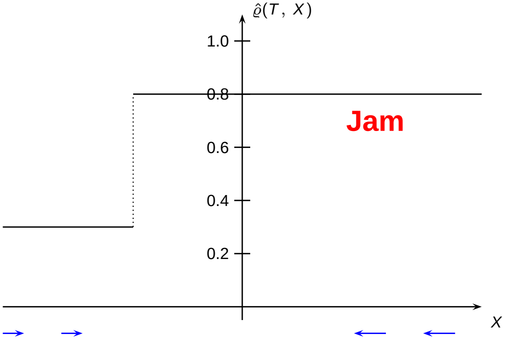

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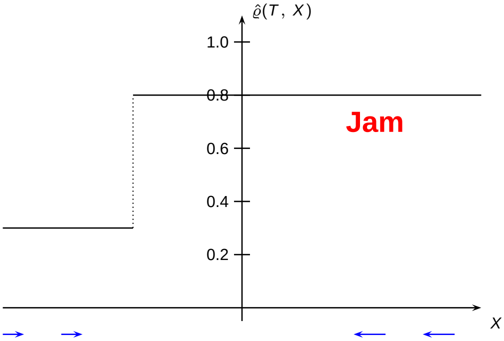

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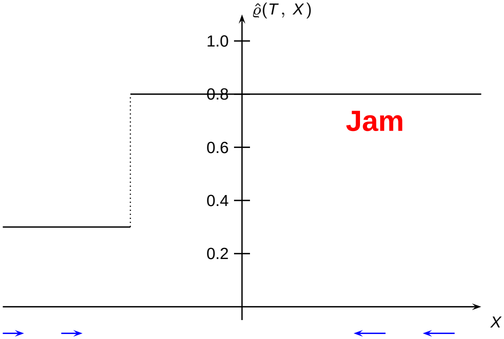
On large scales



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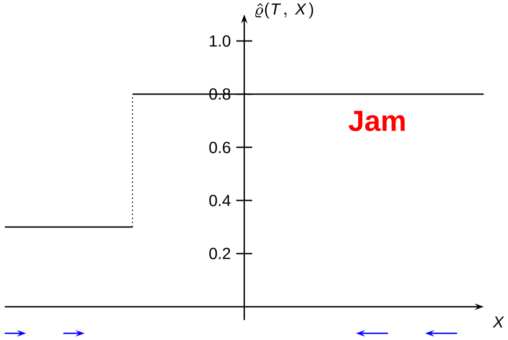
On large scales



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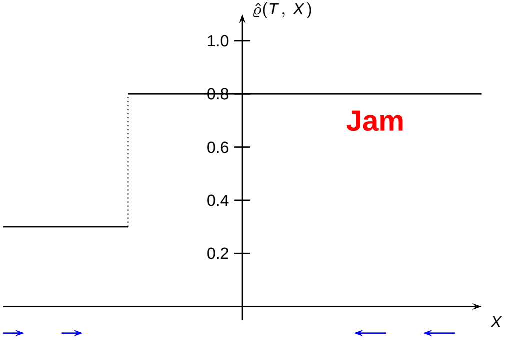

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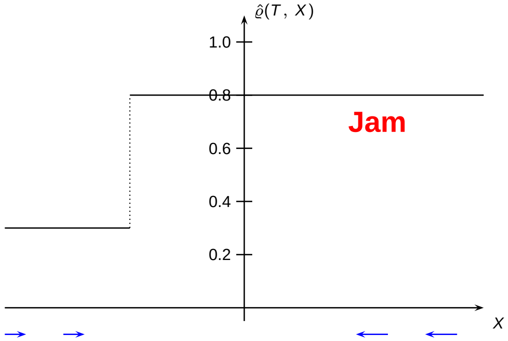

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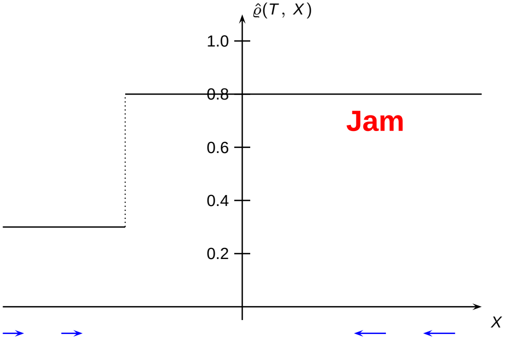

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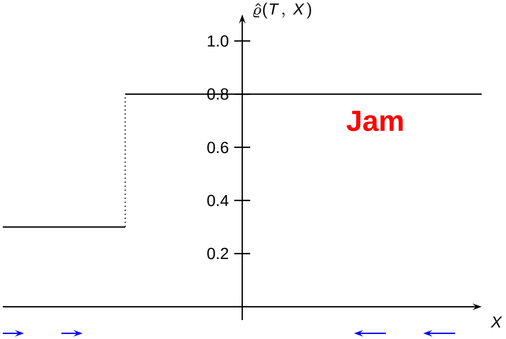

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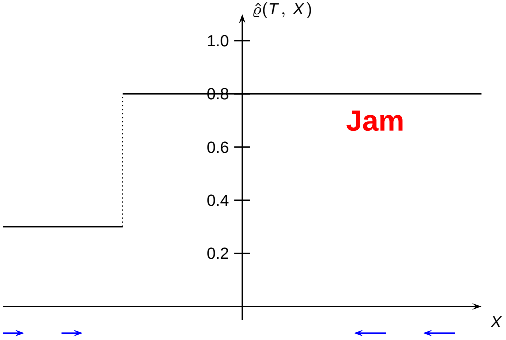

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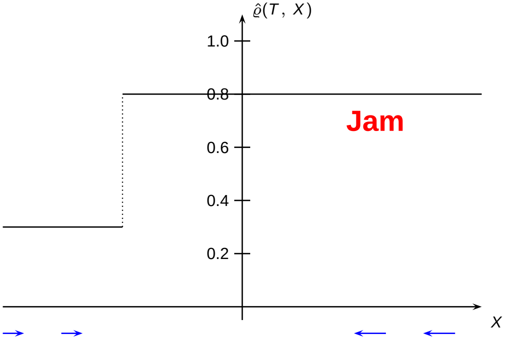

