

# Local convergence in t-PNG

joint with Ruby Bestwick, Artem Borisov, Elnur Emrah, and  
Jessica Jay

Márton Balázs

University of Bristol

Lancaster Probability Days  
March 2026.

## Polynuclear Growth

*t*-PNG

Stationary *t*-PNG

Local convergence

Ingredients

Proof

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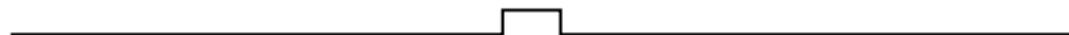
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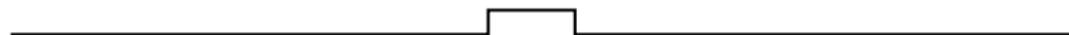
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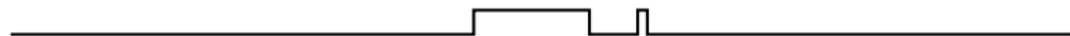
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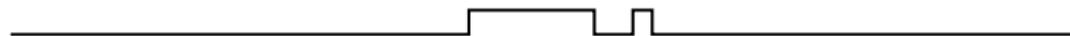
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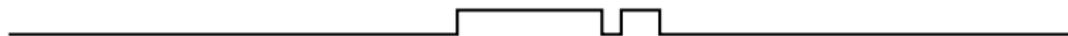
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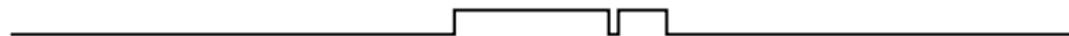
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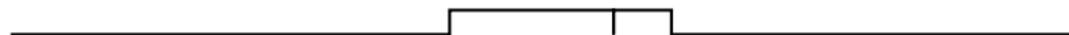
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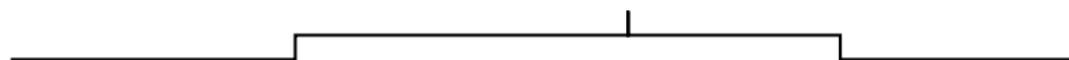
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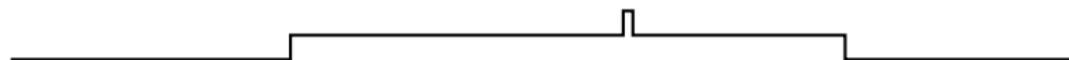
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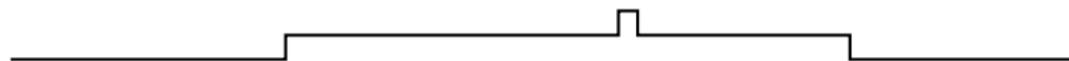
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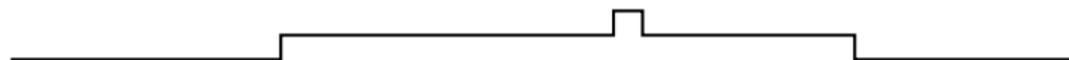
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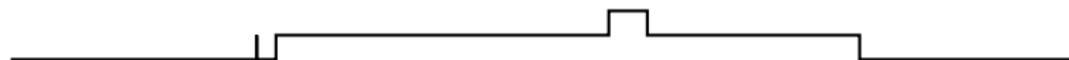
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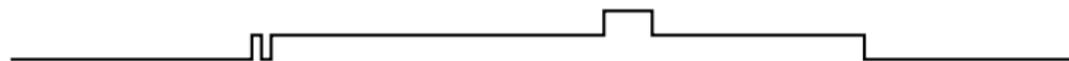
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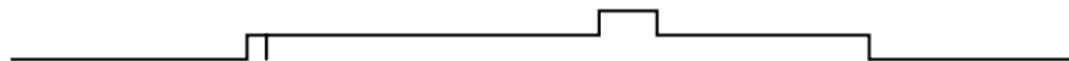
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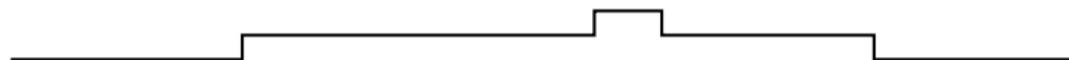
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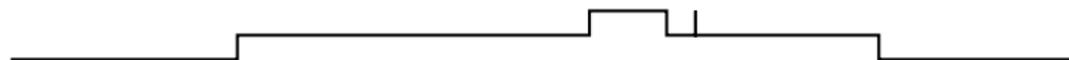
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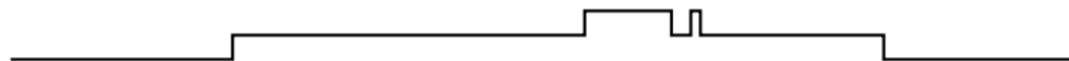
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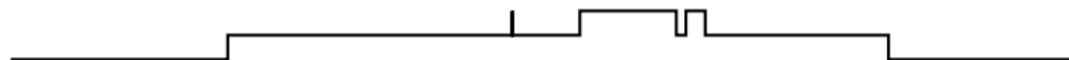
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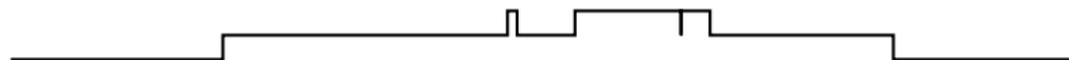
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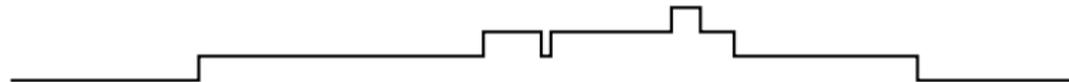
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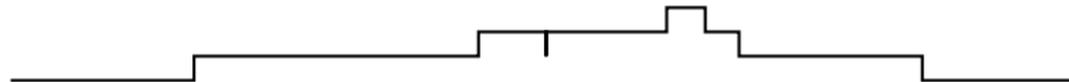
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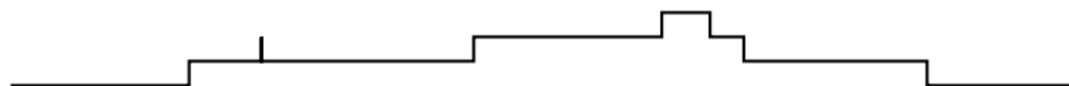
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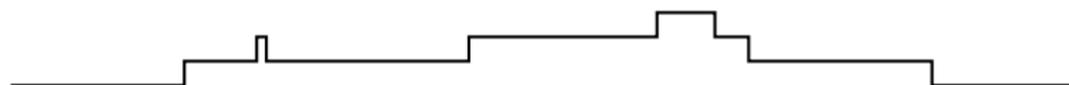
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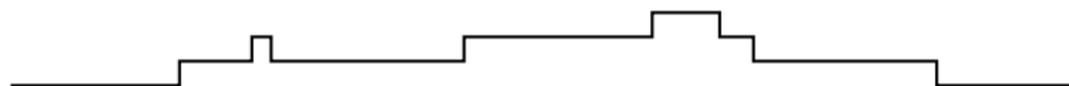
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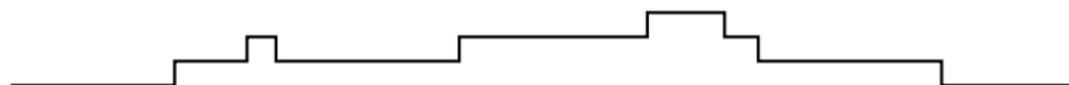
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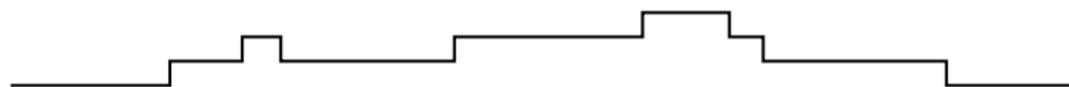
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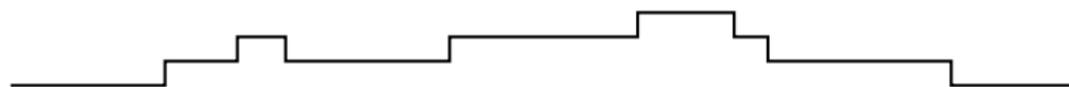
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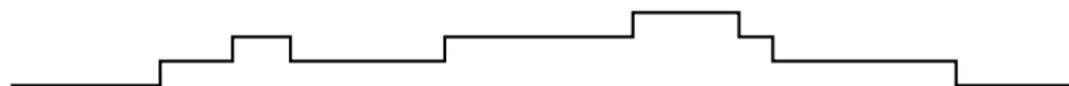
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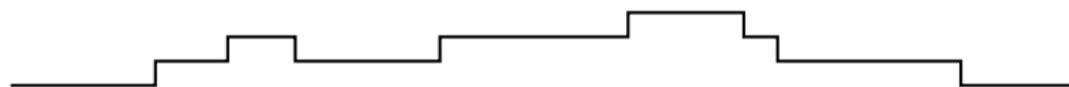
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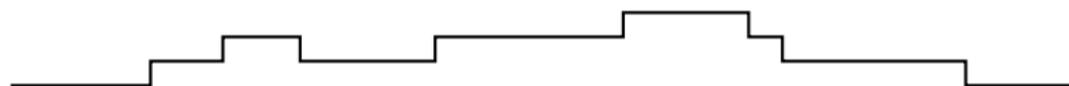
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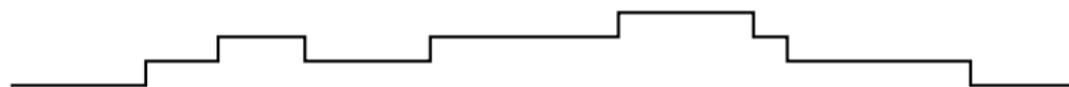
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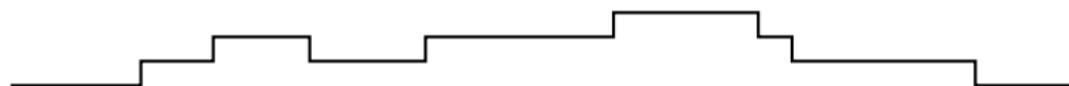
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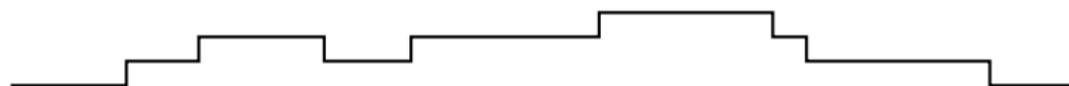
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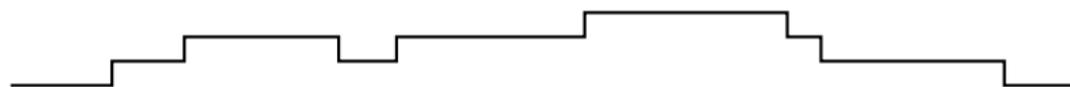
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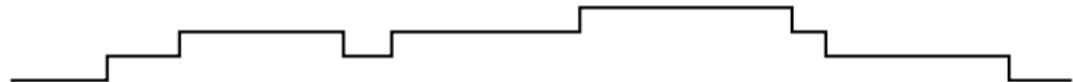
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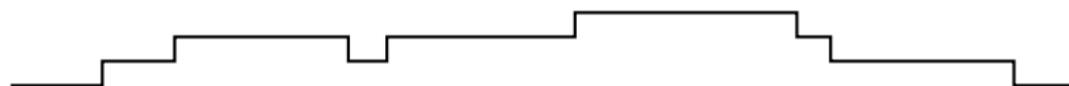
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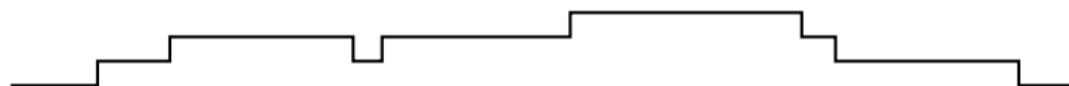
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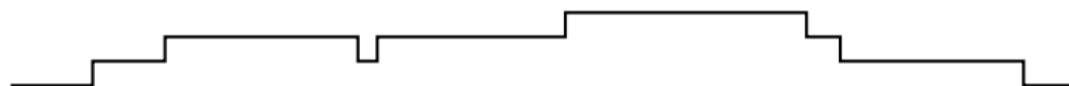
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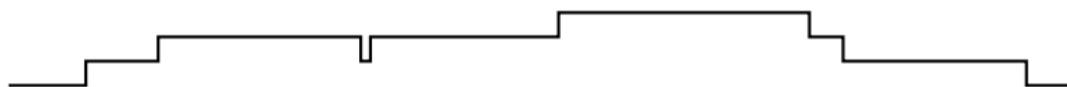
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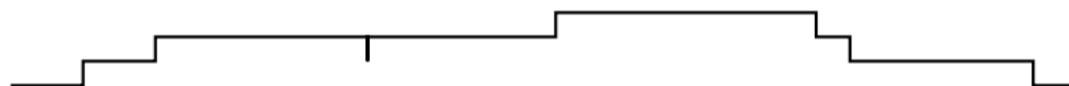
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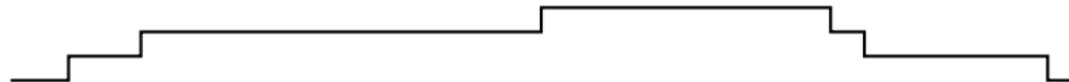
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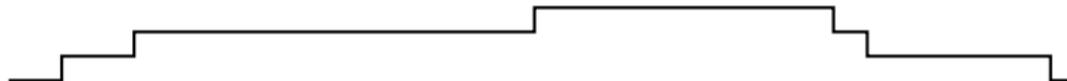
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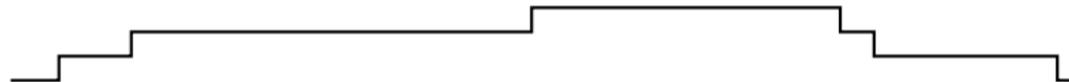
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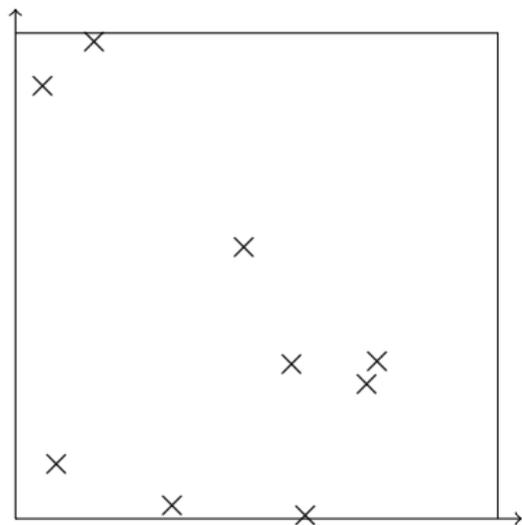
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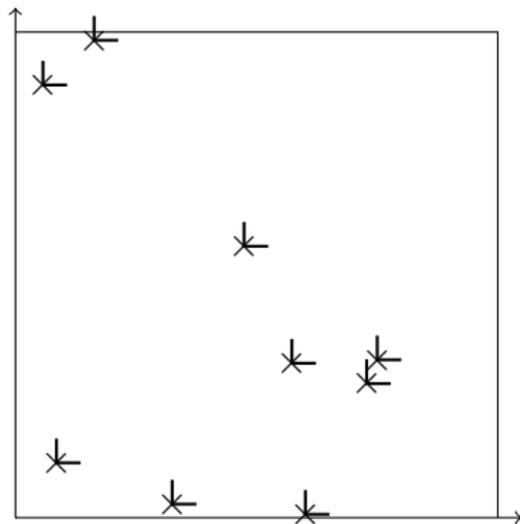


- ▶ Member of the KPZ Universality class ( $\frac{1}{3}$ -power scalings, GO/UE limit distributions) [Krug-Spohn '92]

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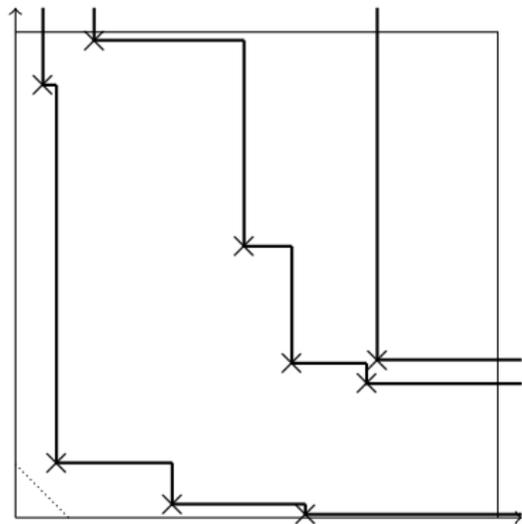




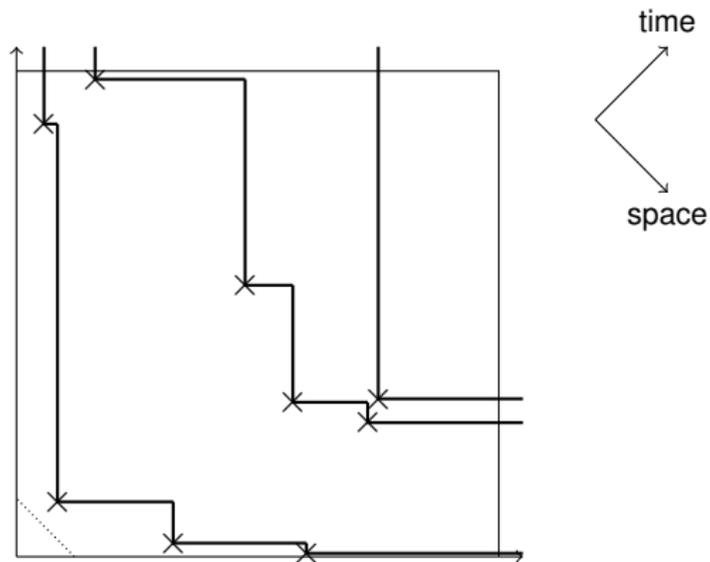




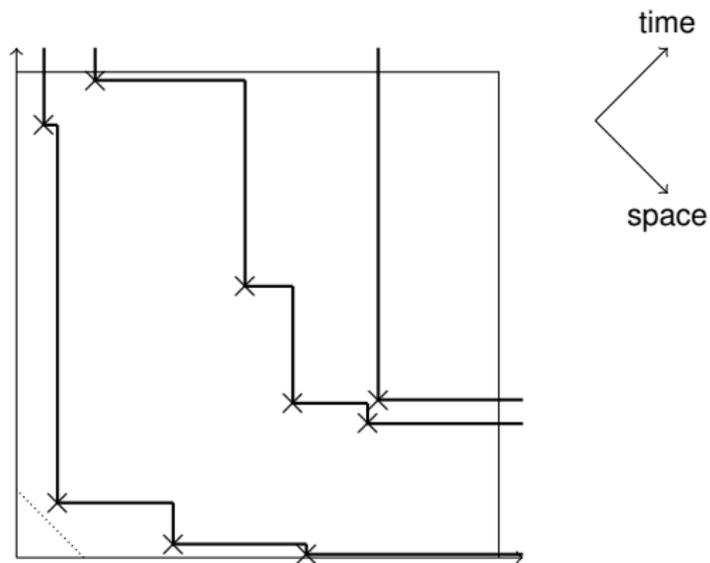
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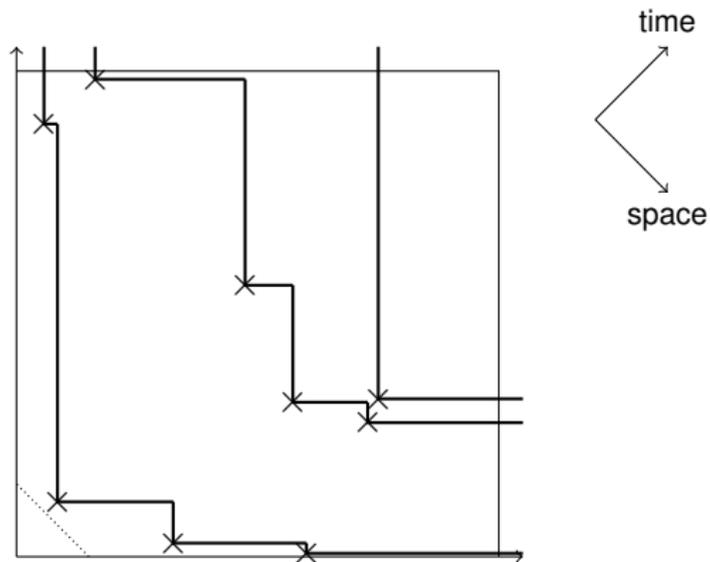
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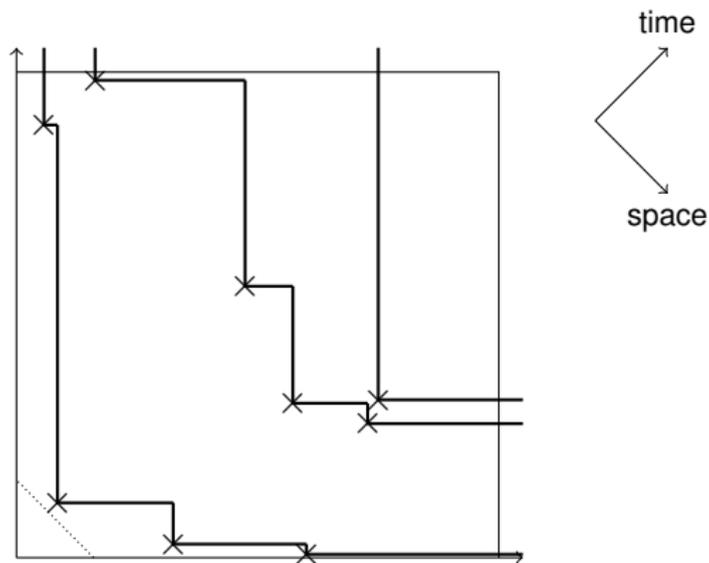
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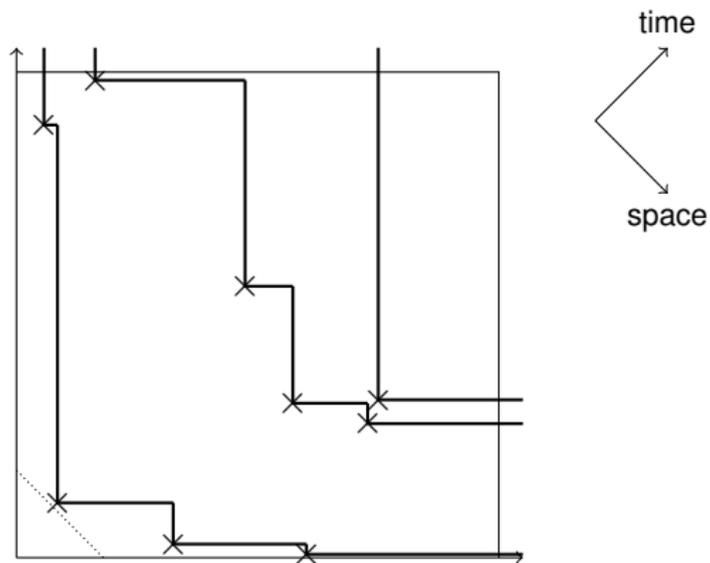
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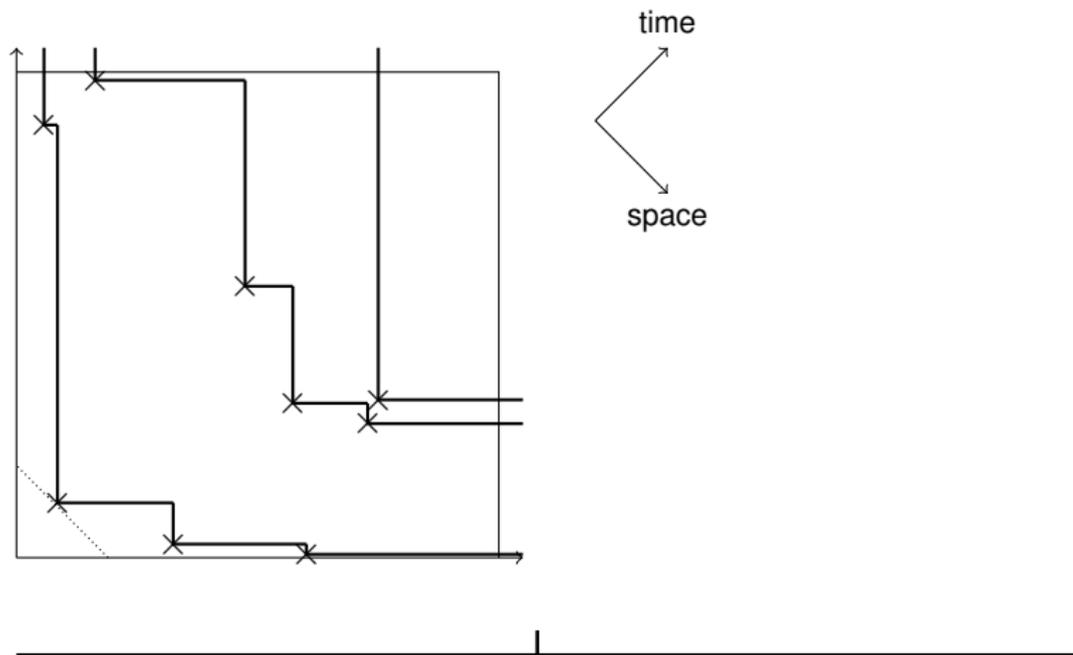
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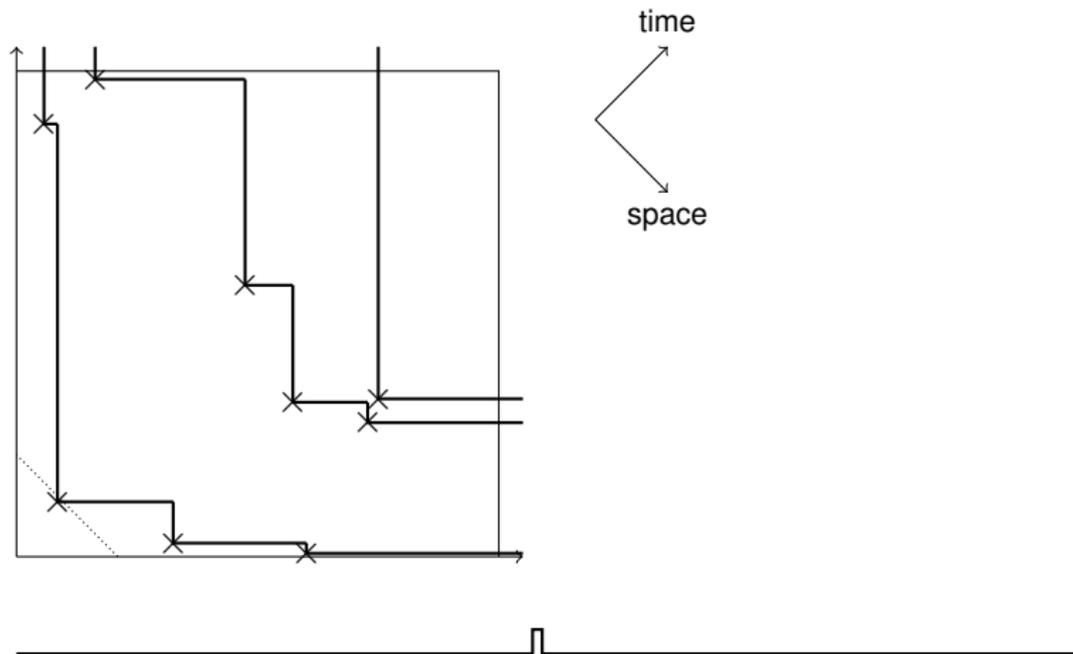
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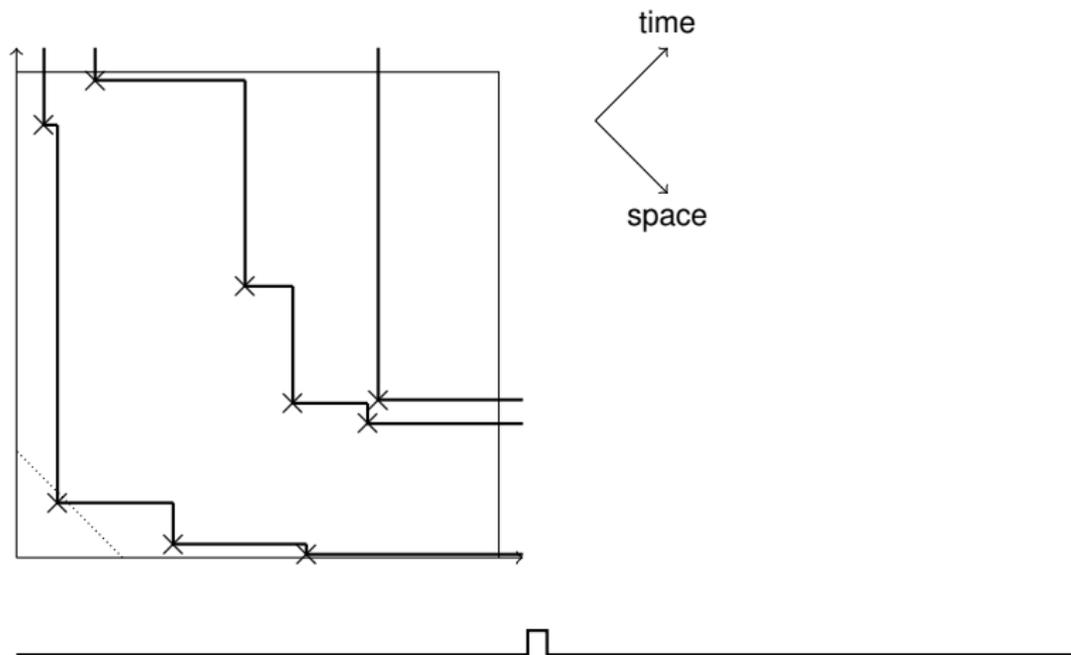
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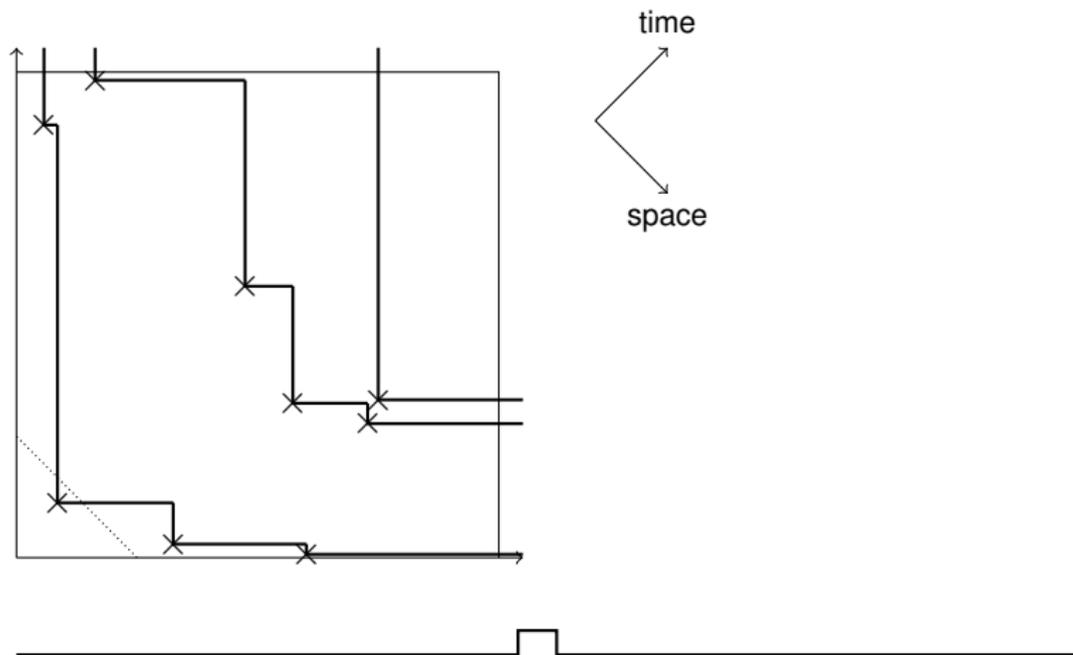


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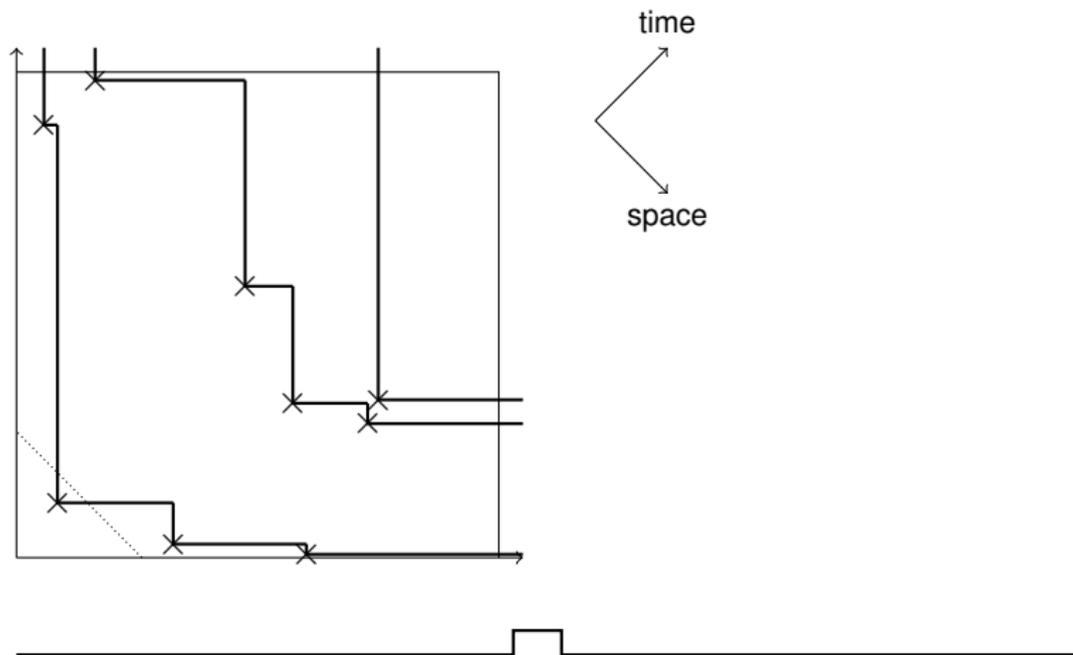




