A microscopic concavity property and $t^{1/3}$ scaling of current fluctuations in particle systems I.

Joint with
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Márton Balázs

Budapest University of Technology and Economics

Interacting Particle Systems and Percolation
IHP October 27, 2008.

The models

- Asymmetric simple exclusion process
- Zero range
- Bricklayers

Hydrodynamics

- Characteristics

Tool: the second class particle

- Single
- Many second class particles

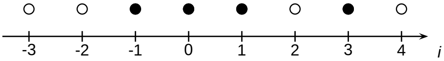
Results

- Normal fluctuations
- Abnormal fluctuations

Proof

- Upper bound
- Microscopic concavity/convexity

Asymmetric simple exclusion



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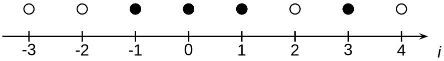

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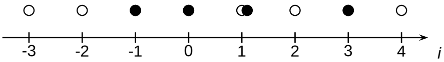

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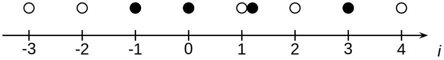

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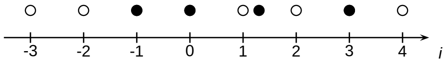

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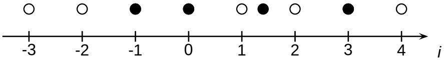

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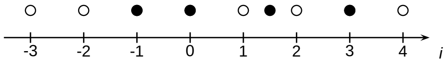

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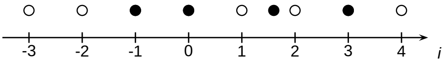

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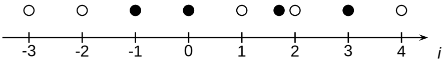

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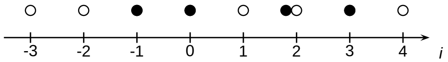

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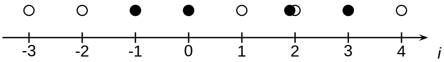

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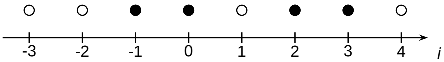

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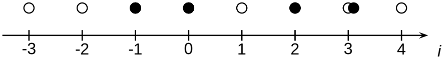

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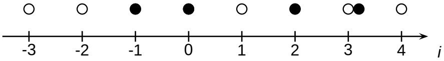

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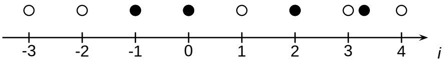

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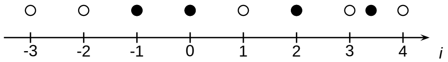

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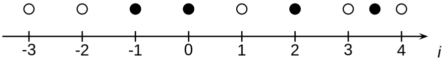

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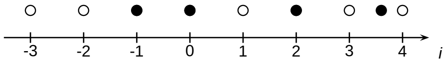

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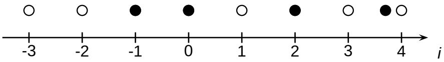

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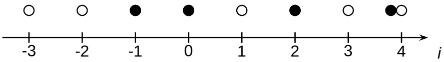

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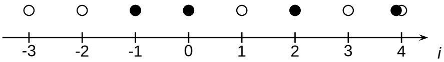

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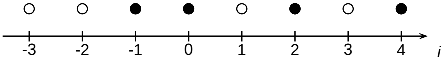

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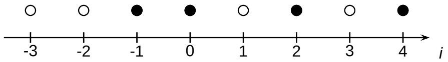

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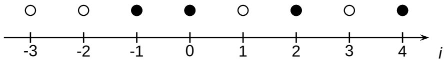

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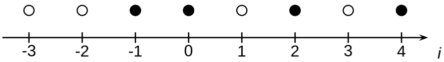

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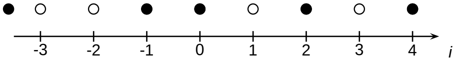

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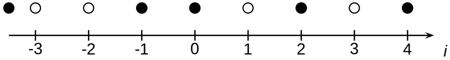

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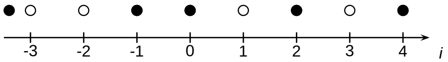

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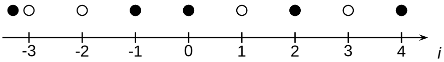

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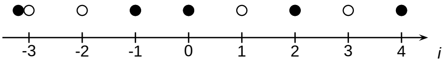

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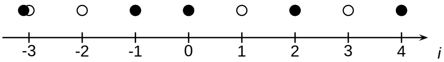

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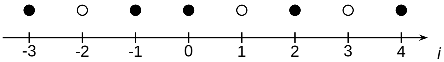

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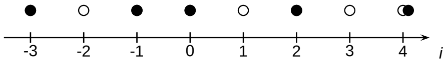

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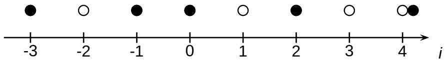

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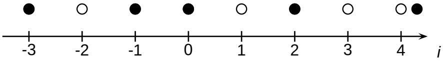

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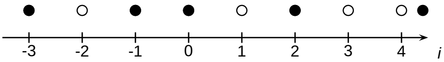

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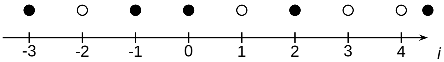

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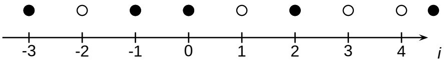

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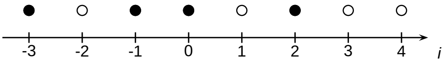

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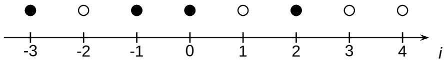

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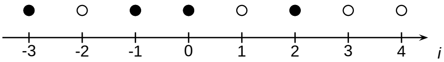

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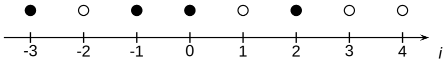

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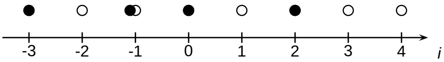

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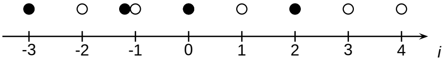

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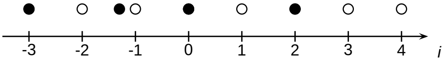

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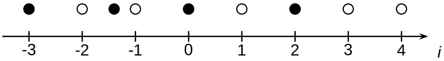

Bernoulli(ϱ) distribution; $\omega_i = 0$ or 1.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

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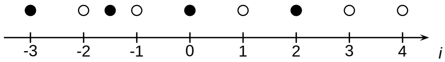

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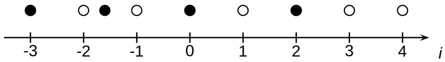

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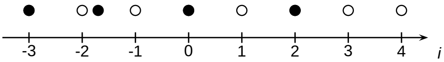

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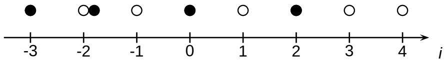

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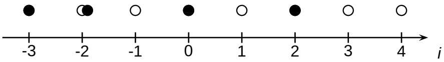

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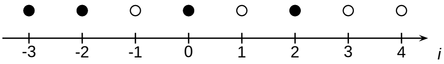

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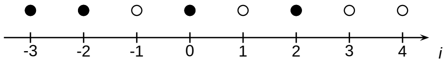

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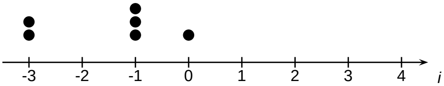
The Bernoulli(ϱ) distribution is time-stationary for any $(0 \leq \varrho \leq 1)$. Any translation-invariant stationary distribution is a mixture of Bernoullis.

The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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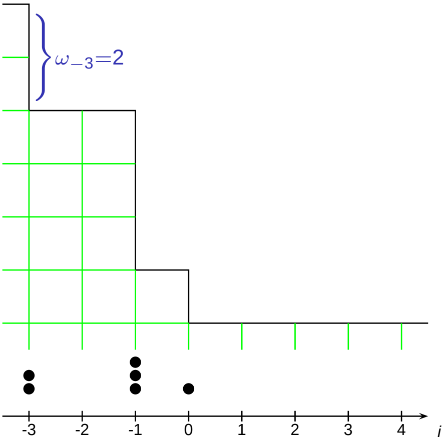

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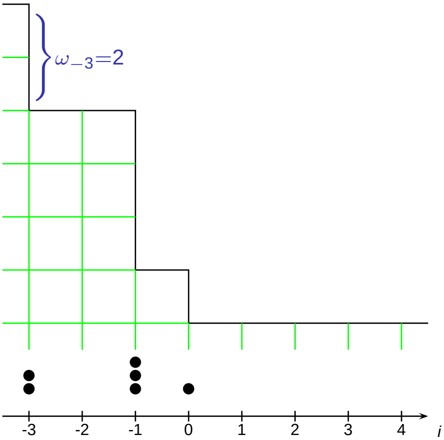

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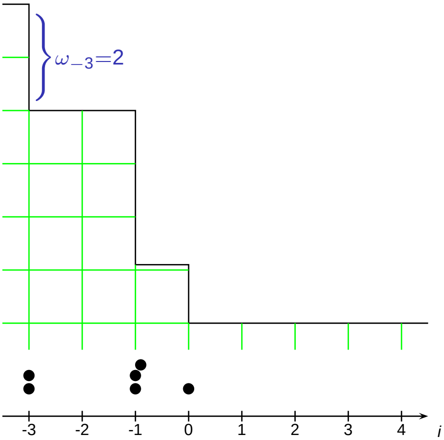

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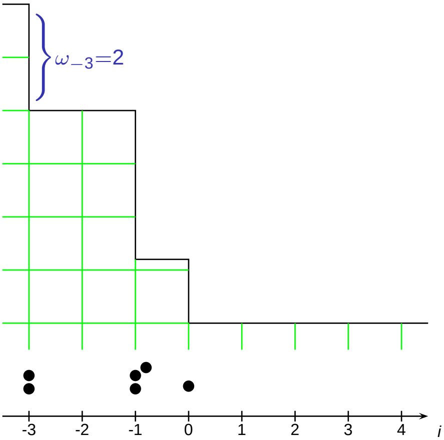

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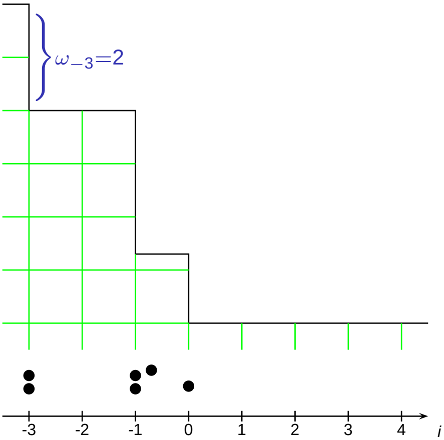

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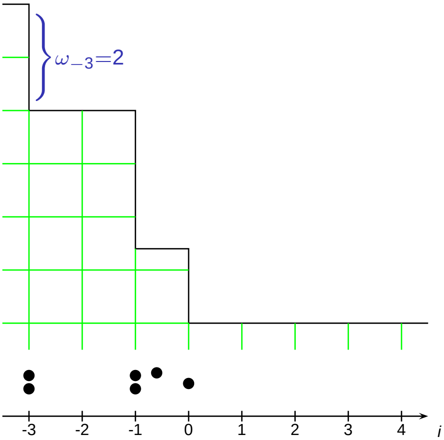

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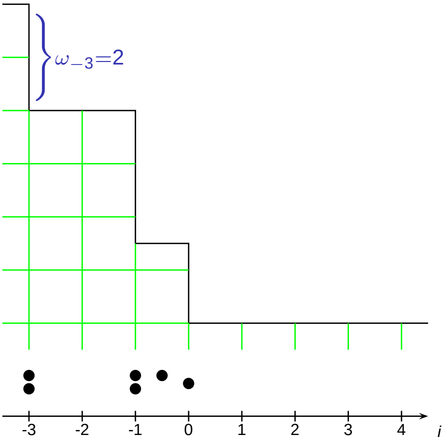

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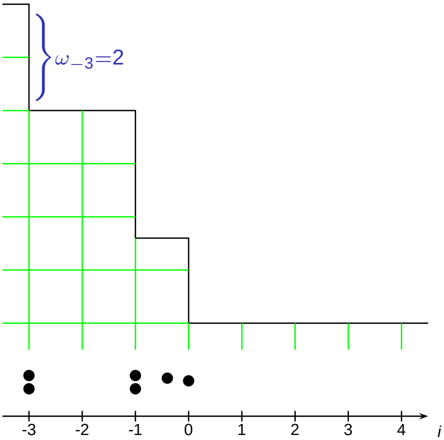

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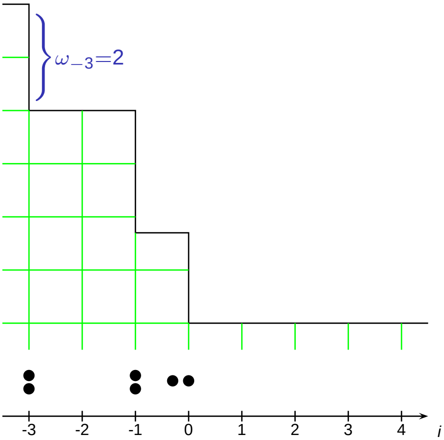

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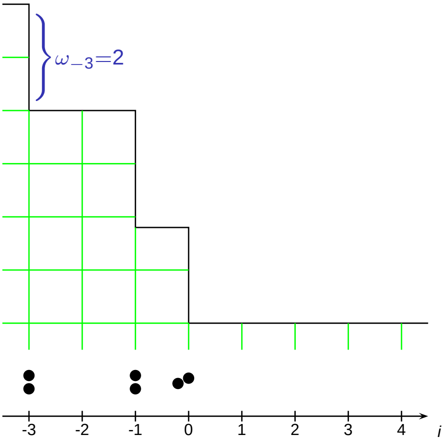

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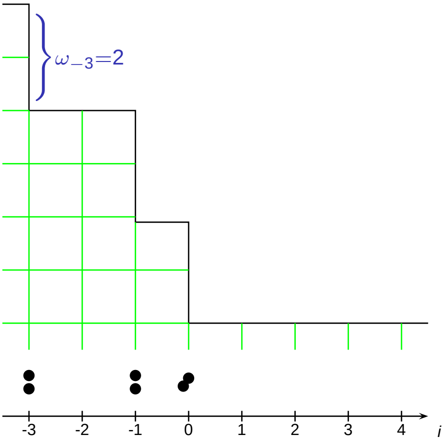

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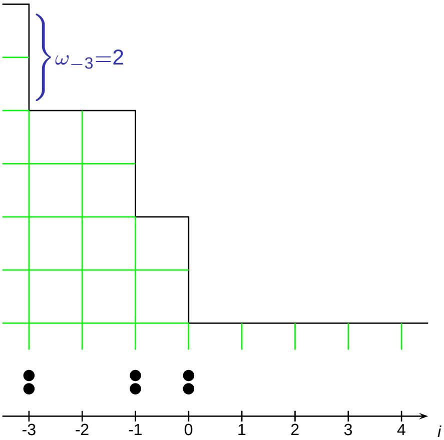

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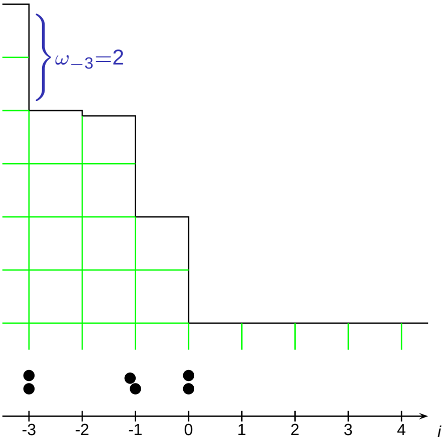

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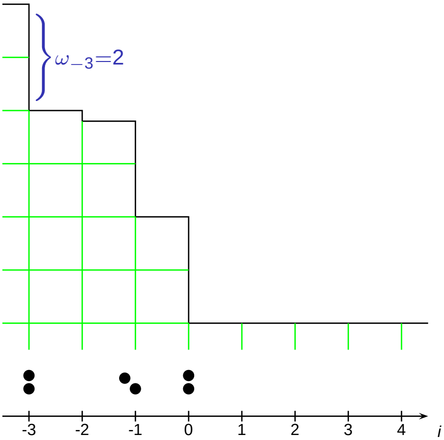

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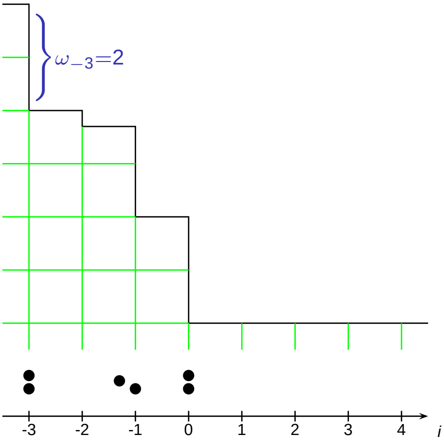

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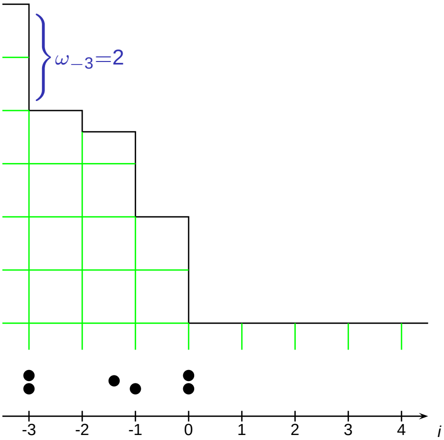

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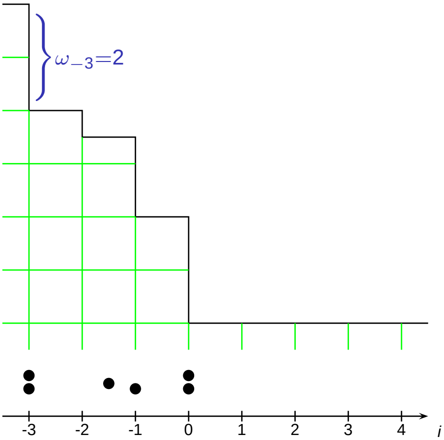

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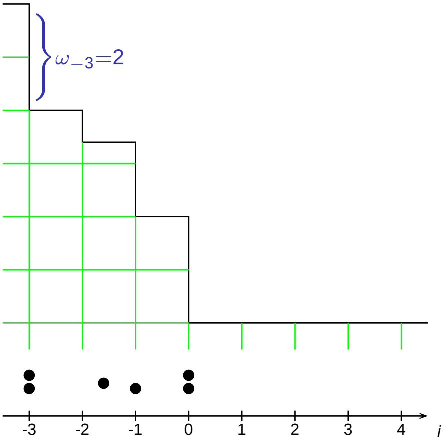

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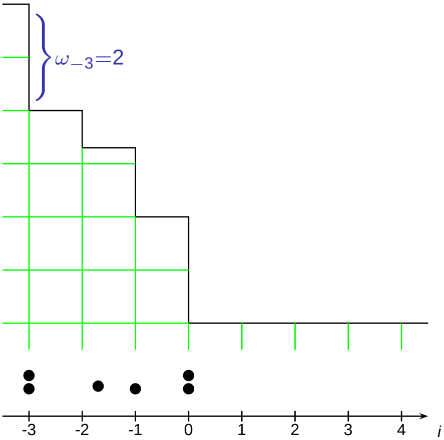

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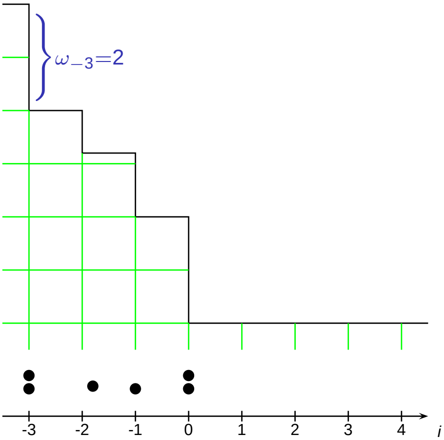

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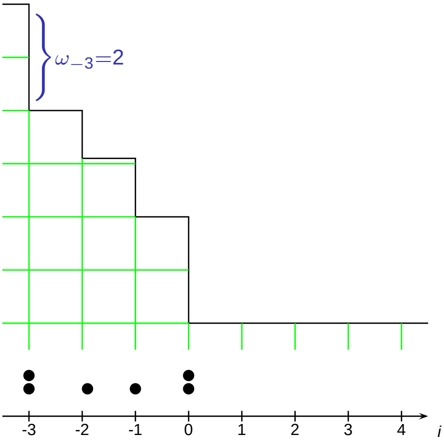

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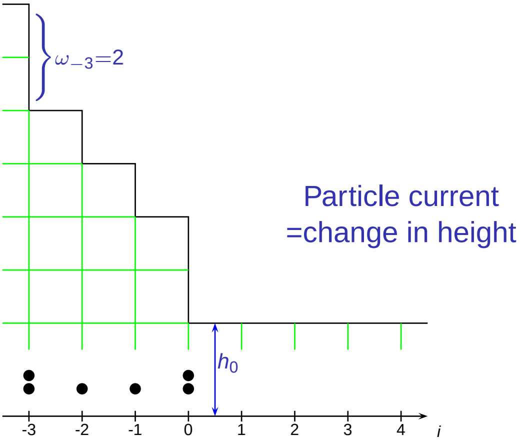

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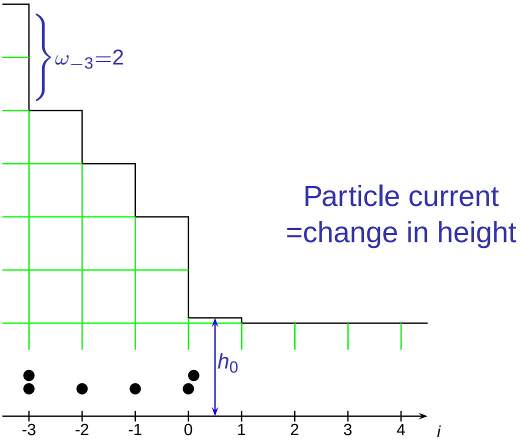

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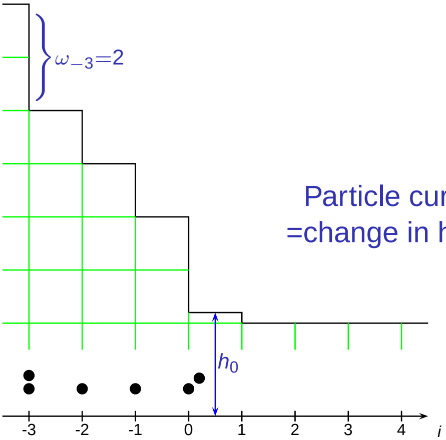

Particle current
=change in height

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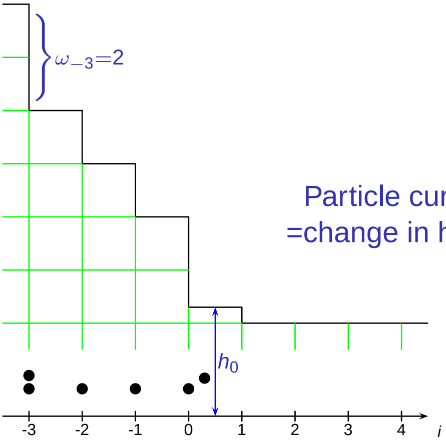

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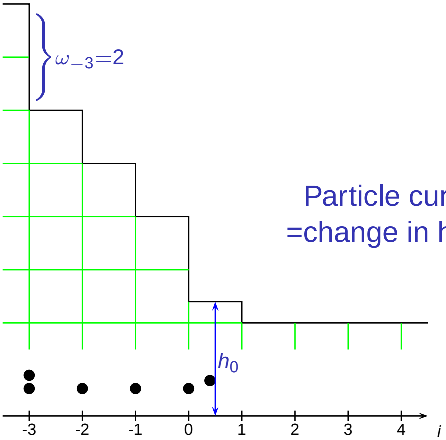

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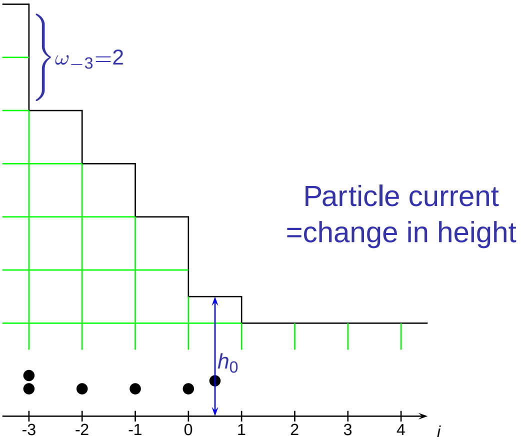

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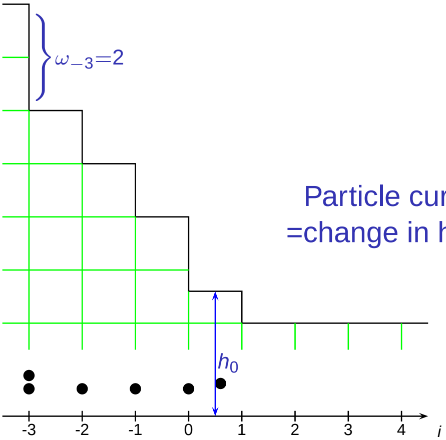

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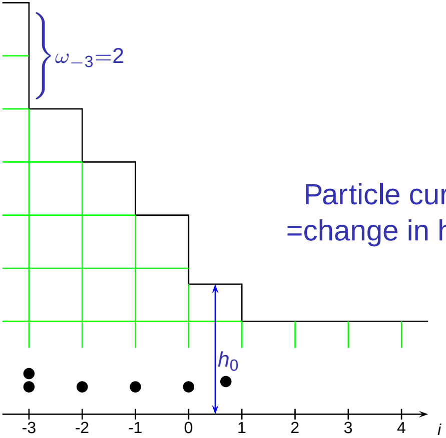

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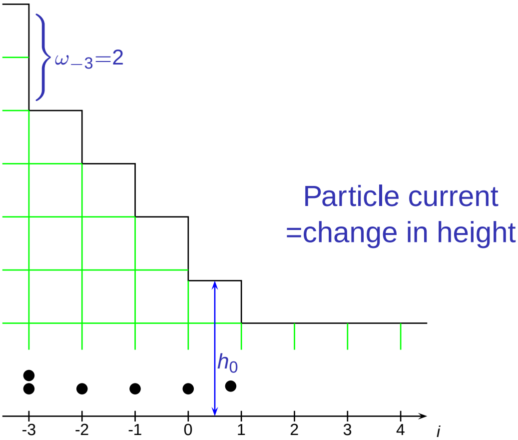

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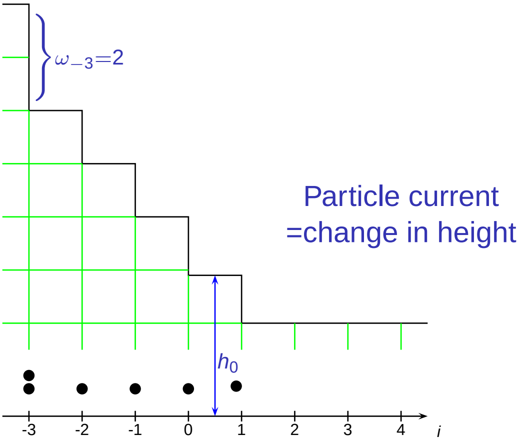

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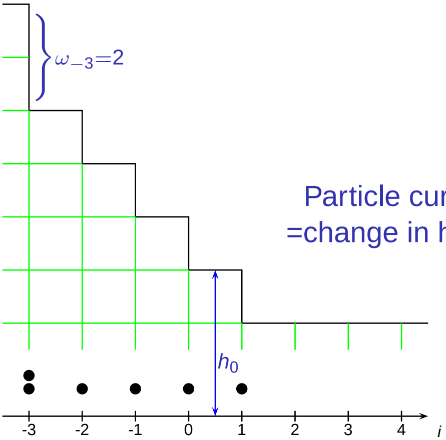

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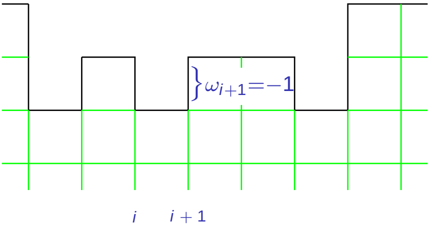
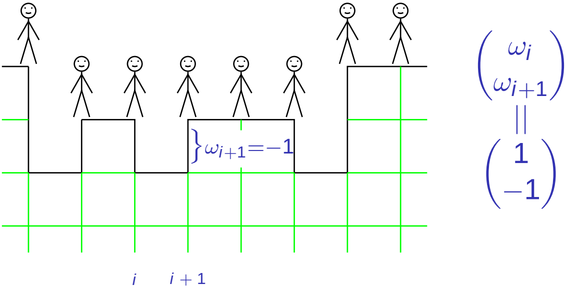
The asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

The asymmetric bricklayers process

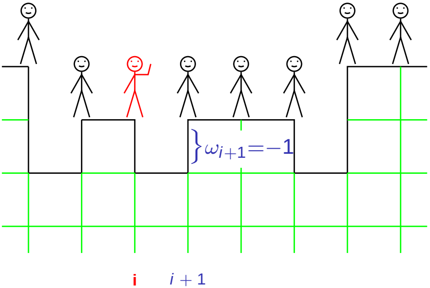


Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

a brick is added with rate $p \cdot [r(\omega_i) + r(-\omega_{i+1})]$
a brick is removed with rate $q \cdot [r(-\omega_i) + r(\omega_{i+1})]$.

$(r(\omega) \cdot r(1-\omega) = 1; \quad r$ non-decreasing; $\quad q = 1 - p < p)$.

The asymmetric bricklayers process



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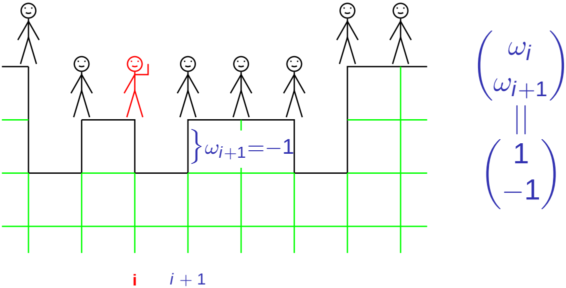
The asymmetric bricklayers process



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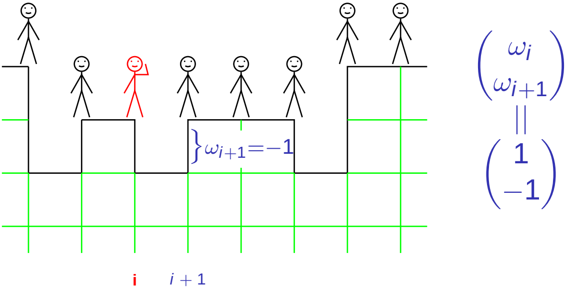

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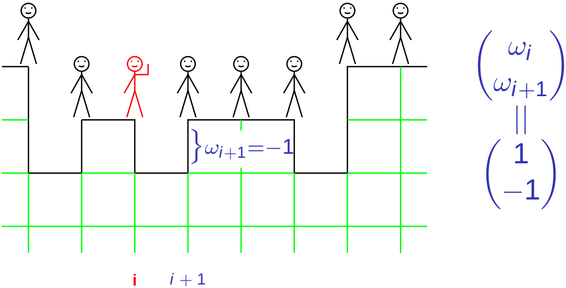

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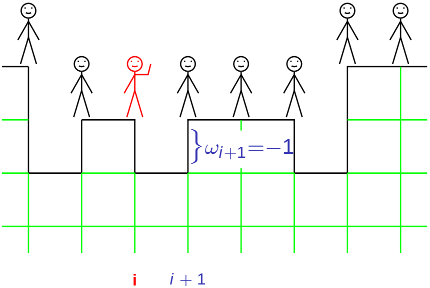

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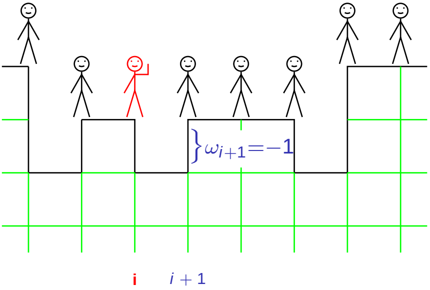

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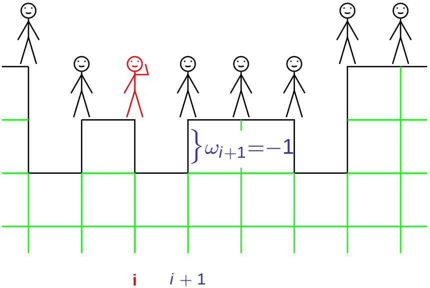

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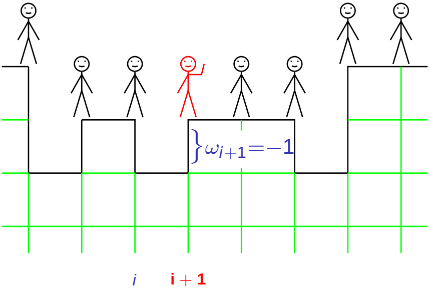

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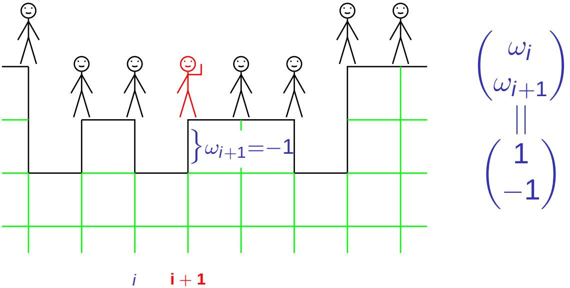

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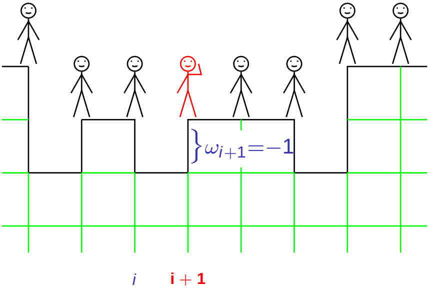

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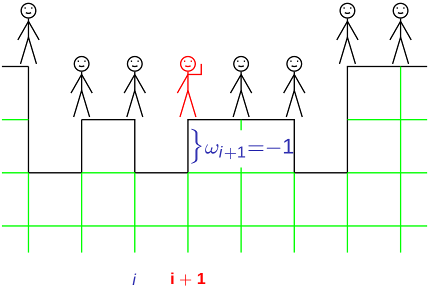

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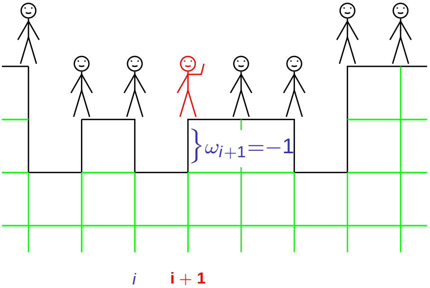

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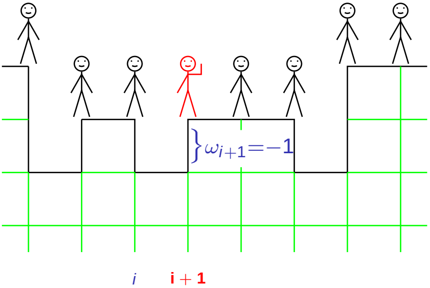

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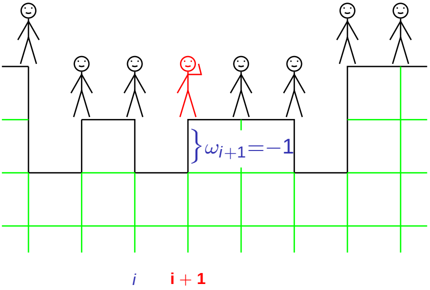

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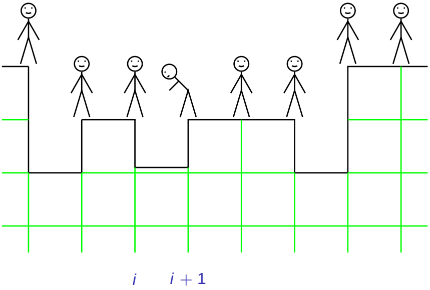

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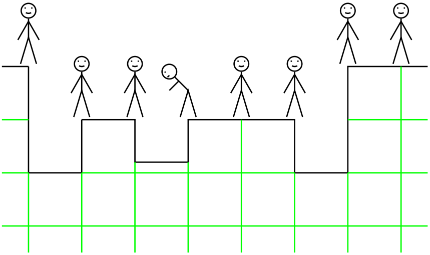

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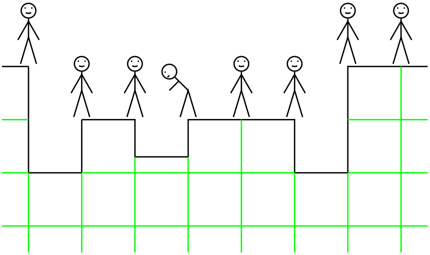

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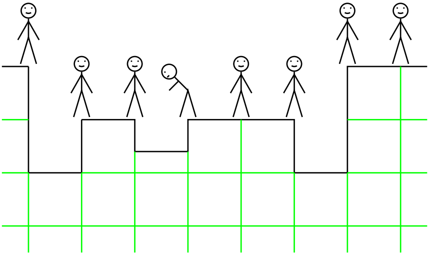

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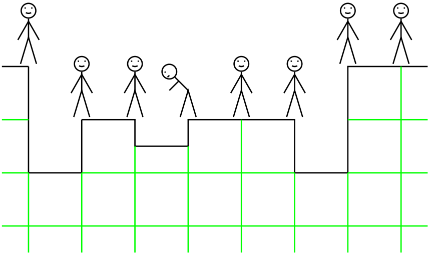

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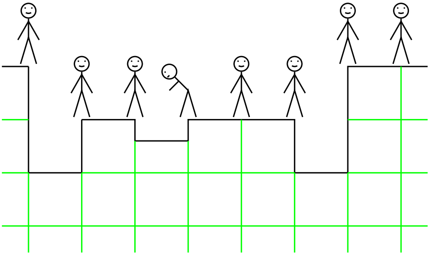

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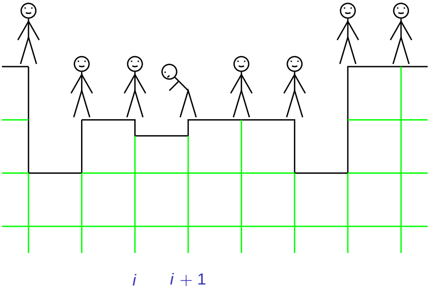

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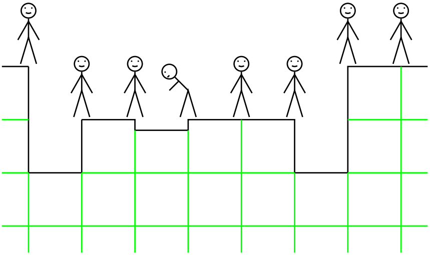

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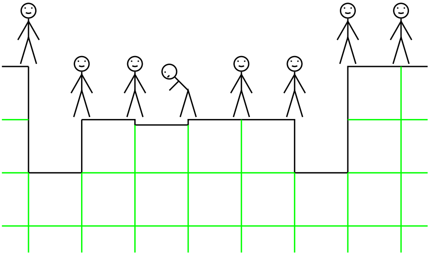

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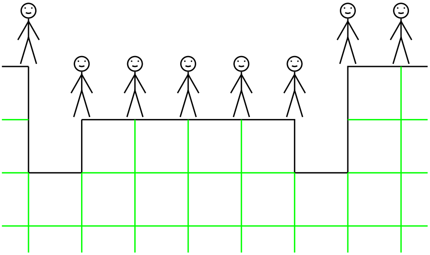

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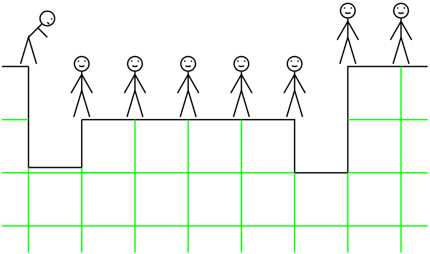

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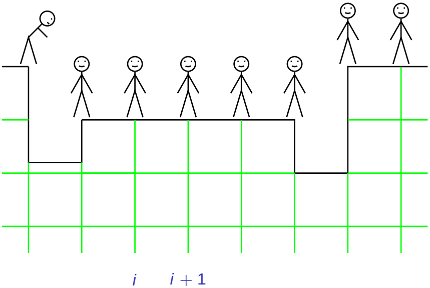

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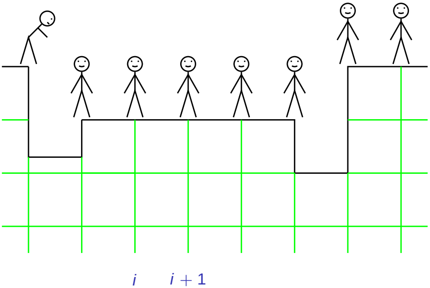

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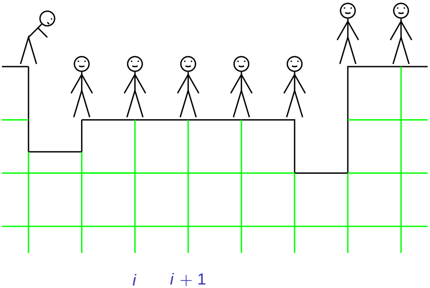

