

Dependent Double Branching Annihilating Random Walk

Joint with
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Márton Balázs

University of Bristol

Pspde iii, University of Minho
17 December, 2014.

Non-attractivity and the second class particle

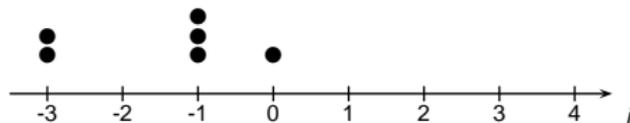
A mean field model

Positive recurrence

Two words on the proof

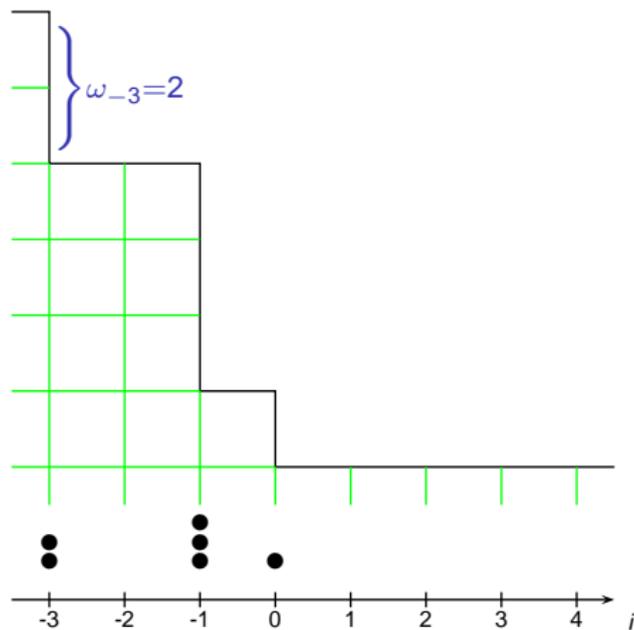
Conservative IPS

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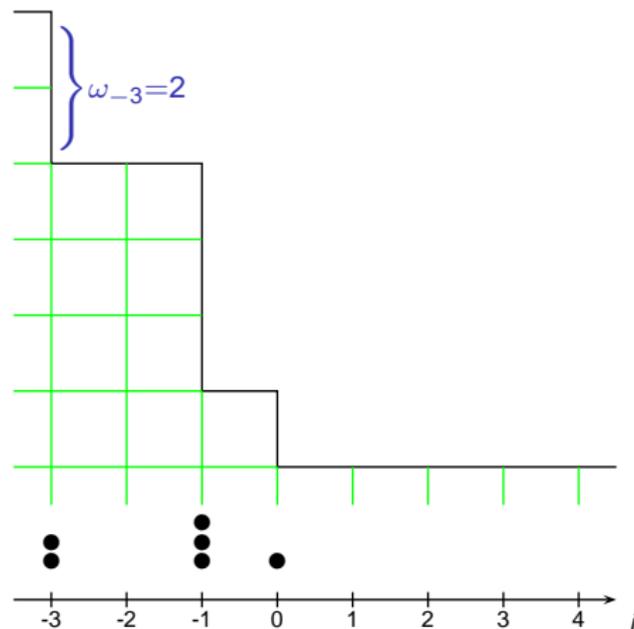
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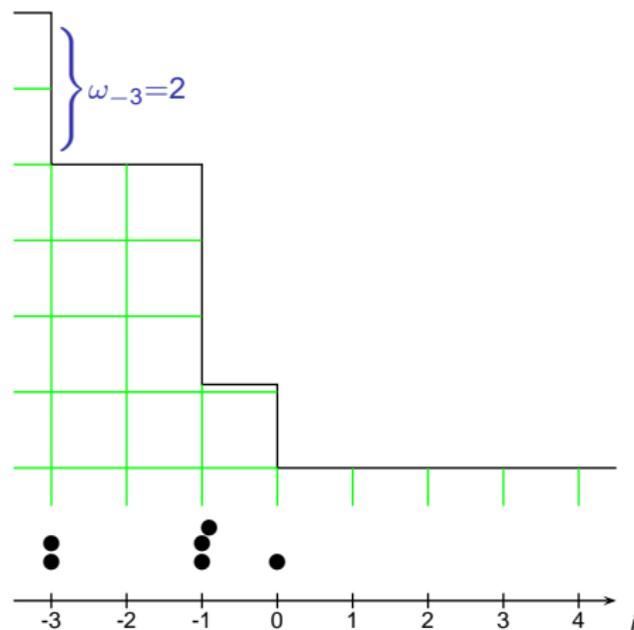
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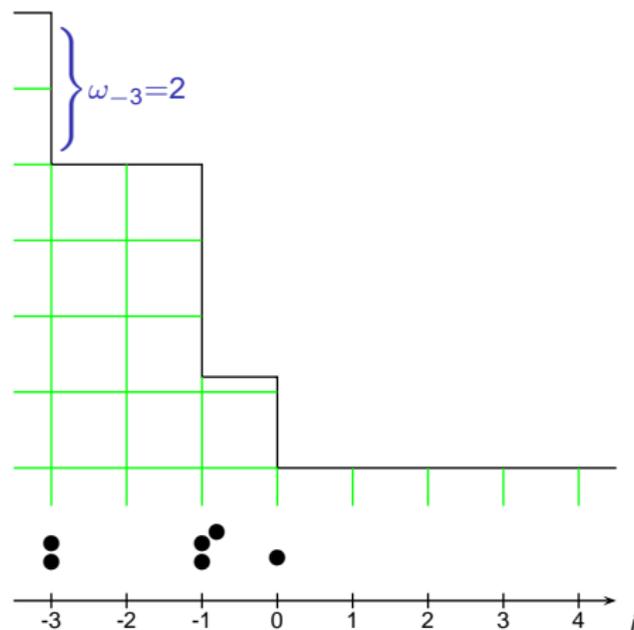
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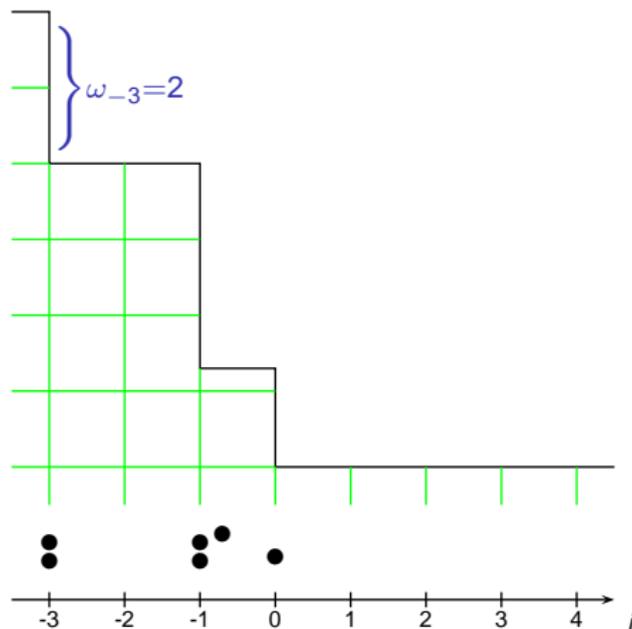
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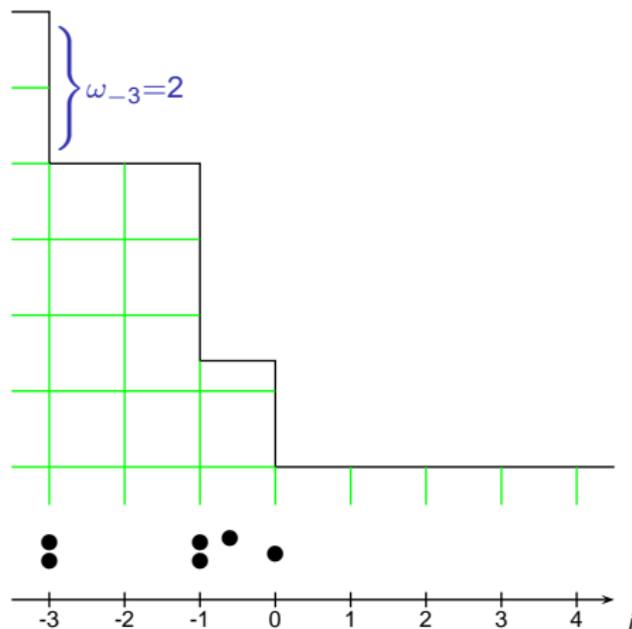
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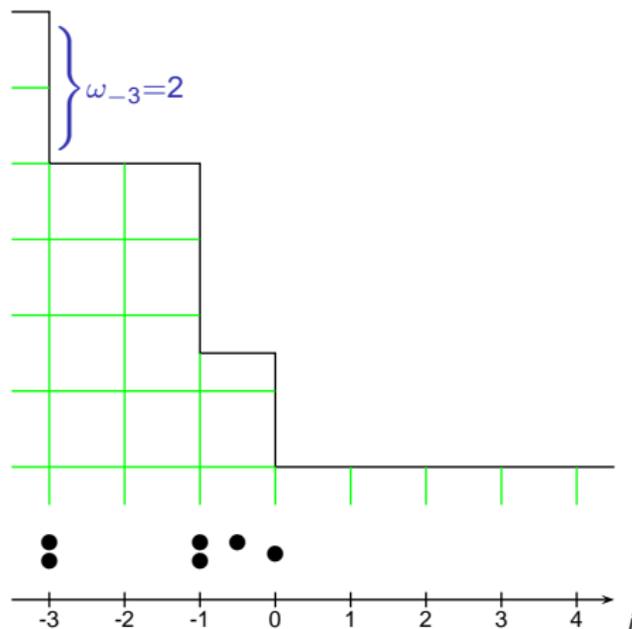
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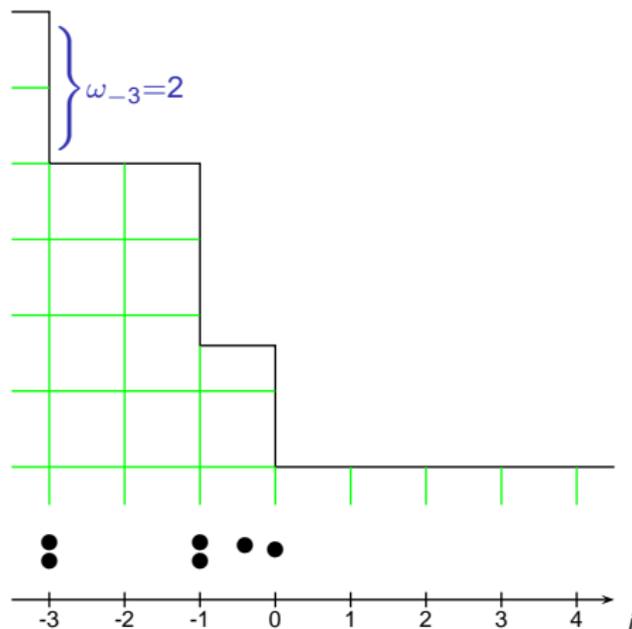
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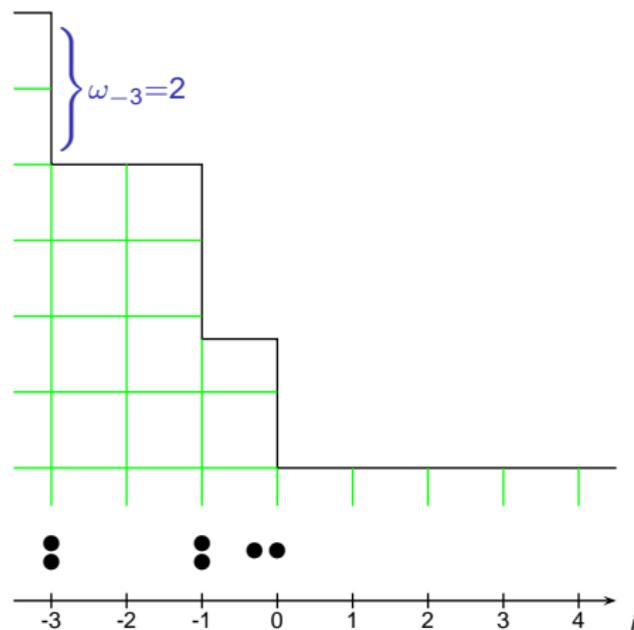