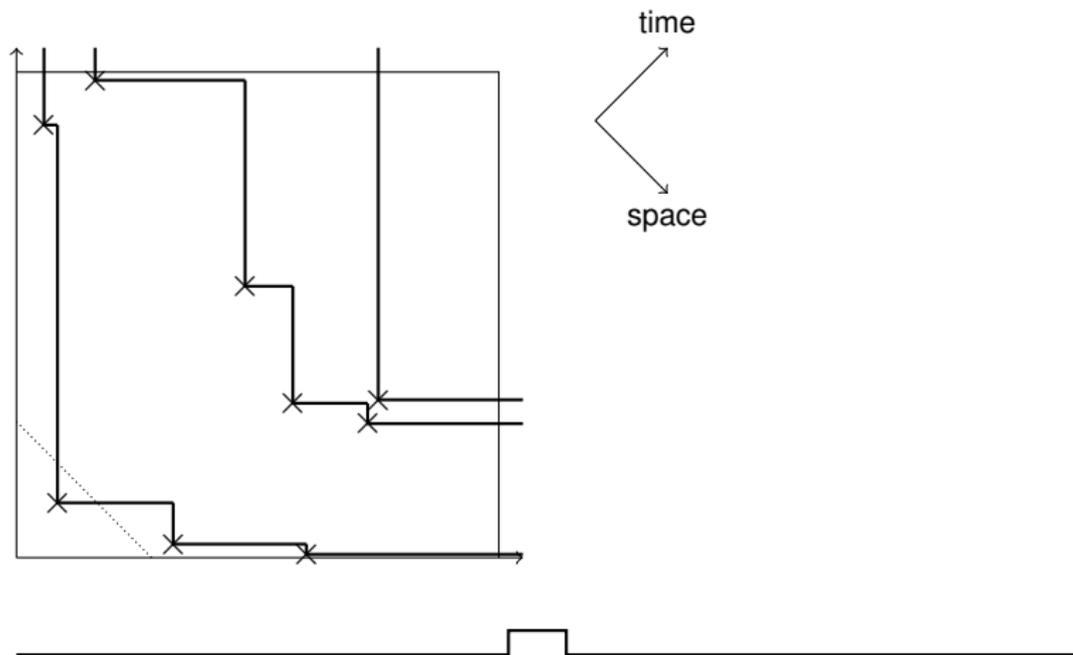
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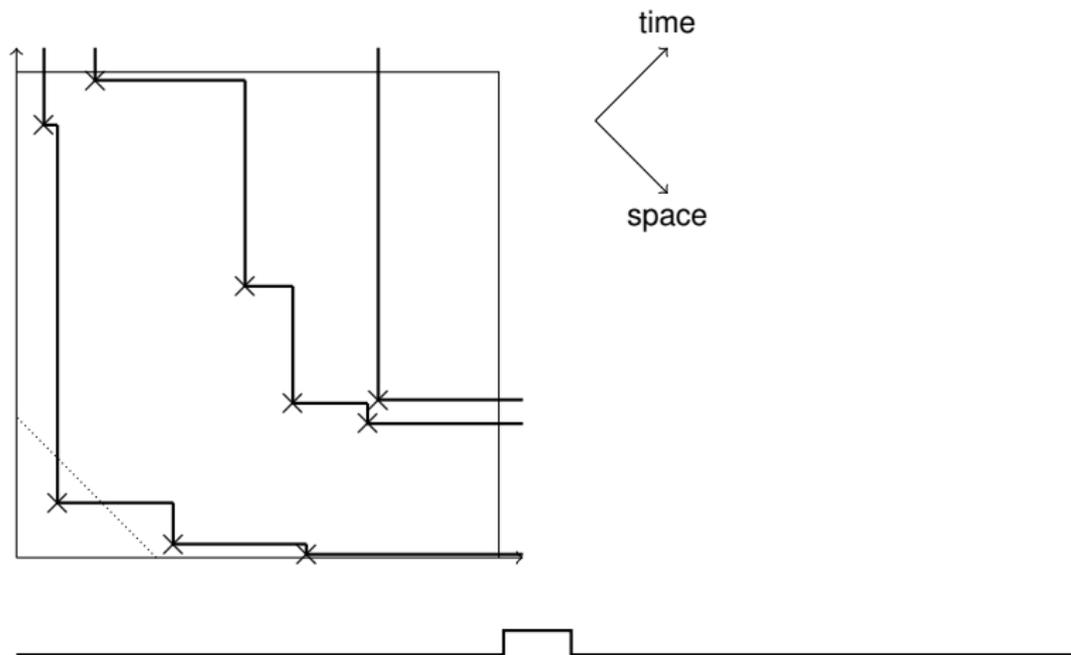
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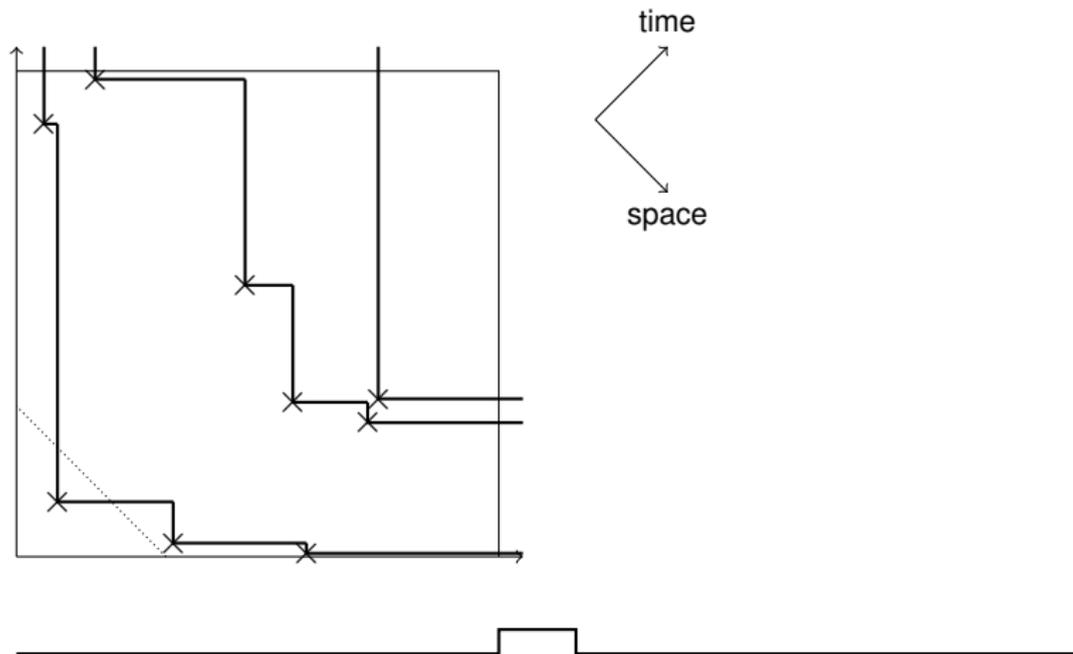
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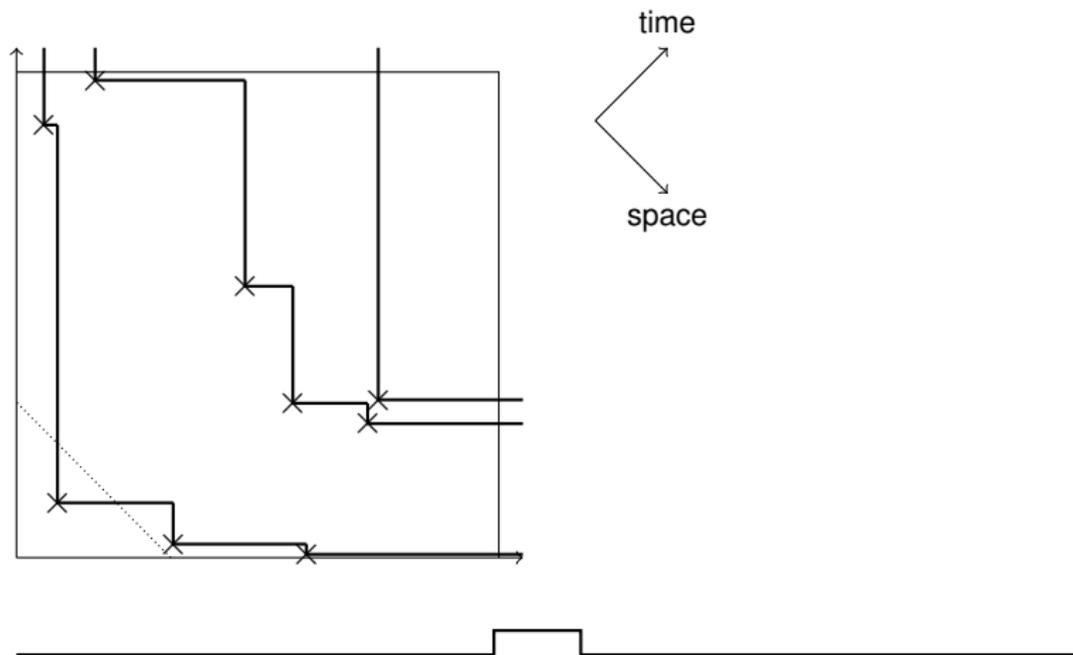
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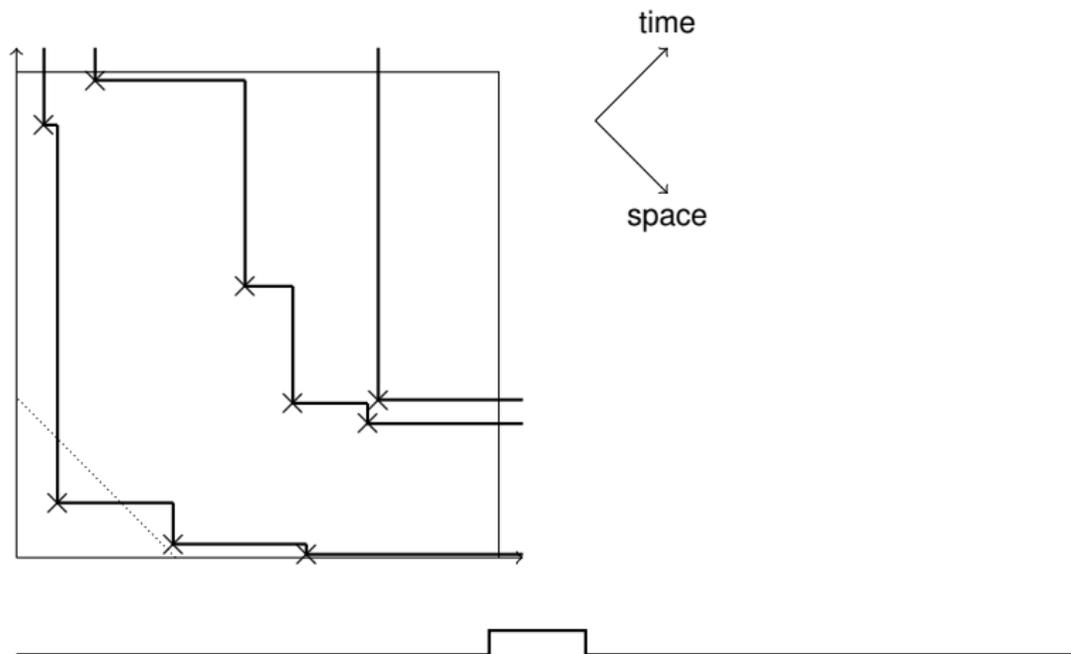
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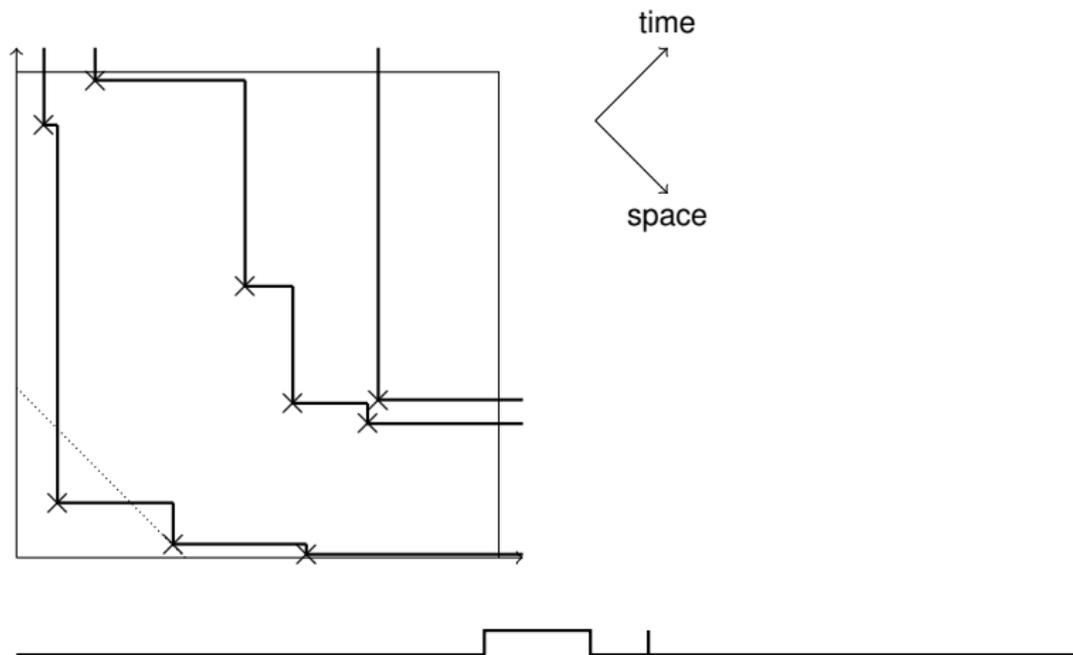
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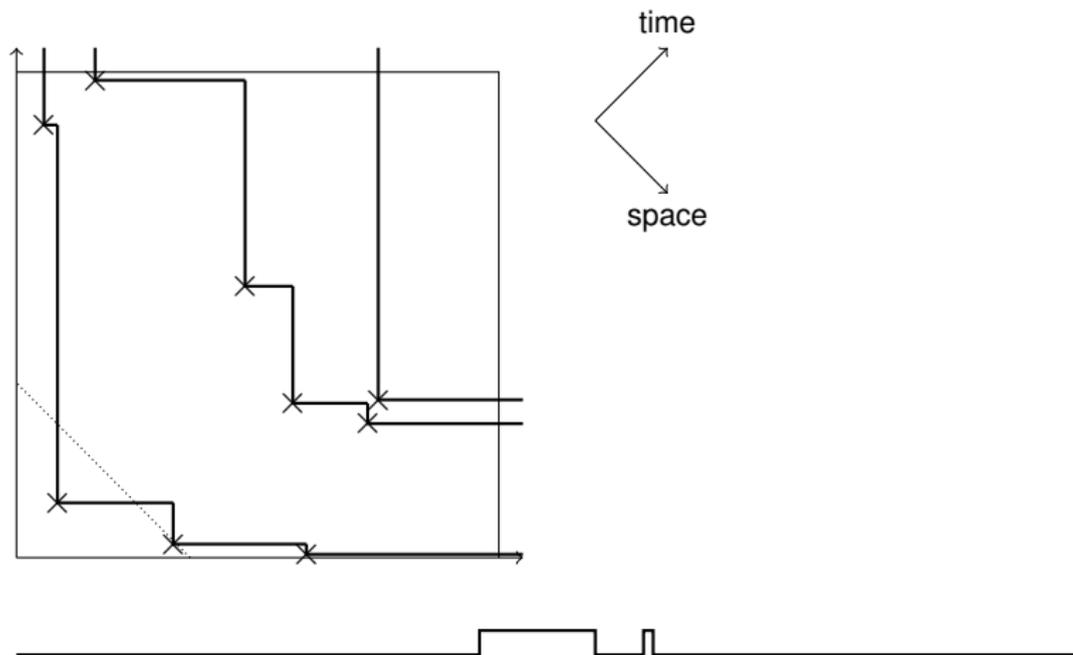
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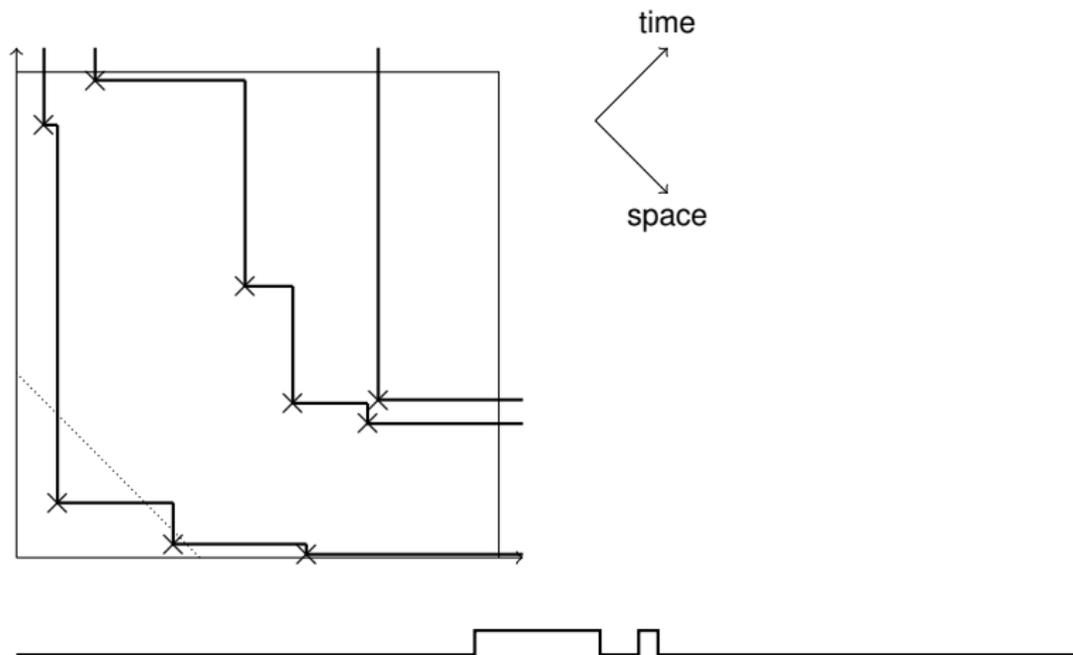
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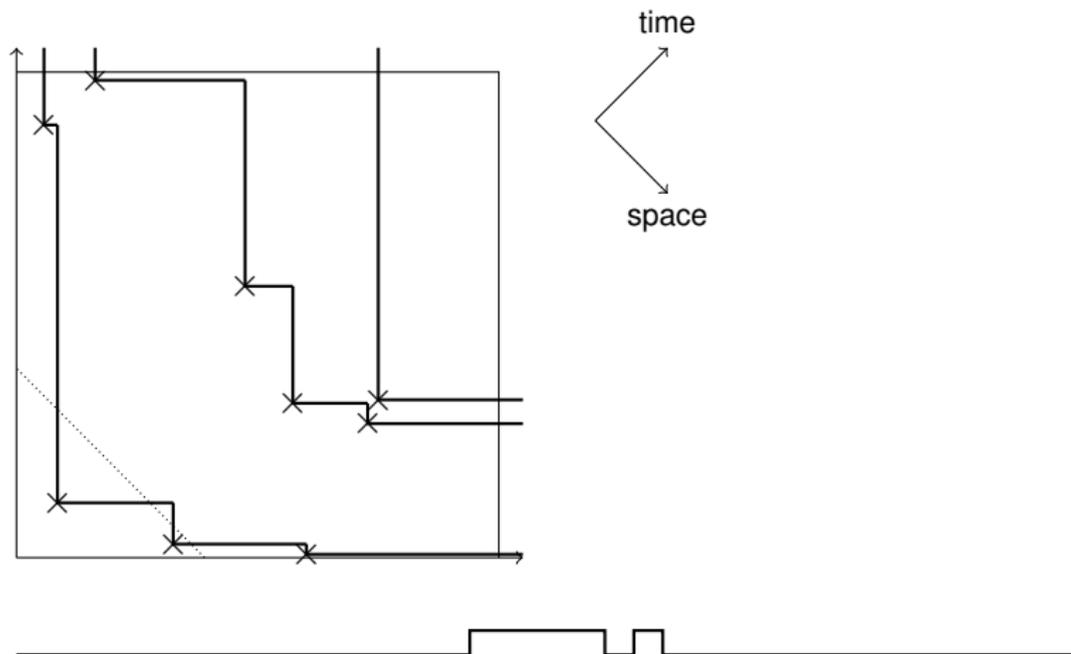
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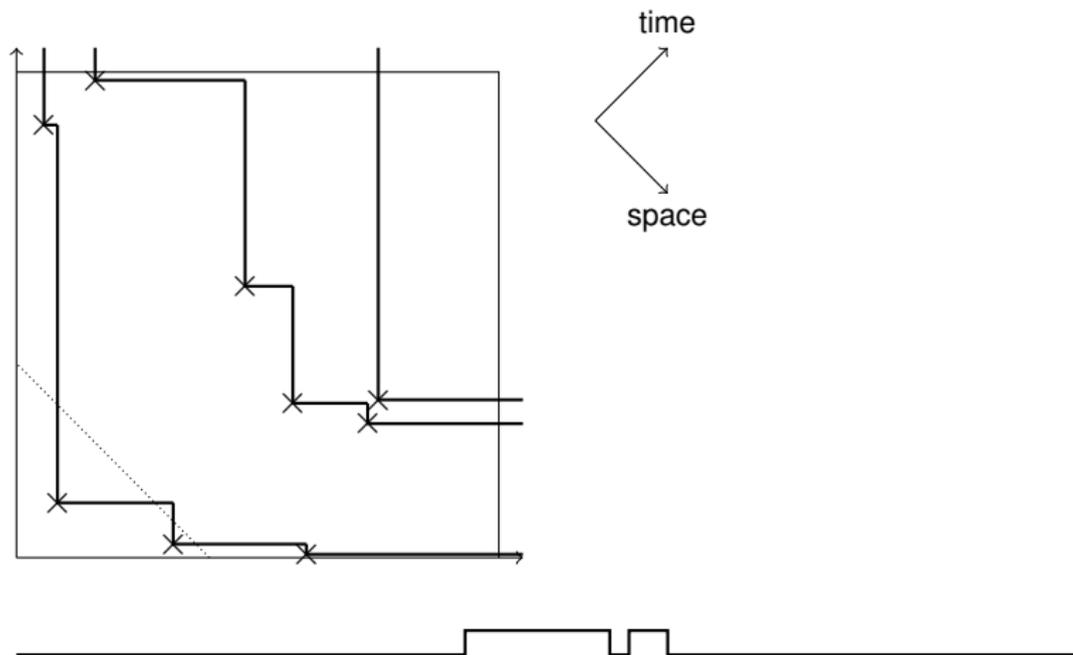
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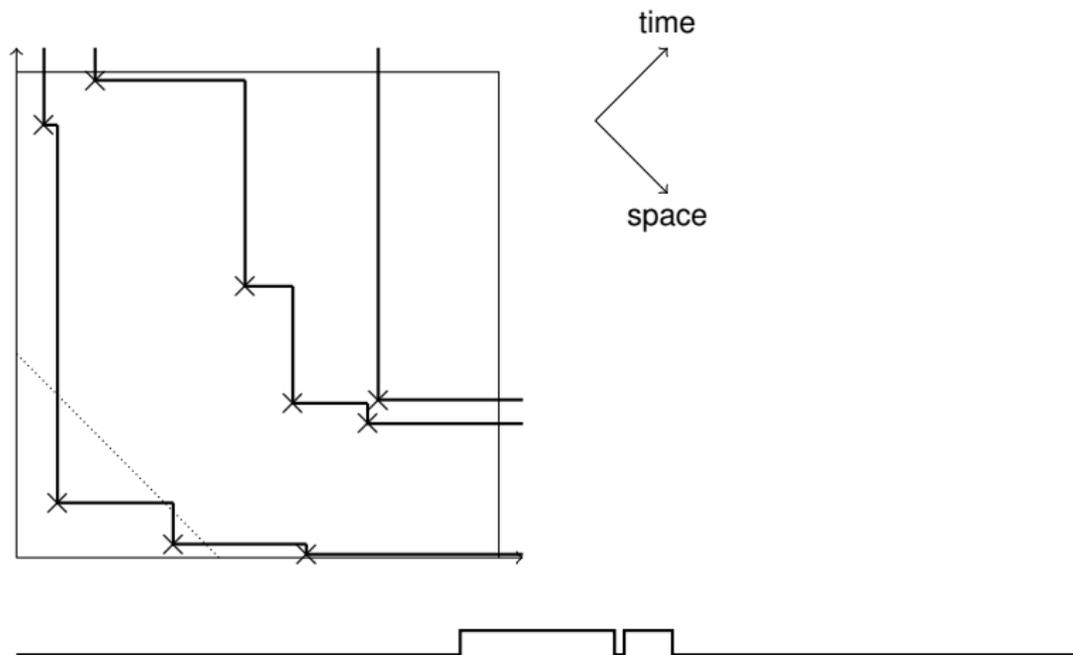
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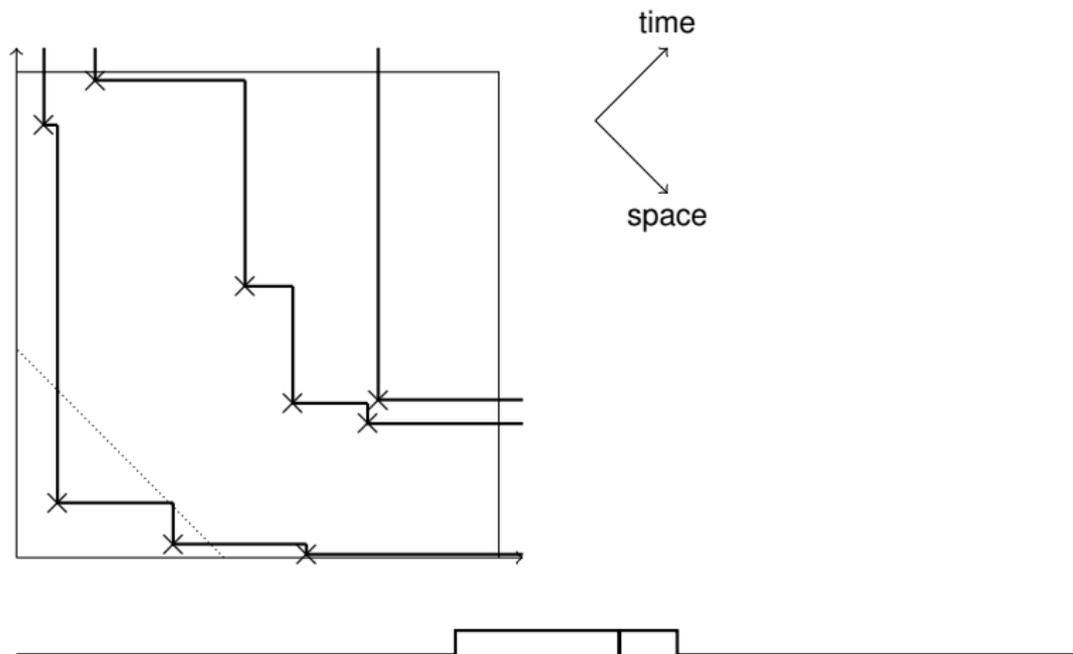
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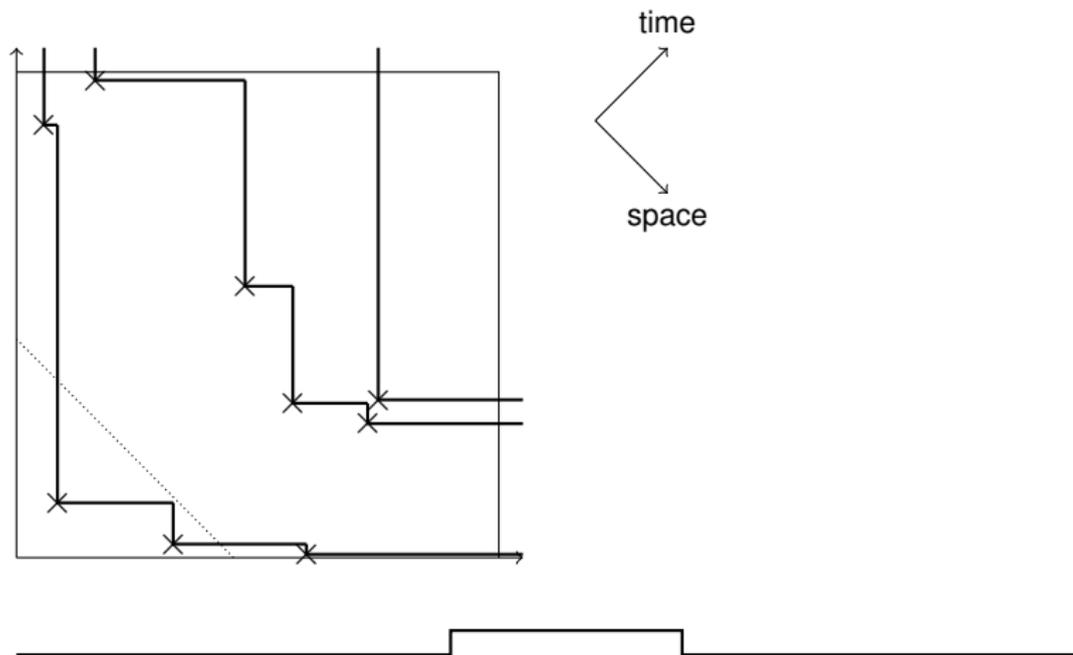
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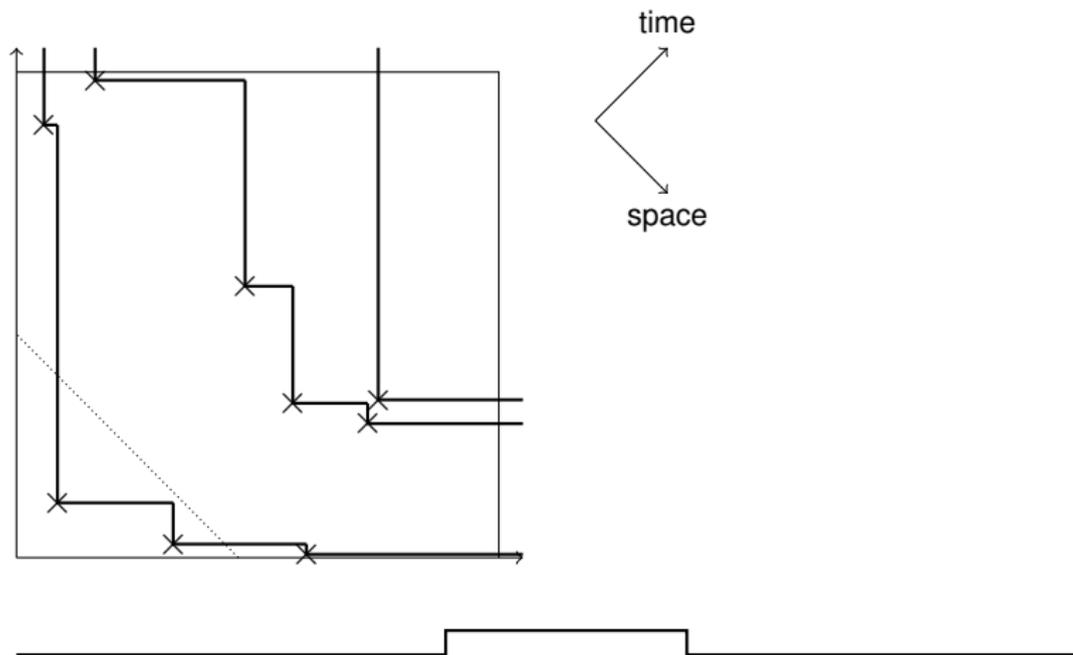
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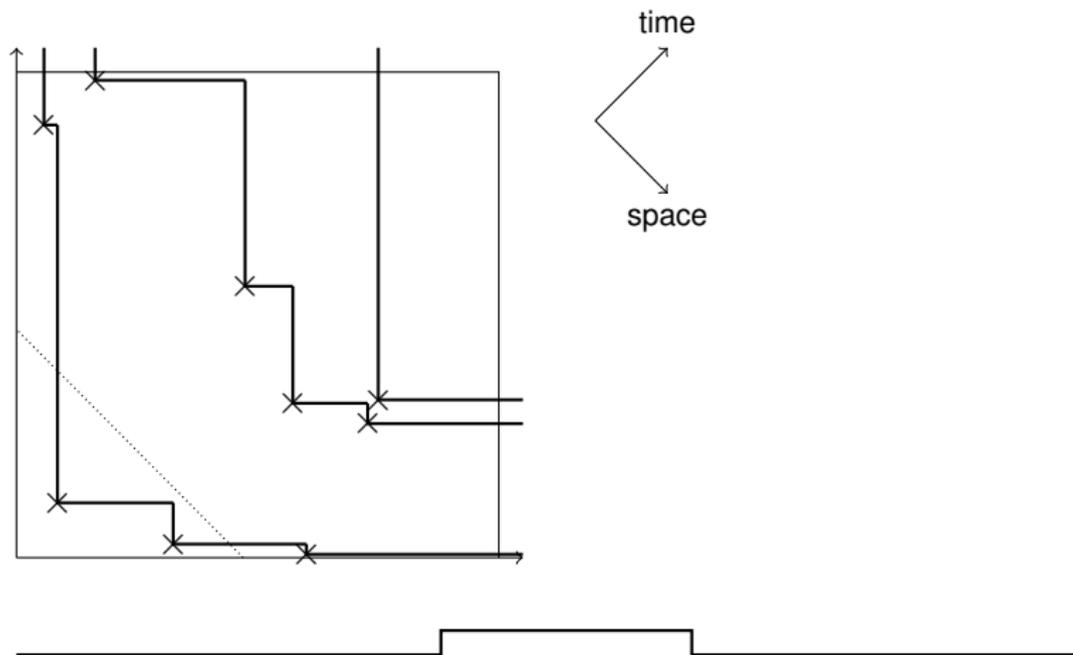
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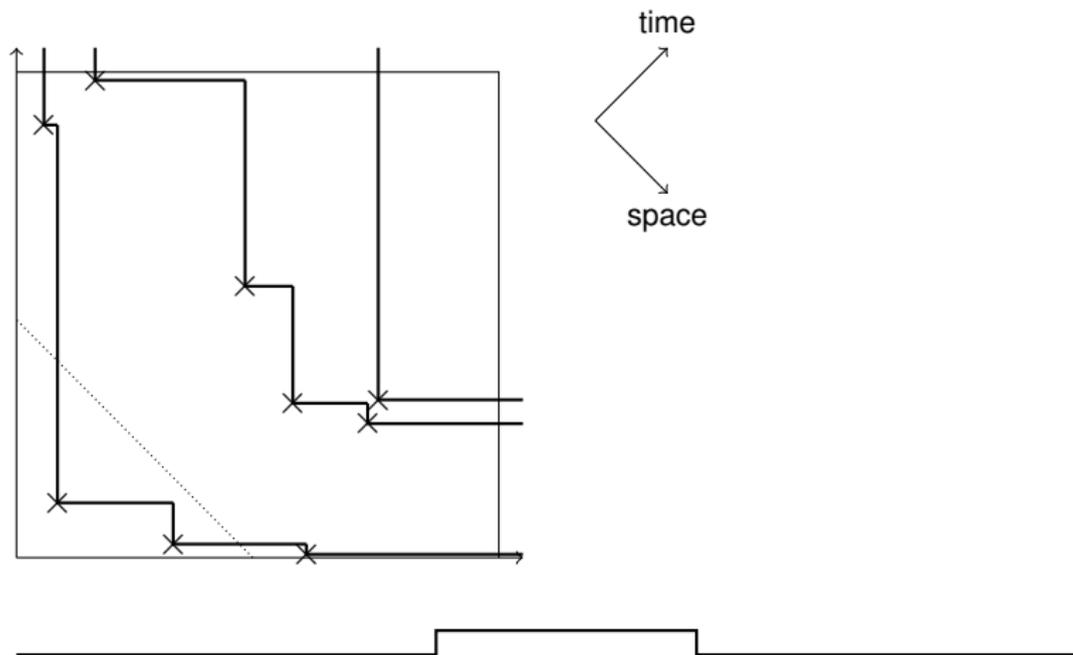
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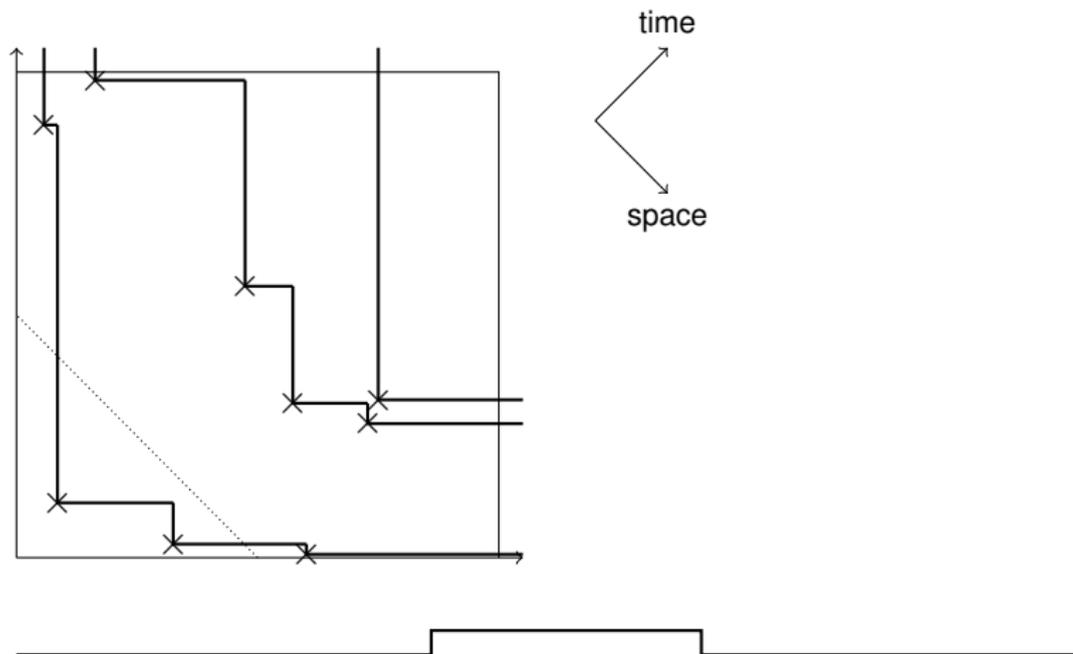
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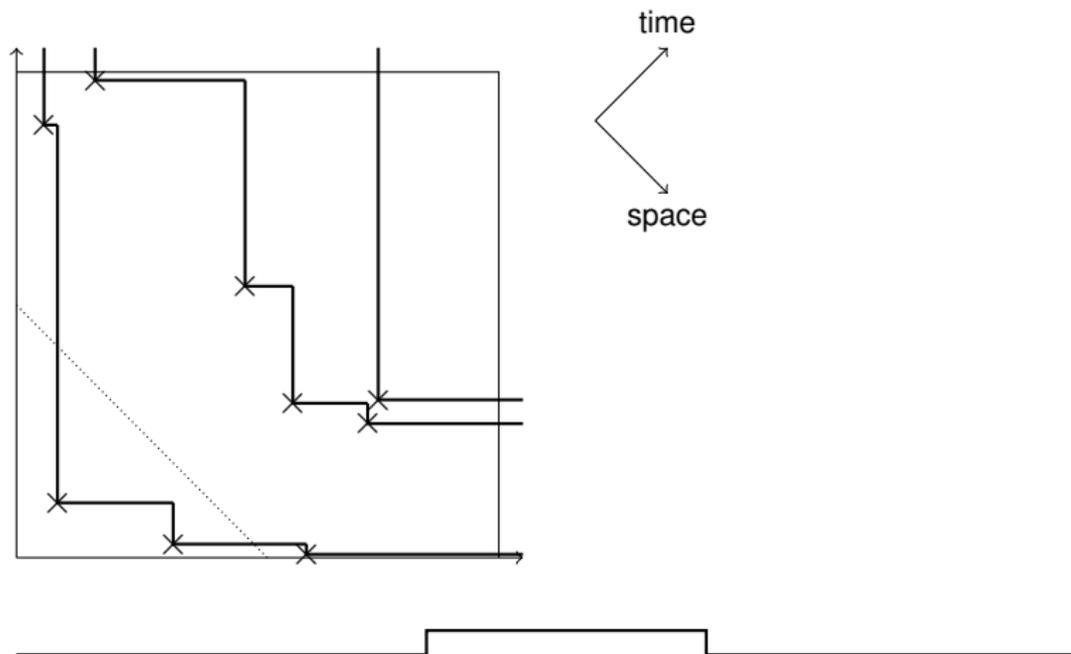
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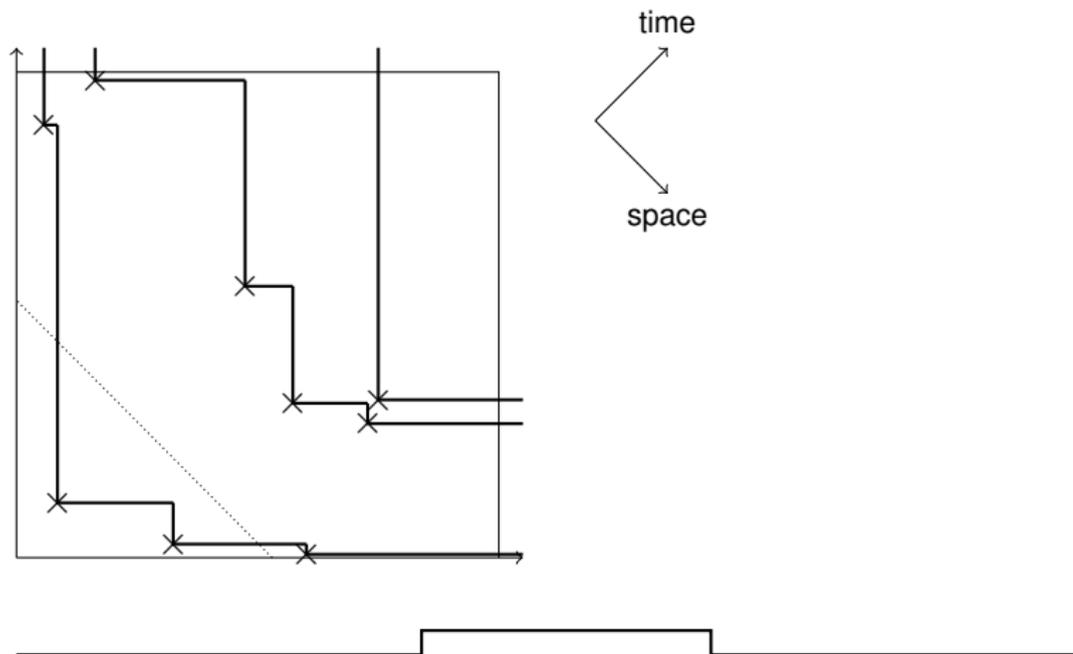
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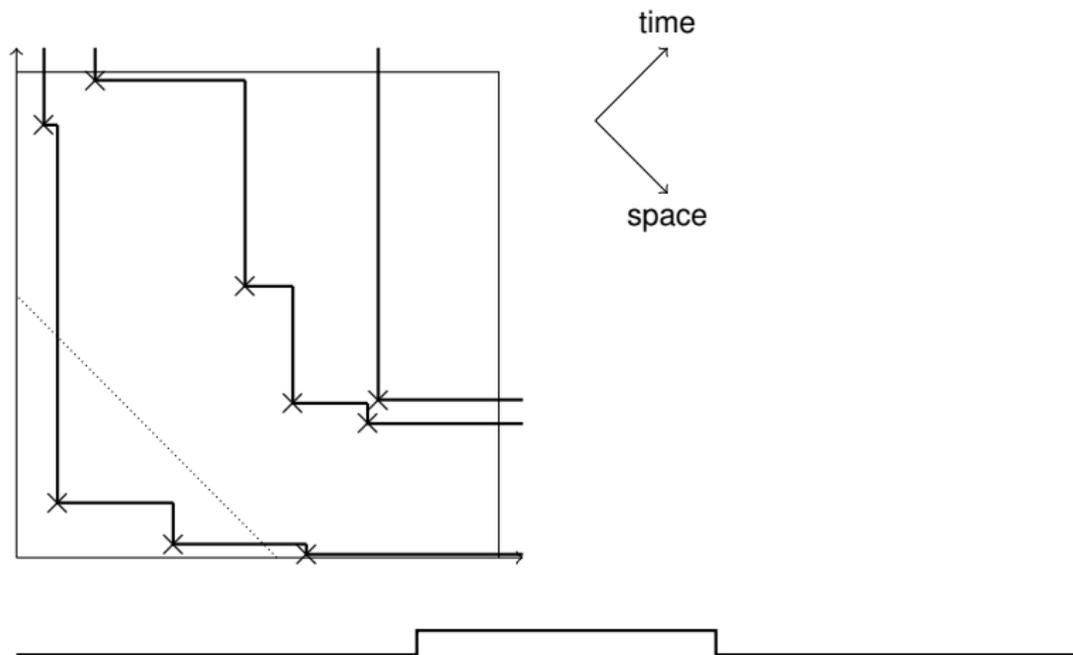
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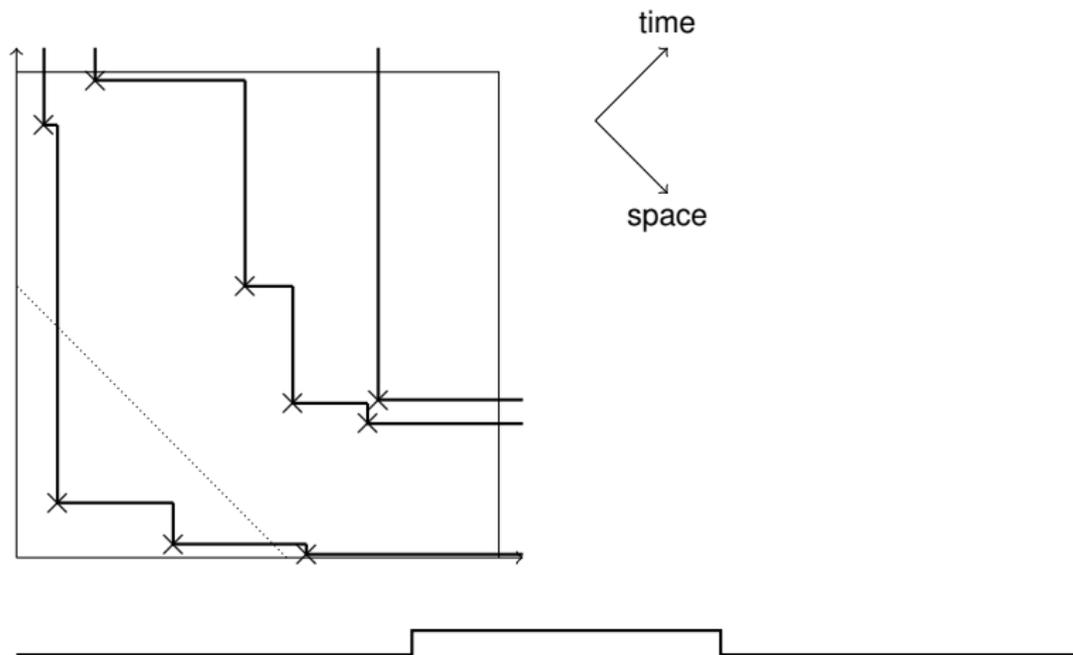
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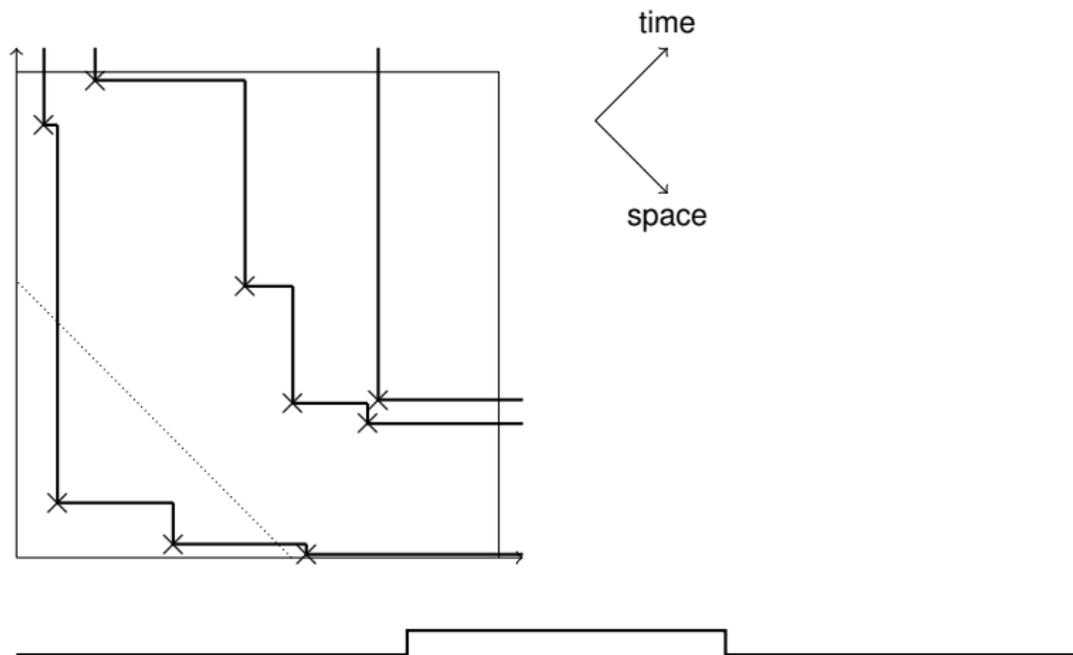
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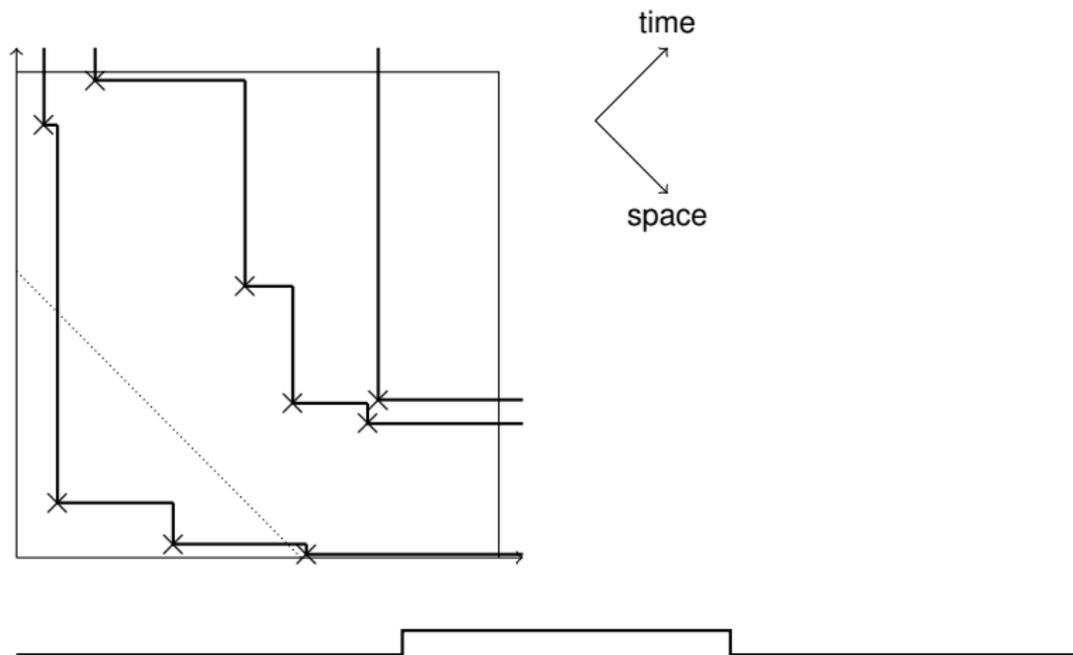
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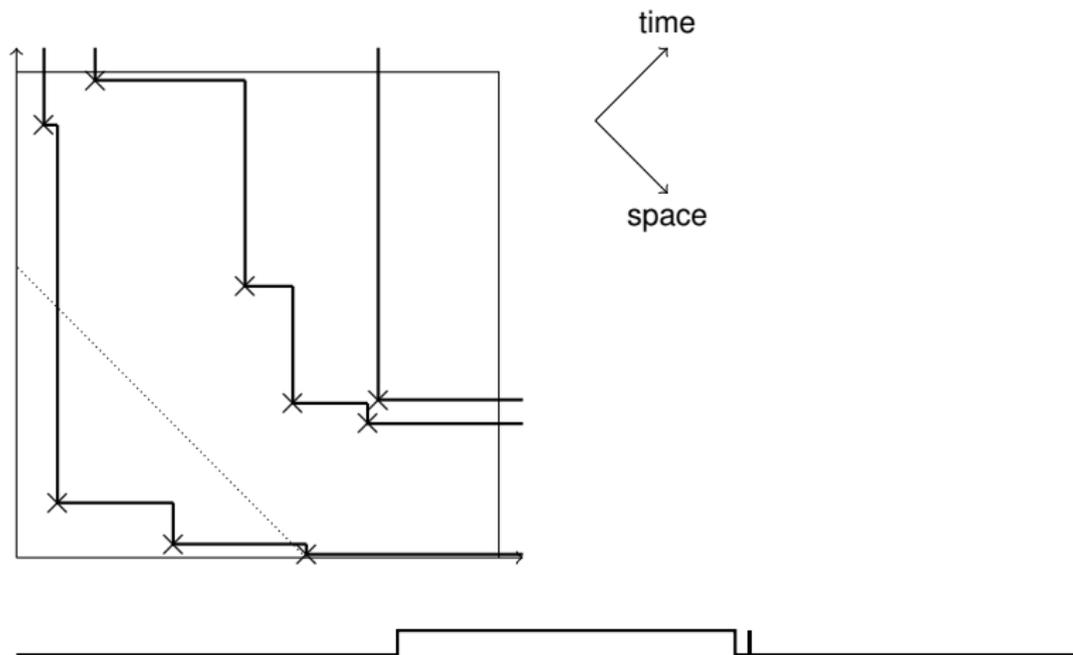
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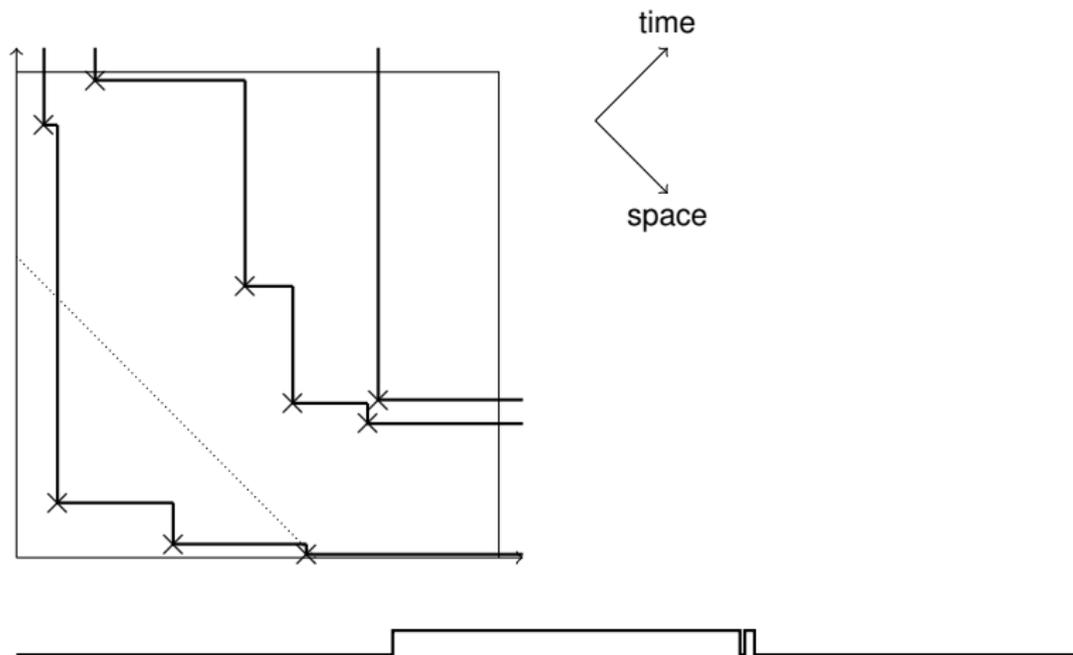
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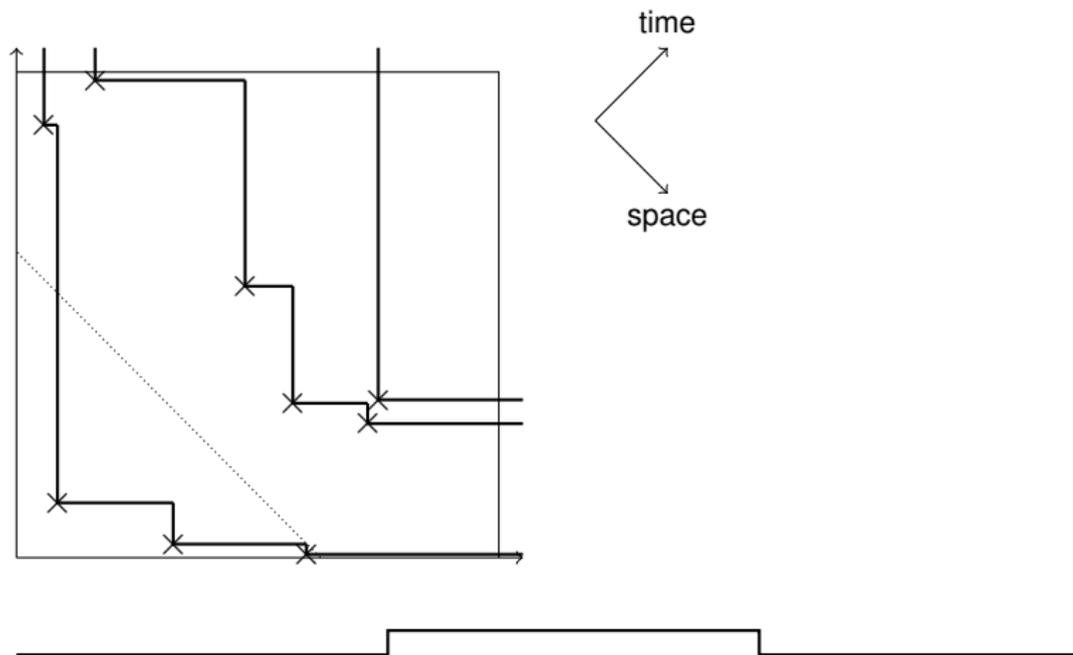
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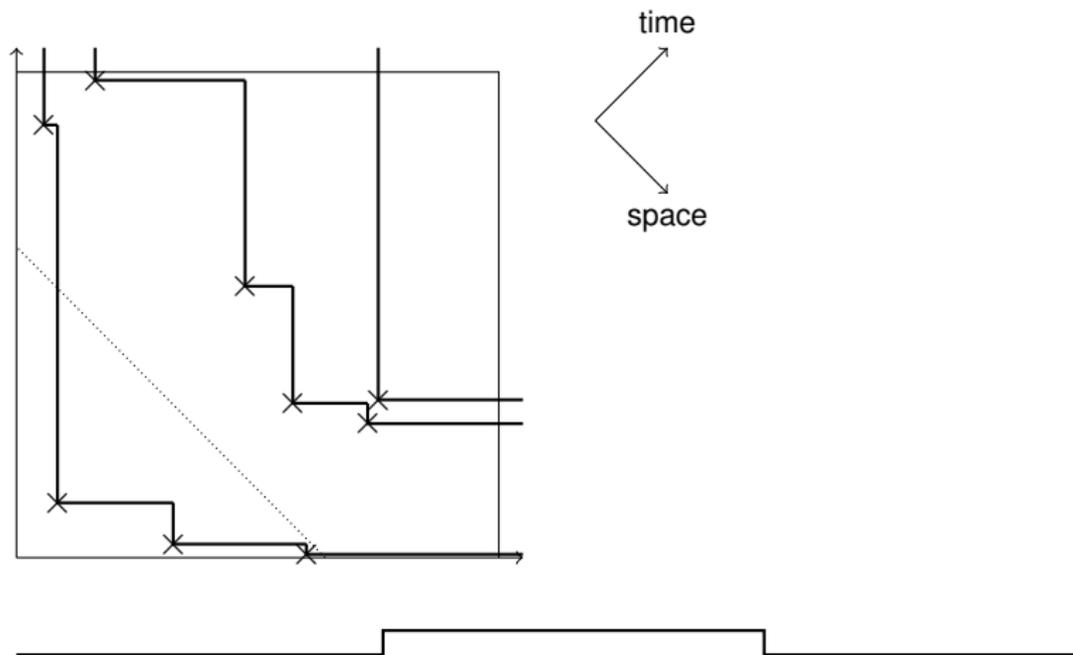
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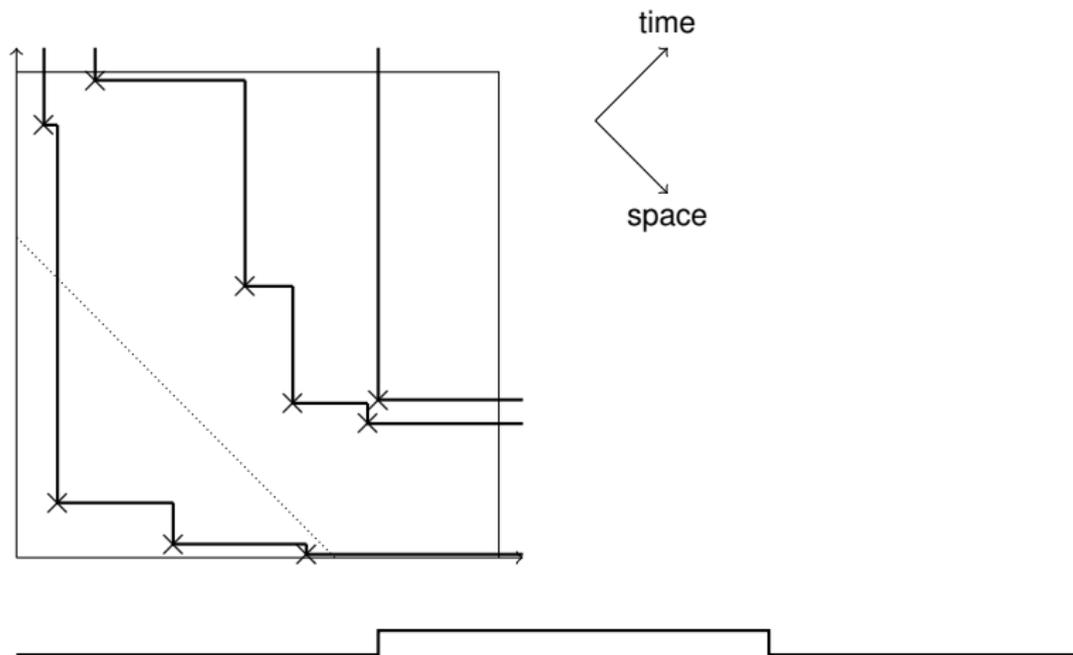
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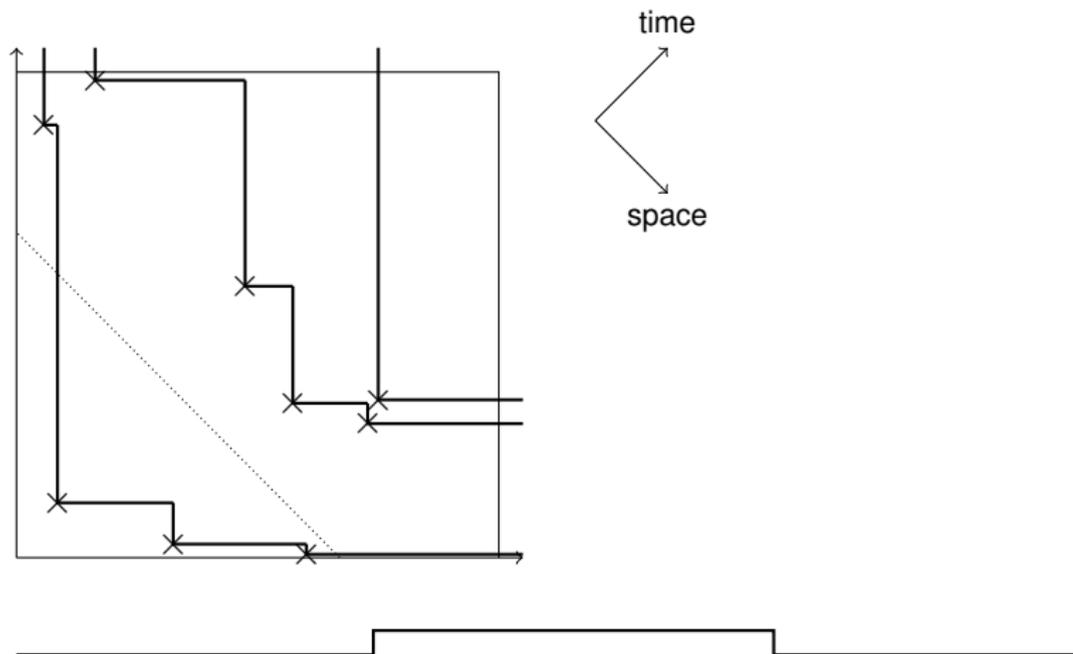
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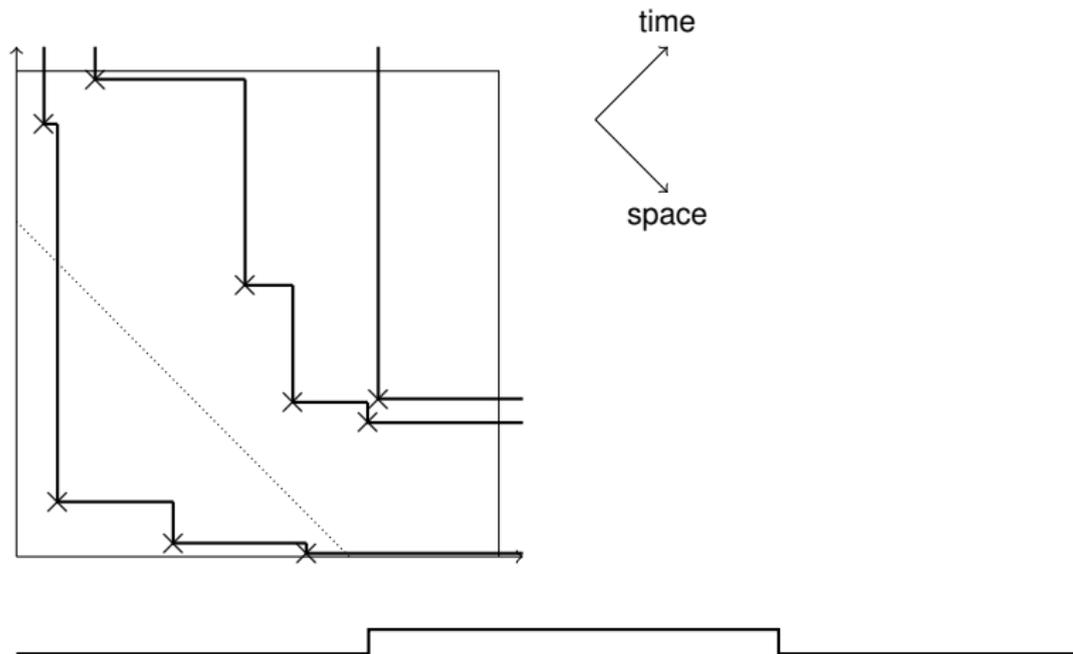
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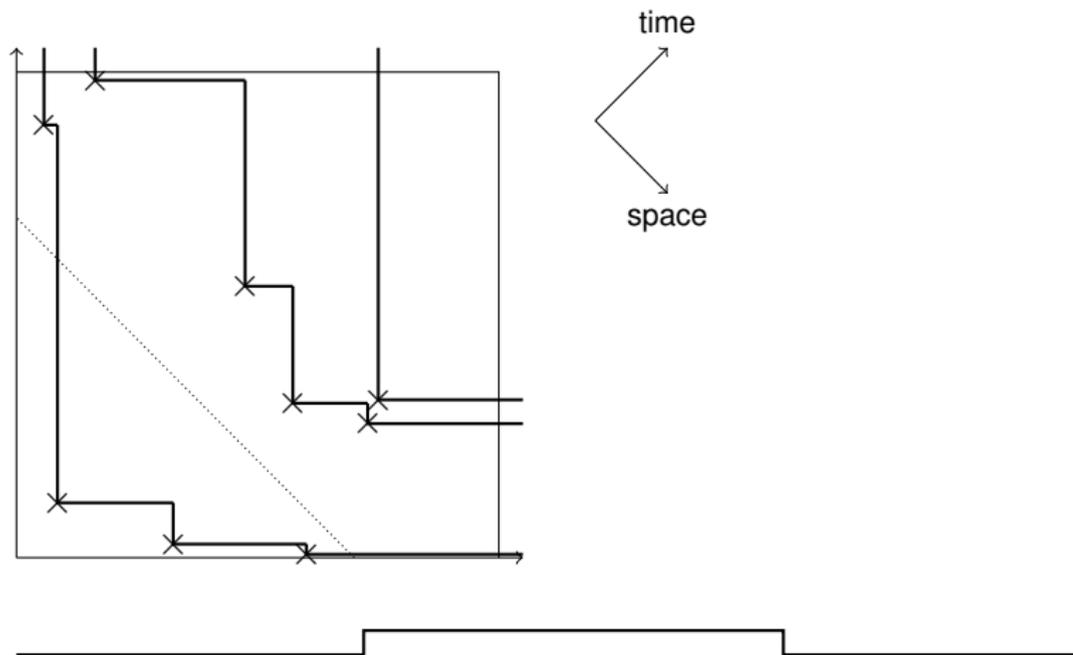
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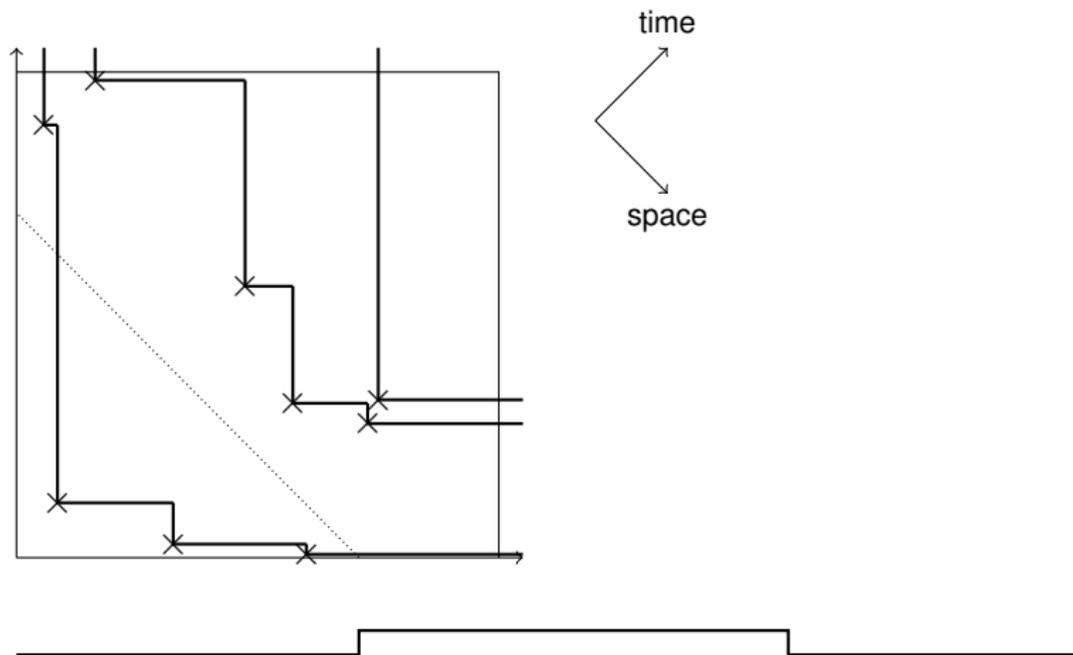
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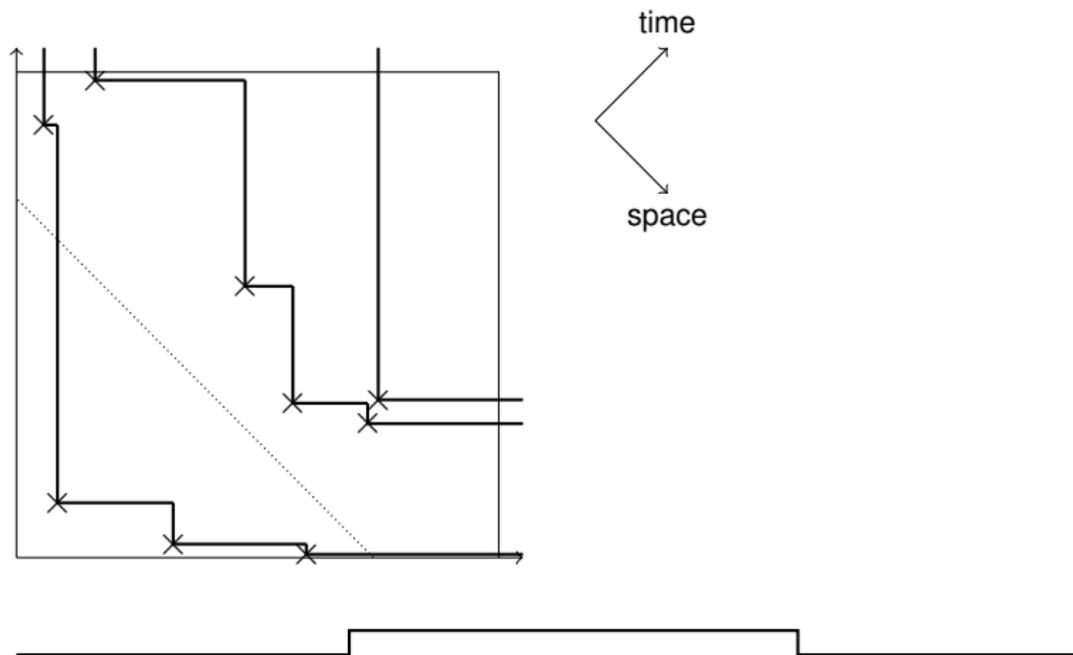