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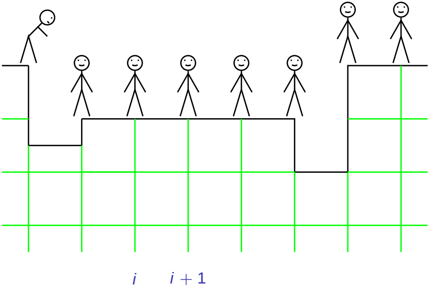

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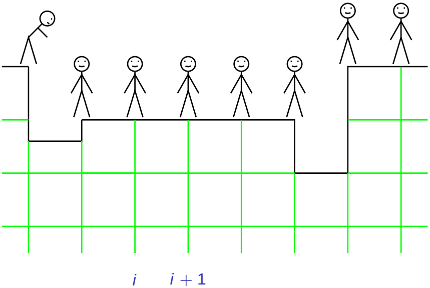

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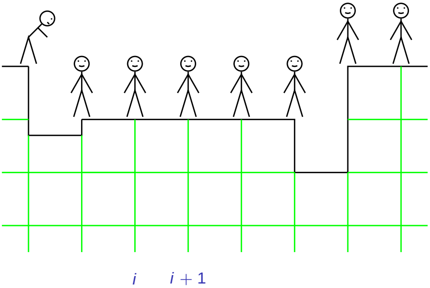

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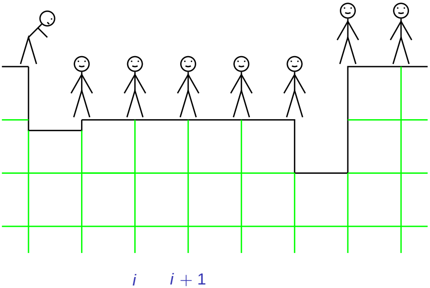

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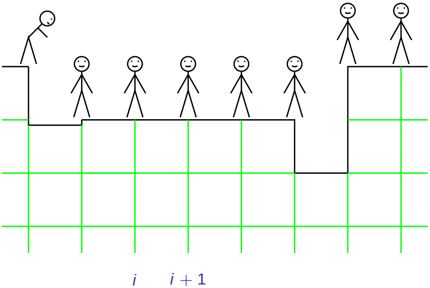

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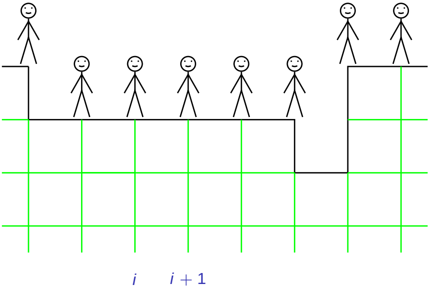

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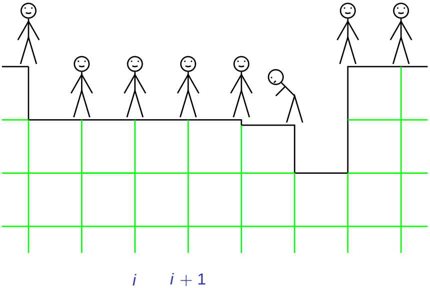

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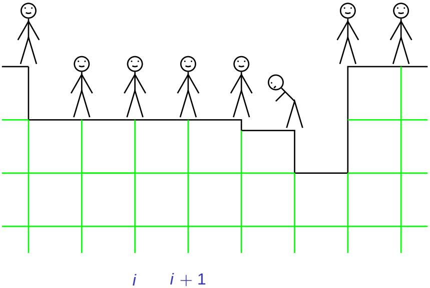

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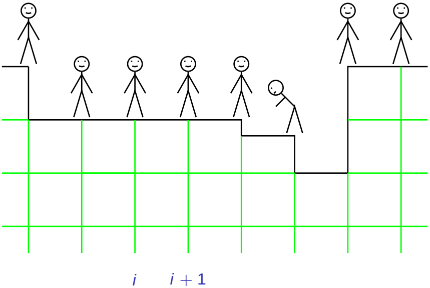

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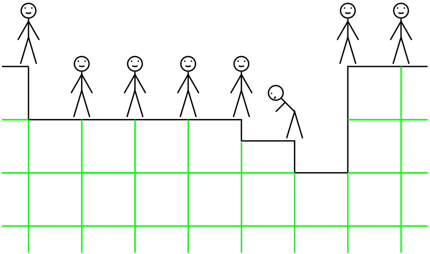

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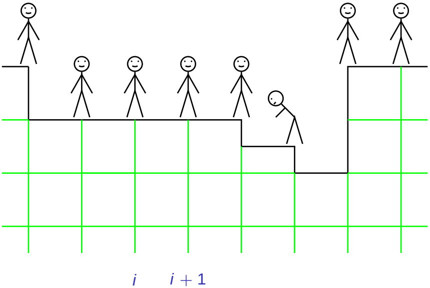

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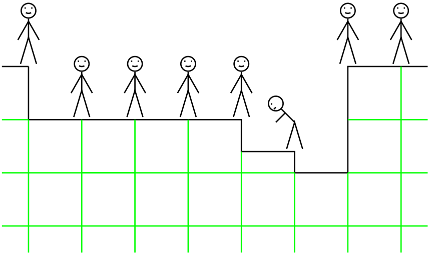

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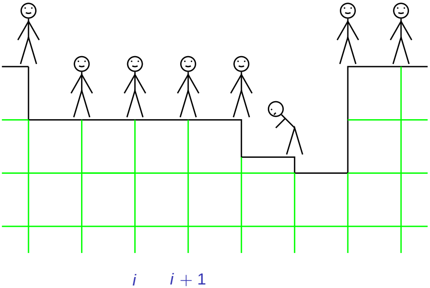

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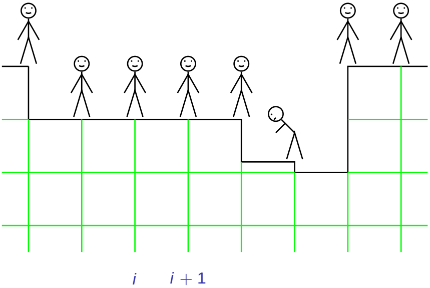

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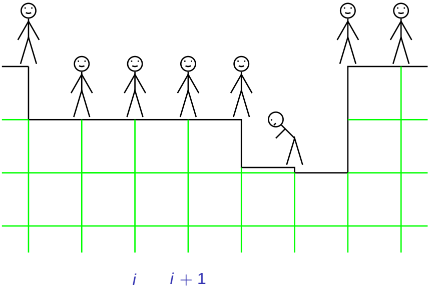

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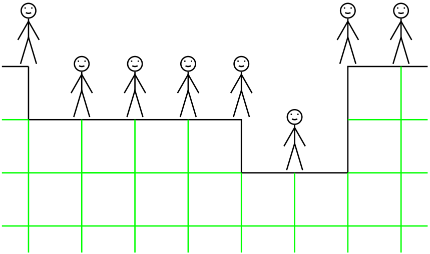

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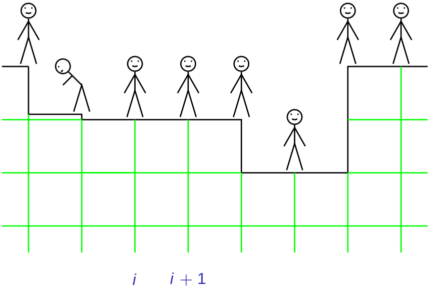

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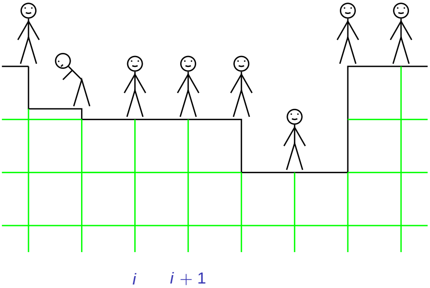

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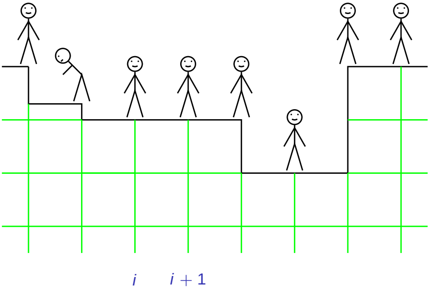

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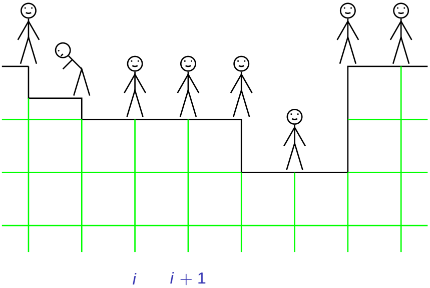

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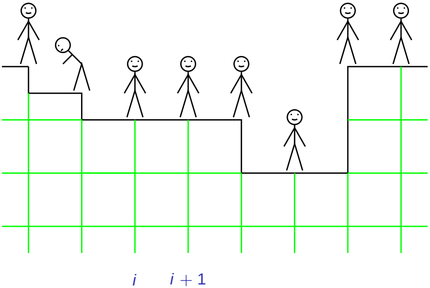

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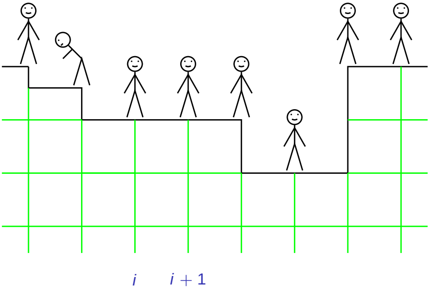

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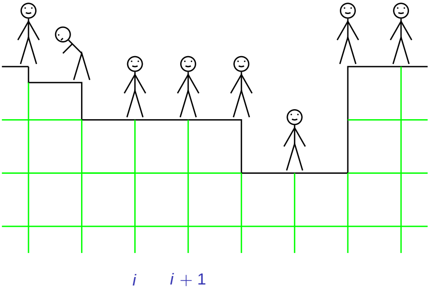

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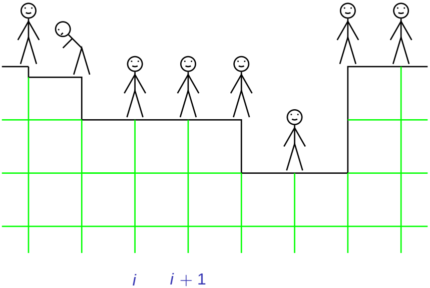

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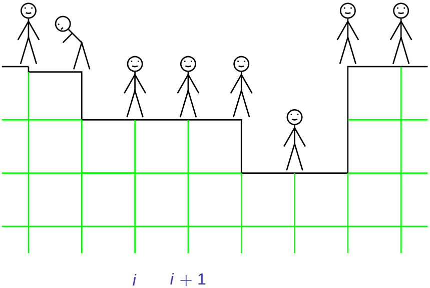

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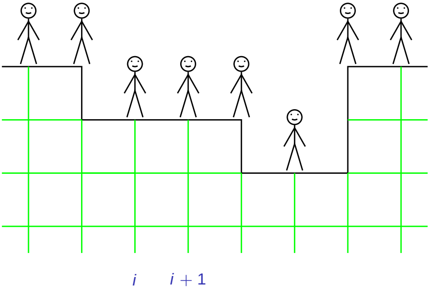

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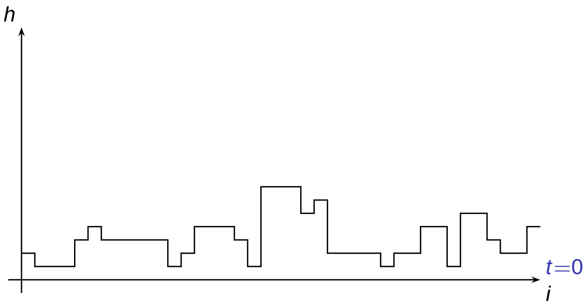

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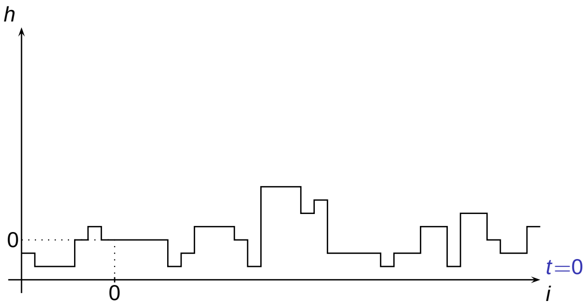

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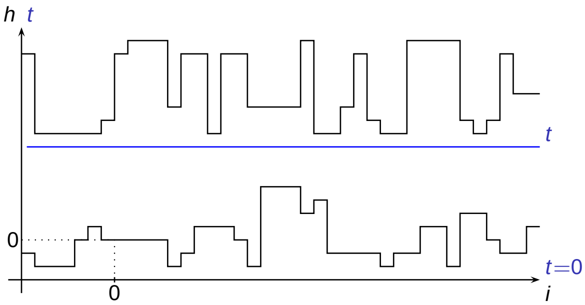

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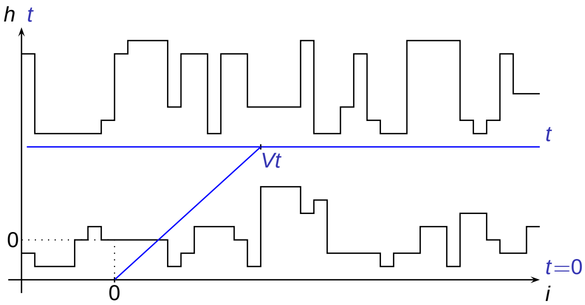

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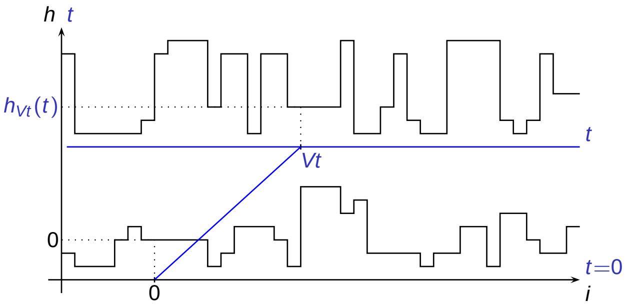

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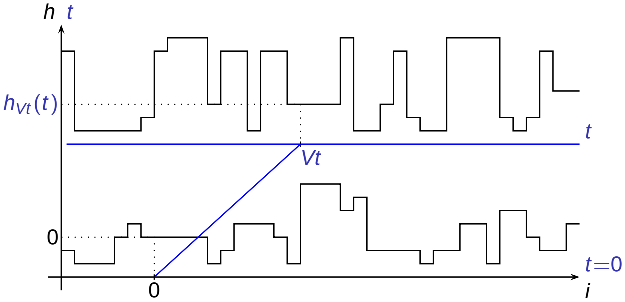

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Question: What is the time-order of $\mathbf{Var}(h_{Vt}(t))$?

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The *density* $\varrho := \mathbf{E}(\omega)$ and the *hydrodynamic flux* $H := \mathbf{E}$[growth rate] both depend on a parameter of the stationary distribution.

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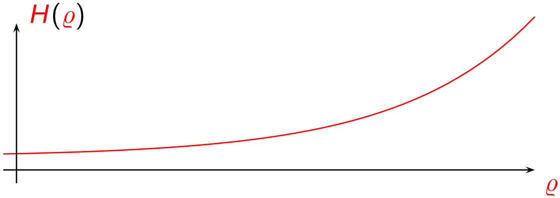
This would be the speed of a shock of densities ϱ and λ.

Characteristics (very briefly)

Convex flux (some cases of AZRP, ABLP):



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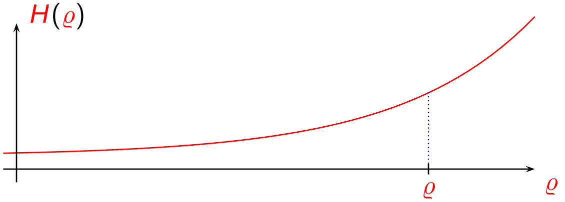

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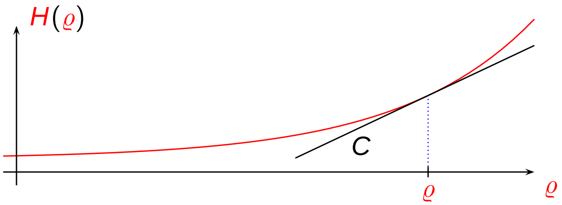
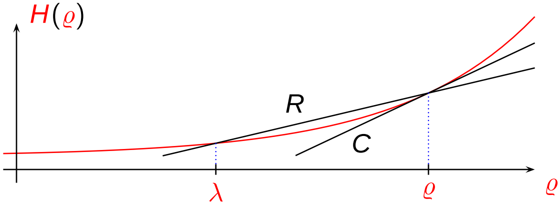


$$C = H'(\varrho)$$

Characteristics (very briefly)

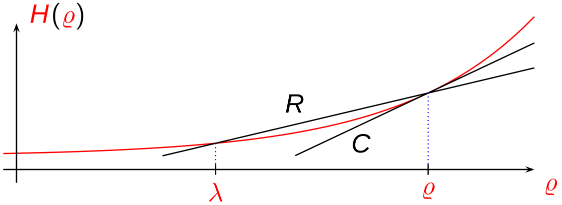
Convex flux (some cases of AZRP, ABLP):



$$C = H'(\varrho) \qquad R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Characteristics (very briefly)

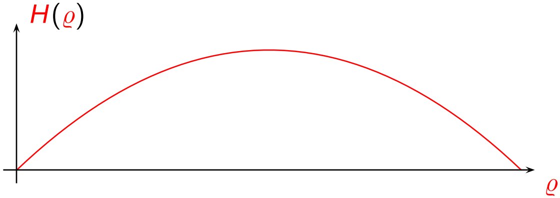
Convex flux (some cases of AZRP, ABLP):



$$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

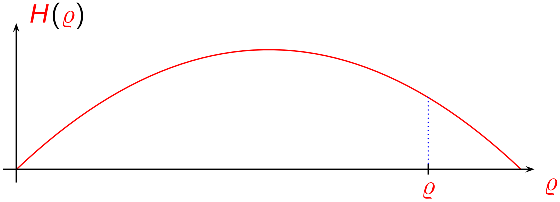
Characteristics (very briefly)

Concave flux (ASEP, AZRP):



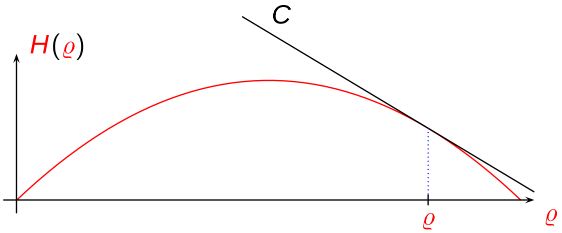
Characteristics (very briefly)

Concave flux (ASEP, AZRP):



Characteristics (very briefly)

Concave flux (ASEP, AZRP):



$$C = H'(\varrho)$$

Characteristics (very briefly)

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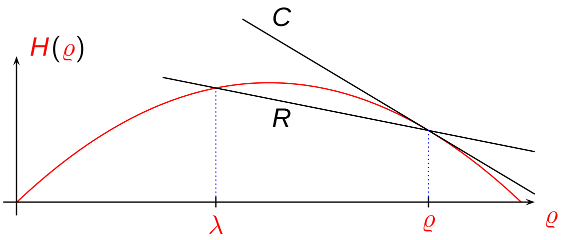

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Characteristics (very briefly)

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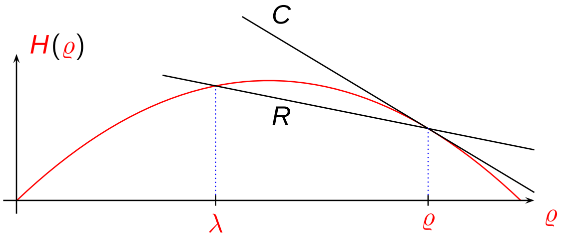
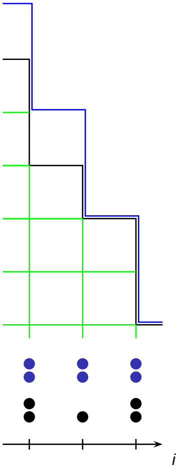


$$C = H'(\varrho) < R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Tool: the second class particle

States ω and ω only differ at one site.



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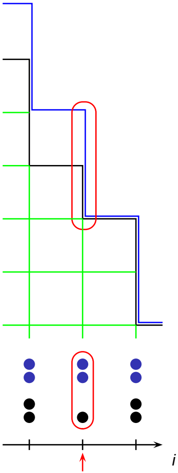

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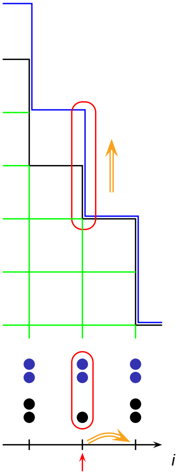

Growth on the right:

rate$\leq$rate

Tool: the second class particle

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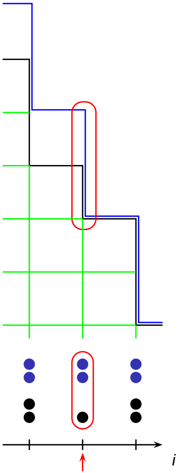

Growth on the right:
rate $\leq$ rate
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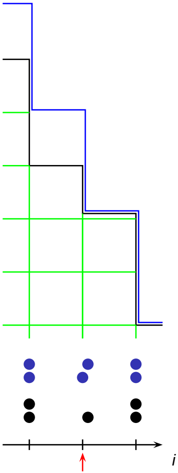

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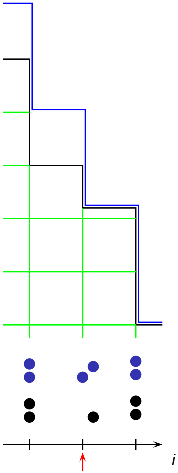

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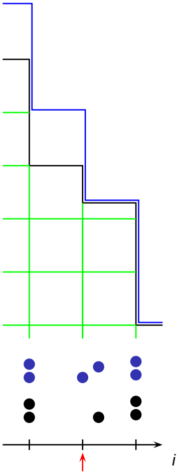

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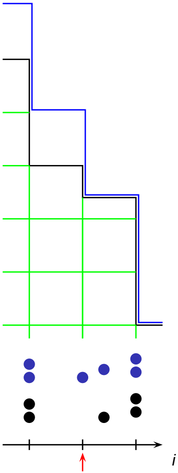

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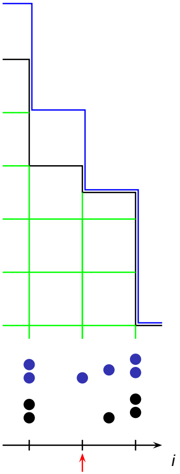

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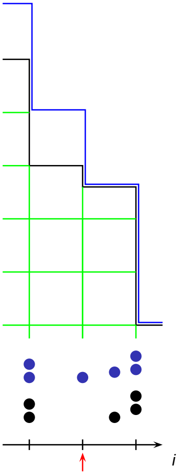

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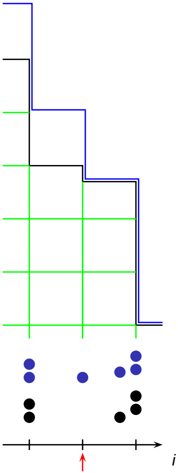

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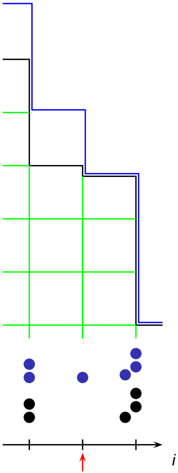

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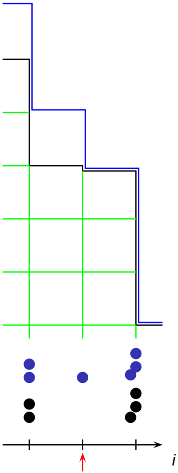

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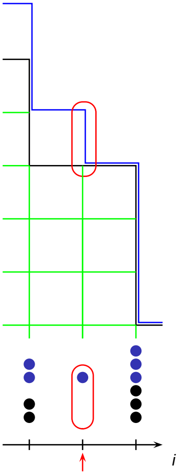

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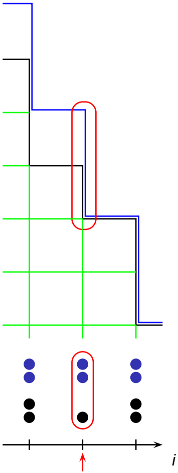

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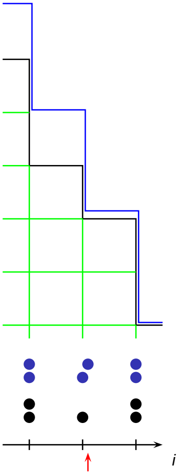

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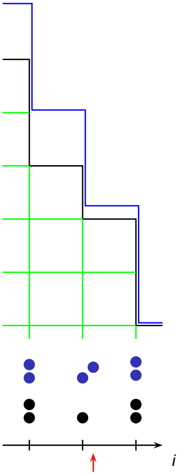

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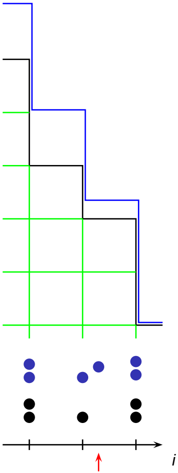

Growth on the right:

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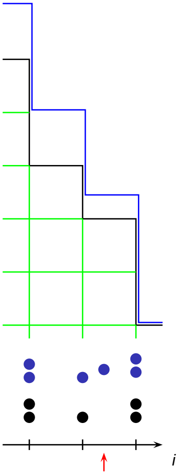

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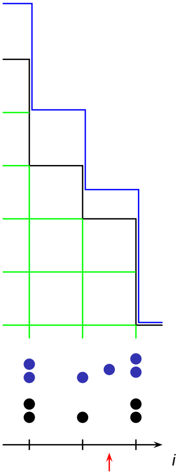

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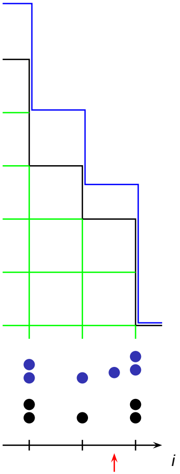

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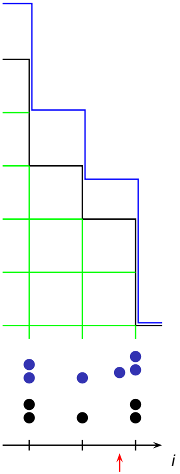

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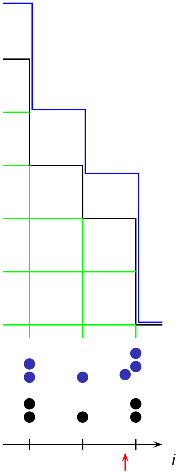

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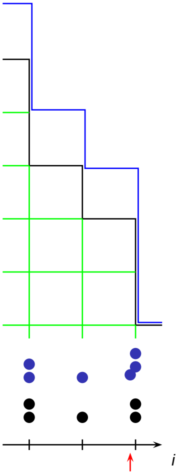


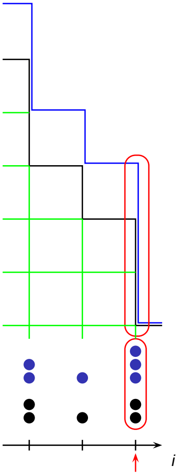
Growth on the right:

rate≤rate

with rate-rate:

Tool: the second class particle

States ω and ω only differ at one site.



Growth on the right:

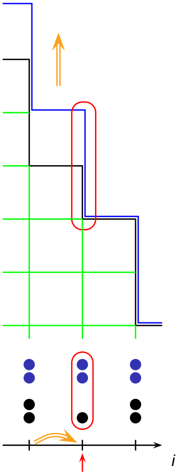
rate$\leq$rate

with rate-rate:

Tool: the second class particle

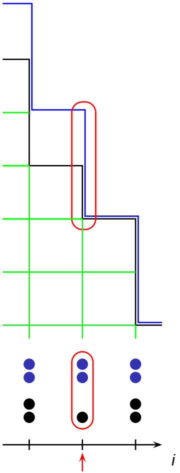
States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate



Tool: the second class particle

States ω and ω only differ at one site.



Growth on the left:

rate $\geq$ rate

with rate:

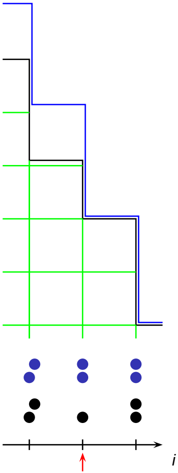
Tool: the second class particle

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

with rate:



Tool: the second class particle

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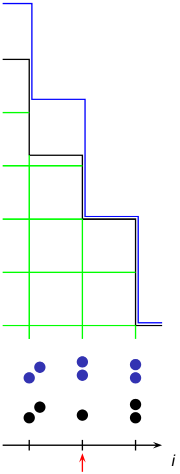
rate$\geq$rate
with rate:



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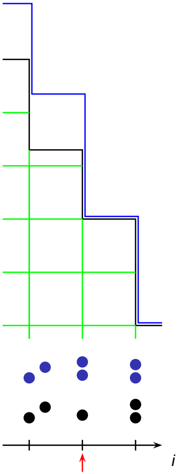

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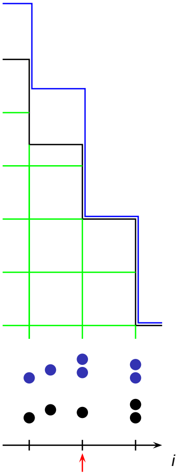

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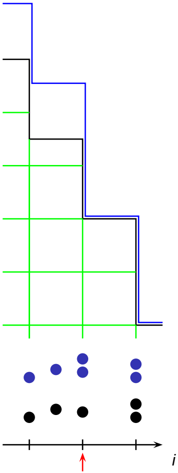

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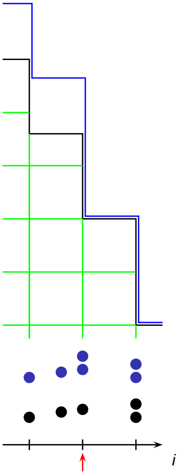

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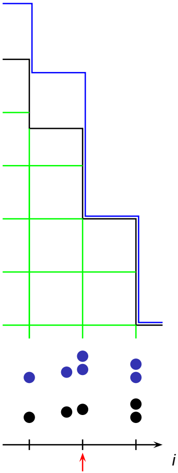

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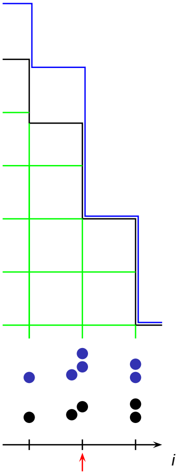

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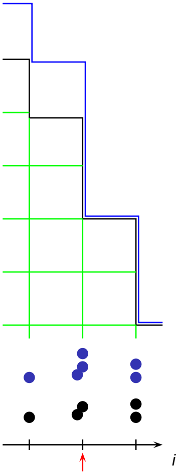

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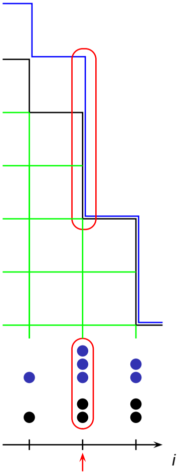

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Growth on the left:
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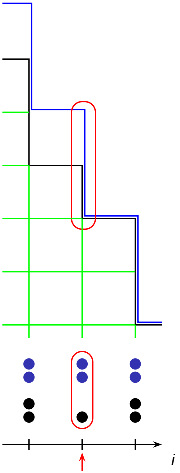

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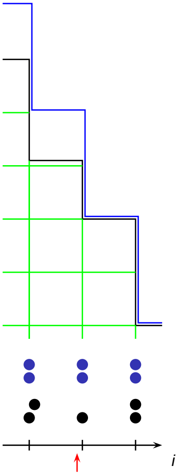

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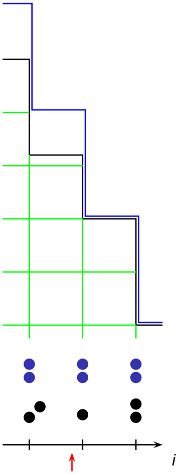

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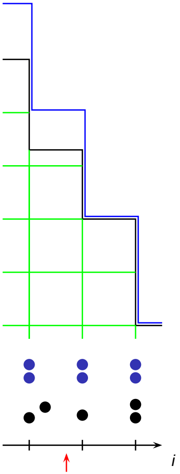

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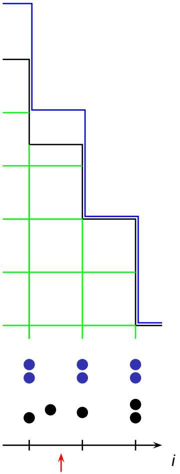

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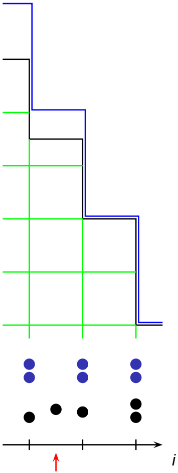

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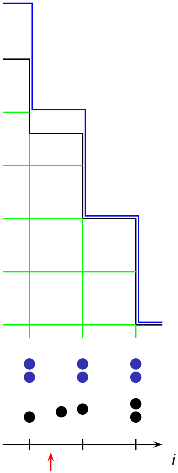

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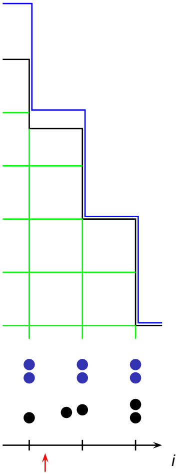

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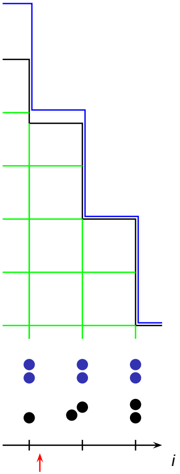

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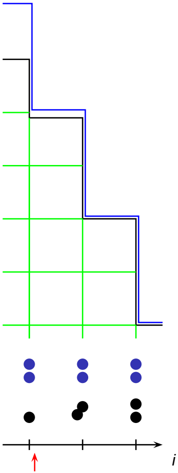
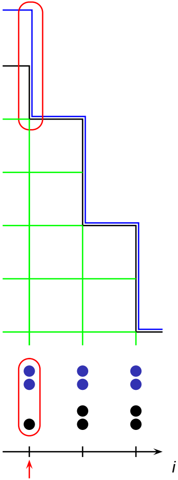


Tool: the second class particle

States ω and ω only differ at one site.



Growth on the left:
rate$\geq$rate
with rate-rate:

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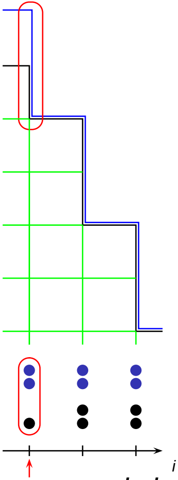

Growth on the left:
rate$\geq$rate
with rate-rate:

A single discrepancy↑, the *second class particle*, is conserved.
Its position at time t is $Q(t)$.

Tool: the second class particle

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{E}(Q(t)) = C \cdot t$$

in the whole family of processes.

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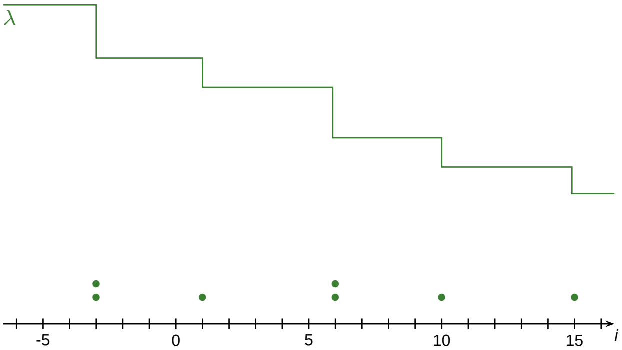
The second class particle follows the characteristics, people have known this for a long time.