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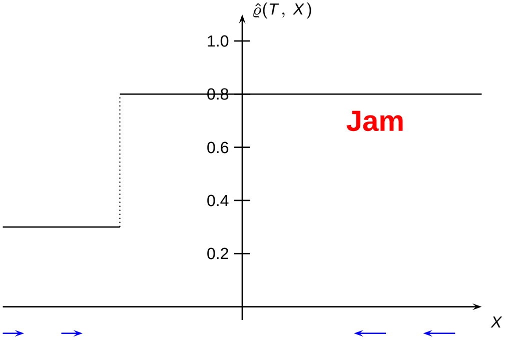

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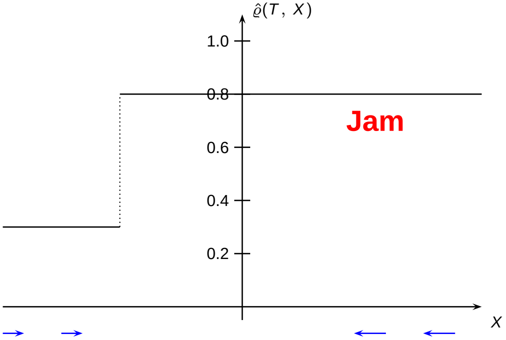
On large scales



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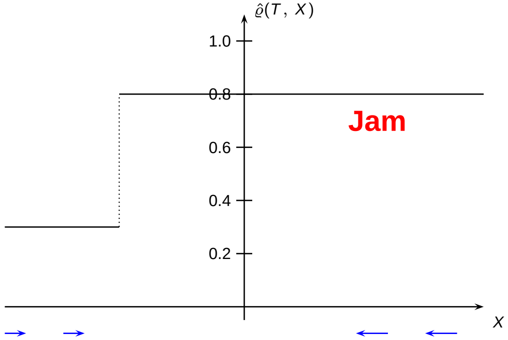
On large scales



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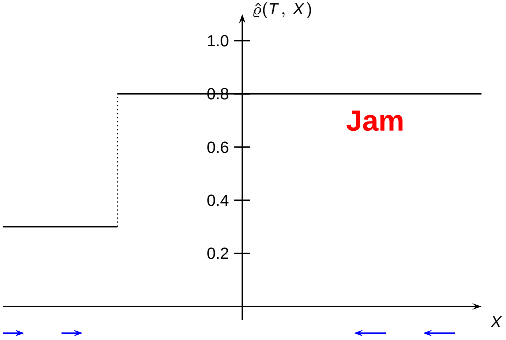

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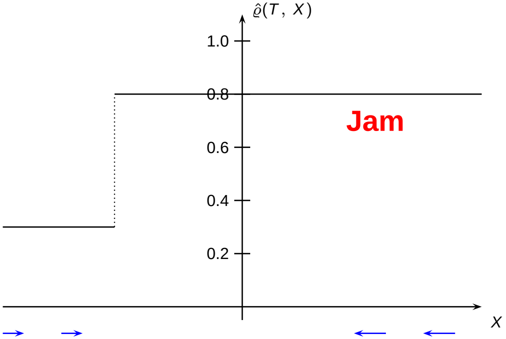

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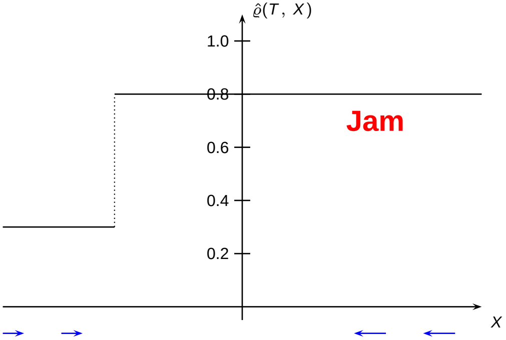

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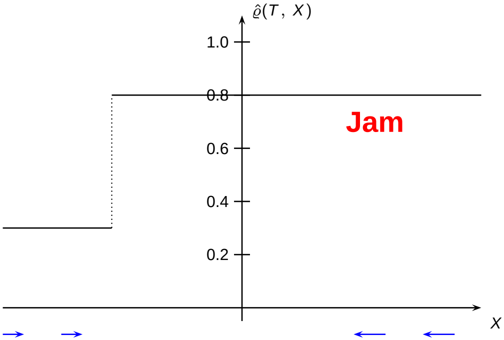

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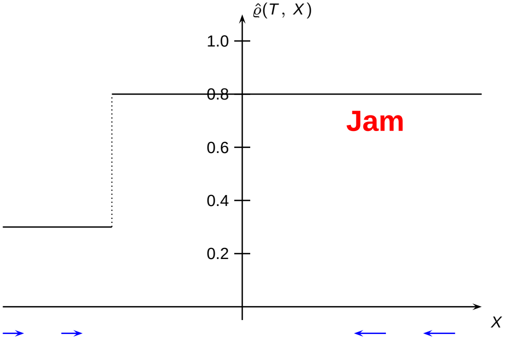

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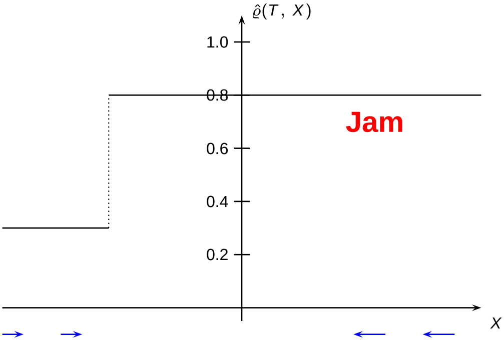
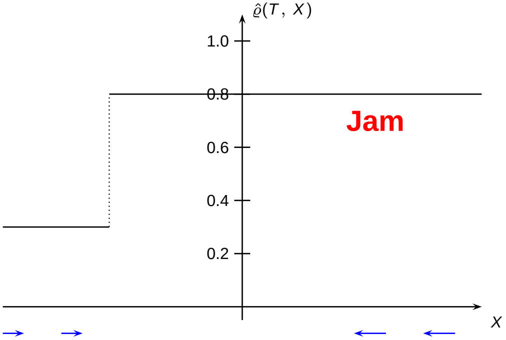
On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

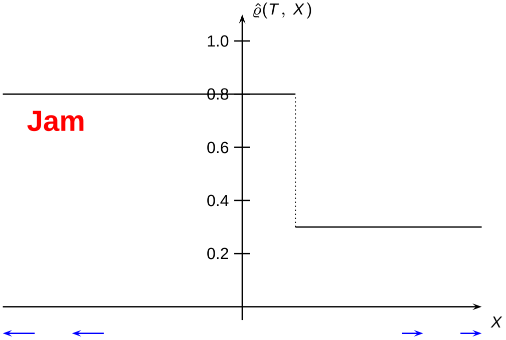
On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