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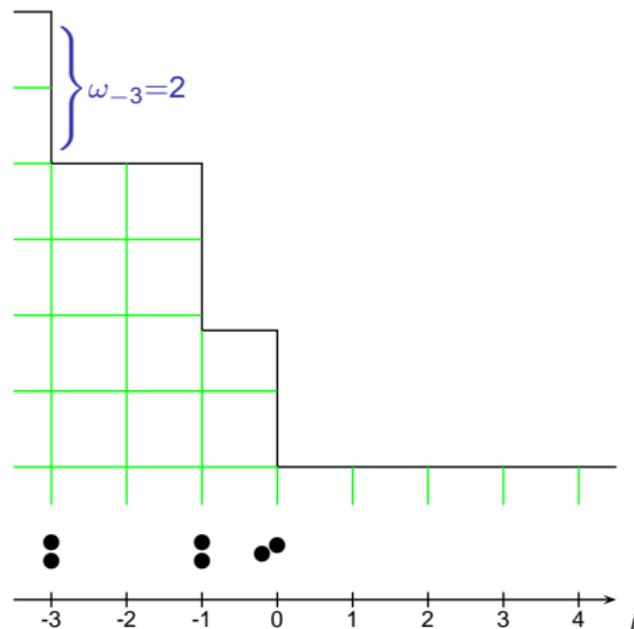
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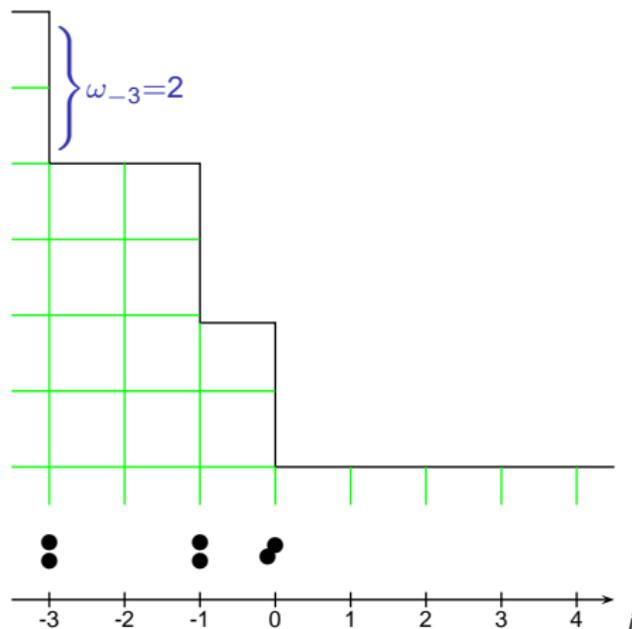
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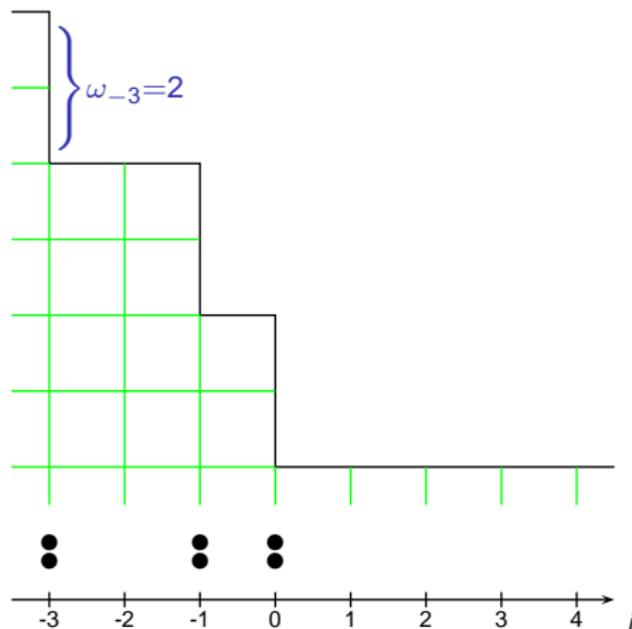
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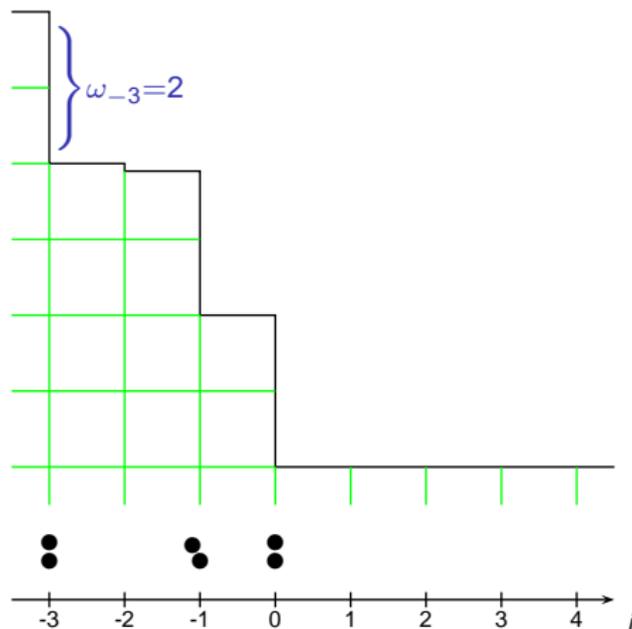
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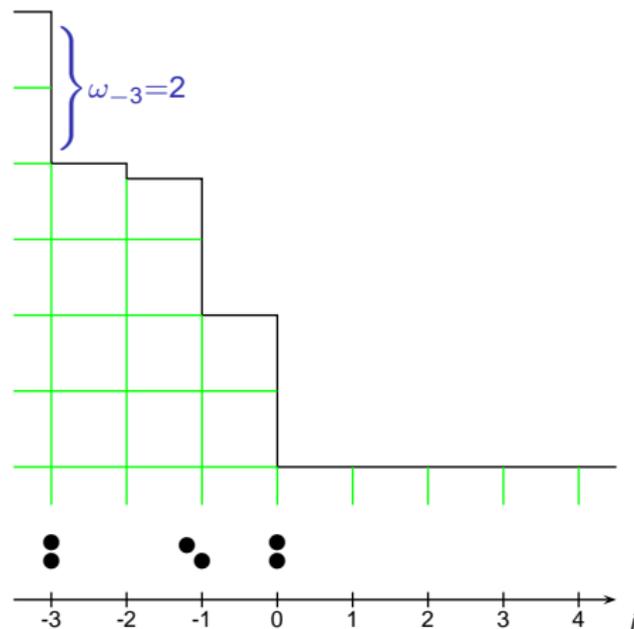
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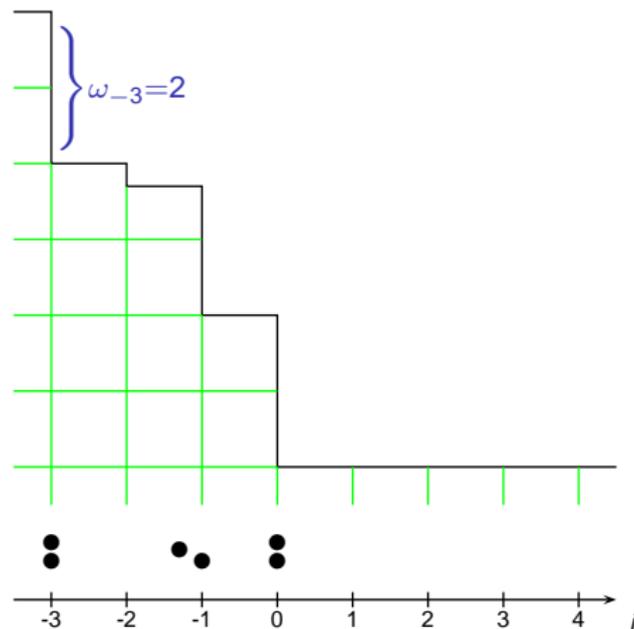
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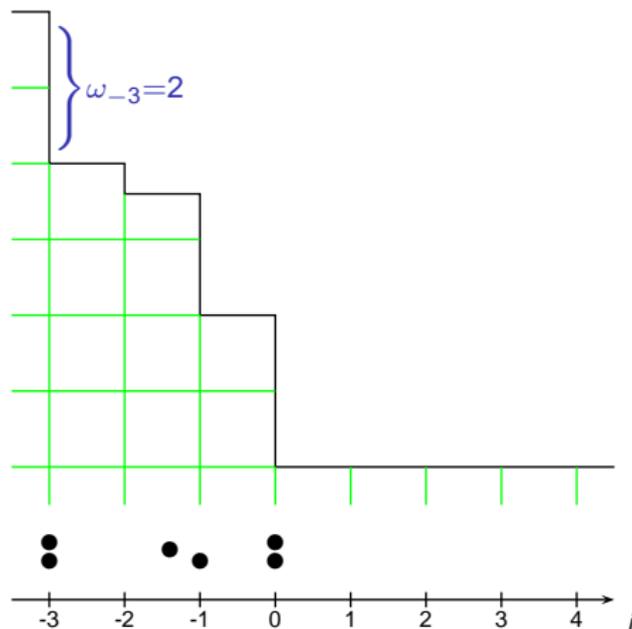
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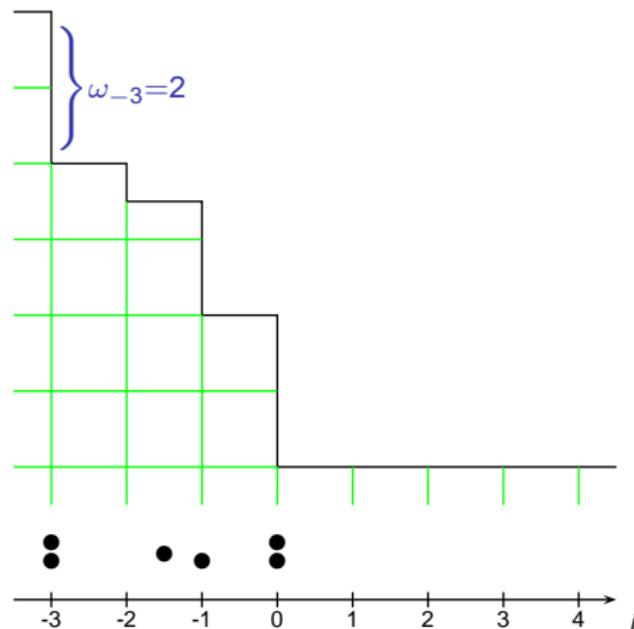
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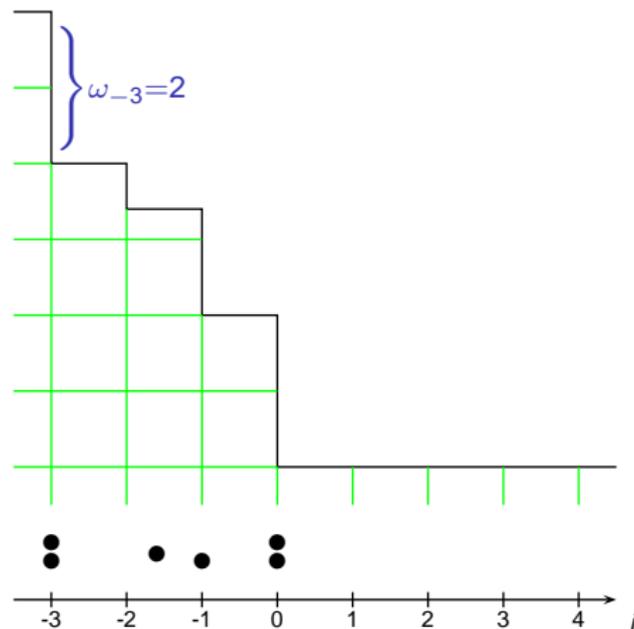
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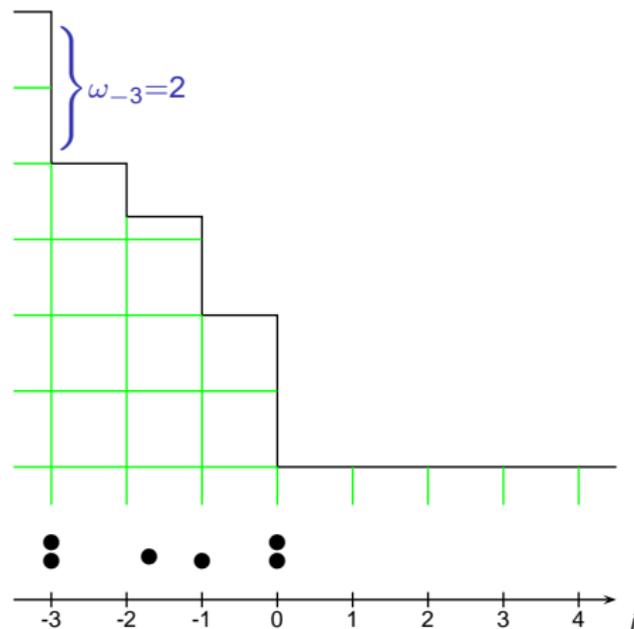
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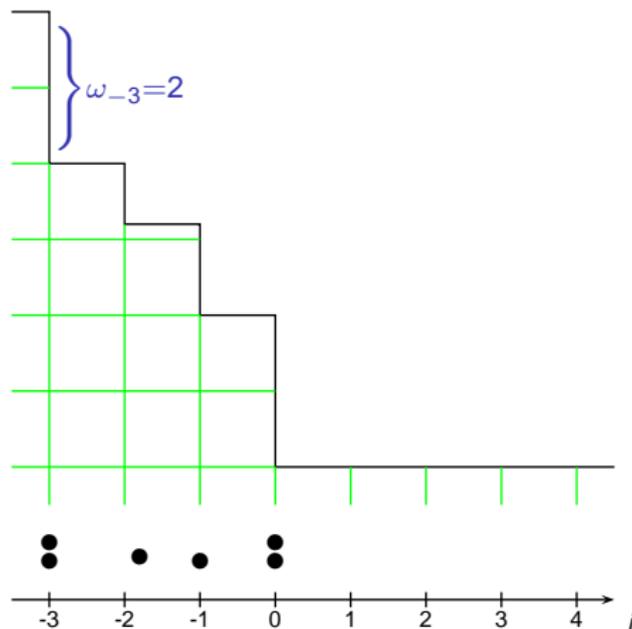
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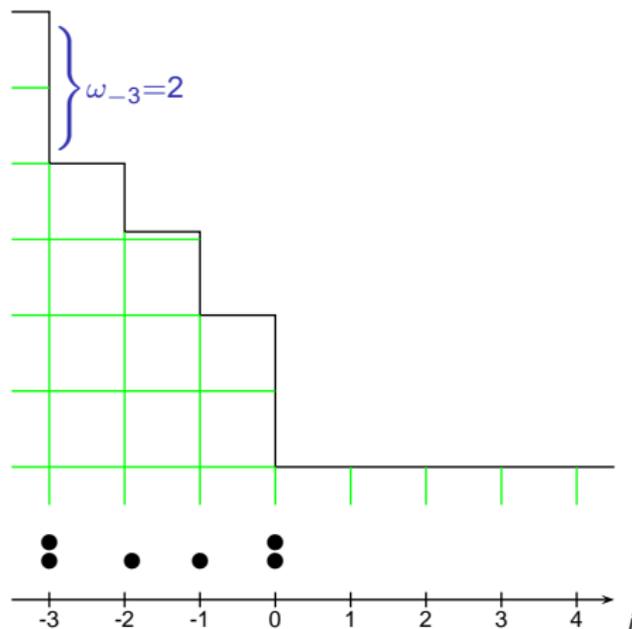
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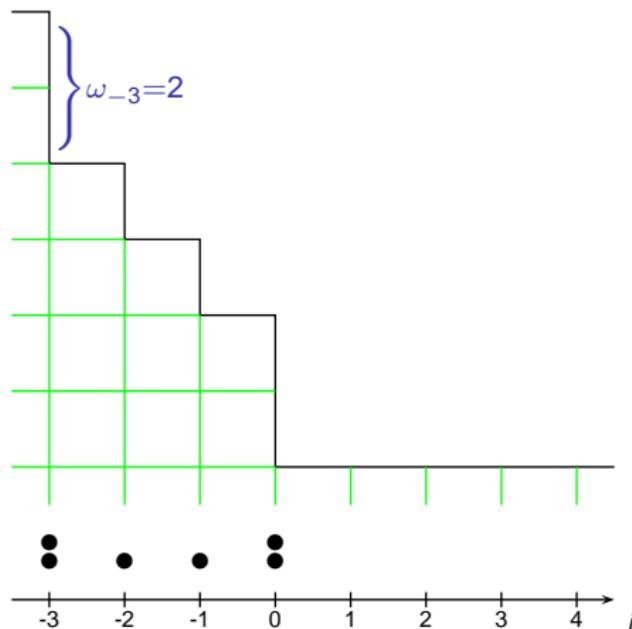