Many second class particles



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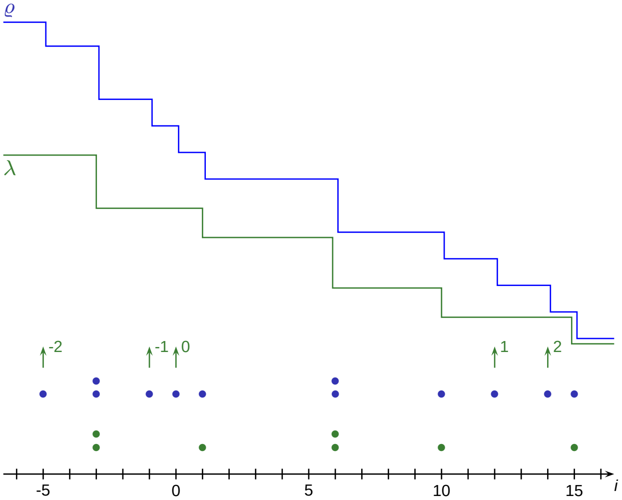

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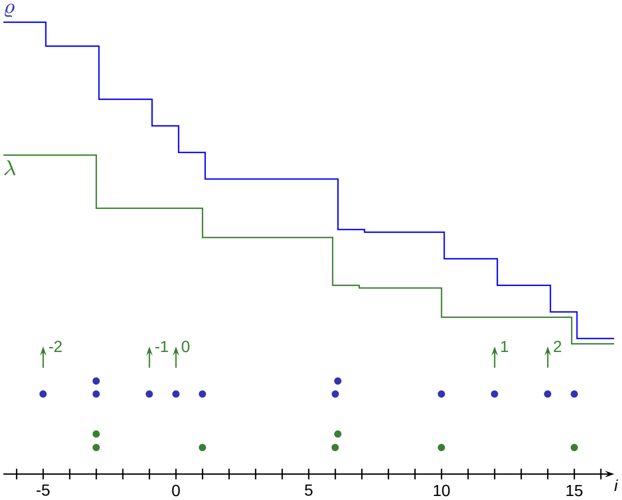

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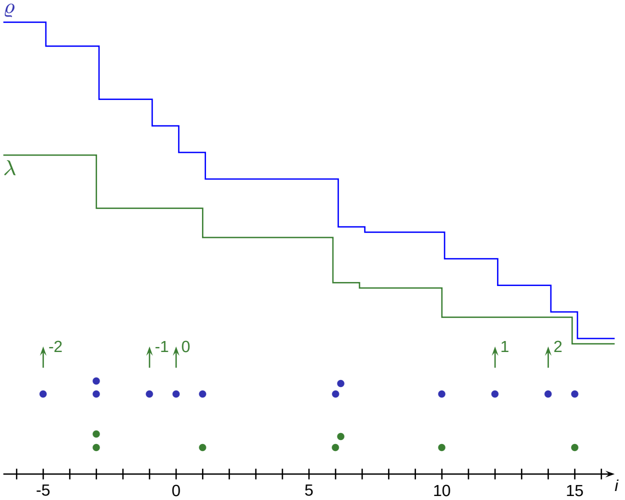

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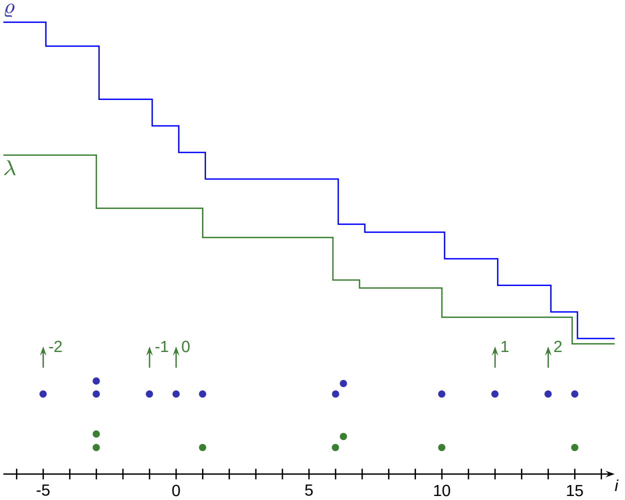

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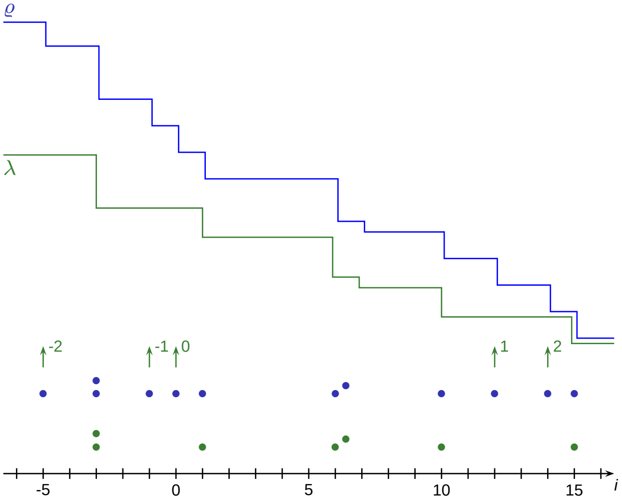

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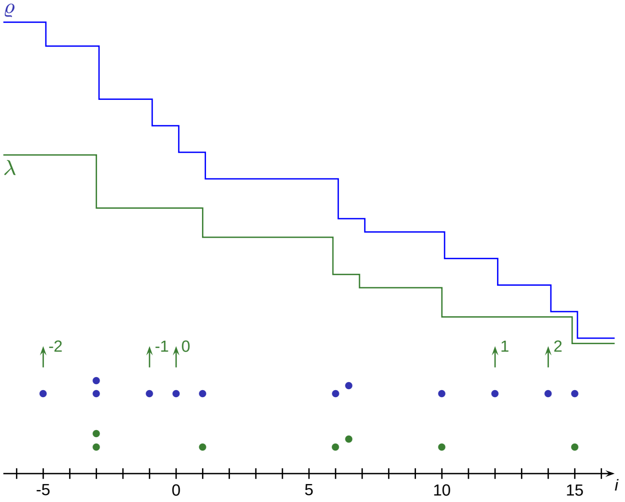

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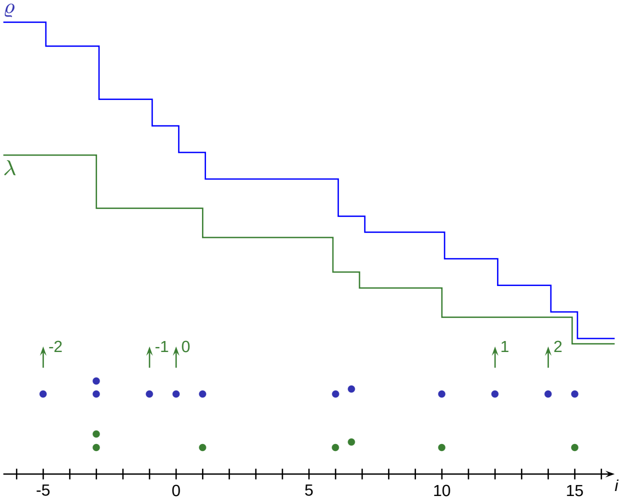

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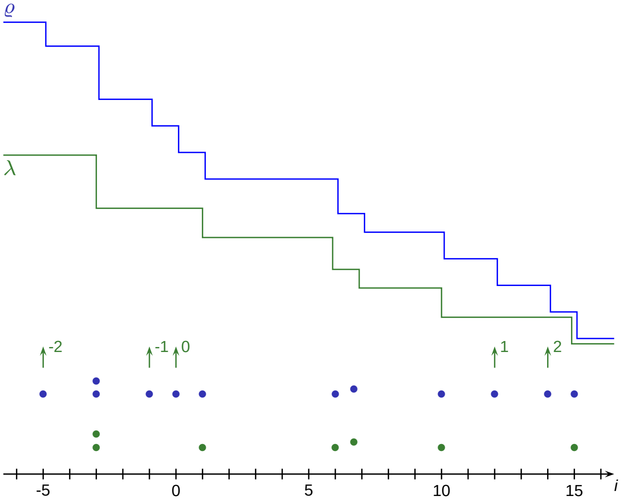

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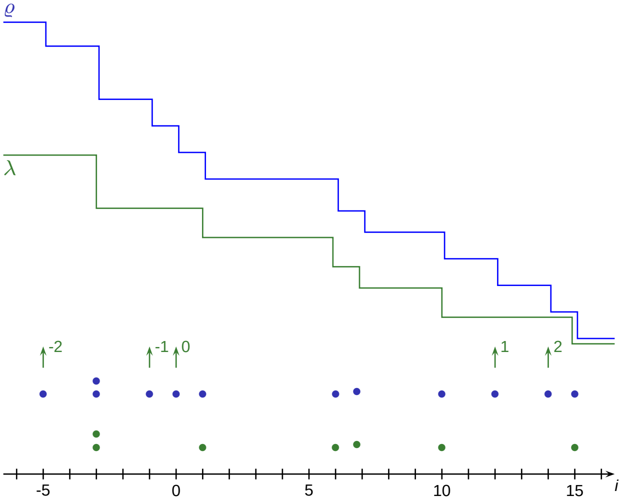

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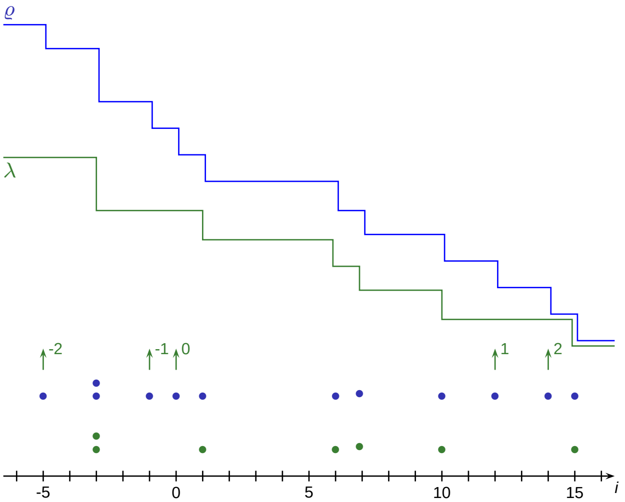

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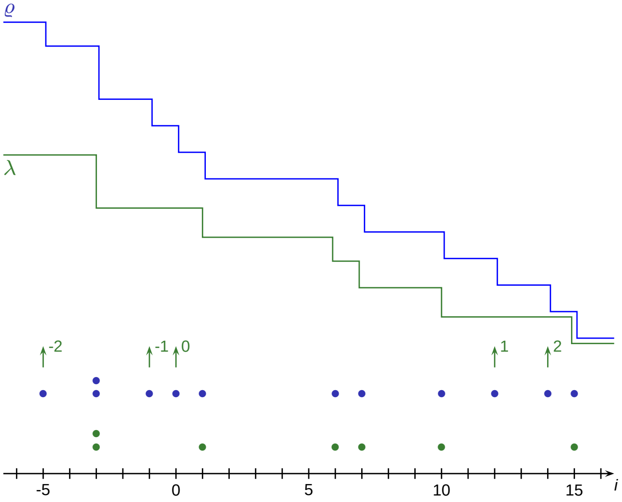

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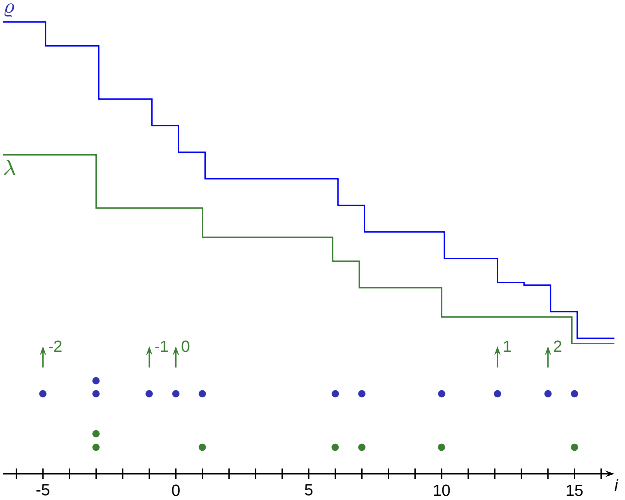

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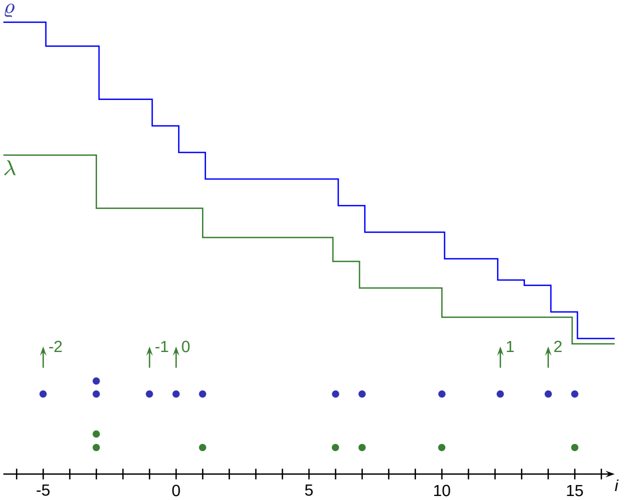

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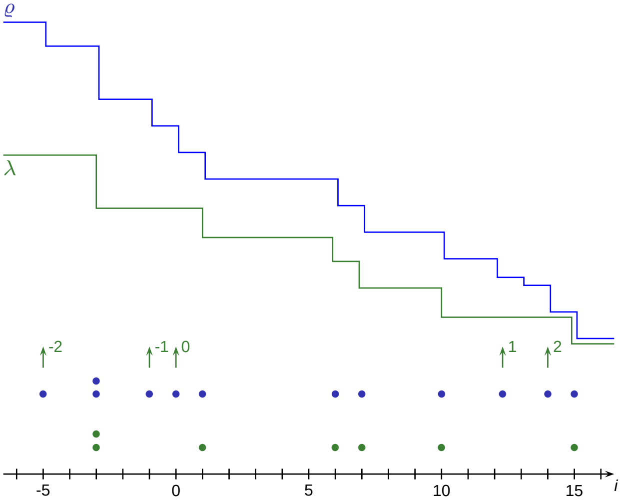

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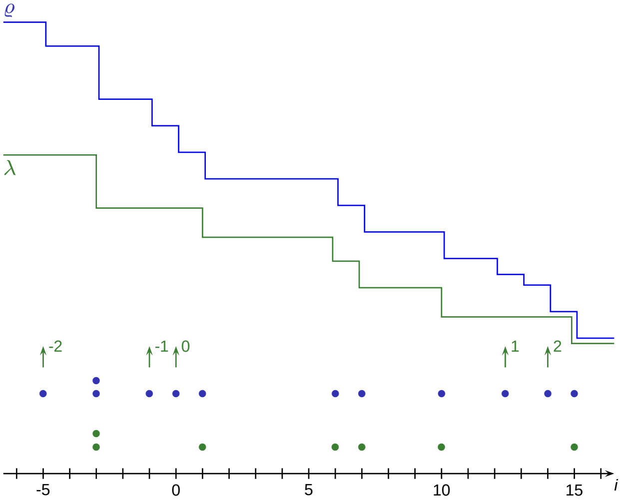

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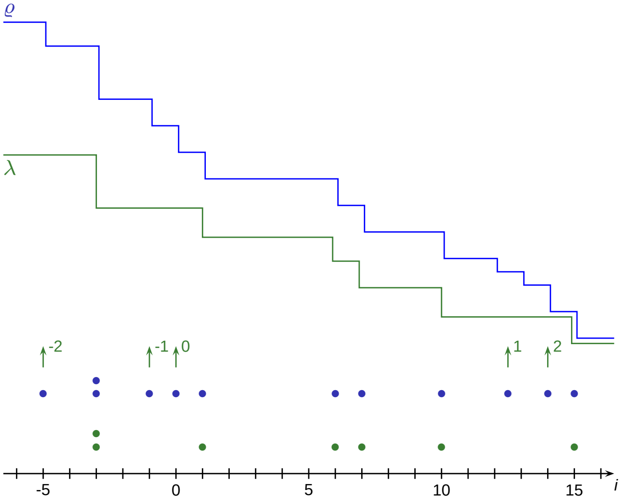

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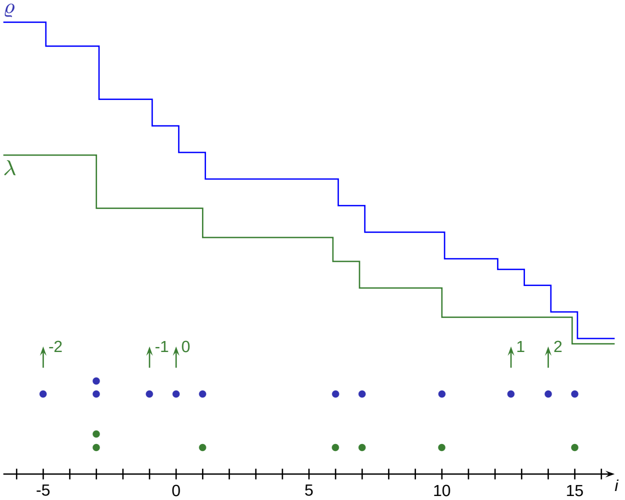

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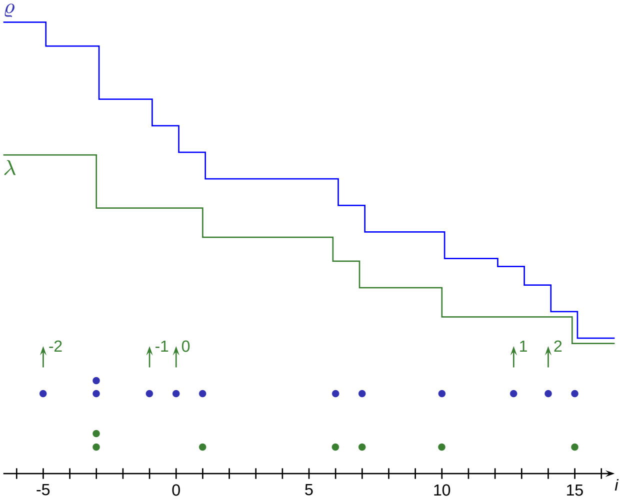

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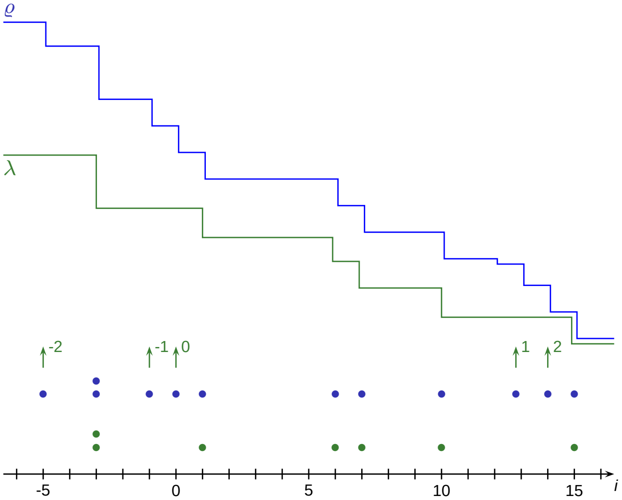

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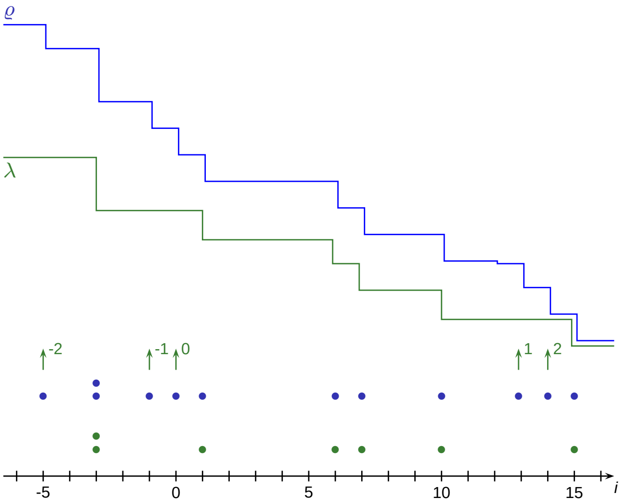

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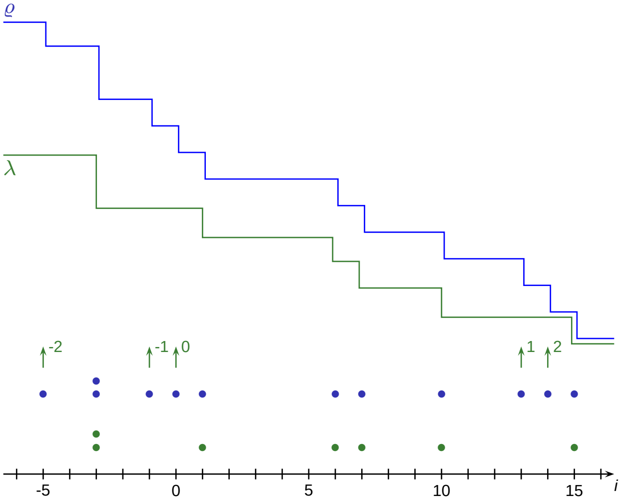

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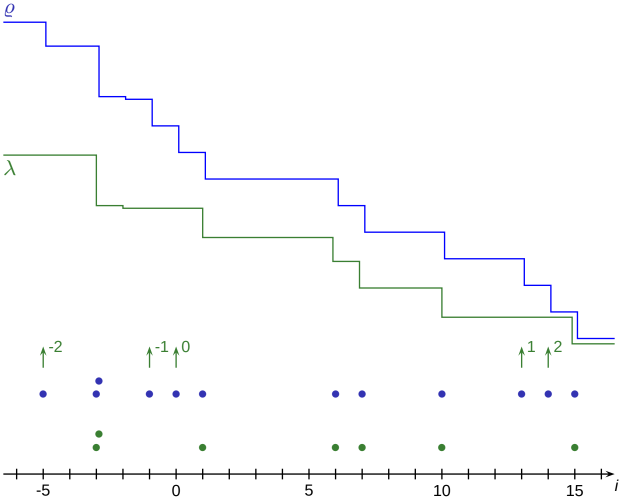

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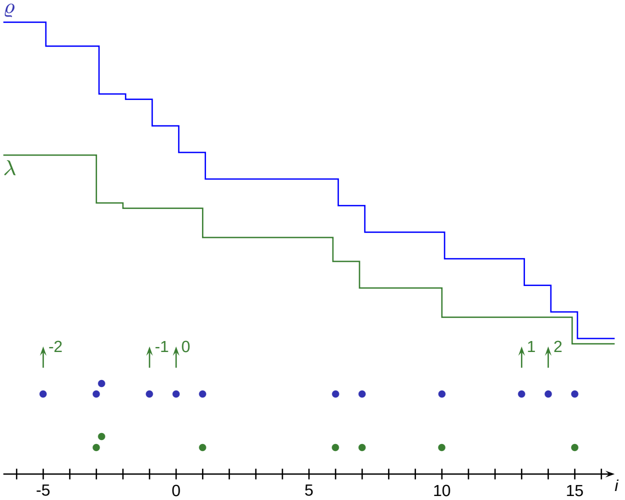

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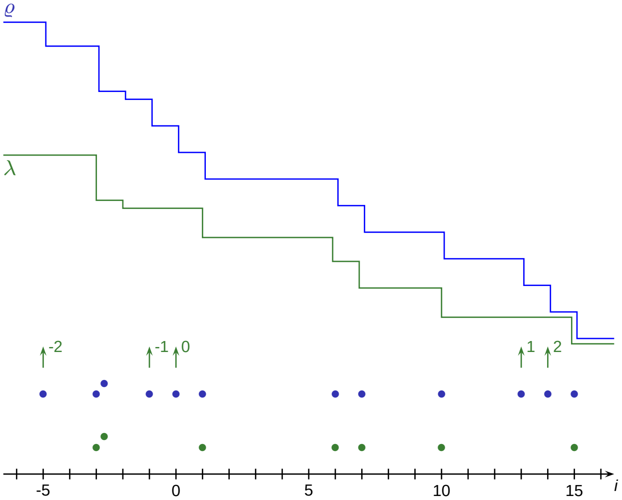

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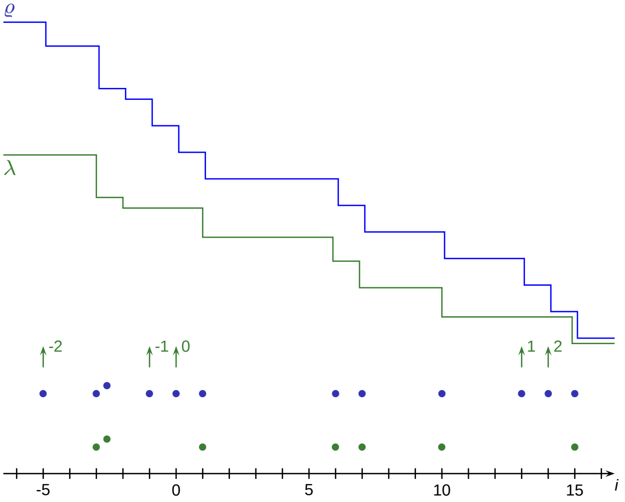

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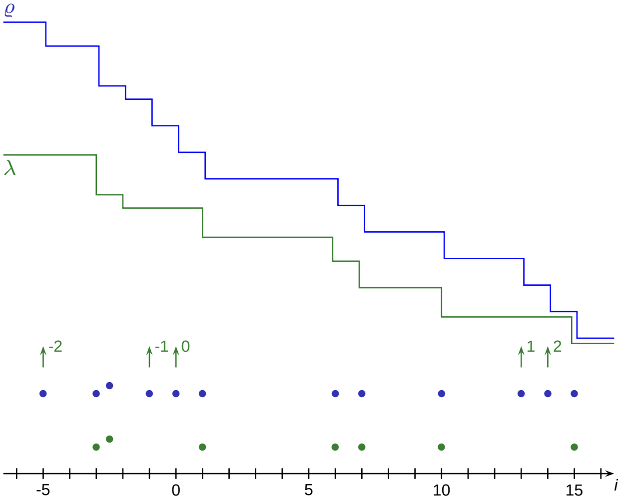

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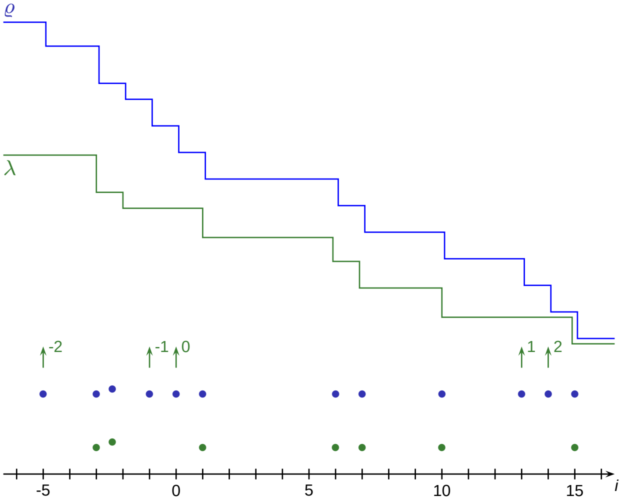

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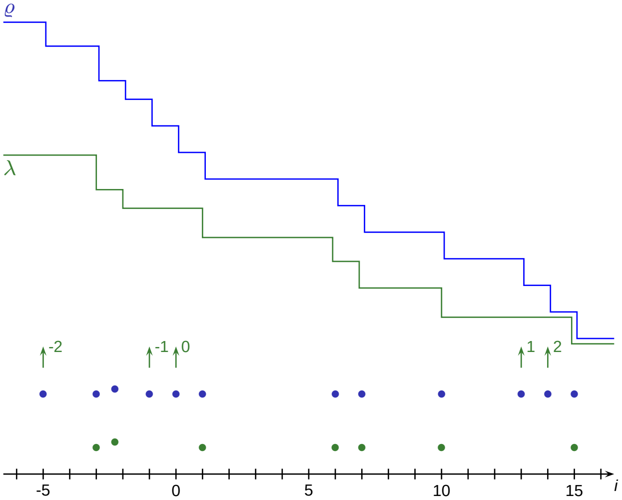

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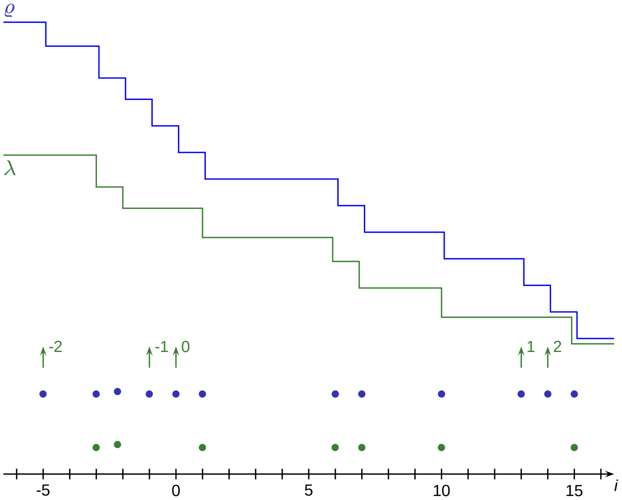

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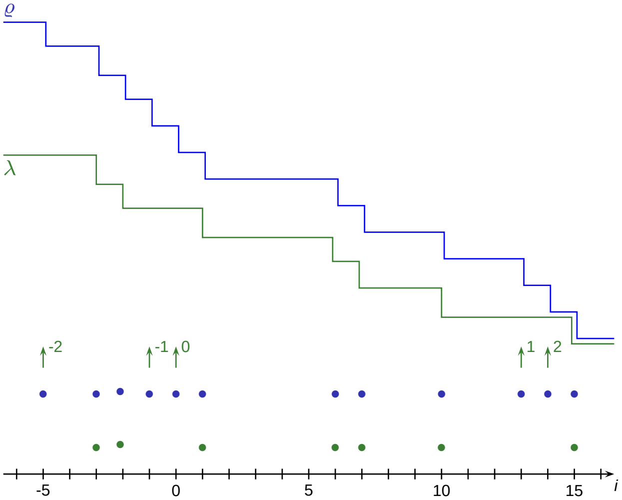

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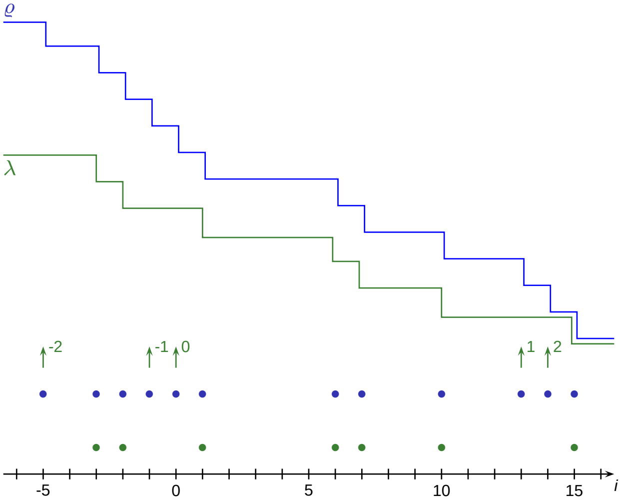

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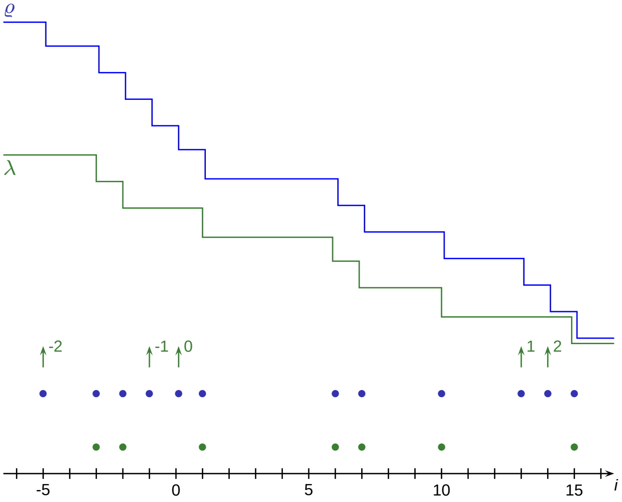

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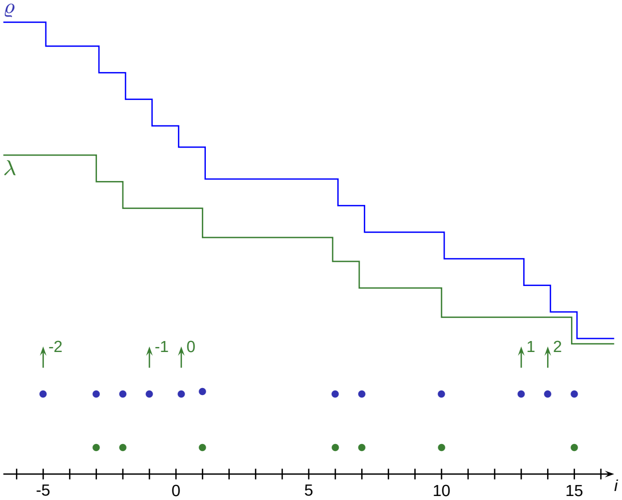

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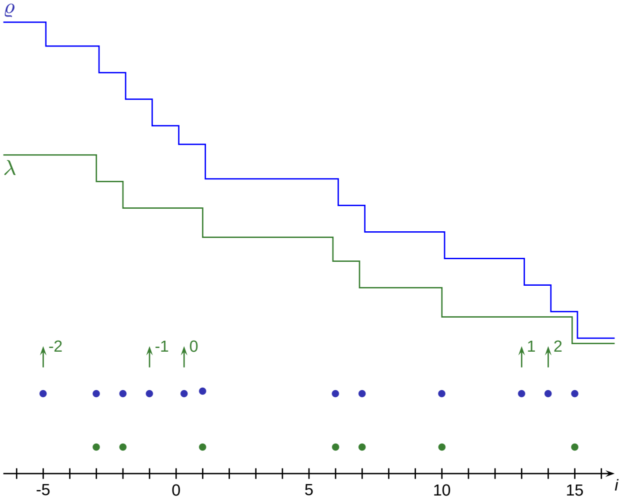

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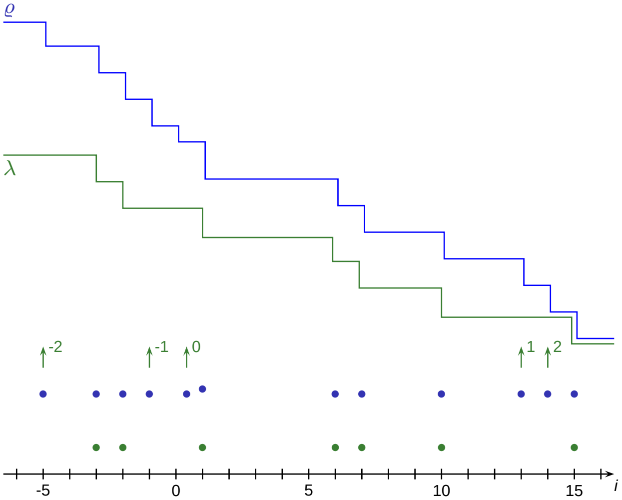

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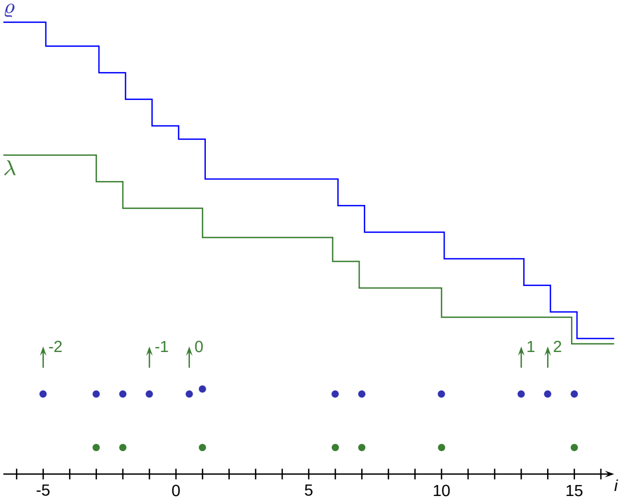

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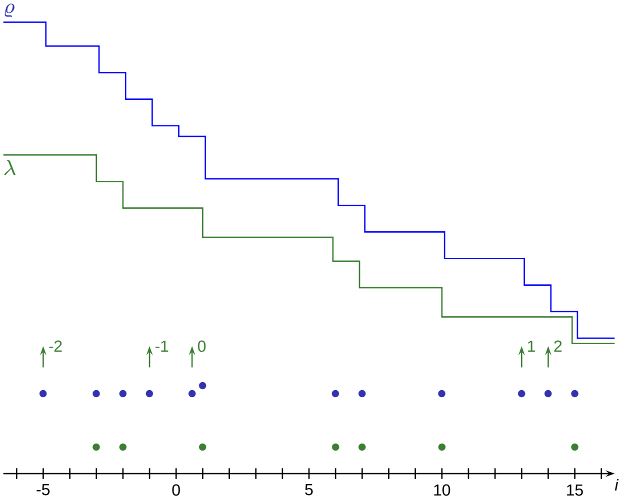

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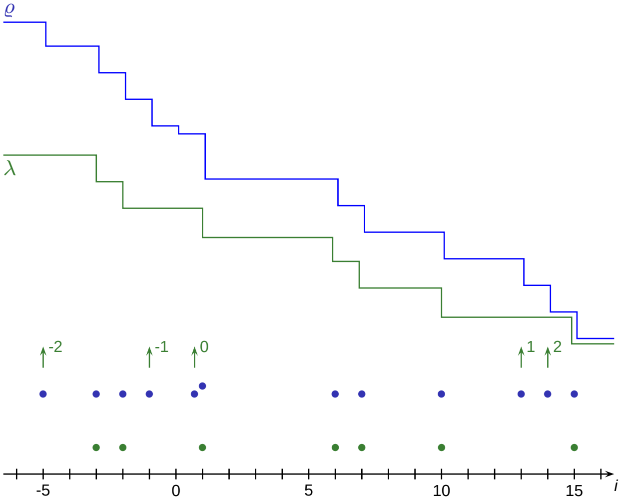

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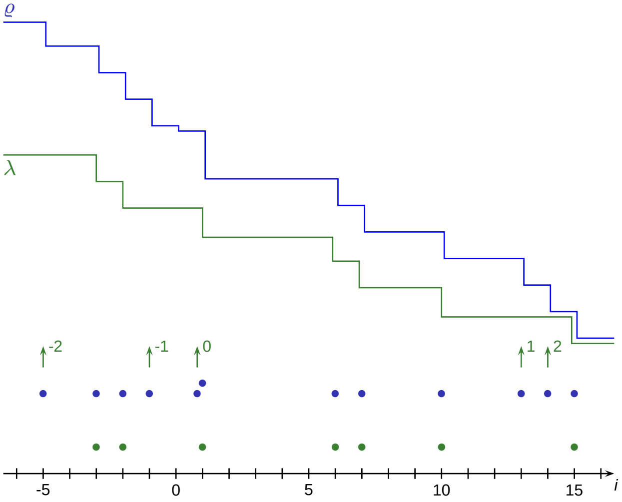

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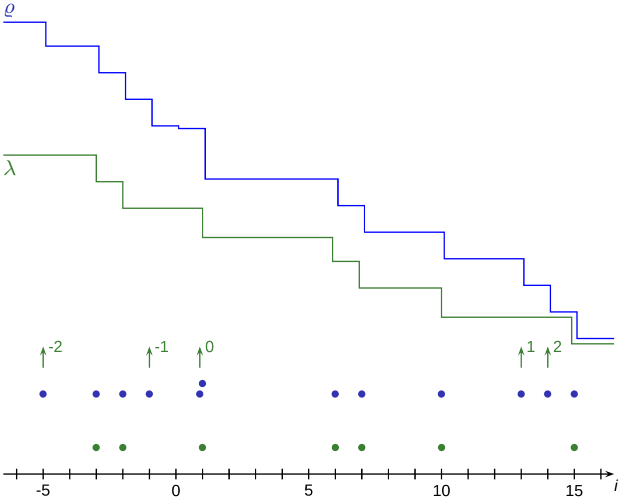

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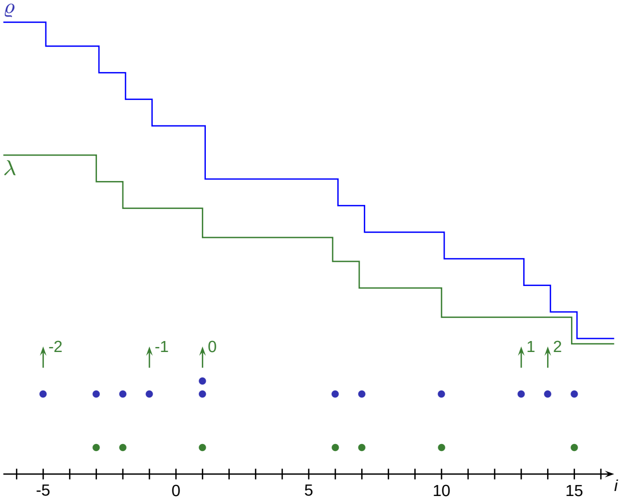

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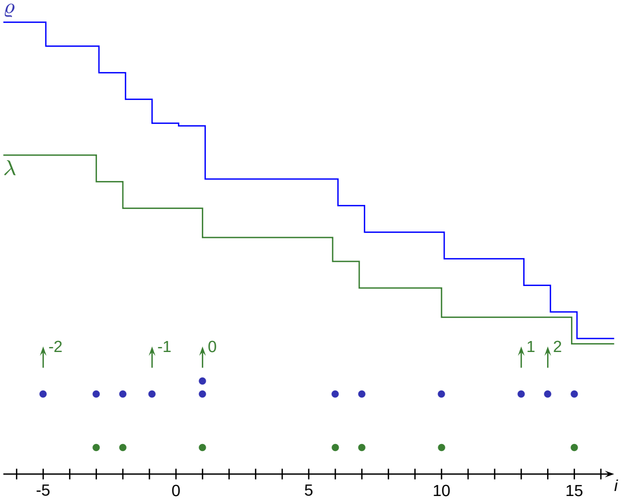

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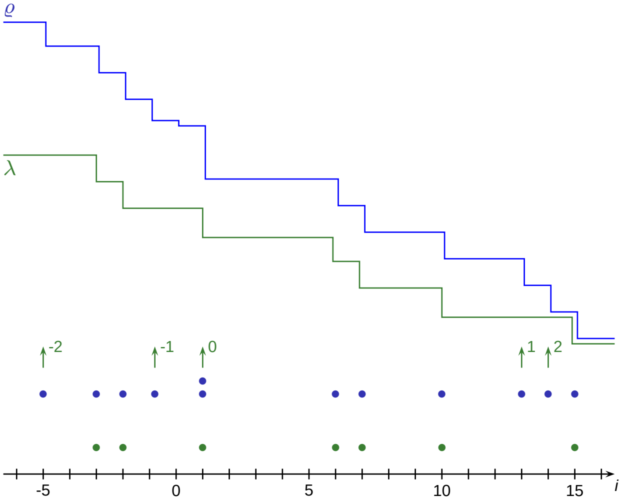

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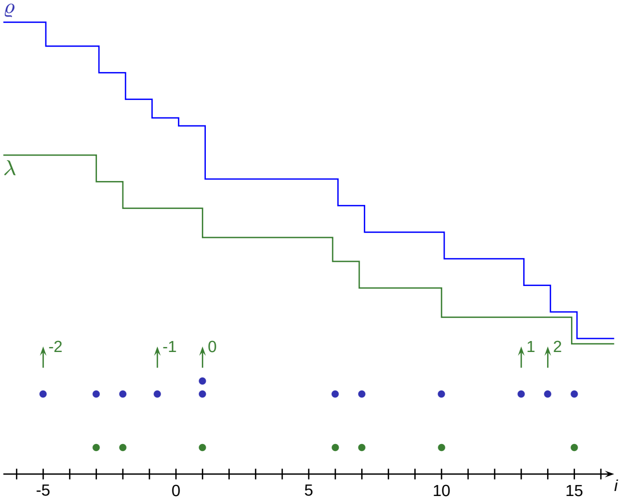

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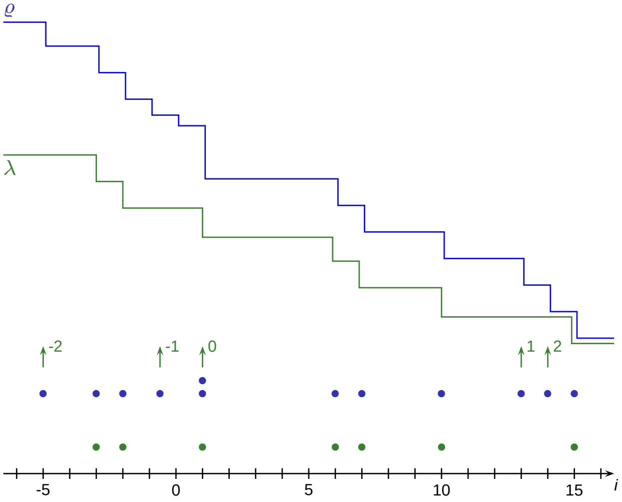

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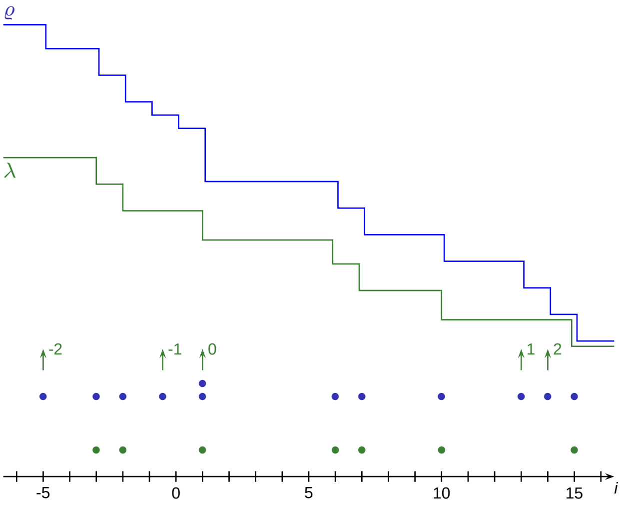

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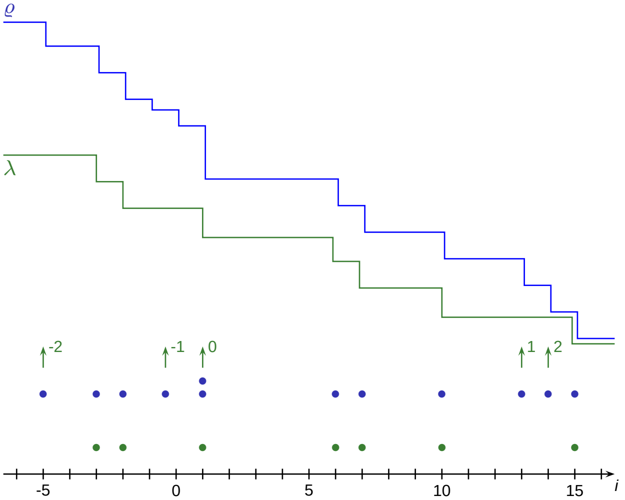