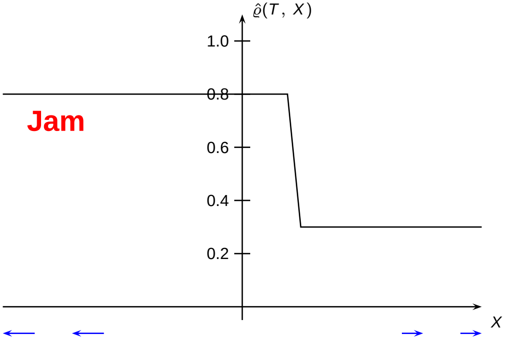
Shock

On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



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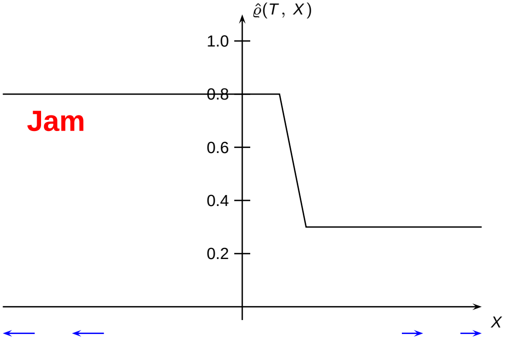
On large scales



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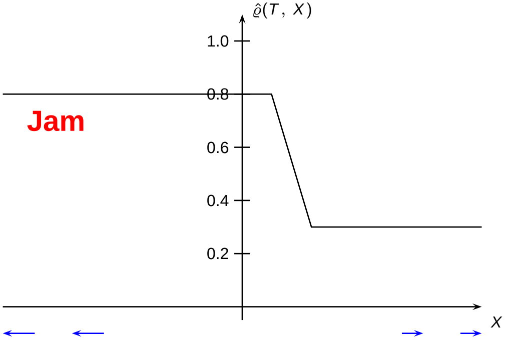
On large scales



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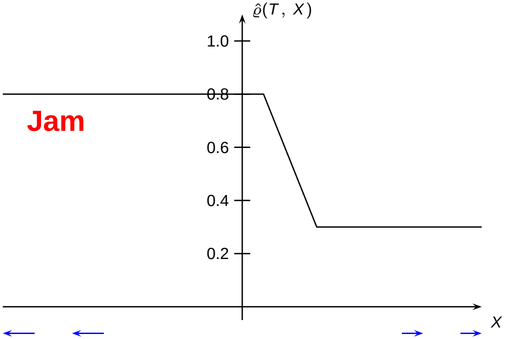

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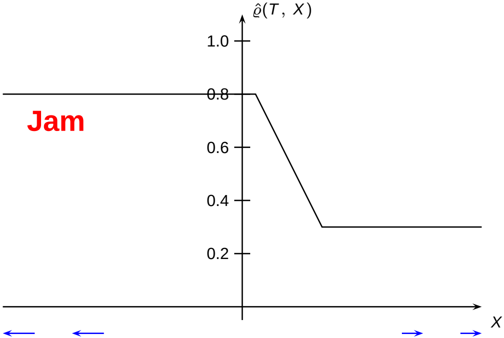

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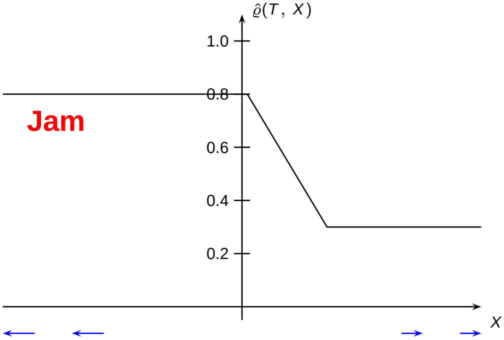

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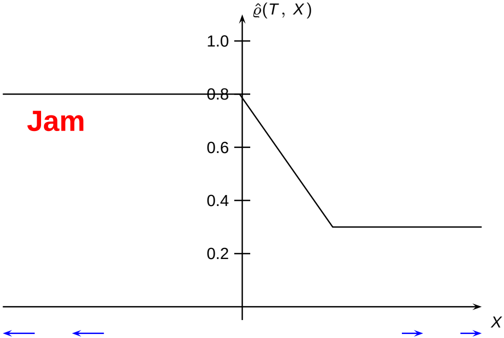

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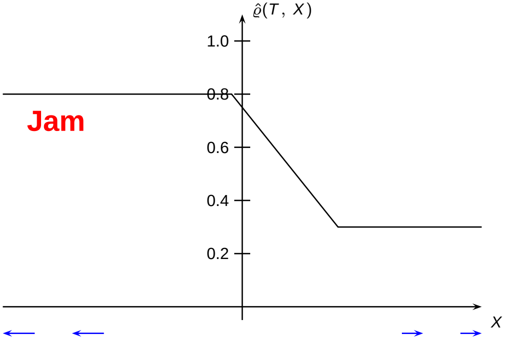

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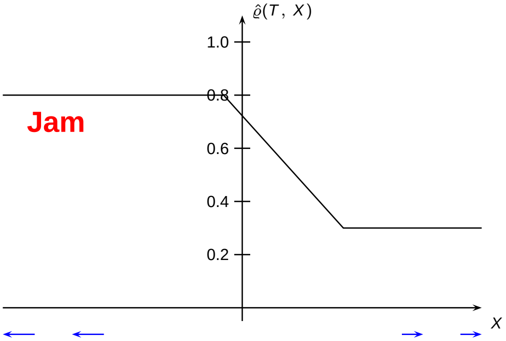

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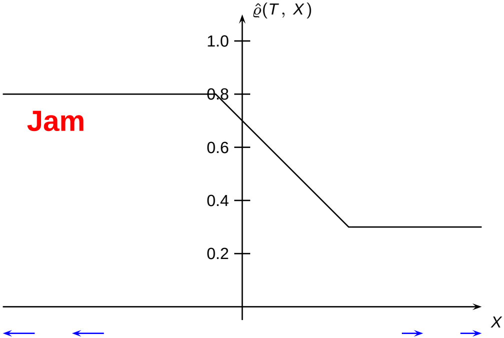