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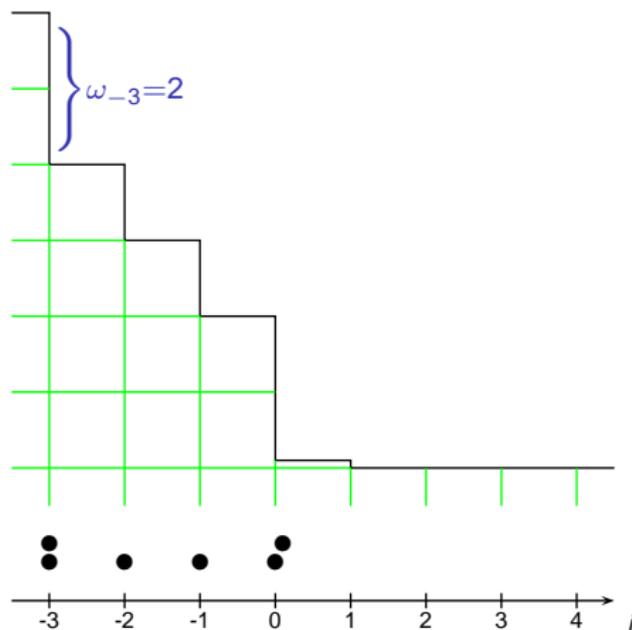
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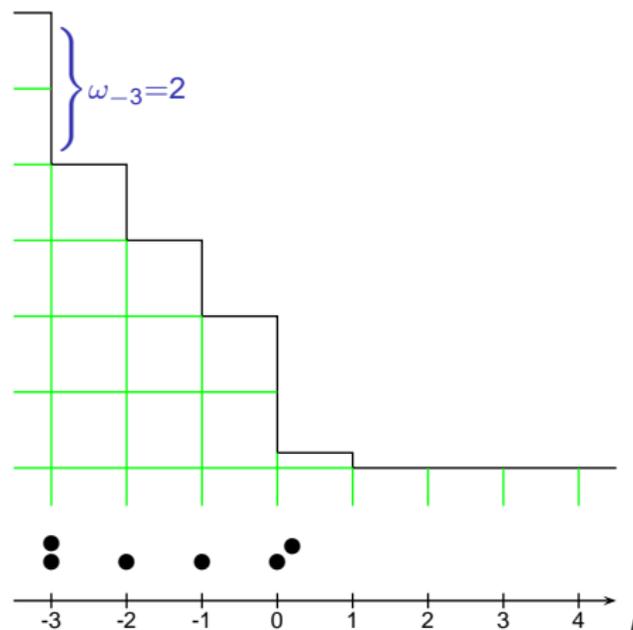
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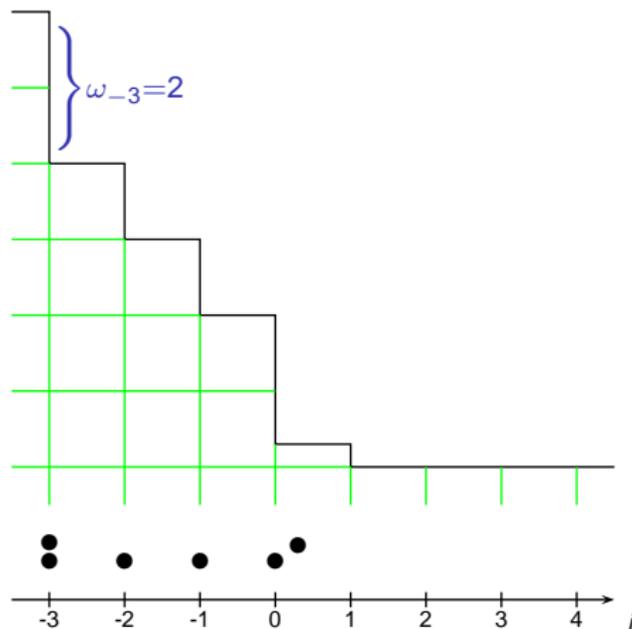
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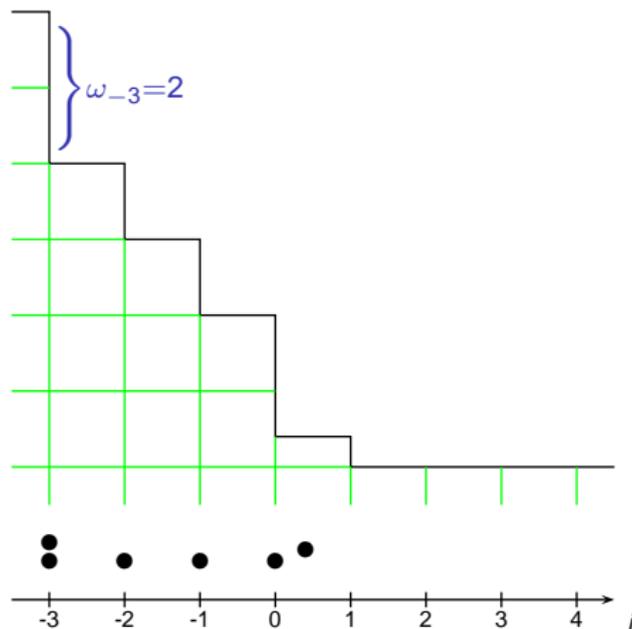
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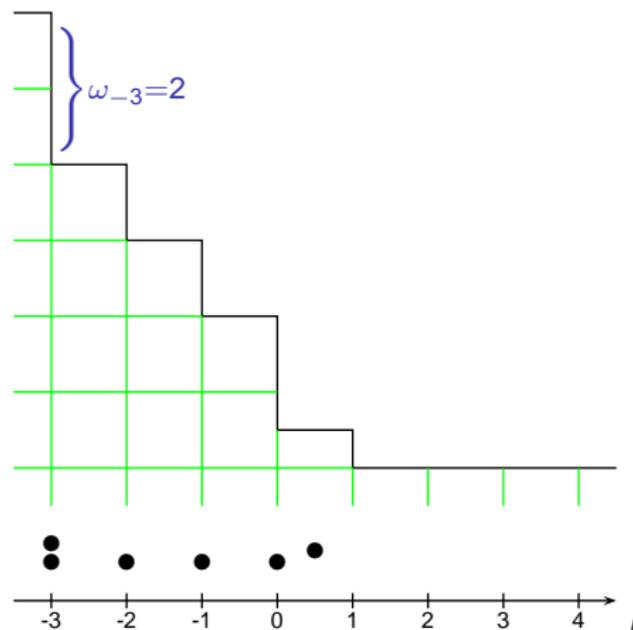
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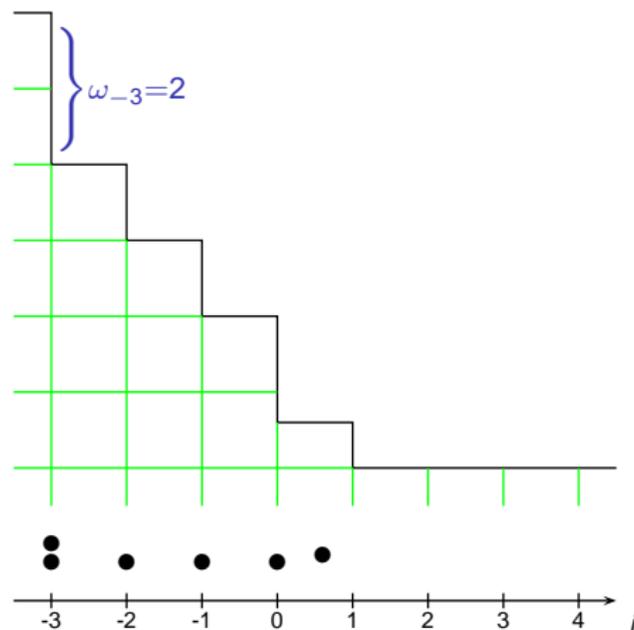
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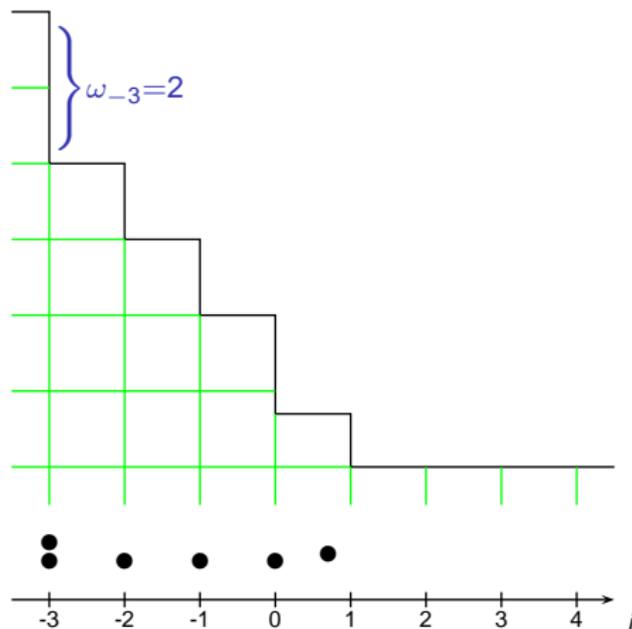
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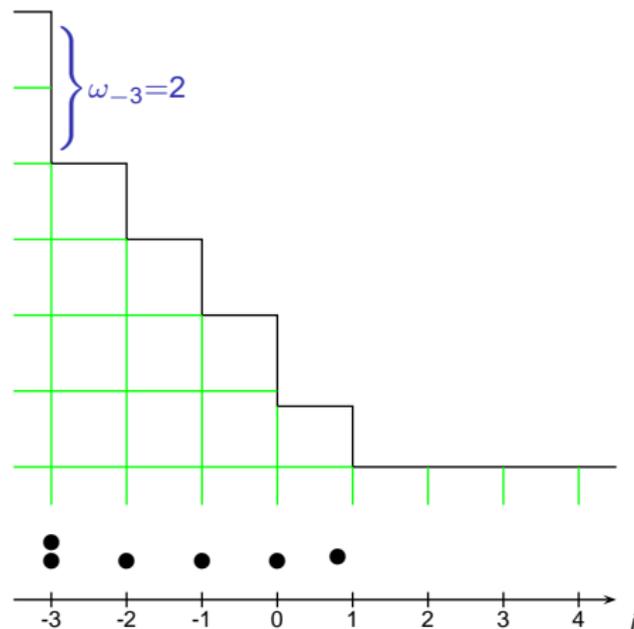
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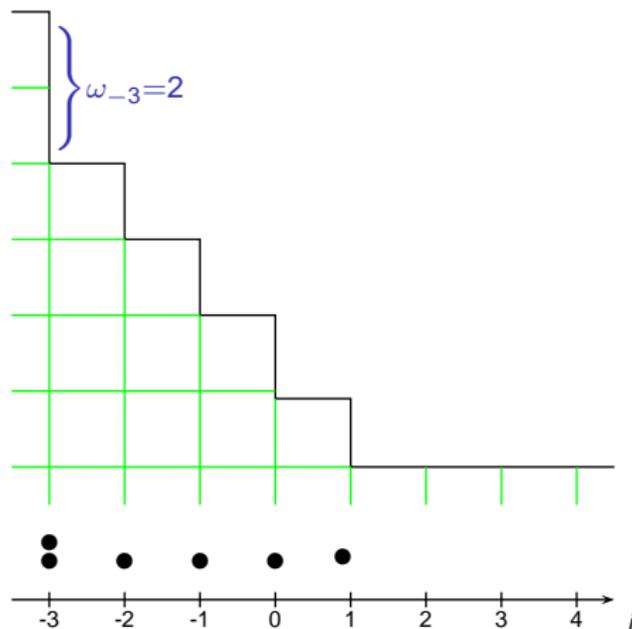
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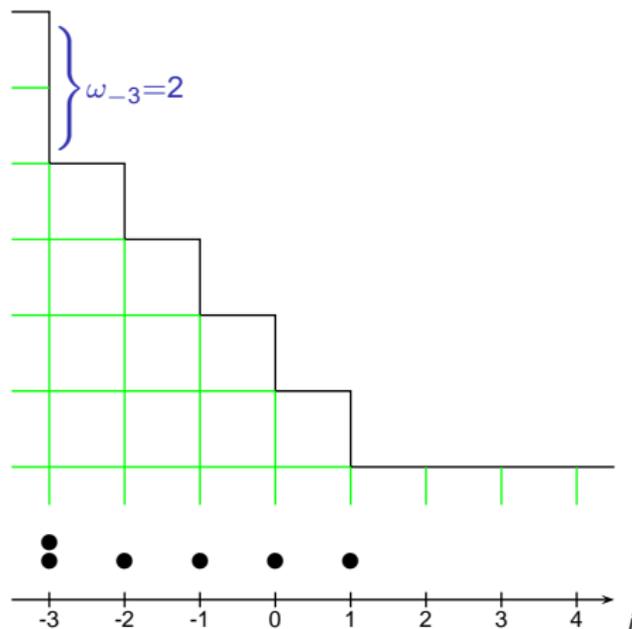
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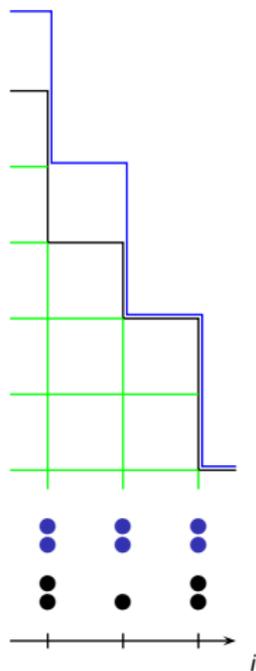
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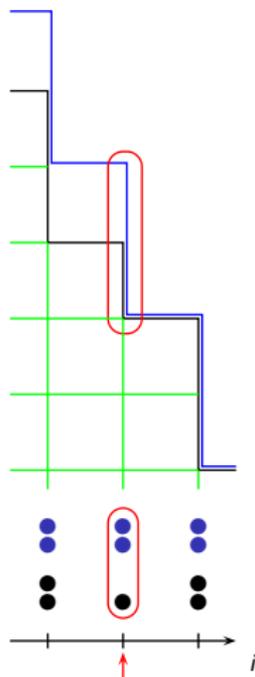
The second class particle: attractive case