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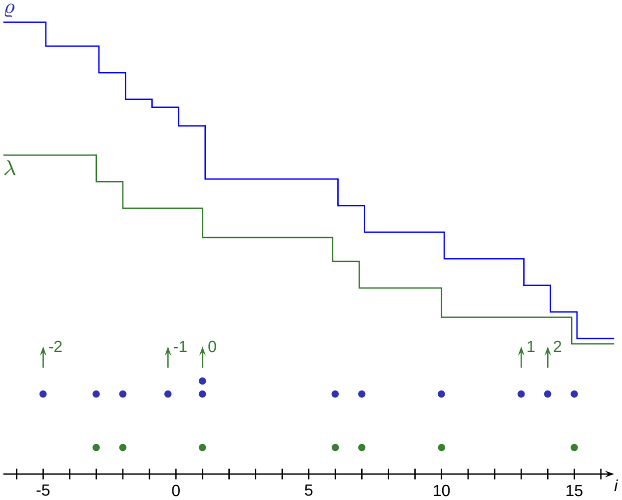

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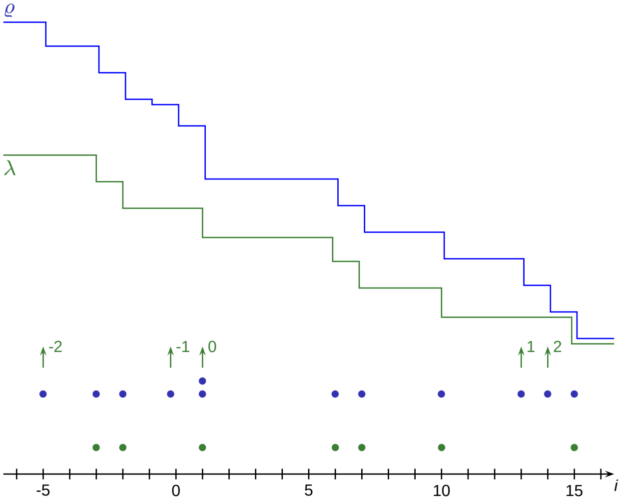

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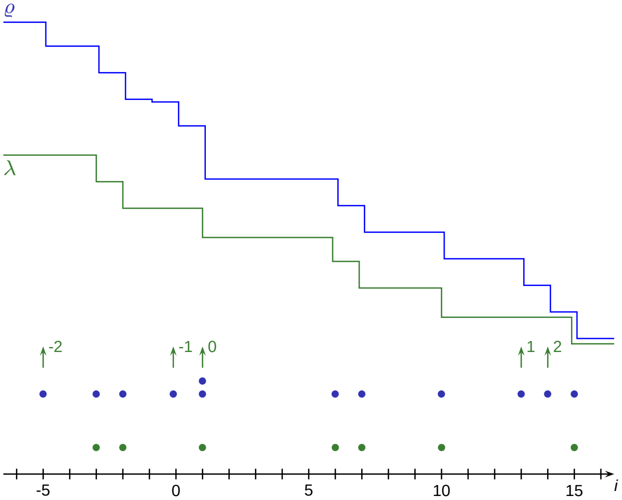

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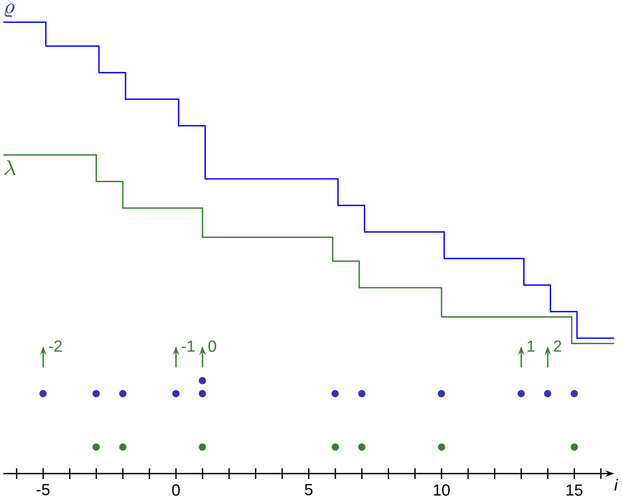

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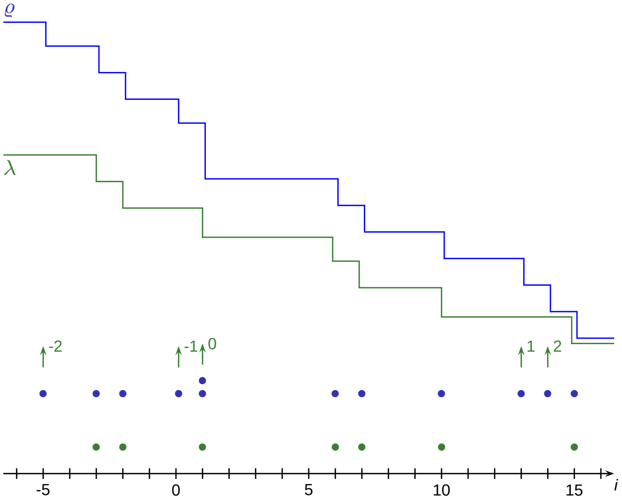

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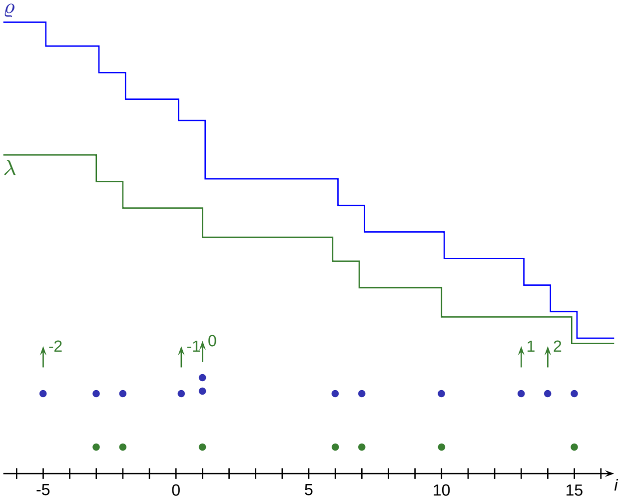

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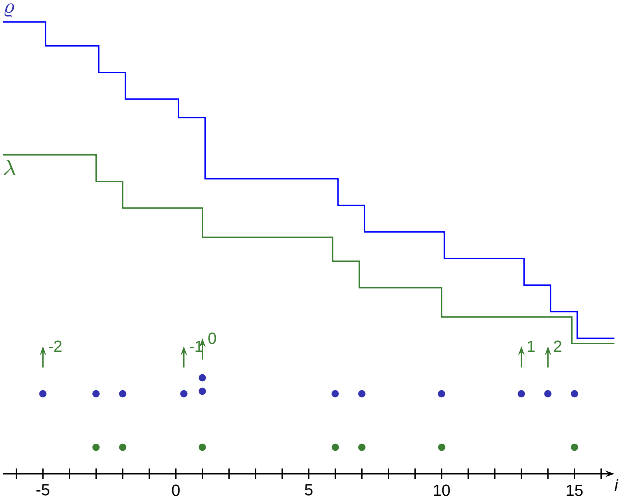

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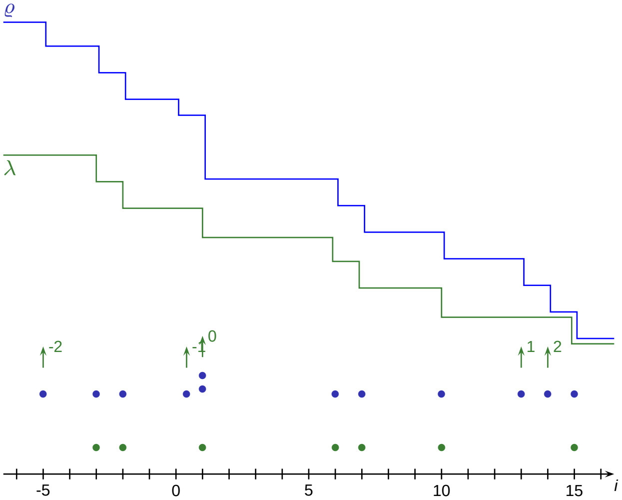

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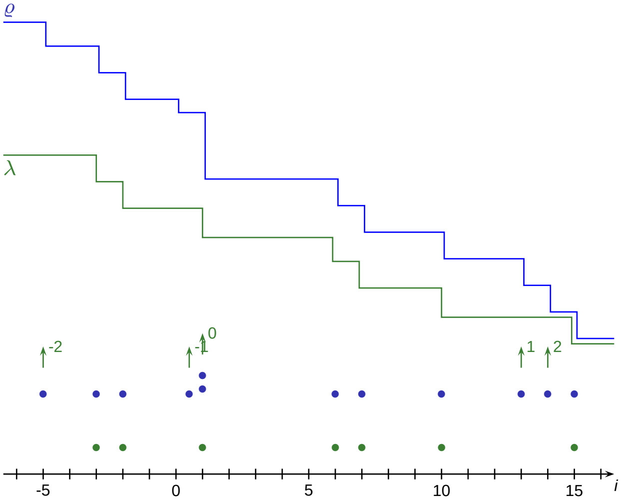

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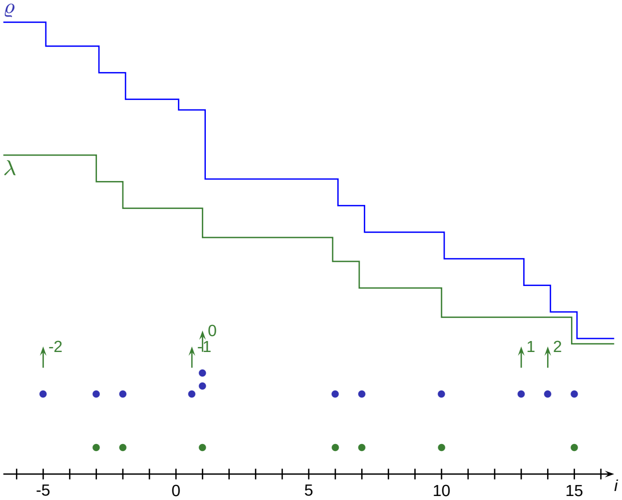

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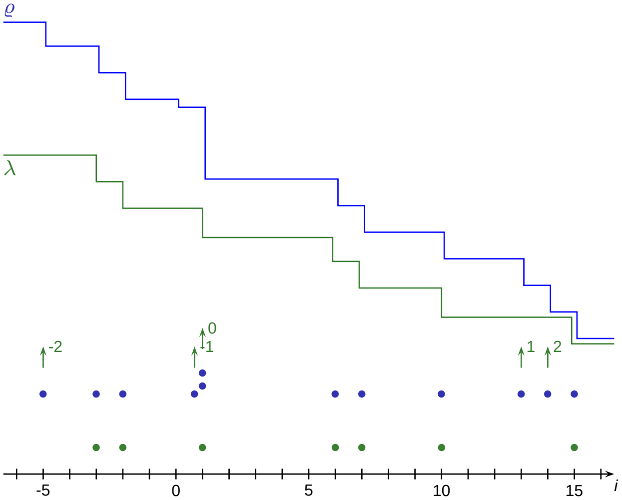

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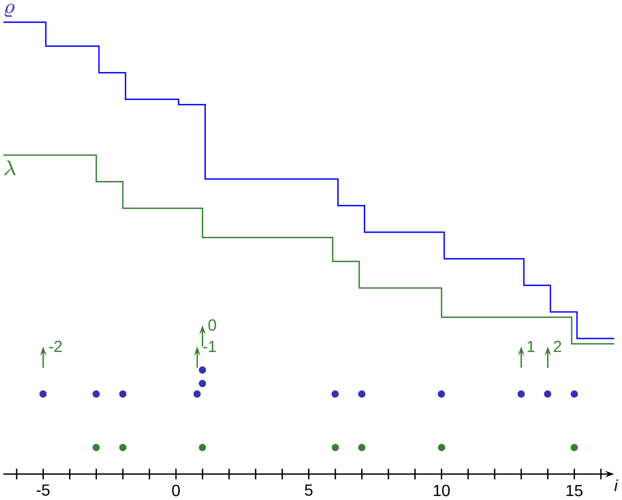

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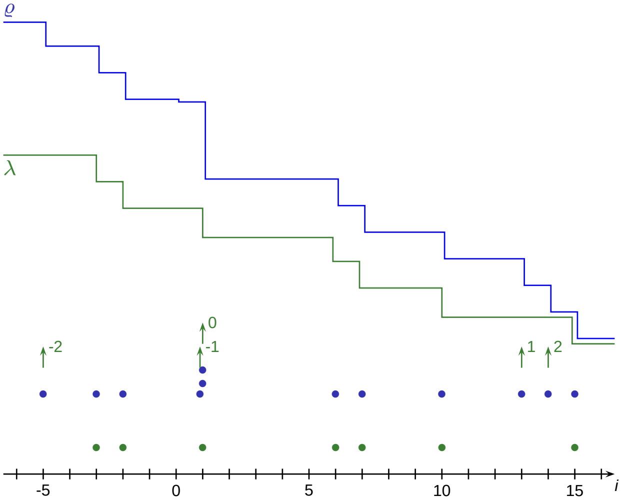

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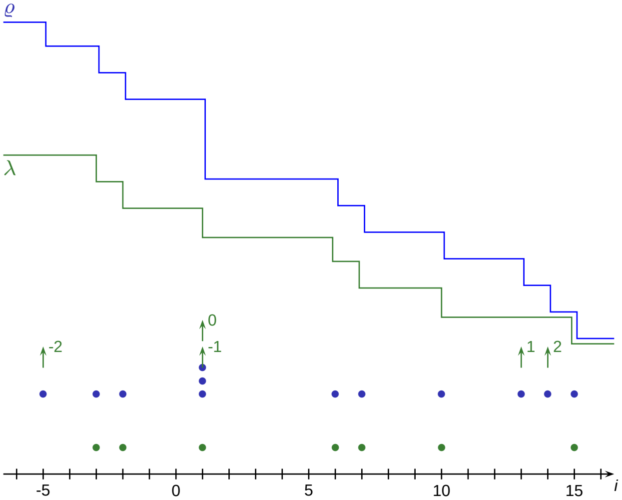

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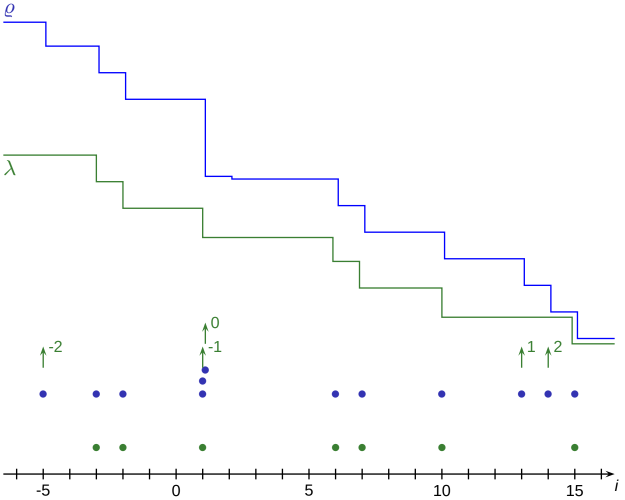

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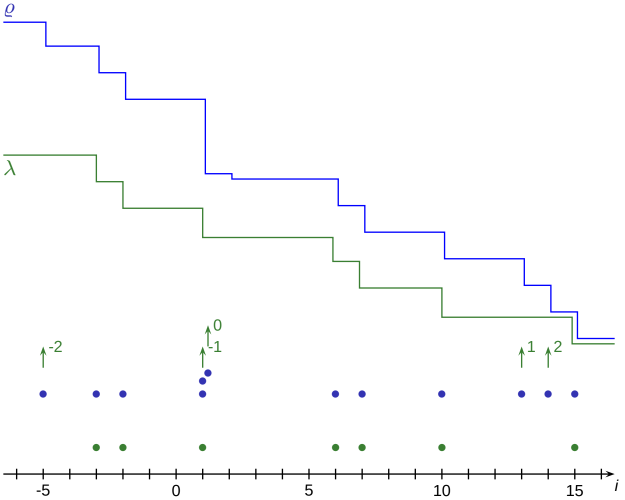

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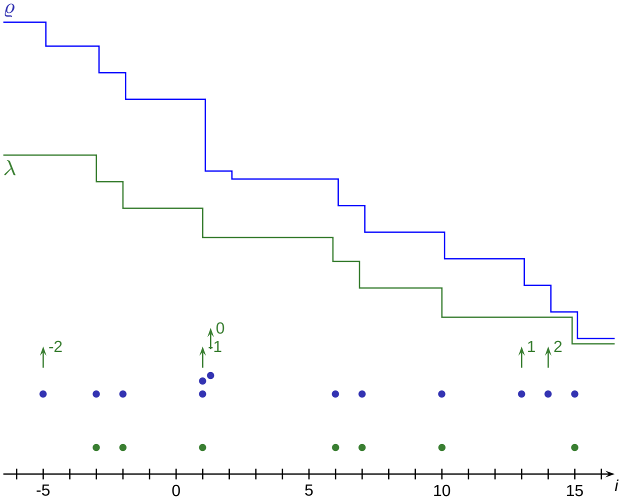

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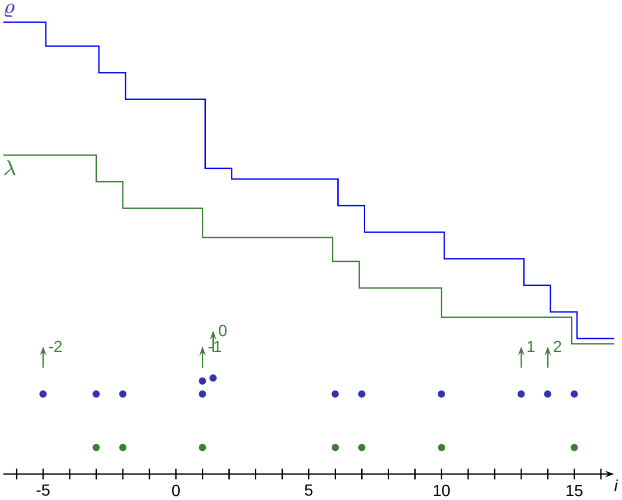

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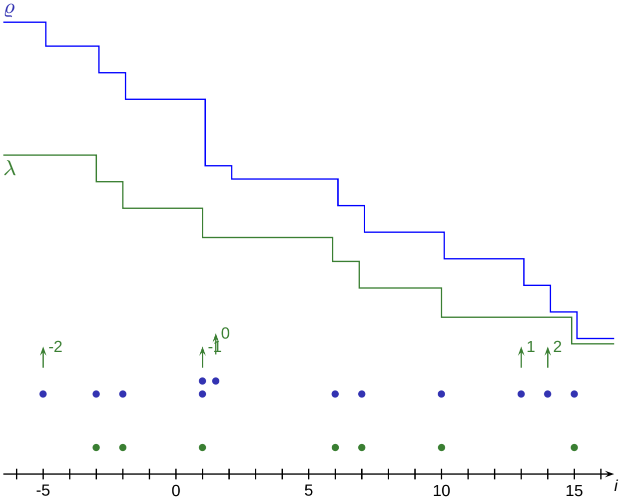

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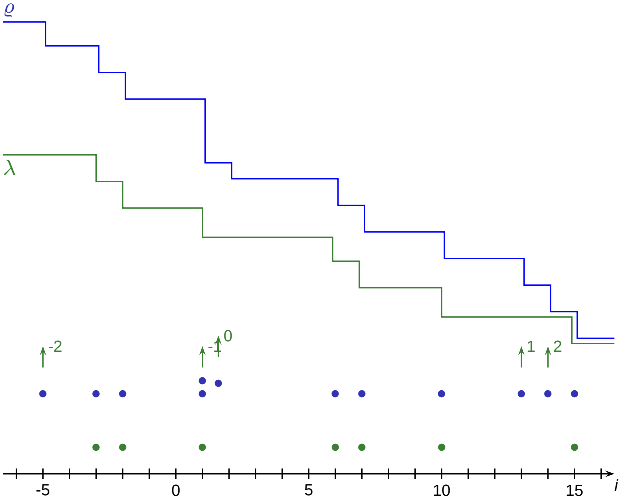

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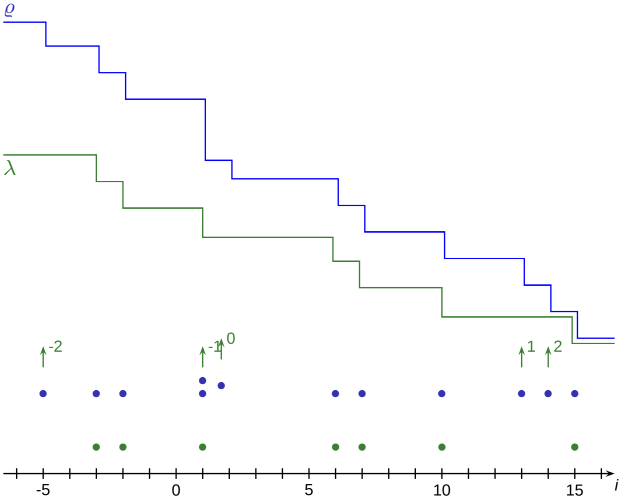

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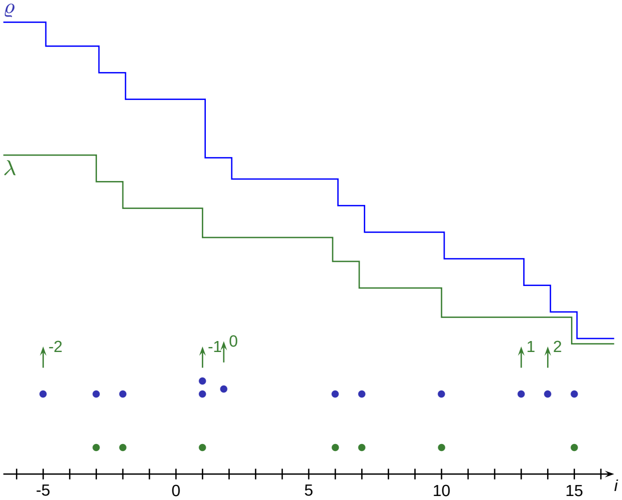

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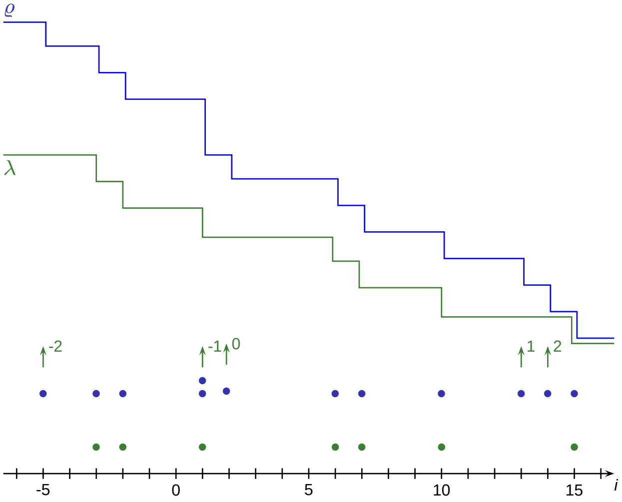

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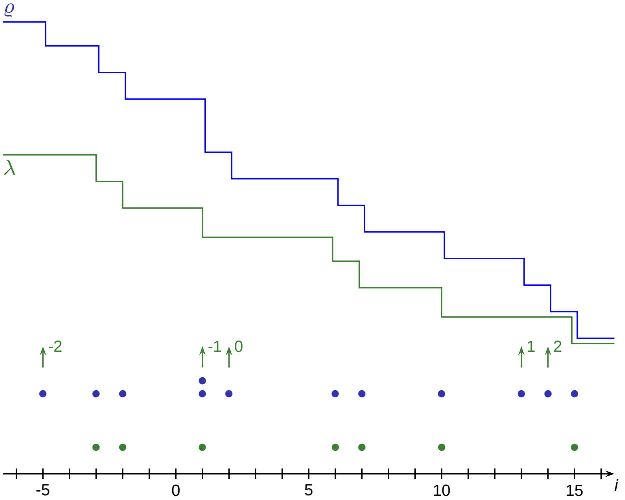

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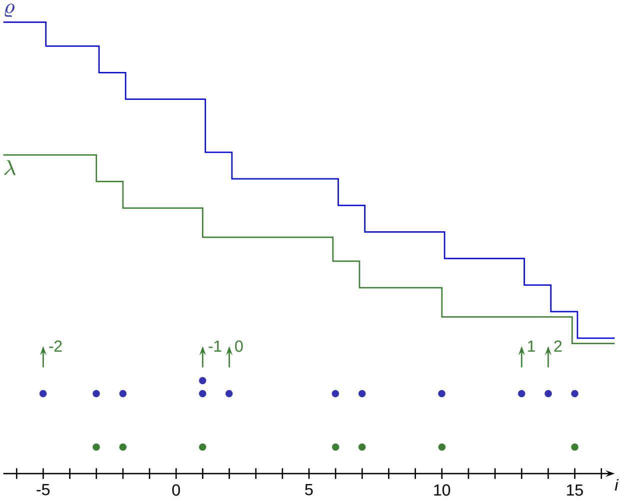
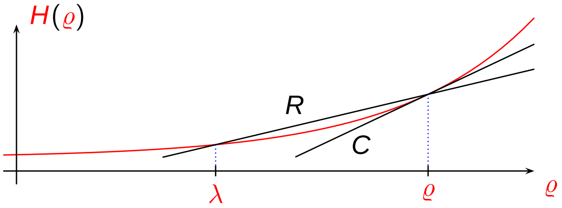


Picture:

The position $X(t)$ of $\uparrow^0$ follows the Rankine-Hugoniot speed R.

Characteristics (very briefly)

Convex flux (some cases of AZRP, ABLP):



Recall $C = H'(\varrho) > R = \dfrac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$

Characteristics (very briefly)

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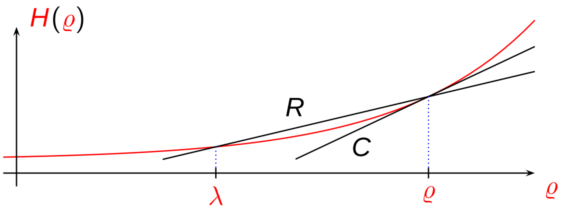

Recall $\quad C = H'(\varrho) > R = \dfrac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$

Do we have $\quad Q(t) \overset{?}{\geq} X(t)$

Characteristics (very briefly)

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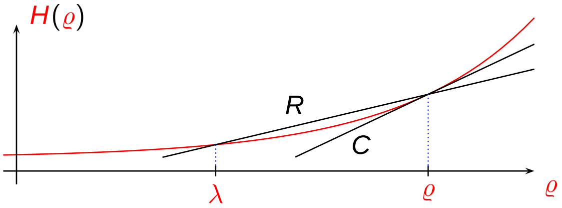
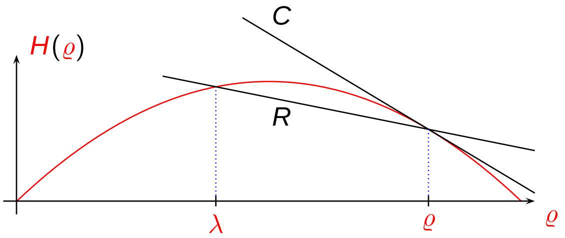


Recall $C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$

Do we have $Q(t) \overset{?}{\geq} X(t) -$ tight error

Characteristics (very briefly)

Concave flux (ASEP, AZRP):



$$C = H'(\varrho) < R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Characteristics (very briefly)

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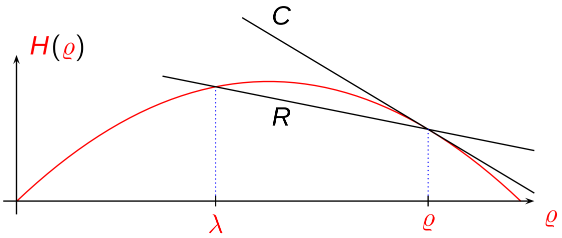

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Do we have $Q(t) \overset{?}{\leq} X(t)$

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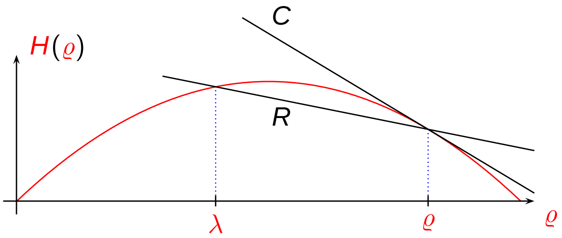
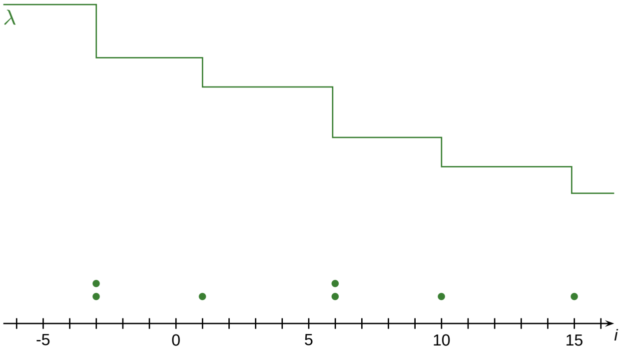


$$C = H'(\varrho) < R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Do we have $Q(t) \overset{?}{\leq} X(t) + \text{tight error}$

Many second class particles



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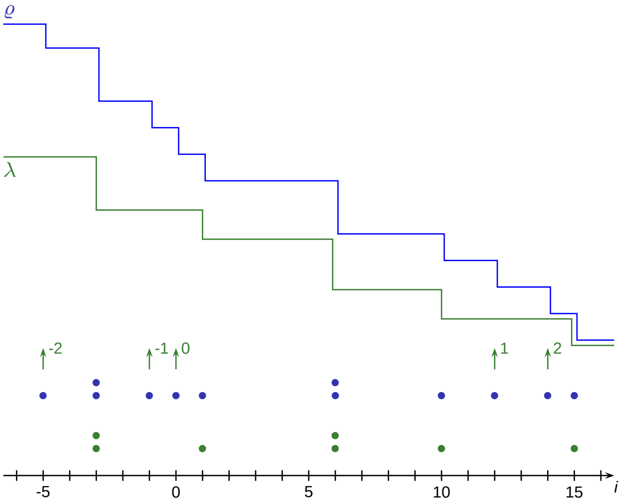
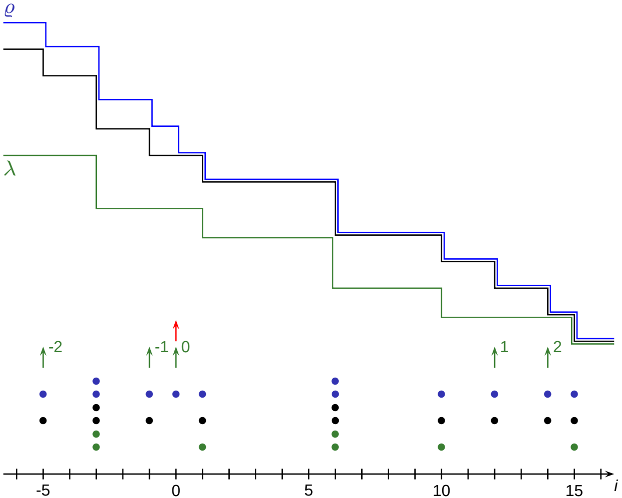
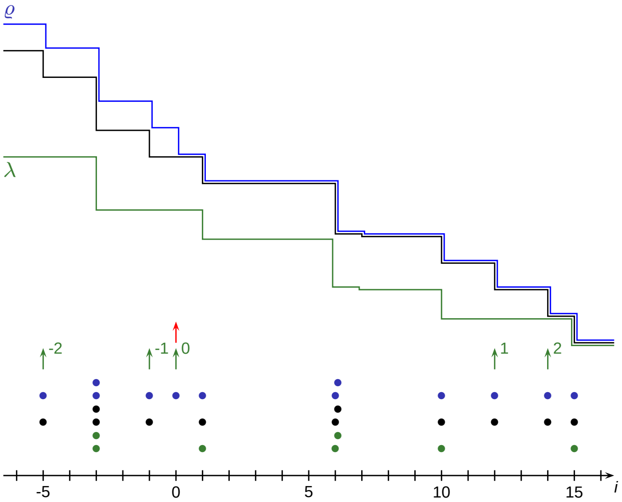


Many second class particles plus one



Couple three processes, and $X(t)$ to $Q(t)$.

Many second class particles plus one



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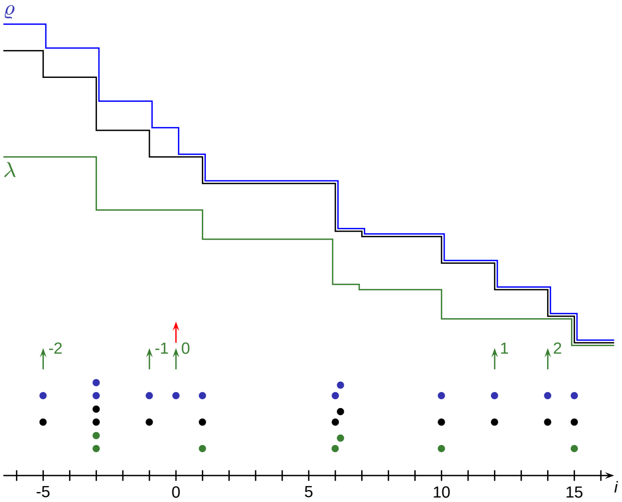

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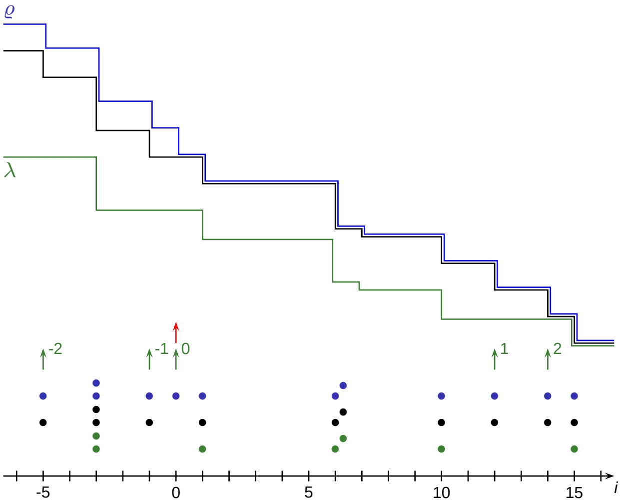

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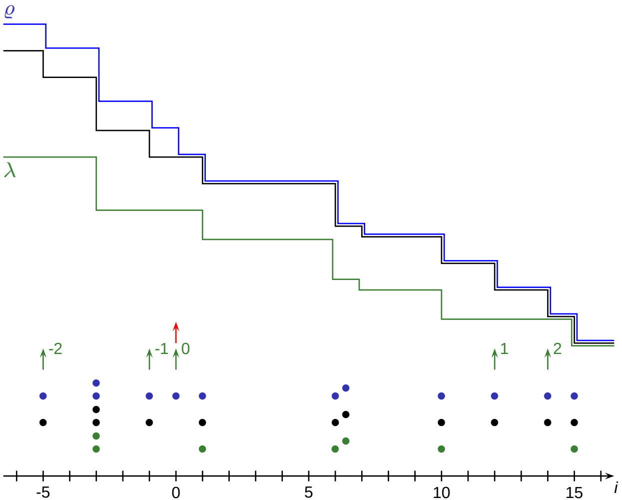

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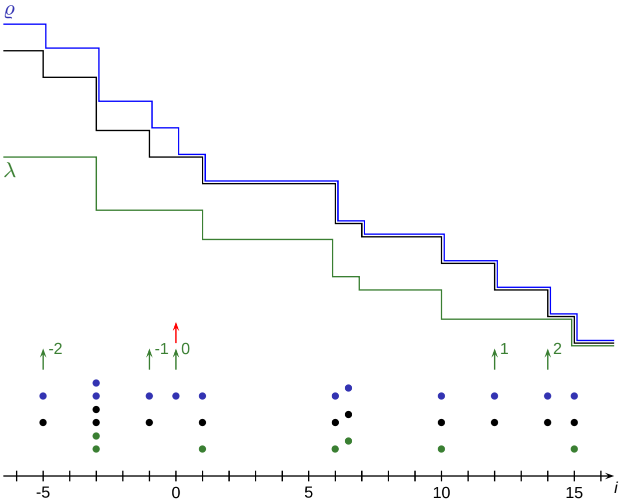

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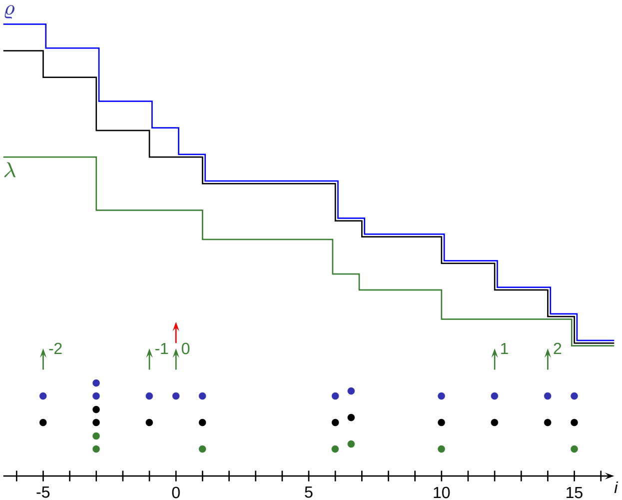

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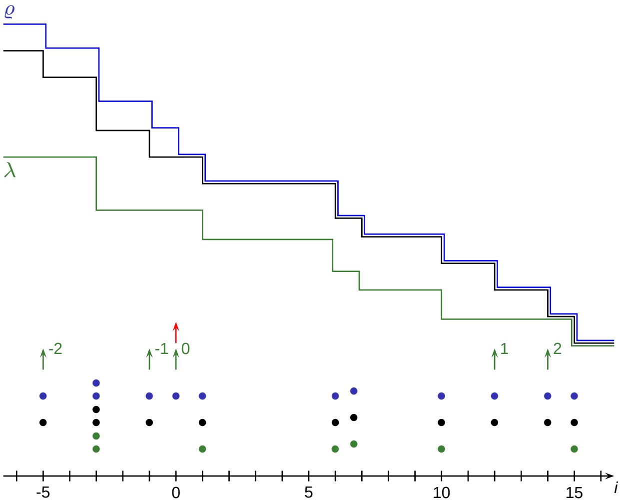

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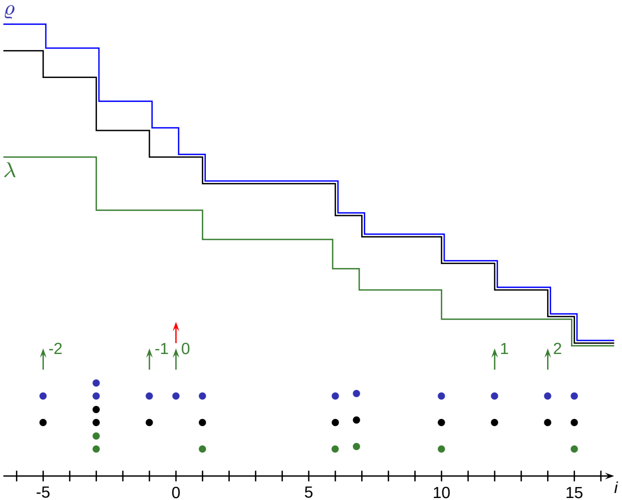

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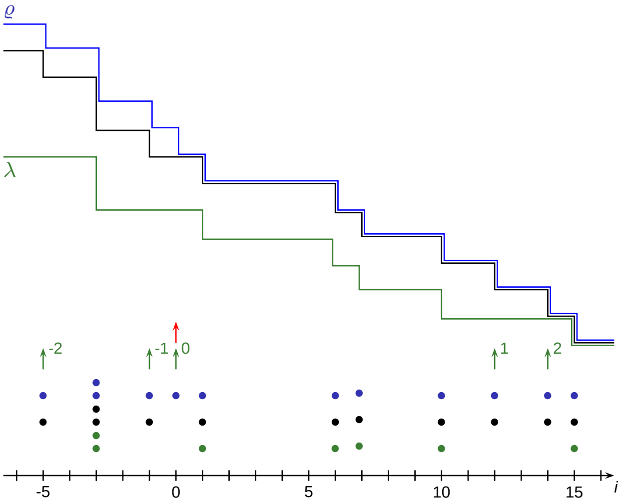

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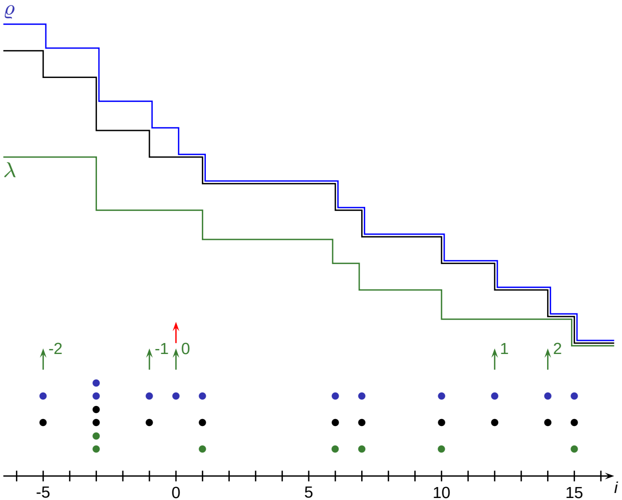

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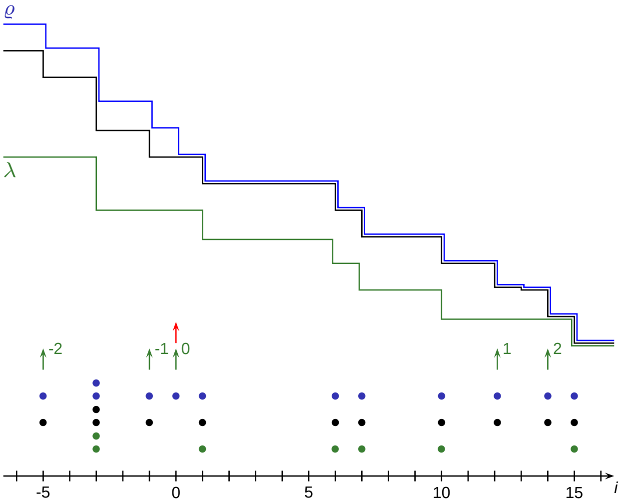