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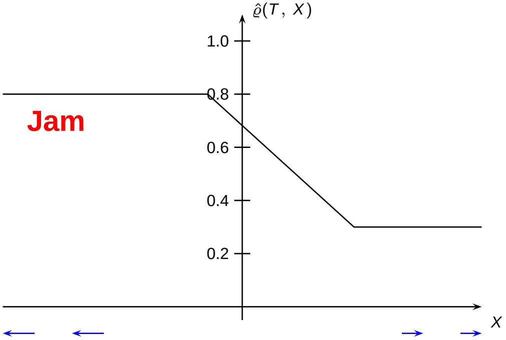

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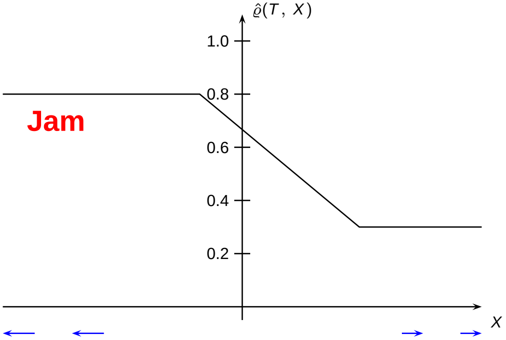

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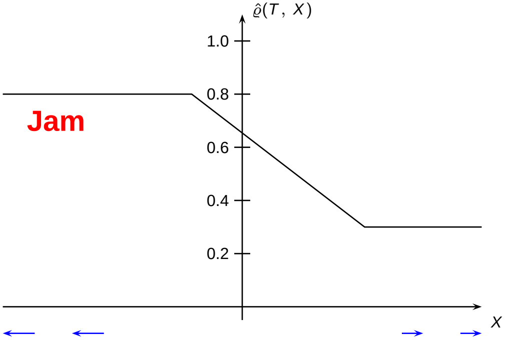

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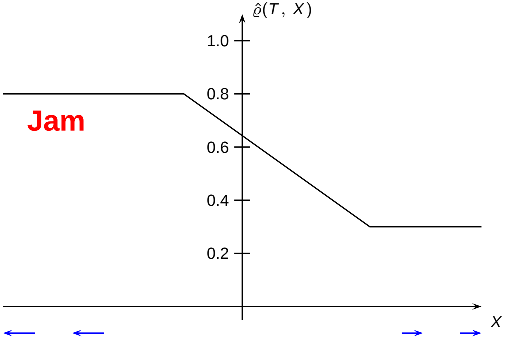
On large scales



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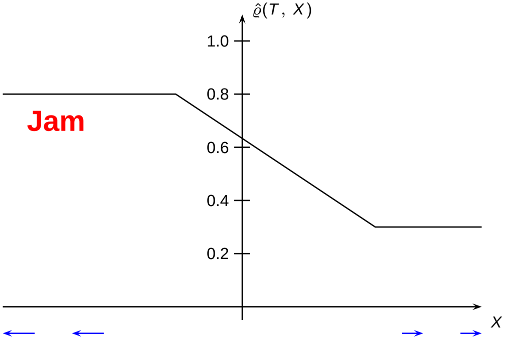

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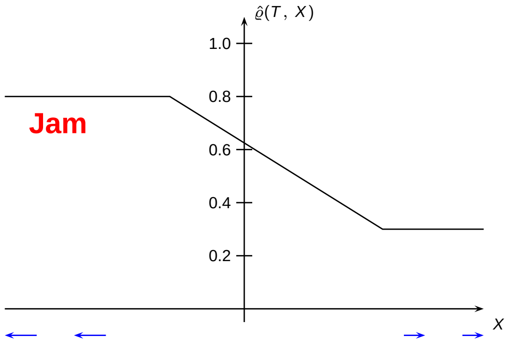

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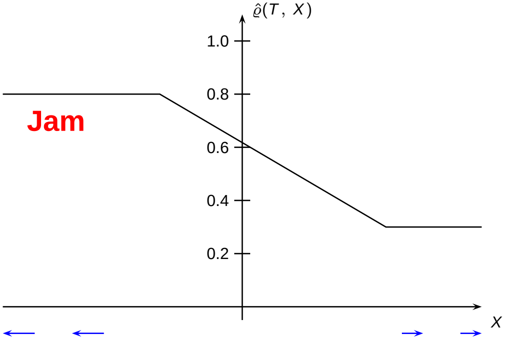

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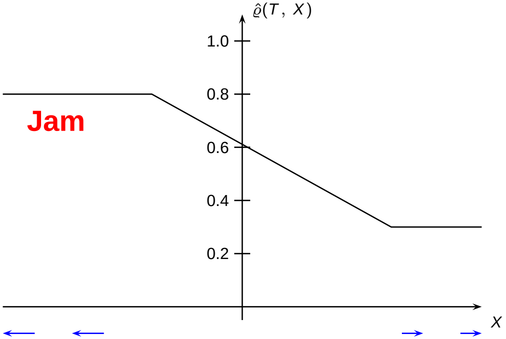

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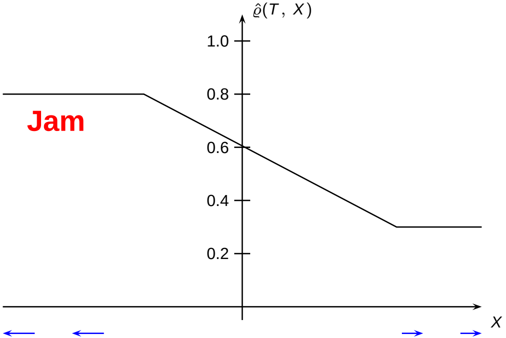

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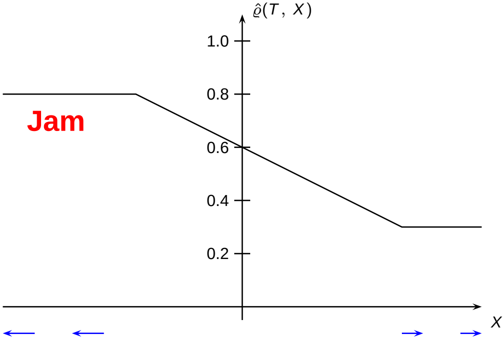

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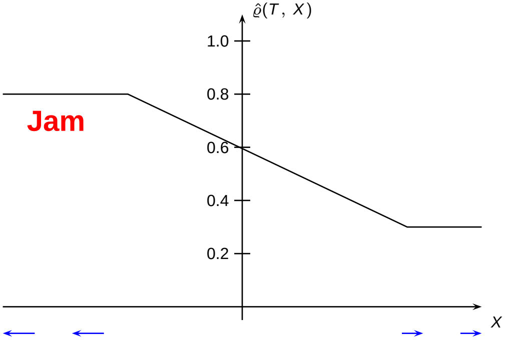

