

# Road layout in the KPZ class

joint with Riddhipratim Basu, Sudeshna Bhattacharjee, Karambir Das, David Harper

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University of Bristol

Cambridge Probability Seminar  
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A naive Poisson model

Last passage percolation

Our model

Questions

Answers

# A naive Poisson model



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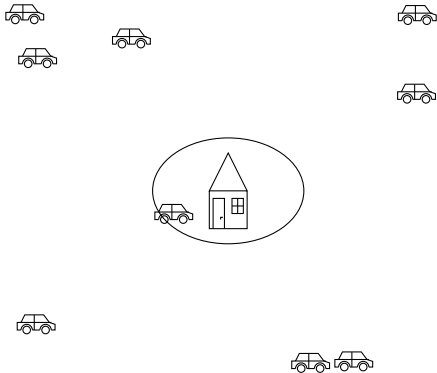
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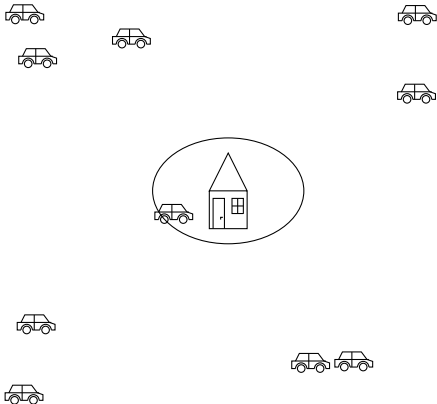
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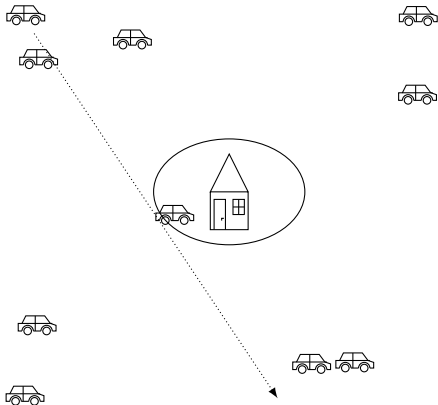
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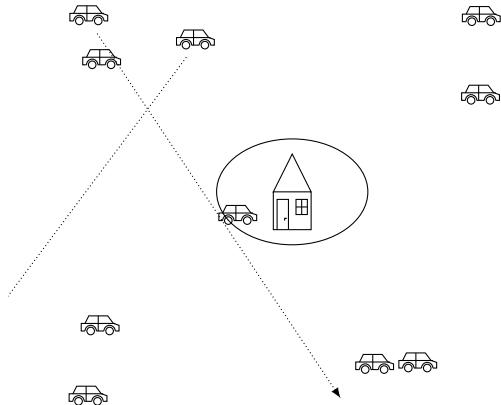
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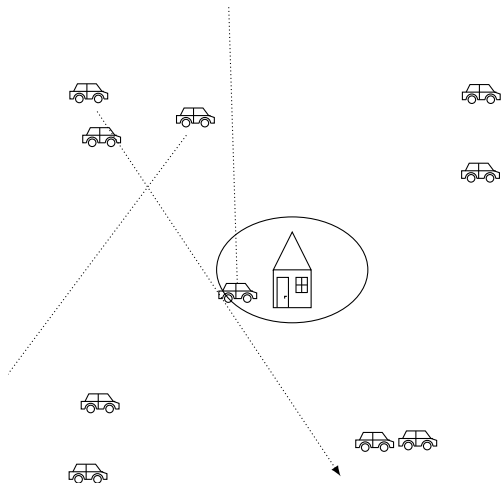
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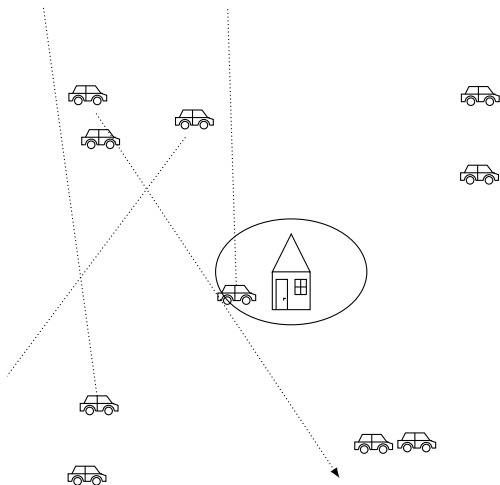