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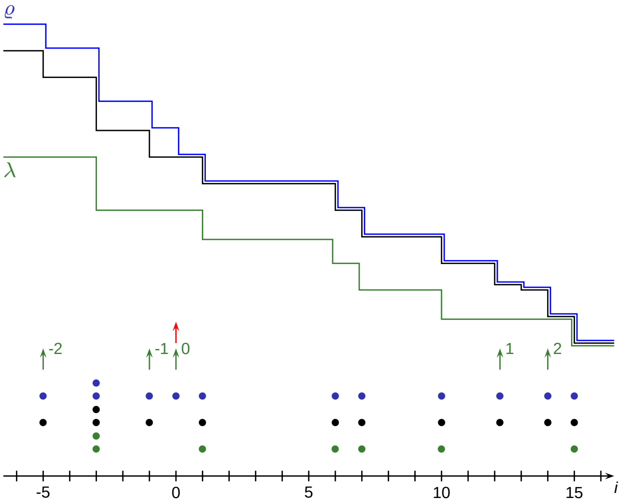

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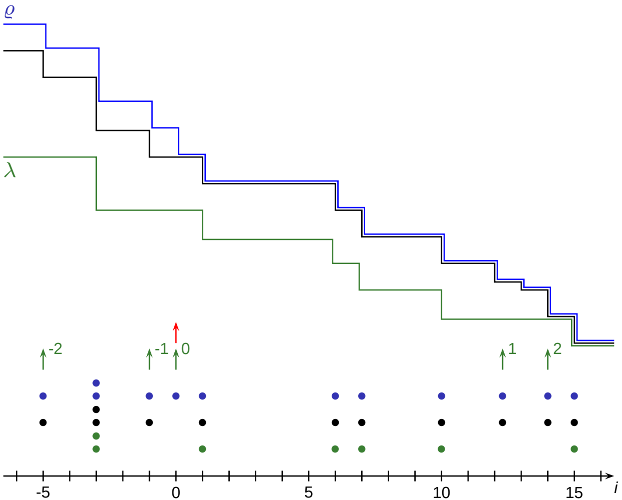

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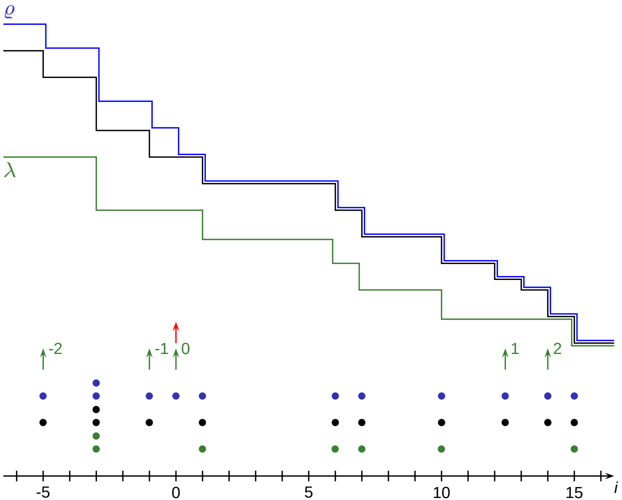

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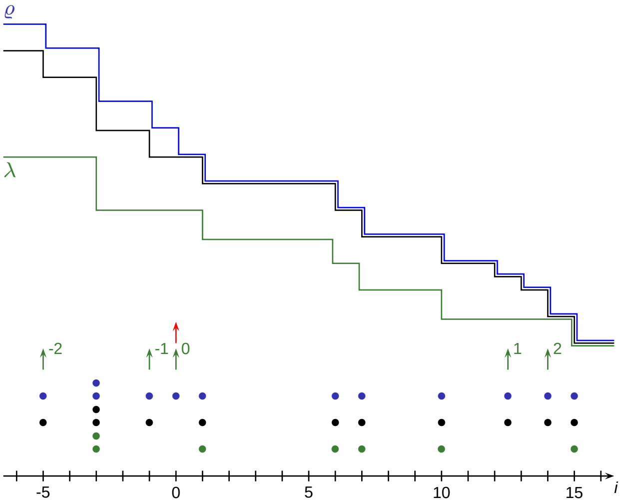

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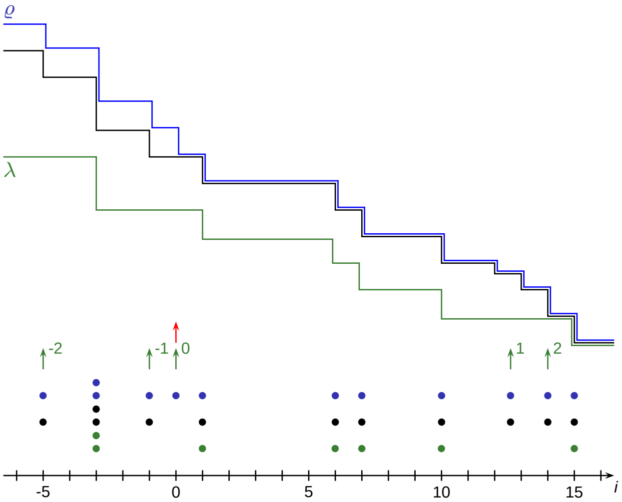

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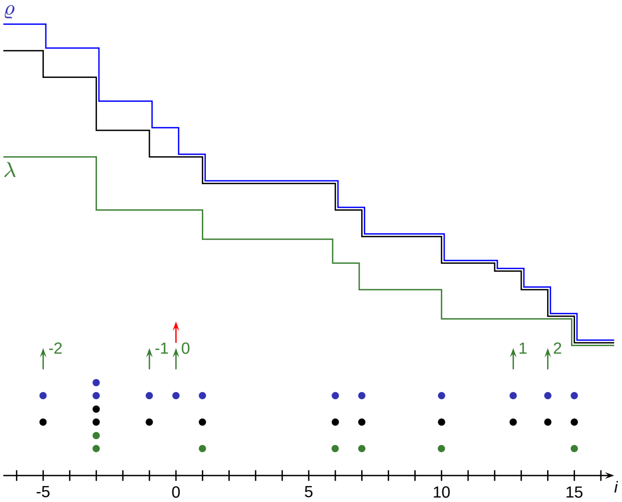

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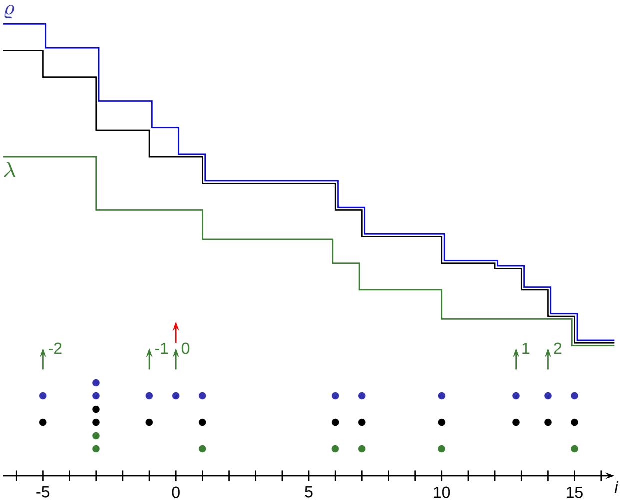

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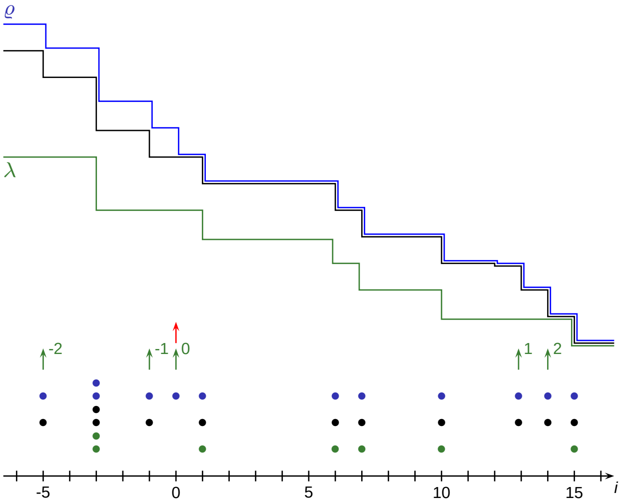

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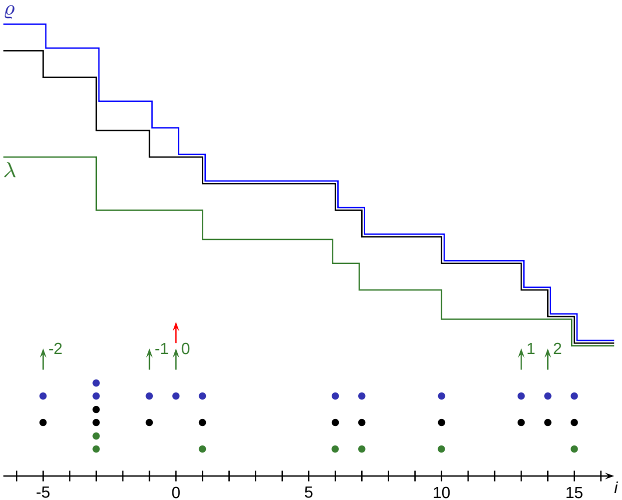

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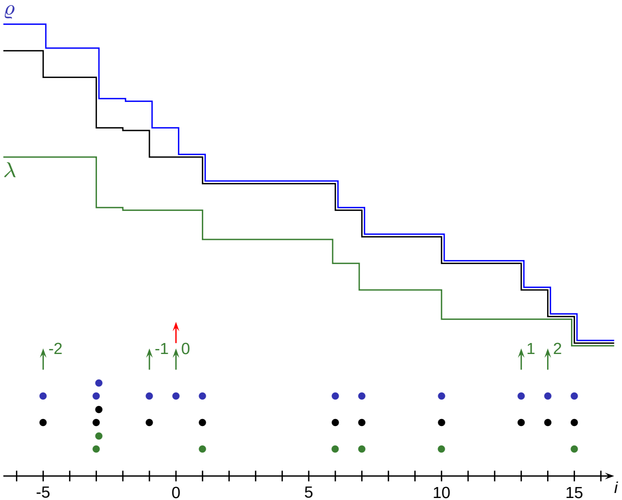

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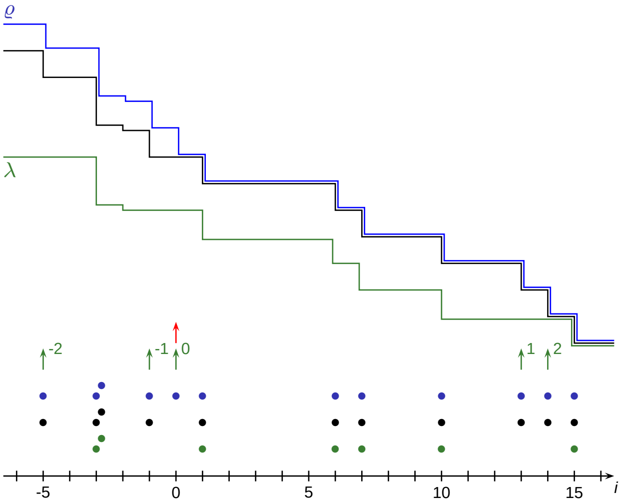

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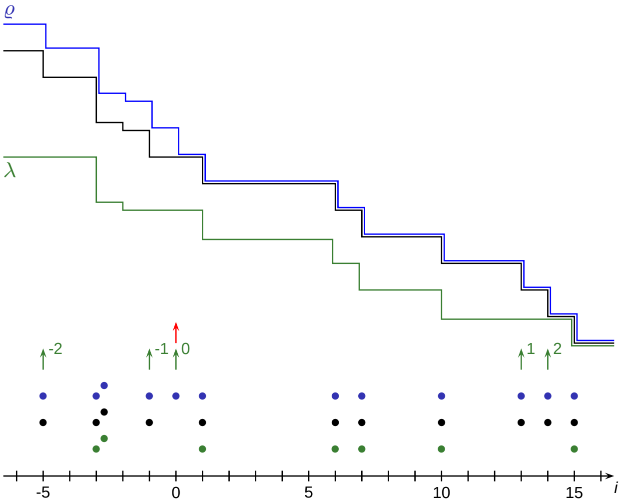

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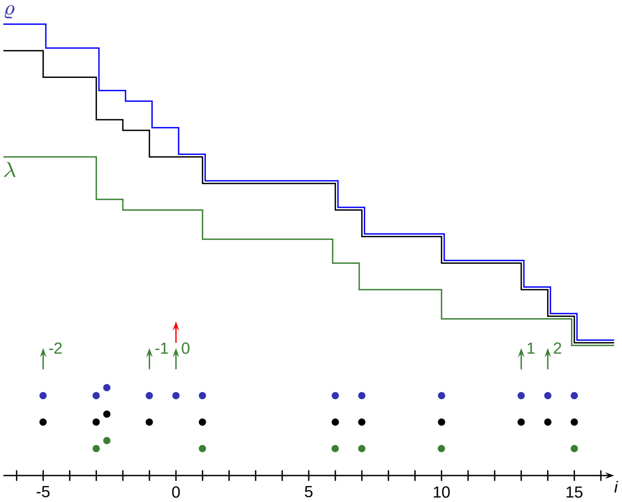

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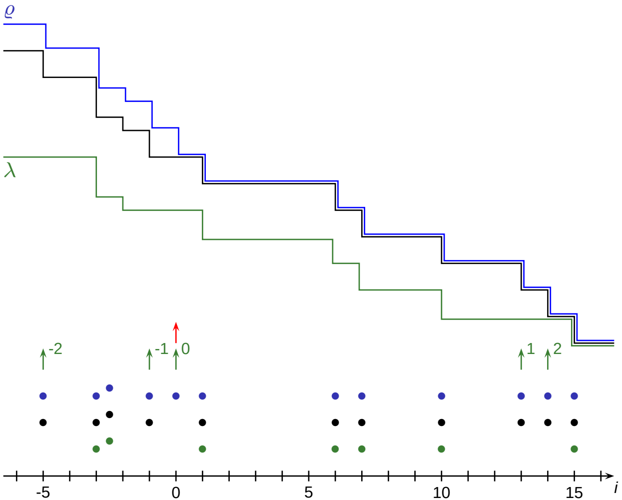

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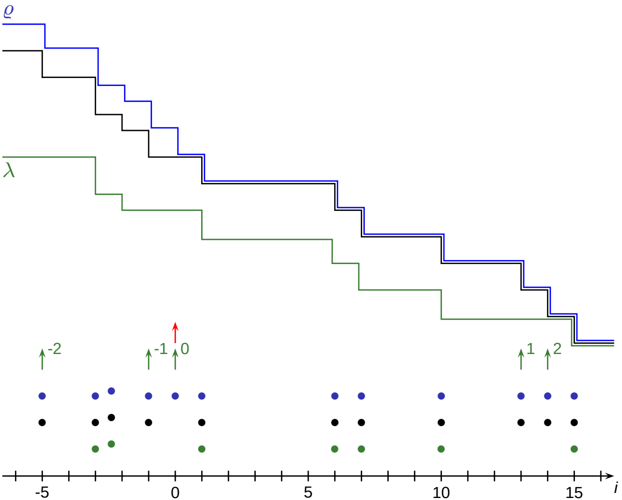

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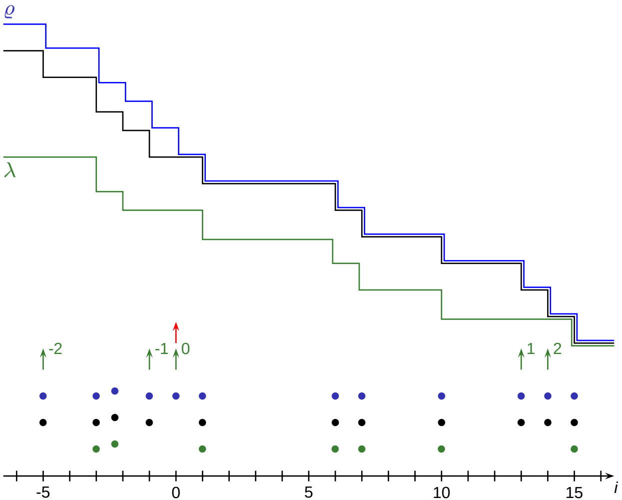

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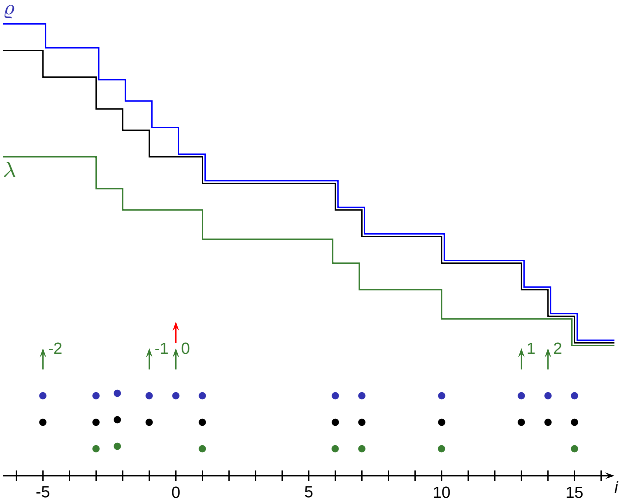

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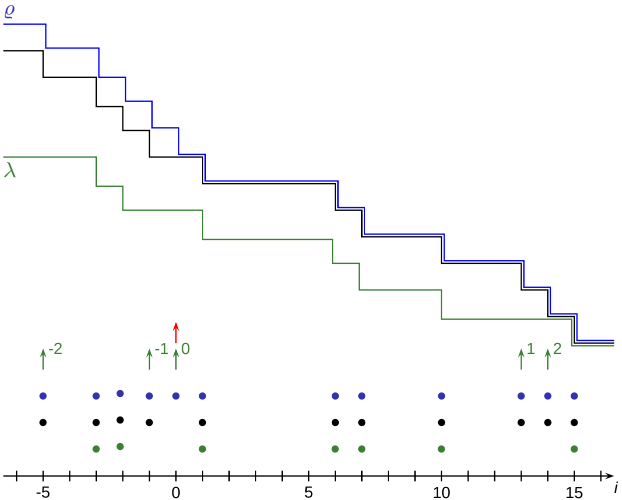

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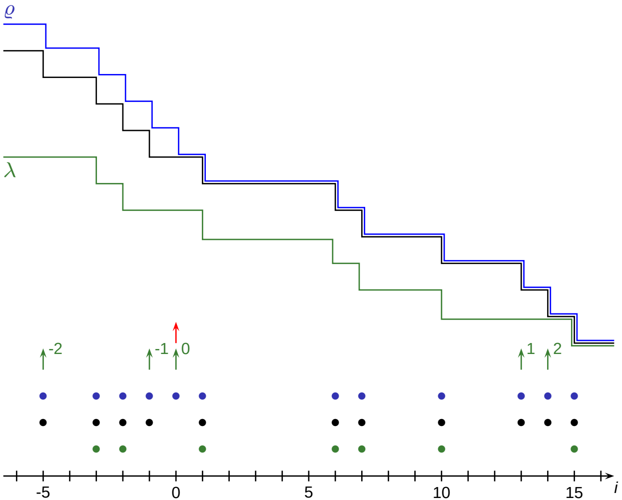

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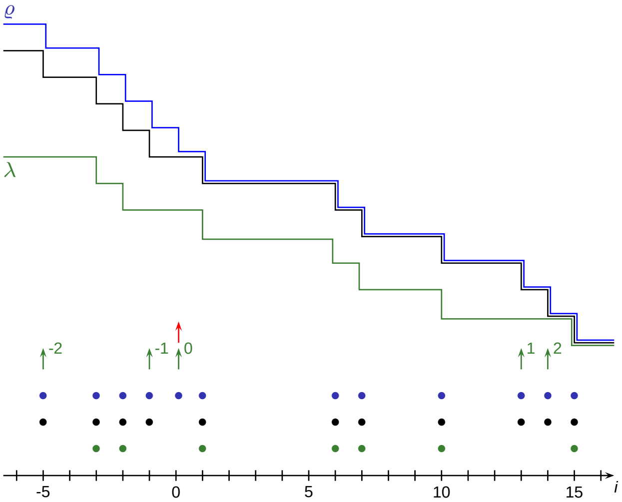

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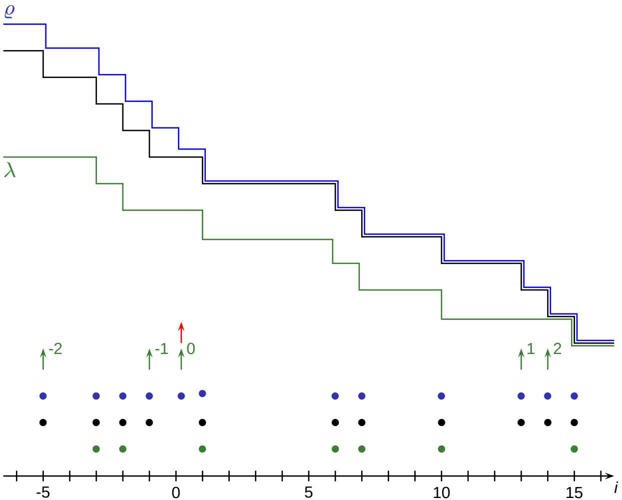

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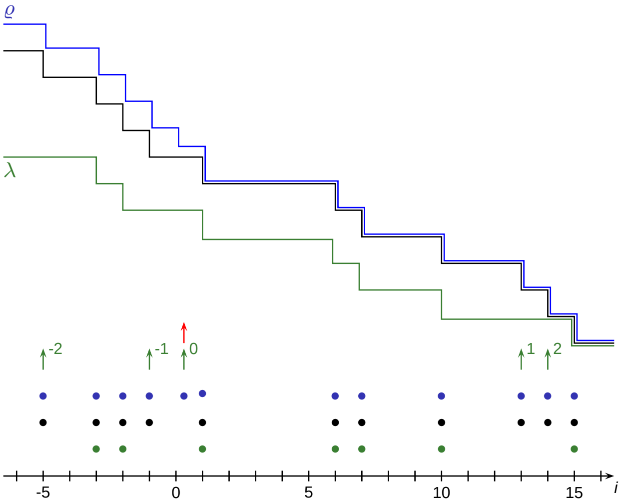

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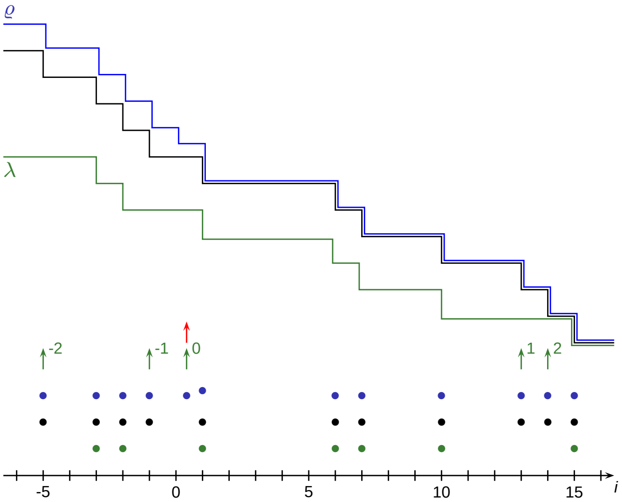

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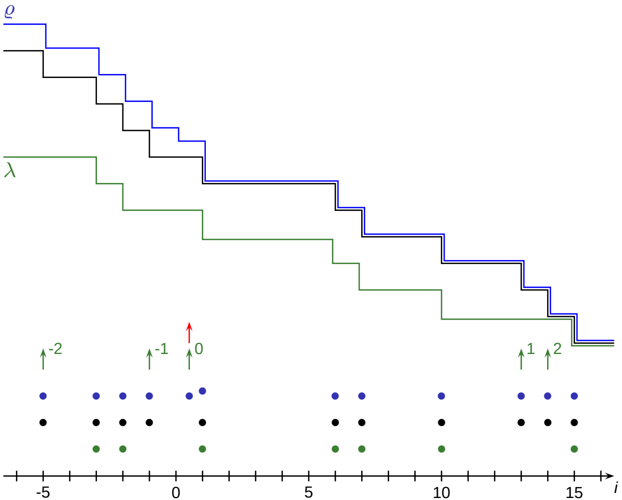

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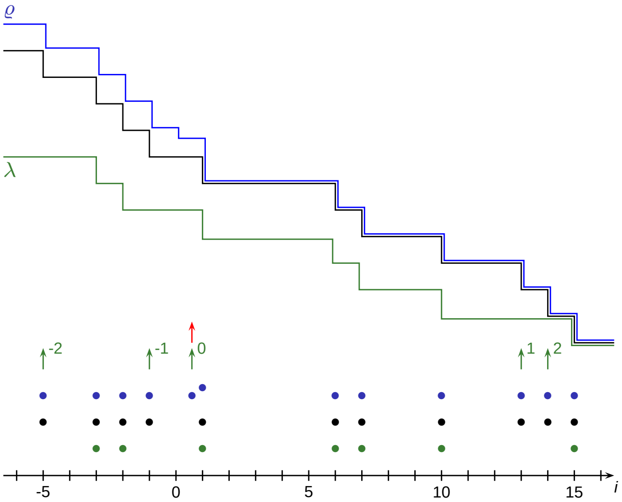

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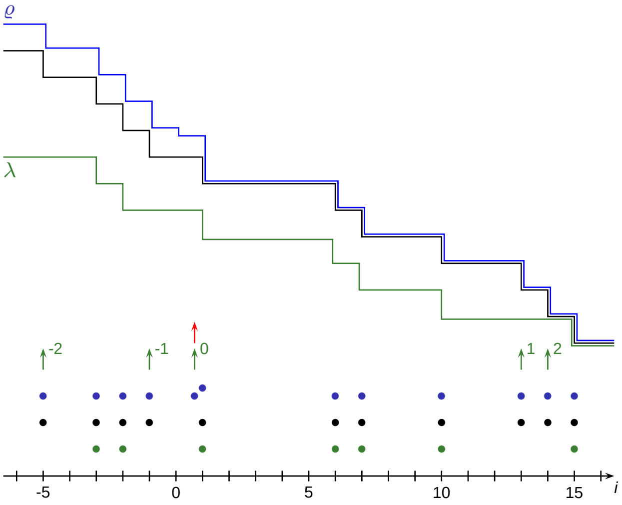

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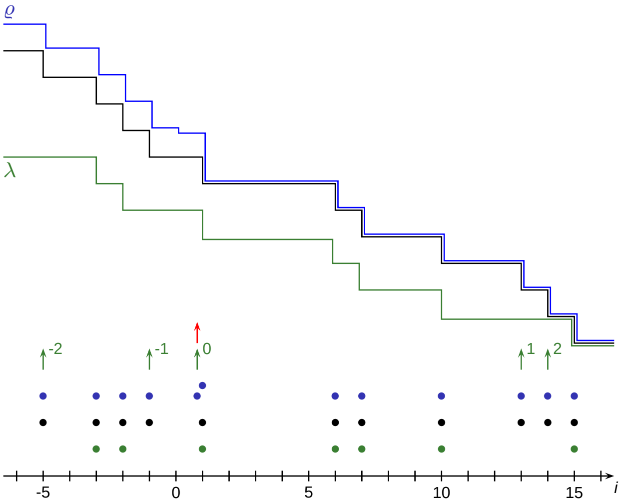

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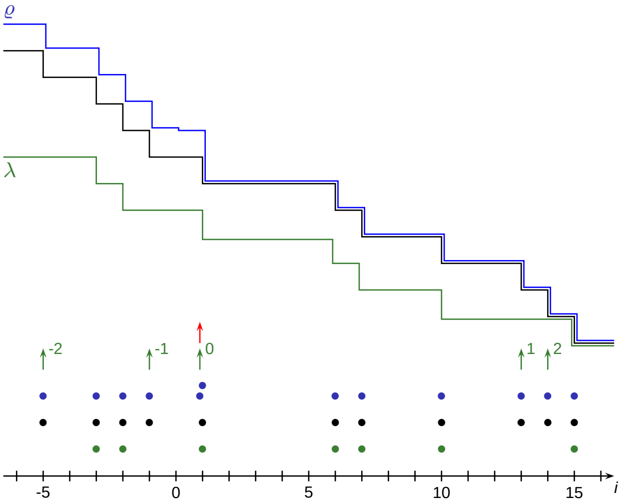

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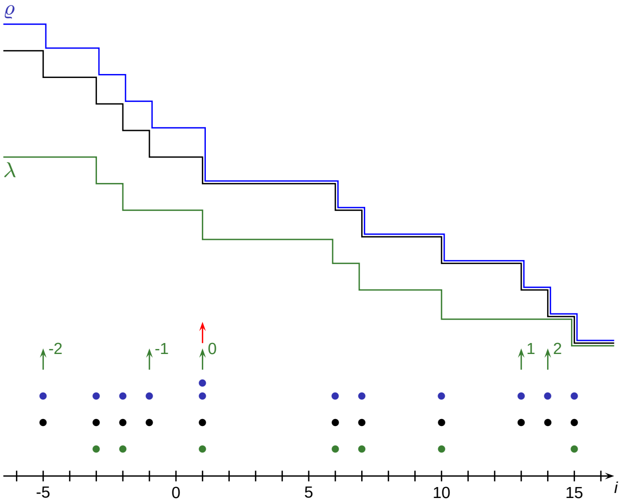

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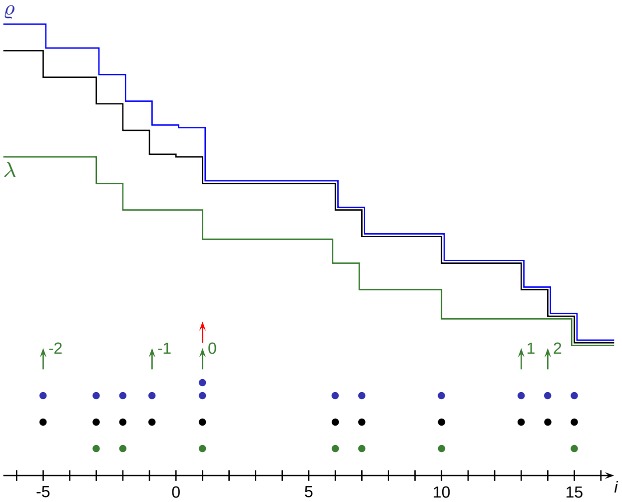

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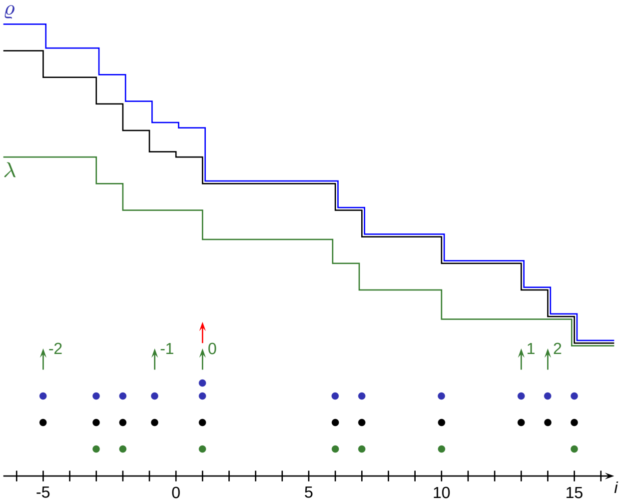

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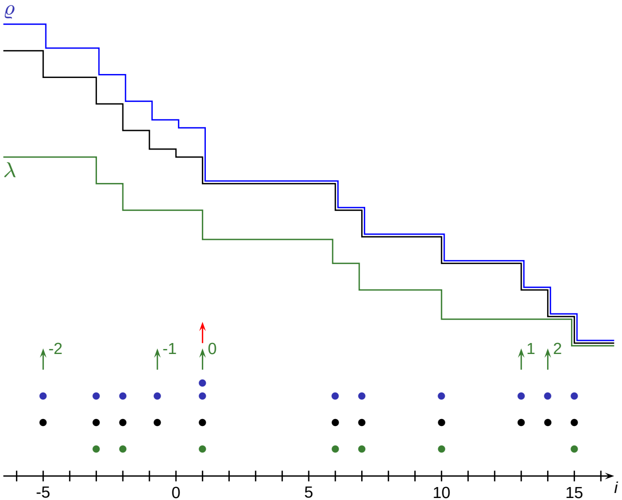

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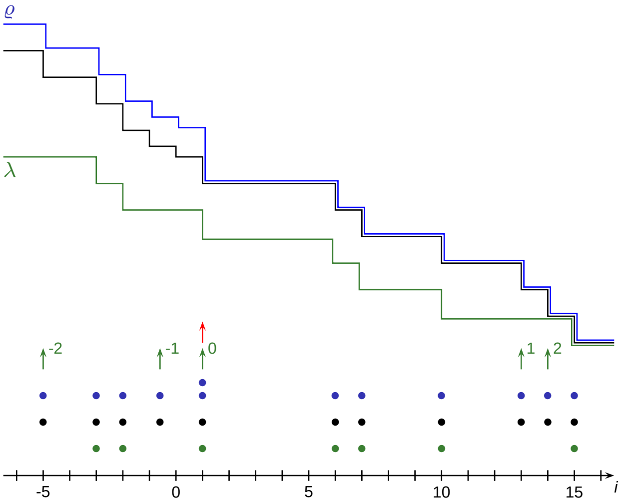

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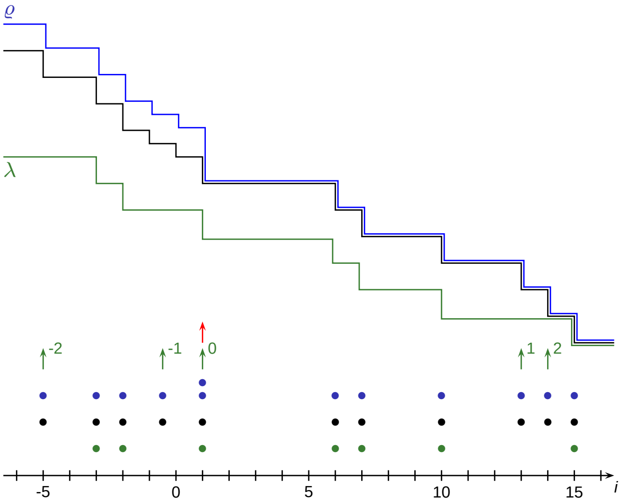

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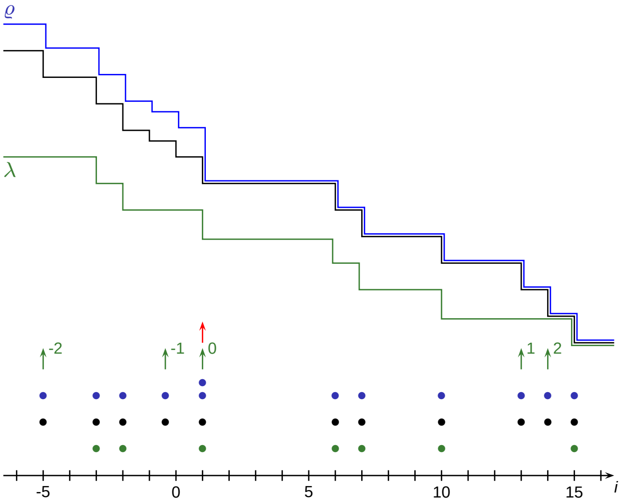

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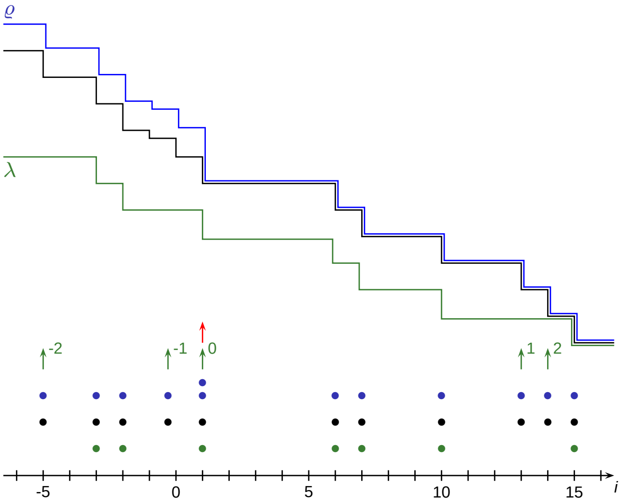

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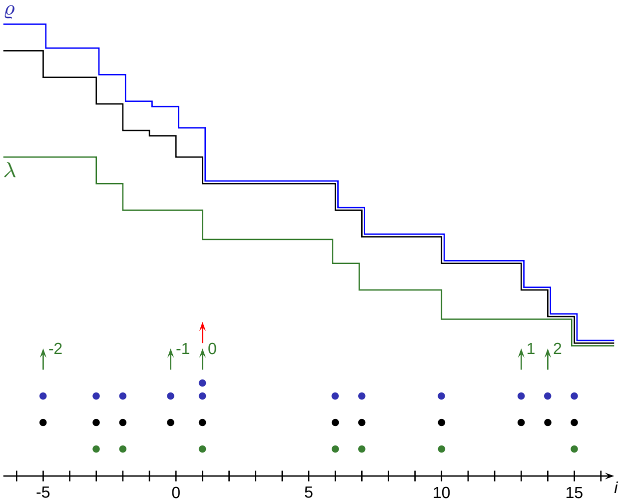

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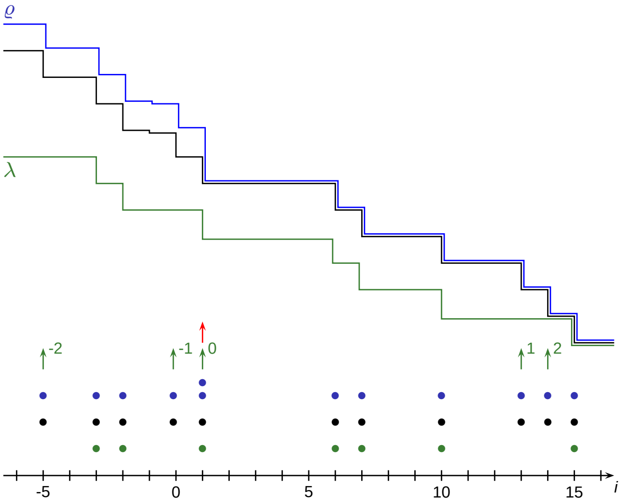

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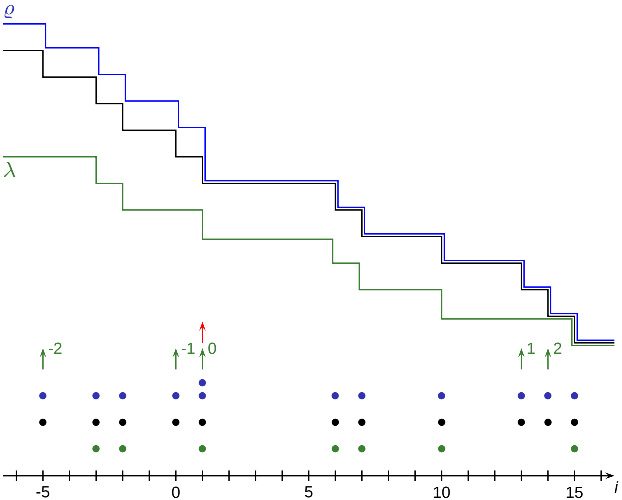

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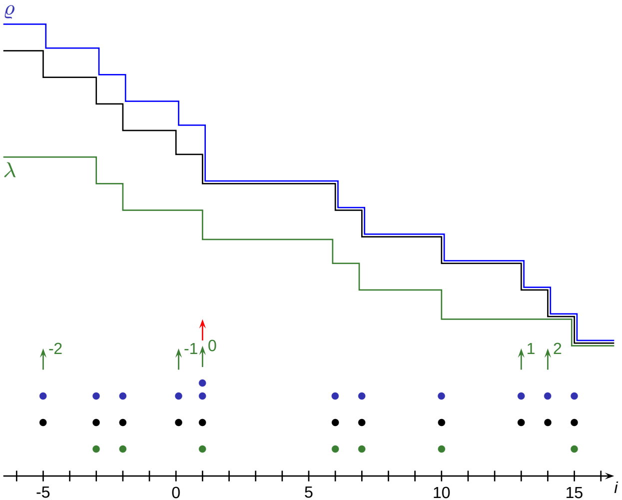

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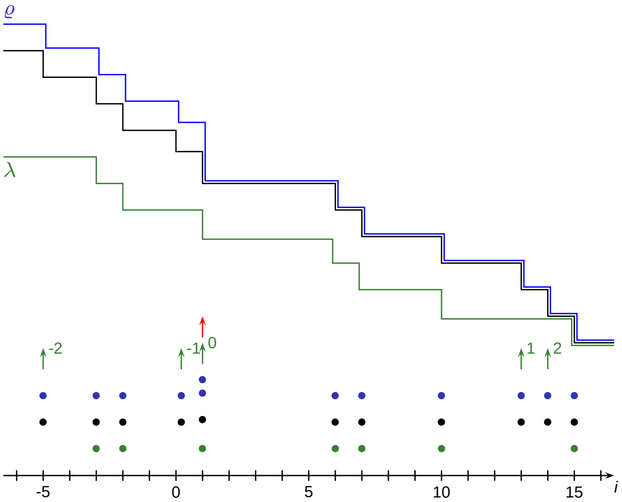