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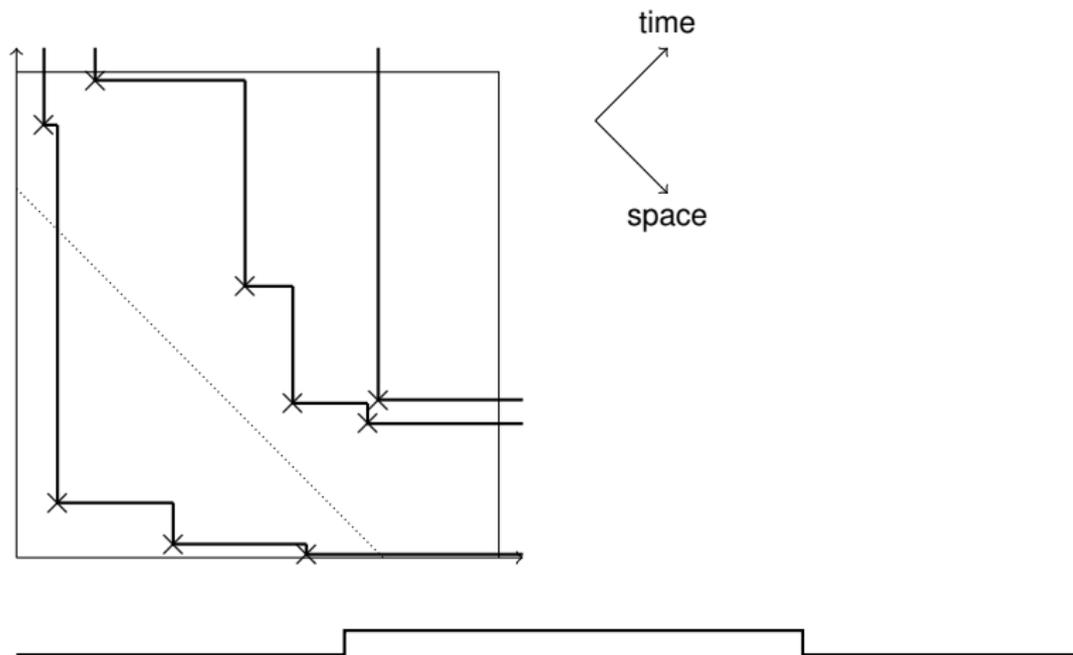




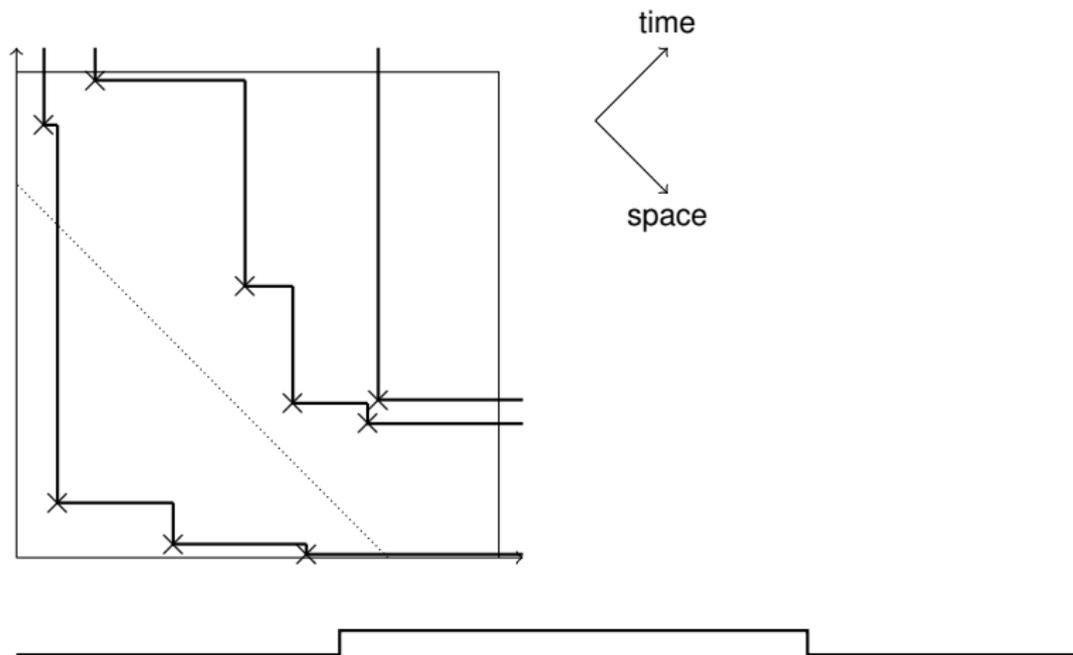
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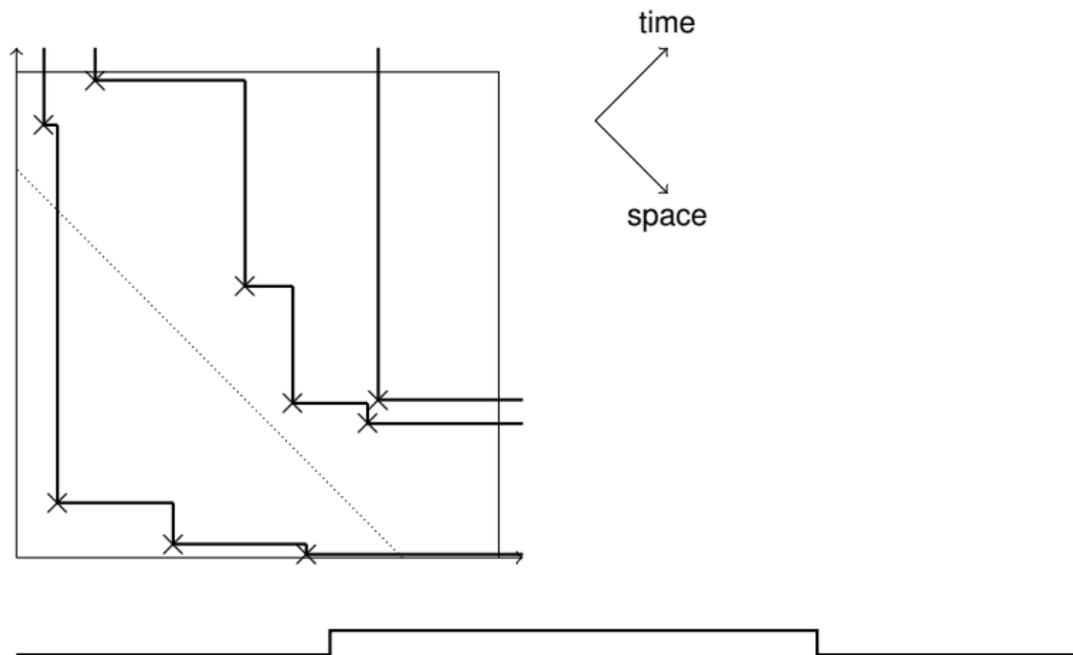


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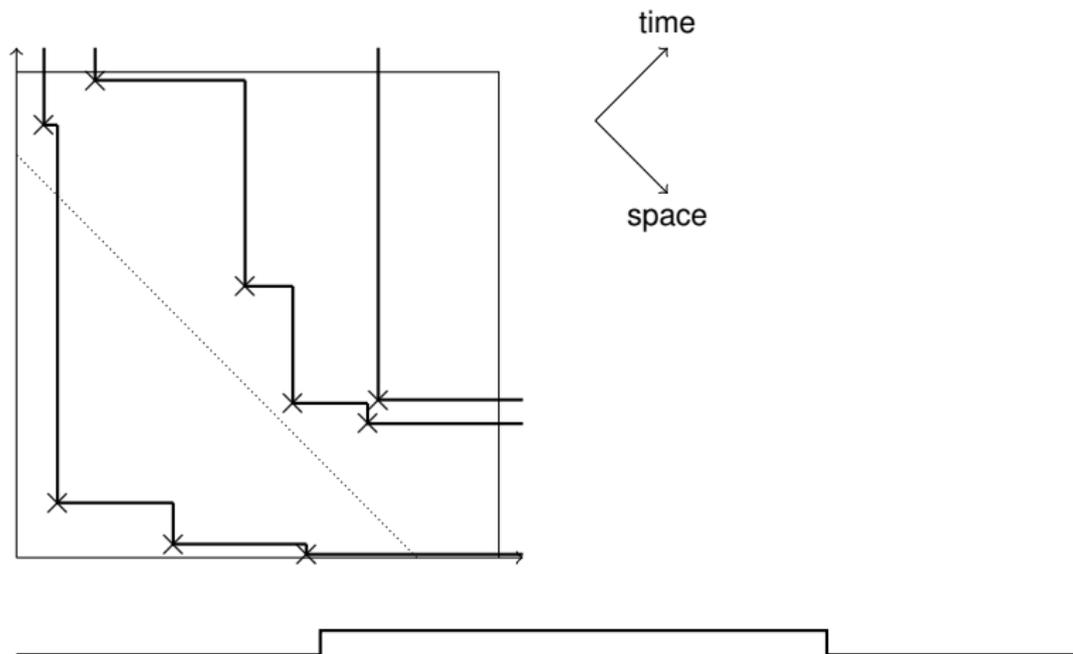


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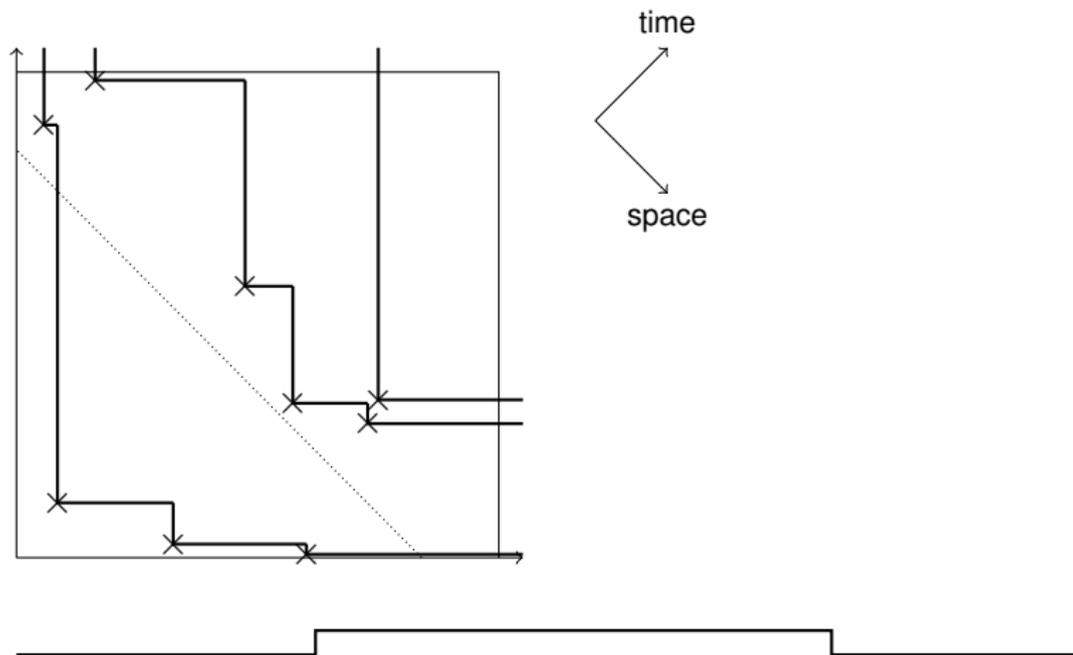