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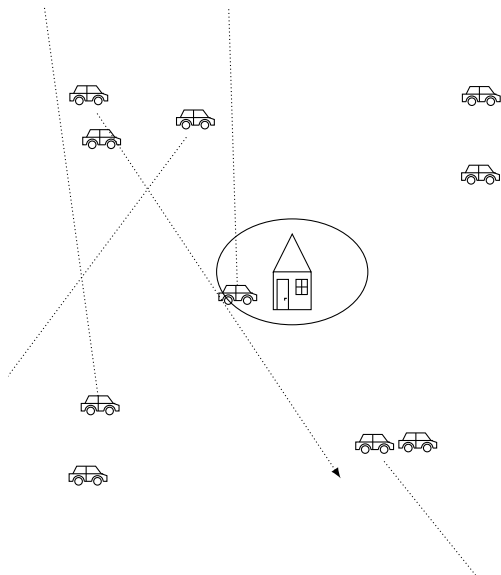




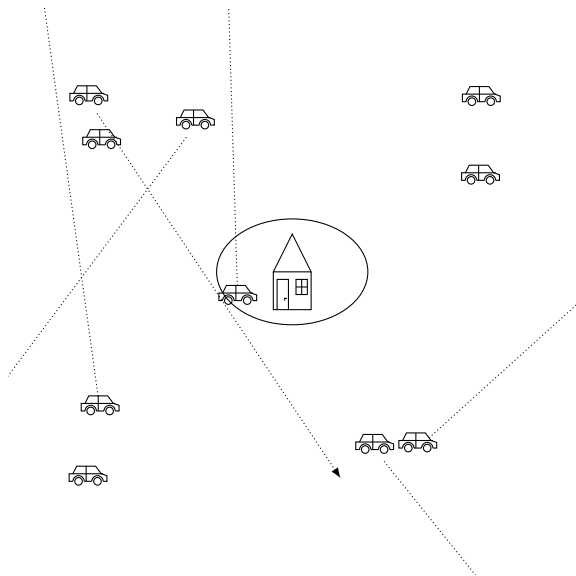
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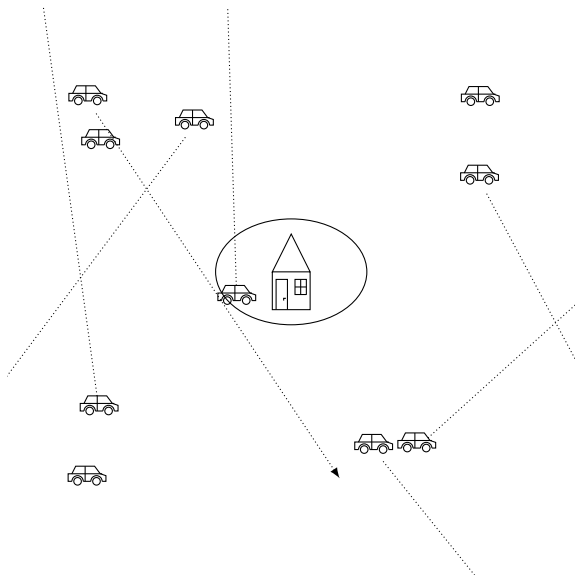
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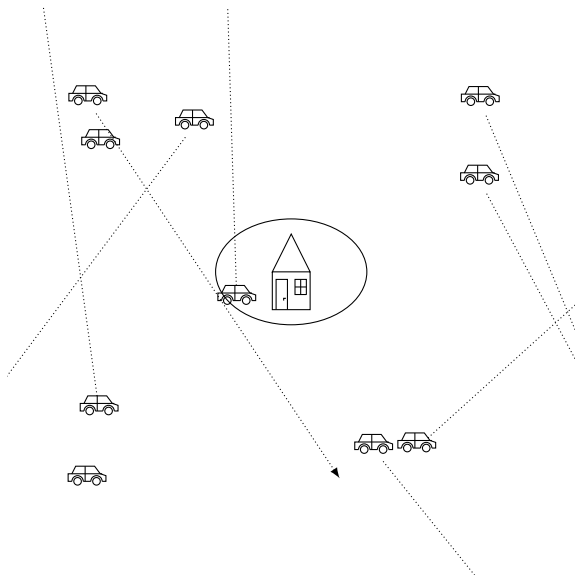