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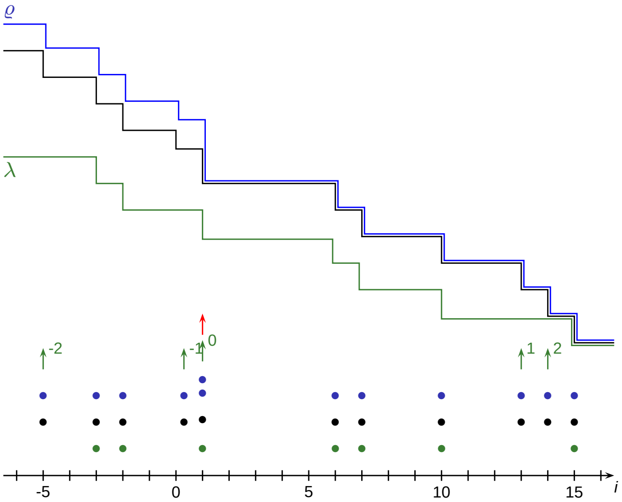

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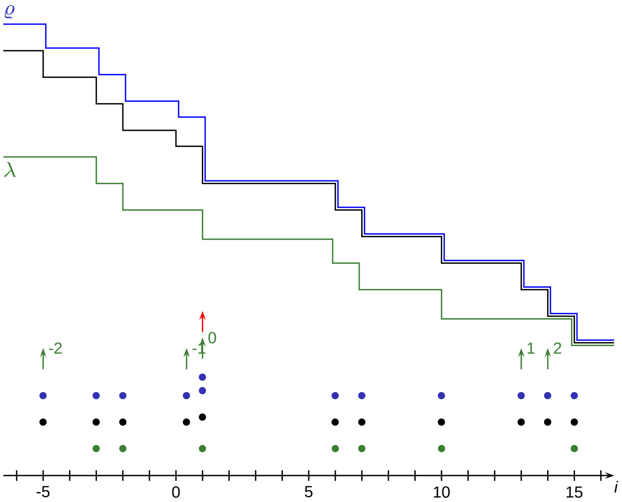

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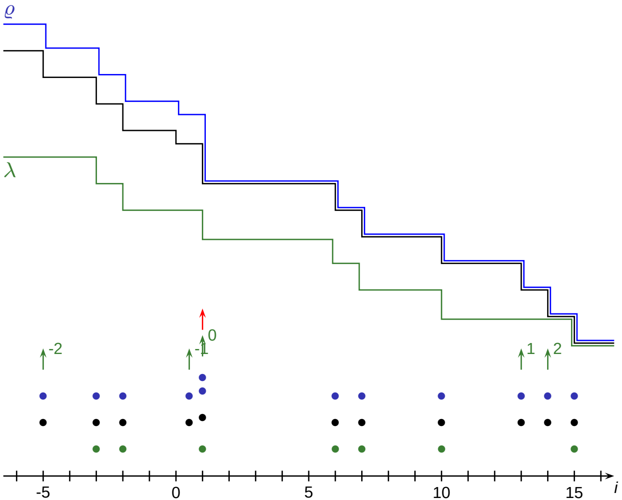

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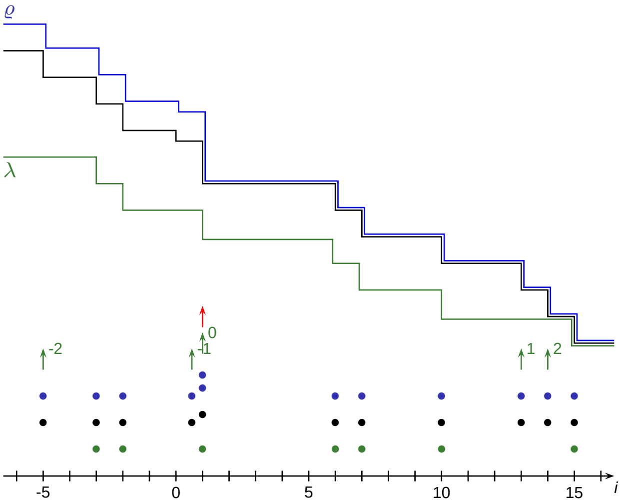

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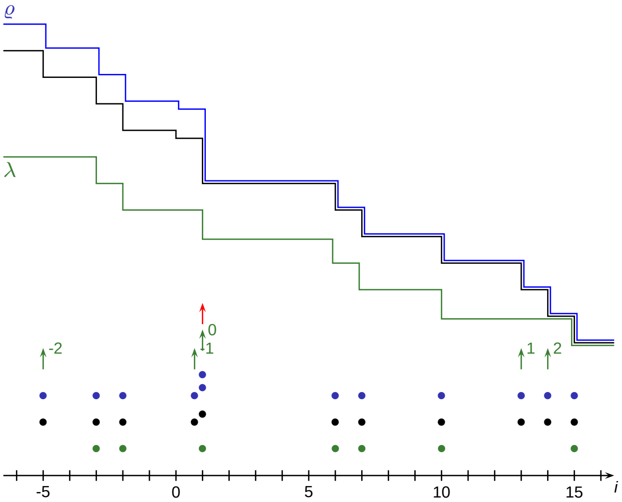

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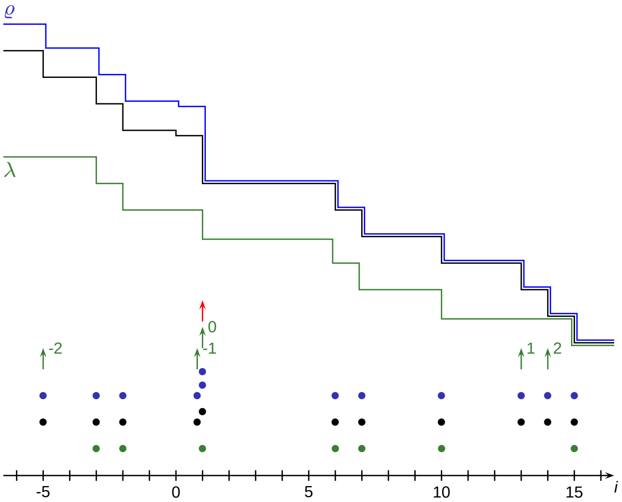

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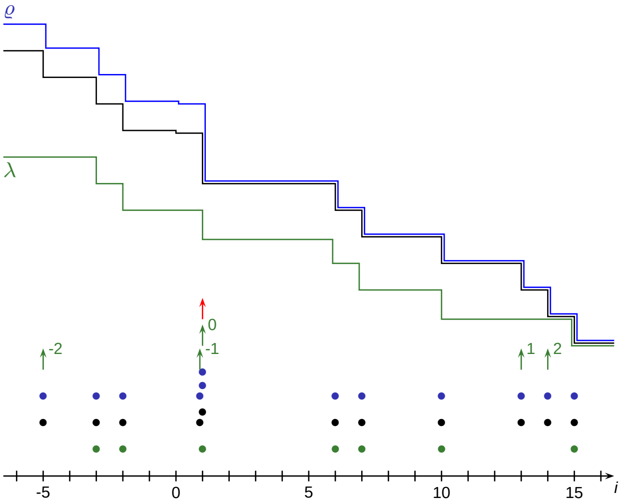

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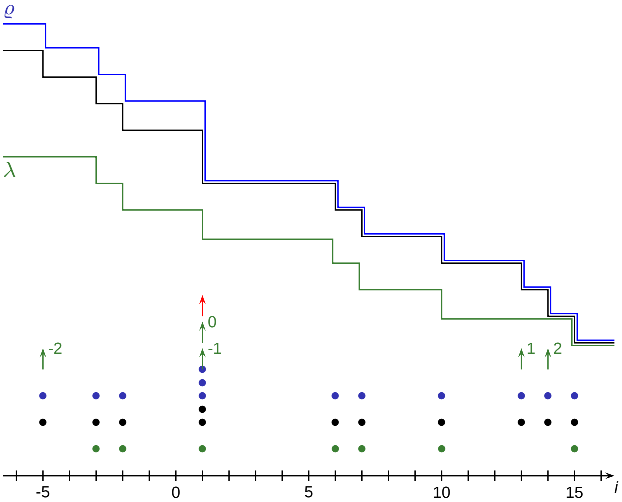

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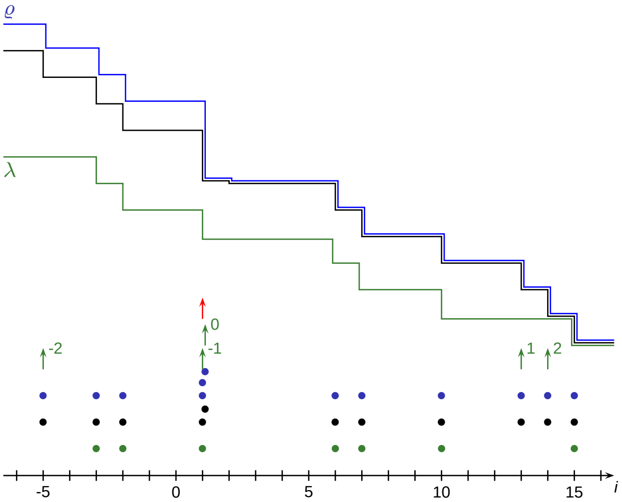

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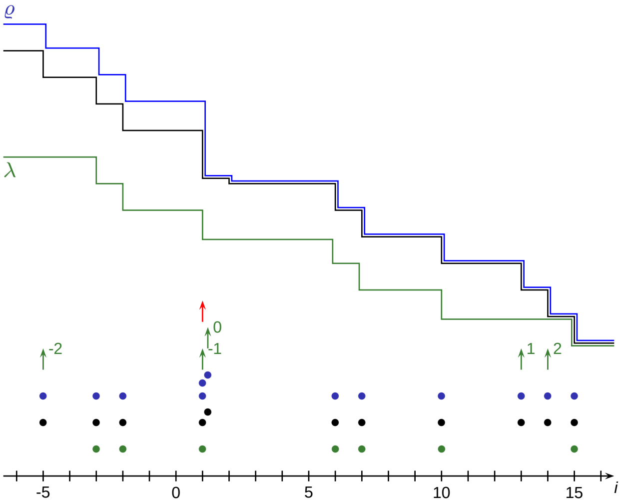

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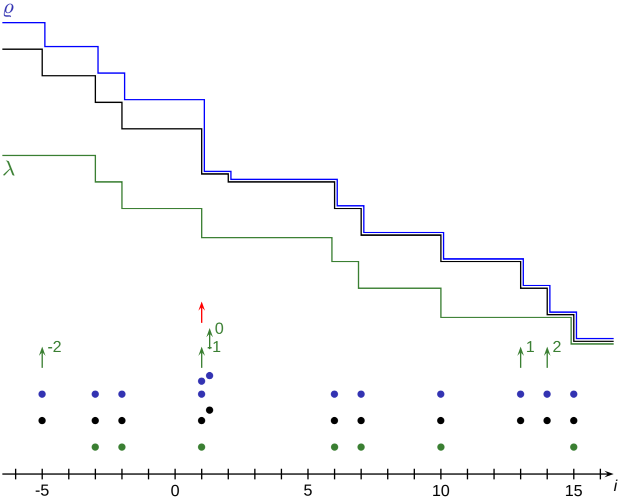

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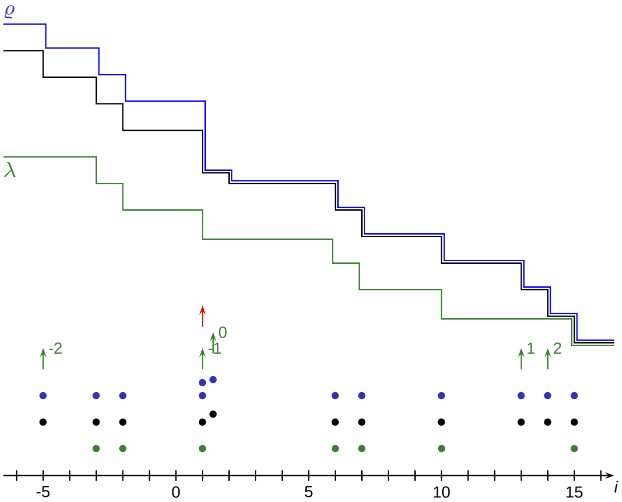

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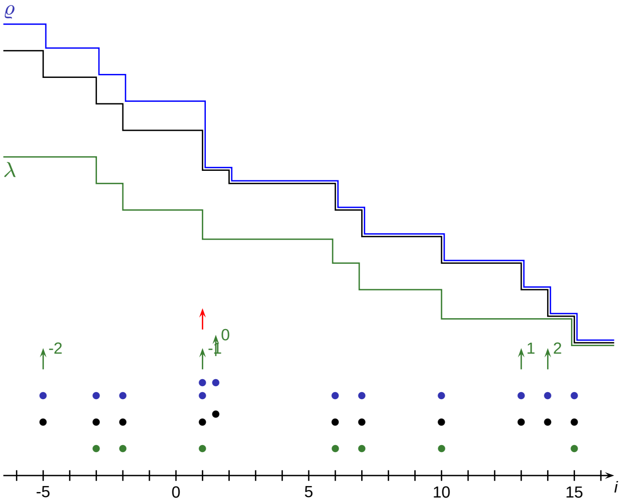

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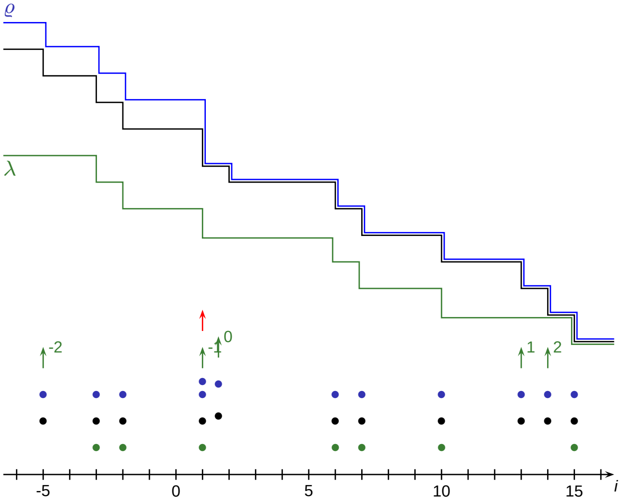

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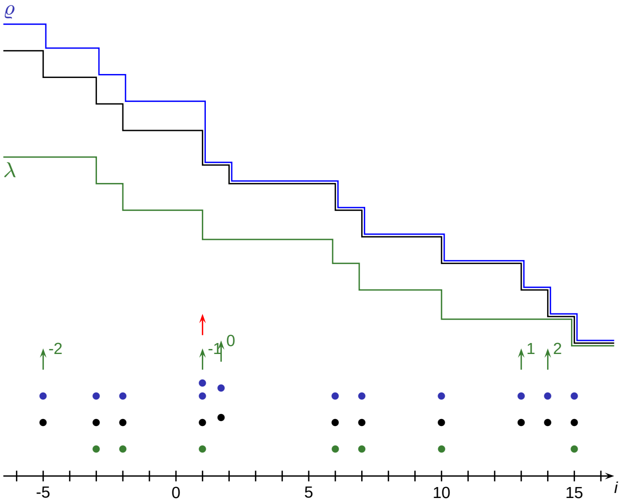

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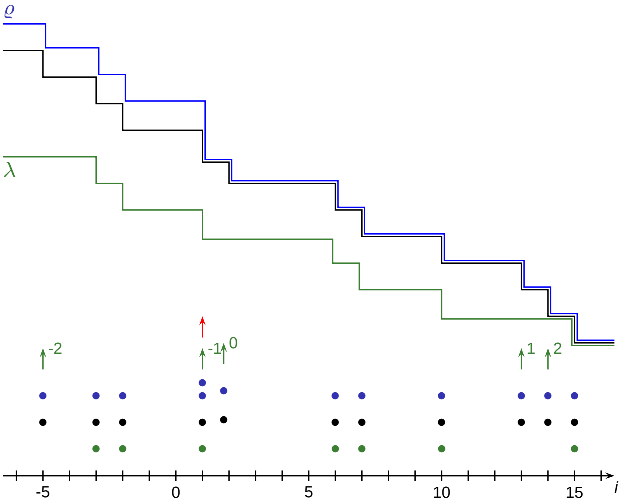

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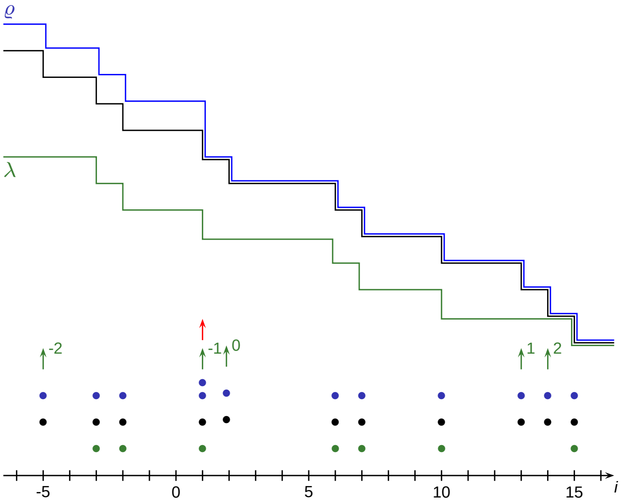

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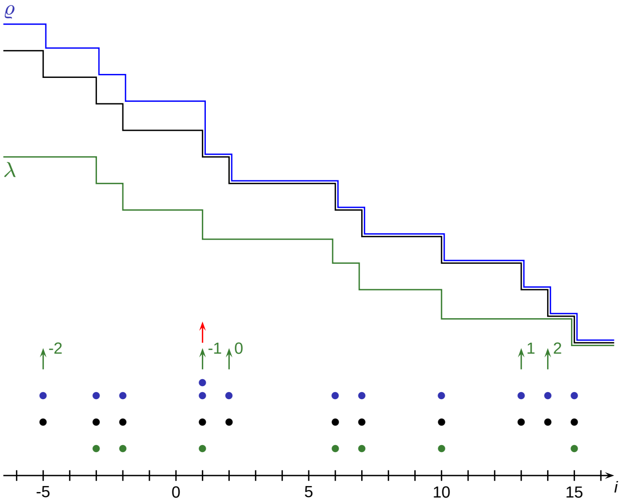

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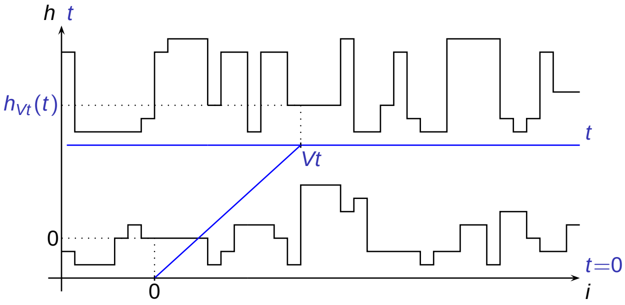

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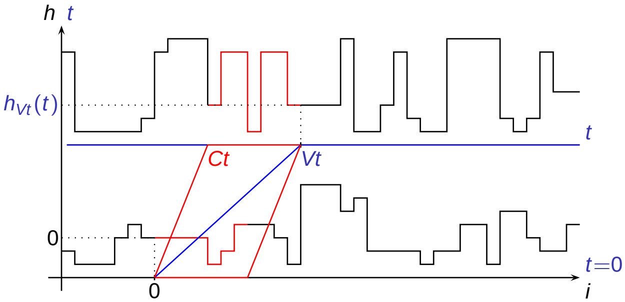

Initial fluctuations are transported along the characteristics on this scale.

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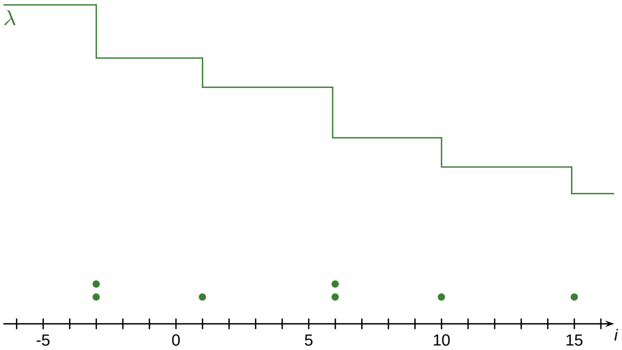
There are limit distribution results for TASEP e.g. by Johansson 2000, Prähofer and Spohn 2001, Ferrari and Spohn 2006. Their methods give limit distributions as well, but are very model-dependent: they rewrite the model as a determinantal process, and perform asymptotic analysis of the determinants.

Proof: many second class particles



Second class particle current: difference in growth.

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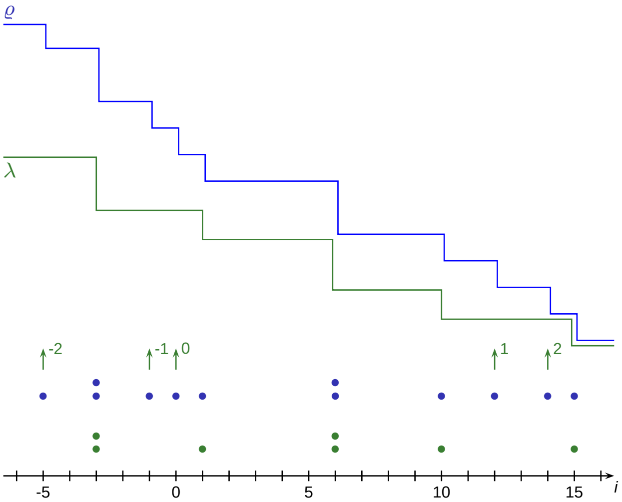

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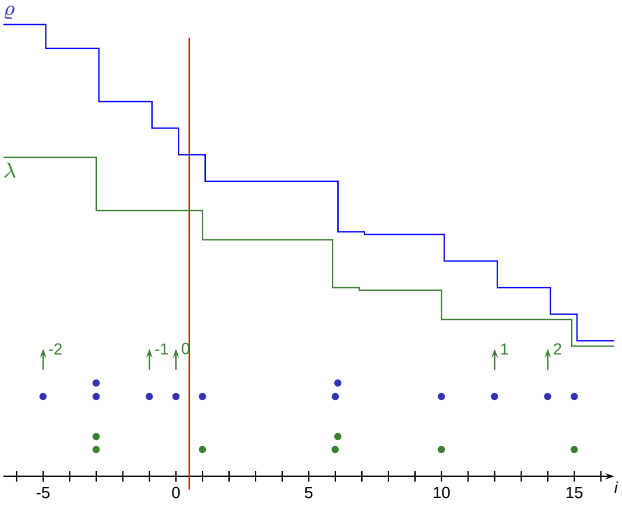

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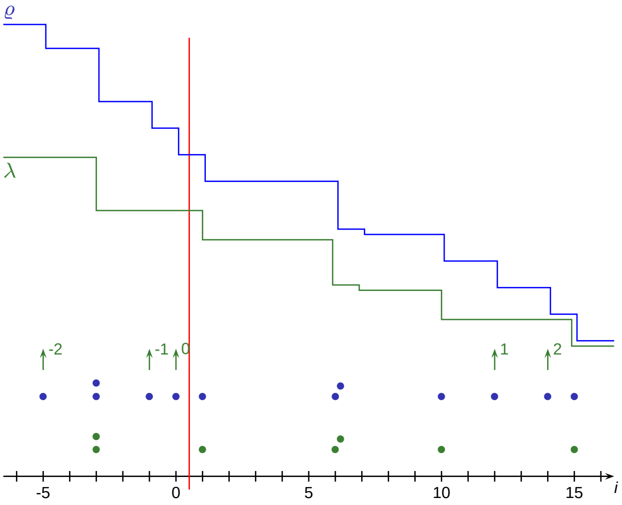

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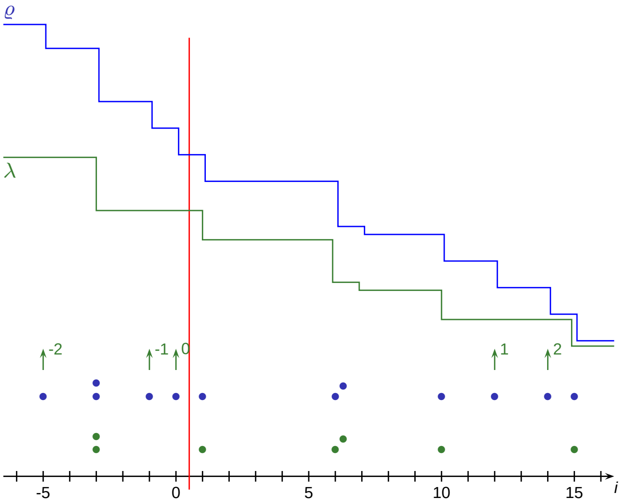

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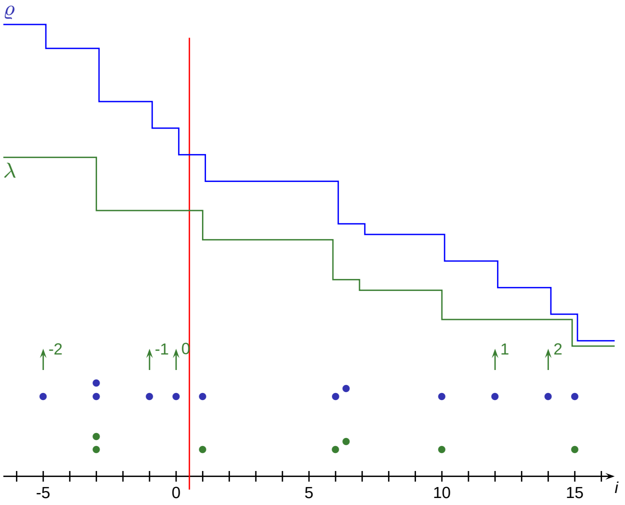

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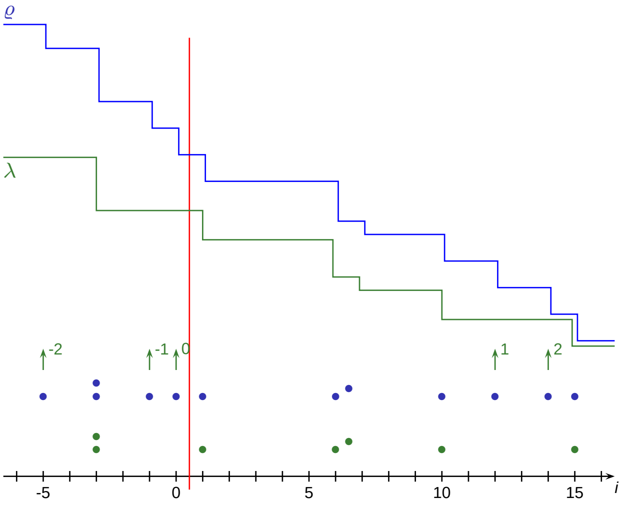

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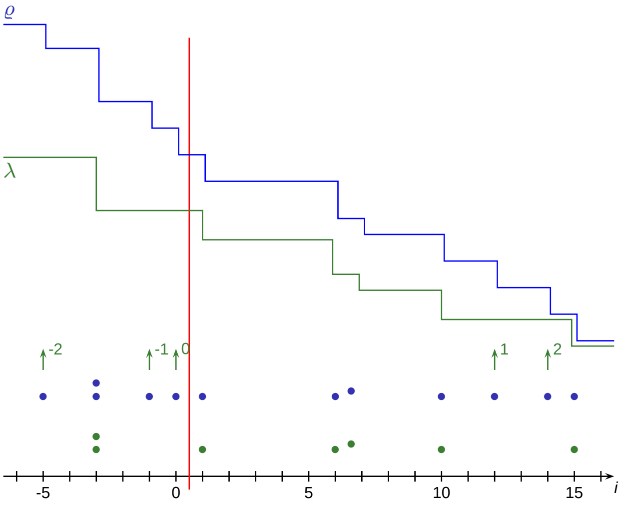

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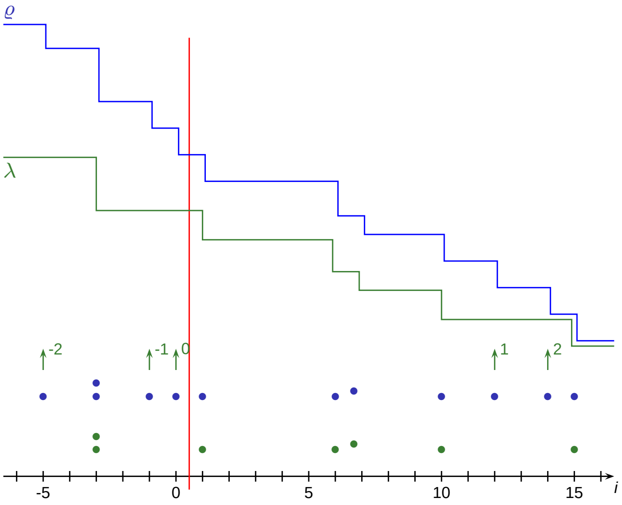

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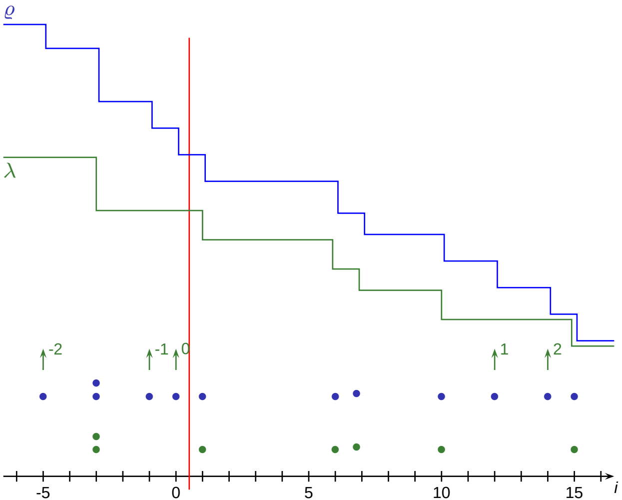

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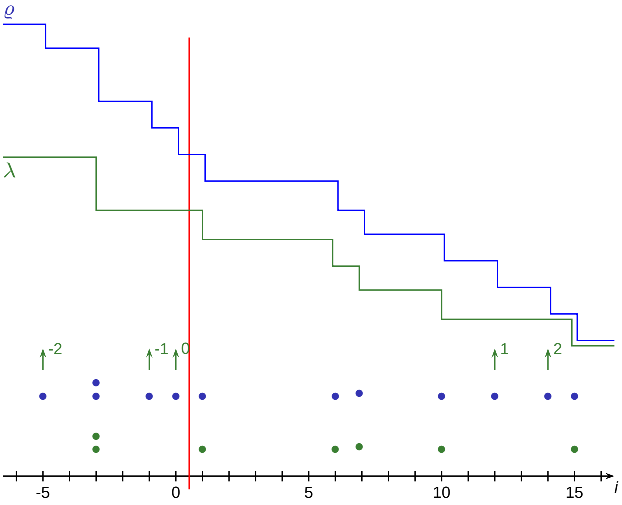

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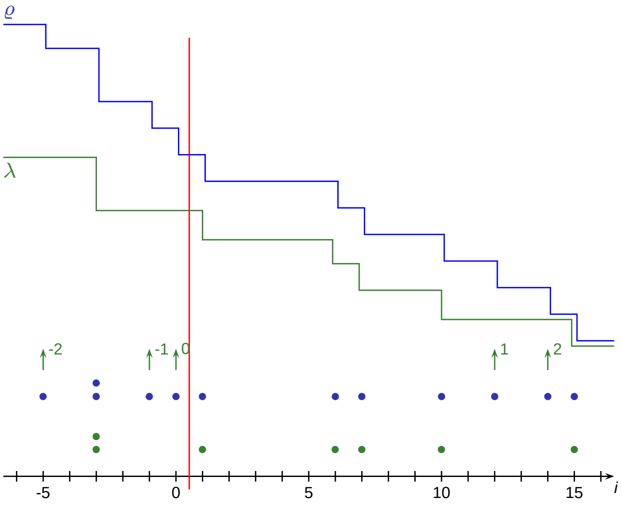

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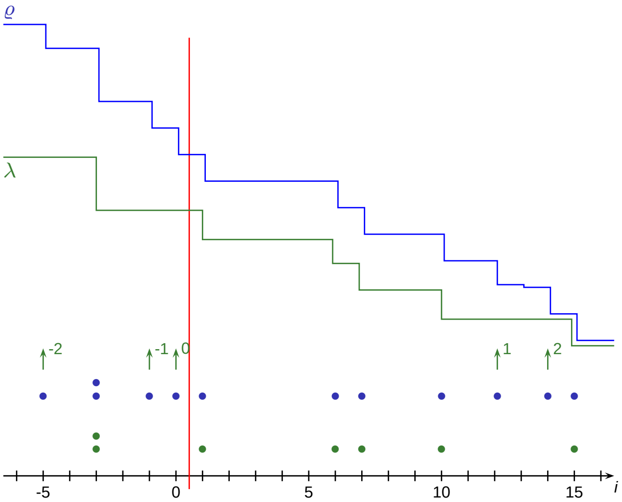

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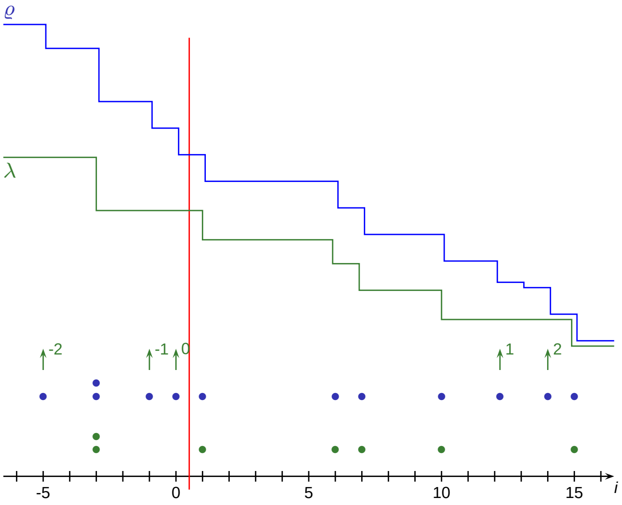

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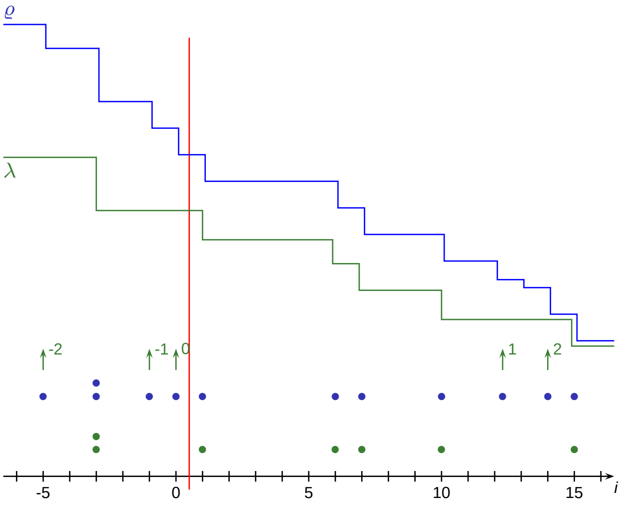

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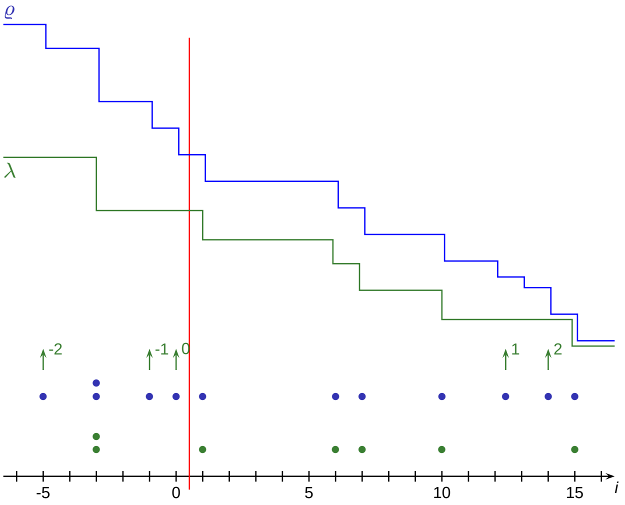

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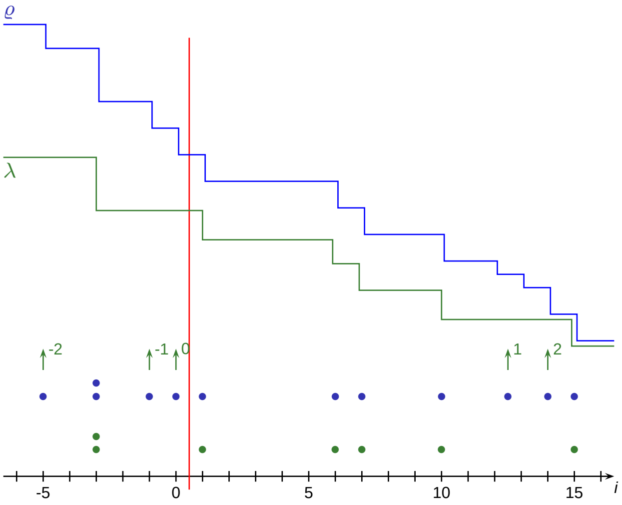

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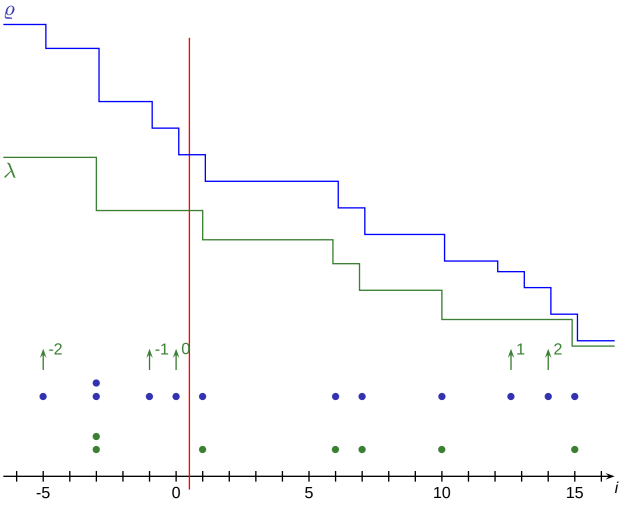

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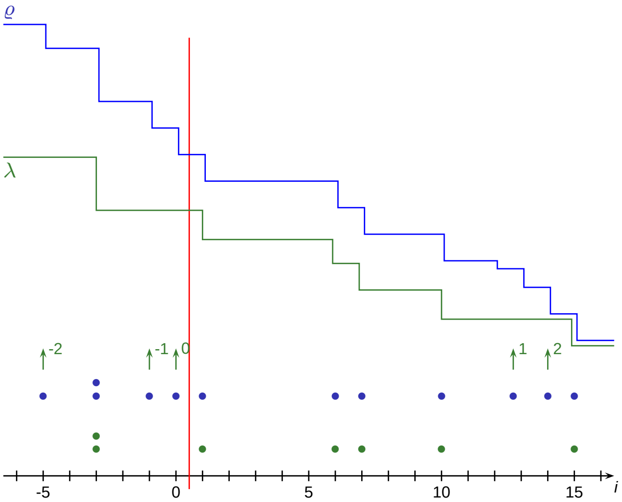

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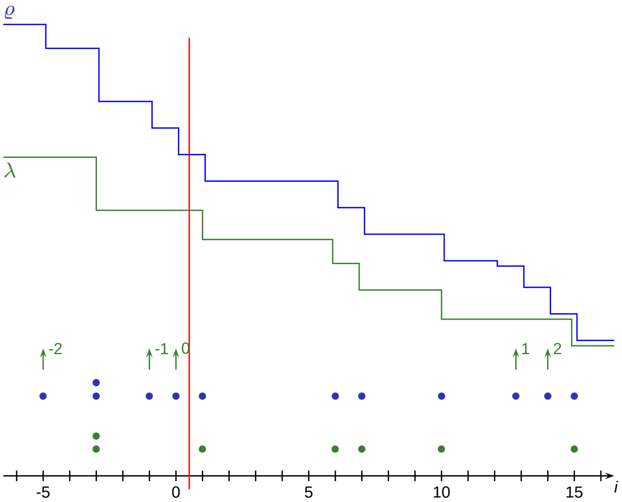

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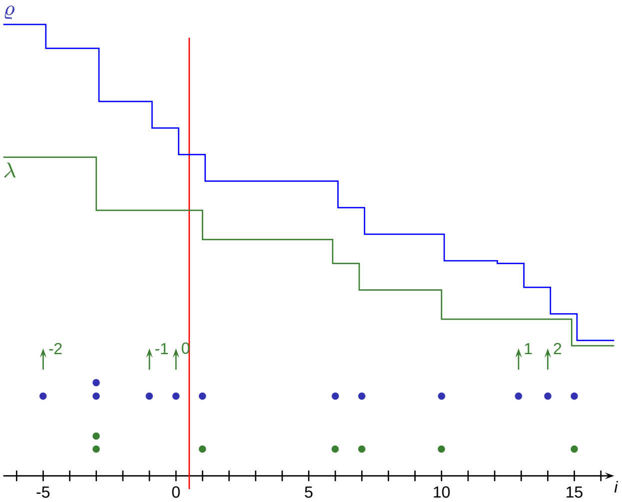

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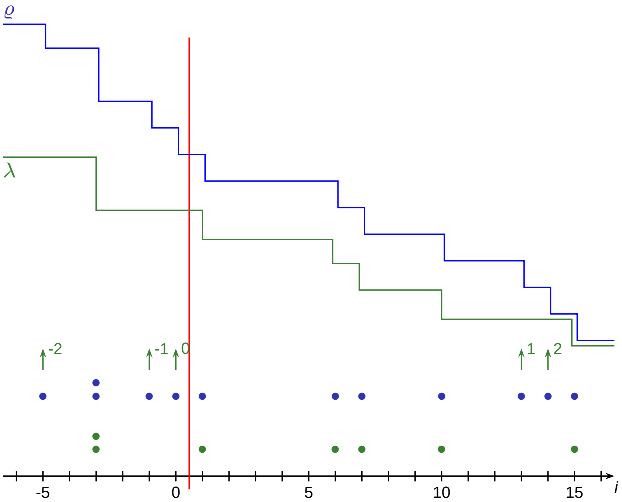