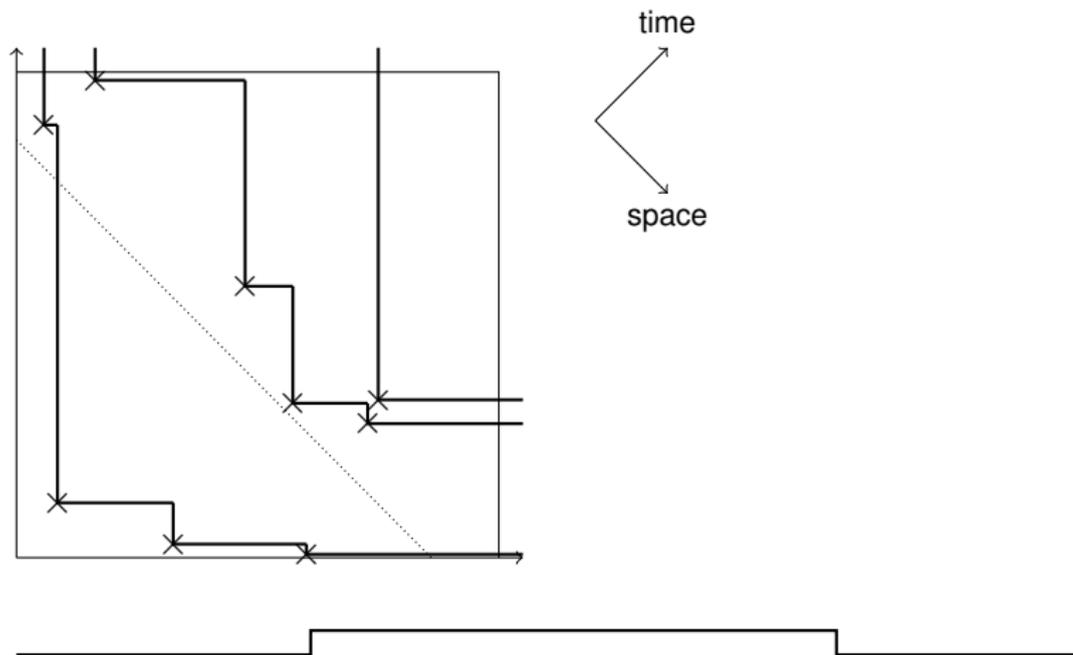
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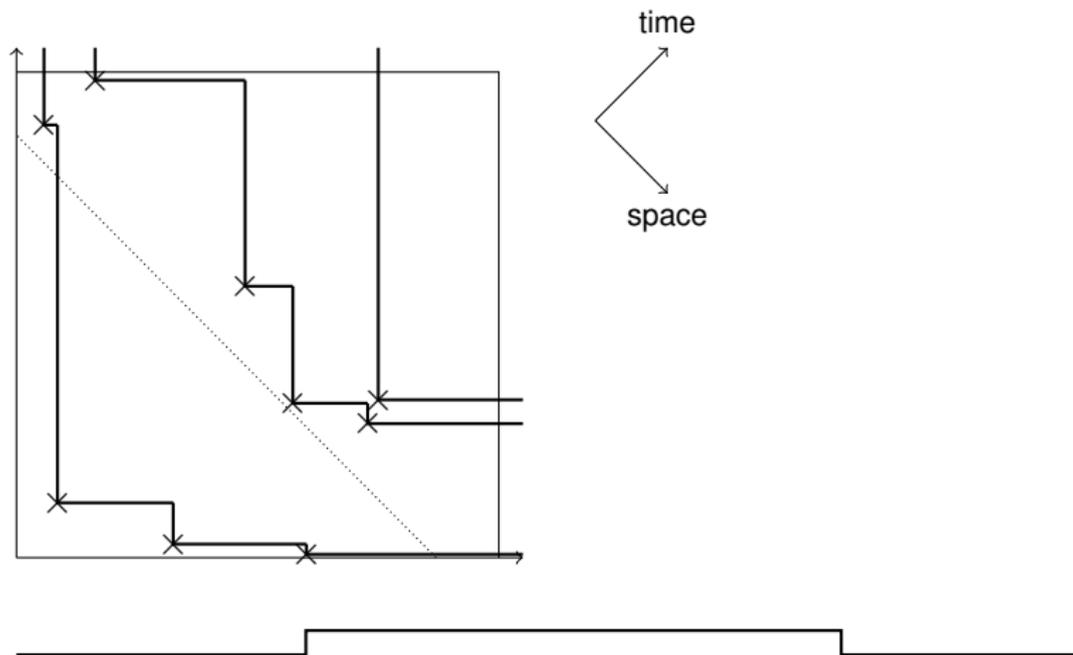
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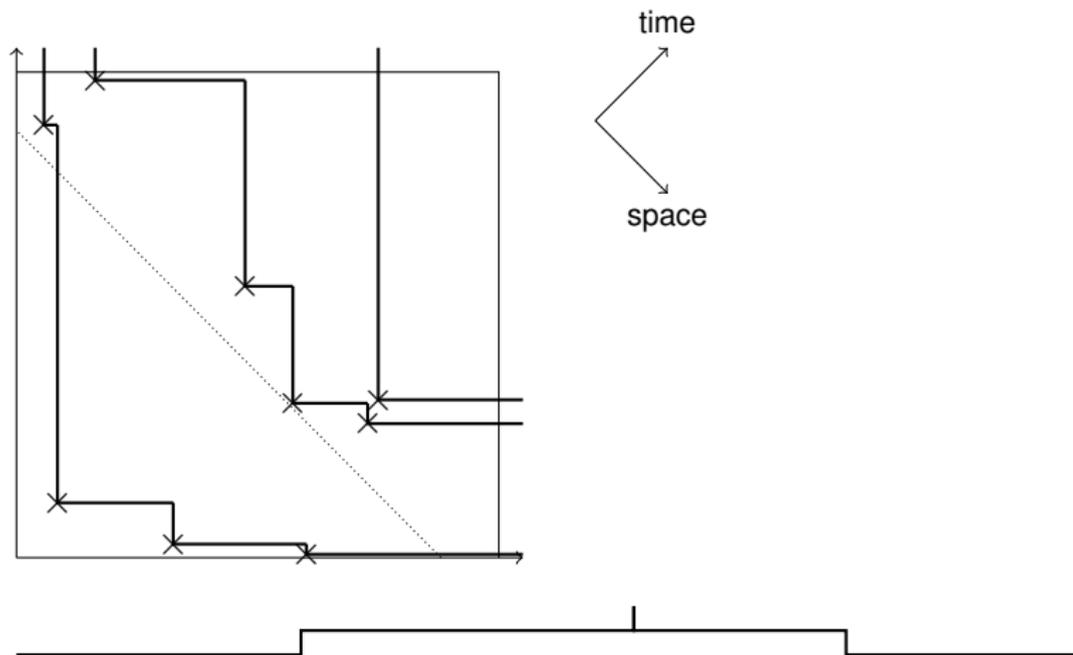
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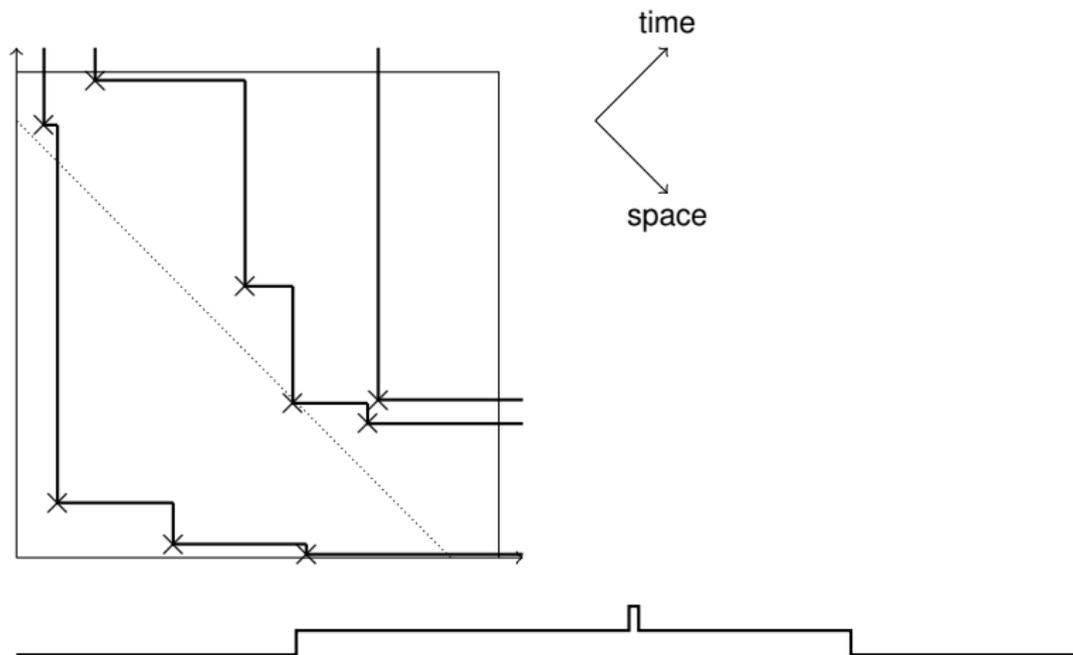
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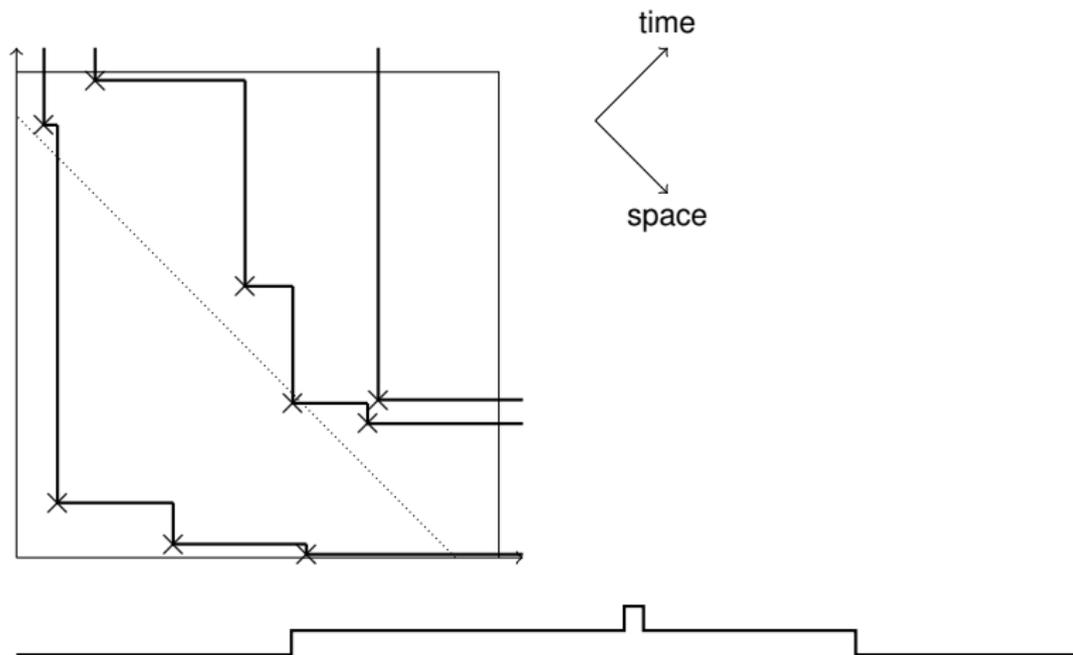
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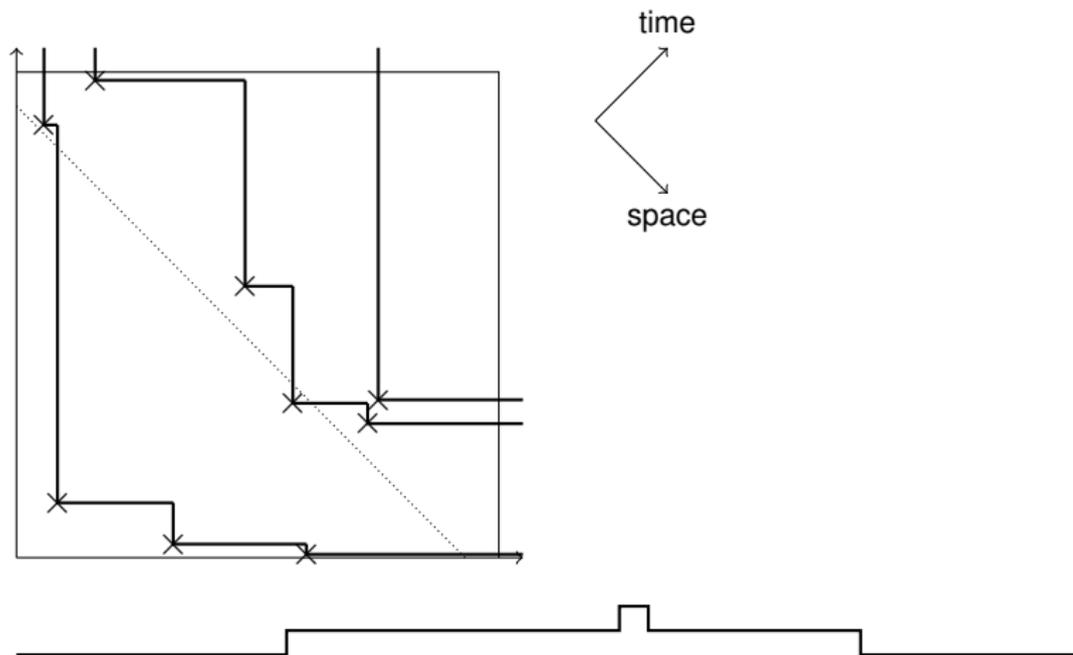
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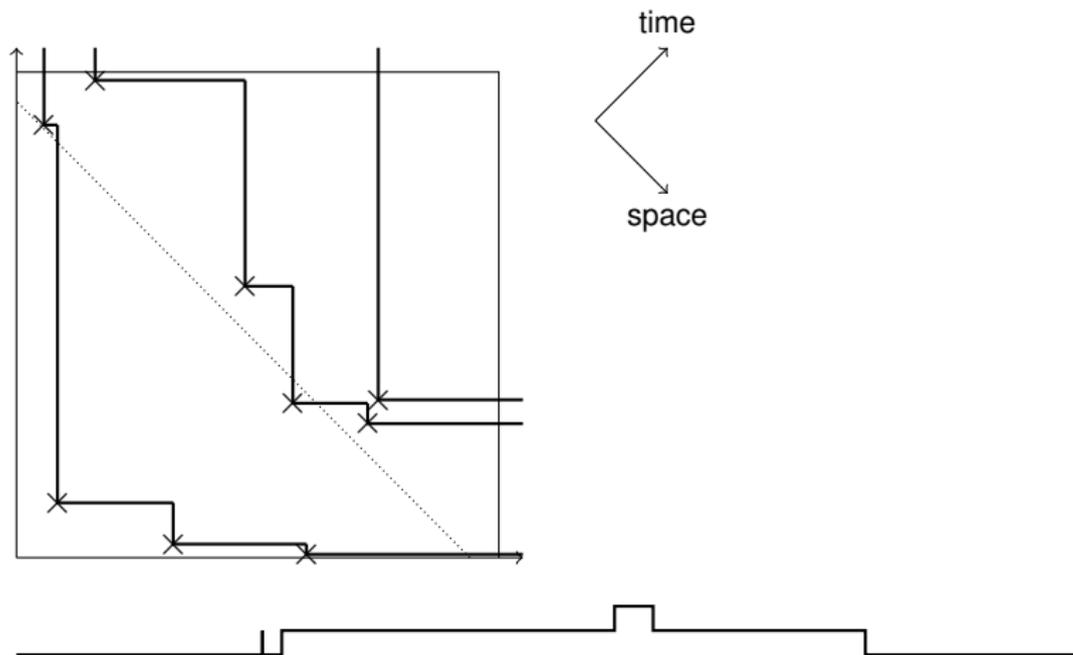
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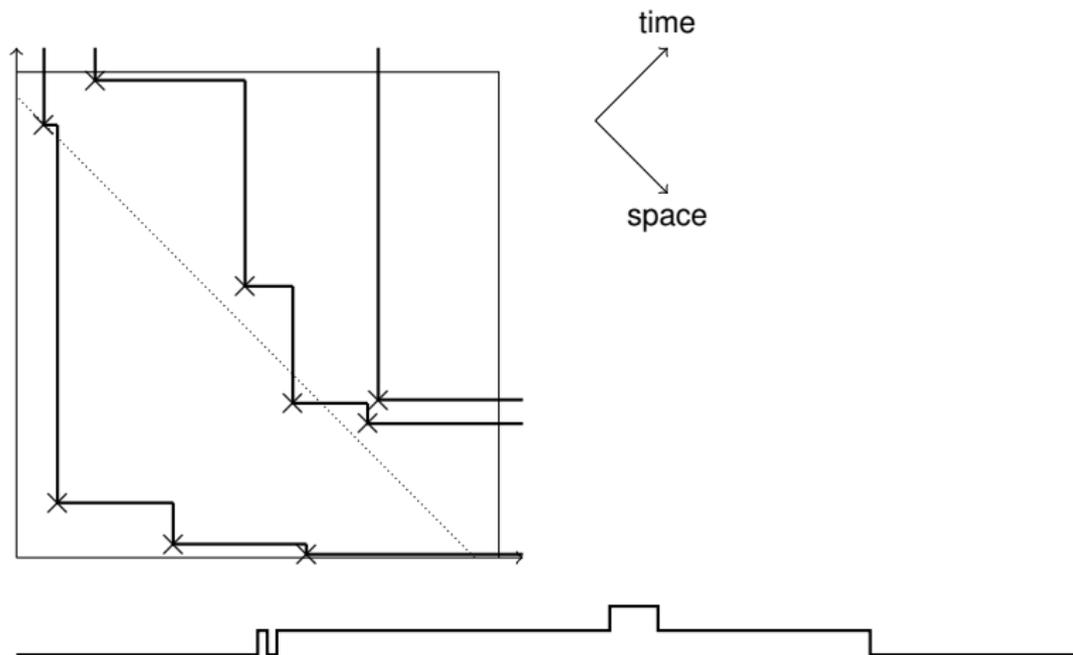
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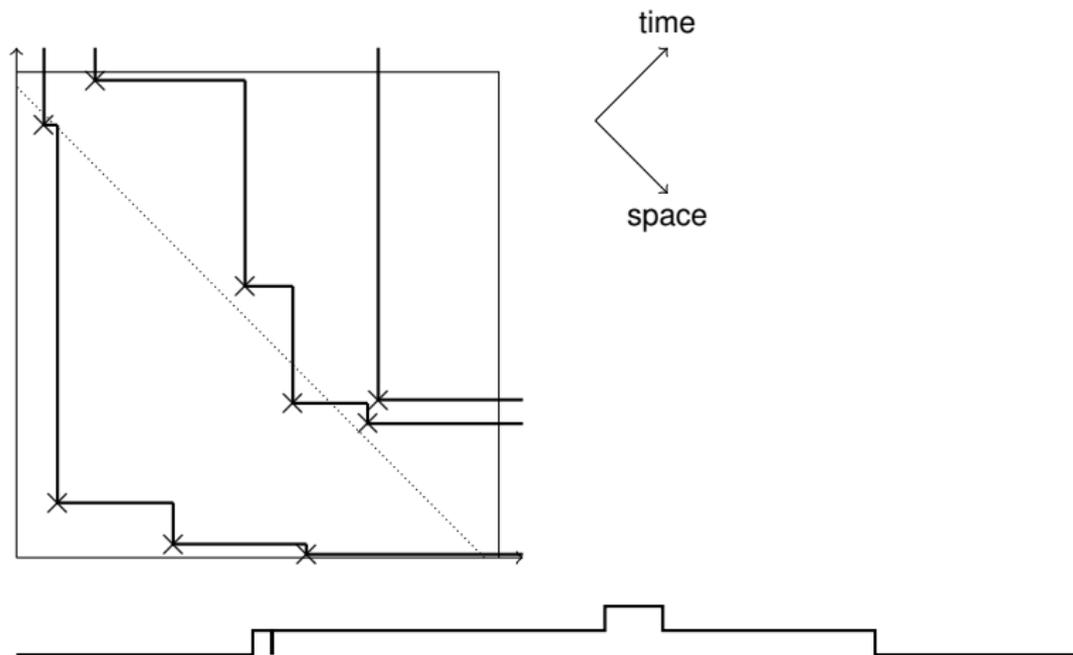
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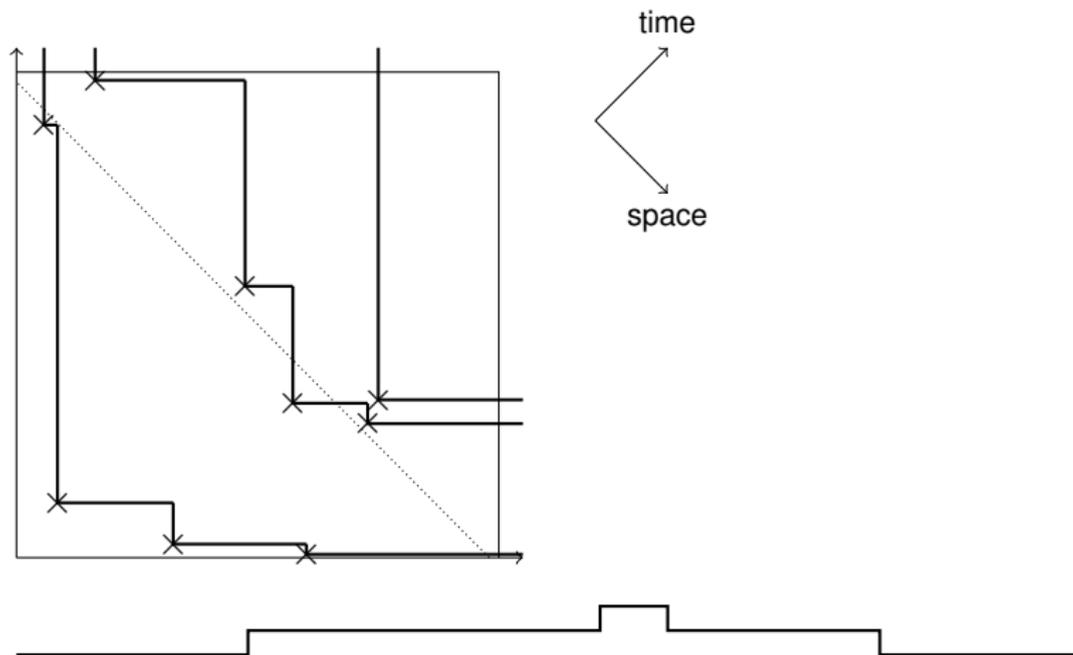
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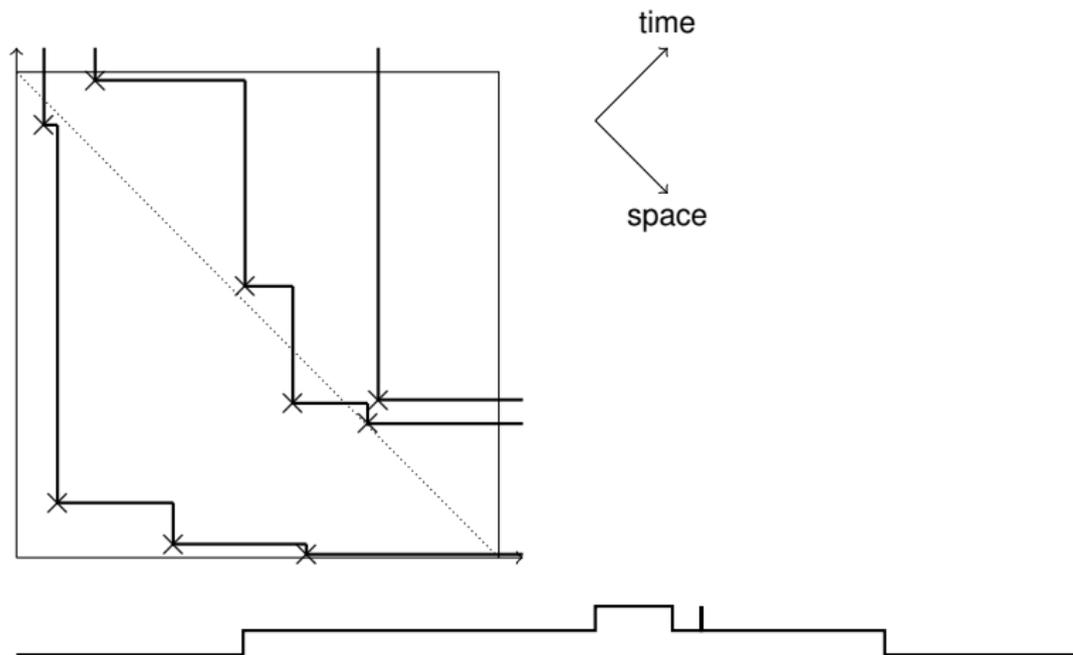
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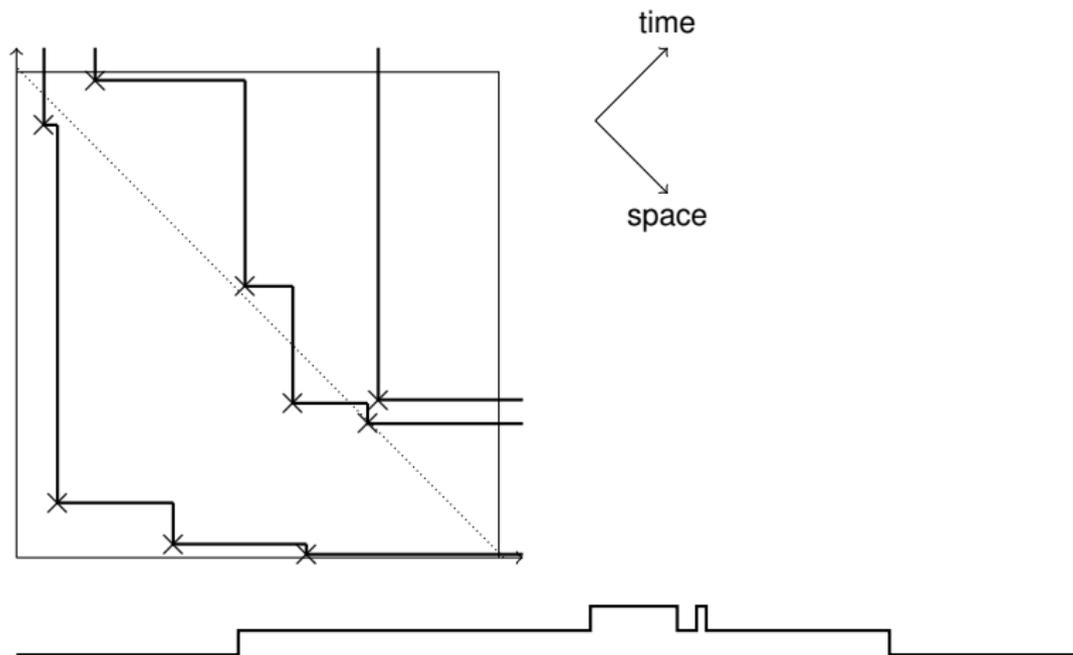
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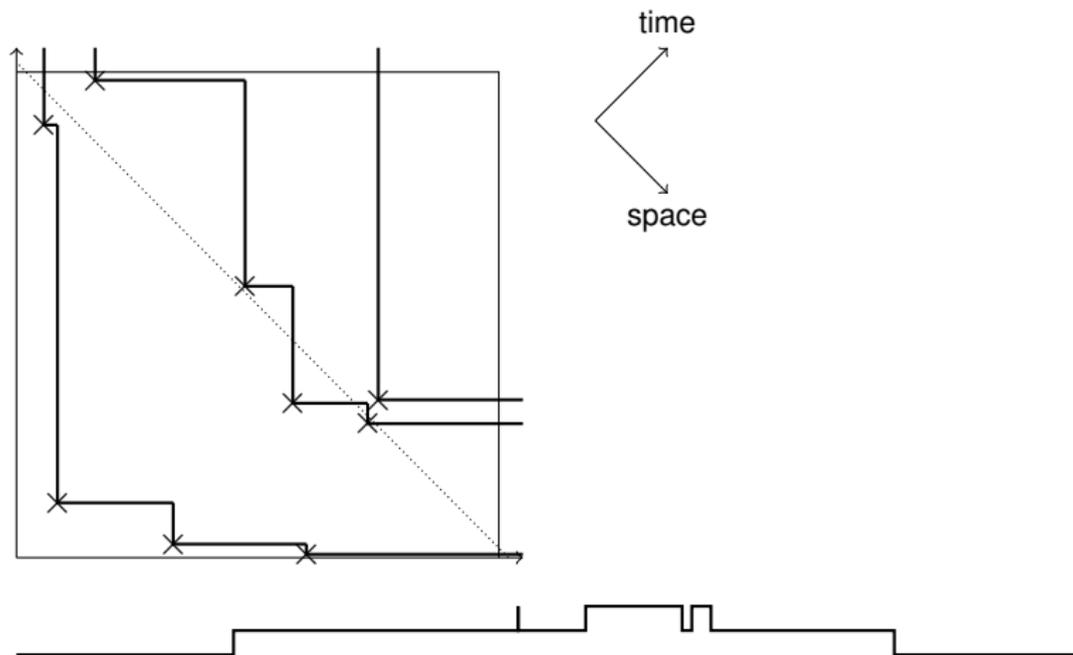
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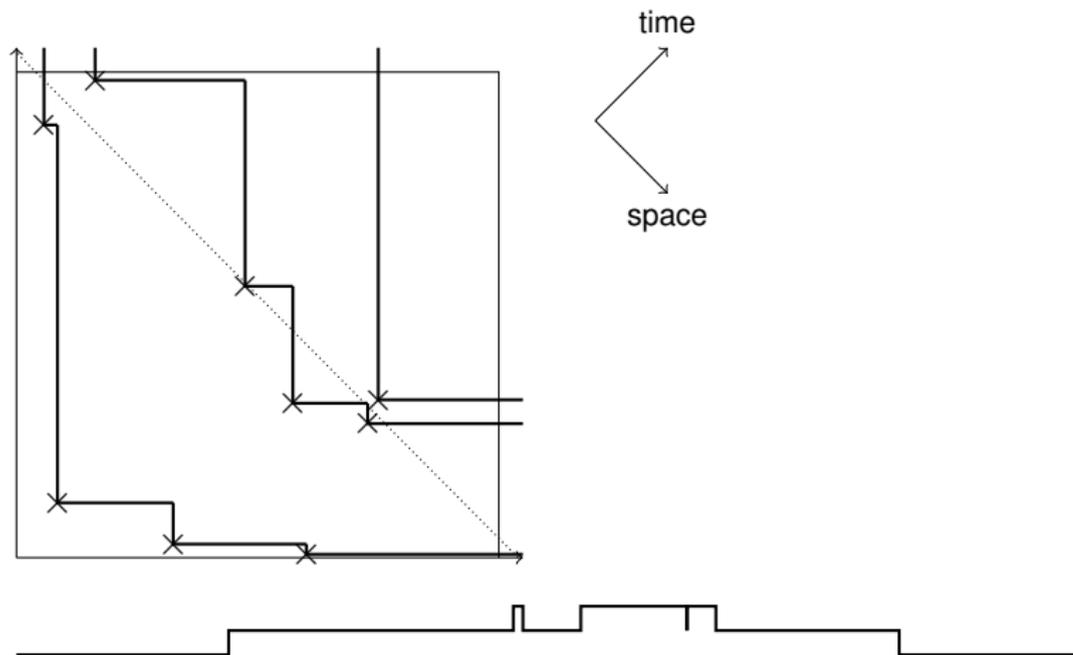
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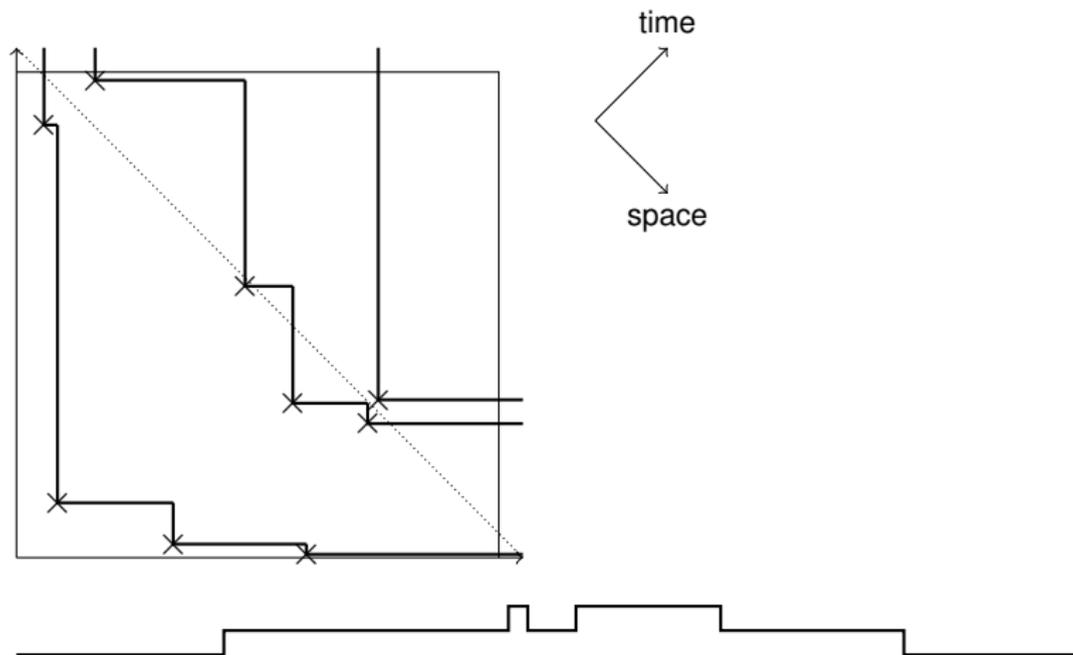
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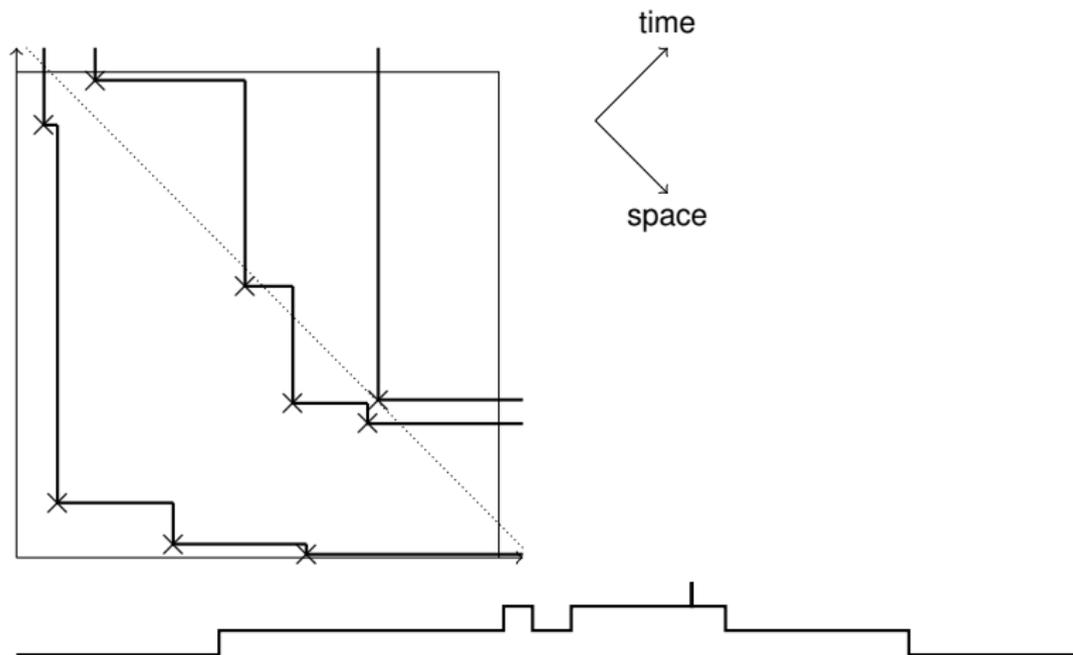
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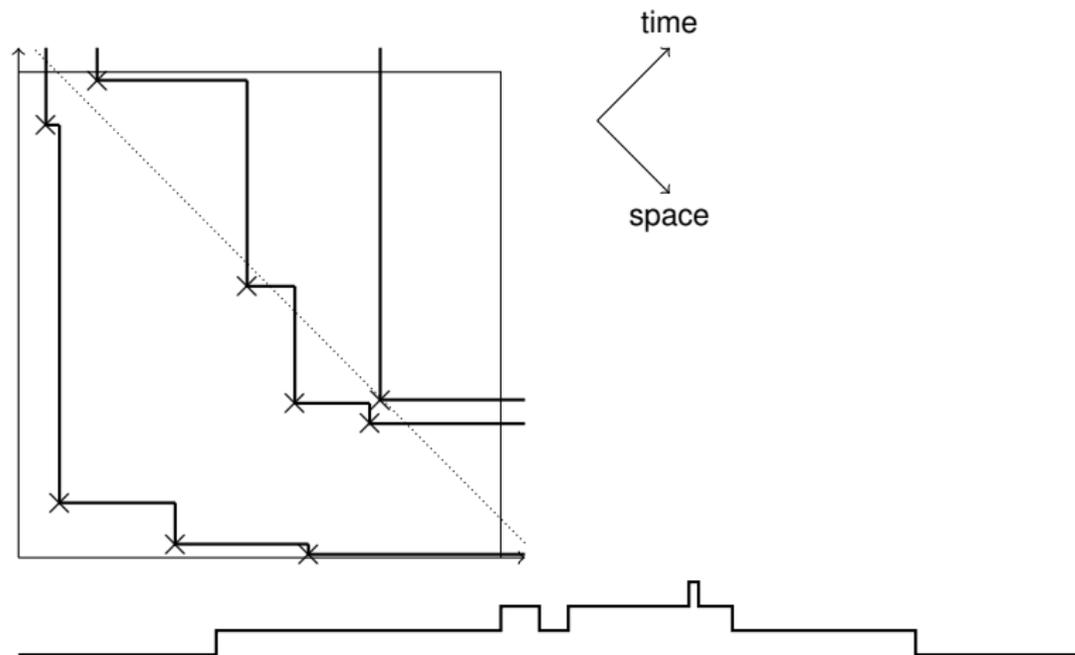
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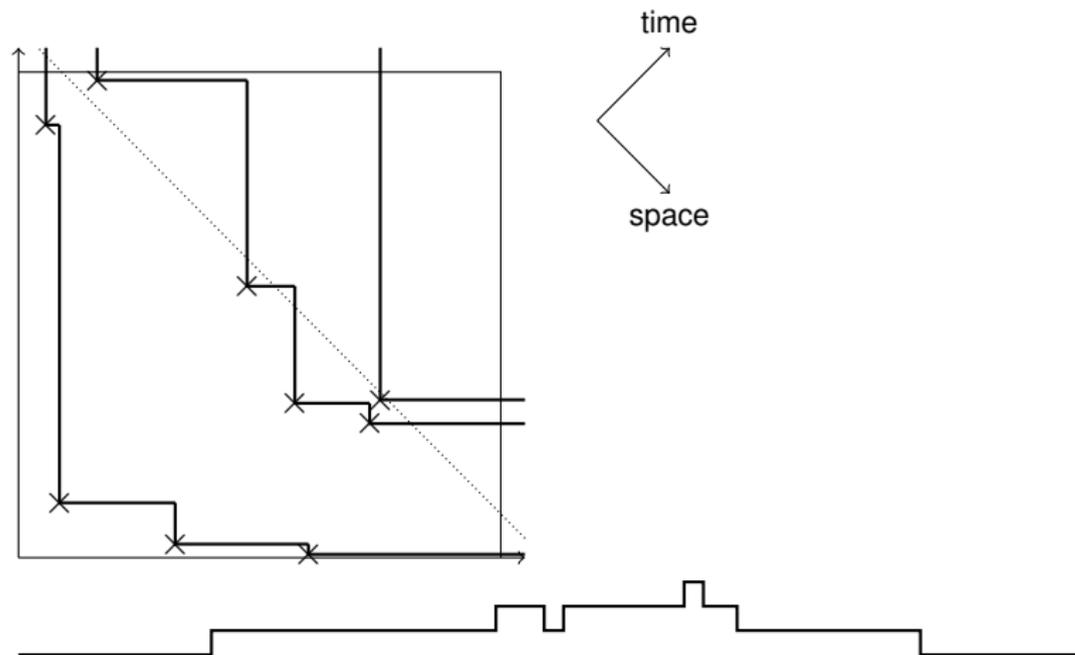
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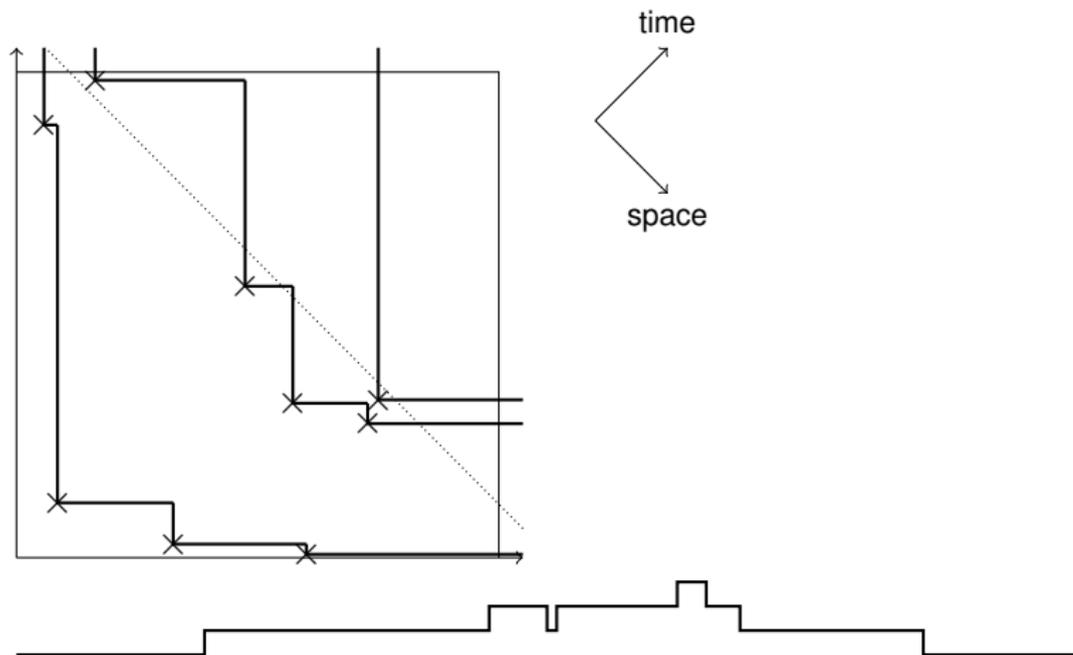
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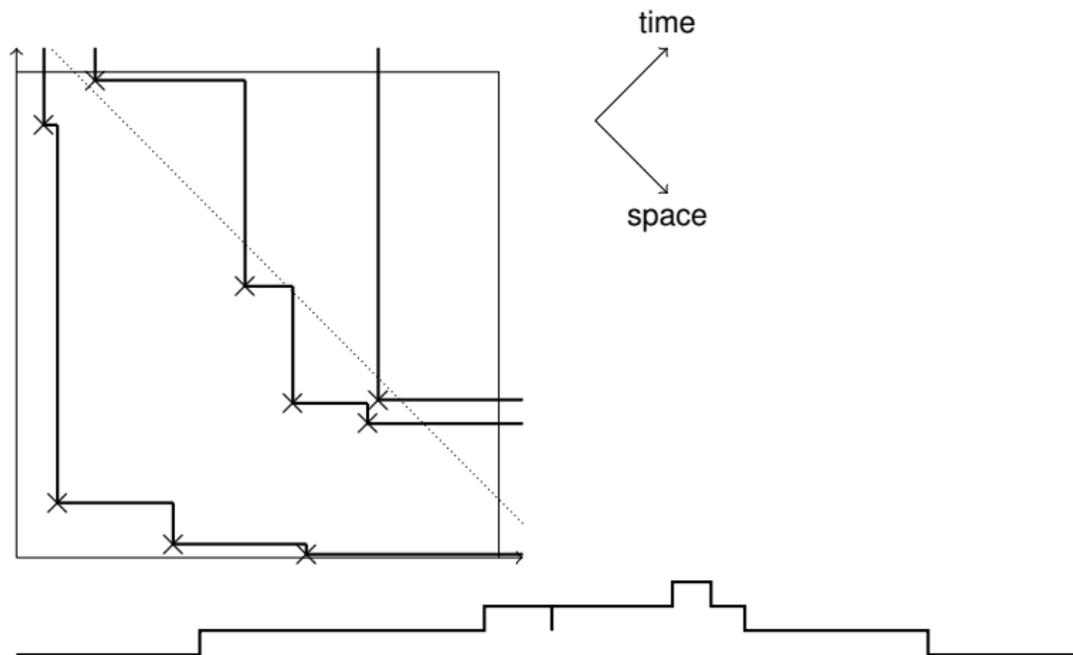
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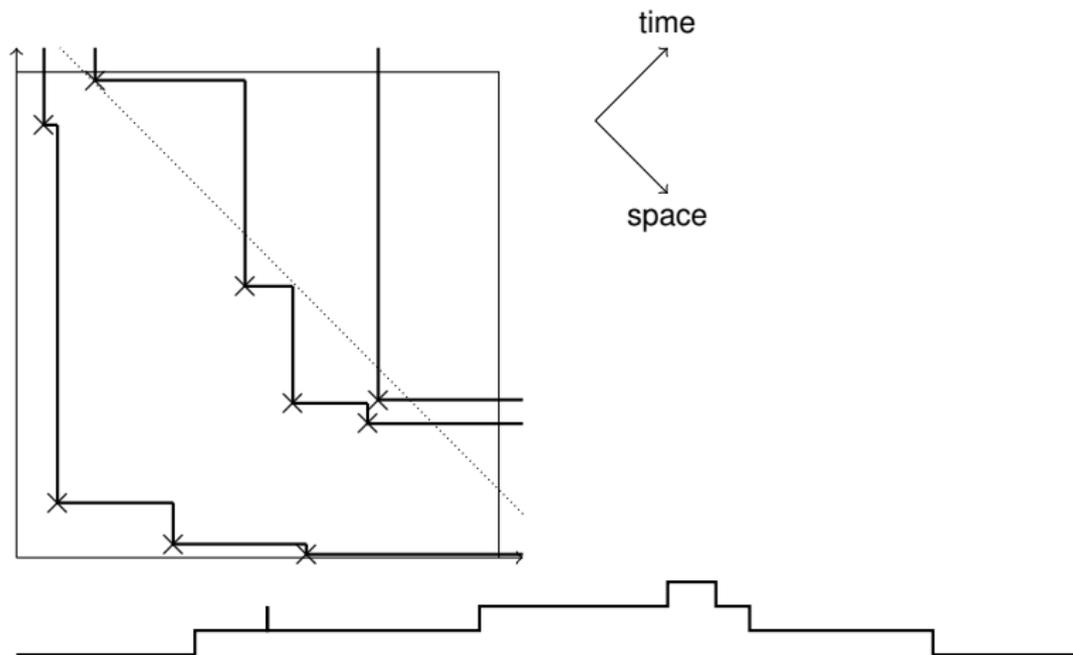
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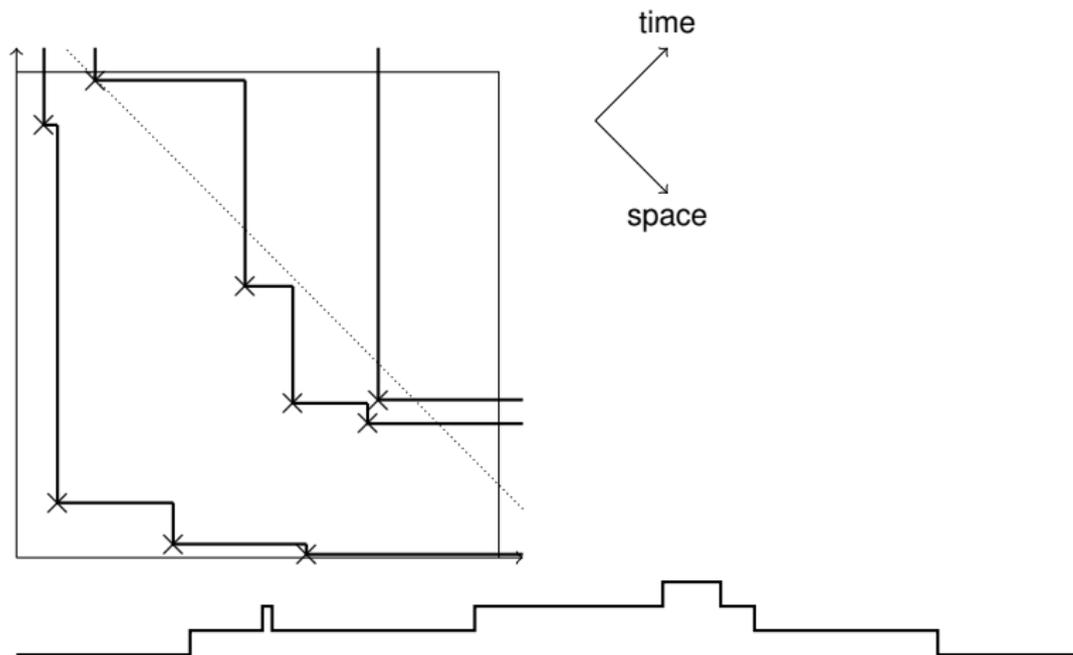
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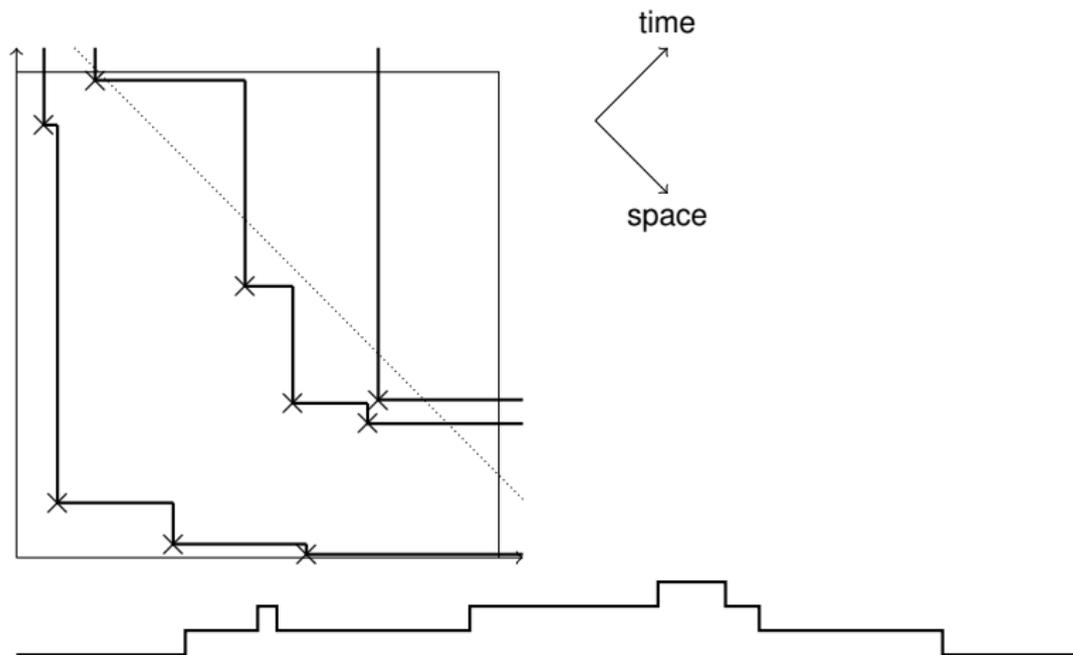
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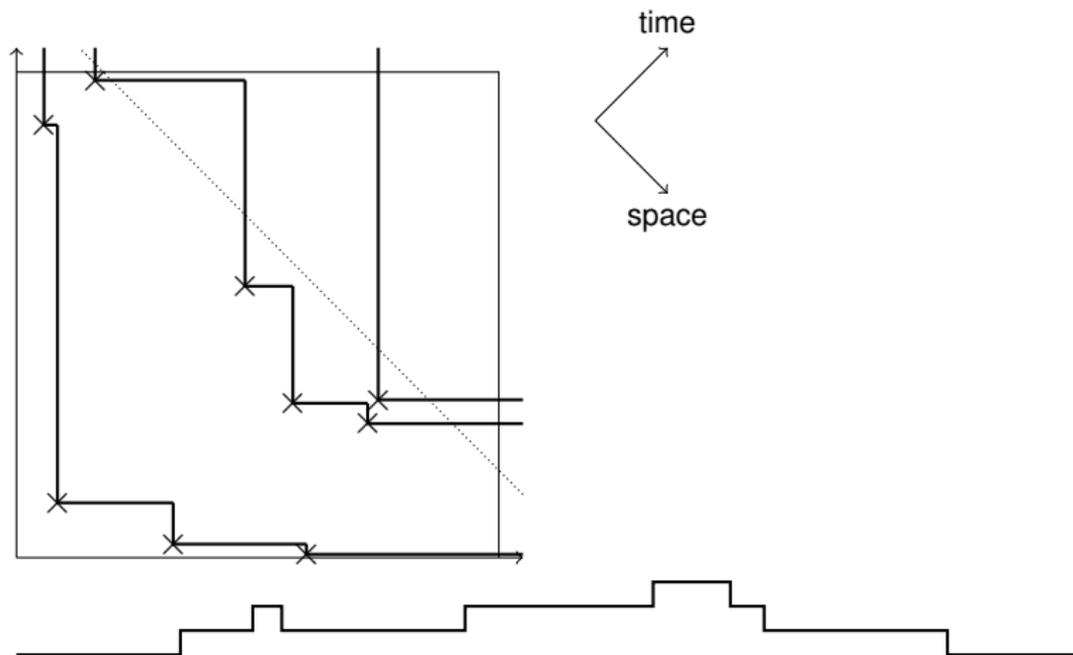
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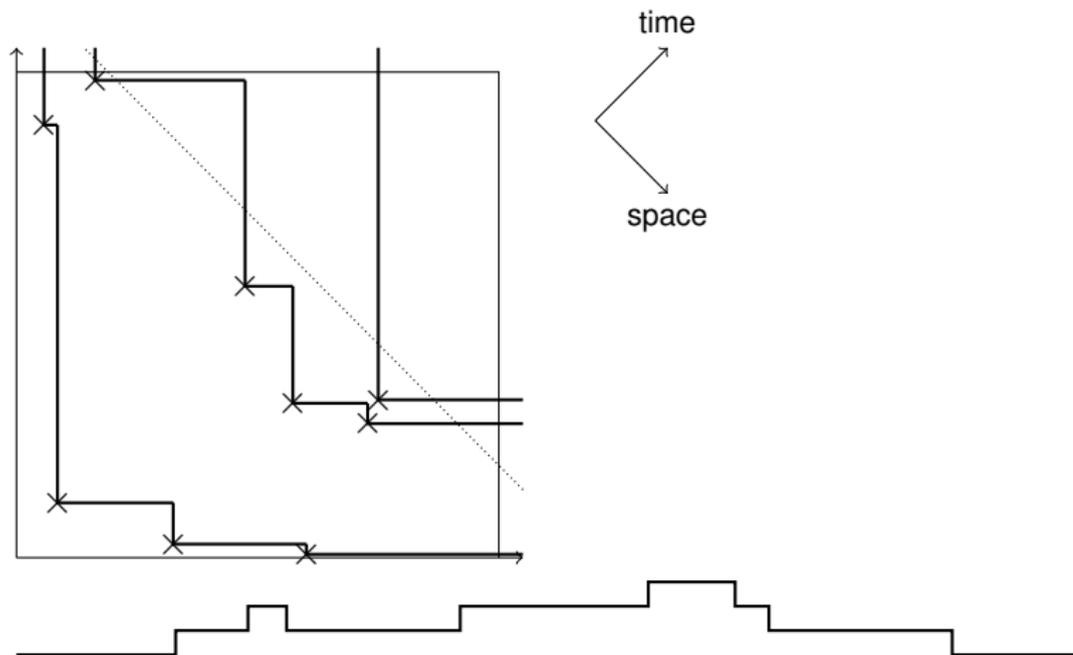
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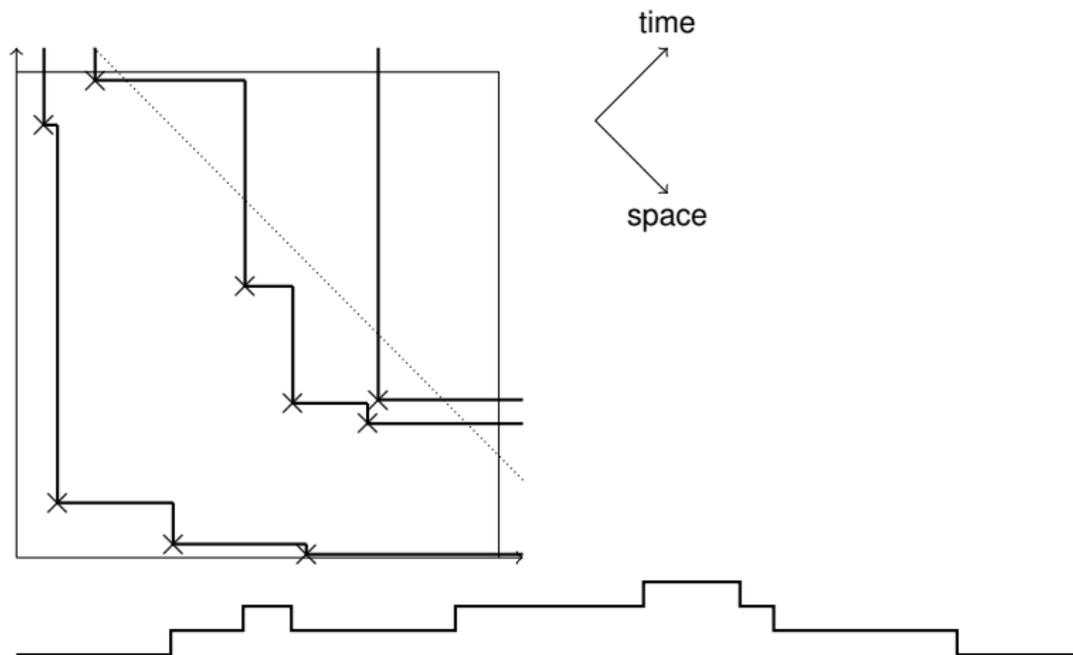
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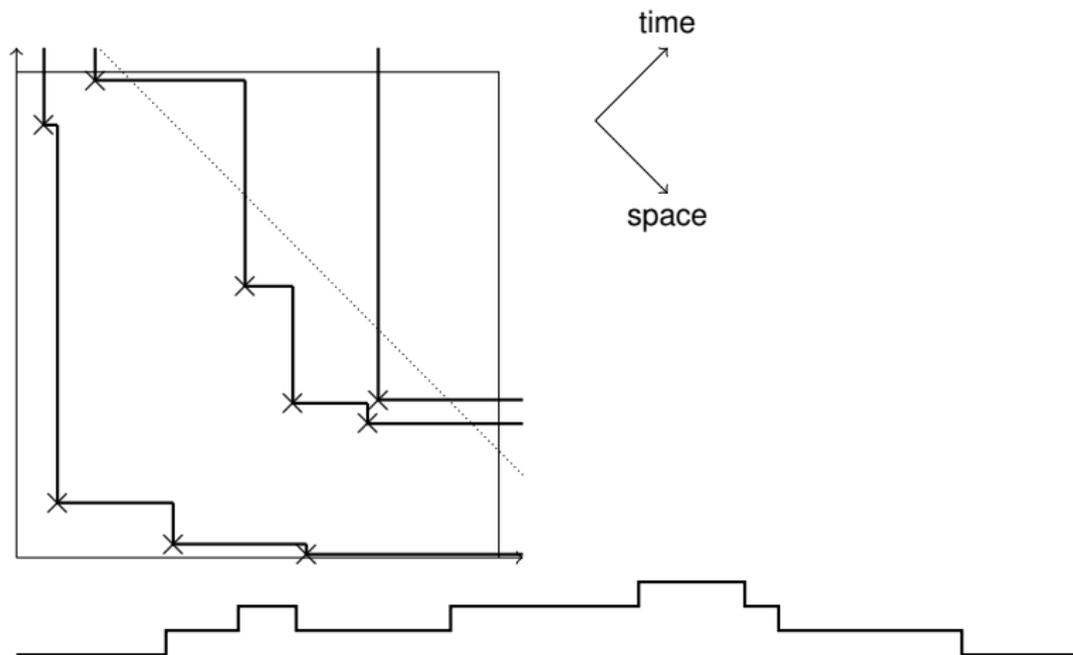
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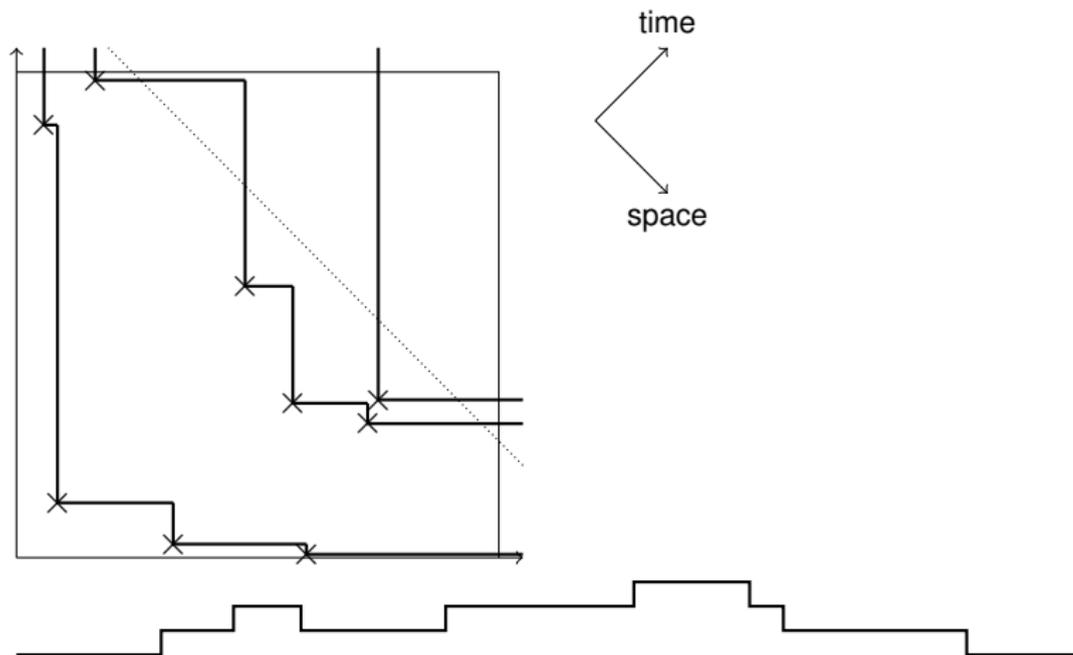
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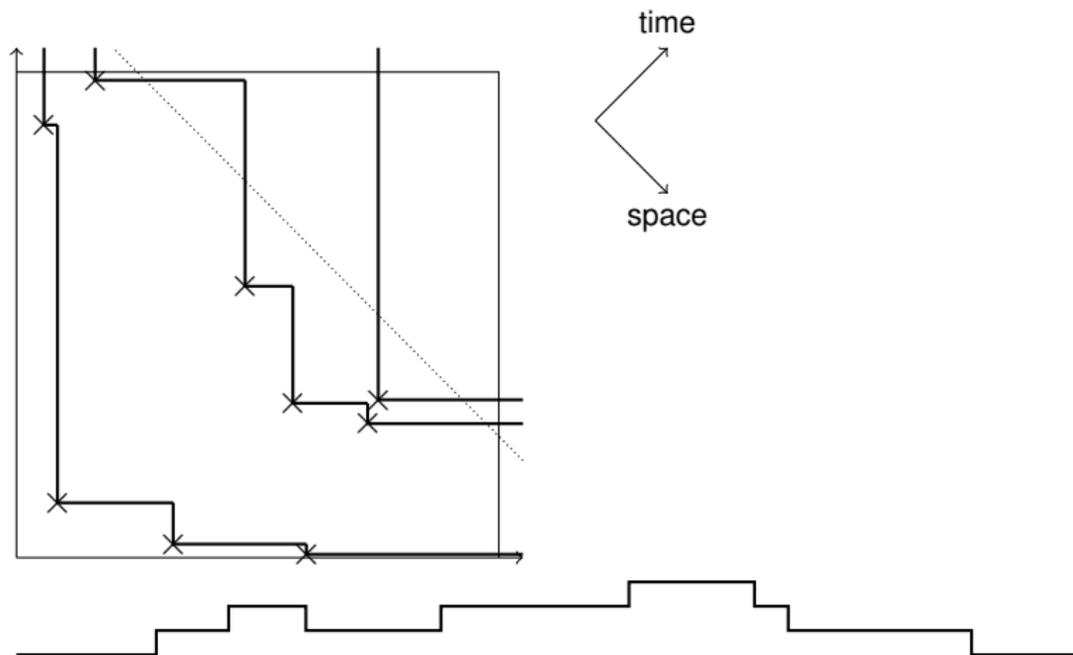
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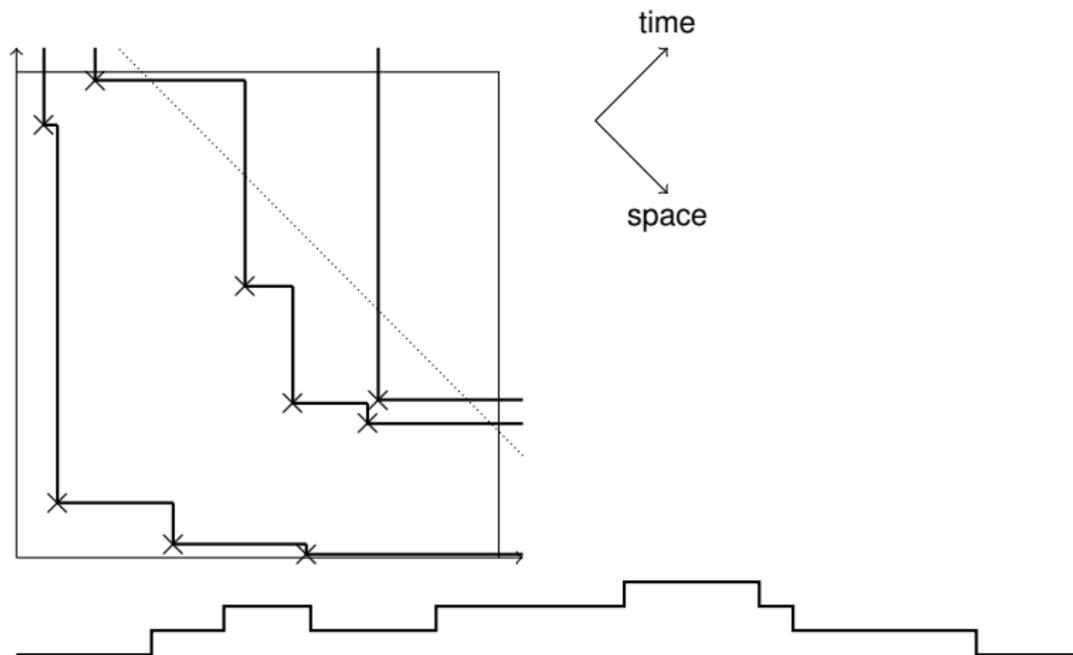
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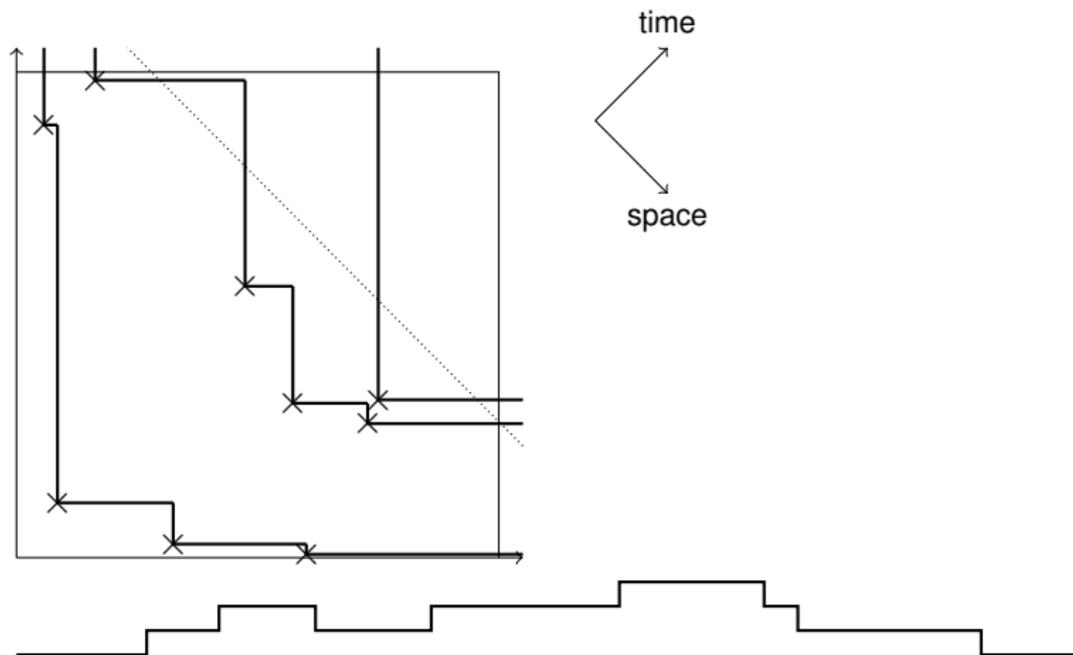
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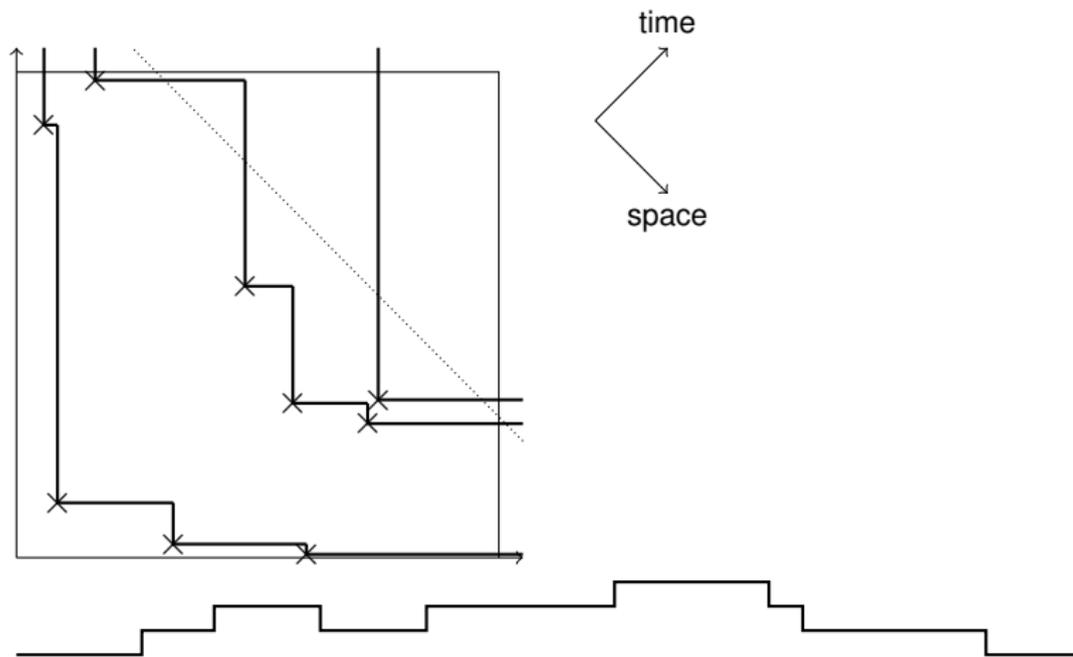
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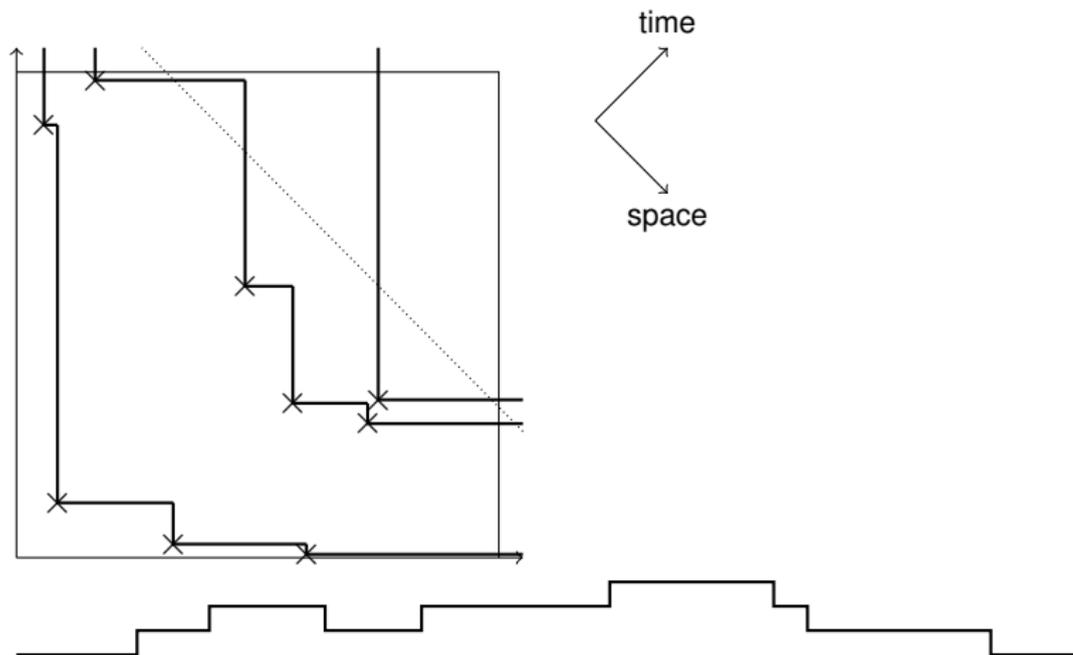
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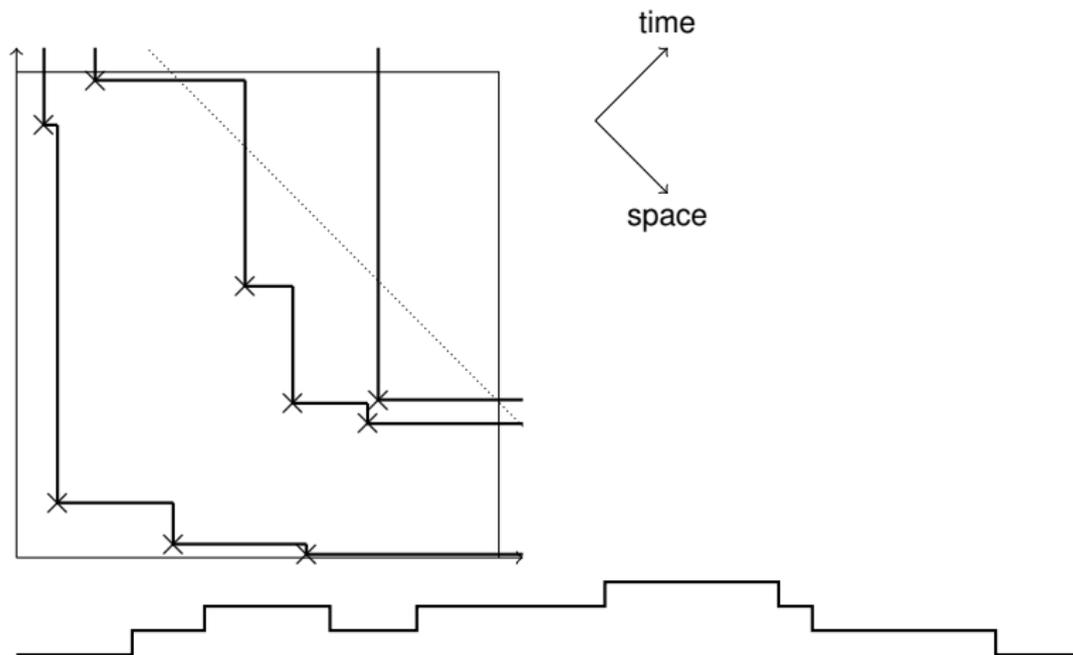
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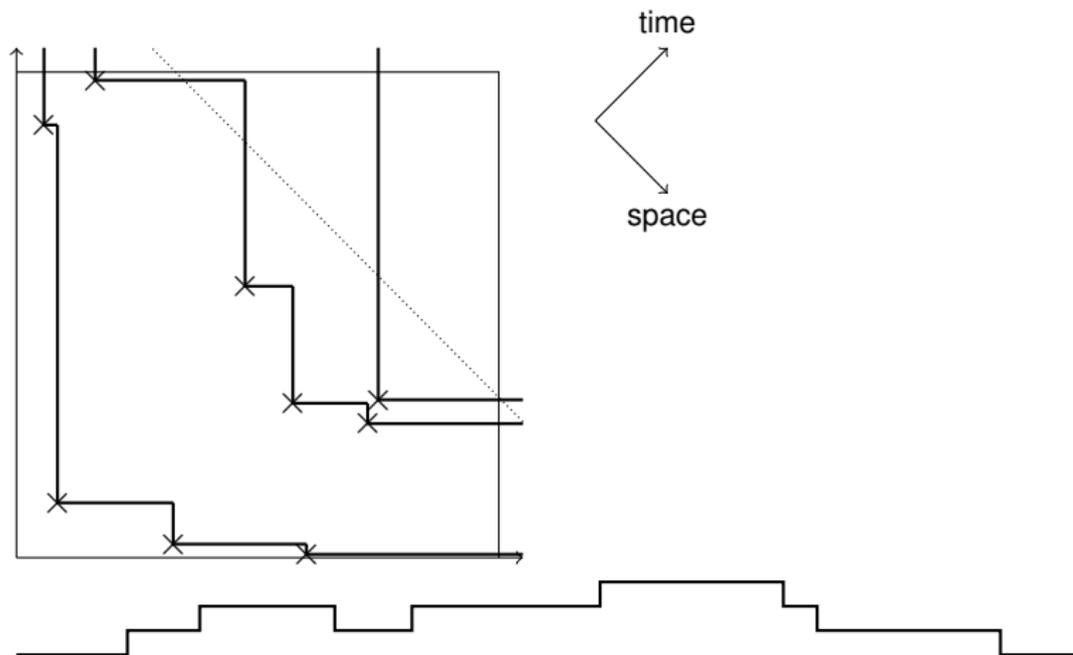
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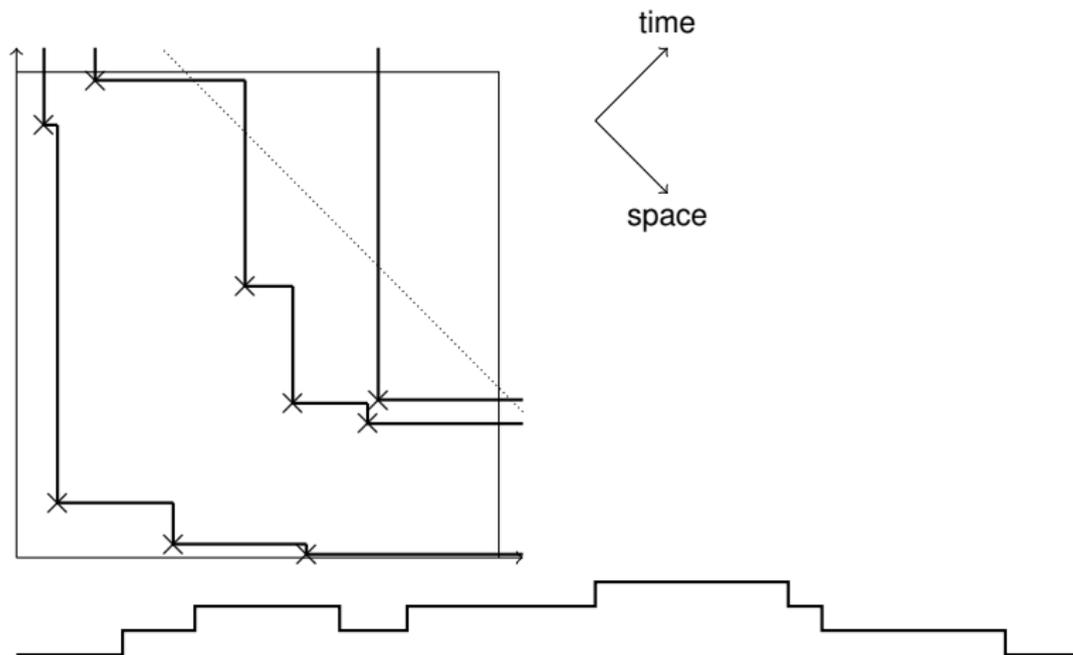
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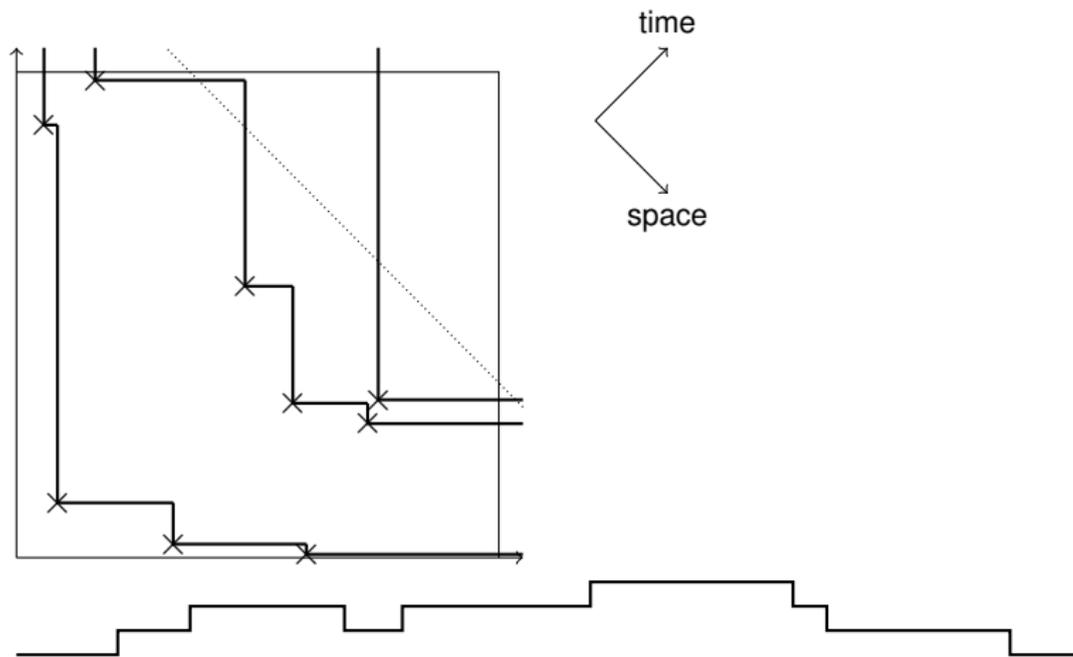
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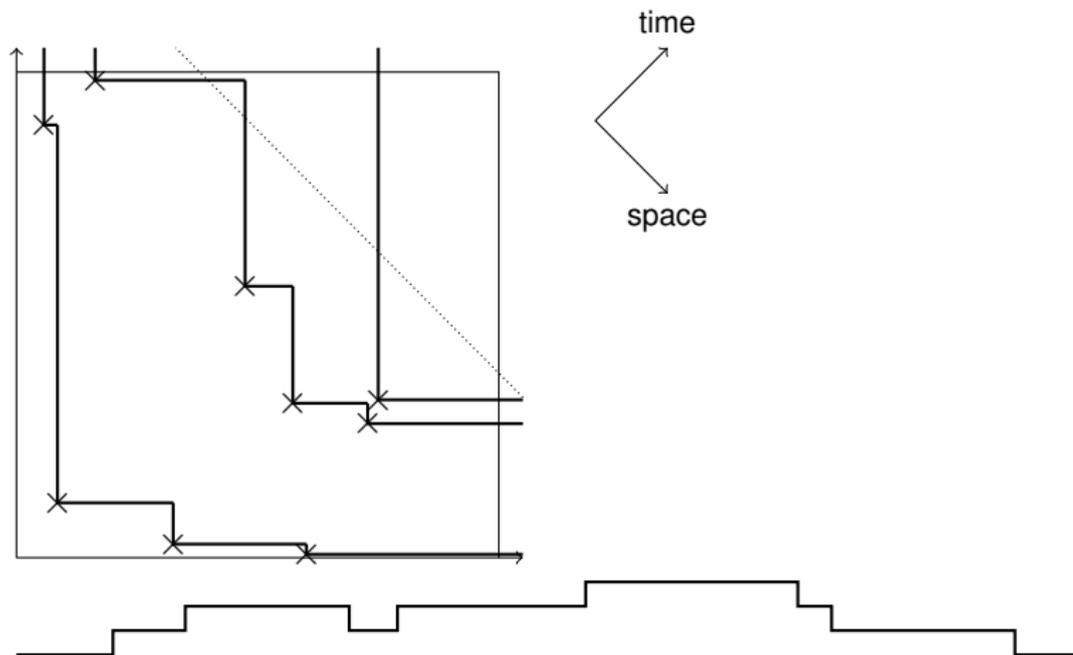
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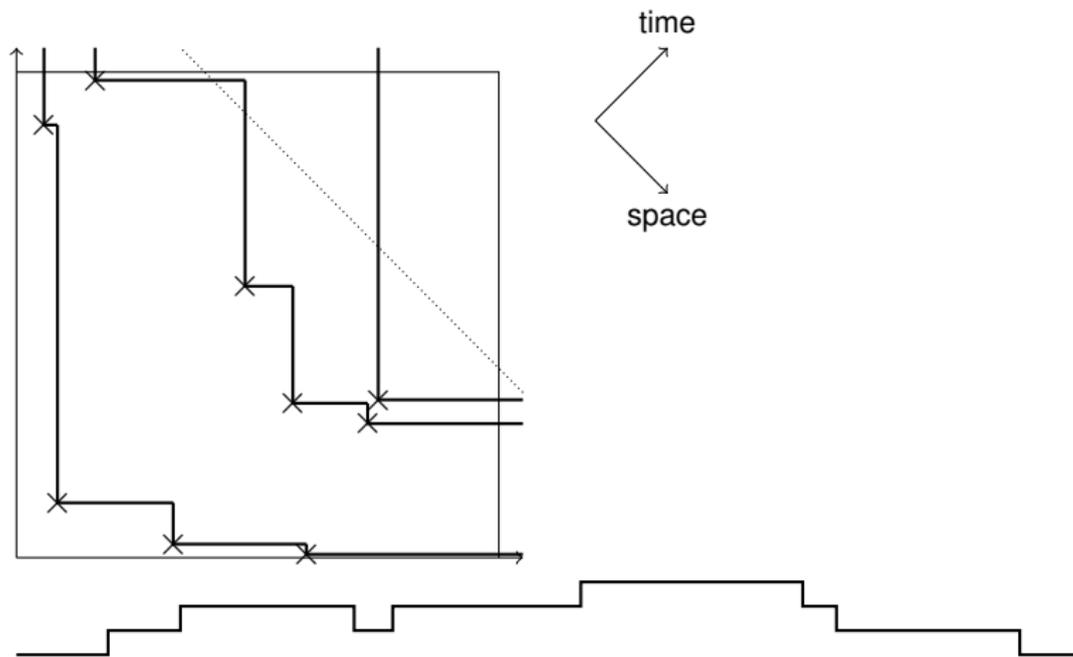
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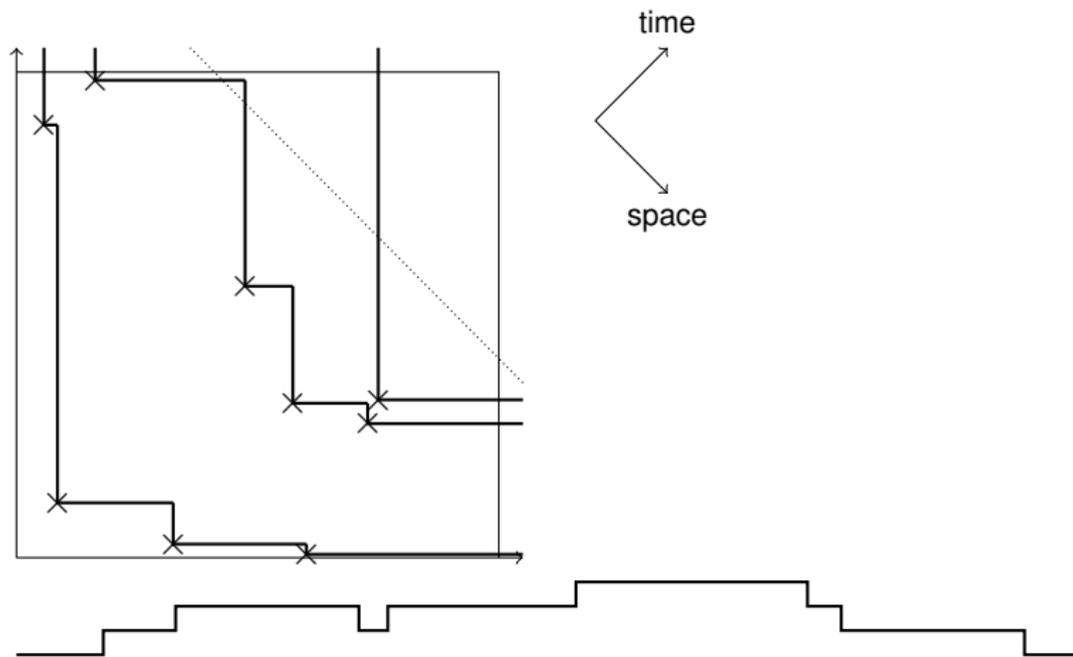
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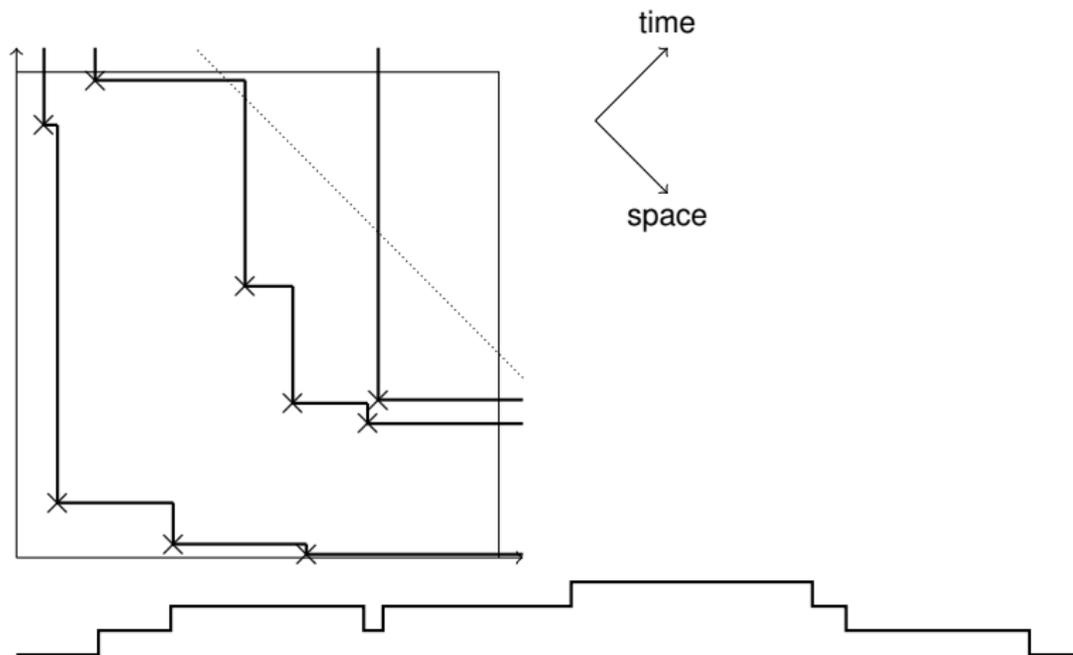
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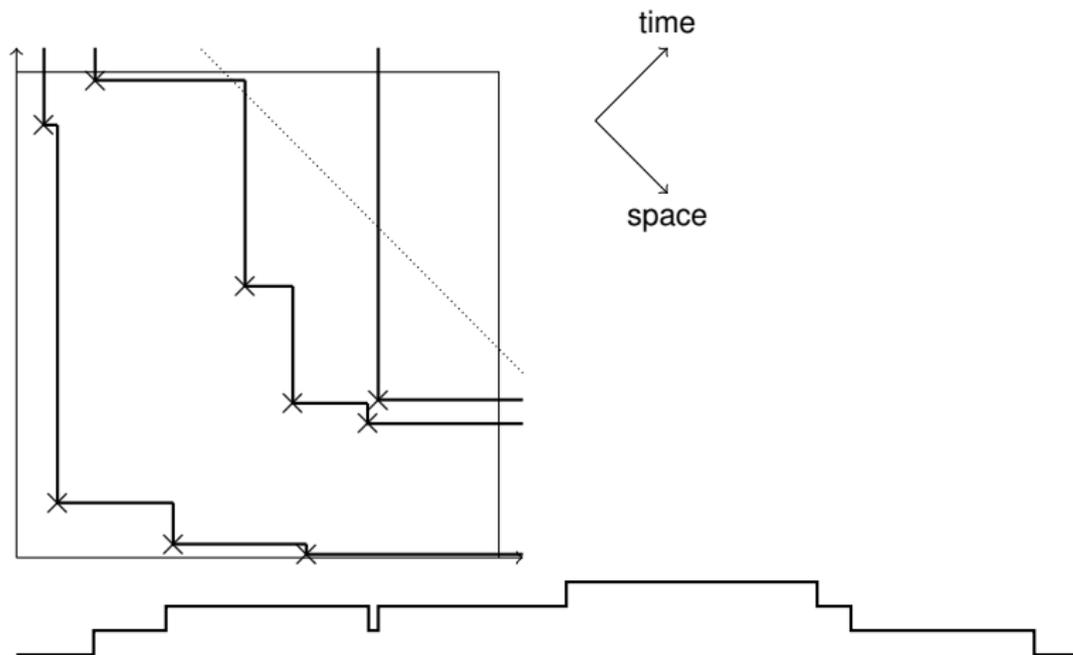
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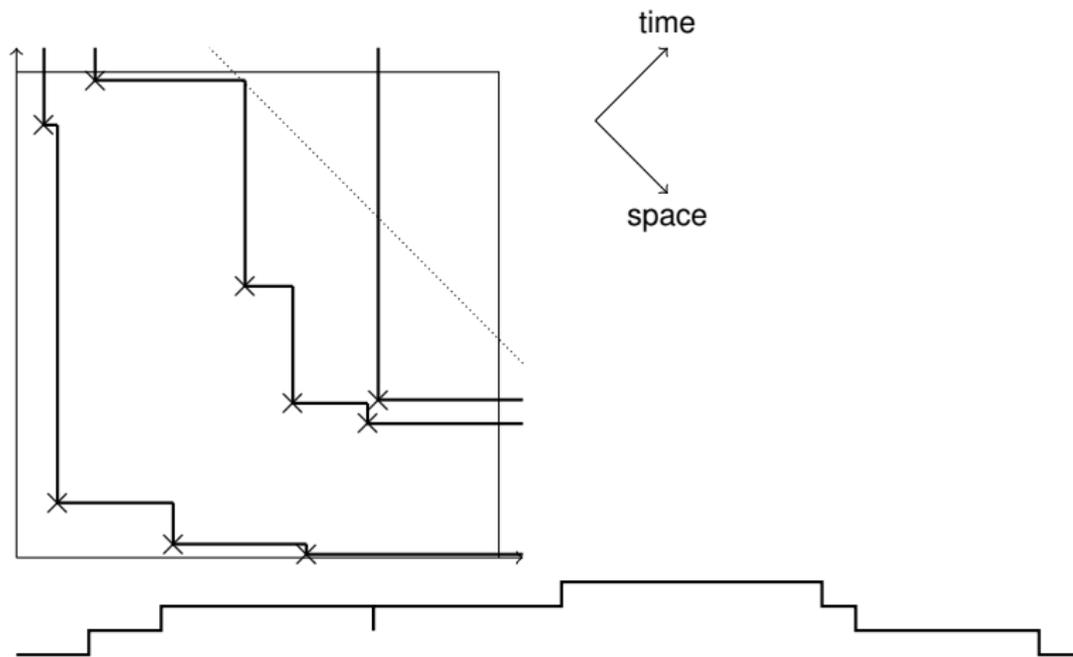
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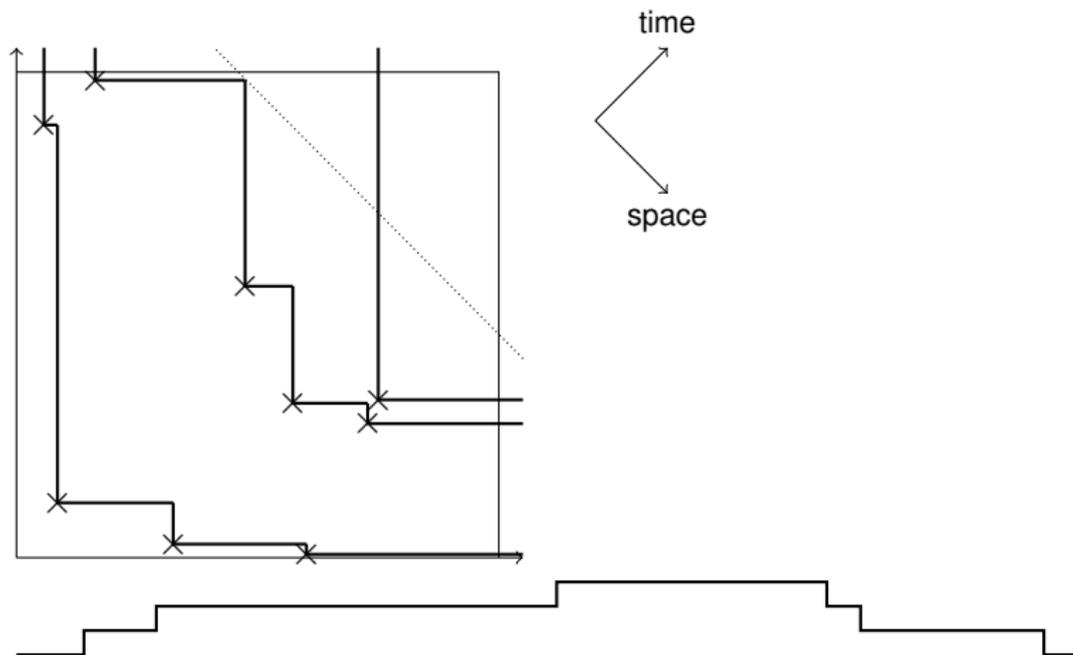
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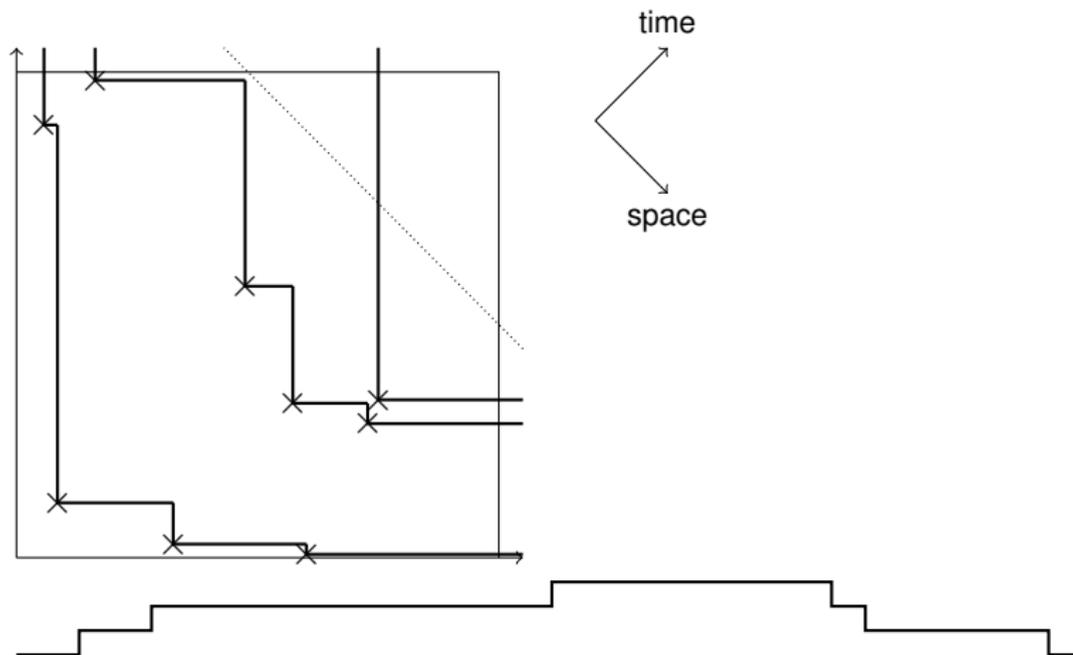
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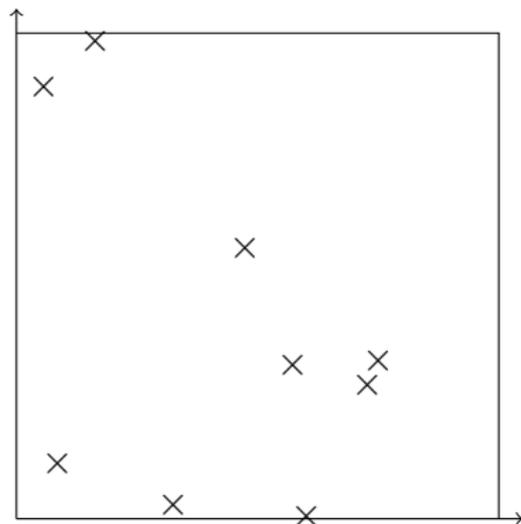