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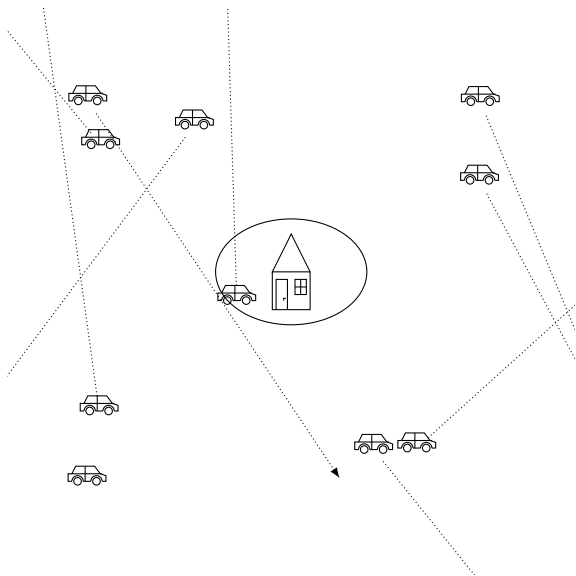
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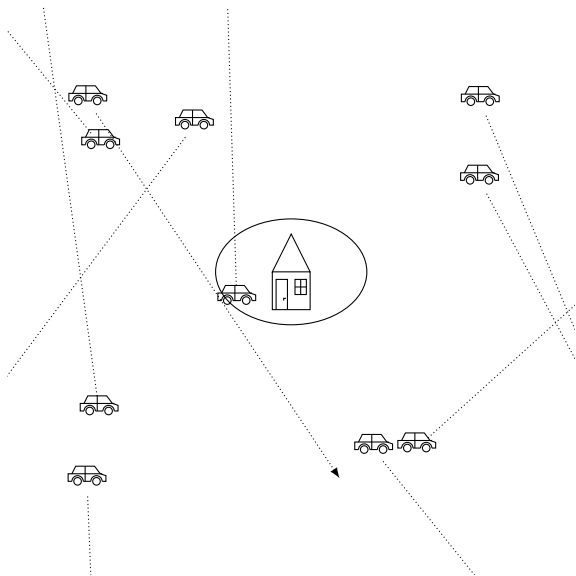
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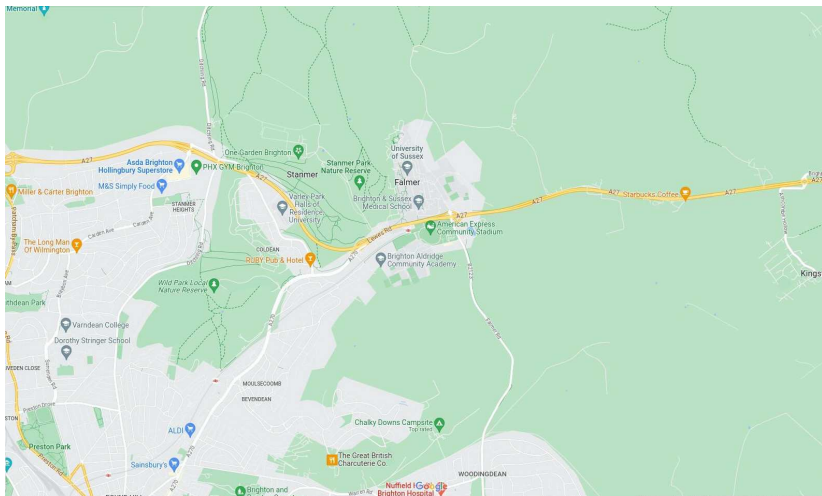
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- ▶ Unfortunately  $D \gg r \dots$