States ω and $\tilde{\omega}$ only differ at one site.



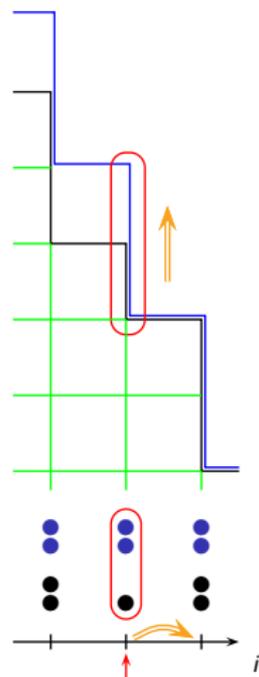
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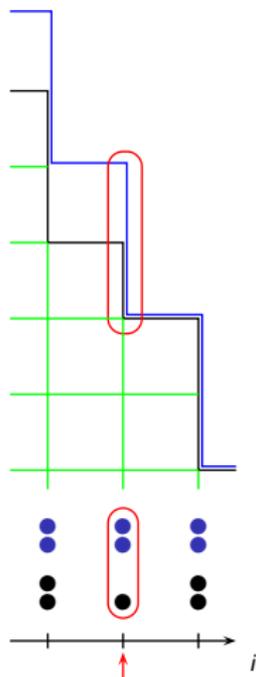
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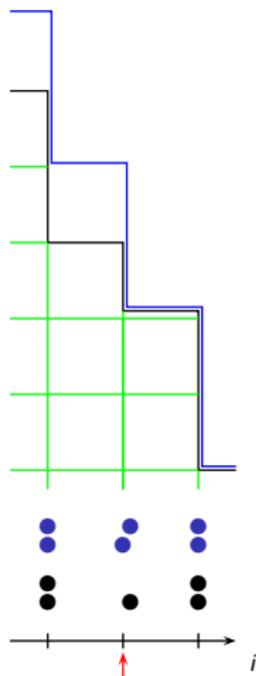
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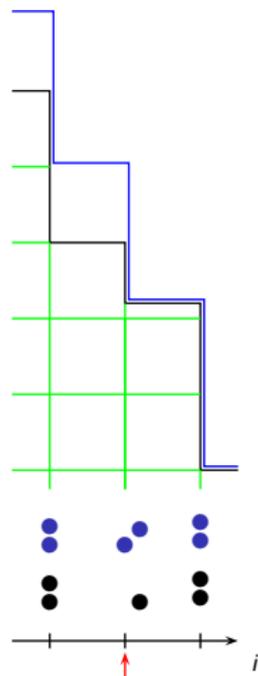
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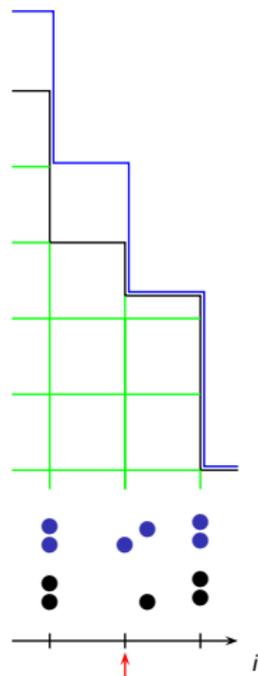
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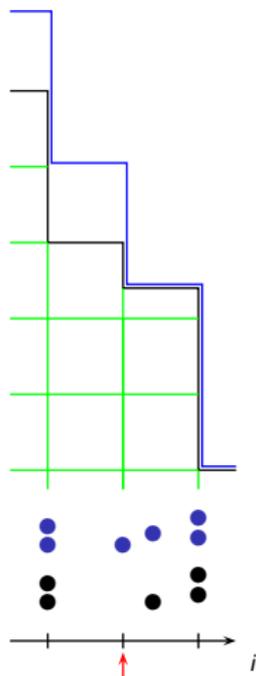
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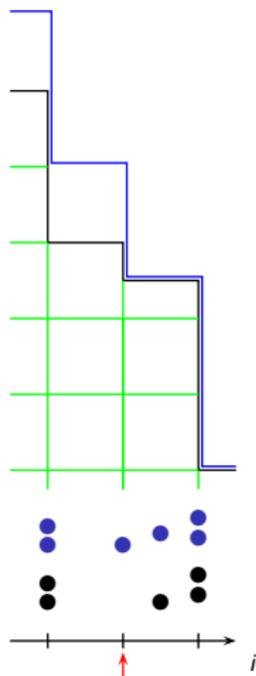
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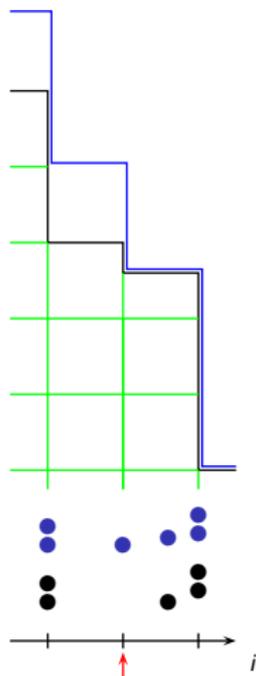
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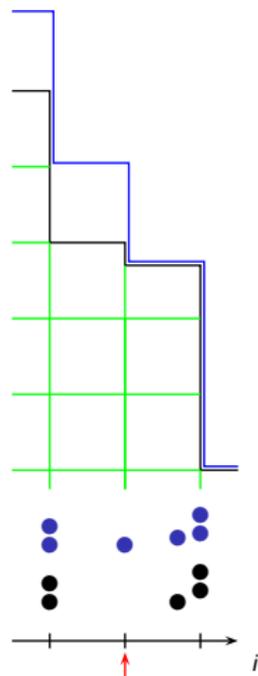
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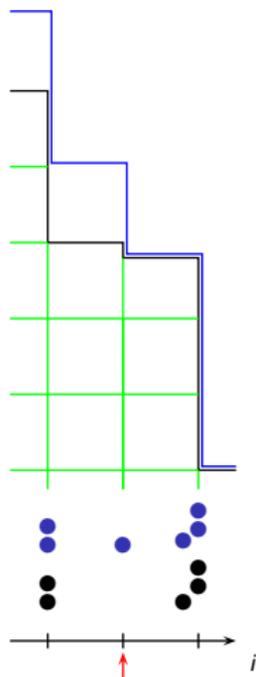
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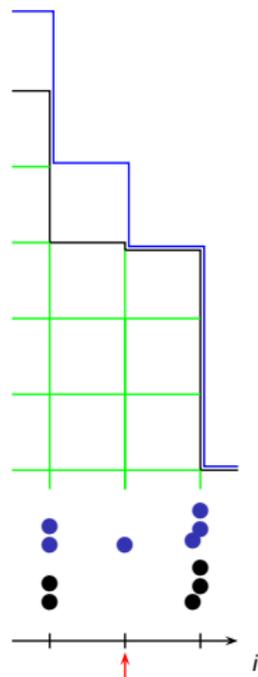
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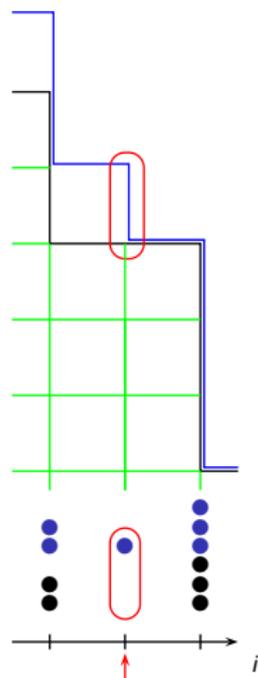
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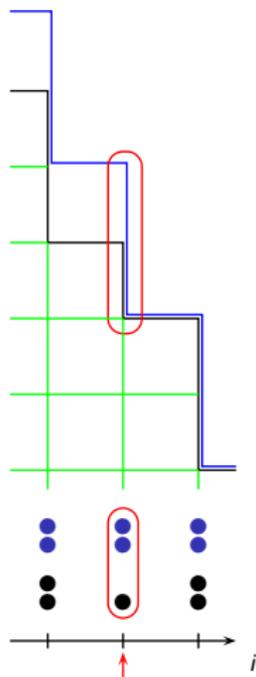
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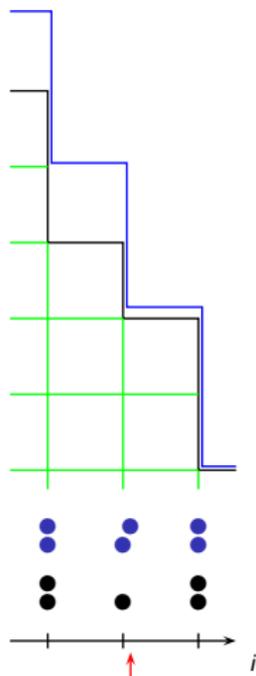
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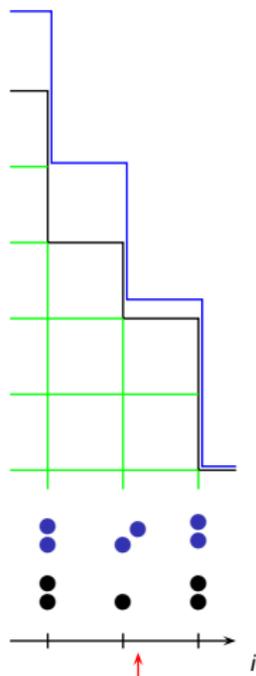
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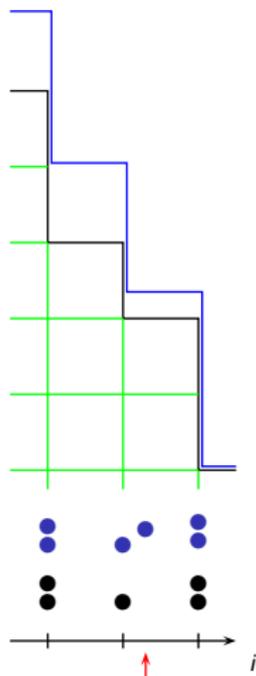
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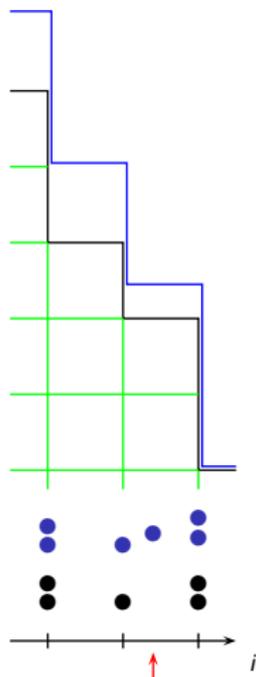
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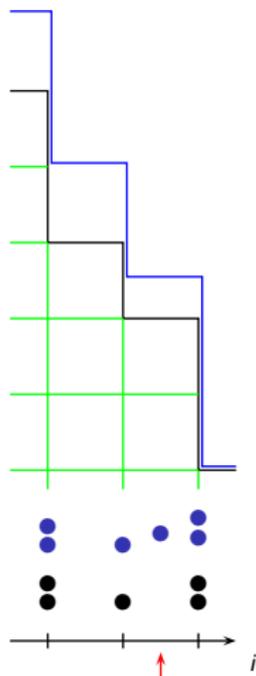
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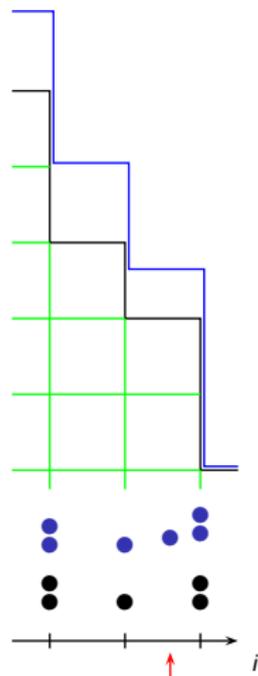
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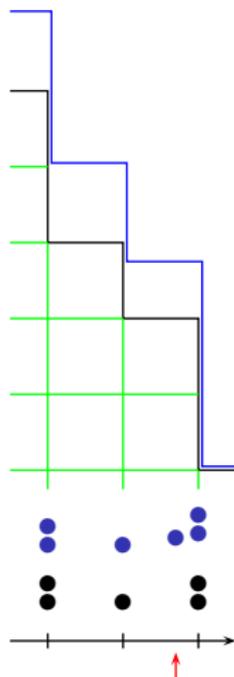
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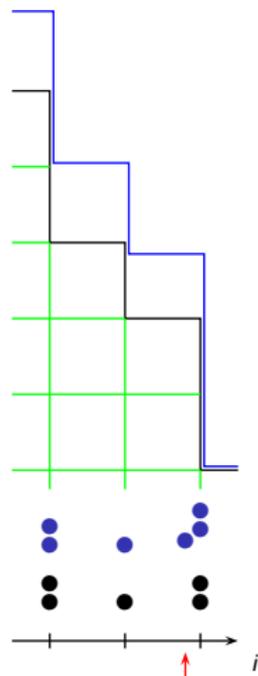
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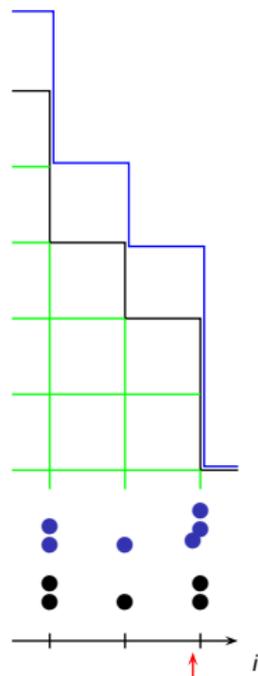
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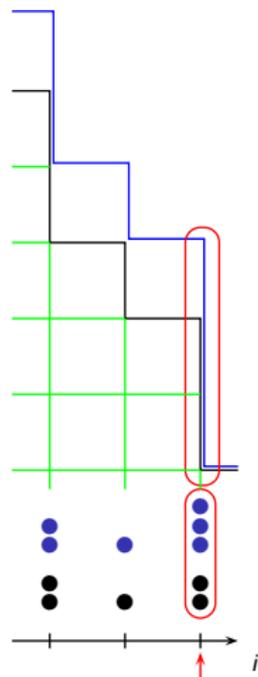
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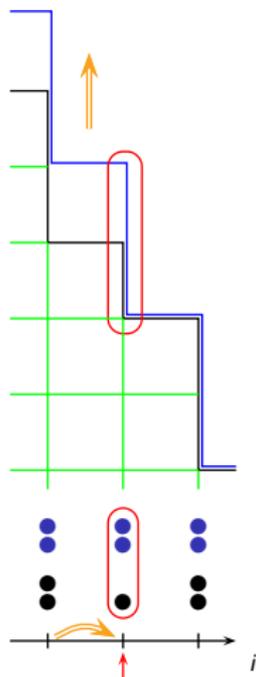