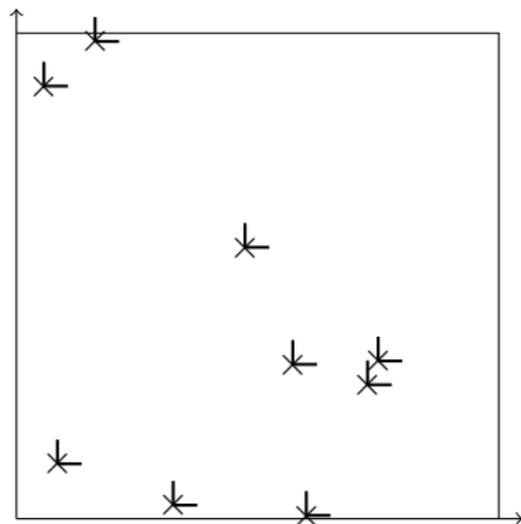
$t$ -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



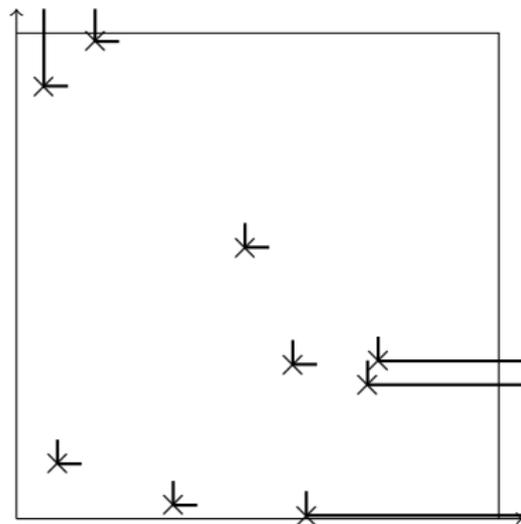
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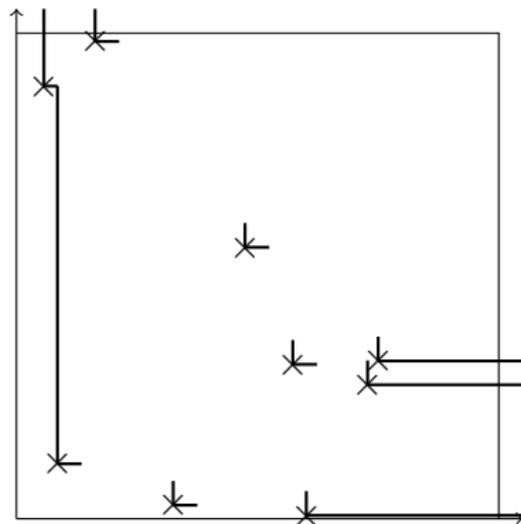
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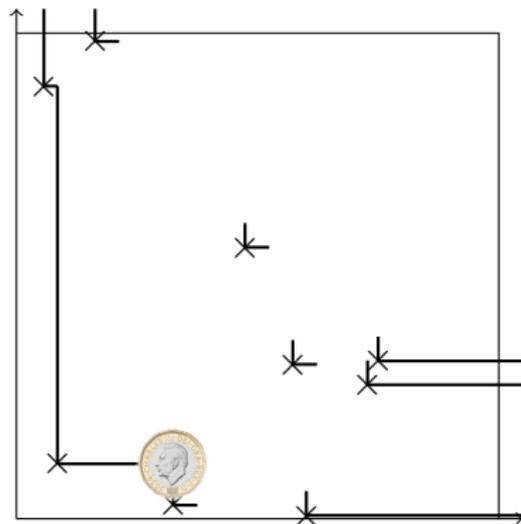






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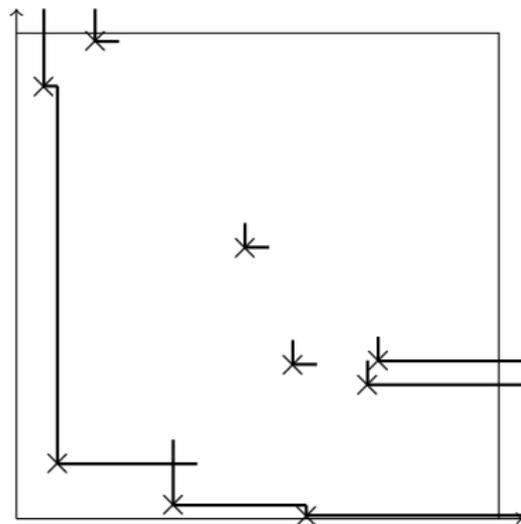
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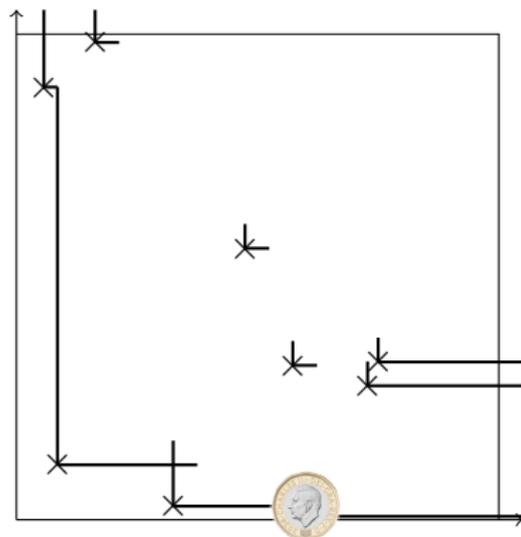
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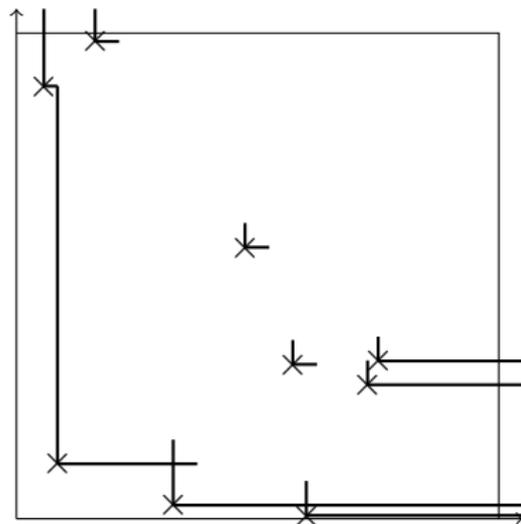
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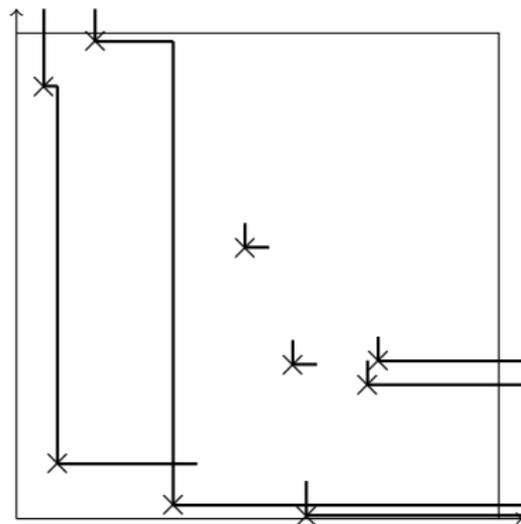
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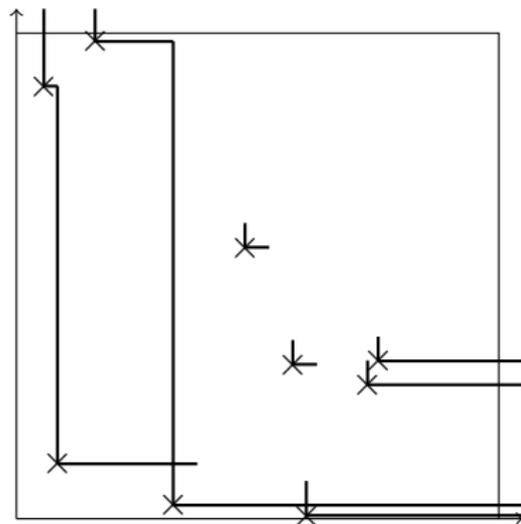
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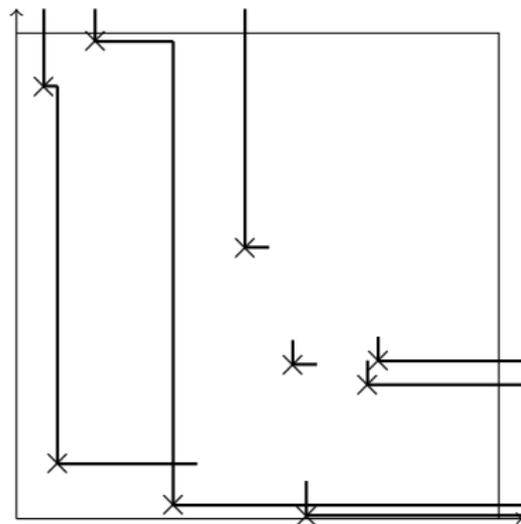
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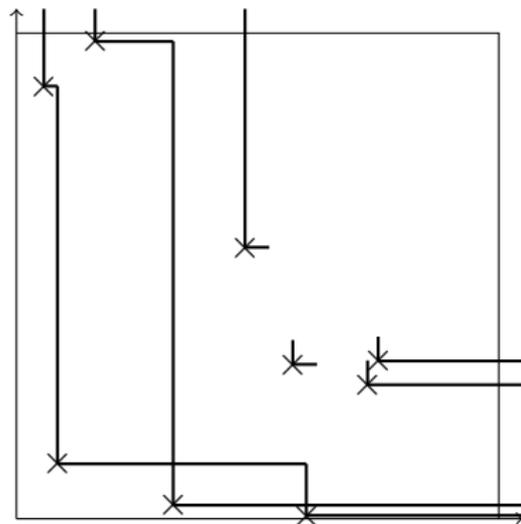
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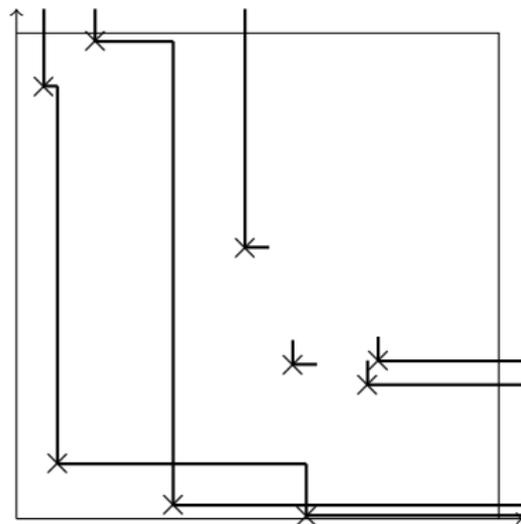
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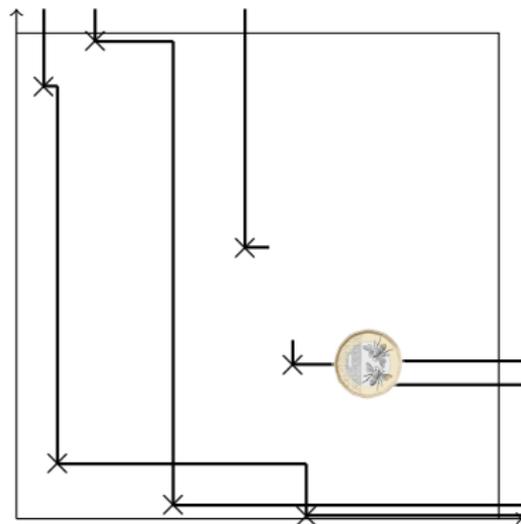
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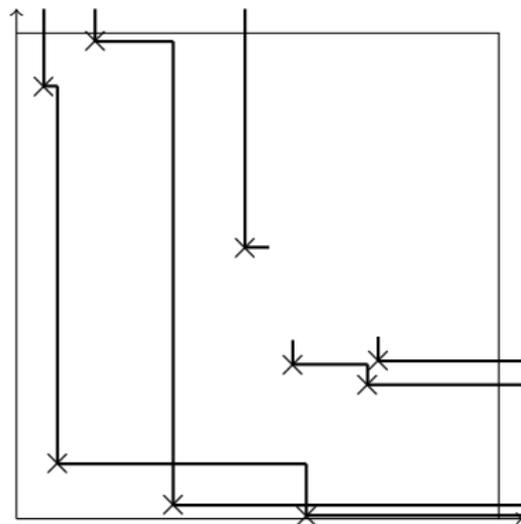
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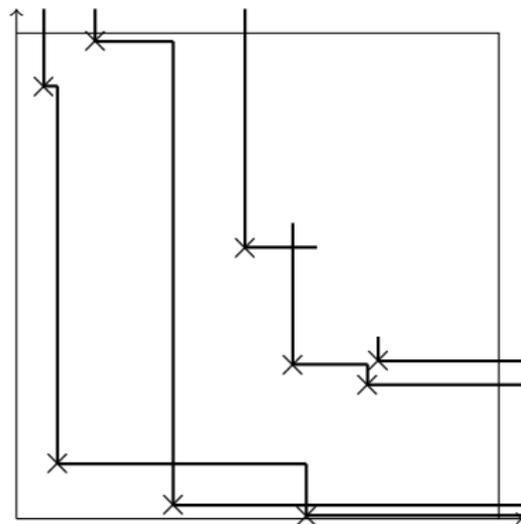






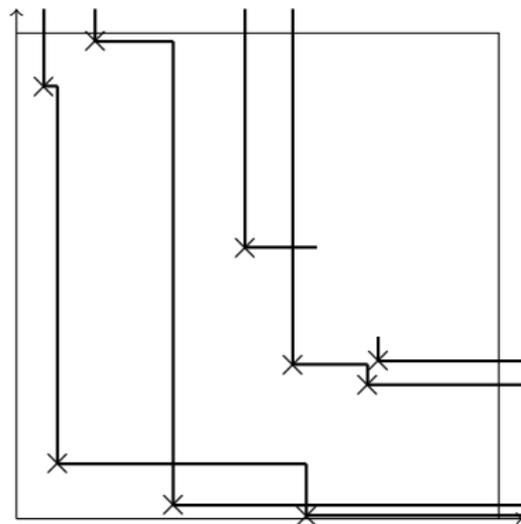
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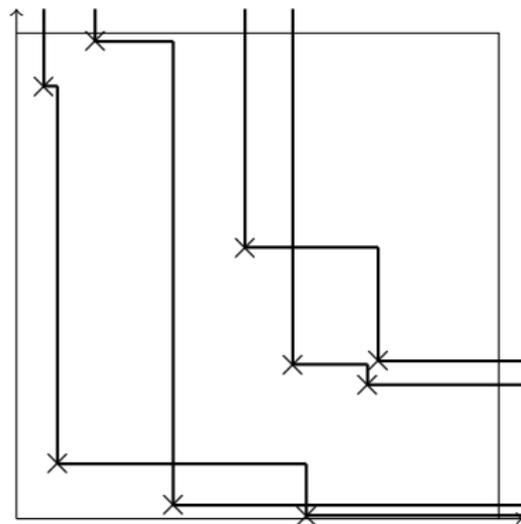
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$t$ -PNG [Aggarwal, Borodin, Wheeler '23]

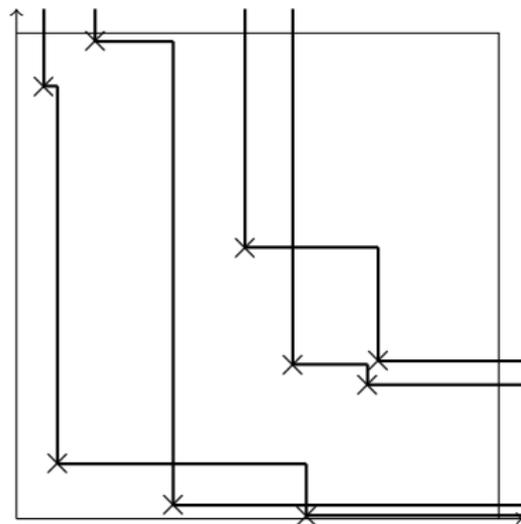
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .





$t$ -PNG [Aggarwal, Borodin, Wheeler '23]

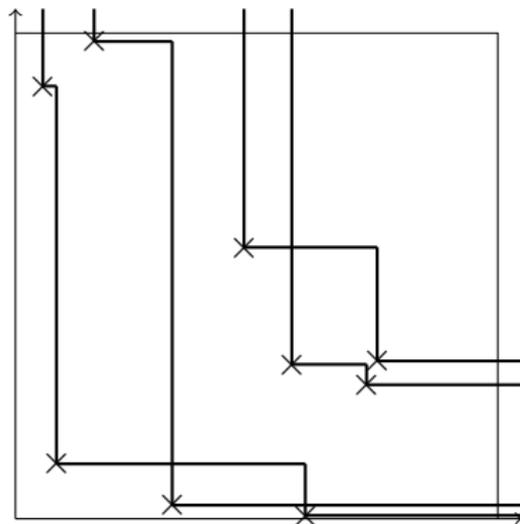
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .