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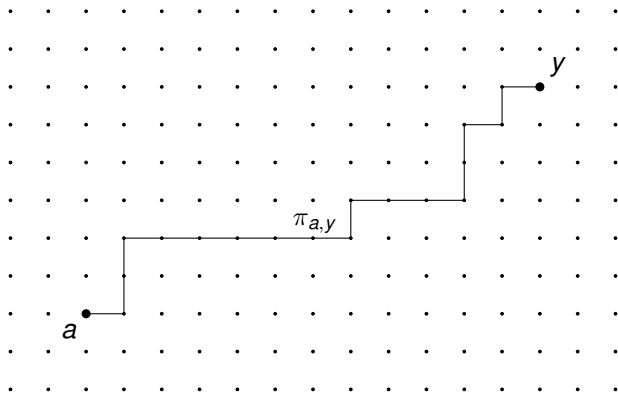
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- ▶ More tools in Exponential last passage percolation (LPP). Should behave similarly to FPP.

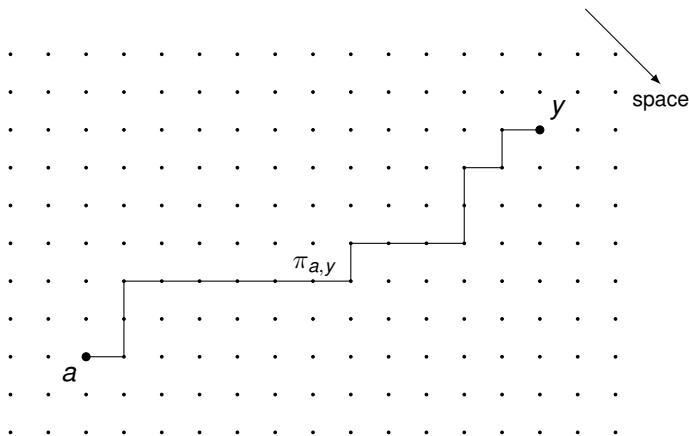
# Last passage percolation

- ▶ Place  $\omega_z$  i.i.d.  $\text{Exp}(1)$  for  $z \in \mathbb{Z}^2$ .
- ▶ The *geodesic*  $\pi_{a,y}$  from  $a$  to  $y$  is the a.s. unique heaviest up-right path from  $a$  to  $y$ . Its weight is  $G_{a,y}$ .



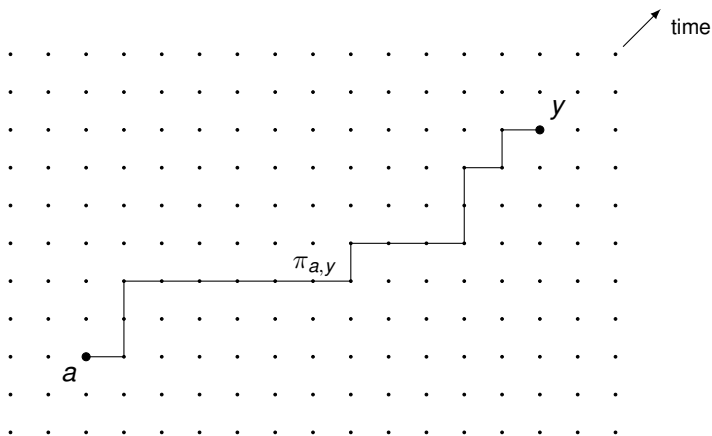
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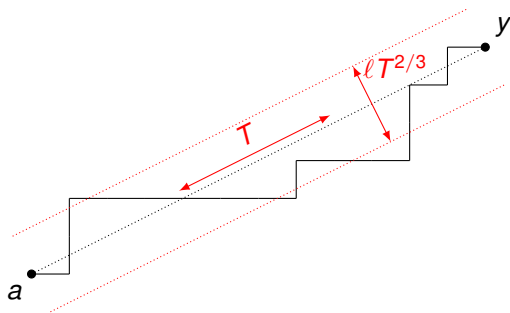


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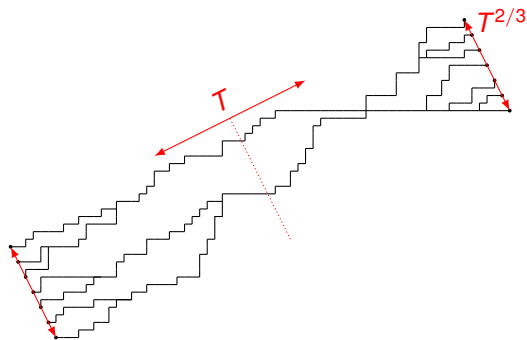
# Last passage percolation: properties



$\mathbb{P}\{\text{geodesic exits width } lT^{2/3}\} \leq \text{const} \cdot e^{-Cl^3}$  [Basu, Sarkar, Sly '19;  
Busani, Ferrari '22]

(KPZ transversal fluctuations).

# Last passage percolation: properties



$\mathbb{P}\{\text{more than } \ell \text{ geodesics at mid-line}\} \leq \text{const} \cdot e^{-C\ell^{1/128}}$

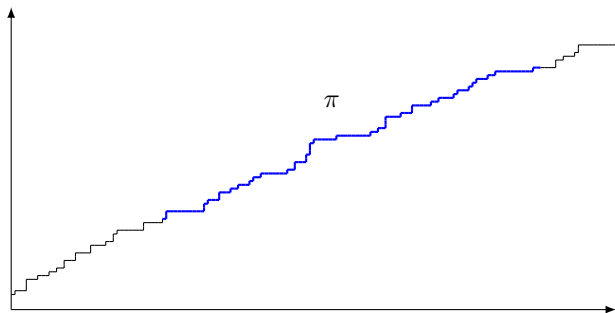
[Basu, Hoffman, Sly '22]

(Midpoint problem).