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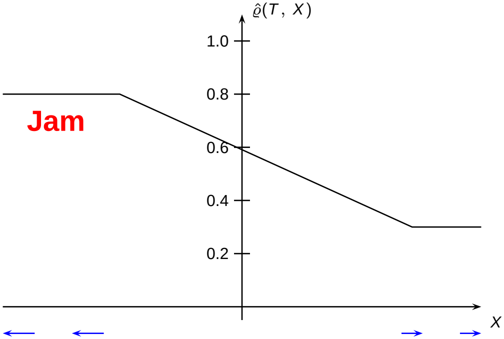

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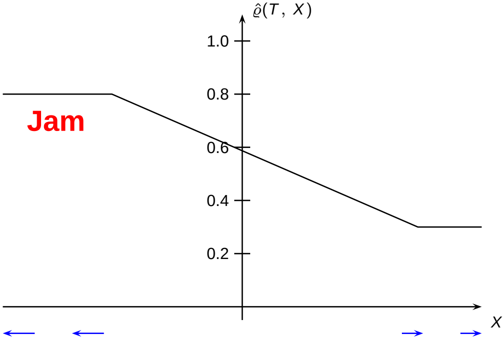

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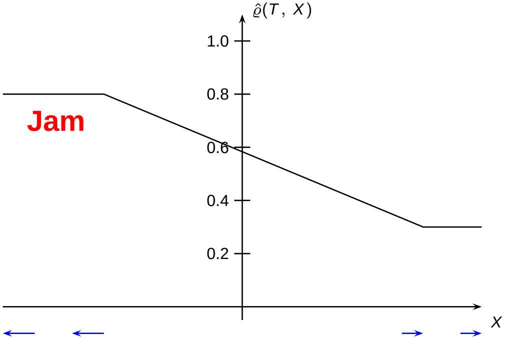

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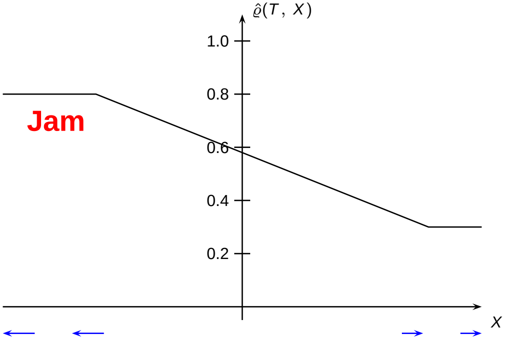

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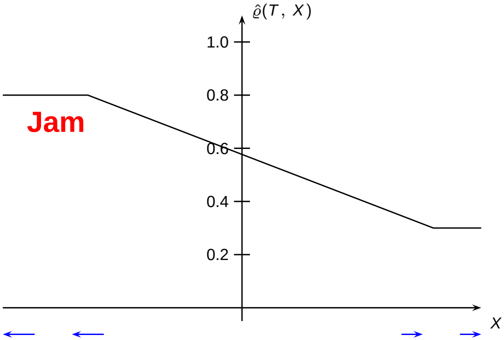

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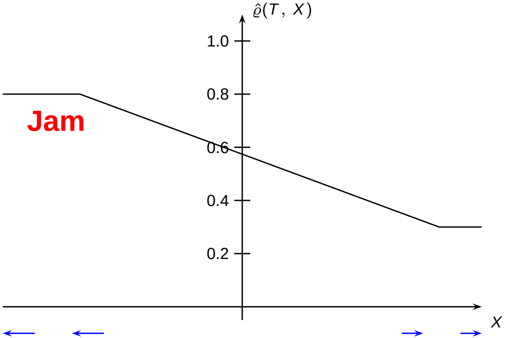

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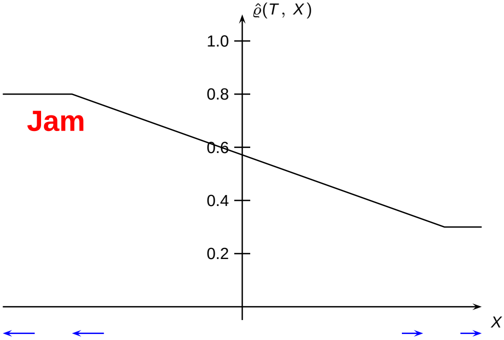

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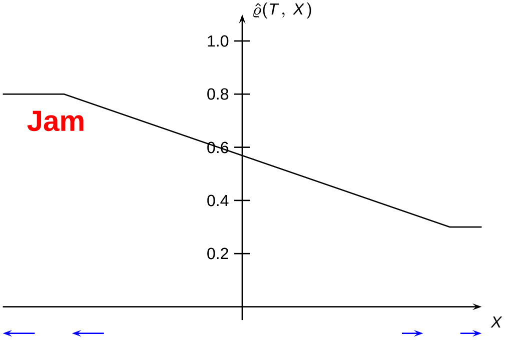

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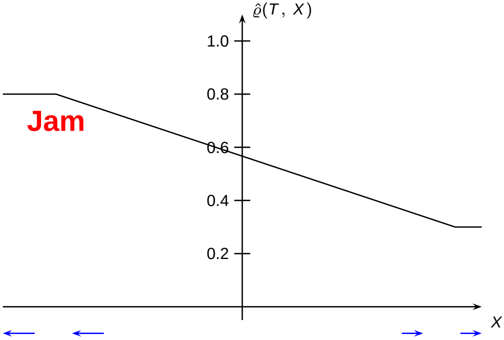

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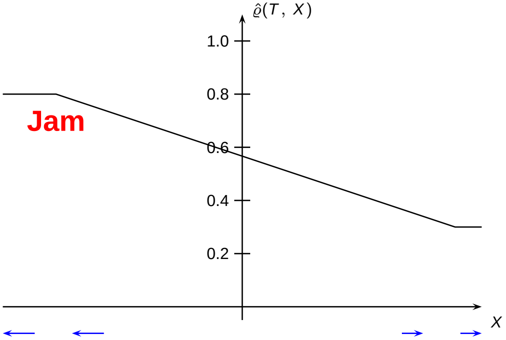
On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

Rarefaction wave

The second class particle: non-attractive case

We are facing a

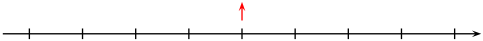
- nearest neighbour
- parity conserving
- branching
- annihilating process
- on the dynamic background of first class particles.