$t$ -PNG [Aggarwal, Borodin, Wheeler '23]

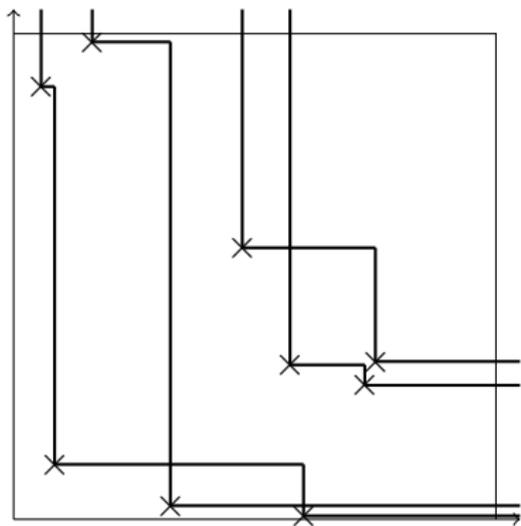
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



- ▶ Fluctuations: in KPZ-class [Aggarwal, Borodin, Wheeler '23]
- ▶ Colliding nucleations don't always annihilate (coin)

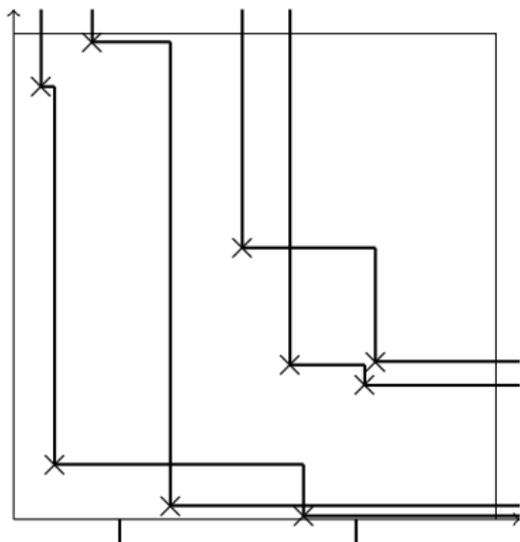
# Stationary $t$ -PNG [Drillick, Lin '24]

Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



# Stationary $t$ -PNG [Drillick, Lin '24]

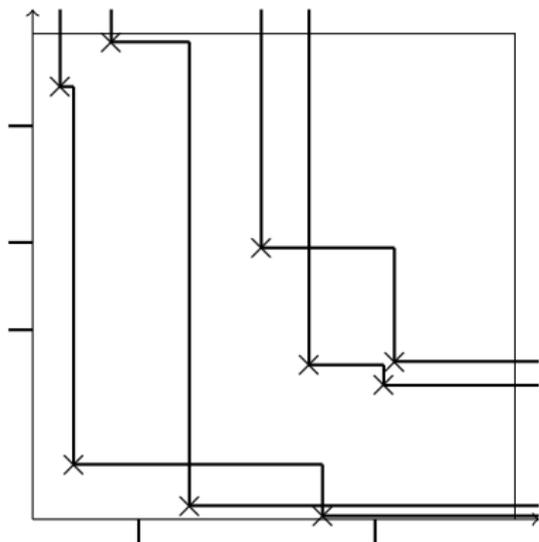
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



- Add **sources** on the bottom, and

# Stationary $t$ -PNG [Drillick, Lin '24]

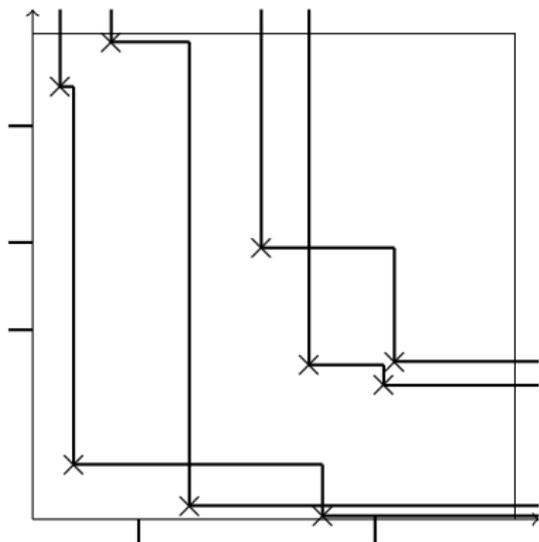
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



- ▶ Add **sources** on the bottom, and
- ▶ **sinks** on the left.

# Stationary $t$ -PNG [Drillick, Lin '24]

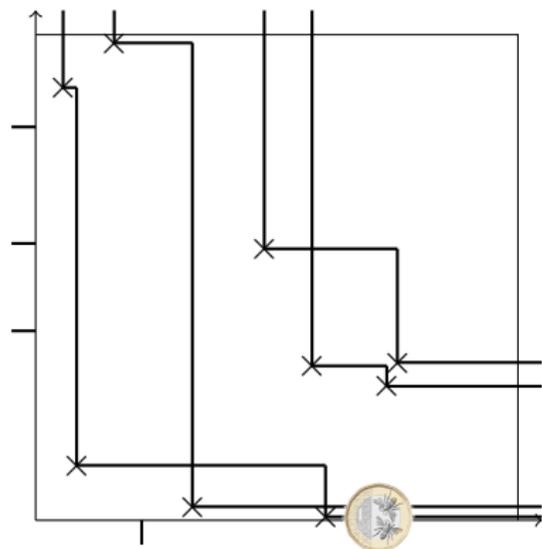
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



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# Stationary $t$ -PNG [Drillick, Lin '24]

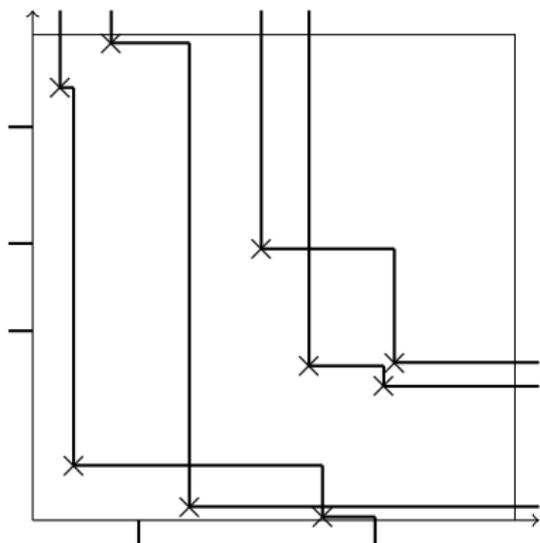
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



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# Stationary $t$ -PNG [Drillick, Lin '24]

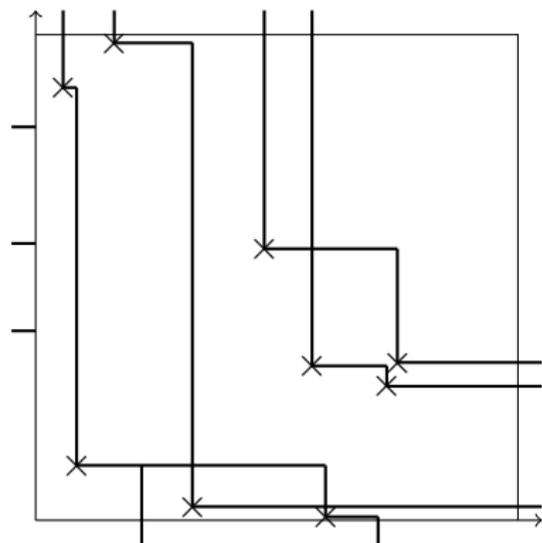
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# Stationary $t$ -PNG [Drillick, Lin '24]

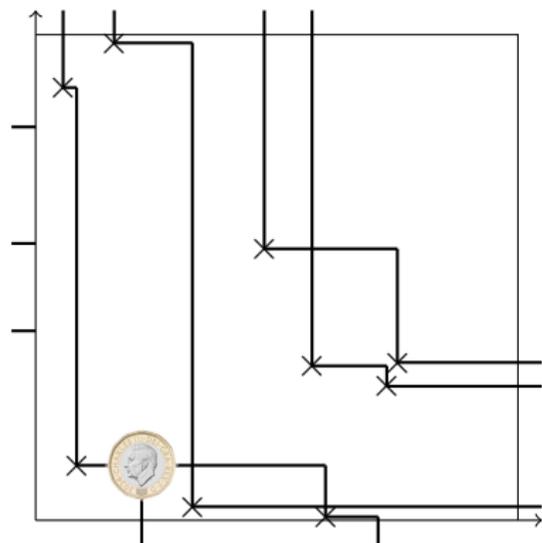
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# Stationary $t$ -PNG [Drillick, Lin '24]

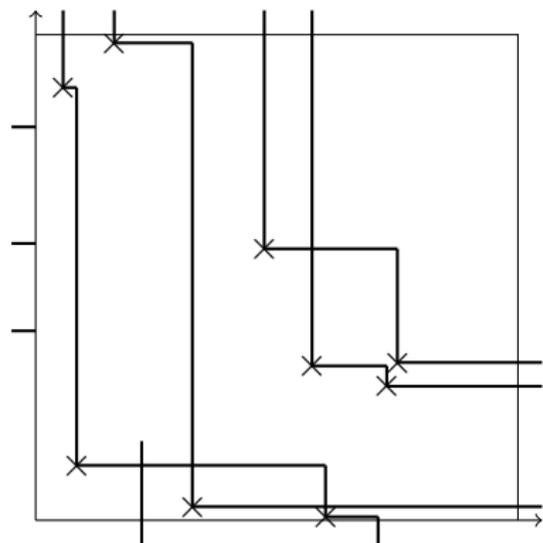
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# Stationary $t$ -PNG [Drillick, Lin '24]

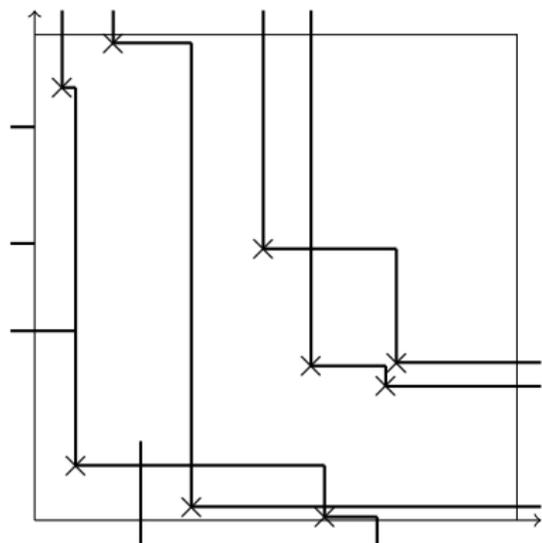
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# Stationary $t$ -PNG [Drillick, Lin '24]

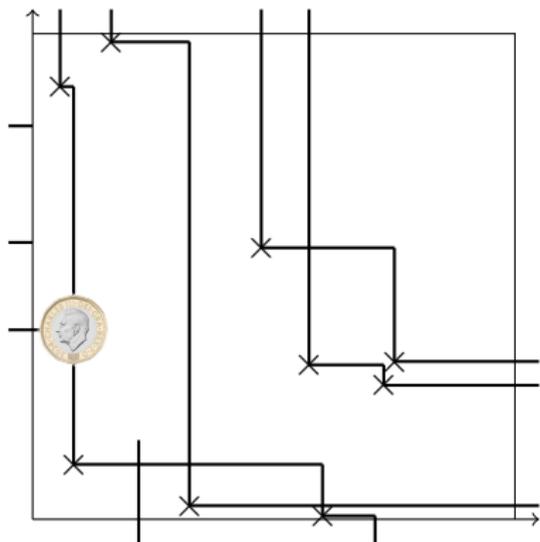
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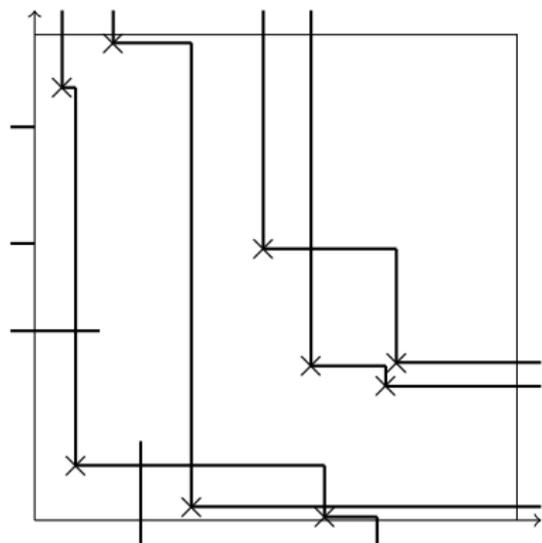
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# Stationary $t$ -PNG [Drillick, Lin '24]

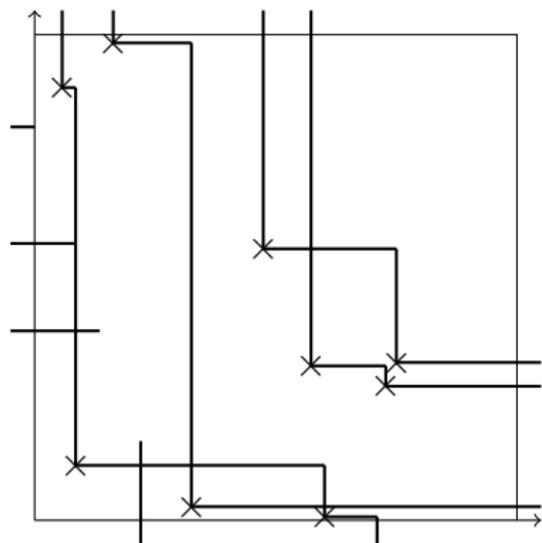
Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



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## Stationary $t$ -PNG [Drillick, Lin '24]

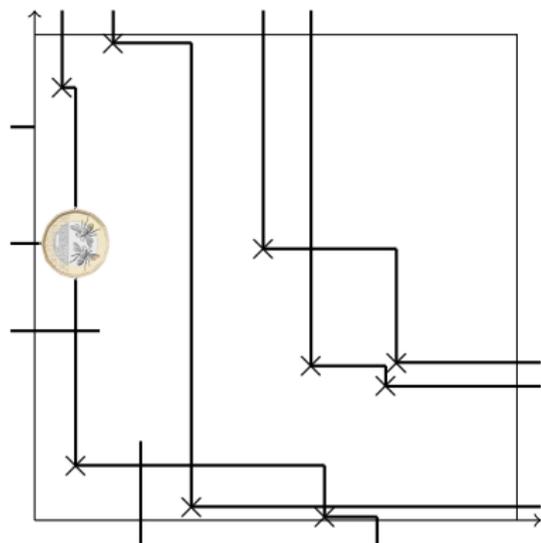
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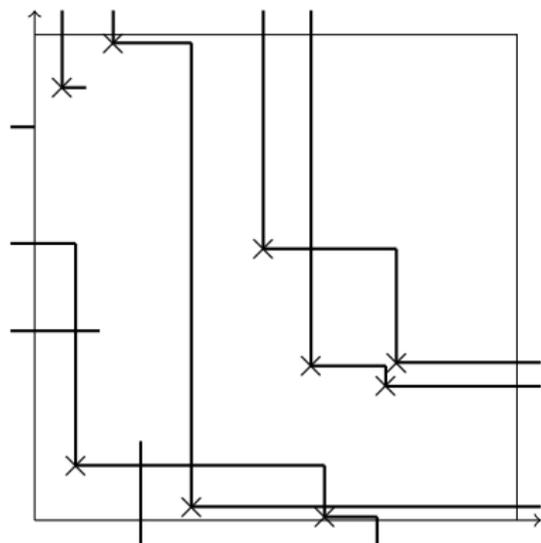
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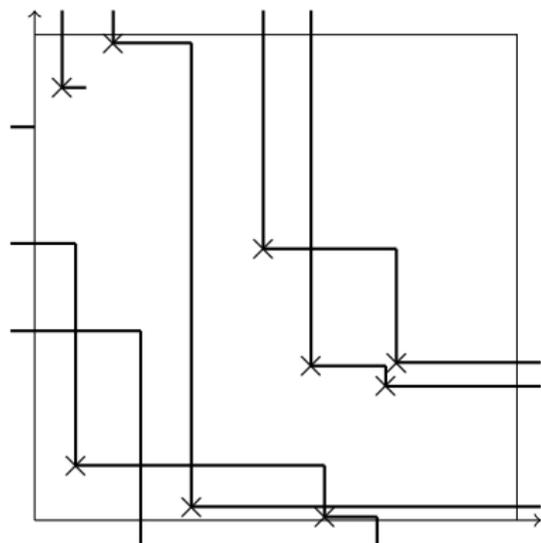
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# Stationary $t$ -PNG [Drillick, Lin '24]

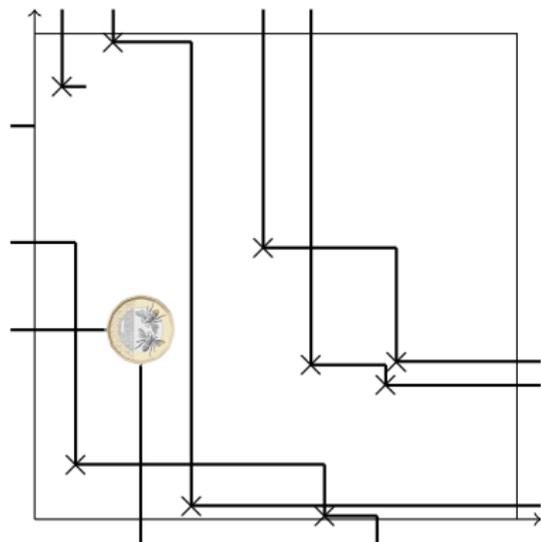
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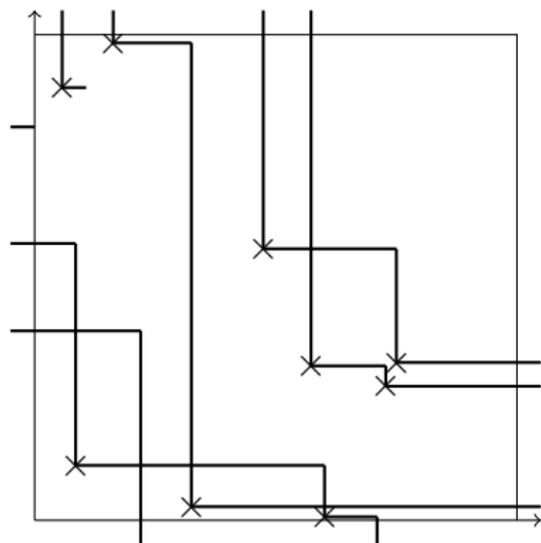
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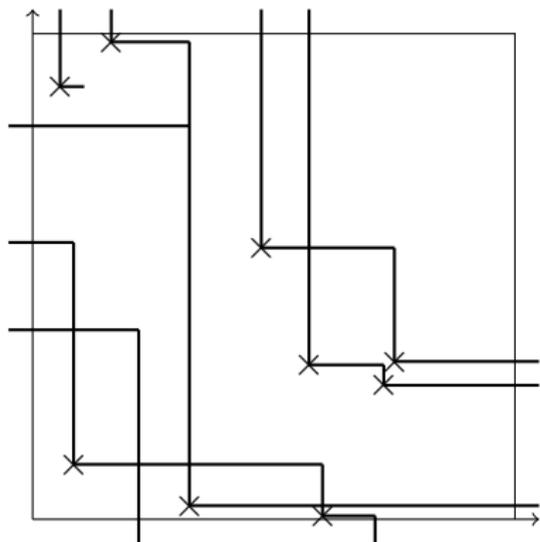
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## Stationary $t$ -PNG [Drillick, Lin '24]

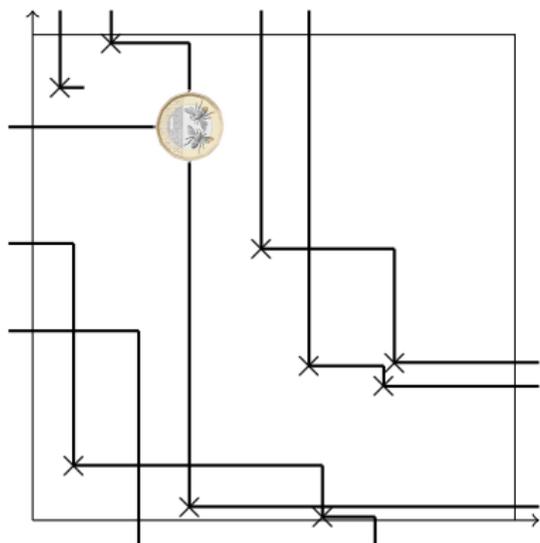
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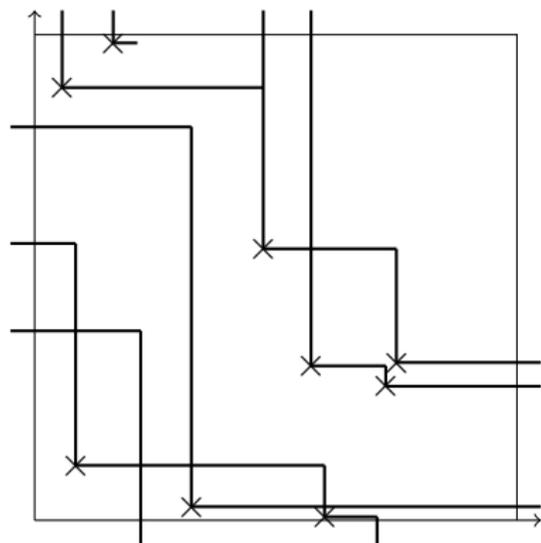


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## Stationary $t$ -PNG [Drillick, Lin '24]

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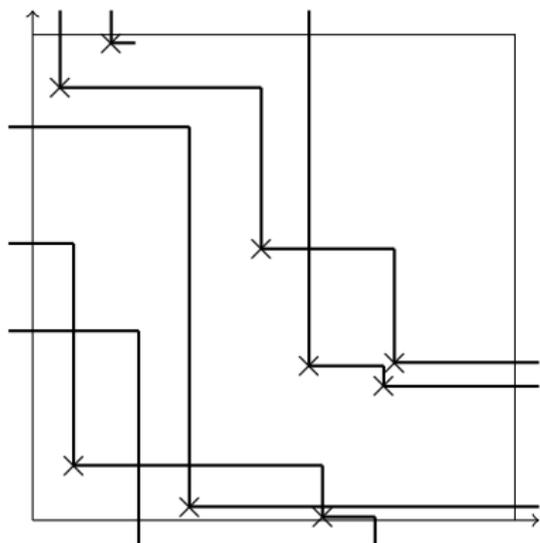


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## Stationary $t$ -PNG [Drillick, Lin '24]

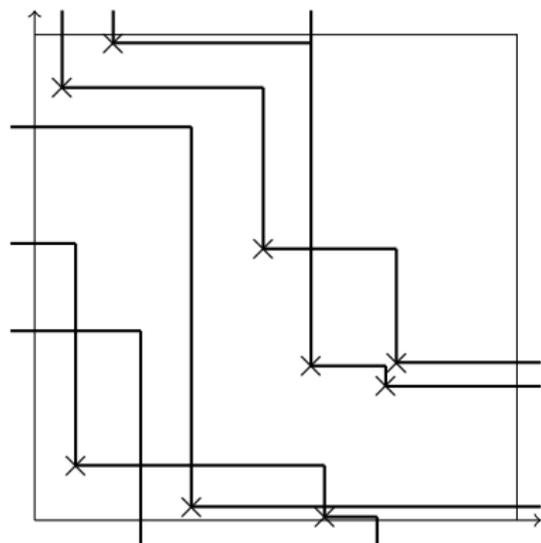
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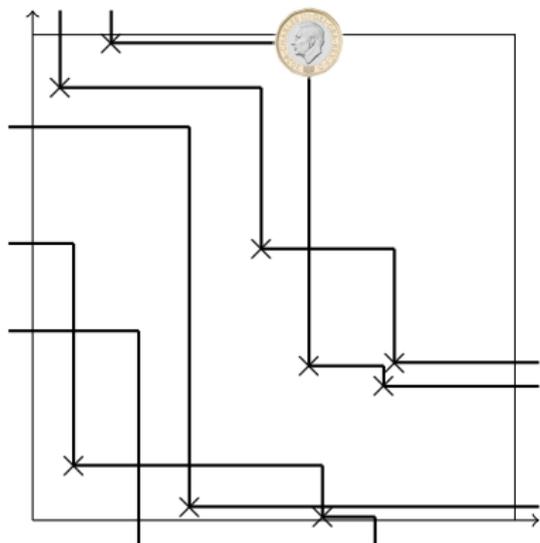
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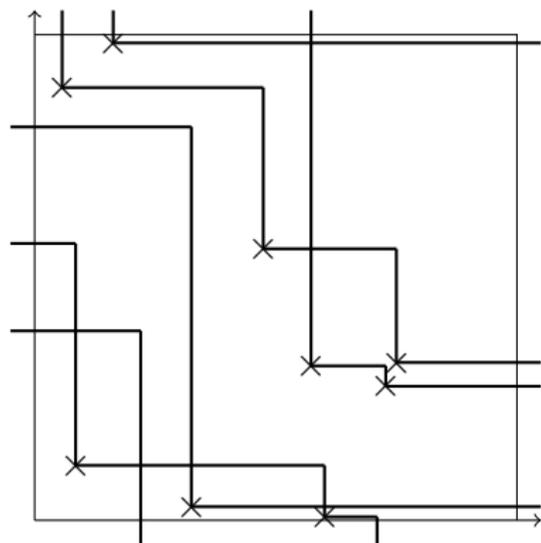
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## Stationary $t$ -PNG [Drillick, Lin '24]

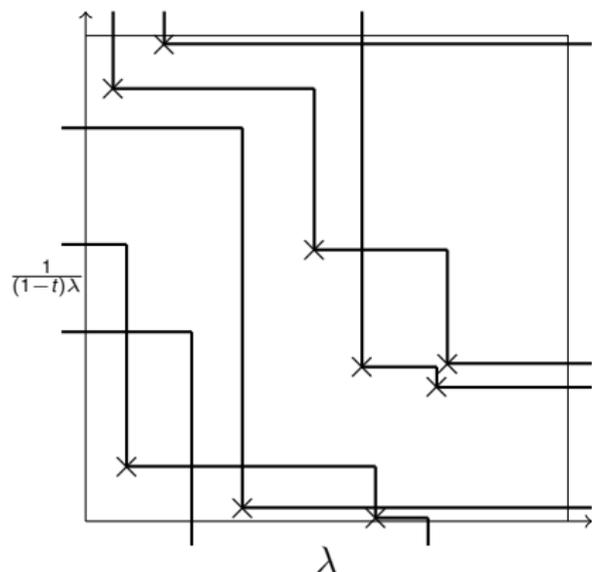
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Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



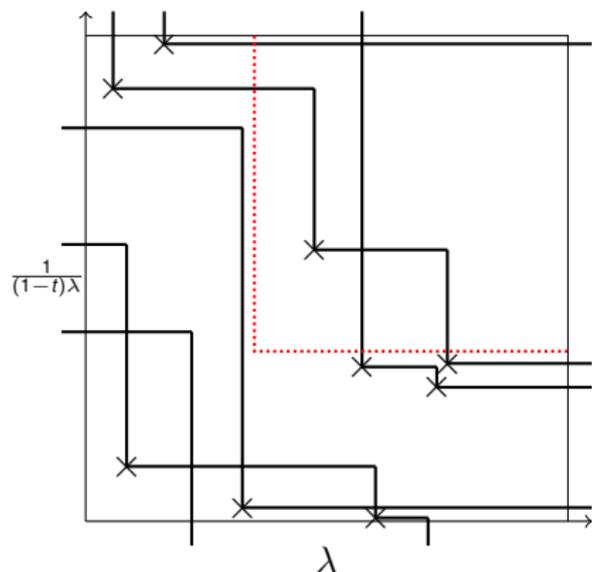
- ▶ Add **sources** on the bottom, and
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## Theorem

- ▶ If *sources*  $\sim \text{Poi}(\lambda)$ , and
- ▶ *sinks*  $\sim \text{Poi}(\frac{1}{(1-t)\lambda})$  independent,

# Stationary $t$ -PNG [Drillick, Lin '24]

Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



- ▶ Add **sources** on the bottom, and
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## Theorem

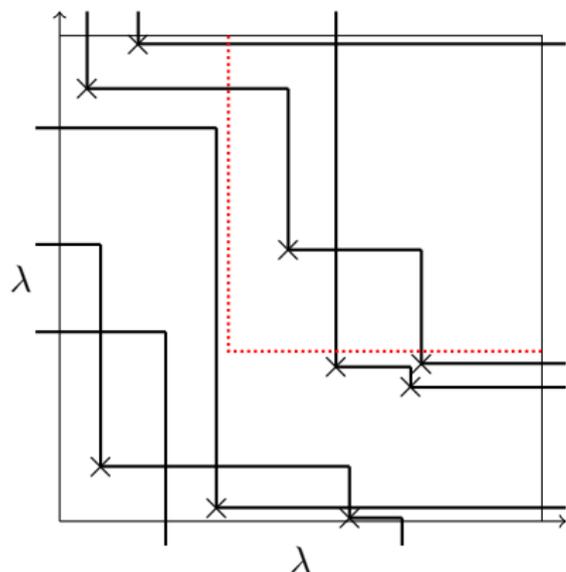
- ▶ If *sources*  $\sim \text{Poi}(\lambda)$ , and
- ▶ *sinks*  $\sim \text{Poi}(\frac{1}{(1-t)\lambda})$  independent,

the no of lines entering

- ▶ on the bottom  $\sim \text{Poi}(\lambda)$
- ▶ on the left  $\sim \text{Poi}(\frac{1}{(1-t)\lambda})$ , independent.

# Stationary $t$ -PNG [Drillick, Lin '24]

Flip coin: cross with prob.  $t$ , corner with prob.  $1 - t$ .



- ▶ Add **sources** on the bottom, and
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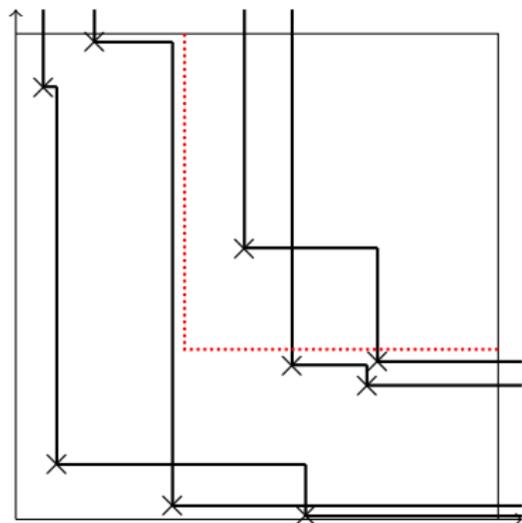
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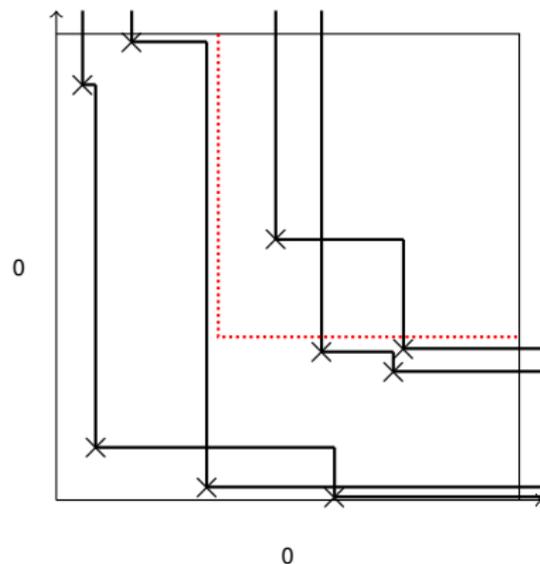
- ▶ on the bottom  $\sim \text{Poi}(\lambda)$
- ▶ on the left  $\sim \text{Poi}(\frac{1}{(1-t)\lambda})$ , independent.

# Local convergence

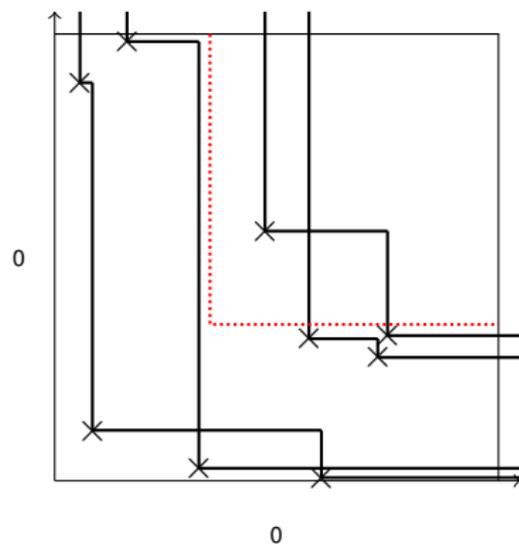




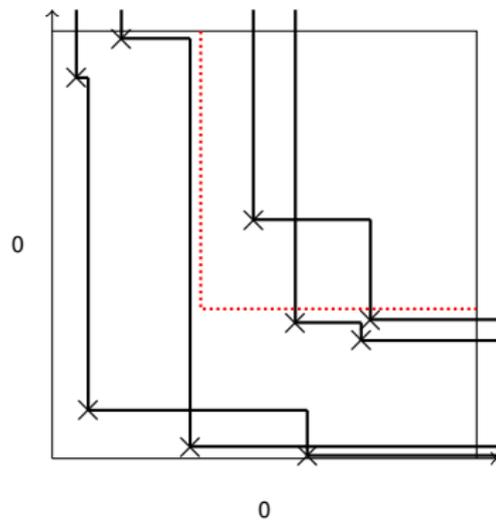
# Local convergence



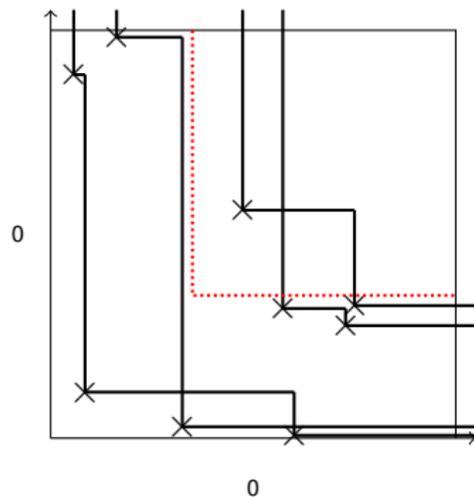
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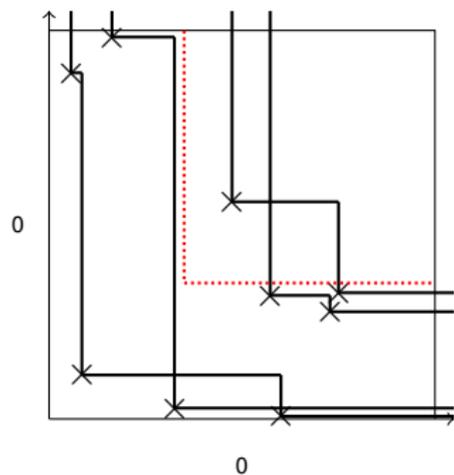
# Local convergence



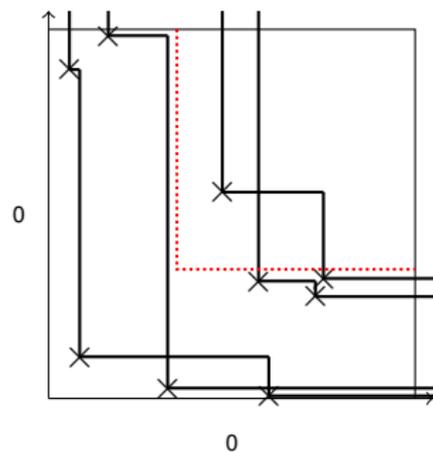
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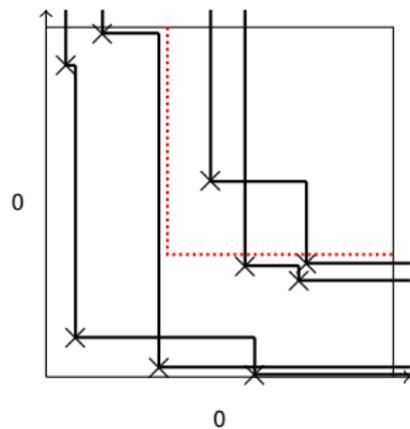
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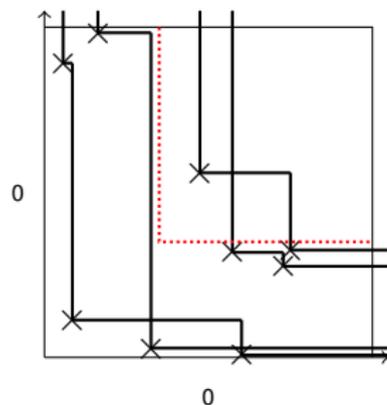
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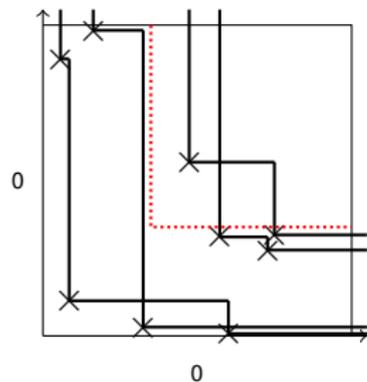
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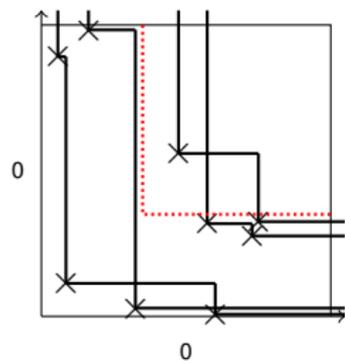
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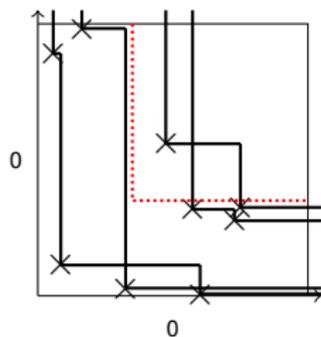
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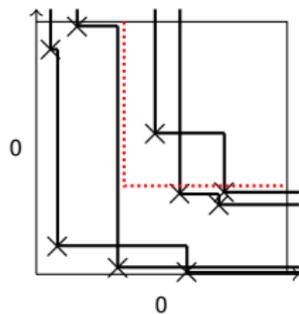
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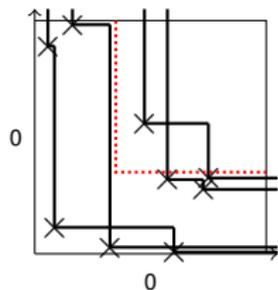
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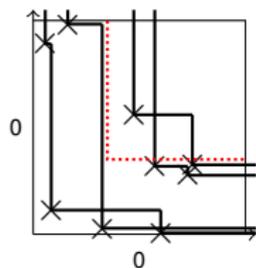
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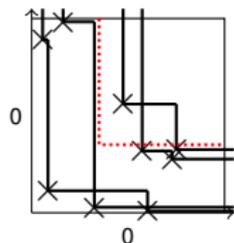
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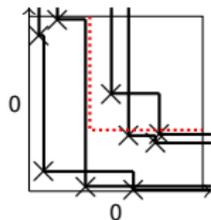
# Local convergence



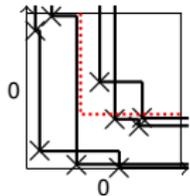
# Local convergence



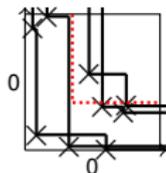
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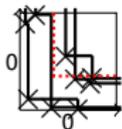
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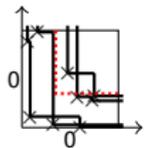
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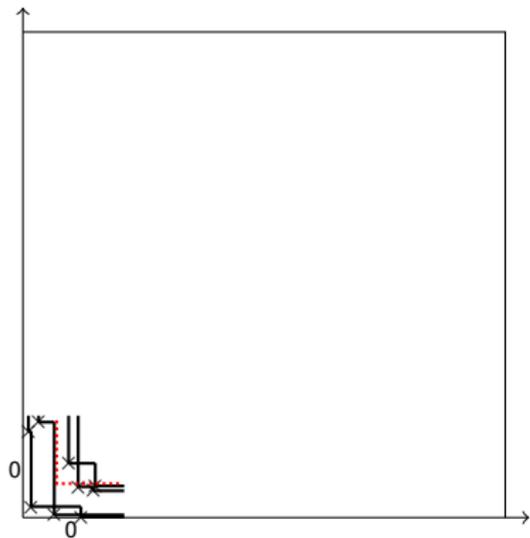
# Local convergence



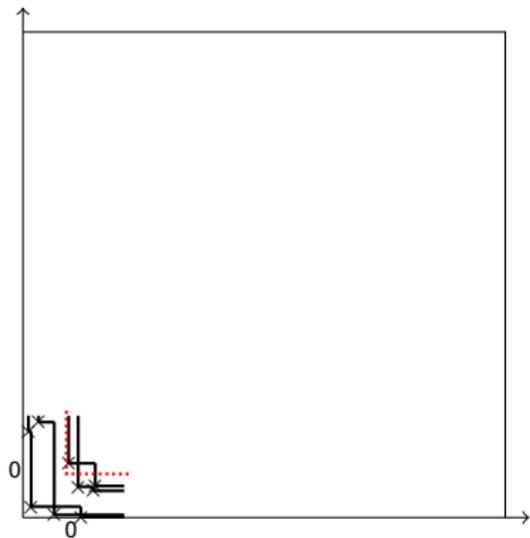
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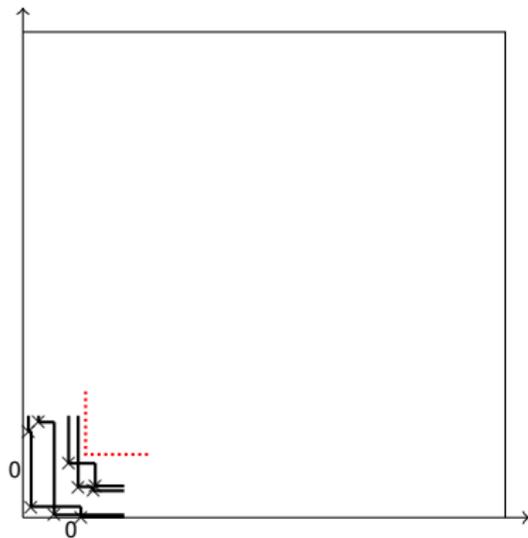


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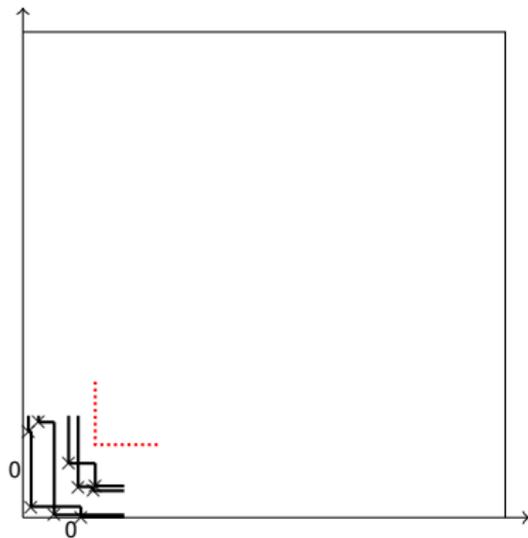




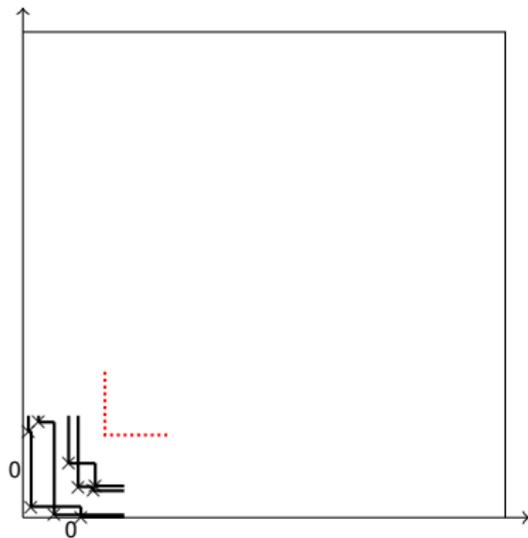
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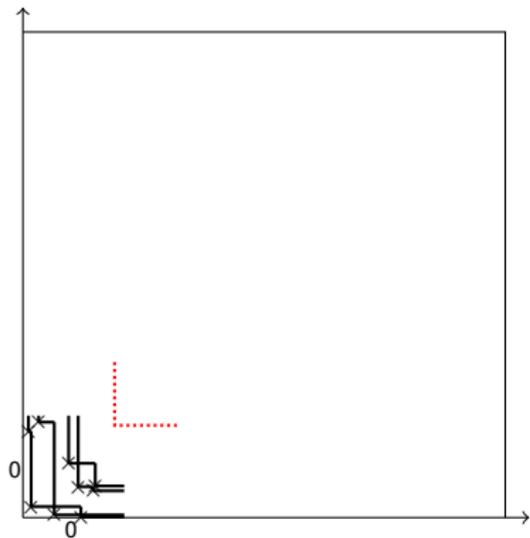
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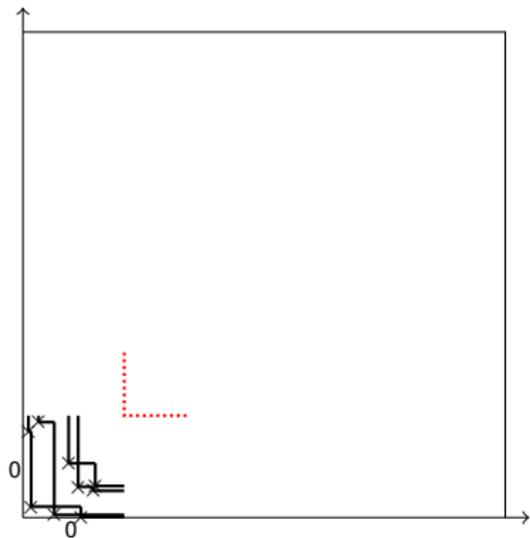
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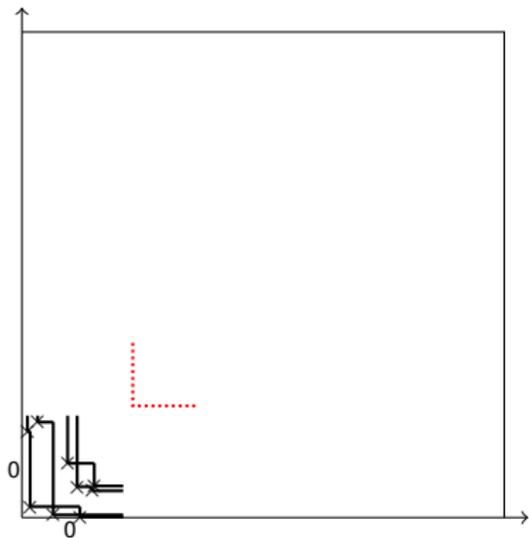
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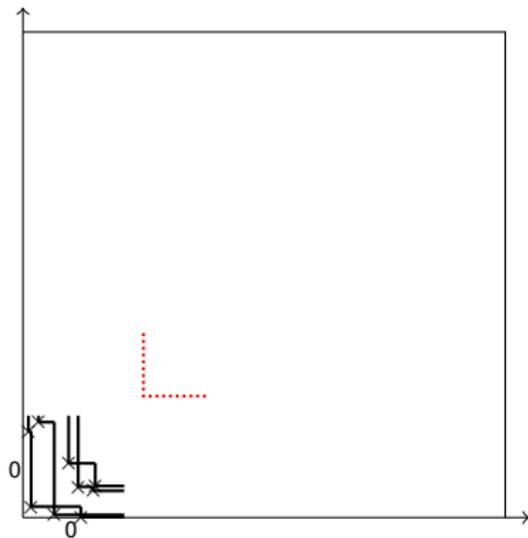
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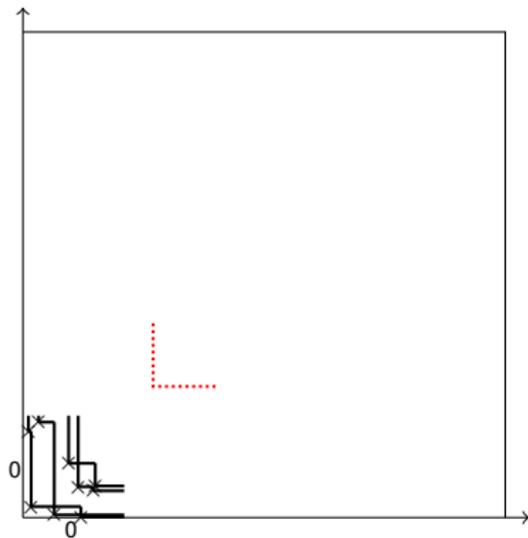
# Local convergence