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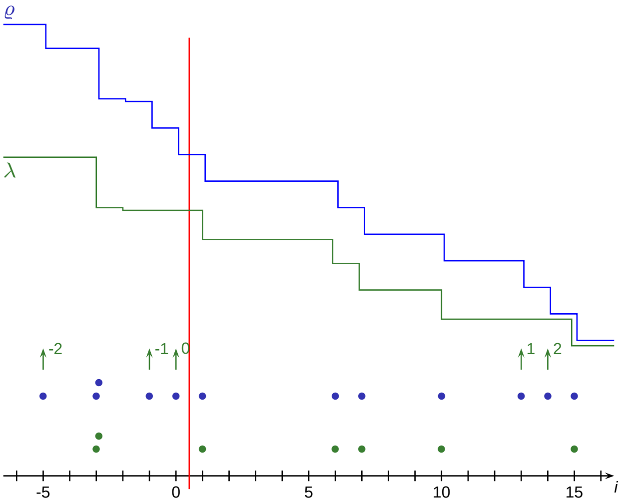

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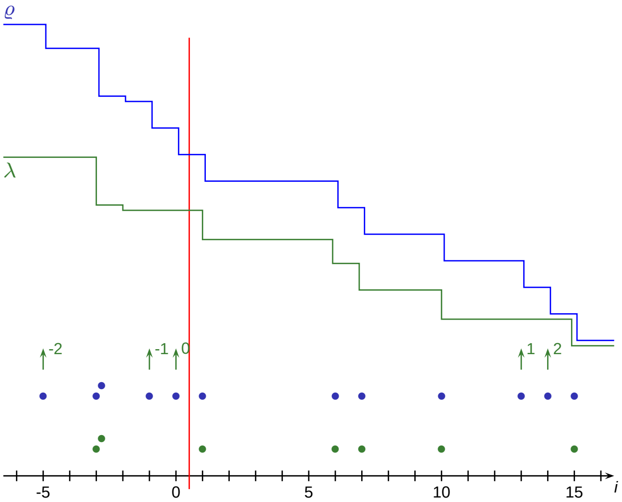

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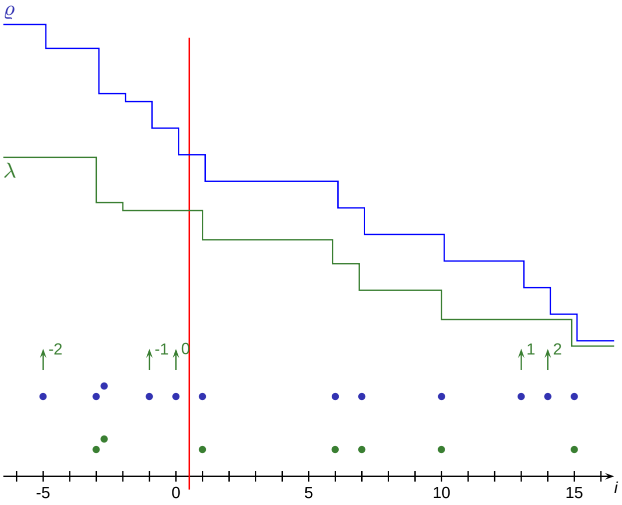

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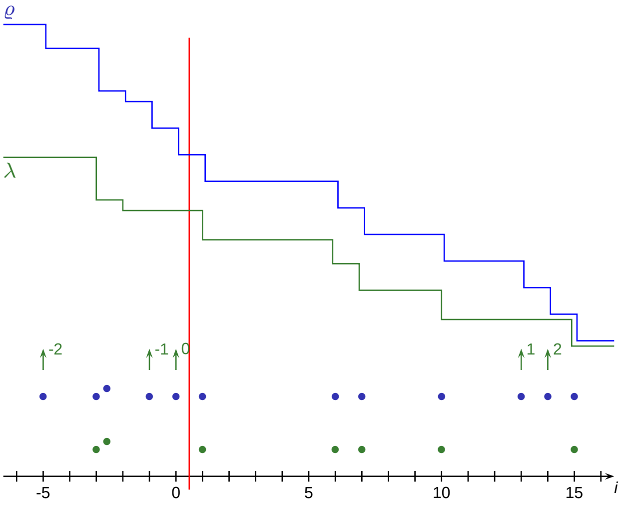

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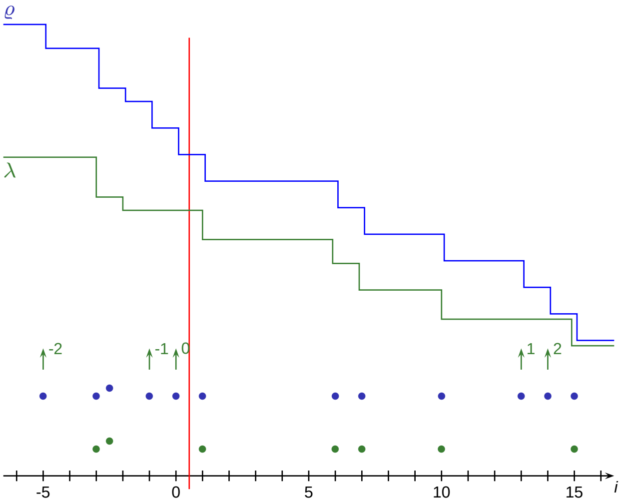

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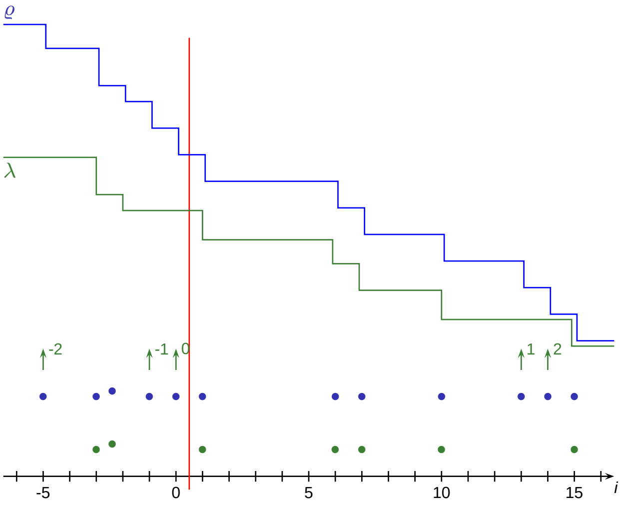

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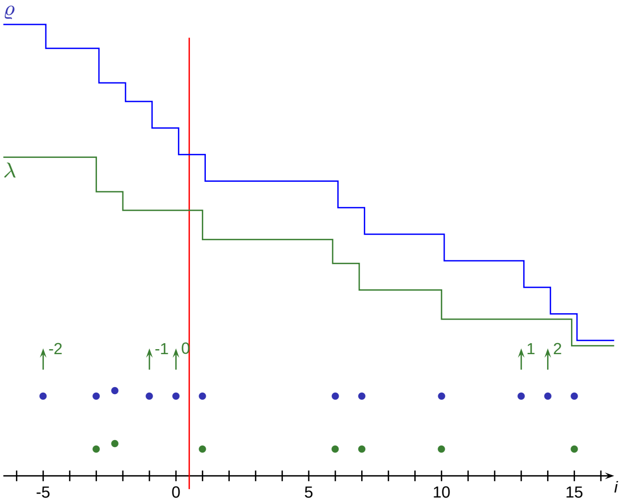

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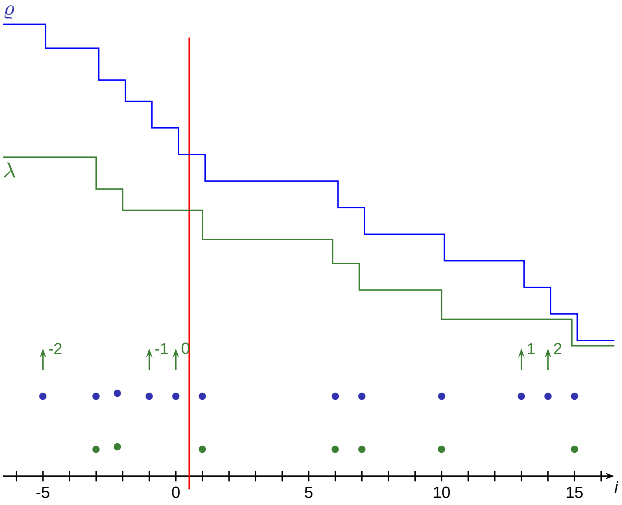

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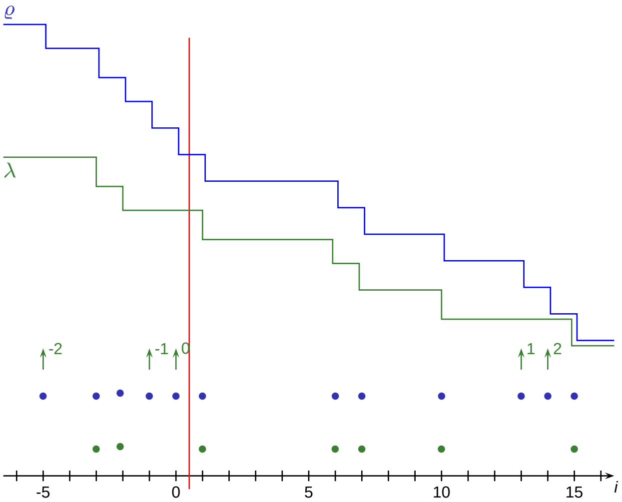

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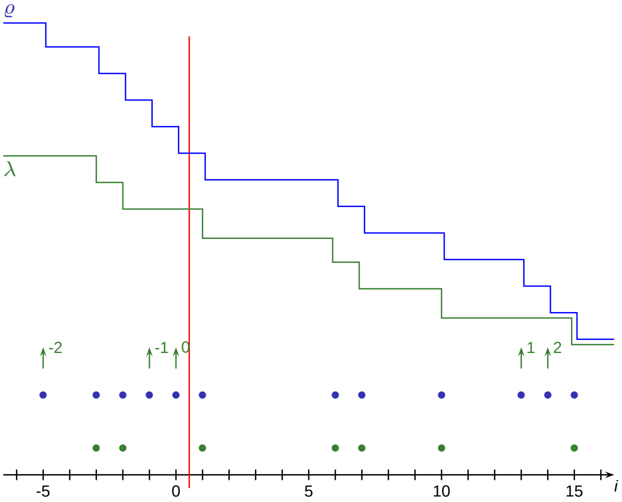

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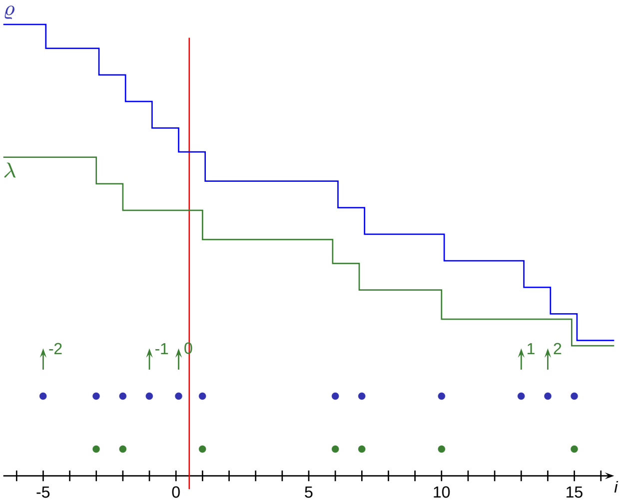

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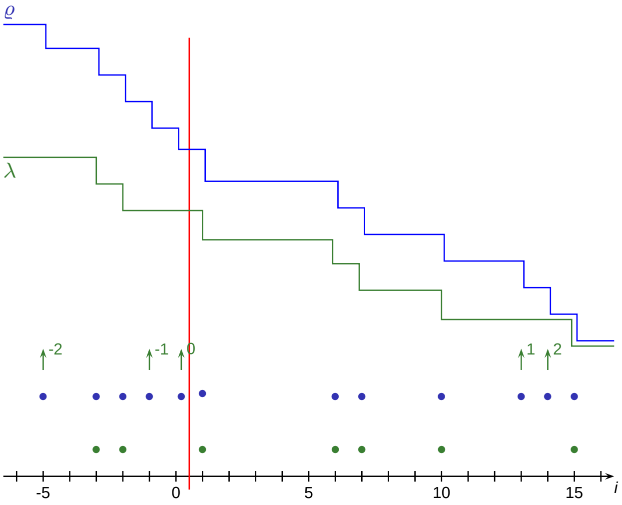

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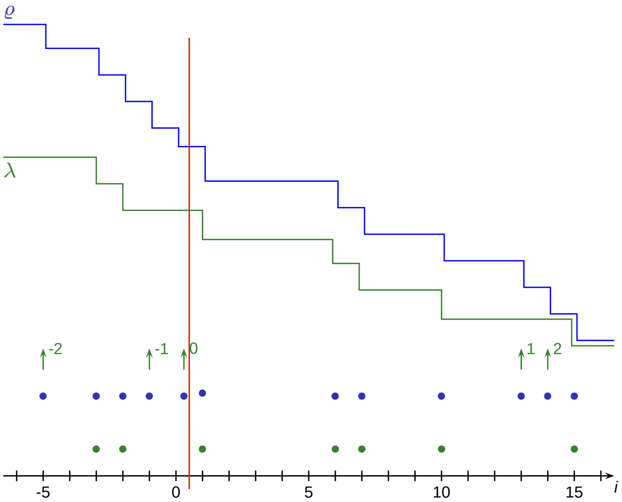

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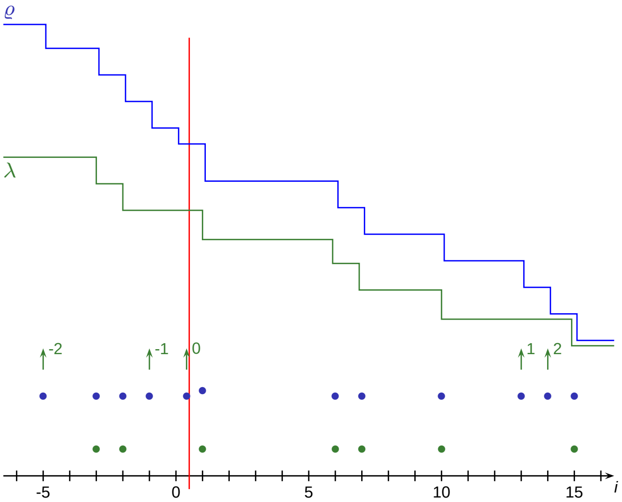

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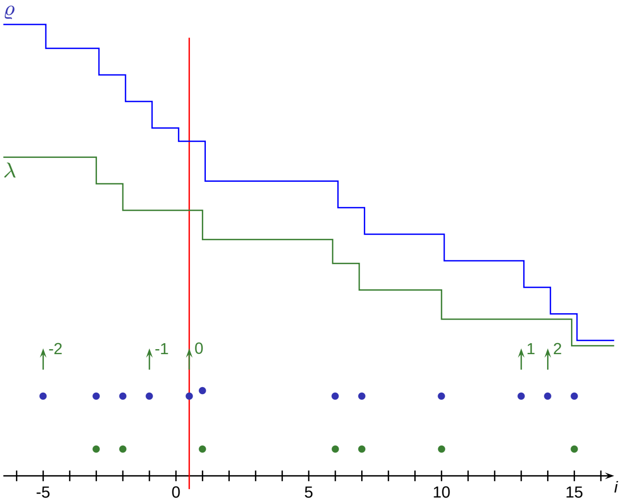

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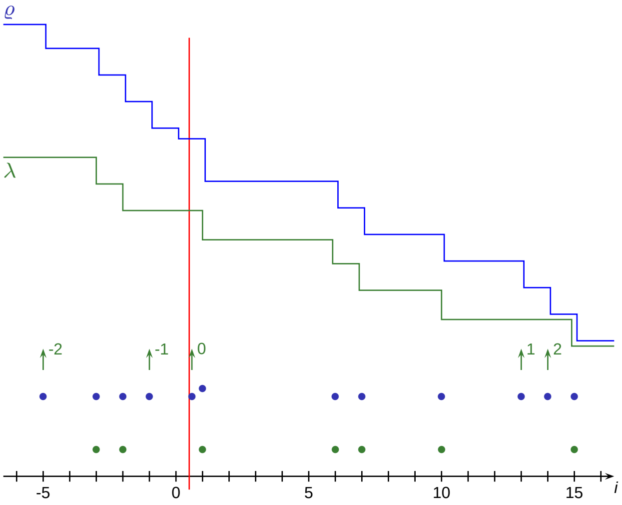

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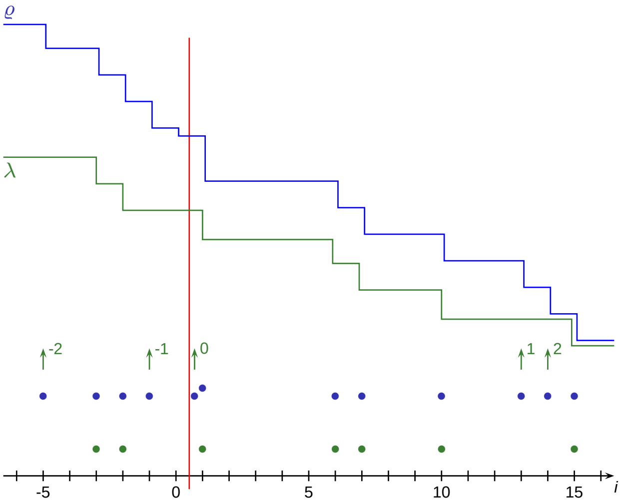

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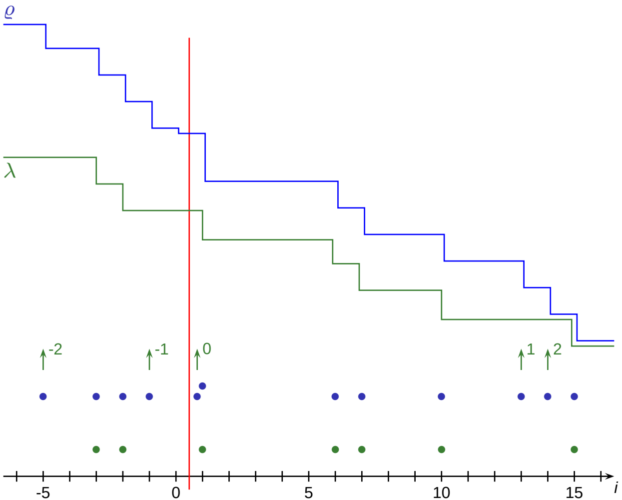

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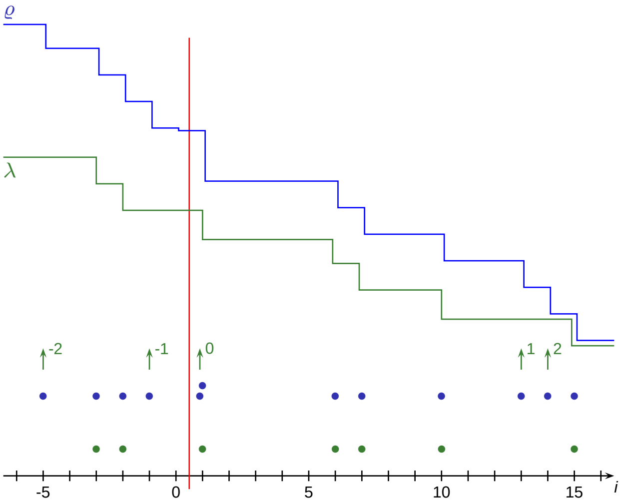

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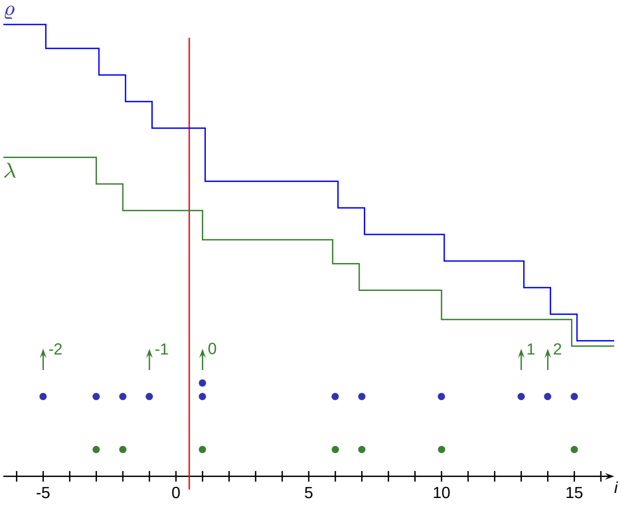

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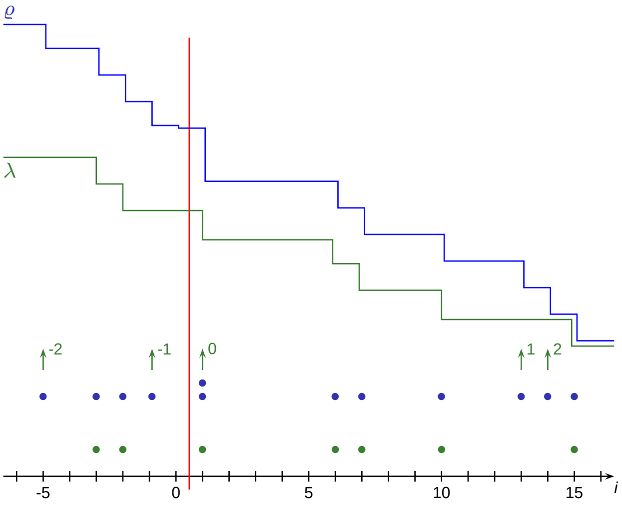

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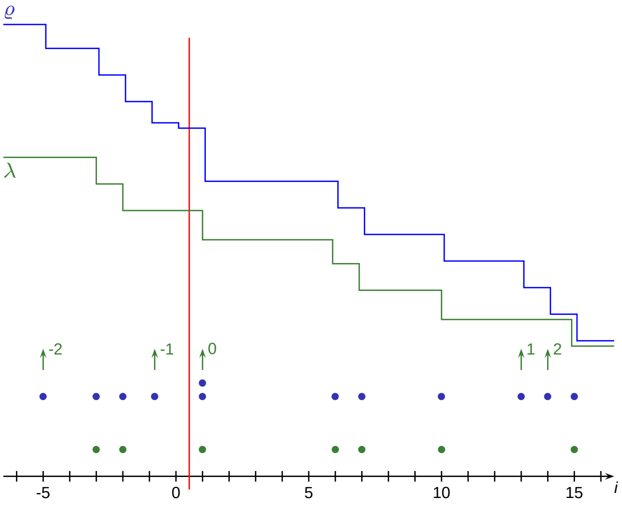

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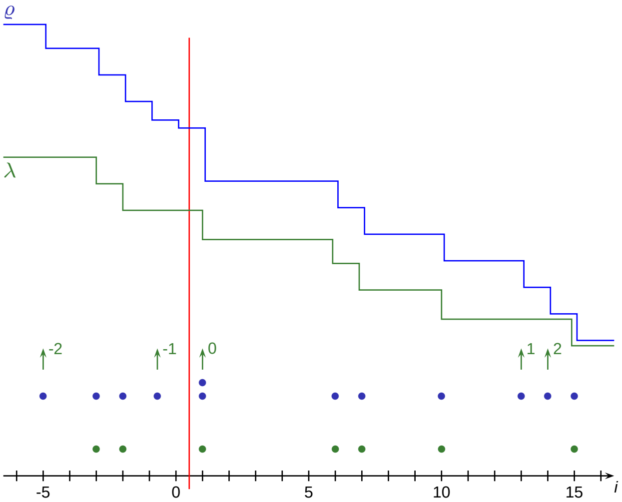

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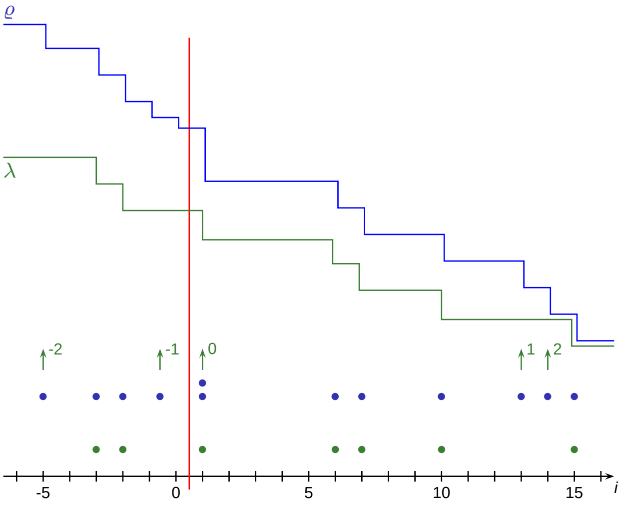

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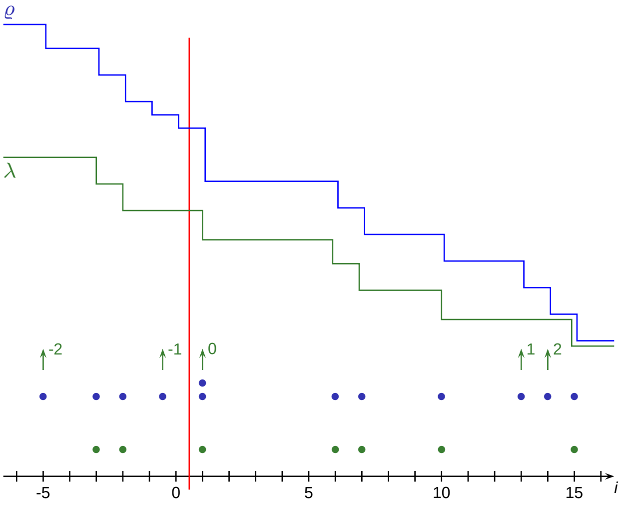

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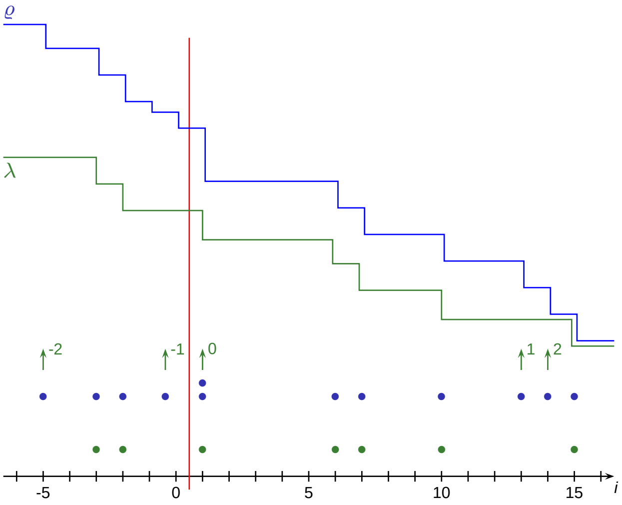

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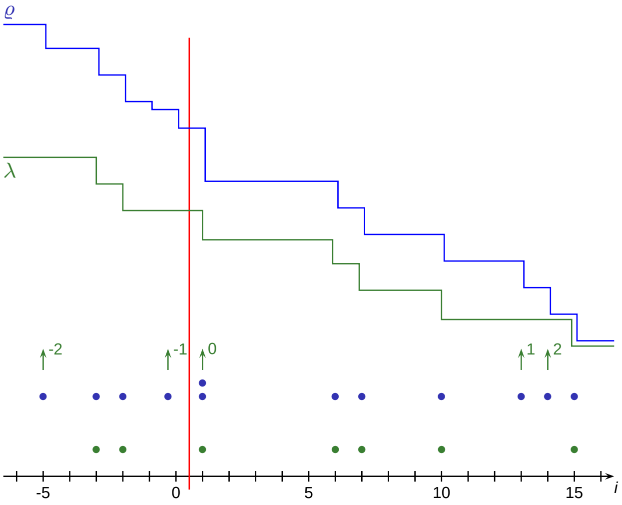

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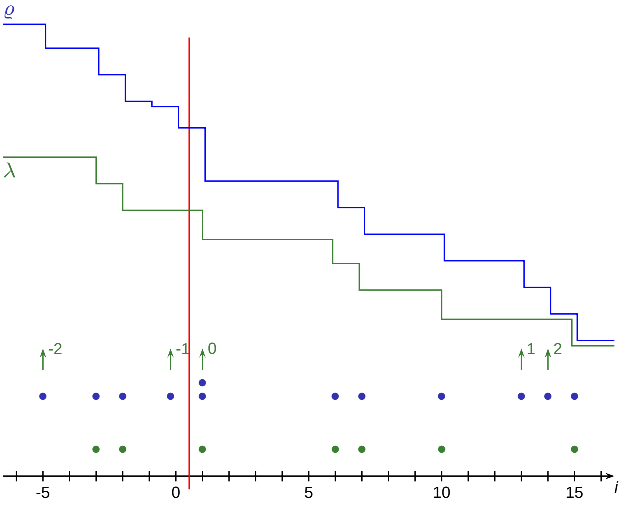

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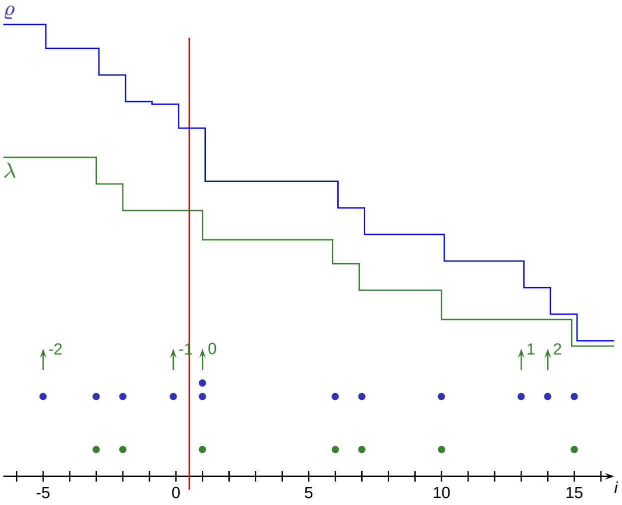

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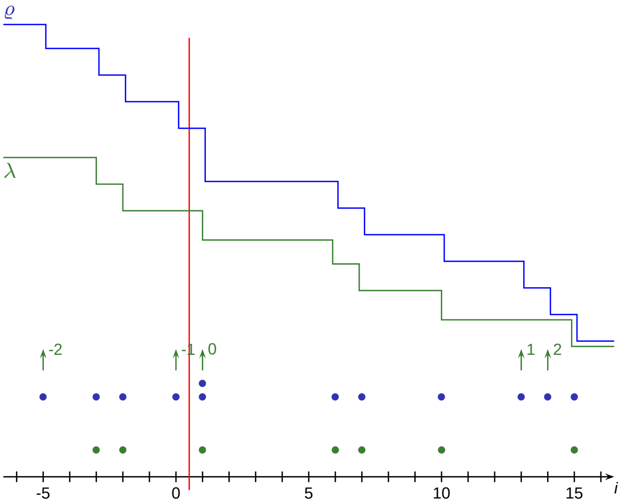

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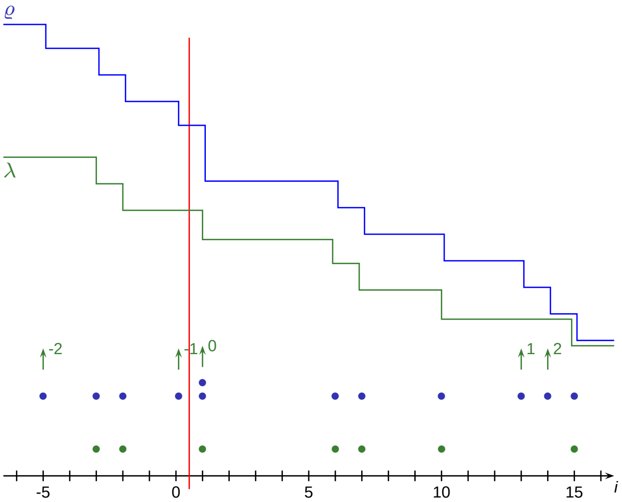

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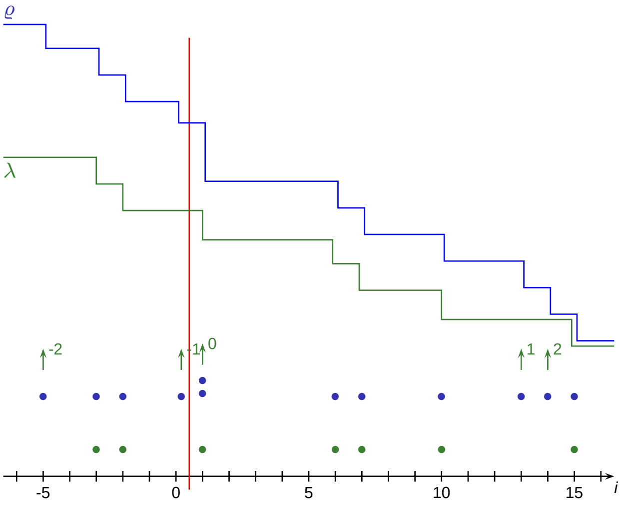

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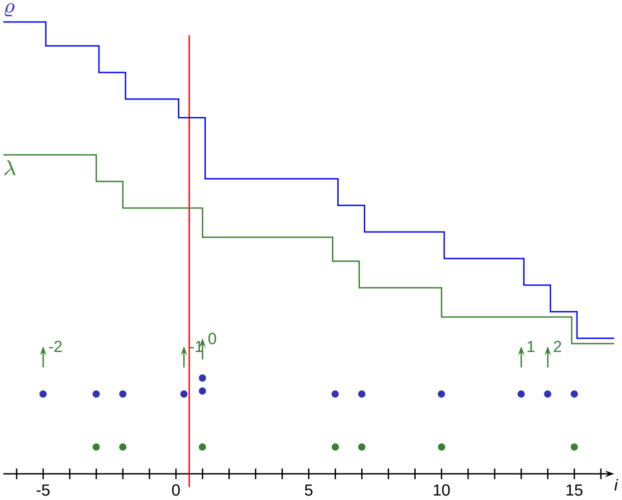

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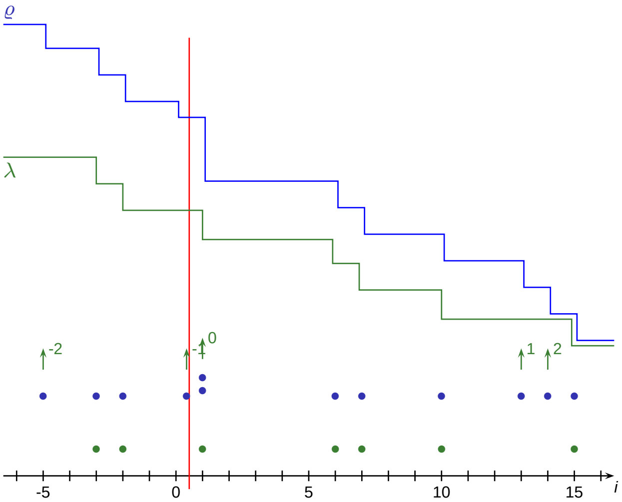

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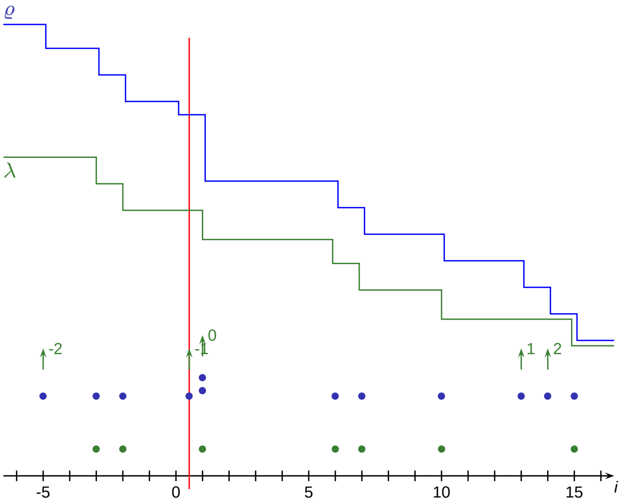

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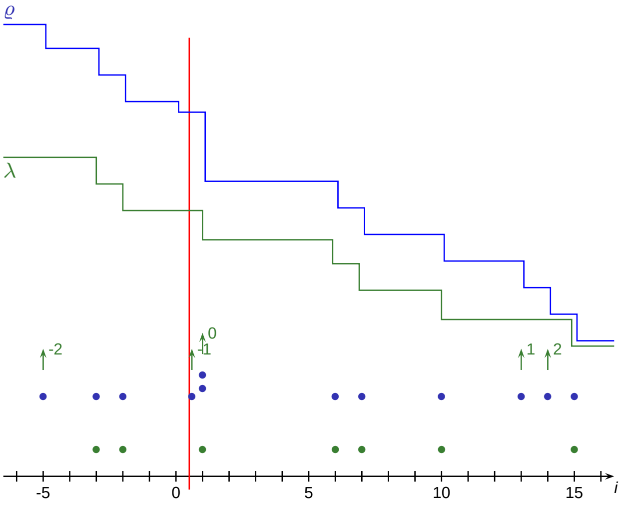

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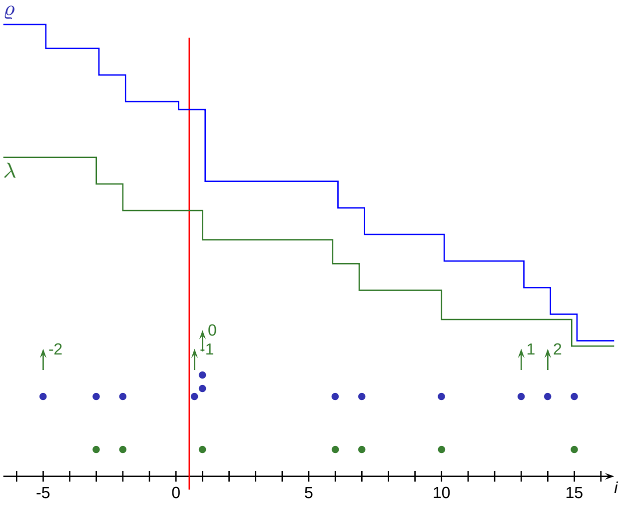

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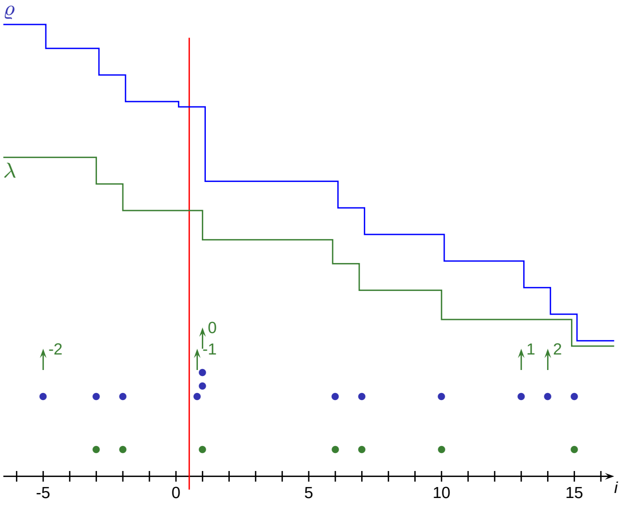

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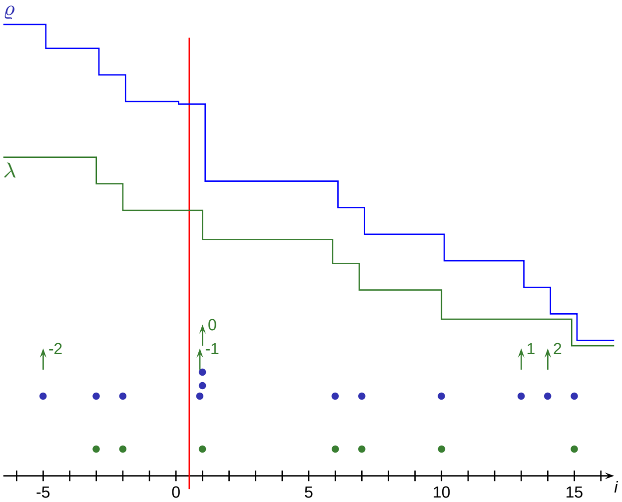

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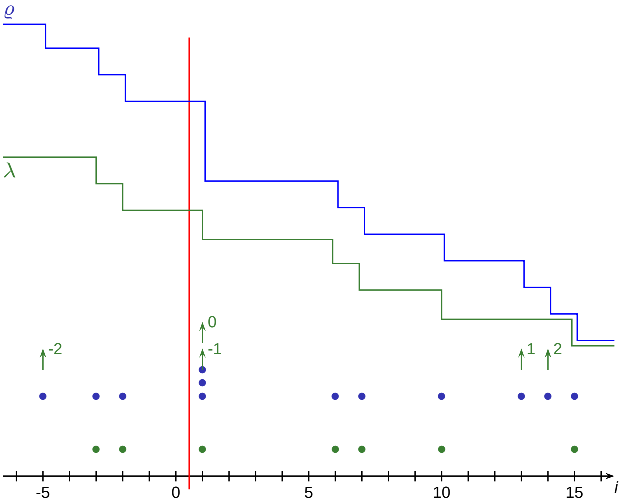