The aim is to control the number of ↑ and ↓'s. Idea from Bálint Tóth.

⇝ homog2.avi

A mean field model

A model we can say something about:



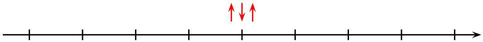
A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:



Branching

A mean field model

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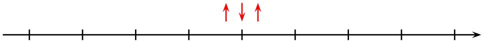

Branching

A mean field model

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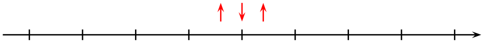

Branching

A mean field model

A model we can say something about:

Branching

A mean field model

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Branching

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A mean field model

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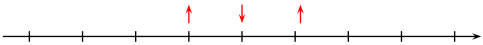

Jump

A mean field model

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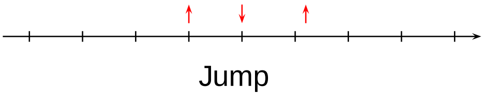

A mean field model

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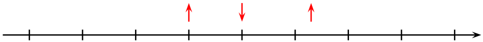

Jump

A mean field model

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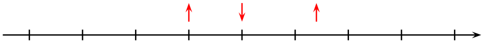

Jump

A mean field model

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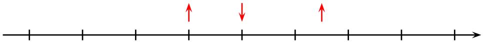

Jump

A mean field model

A model we can say something about:

Jump

A mean field model

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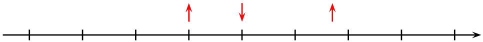

Jump

A mean field model

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A mean field model

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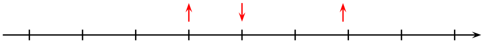

Jump

A mean field model

A model we can say something about:

Jump

A mean field model

A model we can say something about:

Branching

A mean field model

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Branching

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Branching

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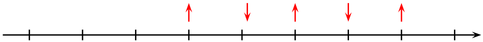
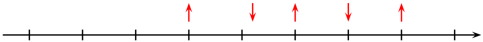


Jump with annihilation

A mean field model

A model we can say something about:



Jump with annihilation

A mean field model

A model we can say something about:

Jump with annihilation

A mean field model

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Jump with annihilation

A mean field model

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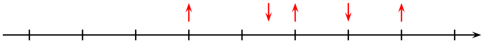
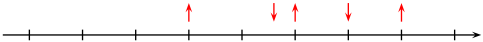


Jump with annihilation

A mean field model

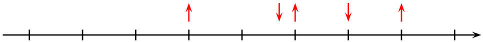
A model we can say something about:



Jump with annihilation

A mean field model

A model we can say something about:



Jump with annihilation

A mean field model

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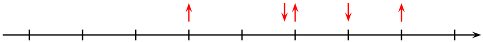
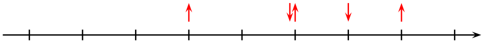


Jump with annihilation

A mean field model

A model we can say something about:



Jump with annihilation

A mean field model

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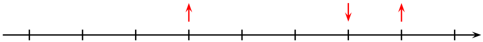
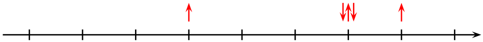


Jump with annihilation

A mean field model

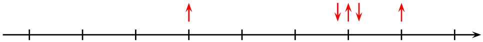
A model we can say something about:



Branching with annihilation

A mean field model

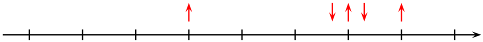
A model we can say something about:



Branching with annihilation

A mean field model

A model we can say something about:



Branching with annihilation

A mean field model

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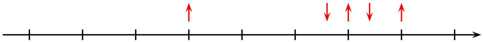
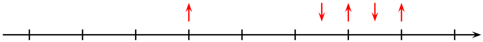


Branching with annihilation

A mean field model

A model we can say something about:



Branching with annihilation

A mean field model

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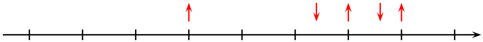

Branching with annihilation

A mean field model

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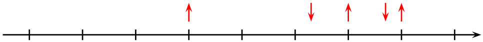

Branching with annihilation

A mean field model

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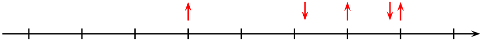
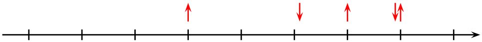


Branching with annihilation

A mean field model

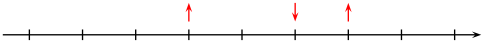
A model we can say something about:



Branching with annihilation

A mean field model

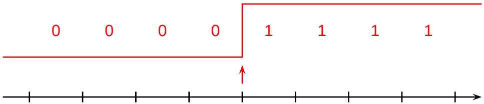
A model we can say something about:



Branching with annihilation

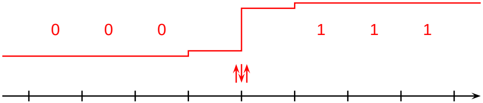
A mean field model

A model we can say something about:



A mean field model

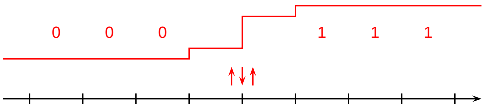
A model we can say something about:



Branching: exclusion

A mean field model

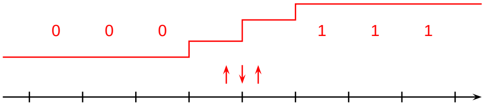
A model we can say something about:



Branching: exclusion

A mean field model

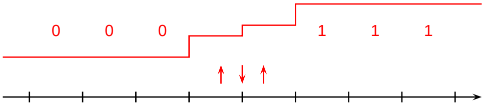
A model we can say something about:



Branching: exclusion

A mean field model

A model we can say something about:



Branching: exclusion

A mean field model

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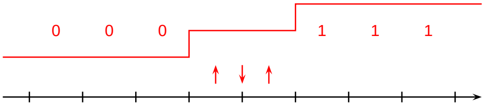

Branching: exclusion

A mean field model

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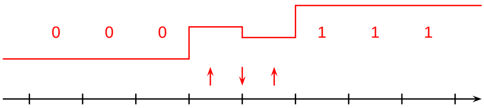
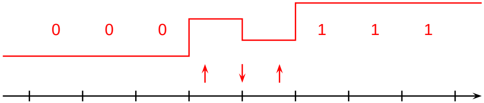