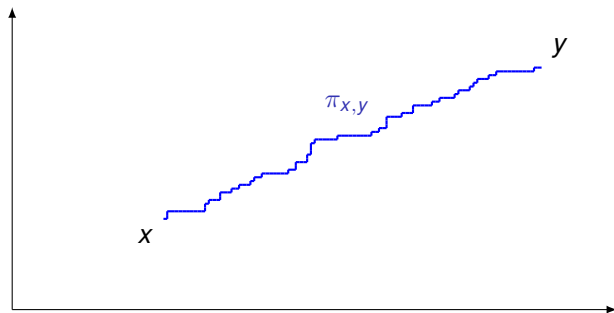
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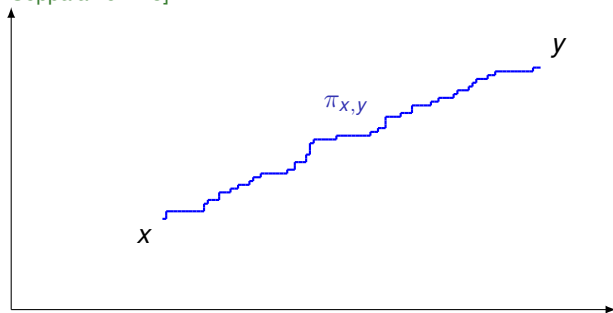
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For any fixed direction this a.s. exists and is unique. [Newman (et al) '96]; [Wüthrich '02]; [Ferrari, Pimentel '05]; [Coupier '11]; [Janjigian, Rassoul-Agha, Seppäläinen '23]



## Our model

- ▶ Throw i.i.d.  $\text{Exp}(1)$  weights on  $\mathbb{Z}^2$ .

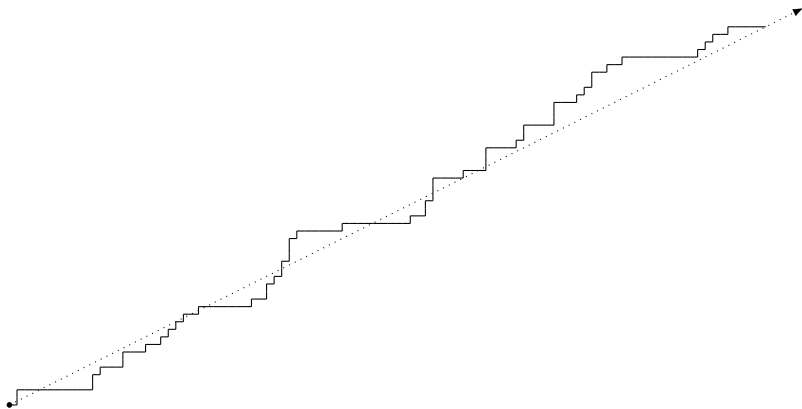
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- ▶ Throw i.i.d.  $\text{Exp}(1)$  weights on  $\mathbb{Z}^2$ .
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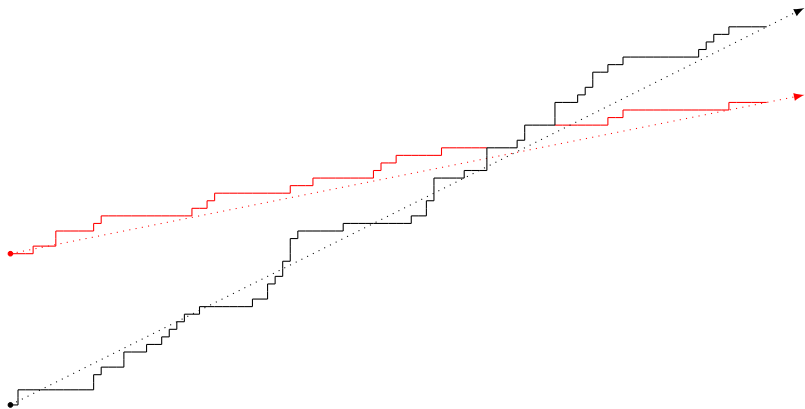
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- ▶ Draw the a.s. semi-infinite geodesic for each point to its chosen direction. Many of these will coalesce when the angles are close. That's our road map with traffic data on it. A road segment is *busy* when many geodesics use that edge.