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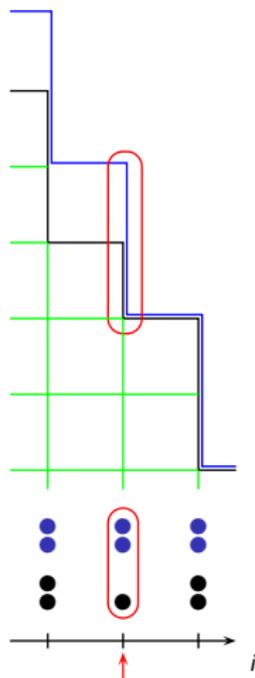
Growth on the left:
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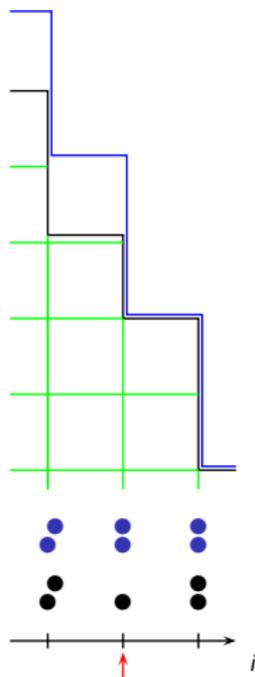
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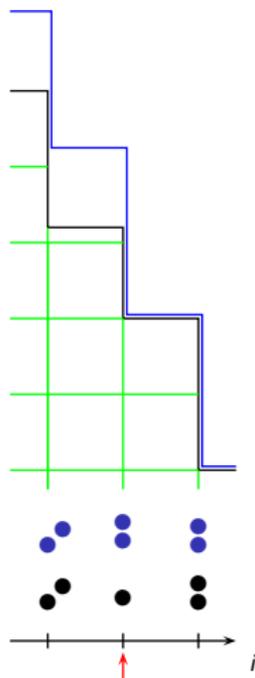
Growth on the left:
 $\text{rate} \geq \text{rate}$
 with rate :



The second class particle: attractive case

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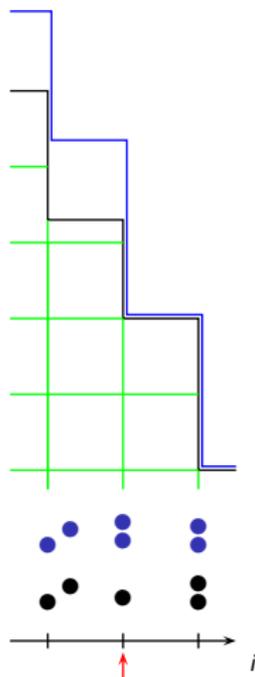
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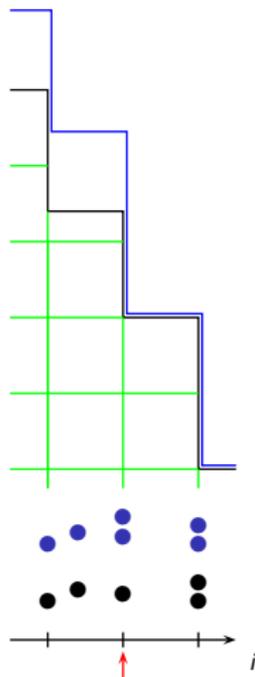
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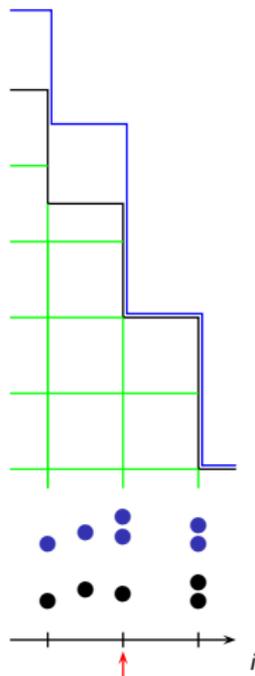
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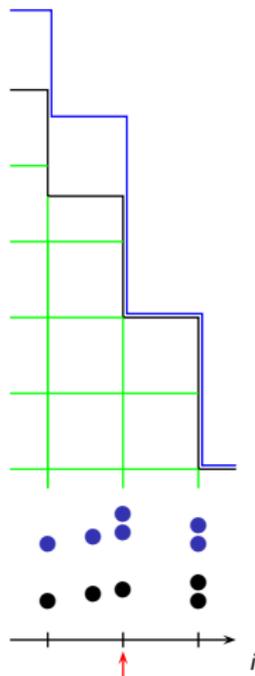
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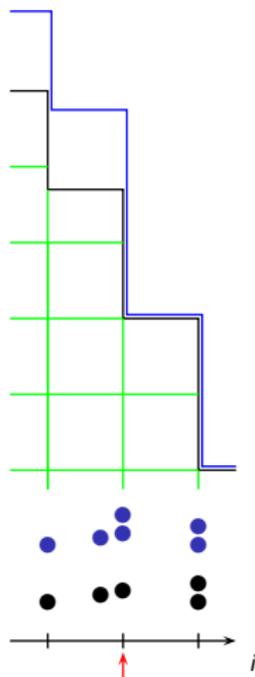
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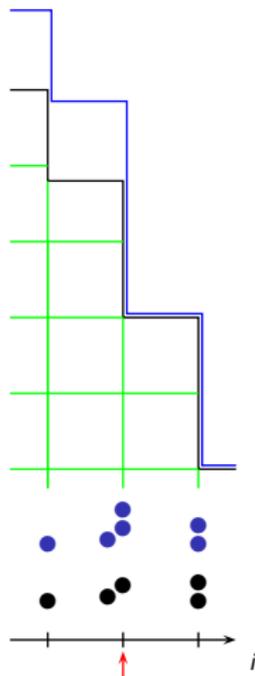
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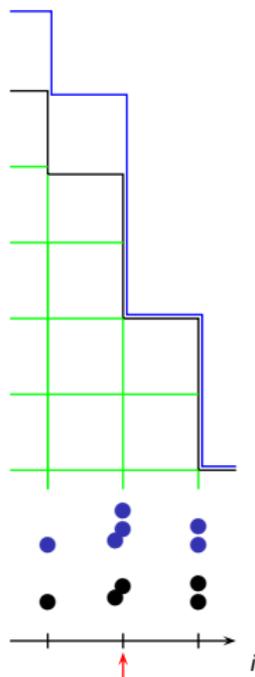
Growth on the left:
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The second class particle: attractive case

States ω and ω' only differ at one site.