Branching: exclusion

A mean field model

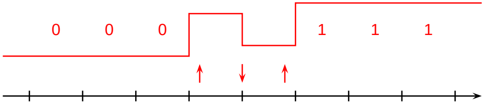
A model we can say something about:



Branching: exclusion

A mean field model

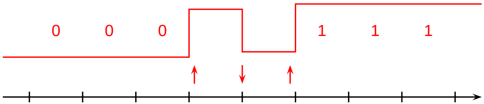
A model we can say something about:



Branching: exclusion

A mean field model

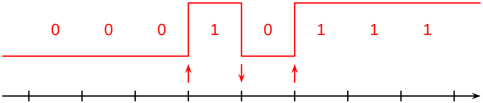
A model we can say something about:



Branching: exclusion

A mean field model

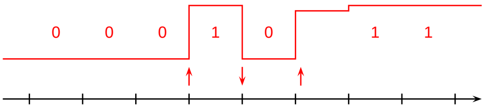
A model we can say something about:



Branching: exclusion

A mean field model

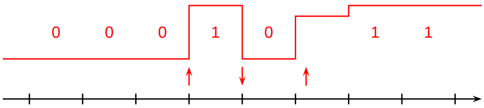
A model we can say something about:



Jump: flip (voting)

A mean field model

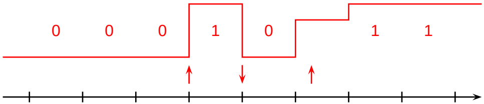
A model we can say something about:



Jump: flip (voting)

A mean field model

A model we can say something about:



Jump: flip (voting)

A mean field model

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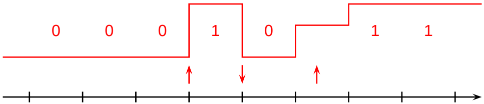
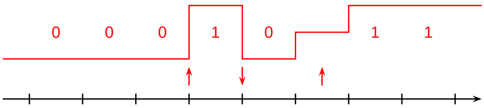


Jump: flip (voting)

A mean field model

A model we can say something about:



Jump: flip (voting)

A mean field model

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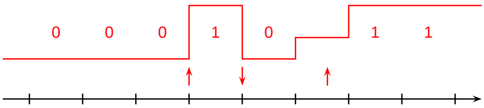

Jump: flip (voting)

A mean field model

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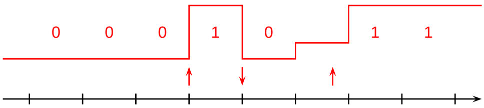

Jump: flip (voting)

A mean field model

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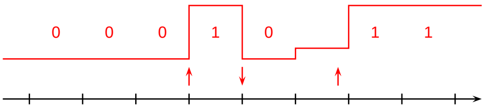

Jump: flip (voting)

A mean field model

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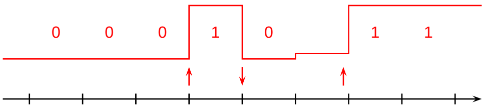

Jump: flip (voting)

A mean field model

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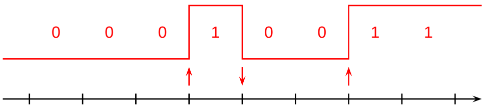
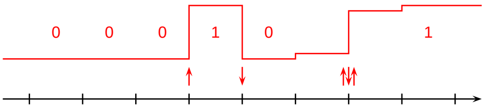


Jump: flip (voting)

A mean field model

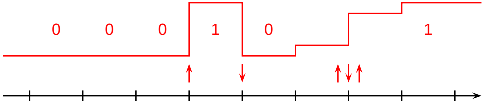
A model we can say something about:



Branching: exclusion

A mean field model

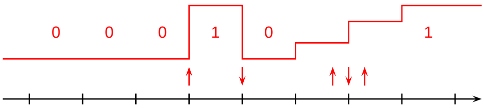
A model we can say something about:



Branching: exclusion

A mean field model

A model we can say something about:



Branching: exclusion

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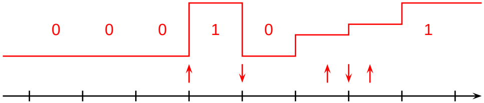
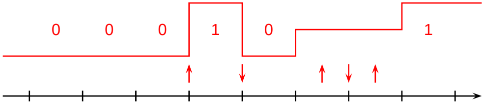
A model we can say something about:



Branching: exclusion

A mean field model

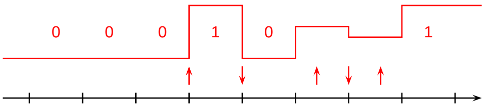
A model we can say something about:



Branching: exclusion

A mean field model

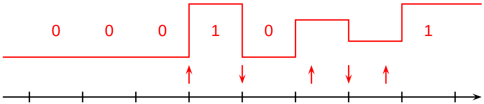
A model we can say something about:



Branching: exclusion

A mean field model

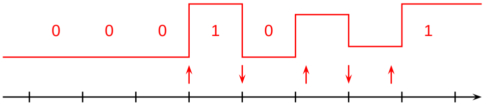
A model we can say something about:



Branching: exclusion

A mean field model

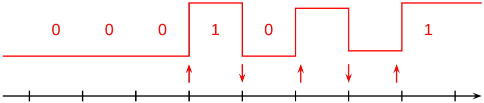
A model we can say something about:



Branching: exclusion

A mean field model

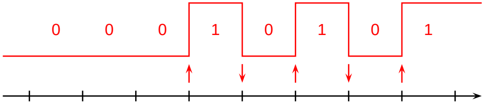
A model we can say something about:



Branching: exclusion

A mean field model

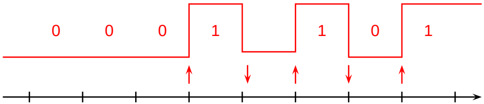
A model we can say something about:



Branching: exclusion

A mean field model

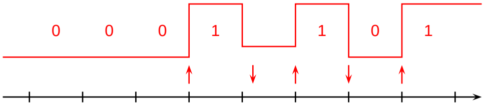
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

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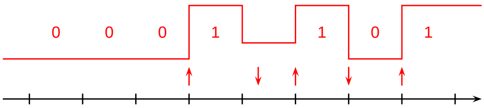

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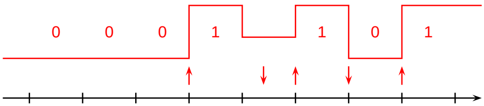