# Our model

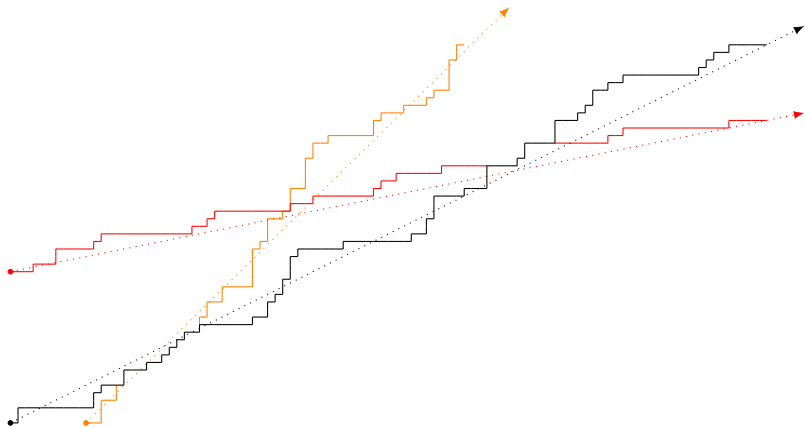


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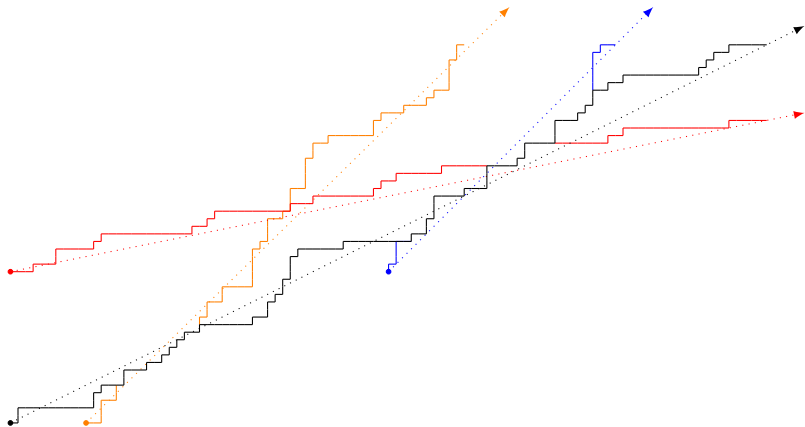




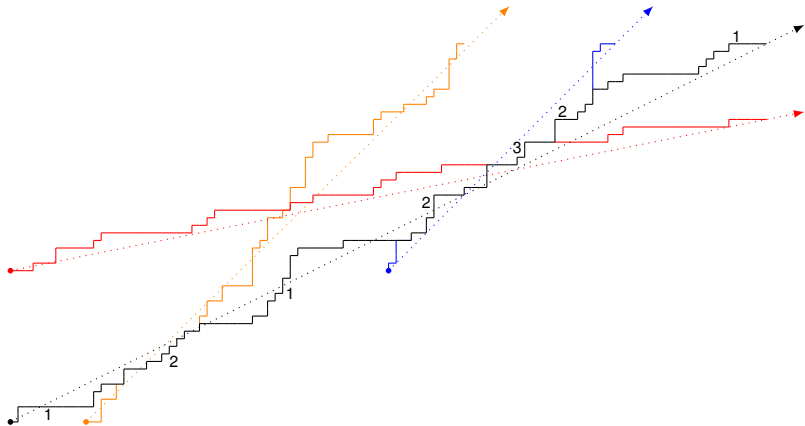
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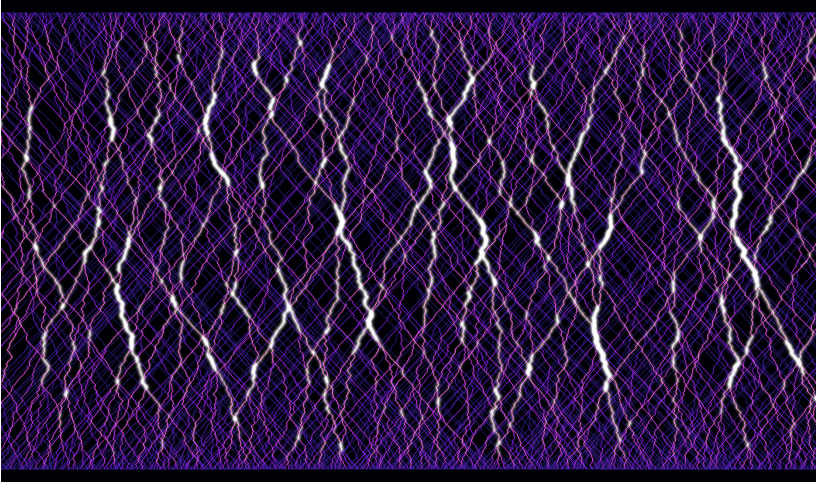
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*Simulation by David Harper*

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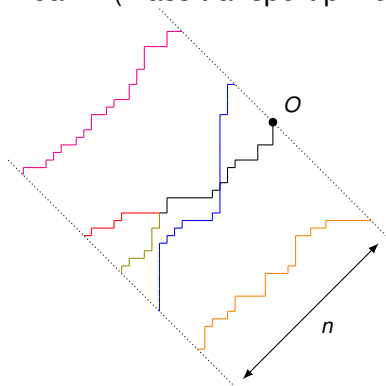
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- ▶ Is this actually a good model of real road networks out there?