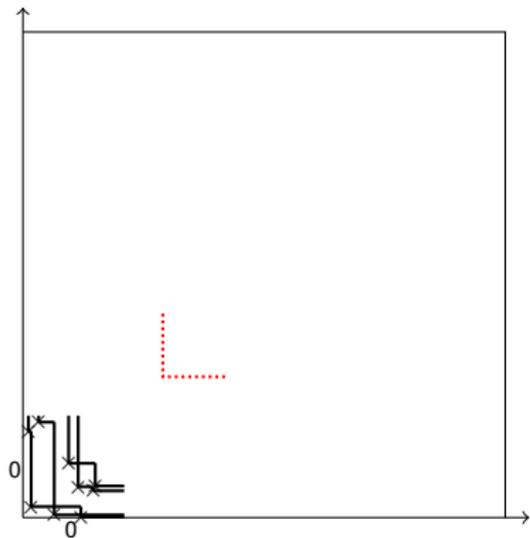
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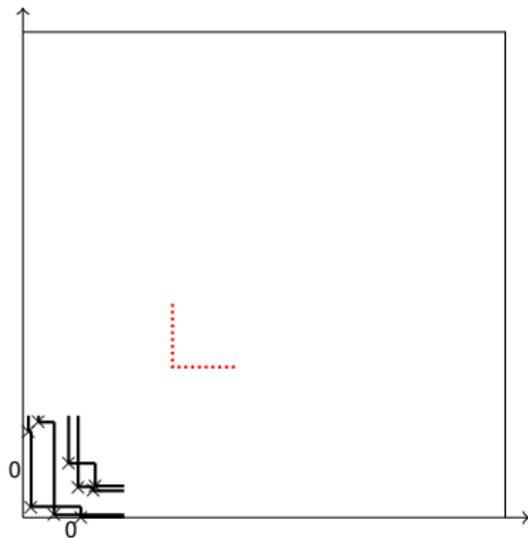
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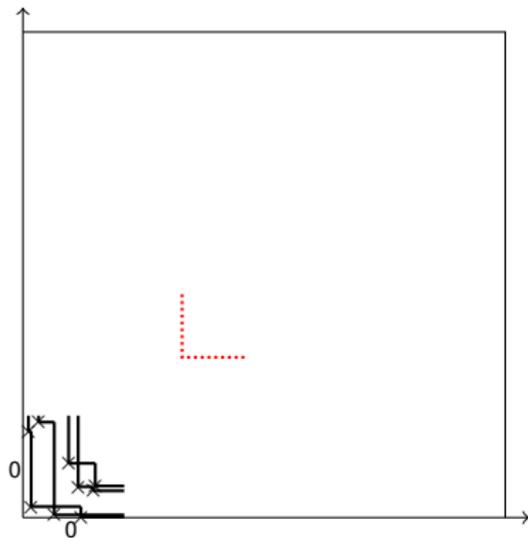
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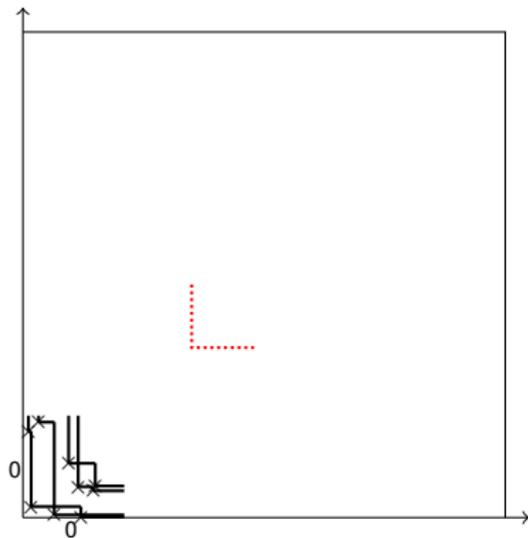
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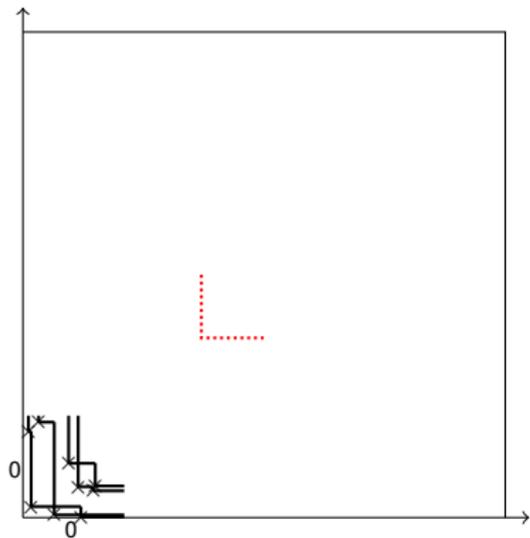
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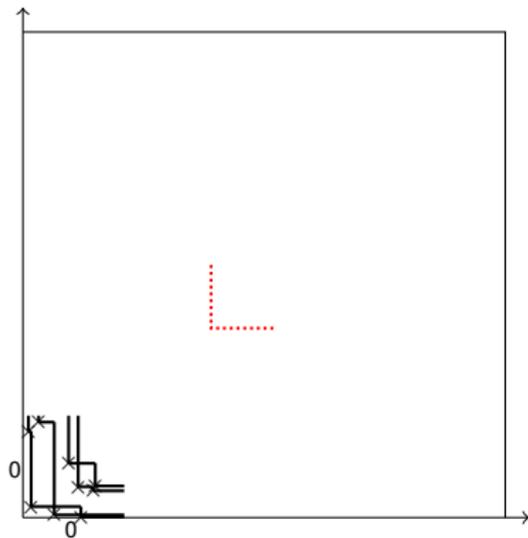
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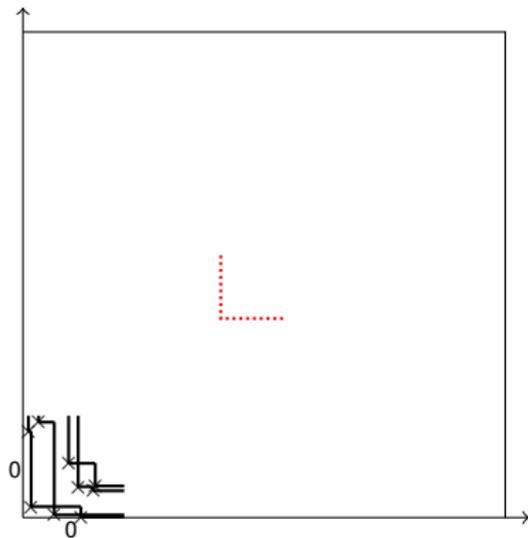
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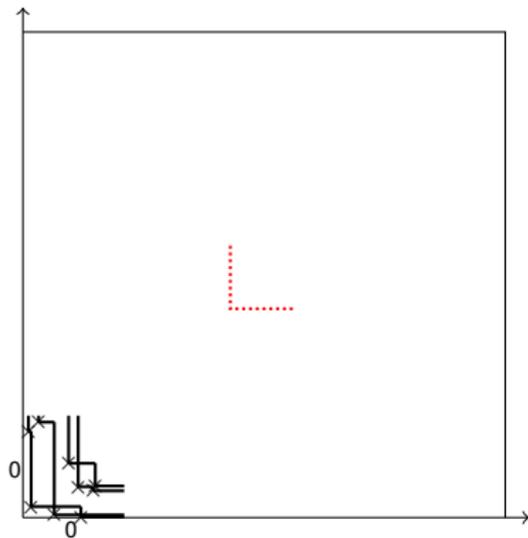
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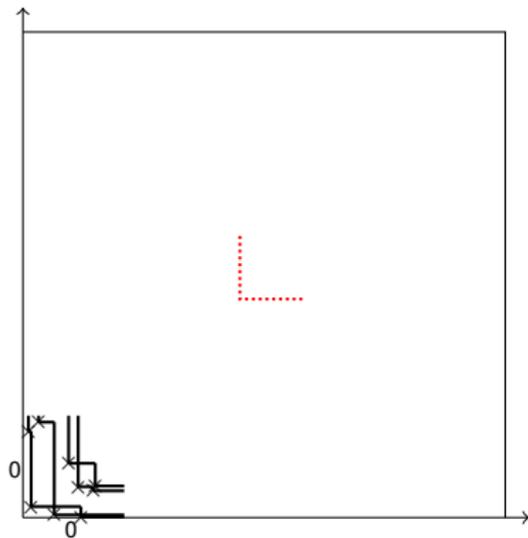
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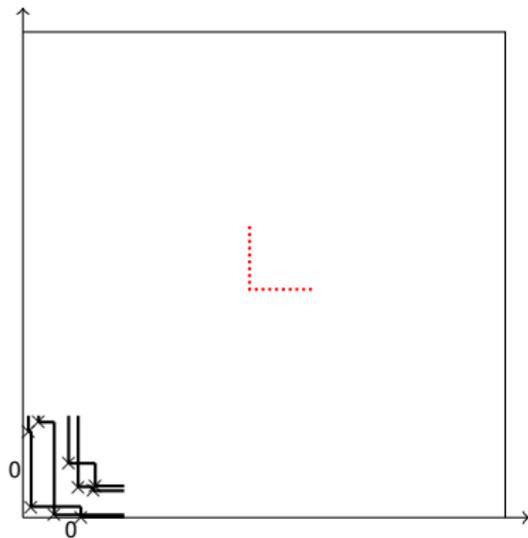
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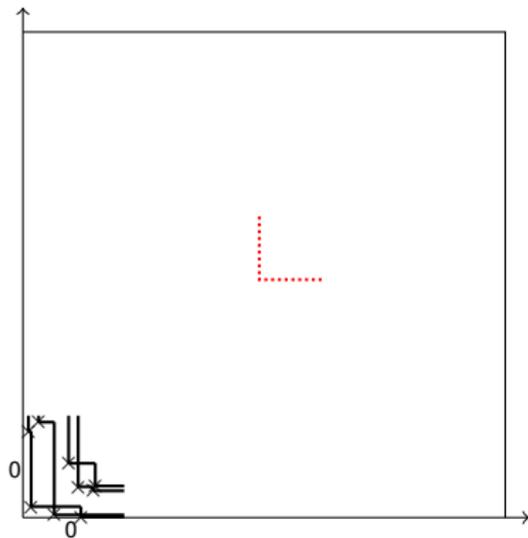
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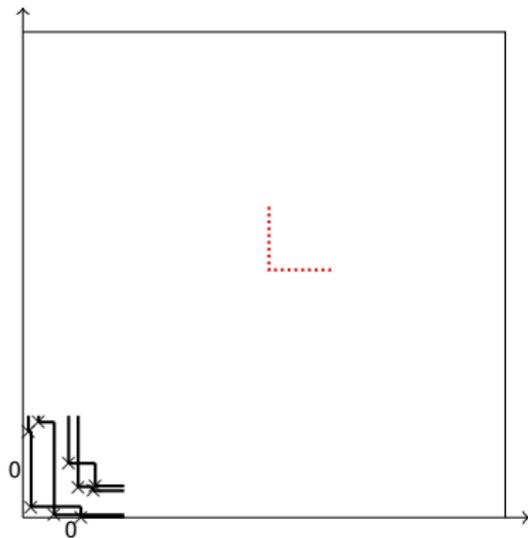
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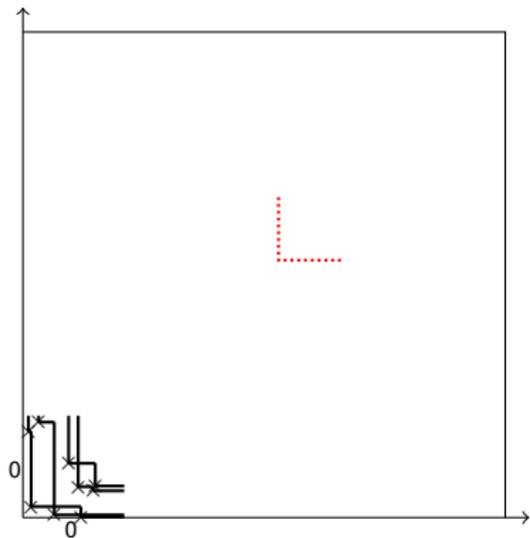
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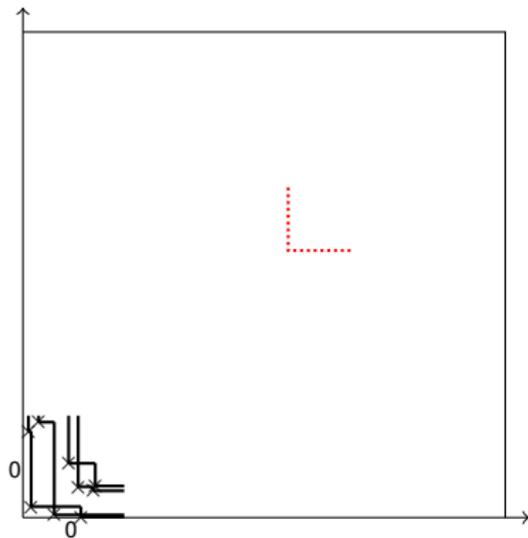
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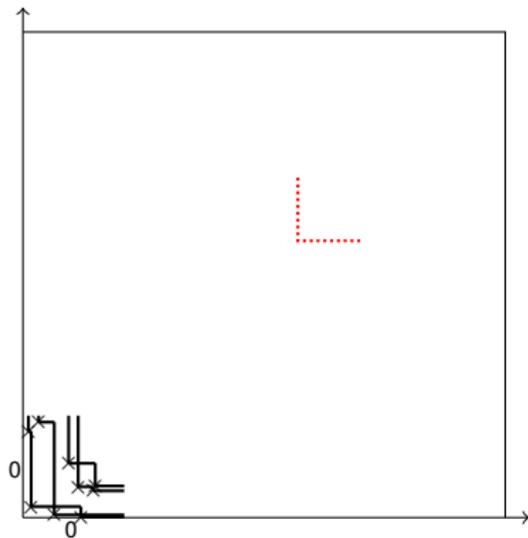
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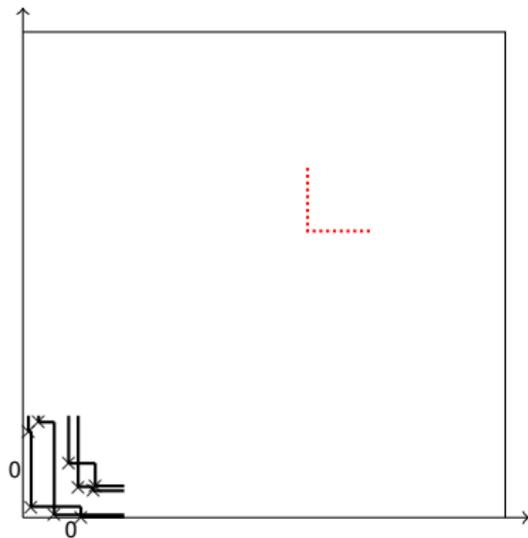
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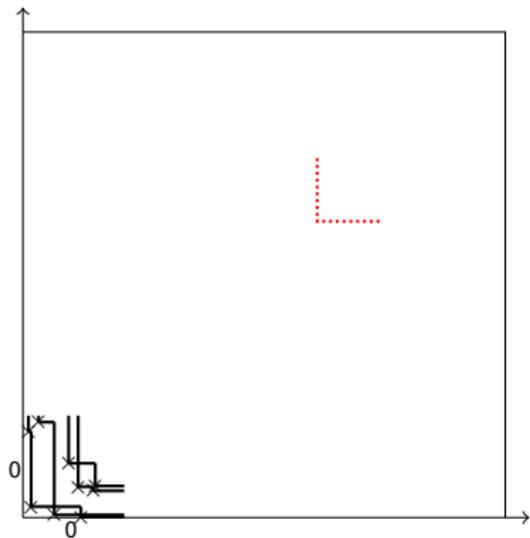
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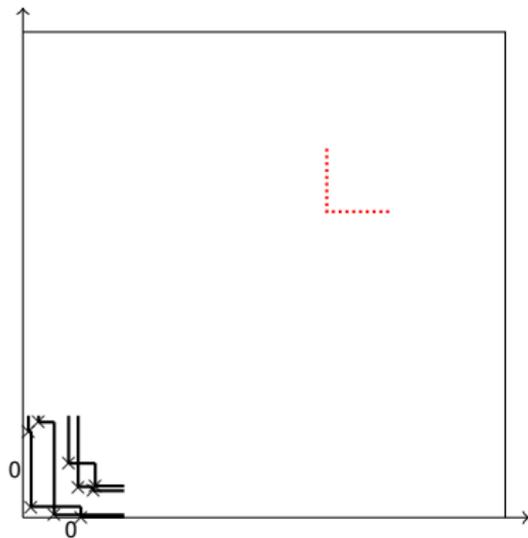
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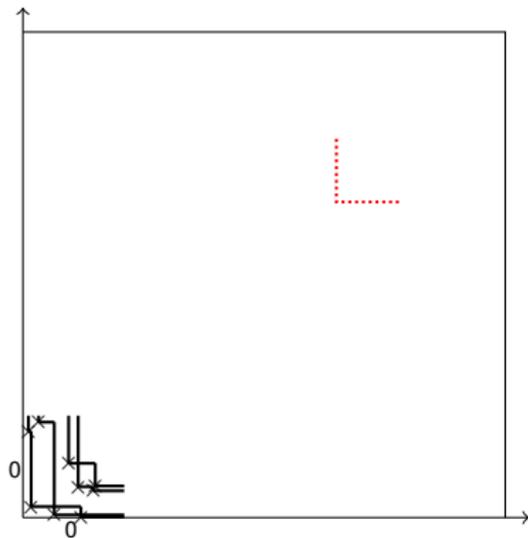
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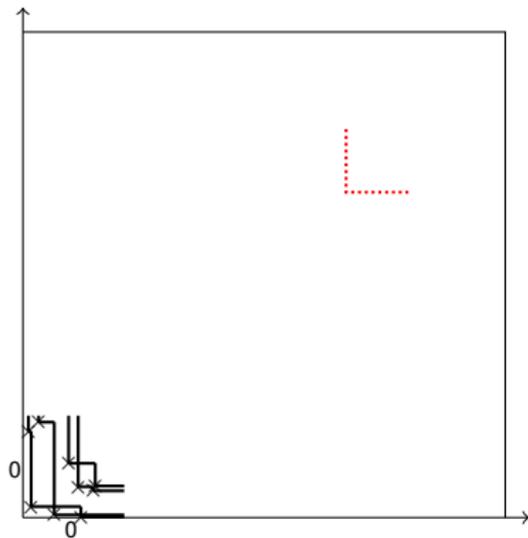
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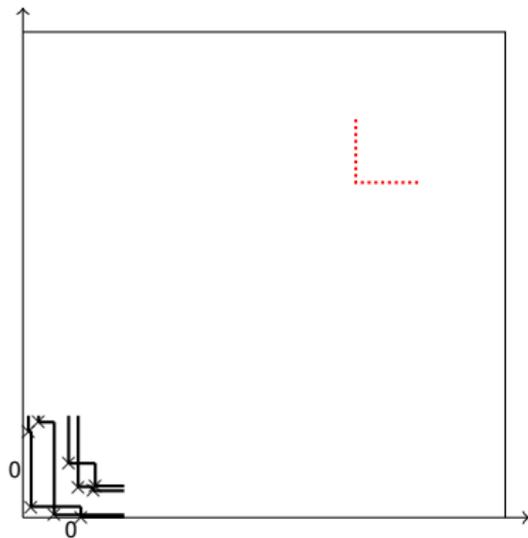
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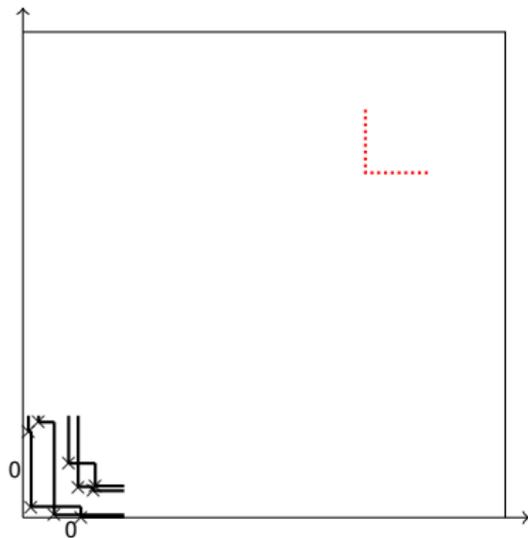
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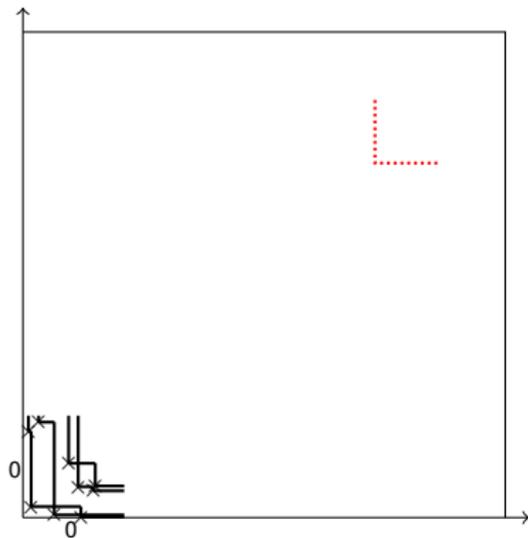
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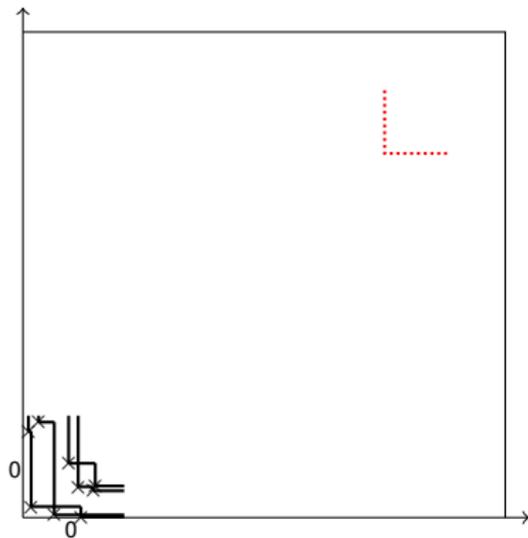
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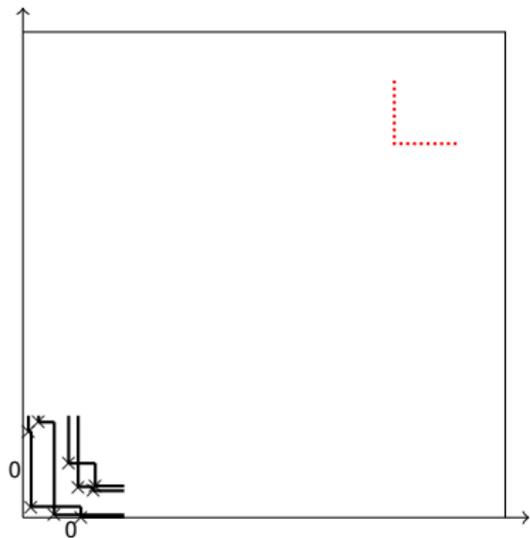
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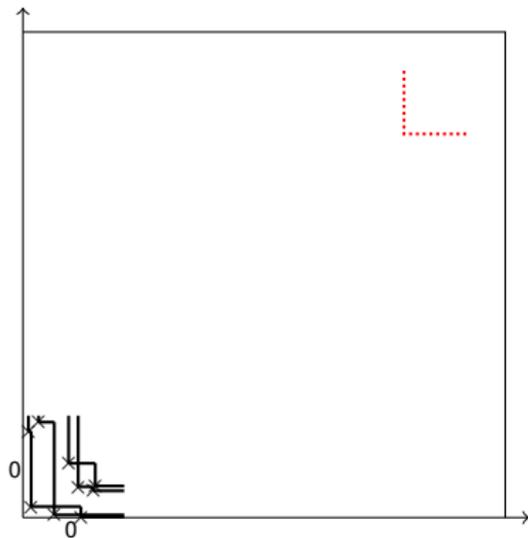
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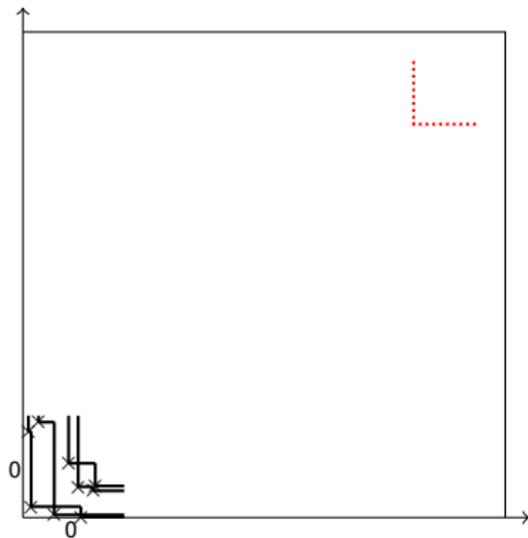
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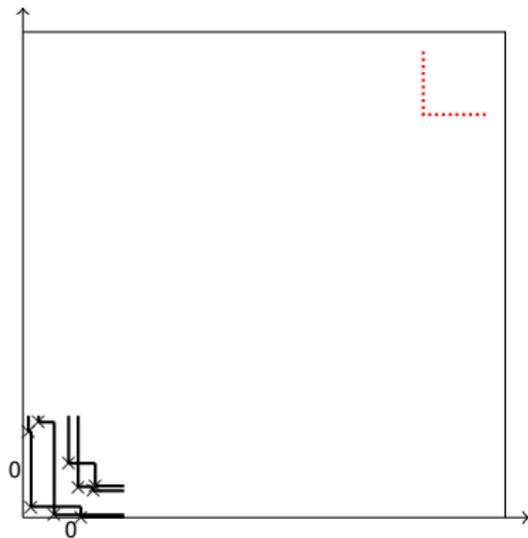
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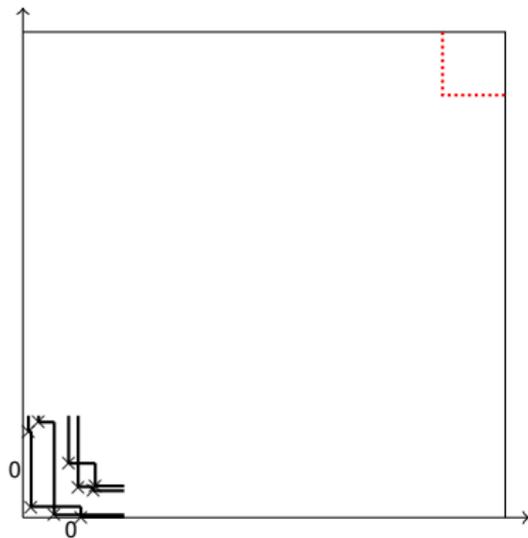
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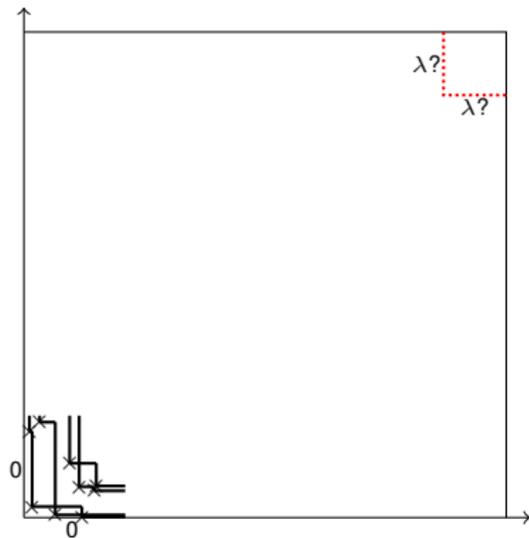
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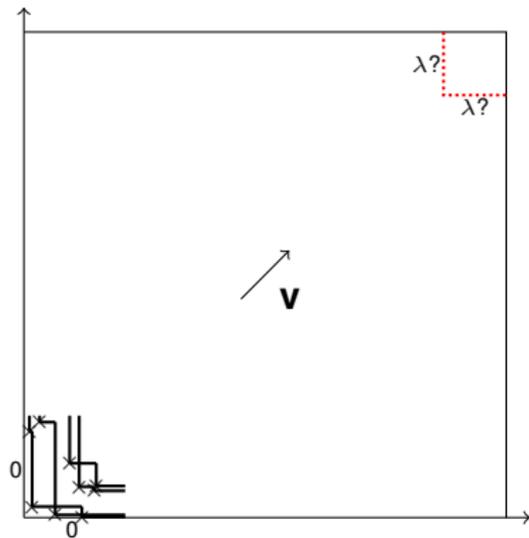
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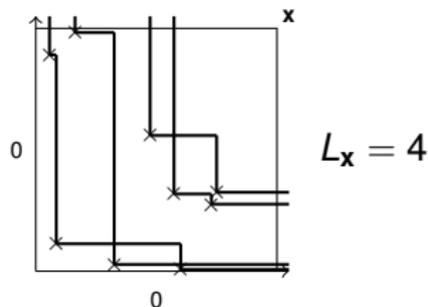
*Yes, the boundaries of the small box converge in distribution to the two independent stationary Poisson processes with an explicit  $\lambda = \lambda(\mathbf{v})$  (direction).*

- ▶ Confirms the conjecture from [Drillick, Lin '24].
- ▶ Hence the full  $t$ -PNG inside the small box converges to stationary  $t$ -PNG.
- ▶ The actual limit intensities are  $\frac{1}{\sqrt{1-t}} \sqrt{\frac{v_2}{v_1}}$  for sources and  $\frac{1}{\sqrt{1-t}} \sqrt{\frac{v_1}{v_2}}$  for sinks.

## Shape theorem [Drillick, Lin '24]

Define  $L_x :=$  no. of lines entering the bottom or right edge.

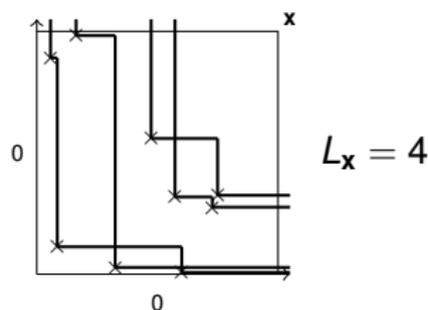
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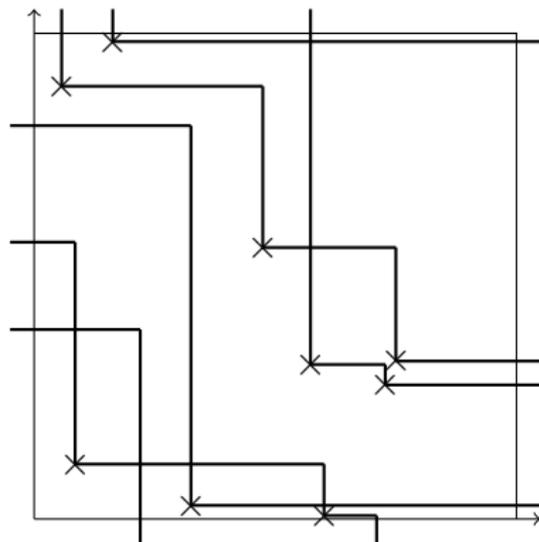


## Theorem

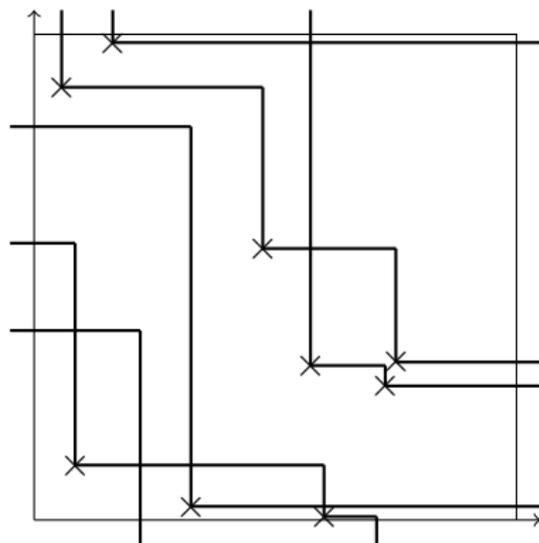
*With zero boundaries,*

$$\lim_{\mathbf{x} \rightarrow \infty} L_{\mathbf{x}} = 2\sqrt{\frac{x_1 x_2}{1-t}}.$$

# Second class particles

 $\alpha$

## Second class particles

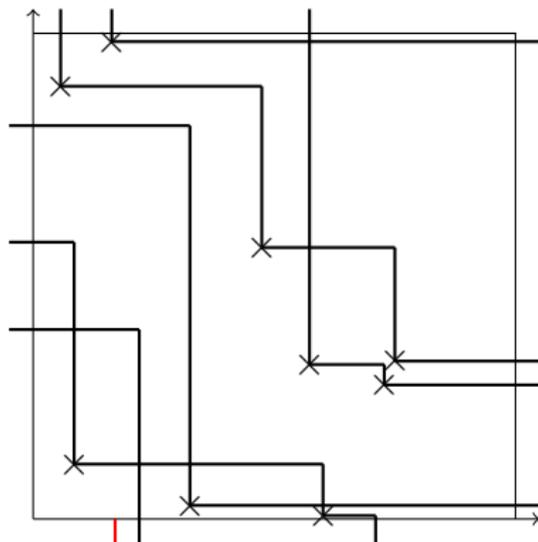


Definition:  $\alpha \leq \omega$ , if

- ▶ more verticals in  $\omega$ ,
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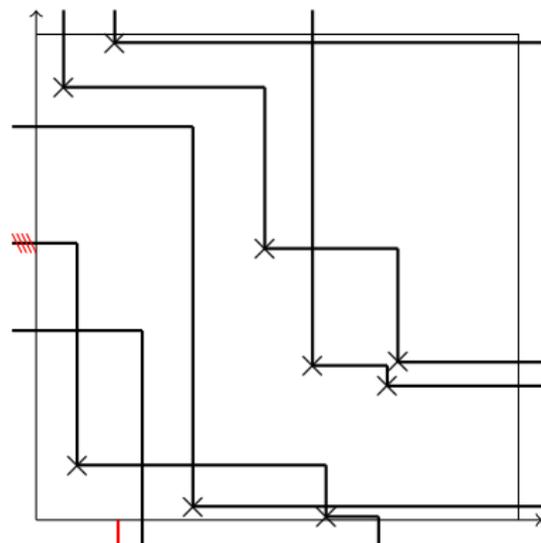
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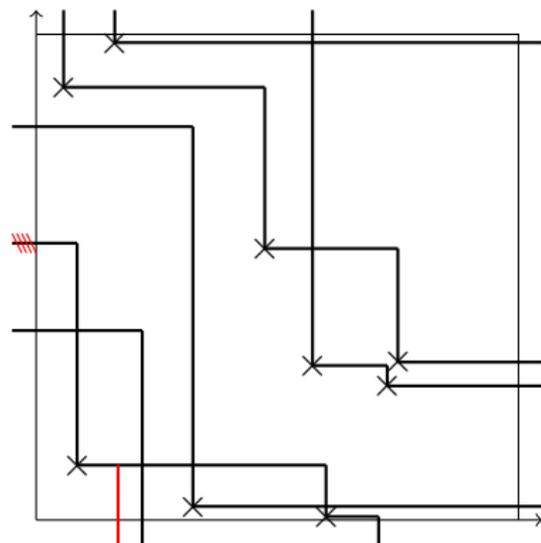
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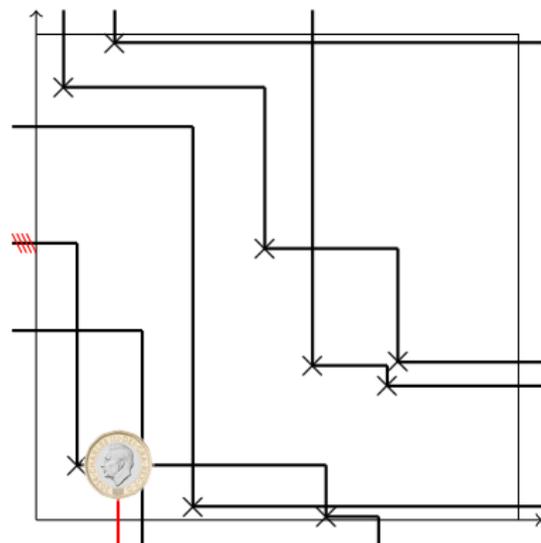
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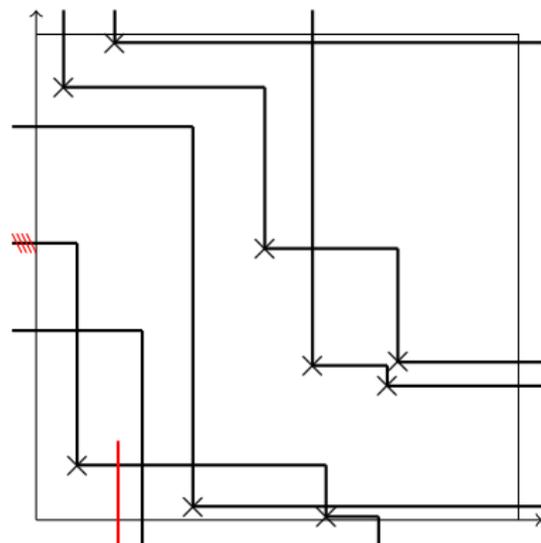
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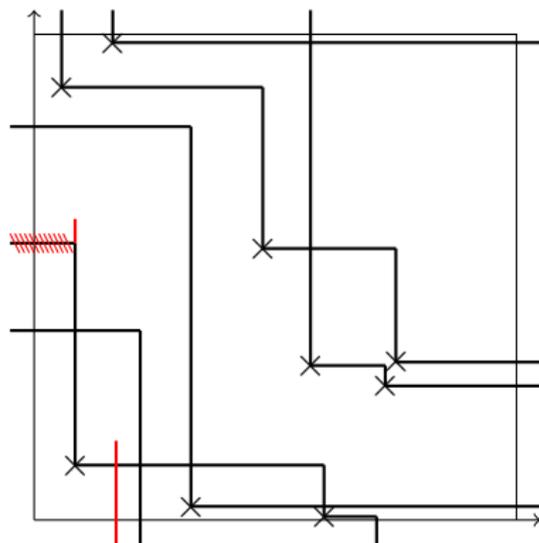
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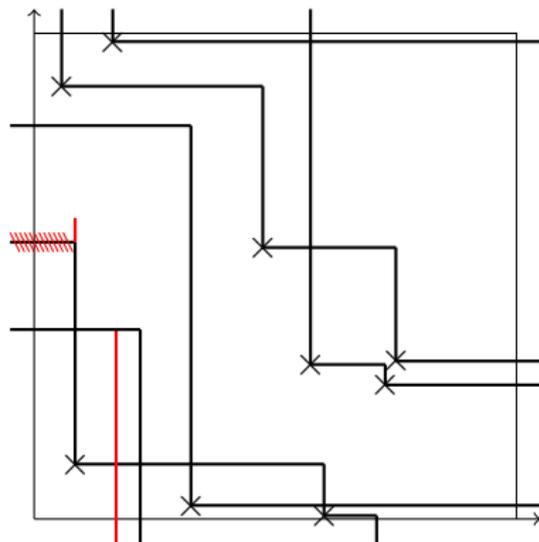
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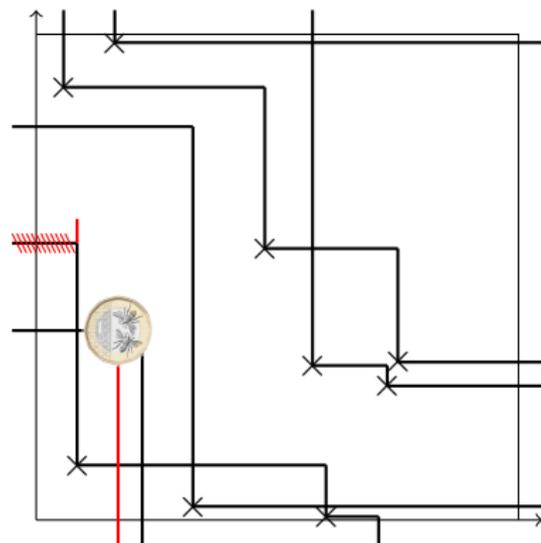
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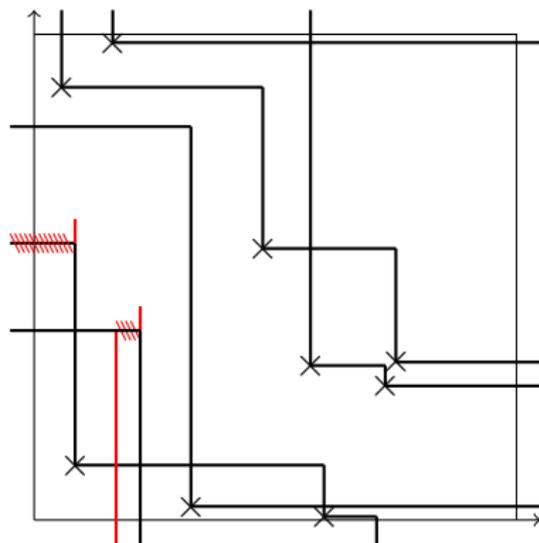
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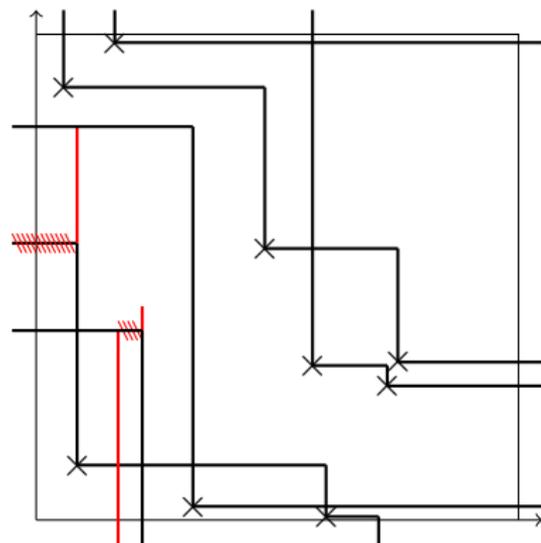
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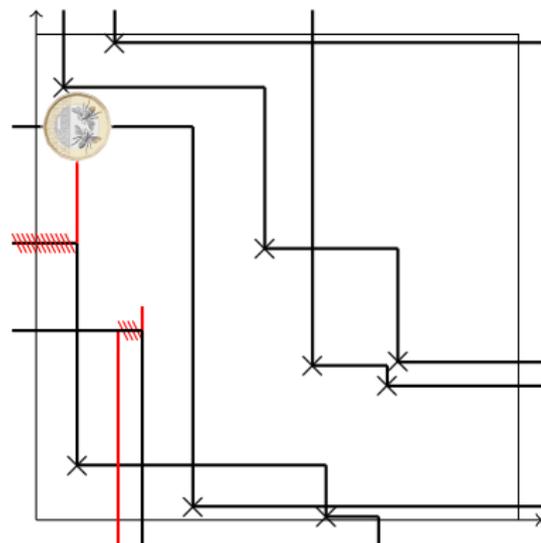
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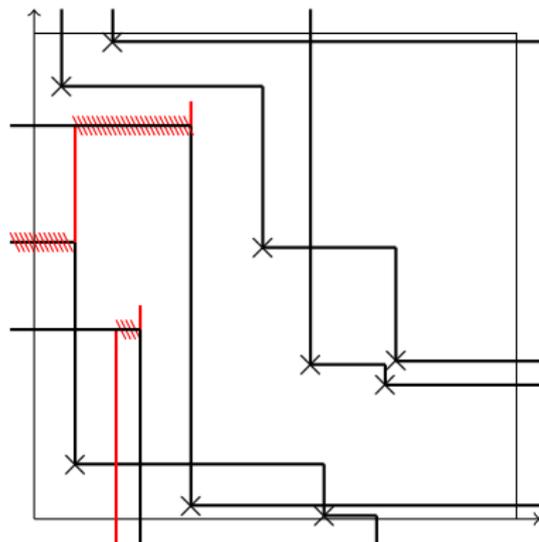
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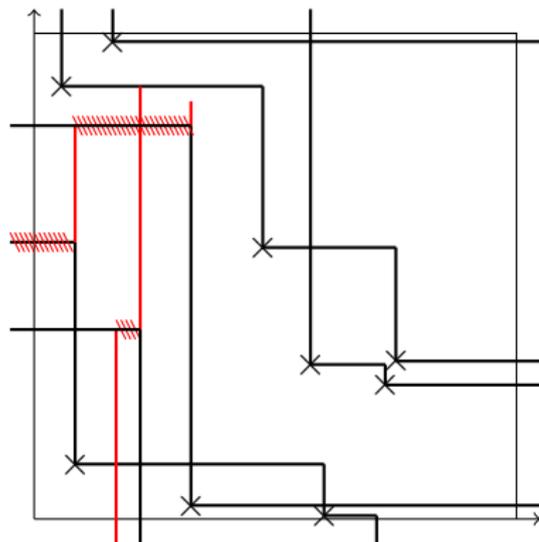
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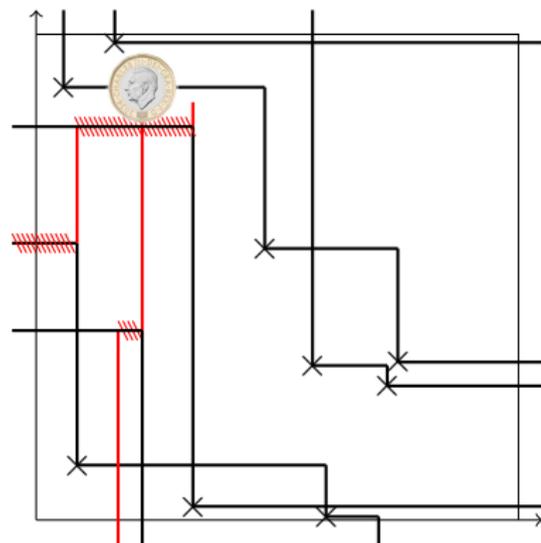
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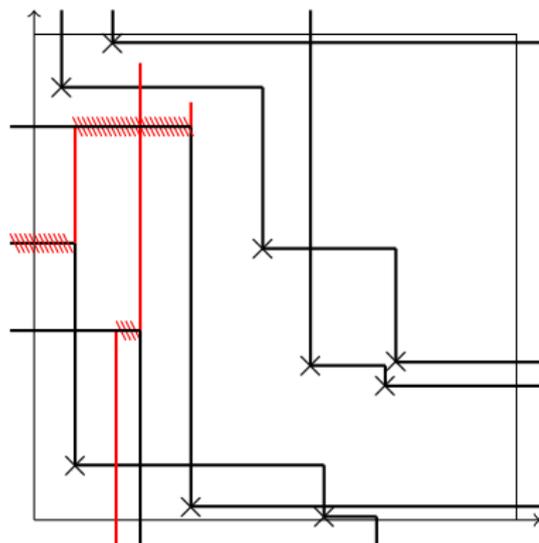
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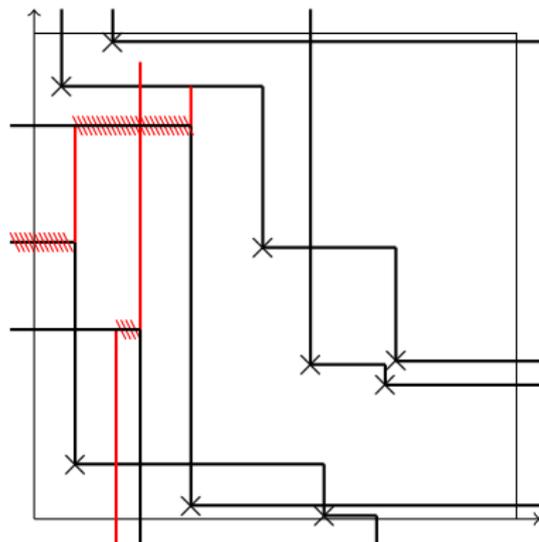
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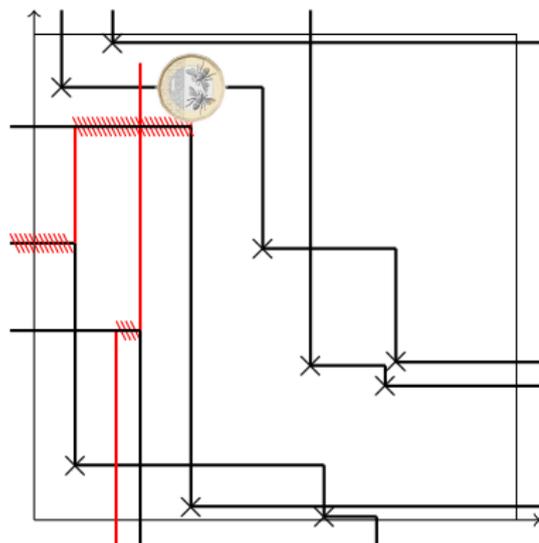
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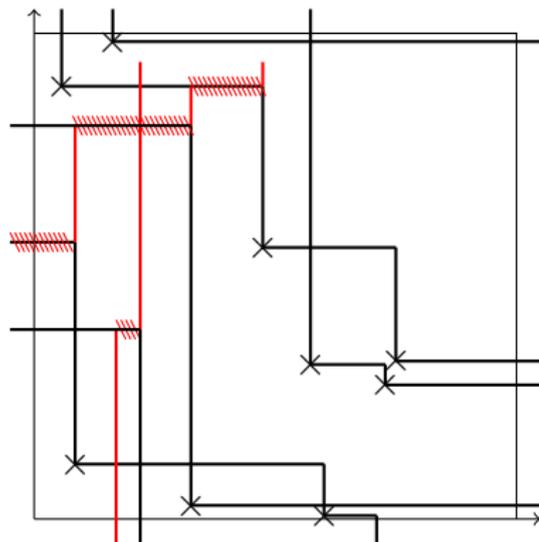
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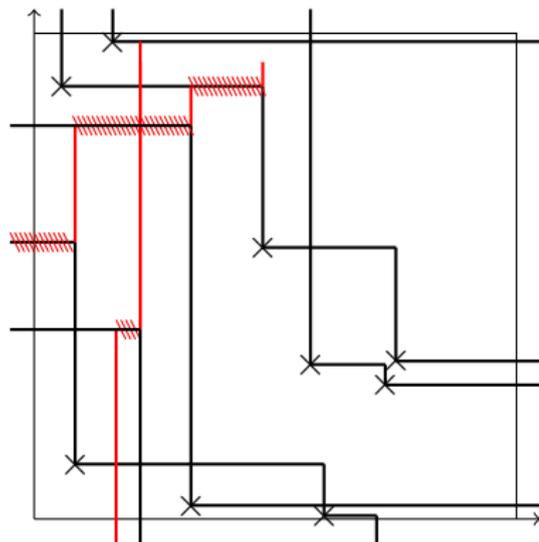
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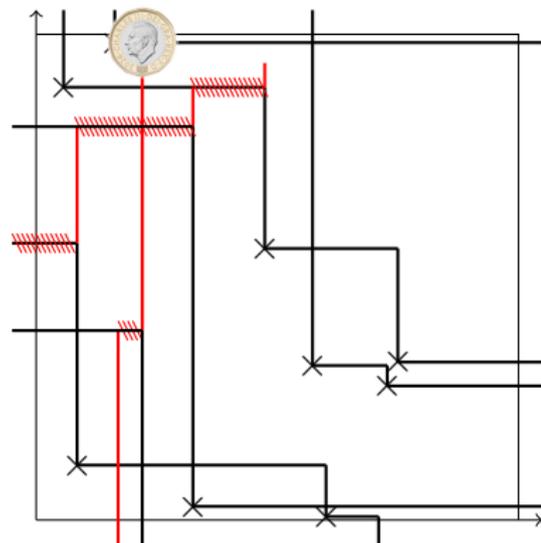
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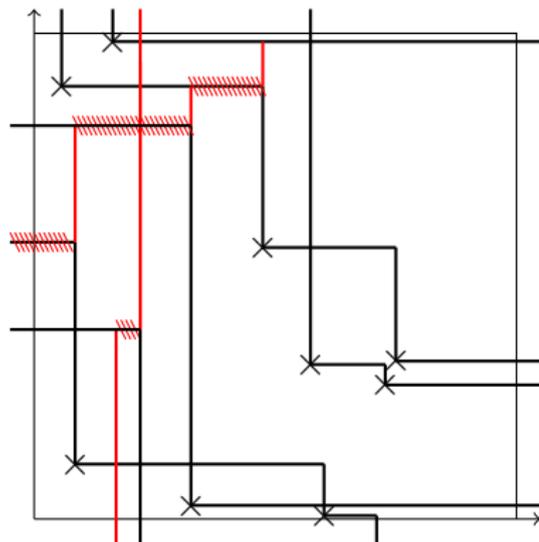