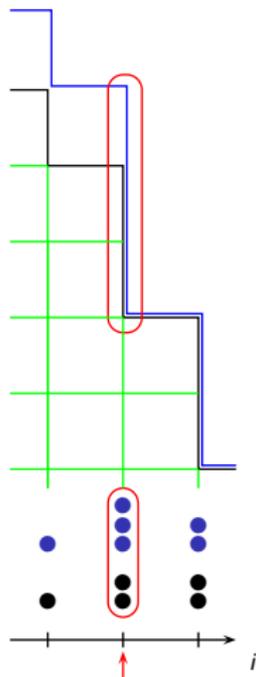
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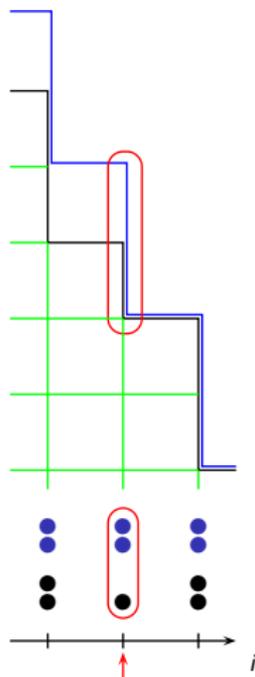
Growth on the left:
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 with rate :



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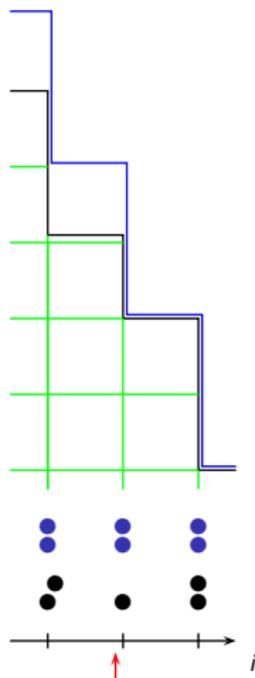
Growth on the left:
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 with $\text{rate} - \text{rate}$:



The second class particle: attractive case

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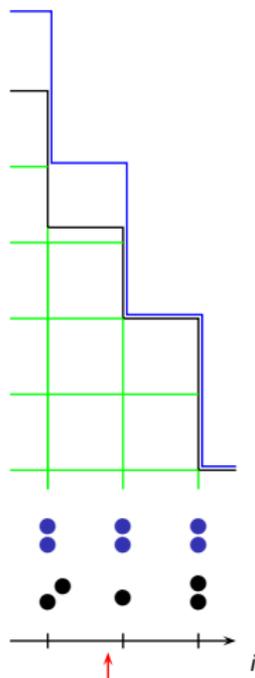
Growth on the left:
 $\text{rate} \geq \text{rate}$
 with rate-rate:



The second class particle: attractive case

States ω and $\tilde{\omega}$ only differ at one site.

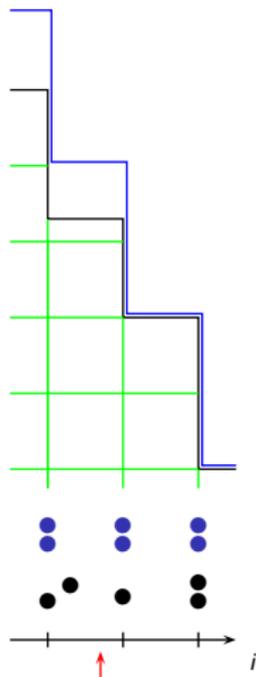
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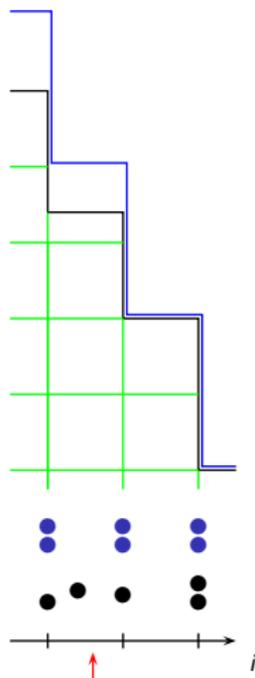
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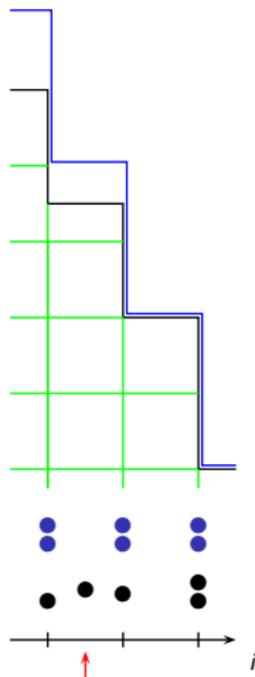
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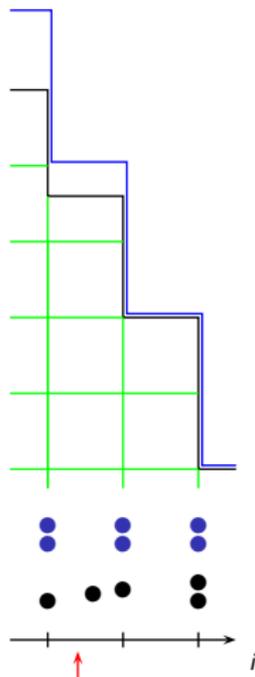
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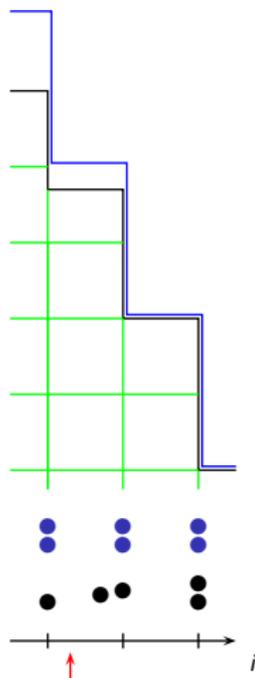
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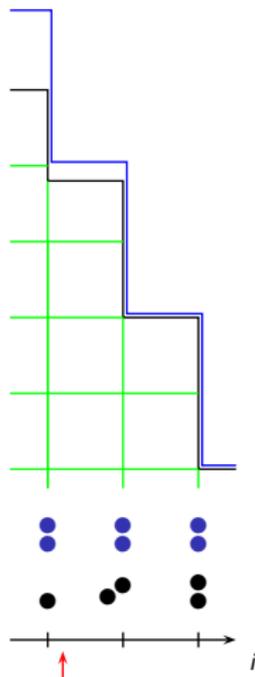
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The second class particle: attractive case

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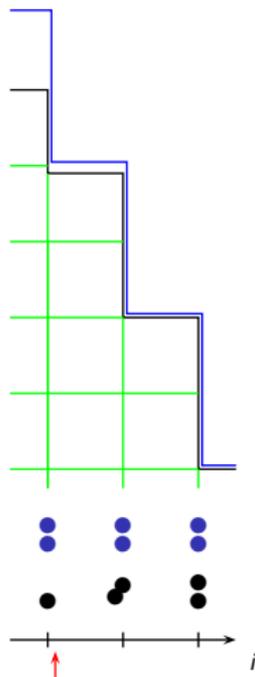
Growth on the left:
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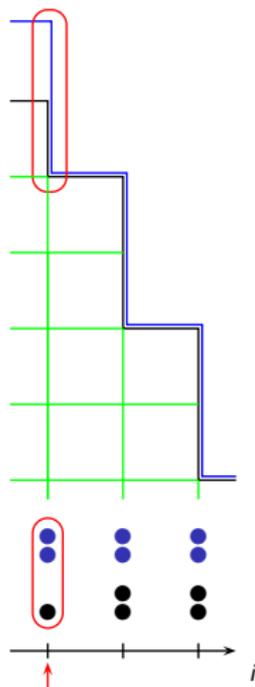
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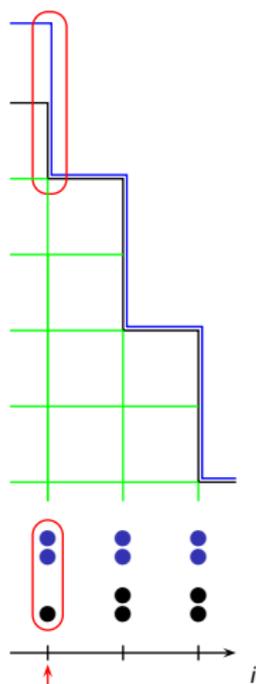
Growth on the left:
 $\text{rate} \geq \text{rate}$
 with rate-rate:



The second class particle: attractive case

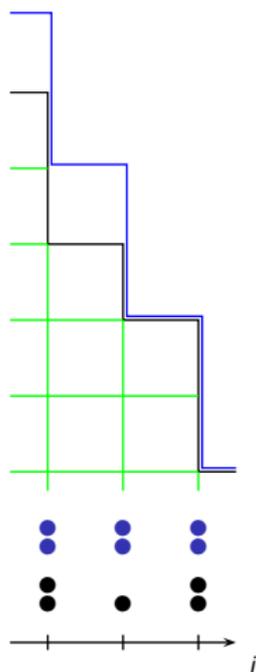
States ω and ω' only differ at one site.

Growth on the left:
 $\text{rate} \geq \text{rate}$
 with rate-rate:

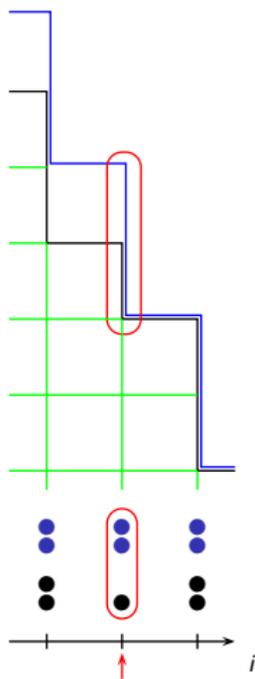


A single discrepancy \uparrow , the *second class particle*, is conserved.

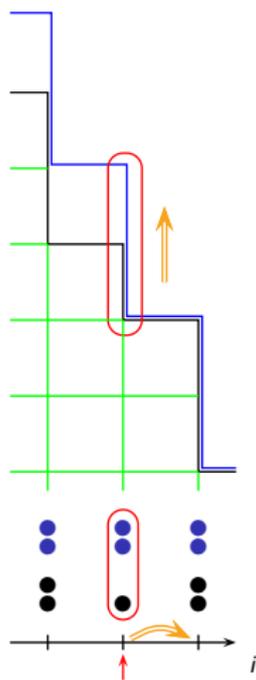
The second class particle: non-attractive case



The second class particle: non-attractive case

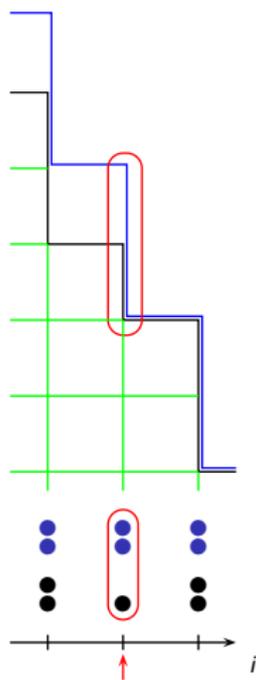


The second class particle: non-attractive case



Growth on the right:
 $\text{rate} > \text{rate}$

The second class particle: non-attractive case

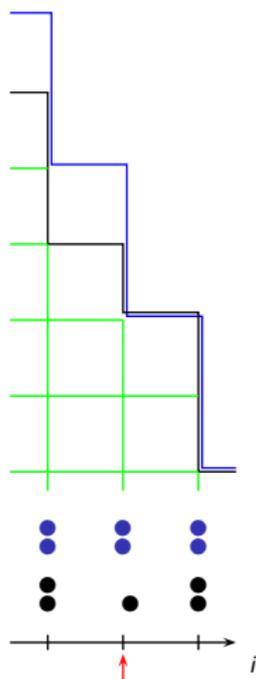


Growth on the right:

rate $>$ rate

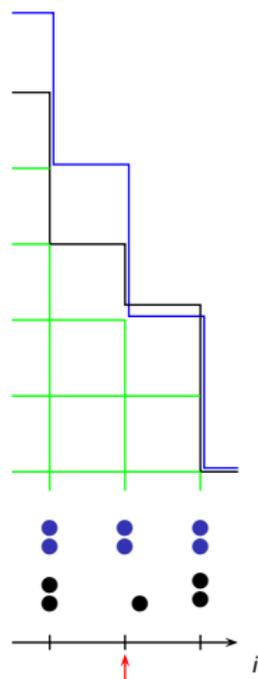
rate-rate:

The second class particle: non-attractive case



Growth on the right:
 $\text{rate} > \text{rate}$
 $\text{rate} - \text{rate}:$

The second class particle: non-attractive case

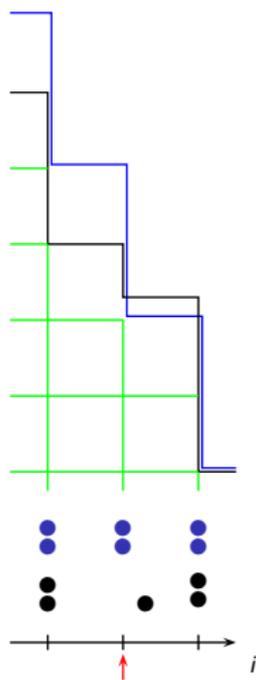


Growth on the right:

$\text{rate} > \text{rate}$

$\text{rate} - \text{rate}:$

The second class particle: non-attractive case

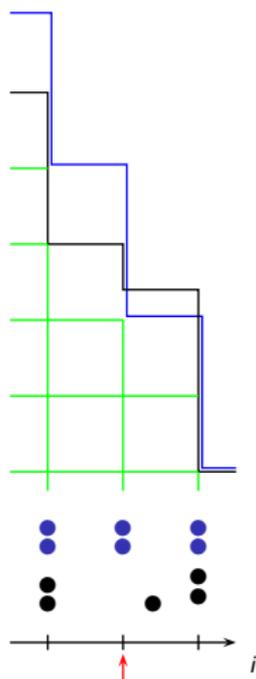


Growth on the right:

$\text{rate} > \text{rate}$

$\text{rate} - \text{rate}:$

The second class particle: non-attractive case

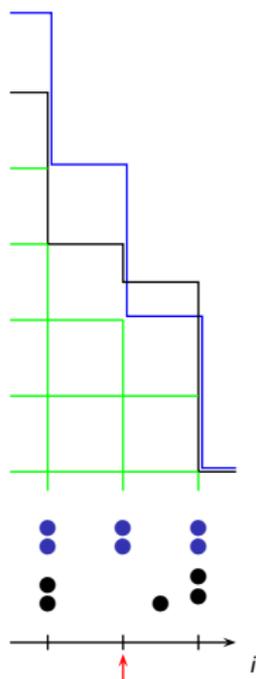


Growth on the right:

$\text{rate} > \text{rate}$

$\text{rate} - \text{rate}$:

The second class particle: non-attractive case

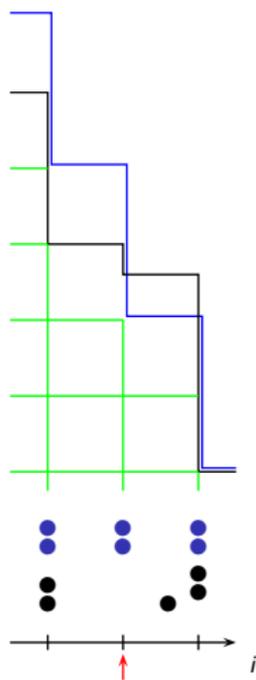


Growth on the right:

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The second class particle: non-attractive case

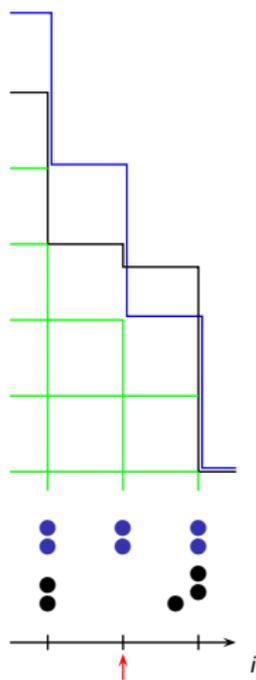


Growth on the right:

$\text{rate} > \text{rate}$

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The second class particle: non-attractive case

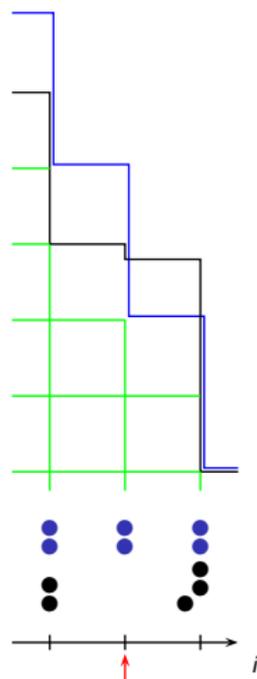


Growth on the right:

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$\text{rate} - \text{rate}$:

The second class particle: non-attractive case

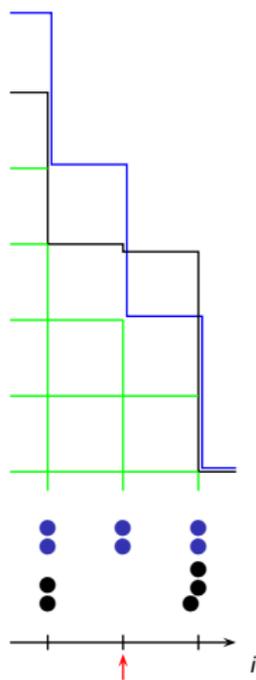


Growth on the right:

$\text{rate} > \text{rate}$

$\text{rate} - \text{rate}:$

The second class particle: non-attractive case

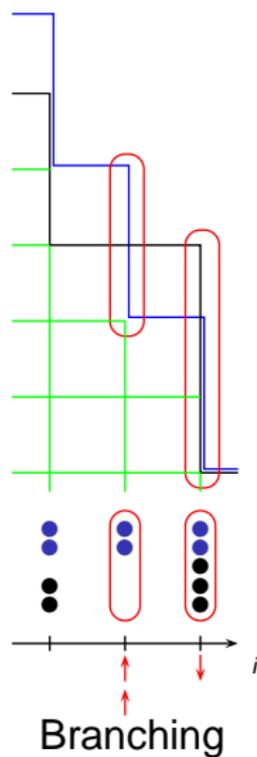


Growth on the right:

$\text{rate} > \text{rate}$

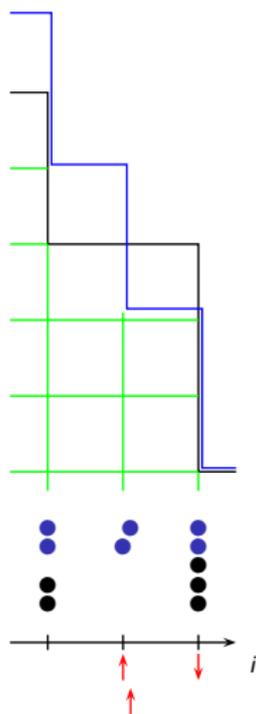
$\text{rate} - \text{rate}:$

The second class particle: non-attractive case

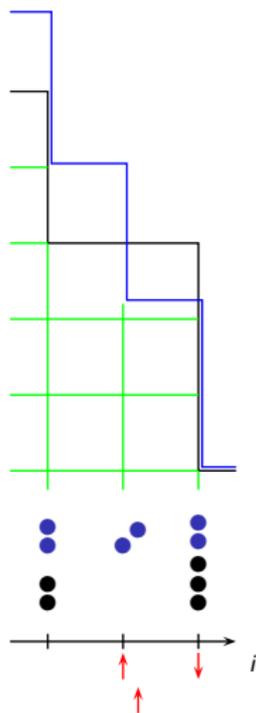


Growth on the right:
 $\text{rate} > \text{rate}$
 $\text{rate} - \text{rate}:$

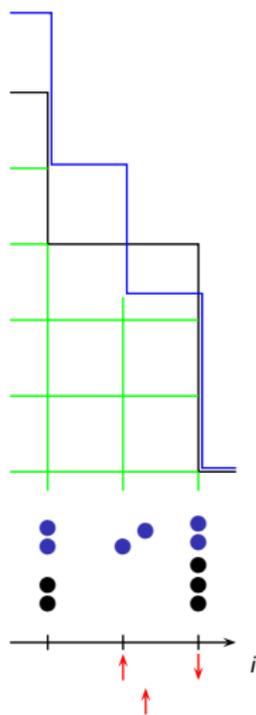
The second class particle: non-attractive case



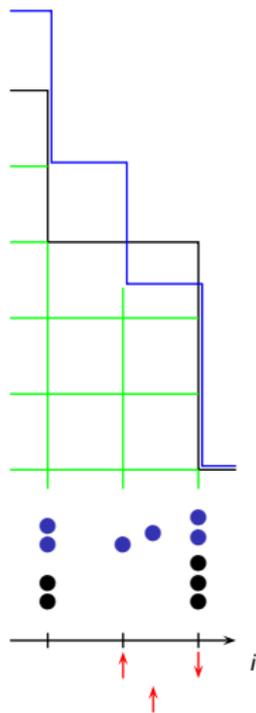
The second class particle: non-attractive case



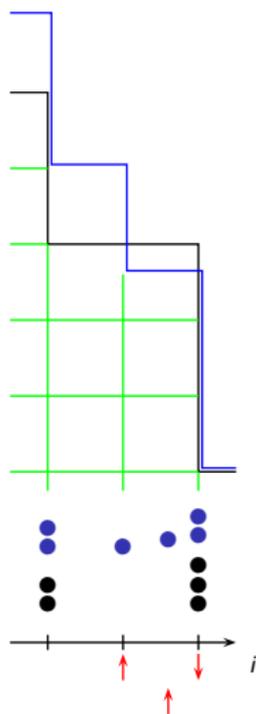
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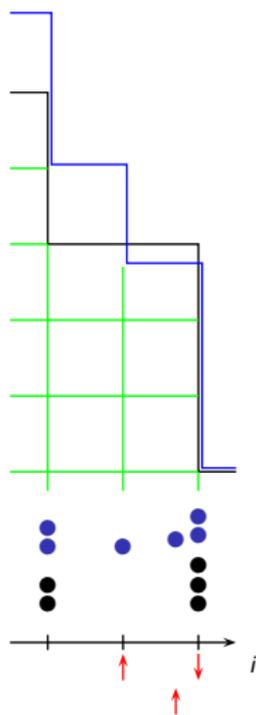
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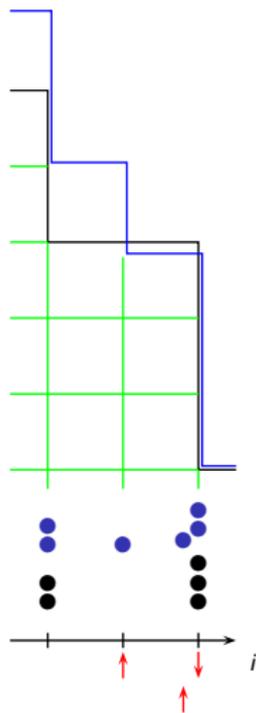
The second class particle: non-attractive case



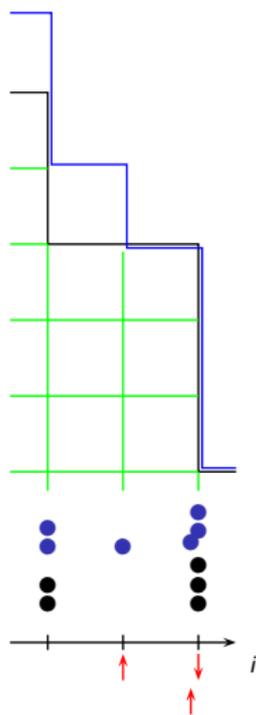
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The second class particle: non-attractive case

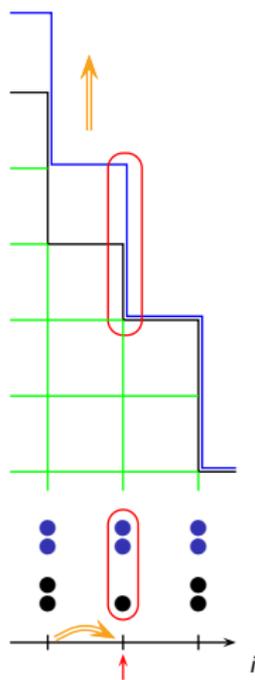


The second class particle: non-attractive case



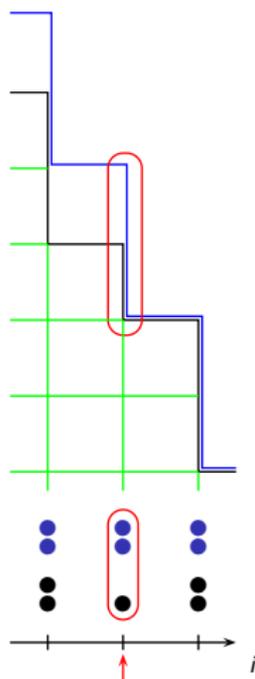
The second class particle: non-attractive case

Growth on the left:
 $\text{rate} < \text{rate}$



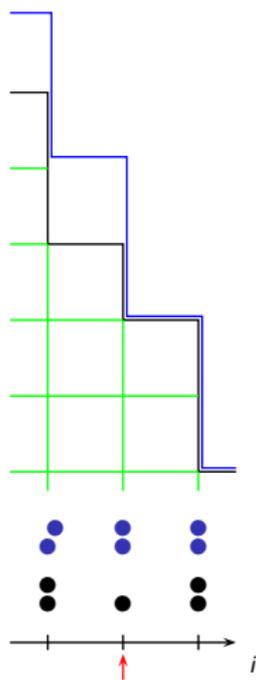
The second class particle: non-attractive case

Growth on the left:
 $\text{rate} < \text{rate}$
 with $\text{rate} - \text{rate}$:



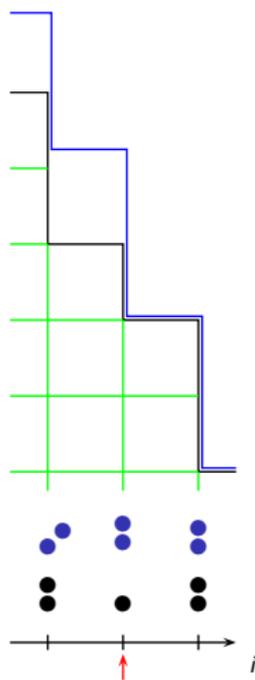
The second class particle: non-attractive case