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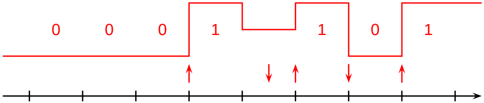

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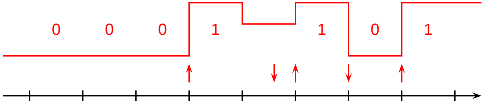

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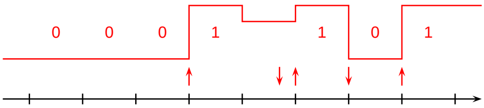

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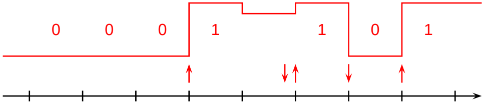

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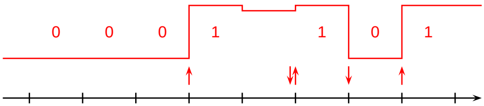

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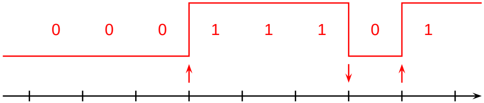

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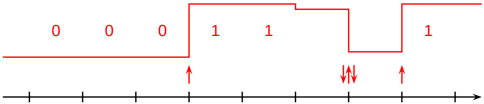

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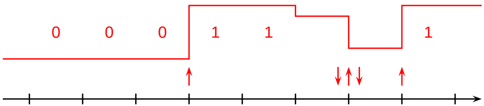

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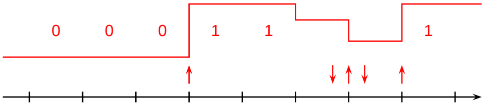

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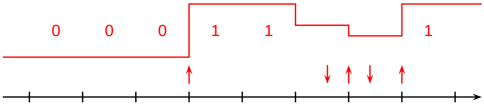

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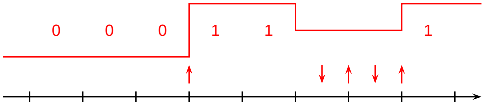

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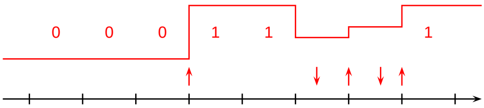

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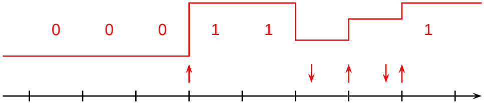

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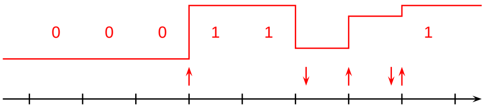

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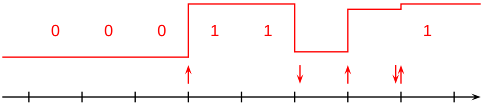

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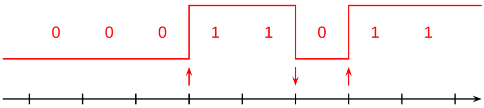

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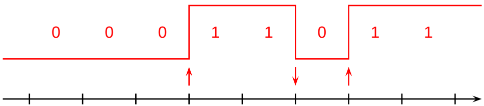

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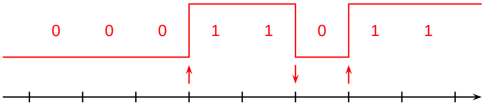

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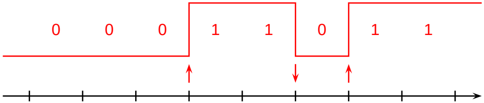

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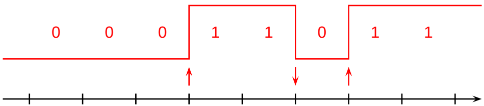

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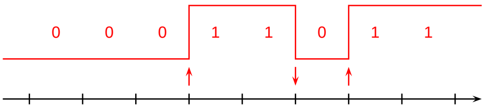

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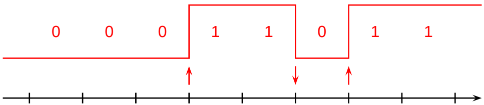
Question: Is the process, as seen by the leftmost ↑, recurrent?

DBARW



First instance of DBARW we could find in the literature: A. Sudbury '90. Positive recurrence: V. Belitsky, P.A. Ferrari, M.V. Menshikov and S.Y. Popov '01; A. Sturm and J.M. Swart '08.

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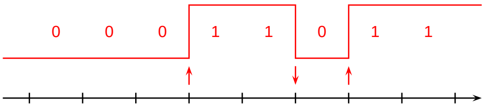

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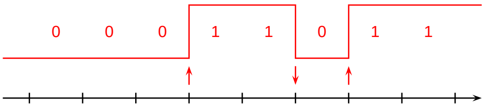

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But: true second class particles interact (*common background of first class particles*).

$\rightsquigarrow$ Repeat the Sturm-Swart proof with configuration dependent jump rates. Jump rates can depend on the whole configuration.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.

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- Bounds on the difference for branching rates of $\uparrow$'s and $\downarrow$'s.
- Weak dependence on particles far away.
- No repulsion in the jumping rates between particles. (*A bit of repulsion locally is still OK.*)

Positive recurrence

Theorem

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- *The stationary distribution sees a finite expected number of particles.*
- *(Extension of all this to non nearest neighbour symmetric branching.)*

An example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{particle on right}} \frac{1}{\text{distance}^{\alpha}},$$

jump rate to the left:

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Unfortunately we do not seem to be there yet... This is not covered at the moment. But a small modification that respects parity in a peculiar way works.

Another example

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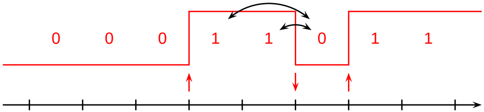
This one is fine.

Two words on the proof



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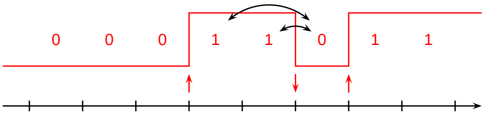

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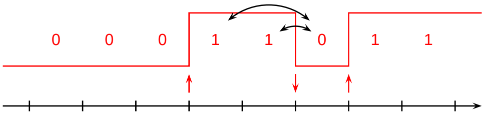

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