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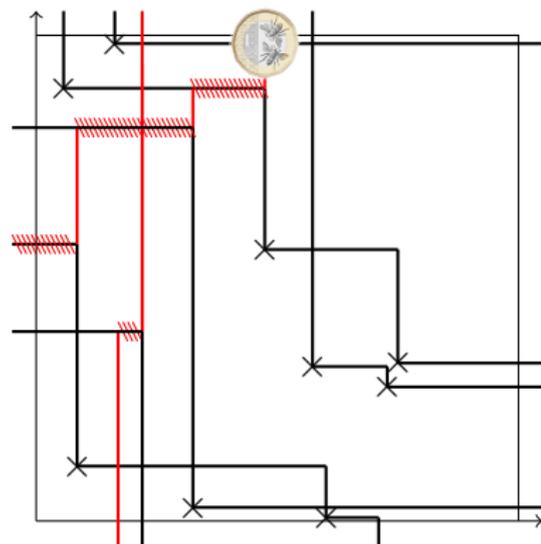
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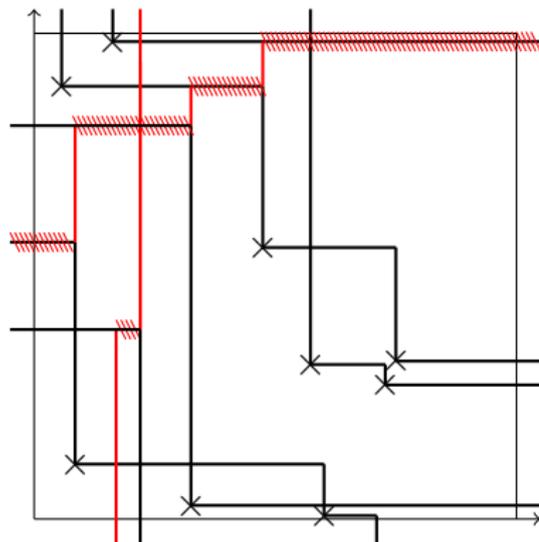
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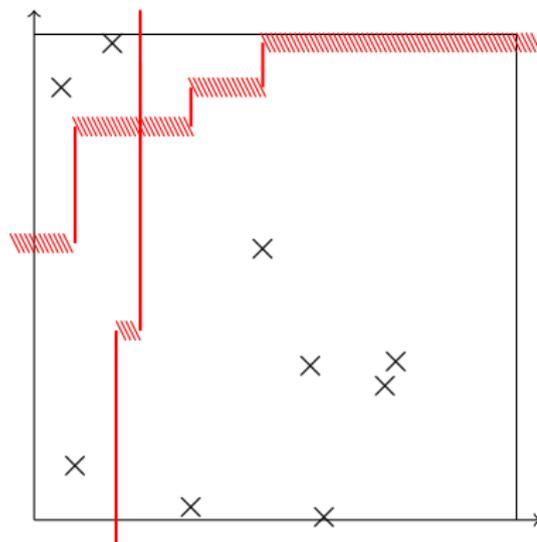
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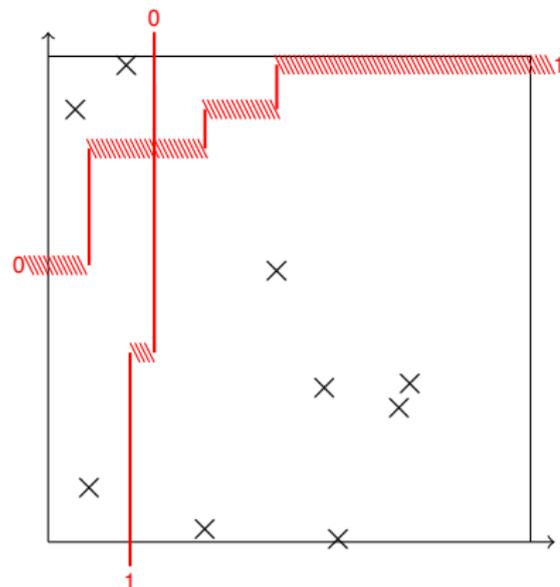


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$\frac{\alpha}{\omega}$  second class particles:  
upright paths

# Second class particles



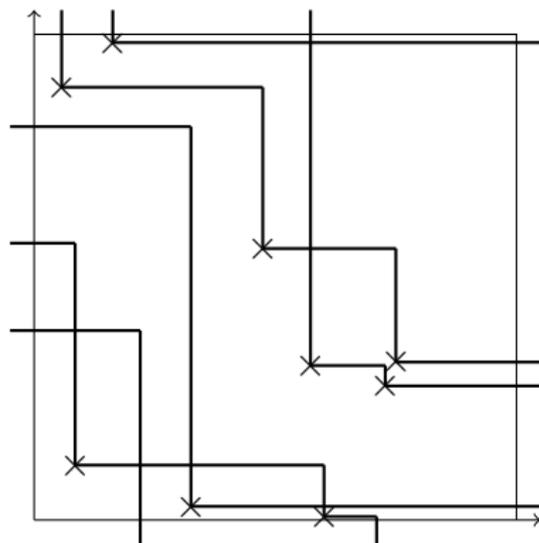
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$\frac{\alpha}{\omega}$  second class particles:  
upright paths  
that stay ordered



## Second class particles

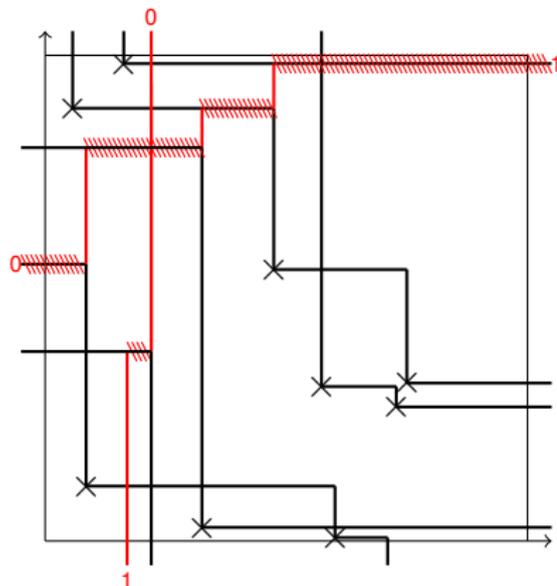


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- ▶ (less horizontals in  $\omega$ .)

$\frac{\omega}{\alpha}$  second class particles:  
upright paths  
that stay ordered

# Second class particles

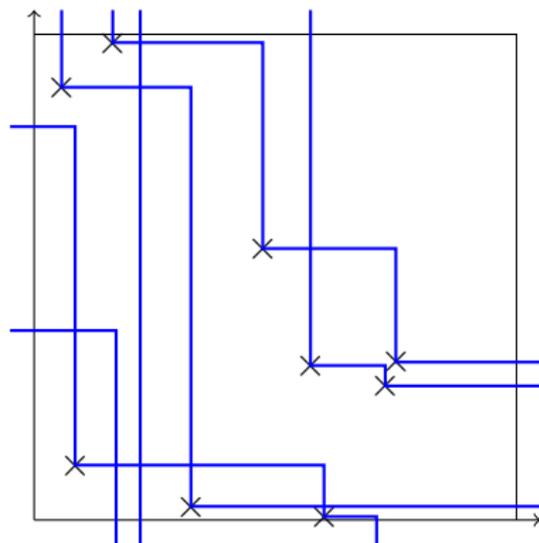


Definition:  $\alpha \leq \omega$ , if

- ▶ more verticals in  $\omega$ ,
- ▶ more horizontals in  $\alpha$ .
- ▶ (less horizontals in  $\omega$ .)

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## Second class particles

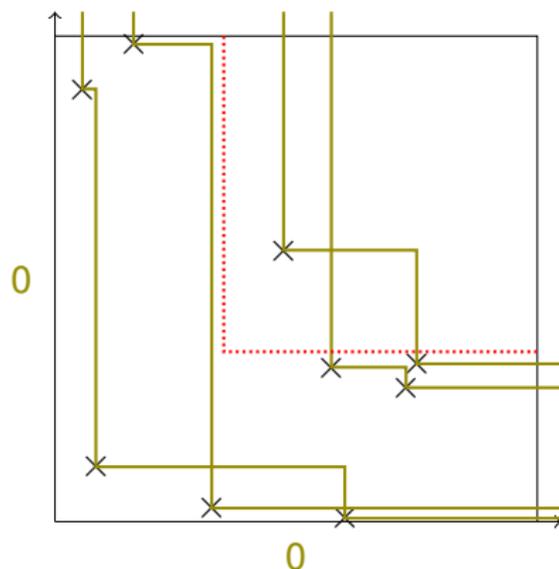


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that stay ordered

# The main actors

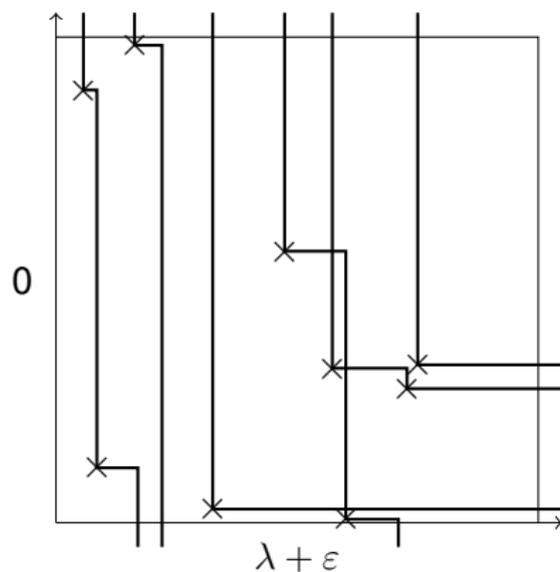


WTS: no. of lines crossing

→  $\text{stati}(\lambda(\mathbf{v}))$  Poisson

$\theta^{0,0}$

# The main actors



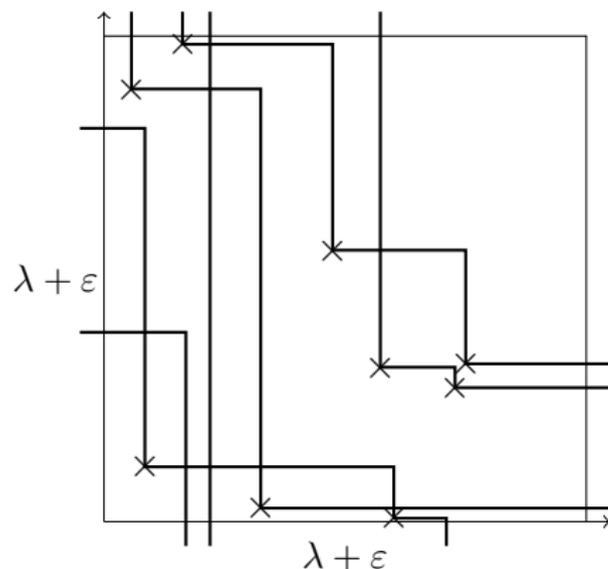
WTS: no. of lines crossing

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$$\theta^{0,0} \leq \omega^{\lambda+\epsilon,0}$$

$$\omega^{\lambda+\epsilon,0}$$

# The main actors

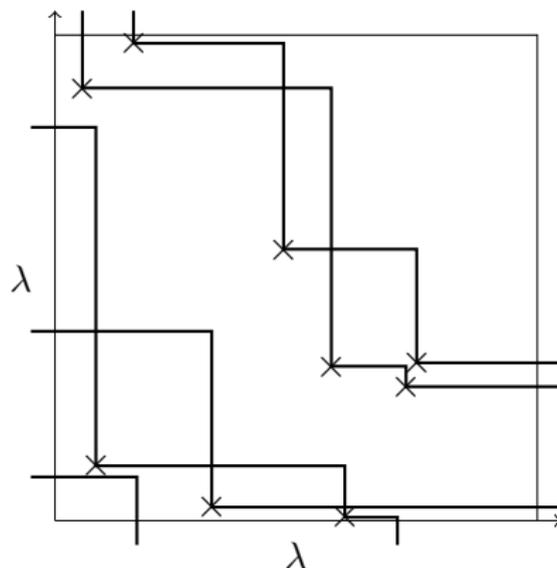


WTS: no. of lines crossing  
 .....  $\rightarrow$   $\text{stati}(\lambda(\mathbf{v}))$  Poisson

$$\theta^{0,0} \leq \omega^{\lambda+\epsilon,0}$$

$$\omega^{\lambda+\epsilon,0} \geq \eta^{\lambda+\epsilon,\lambda+\epsilon}$$

# The main actors

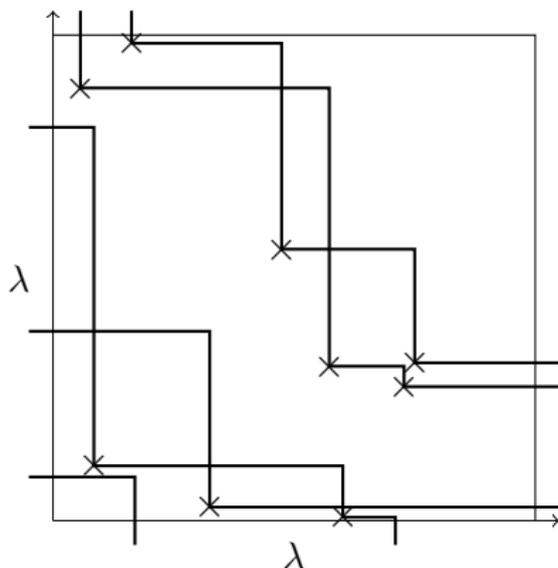


WTS: no. of lines crossing  
 $\text{---} \rightarrow \text{stati}(\lambda(\mathbf{v}))$  Poisson

$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0}$$

$$\omega^{\lambda+\varepsilon,0} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \alpha^{\lambda,\lambda}$$

# The main actors



WTS: no. of lines crossing  
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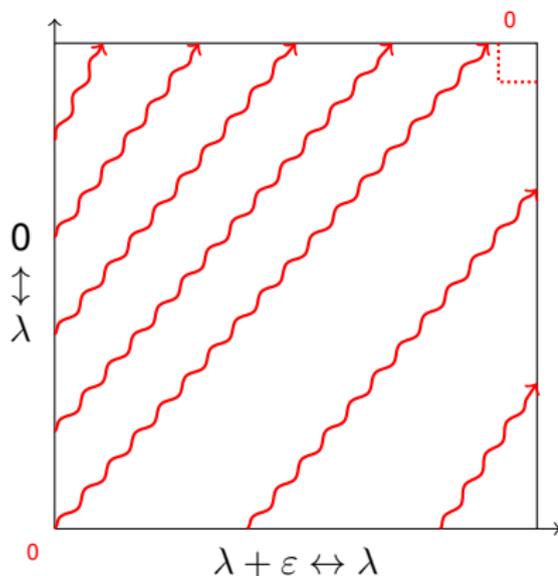
$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0}$$

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$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \alpha^{\lambda,\lambda}$$

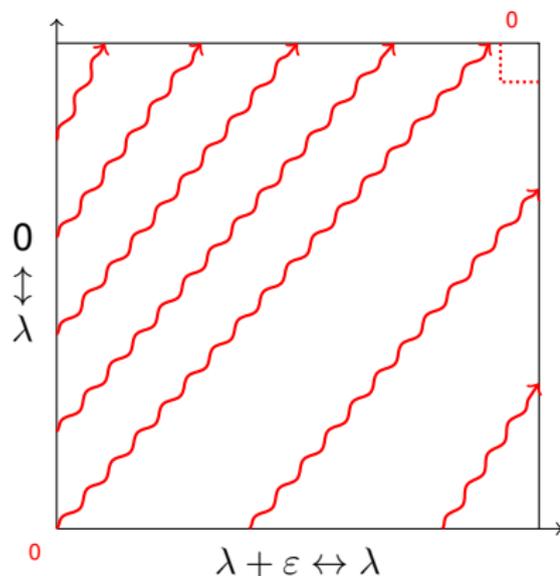
# 1. $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$ 2nd class particles and the small box



Second class particle 0 goes above the small box.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \underline{\alpha^{\lambda,\lambda}}$$

# 1. $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$ 2nd class particles and the small box

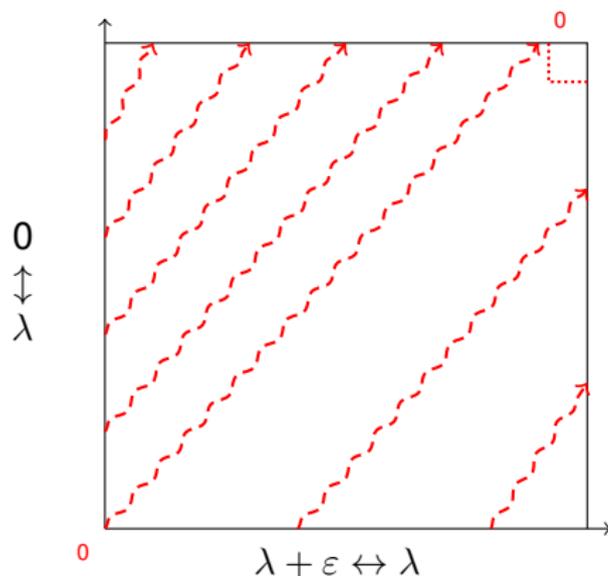


Second class particle 0 goes above the small box.

That's because of the shape  
thm for each  $\omega^{\lambda+\varepsilon,0}$  and  $\alpha^{\lambda,\lambda}$ ,  
and the  $\varepsilon$  leeway.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \underline{\alpha^{\lambda,\lambda}}$$

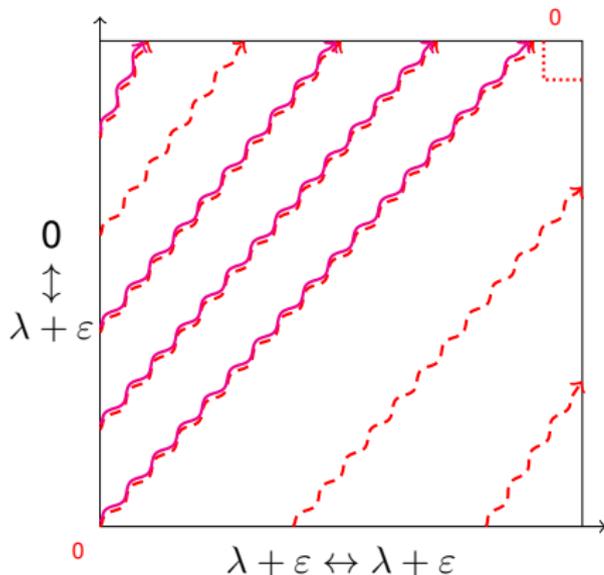
2.  $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$  on  $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$



---


$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \underline{\alpha^{\lambda,\lambda}}$$

2.  $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$  on  $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$



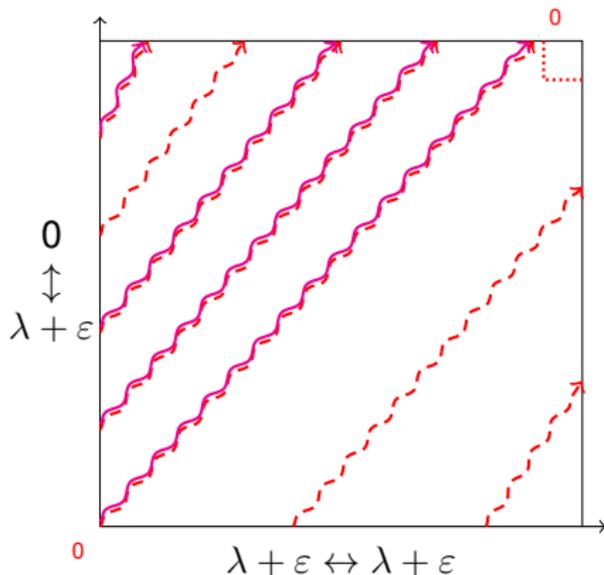
$$\left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \right\} \subseteq \left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}} \right\}$$

and only start from the left boundary.

---


$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \alpha^{\lambda,\lambda}$$

2.  $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$  on  $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$



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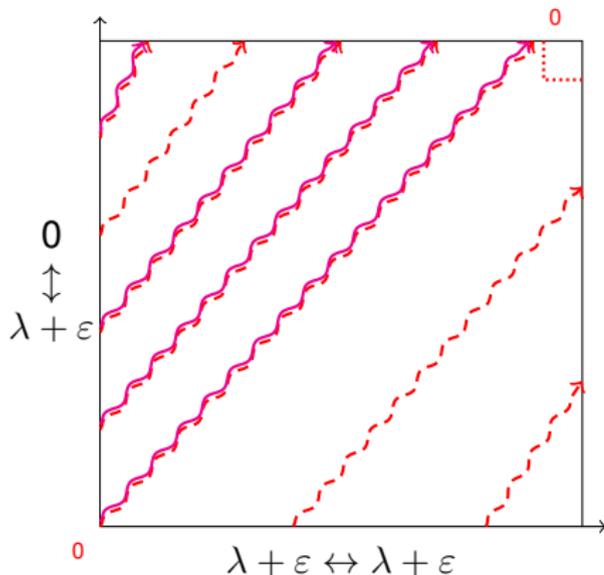
and only start from the left boundary.

$\rightsquigarrow$  there is a rightmost  $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$  second class particle, jumping on  $\left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}} \right\}$ .

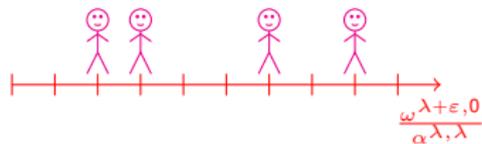
---


$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \underline{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

3.  $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$  on  $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$ : we have drift!



$$\left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \right\} \subseteq \left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}} \right\}$$

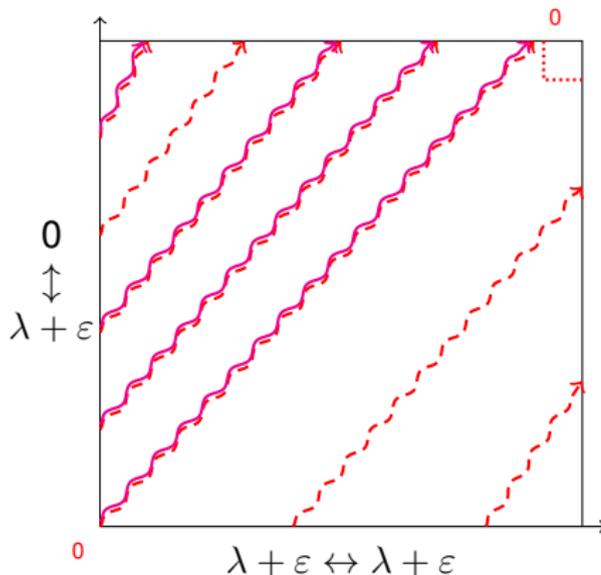


↪: prob. 1

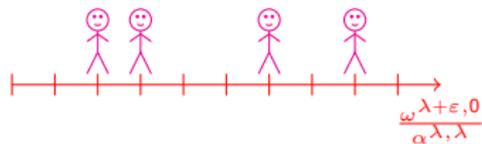
↪: prob.  $t$

$$\theta^{0,0} \leq \frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

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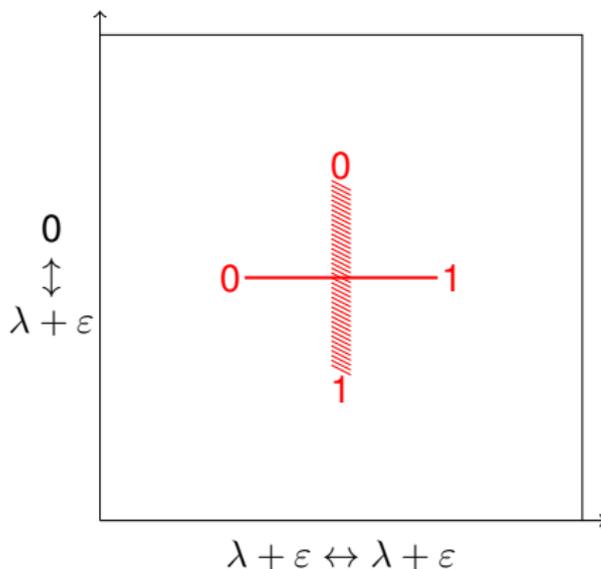
↶: prob. 1

↷: prob.  $t$

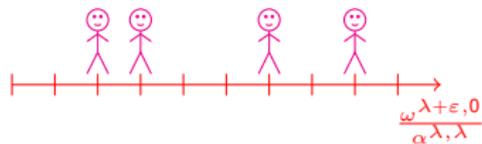
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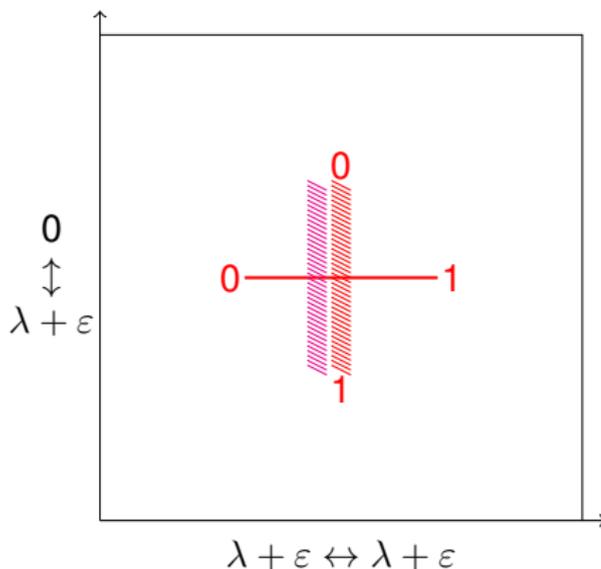
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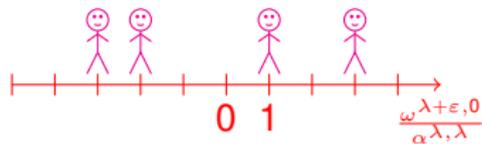
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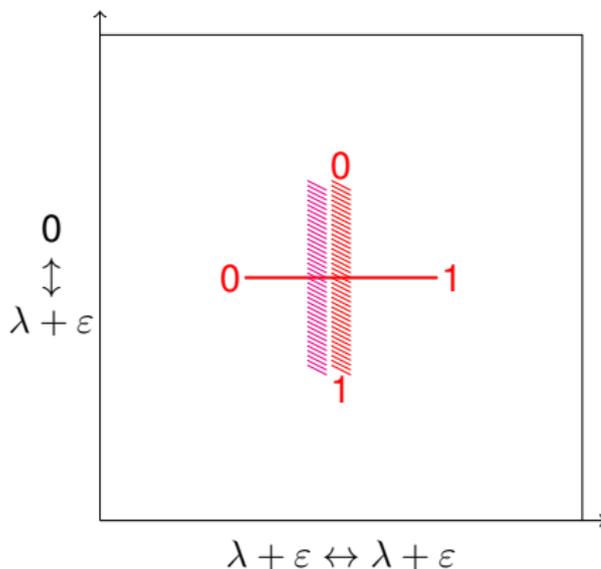
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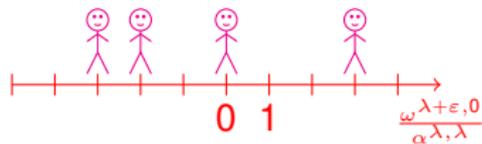
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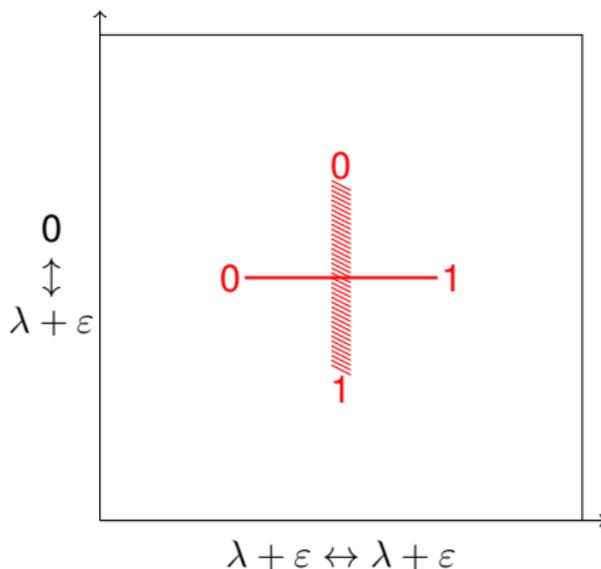
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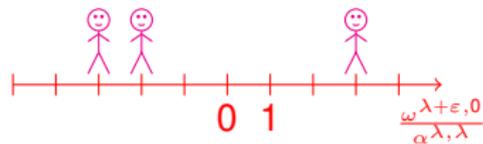
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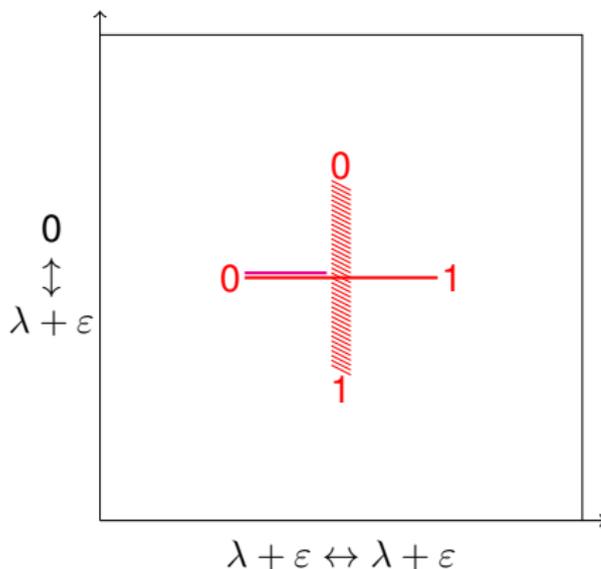
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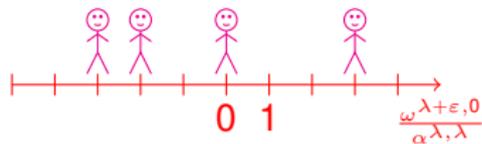
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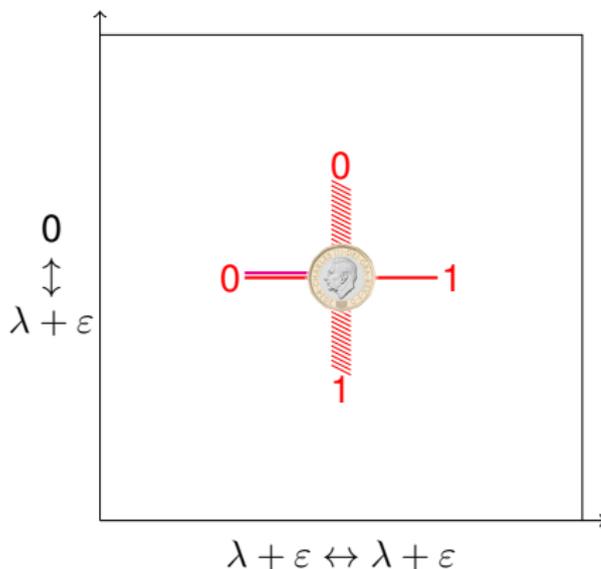
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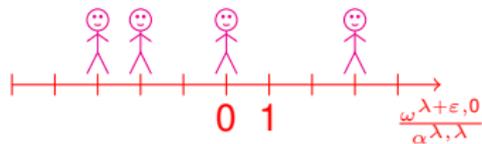
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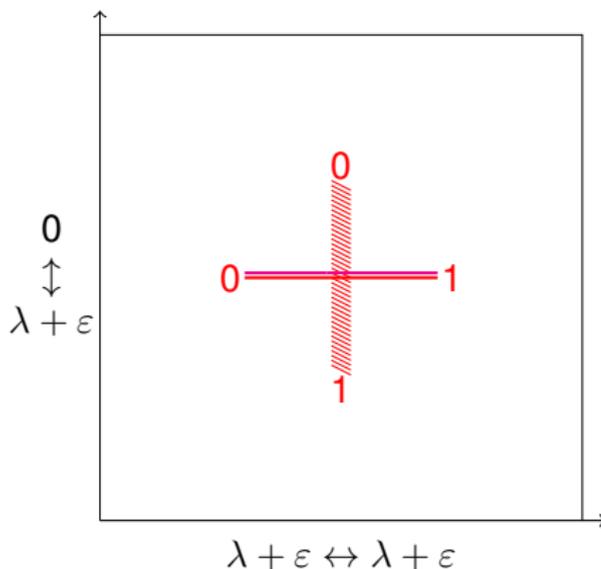
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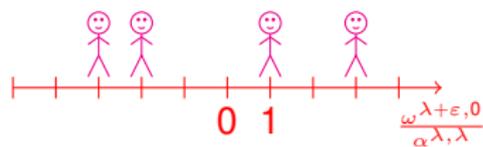
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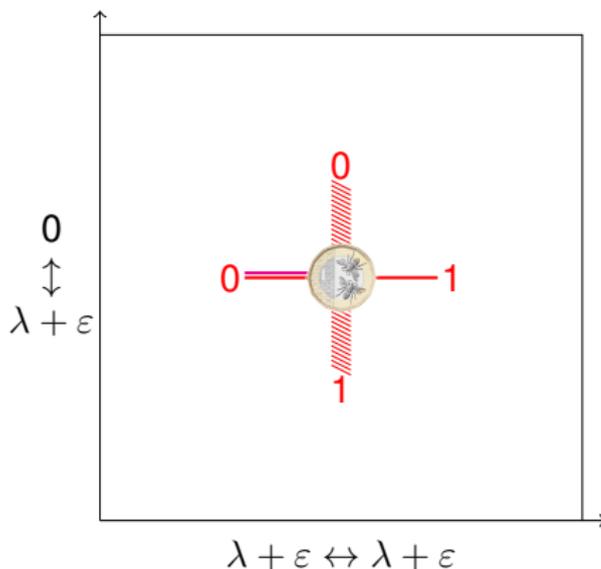
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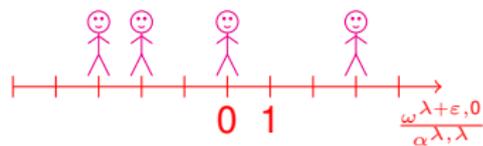
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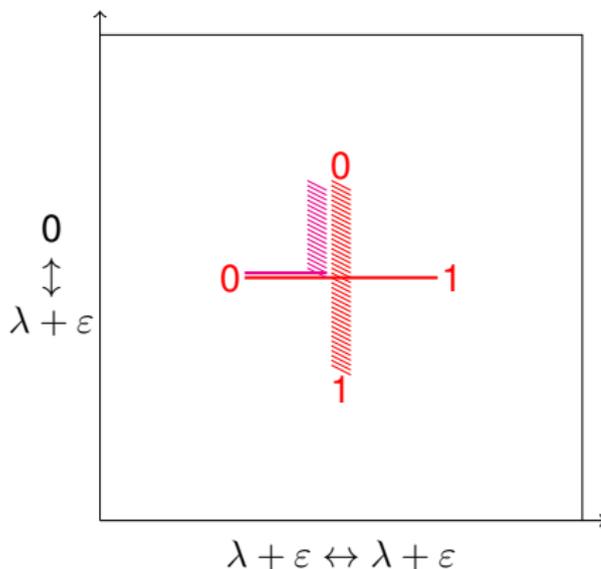
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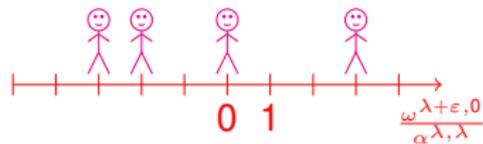
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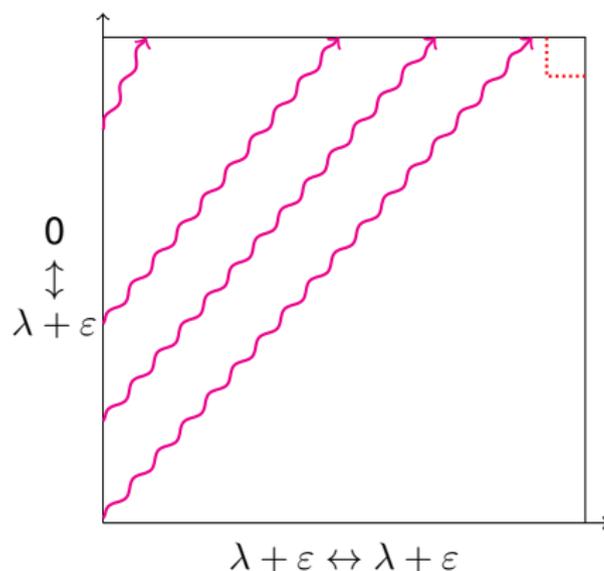
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## 4. Conclusion

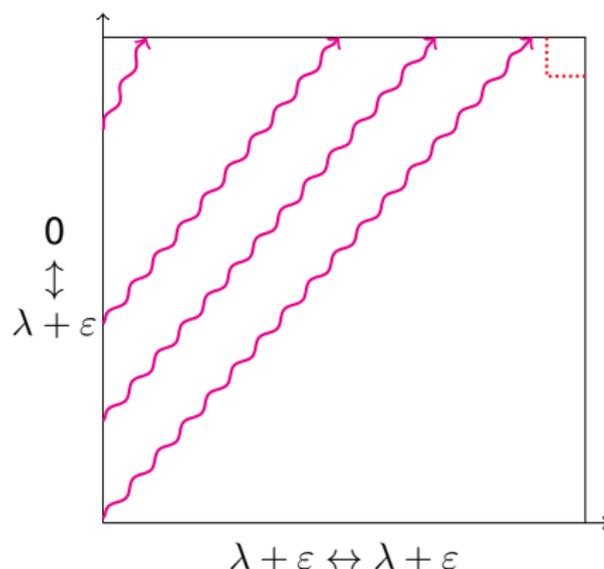


Rightmost  $\frac{\omega^{\lambda+\epsilon,0}}{\eta^{\lambda+\epsilon,\lambda+\epsilon}}$  second class still left of the small box whp.

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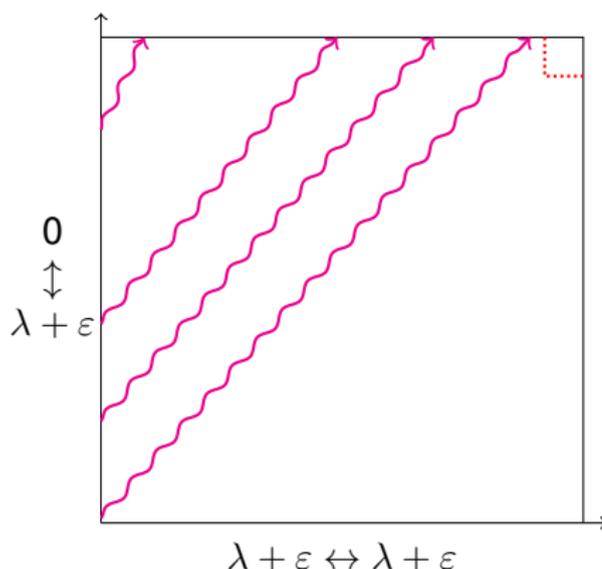
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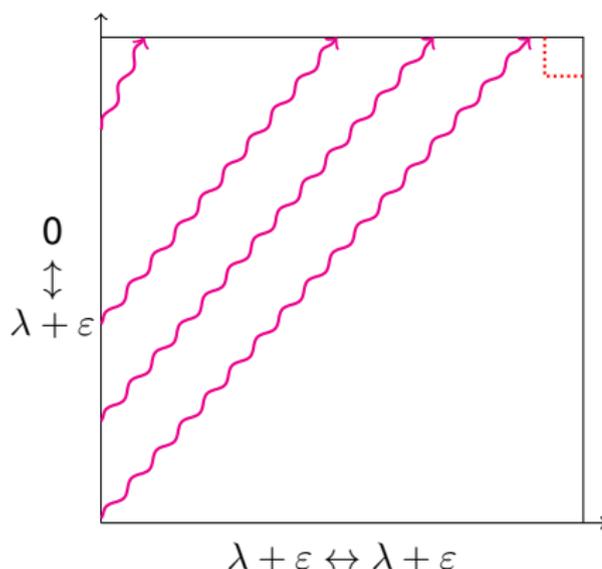
$\rightsquigarrow \omega^{\lambda+\epsilon,0} = \eta^{\lambda+\epsilon,\lambda+\epsilon}$  in the small box whp.

$\rightsquigarrow$  but  $\eta^{\lambda+\epsilon,\lambda+\epsilon}$  is stationary so has  $\text{Poi}(\lambda + \epsilon)$  number of lines entering the box whp.

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$$\theta^{0,0} \leq \underline{\omega^{\lambda+\epsilon,0}} \geq \underline{\eta^{\lambda+\epsilon,\lambda+\epsilon}} \geq \alpha^{\lambda,\lambda}$$

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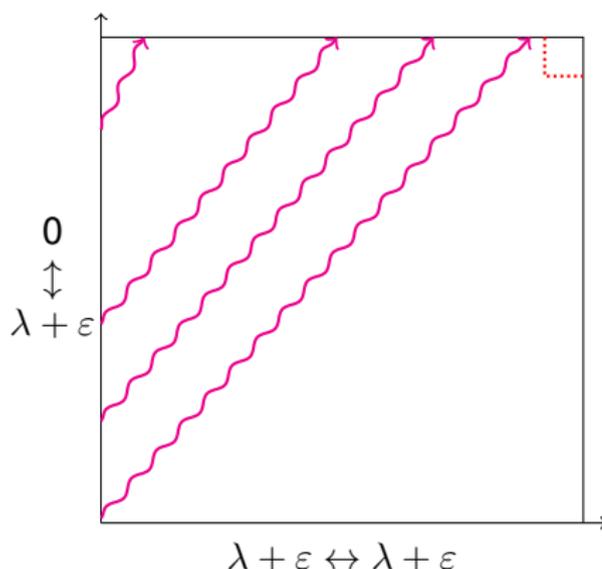
$\rightsquigarrow$  but  $\eta^{\lambda+\epsilon,\lambda+\epsilon}$  is stationary so has  $\text{Poi}(\lambda + \epsilon)$  number of lines entering the box whp.

$\rightsquigarrow$  whp.  $\theta^{0,0} \leq \text{Poi}(\lambda + \epsilon)$   
 $\forall \epsilon > 0$ .

---


$$\theta^{0,0} \leq \underline{\omega^{\lambda+\epsilon,0}} \geq \underline{\eta^{\lambda+\epsilon,\lambda+\epsilon}} \geq \alpha^{\lambda,\lambda}$$

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*Thank you.*

Rightmost  $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$  second class still left of the small box whp.

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