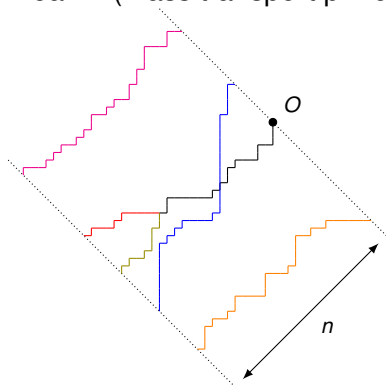
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From all layers:  $N = \sum_{n=1}^{\infty} N_n$  is of infinite mean.

# Answers

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$$cn^{-1/3} \leq \mathbb{P}\{\text{a car from distance } \geq n \text{ visits } O\} \leq Cn^{-1/3}.$$

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$$\frac{C}{k} \leq \mathbb{P}\{N \geq k^4\} \leq \frac{C \log k}{k}.$$

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$$\frac{c}{k} \leq \mathbb{P}\{N \geq k^4\} \leq \frac{C \log k}{k}.$$

- ▶ Break up the angle space into intervals of length  $n^{-1/3}$ : only need to deal with geodesic trees of a fixed direction.
- ▶ Averaging argument: if  $\mathbb{P}\{\cdot\}$  is too small then the event is not expected to happen much in a segment of the space-line.
- ▶ Then either no cars picked this direction (independent choices) or they wandered in wrong directions (transversal fluctuations) or they haven't coalesced enough (midpoint problem).
- ▶ Complications with inclusion-exclusion; independence of different enough directions...

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- ▶  $\rightsquigarrow$  lower bound.

## Busy road close by?

With similar methods,

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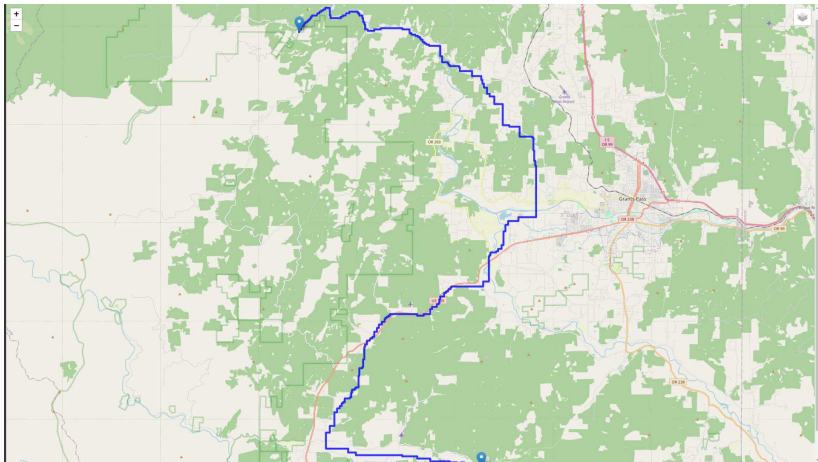
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### Theorem

$$\mathbb{P}\{\text{yes, road with } \geq \text{const} \cdot k^4 \text{ cars within distance } k\} \geq c.$$

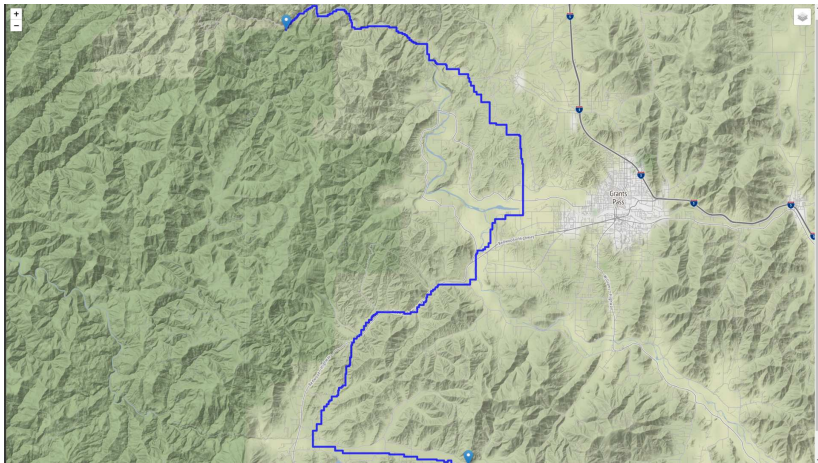
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*Simulation by David Harper*