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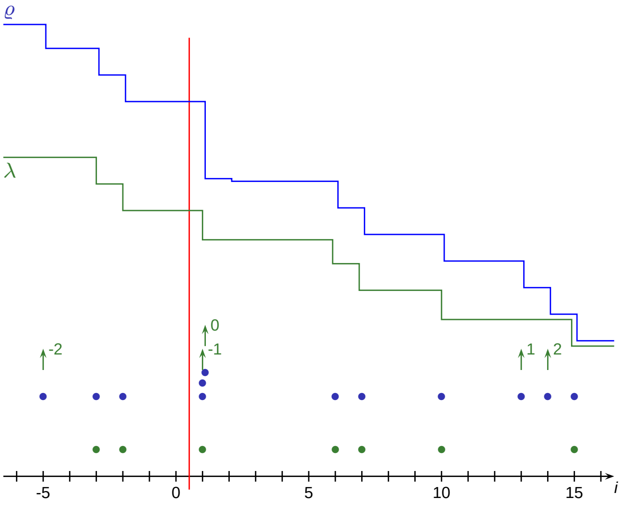

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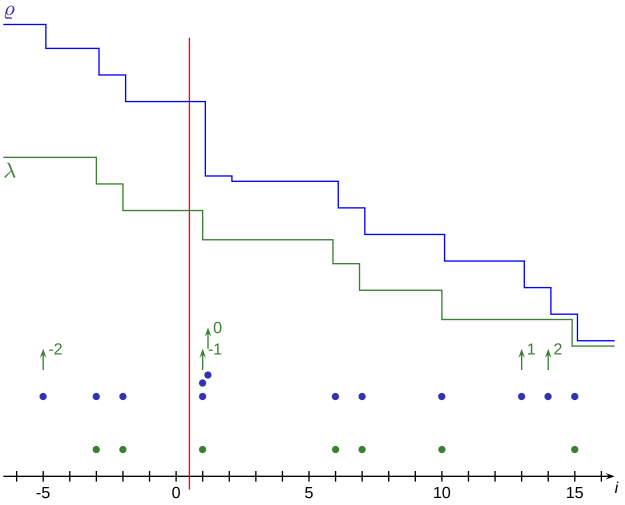

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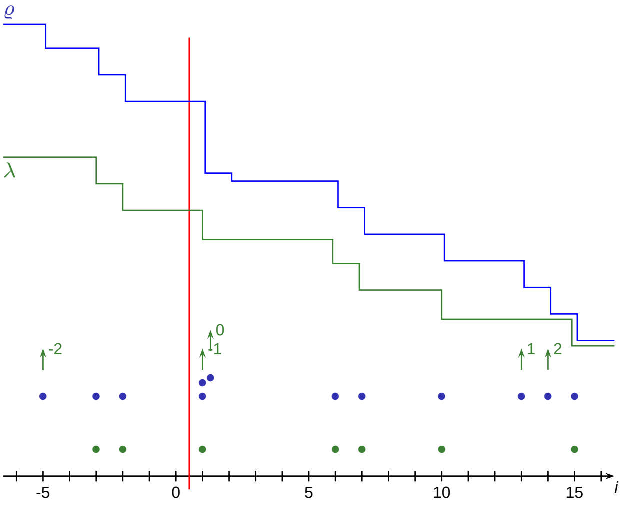

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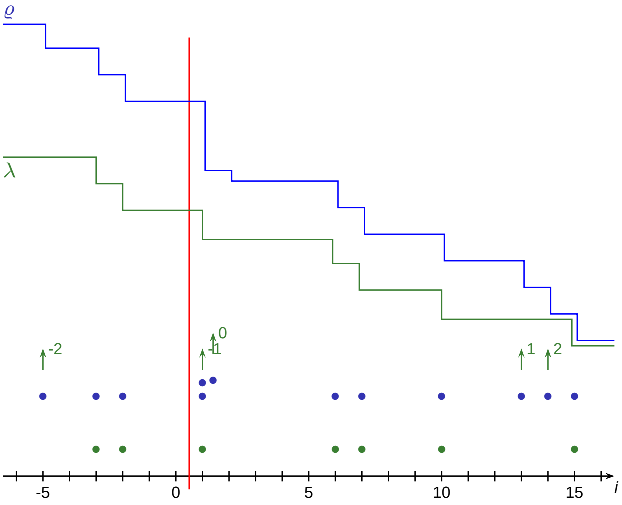

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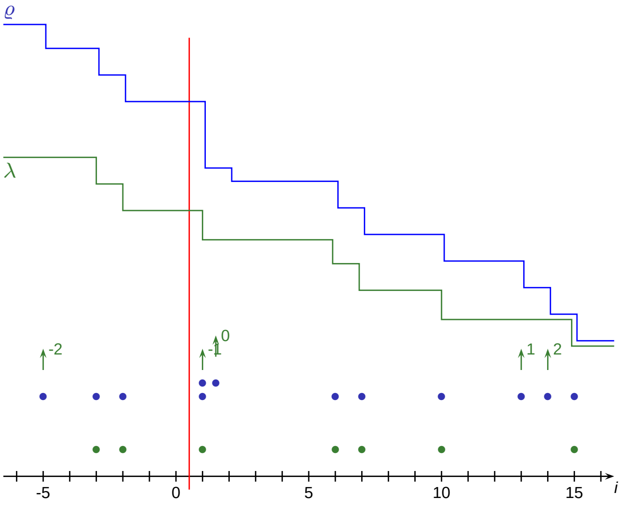

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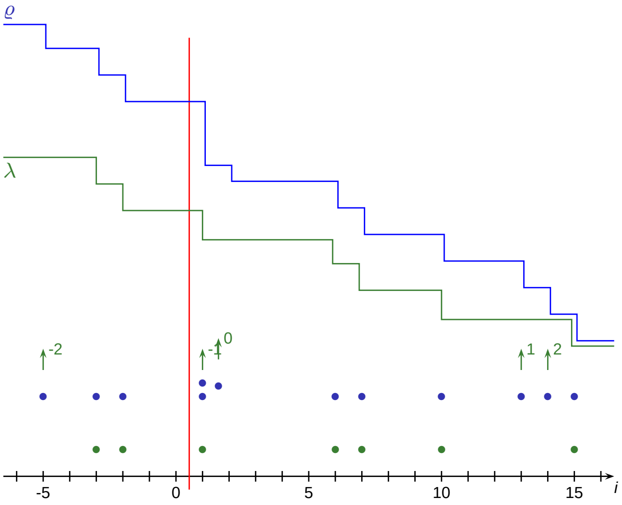

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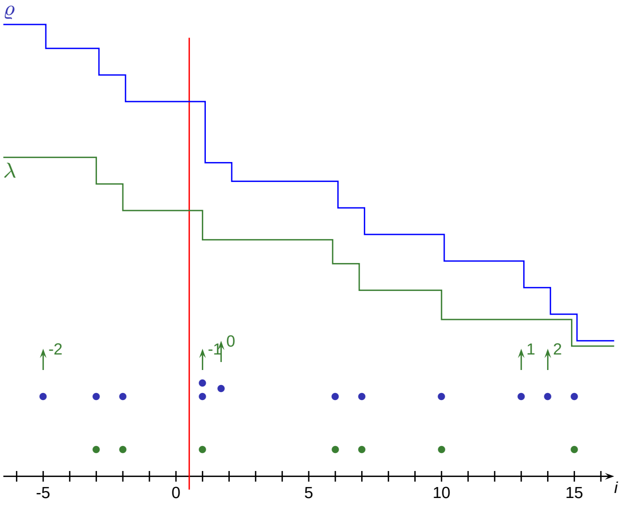

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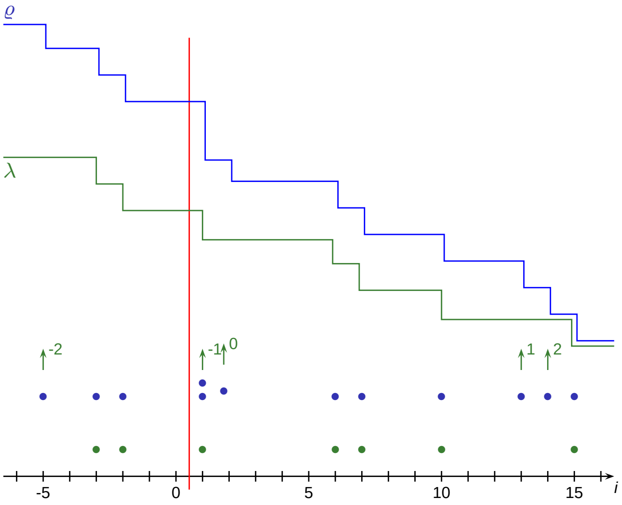

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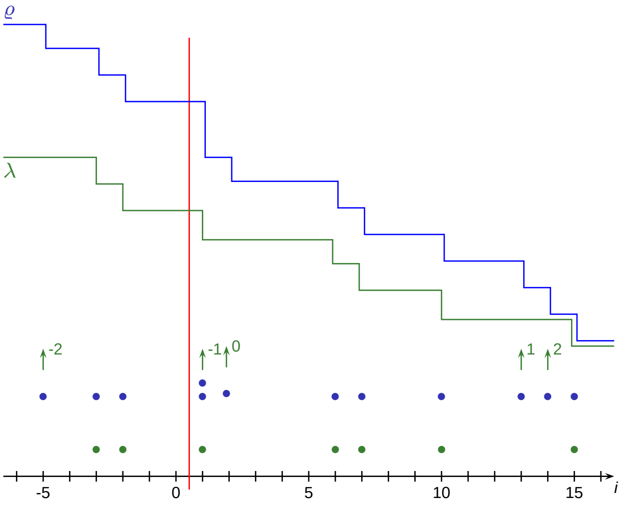

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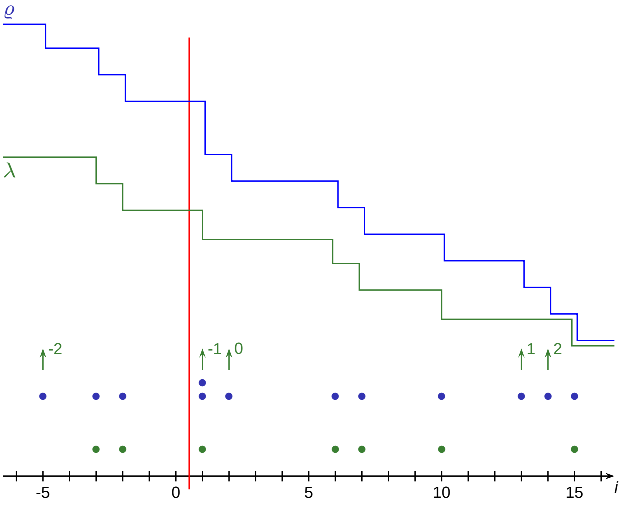

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The computations result in (remember $\mathbf{E}(Q(t)) = Ct$)

$$\mathbf{P}\{Q(t) - Ct \ge u\} \le c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t)).$$

Upper bound

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{Var}(h_{Ct}(t)) = c \cdot \mathbf{E}|Q(t) - C \cdot t|$$

in the whole family of processes.

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Hence proceed with

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$$= c \cdot \frac{t^2}{u^4} \cdot \mathbf{E}|Q(t) - C \cdot t|.$$

Upper bound

With

$$\widetilde{Q}(t) := Q(t) - Ct \qquad \text{and} \qquad E := \mathbf{E}|\widetilde{Q}(t)|,$$

we have (with a similar lower deviation bound)

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Claim: this already implies the $t^{2/3}$ upper bound:

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that is, $E^3 \le c \cdot t^2$.

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that is, $E^3 \leq c \cdot t^2$.

$$\begin{aligned} \mathbf{Var}(h_{Ct}(t)) &\overset{\text{Thm}}{=} \text{const.} \cdot \mathbf{E}|Q(t) - Ct| \\ &= \text{const.} \cdot E \leq c \cdot t^{2/3}. \end{aligned}$$

■

Microscopic convexity/concavity

Model	$H(\varrho)$ is	Micro c.?	$t^{2/3}$ law

Microscopic convexity/concavity

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Microscopic convexity/concavity

Model	$H(\varrho)$ is	Micro c.?	$t^{2/3}$ law
TASEP	concave	$Q(t) \leq X(t)$	proved (B.-S.)

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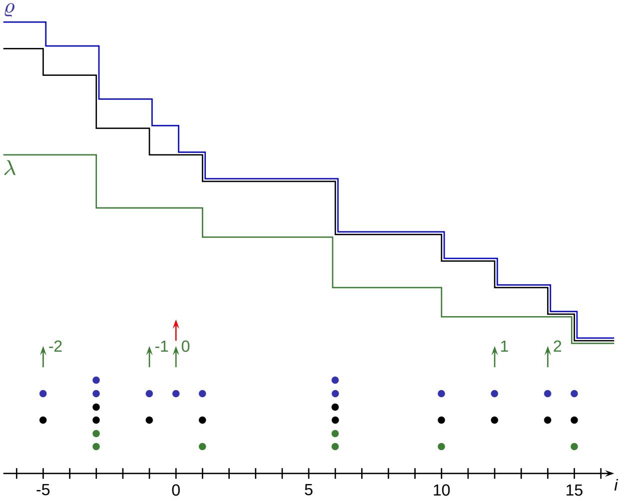

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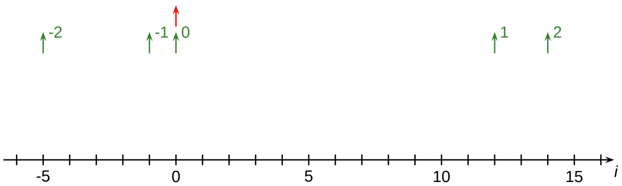

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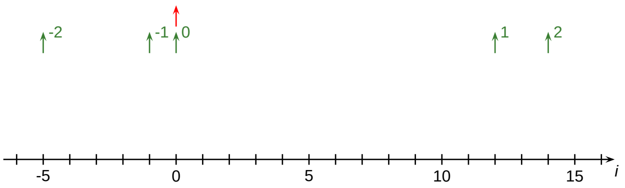

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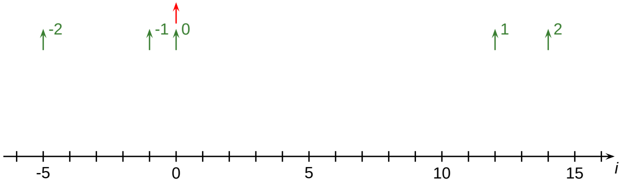

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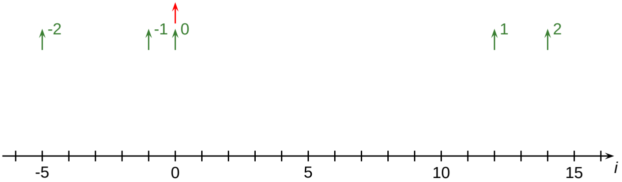

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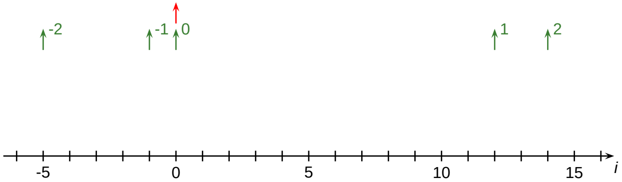

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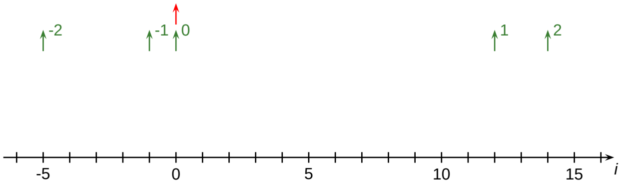

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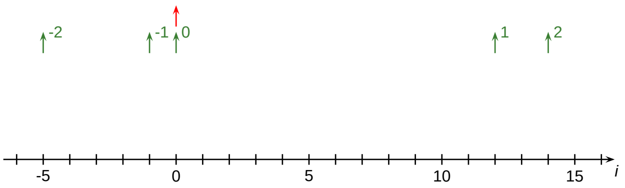

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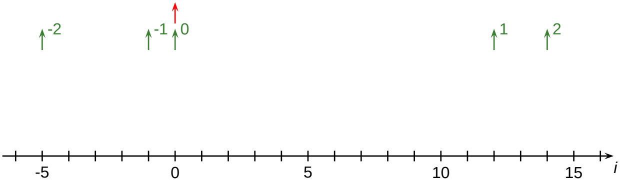

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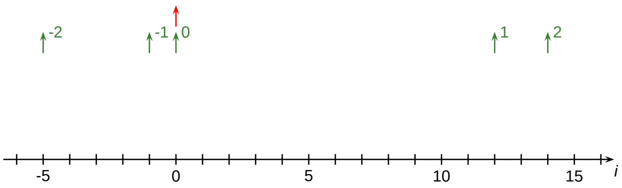

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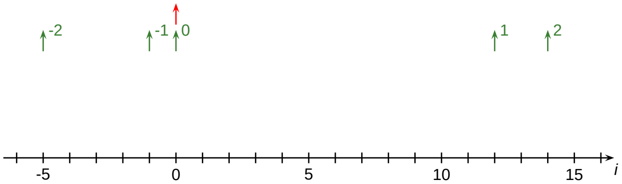

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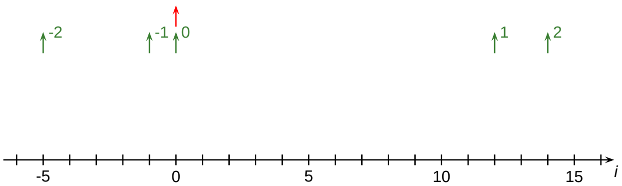

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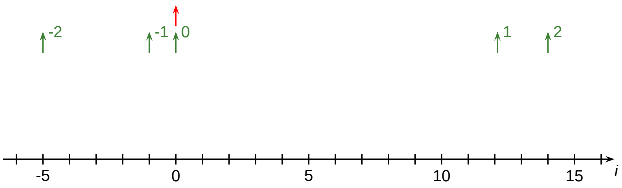

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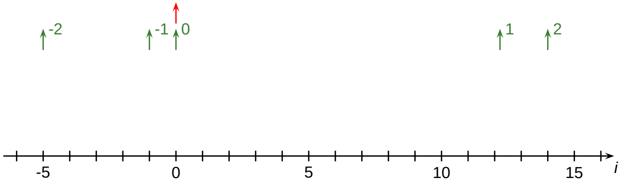

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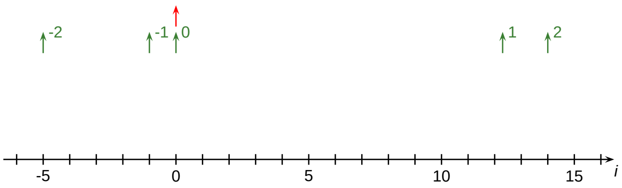

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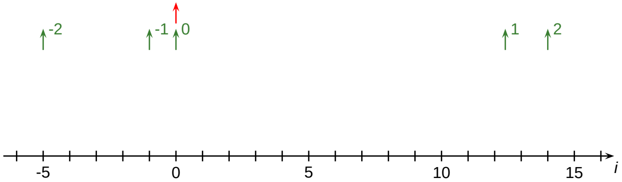

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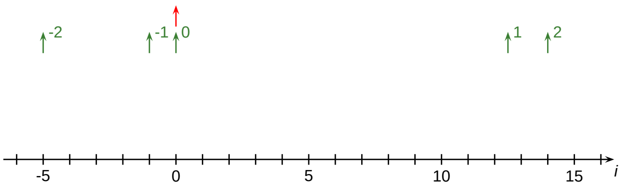

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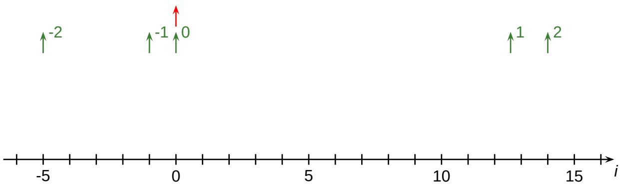

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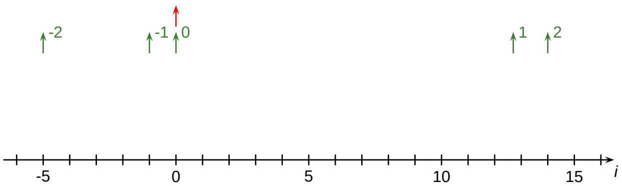

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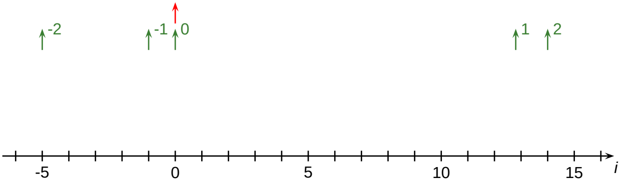

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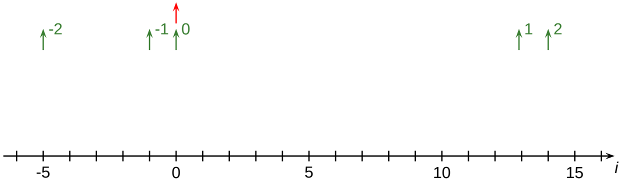

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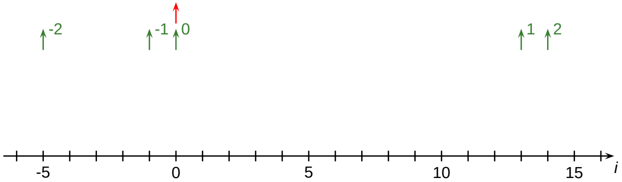

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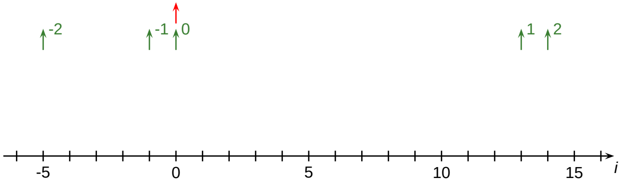

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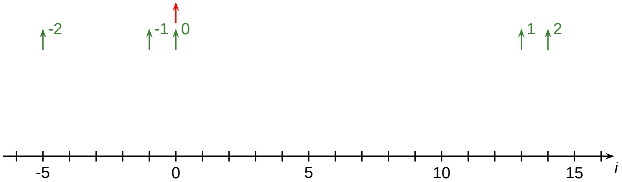

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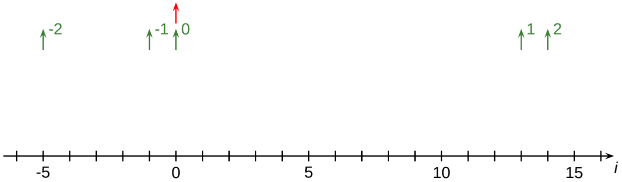

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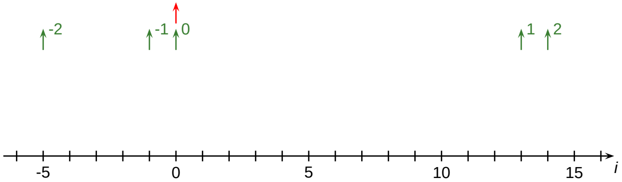

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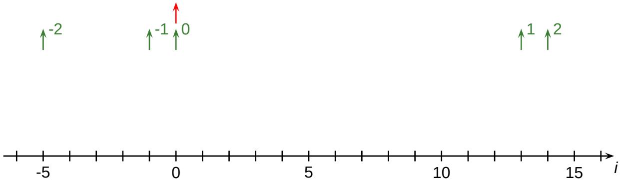

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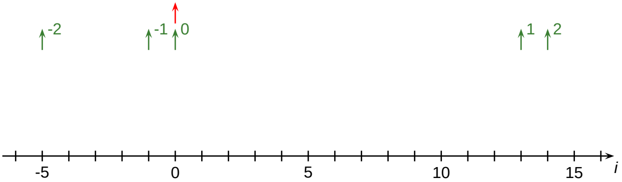

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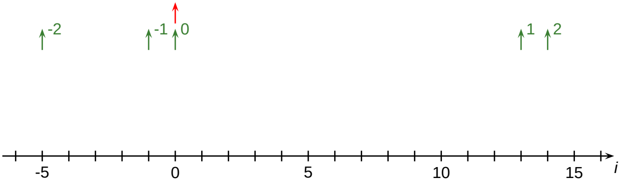

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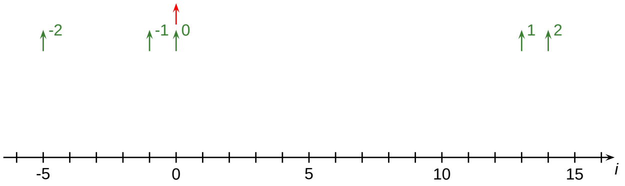

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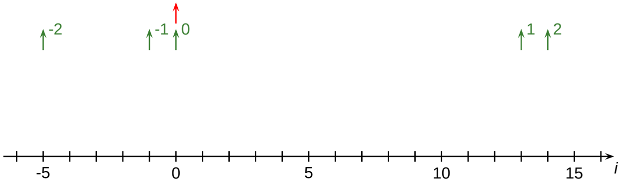

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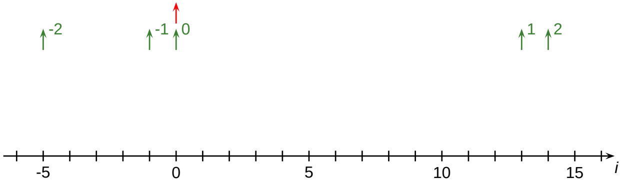

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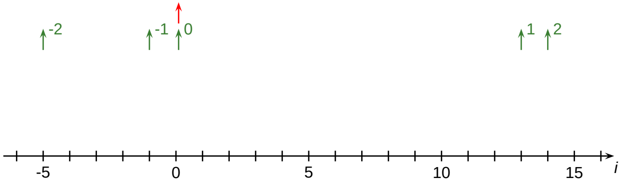

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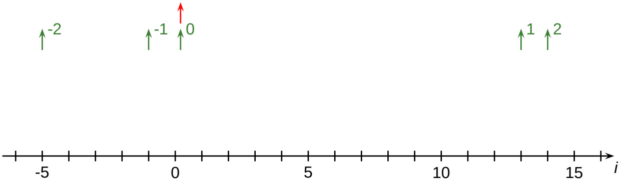

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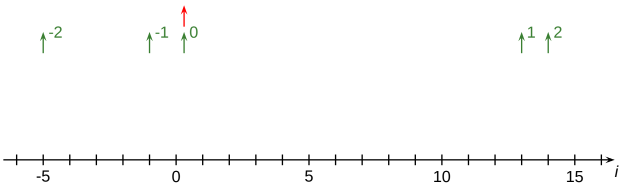

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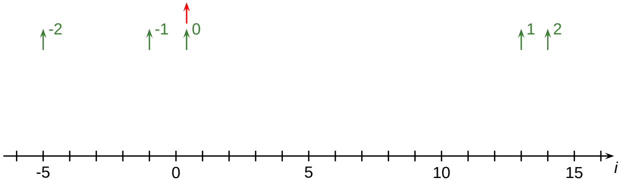

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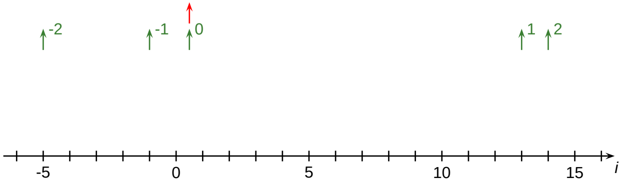

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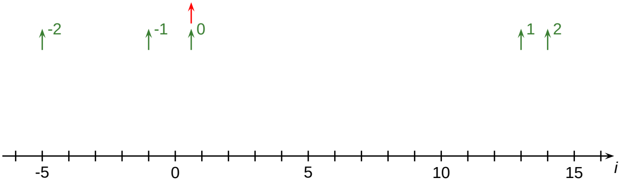

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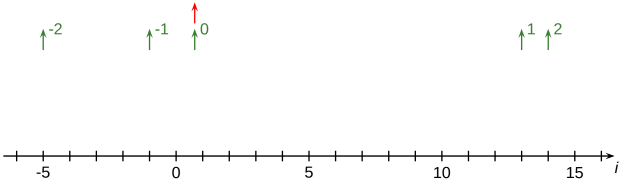

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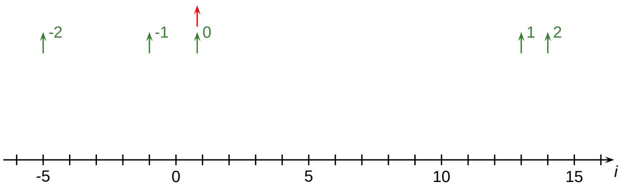

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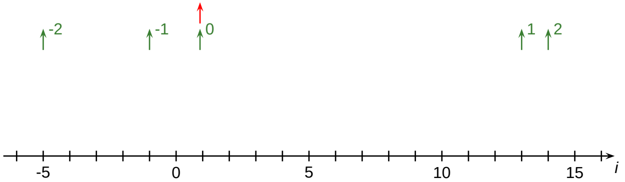

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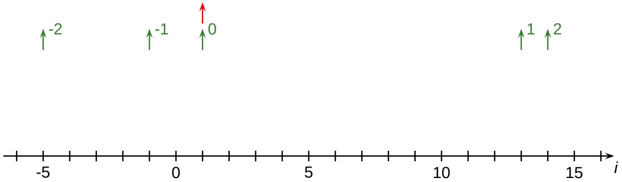

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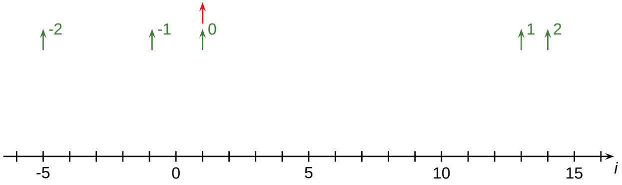

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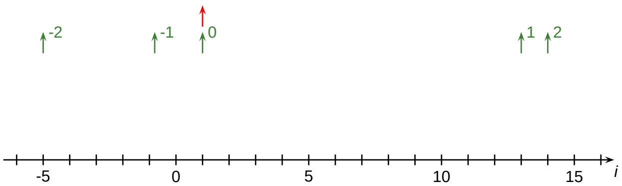

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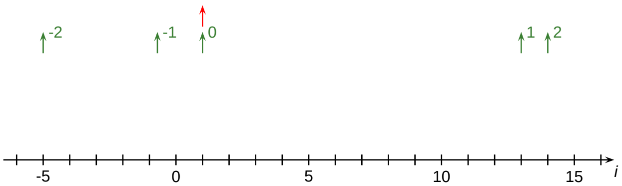

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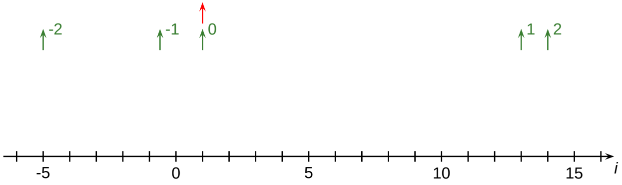

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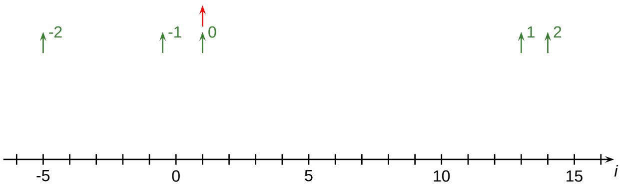

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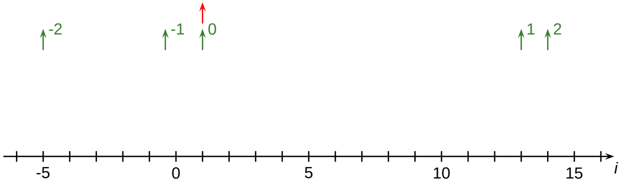

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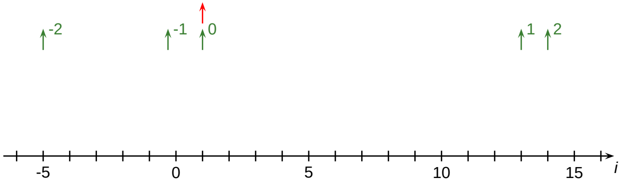

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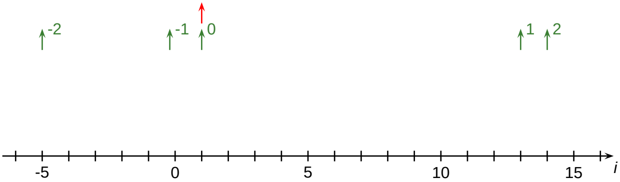

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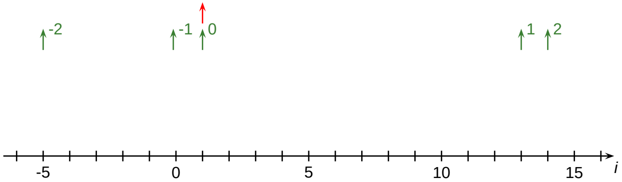

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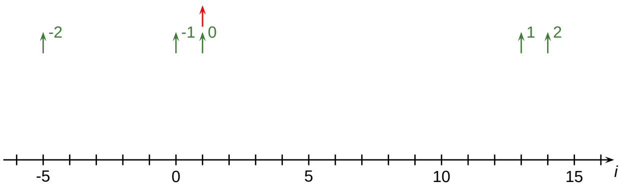

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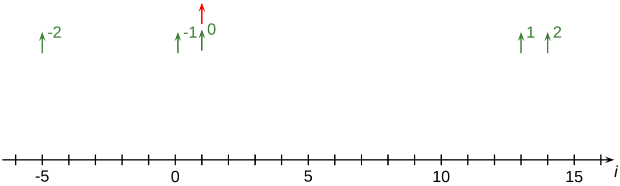

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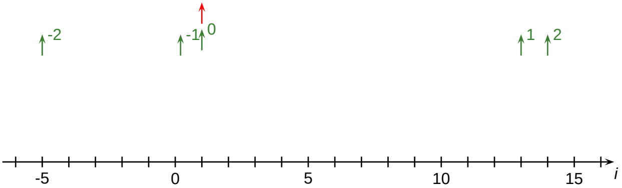

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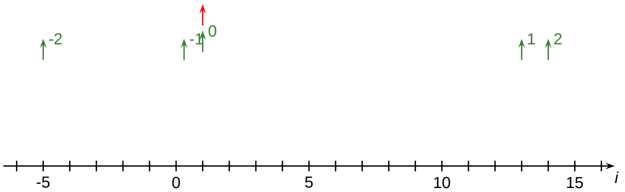

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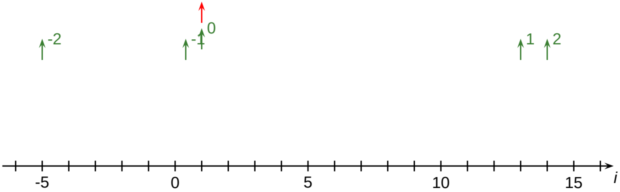

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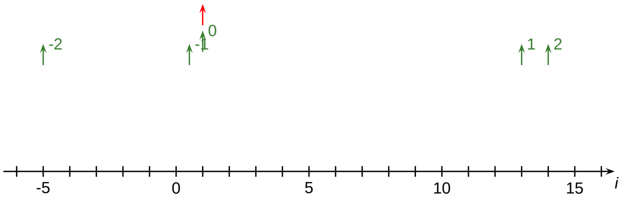

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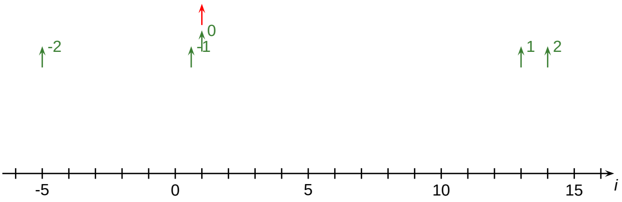

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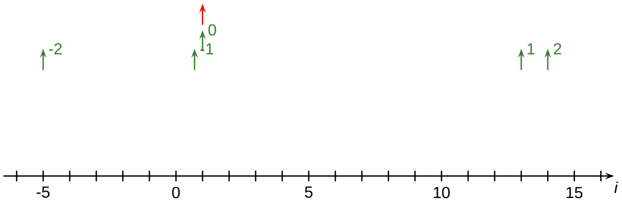

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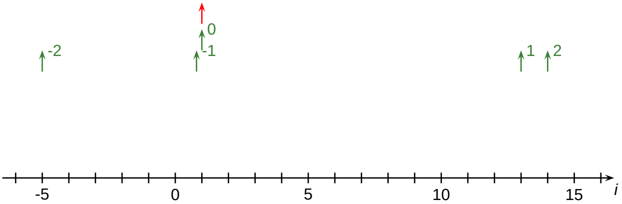

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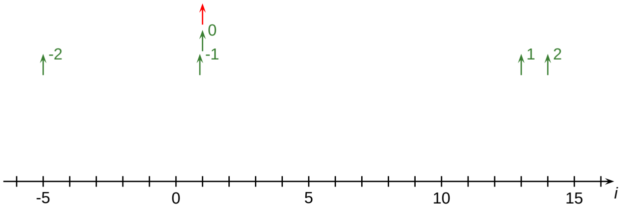

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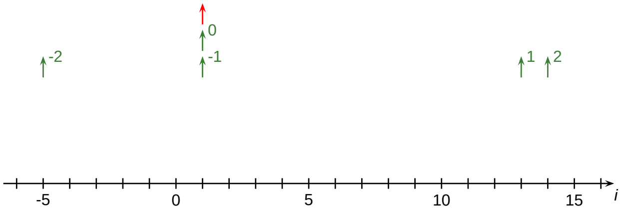

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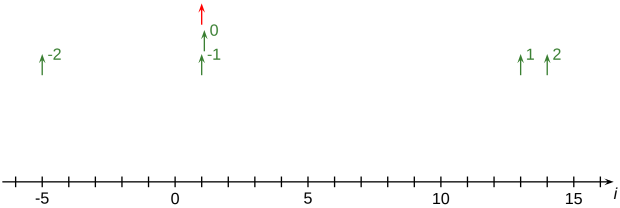

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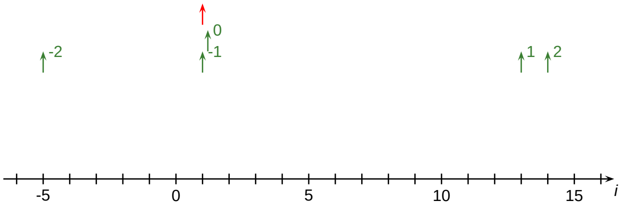

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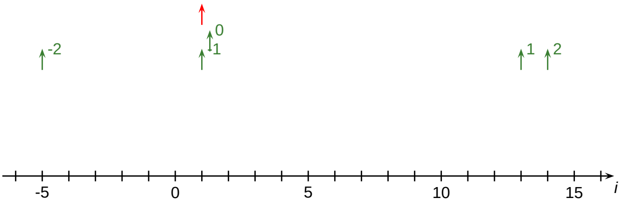

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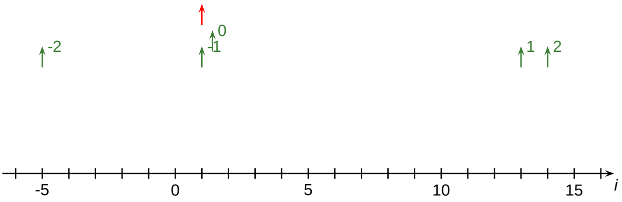

Goal: to understand $Q(t)$ on the background process of the ↑'s.

The critical feature: microscopic concavity

$Q(t) \leq X(t) +$ tight error

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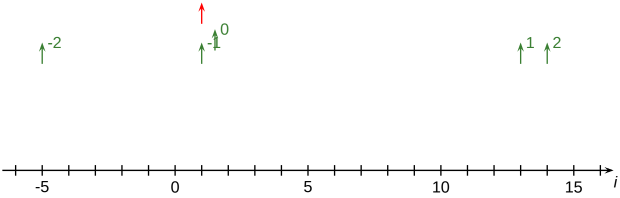

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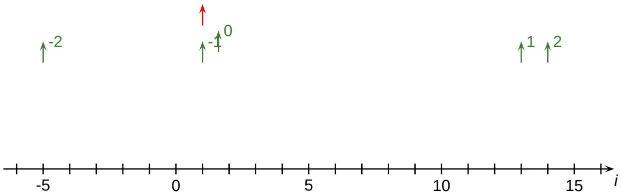

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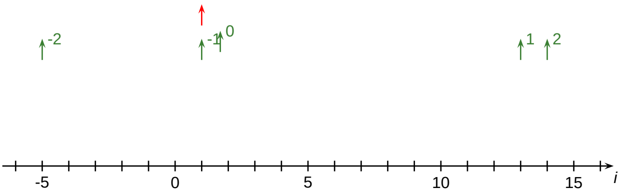

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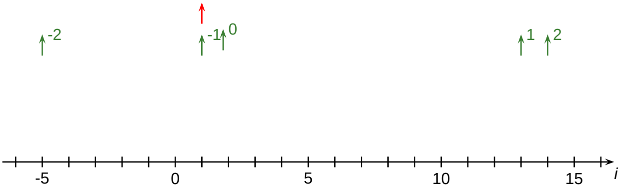

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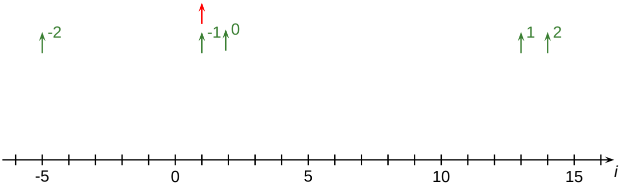

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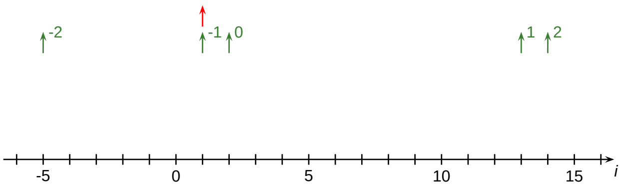

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This is the form of microscopic concavity we currently use: $m_Q(t)$ is dominated by a time-independent distribution with finite variance.

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Once this is done, we could proceed with less and less convex/concave models to see how $t^{1/3}$ scaling turns to $t^{1/4}$ for linear models (random average process, linear rate AZRP)...

Thank you.