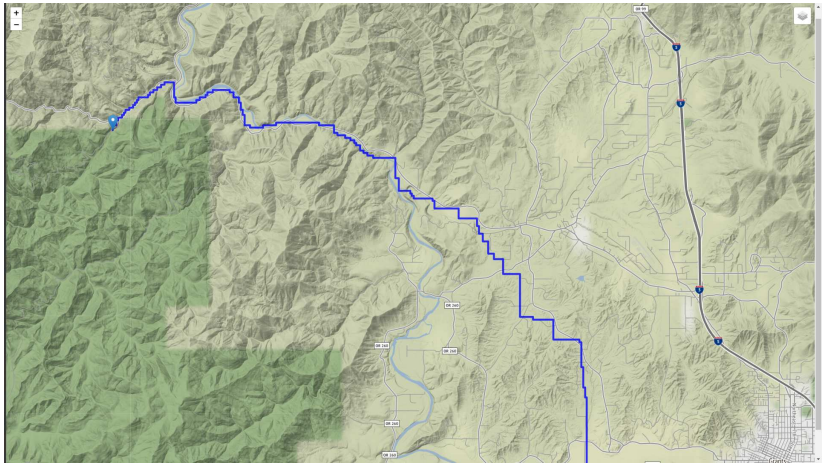


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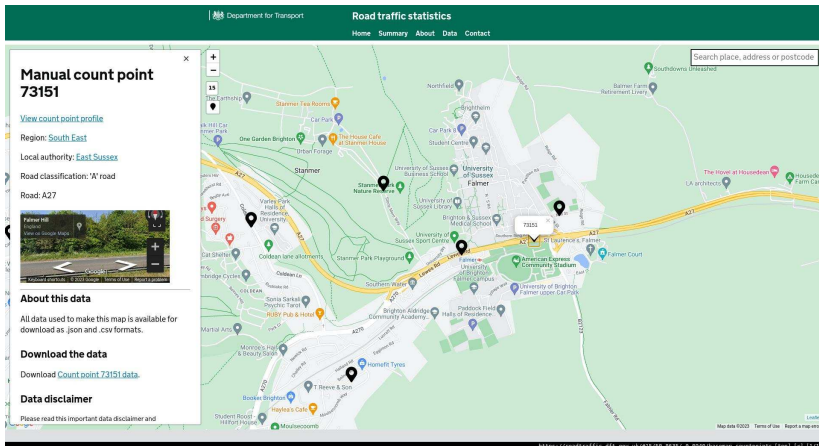
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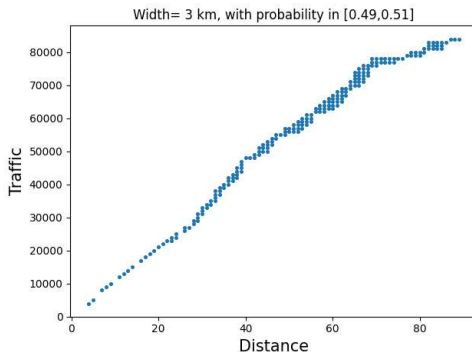


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$\mathbb{P}\{\text{road with } \geq \ell \text{ cars within distance } k\} \dots ?$

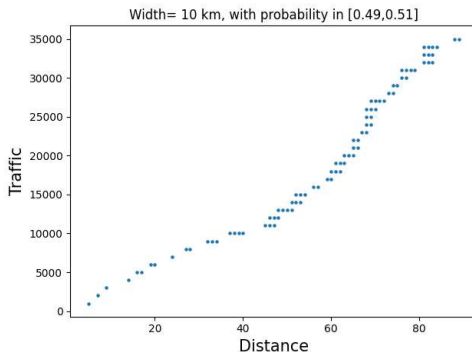
# Is this all any good? On the South:



*Between 49% and 51% of startpoints have at least this much traffic within the distance shown.*

**Thm:**  $\mathbb{P}\{\text{road with } \geq k^4 \text{ cars within distance } k\} \sim \mathcal{O}\left(\frac{1}{2}\right)$ .

# Is this all any good? On the North and the West:



*Between 49% and 51% of startpoints have at least this much traffic within the distance shown.*

**Thm:**  $\mathbb{P}\{\text{road with } \geq k^4 \text{ cars within distance } k\} \sim \mathcal{O}\left(\frac{1}{2}\right)$ .

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Thank you.