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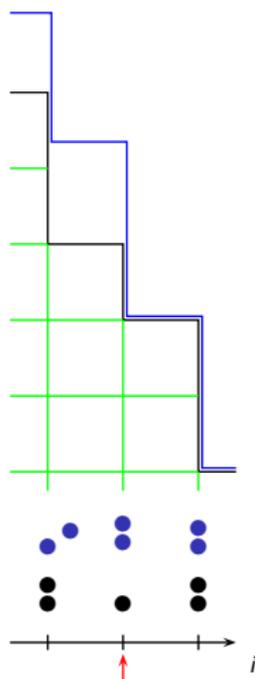
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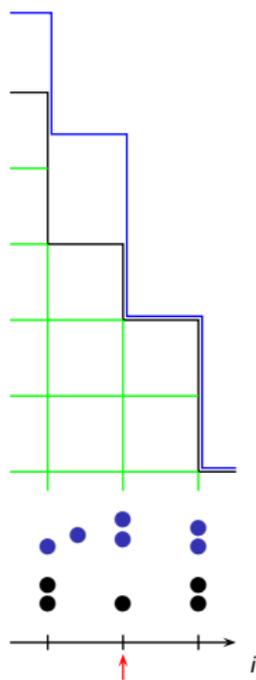
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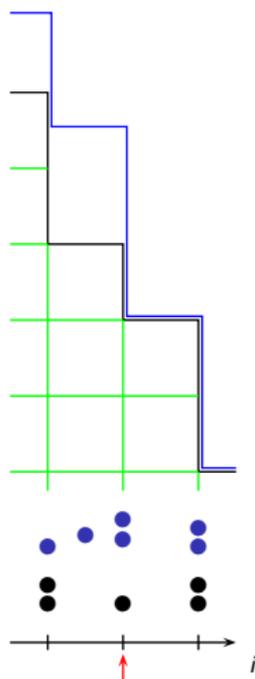
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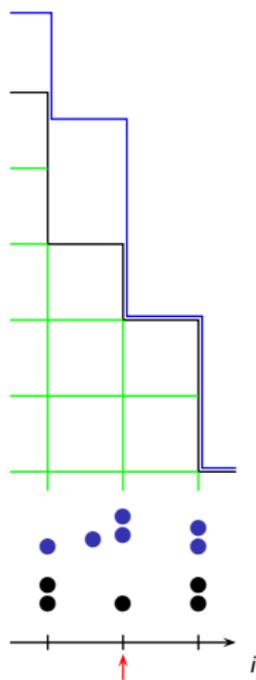
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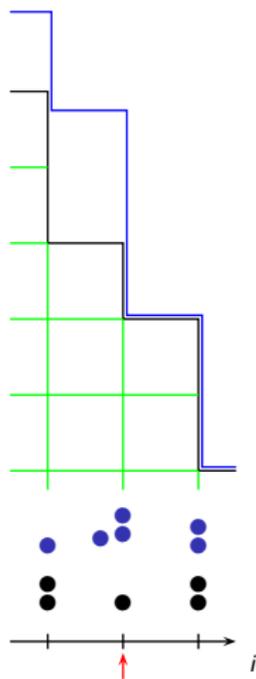
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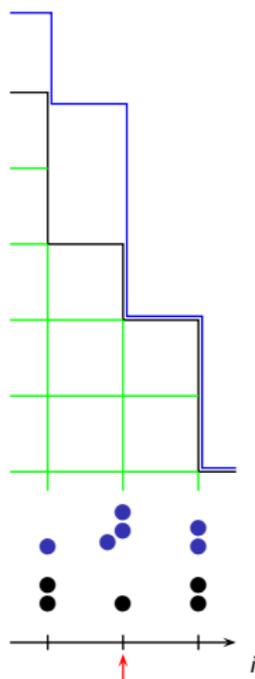
The second class particle: non-attractive case

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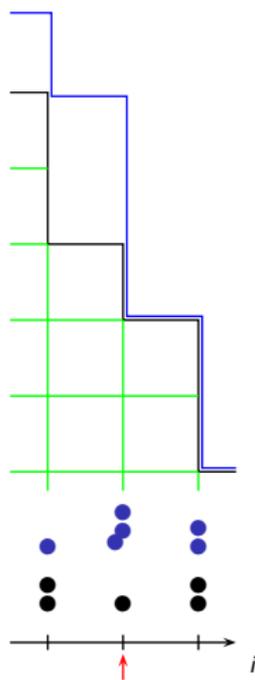
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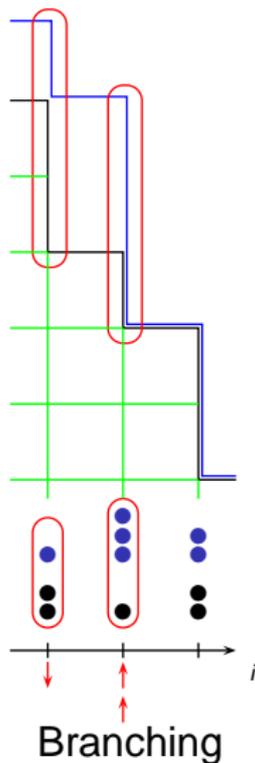
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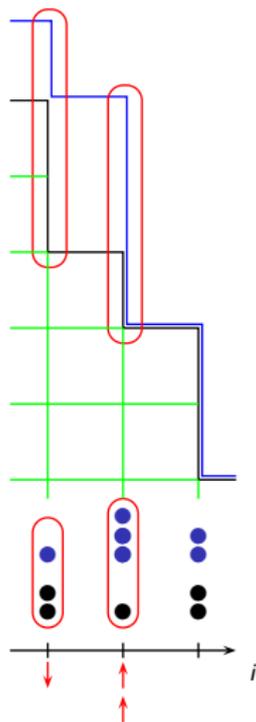


The second class particle: non-attractive case

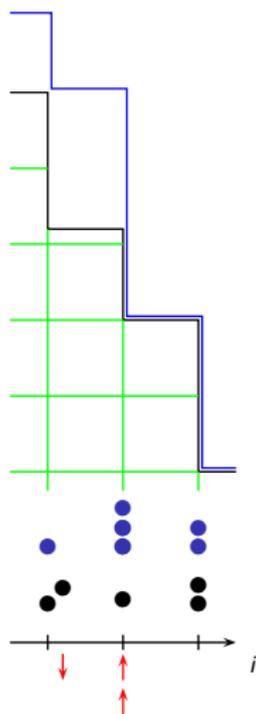
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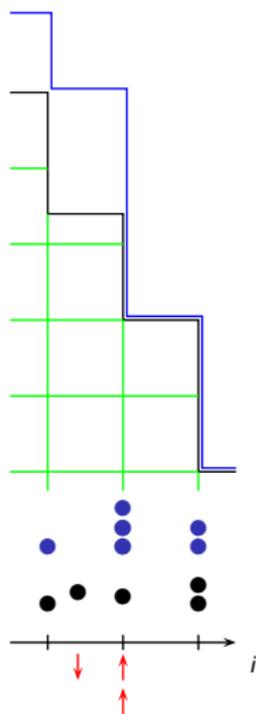
The second class particle: non-attractive case



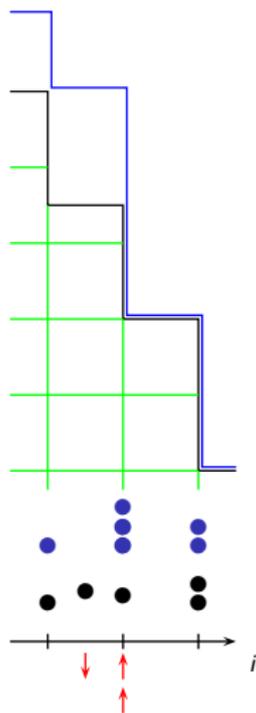
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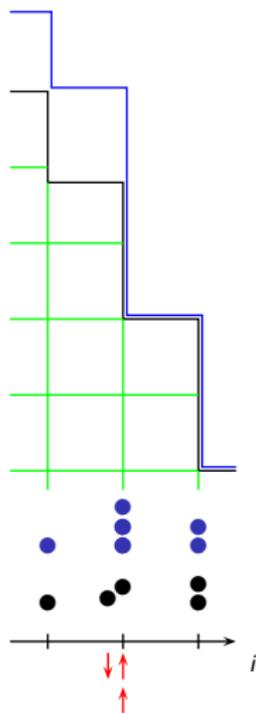
The second class particle: non-attractive case



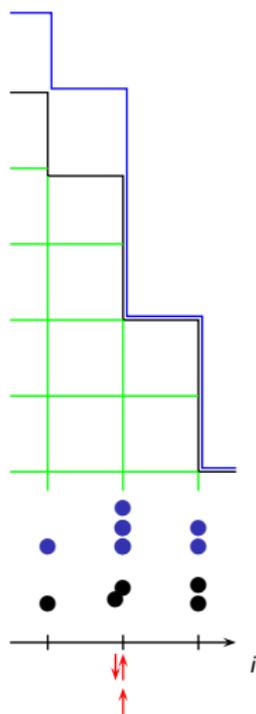
The second class particle: non-attractive case



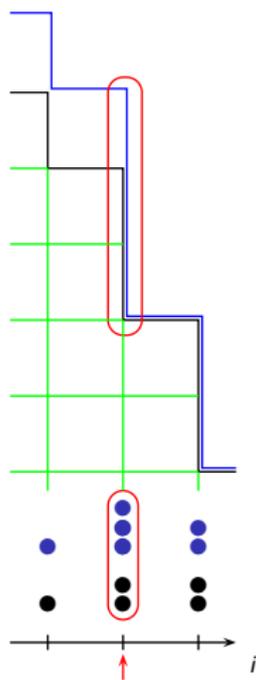
The second class particle: non-attractive case



The second class particle: non-attractive case



The second class particle: non-attractive case



Annihilating

The second class particle: non-attractive case

We are facing a

- ▶ nearest neighbour
- ▶ parity conserving
- ▶ branching
- ▶ annihilating process
- ▶ on the dynamic background of first class particles.

The aim is to control the number of \uparrow and \downarrow 's. Idea from Bálint Tóth.

\rightsquigarrow homog2.avi

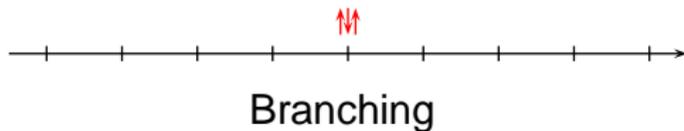
A mean field model

A model we can say something about:



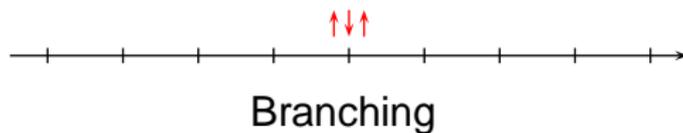
A mean field model

A model we can say something about:



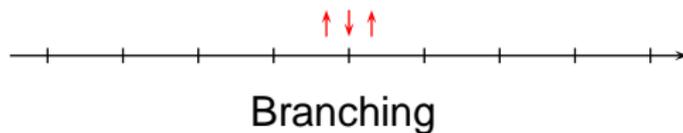
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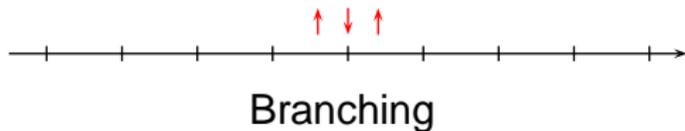
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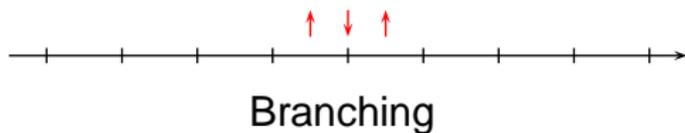
A mean field model

A model we can say something about:



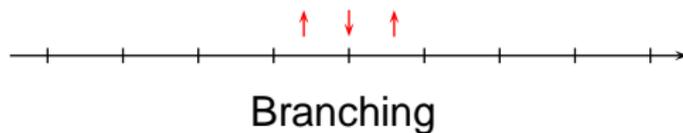
A mean field model

A model we can say something about:



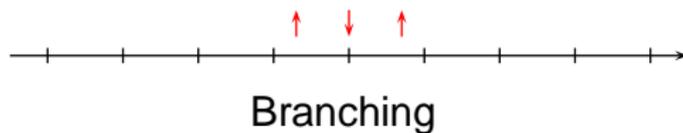
A mean field model

A model we can say something about:



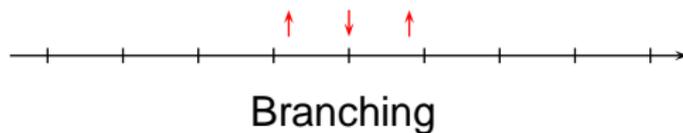
A mean field model

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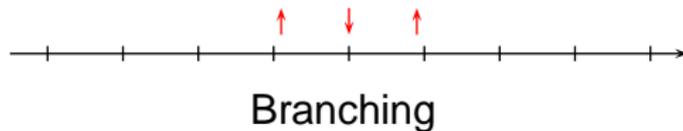
A mean field model

A model we can say something about:



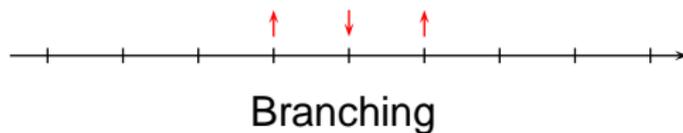
A mean field model

A model we can say something about:



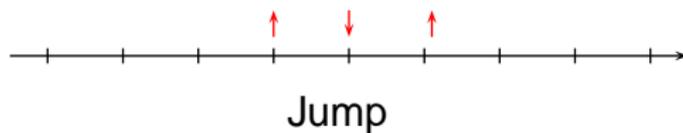
A mean field model

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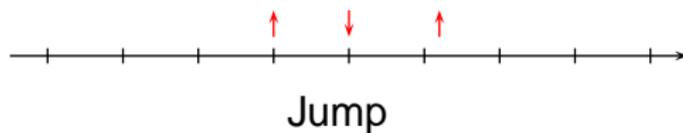
A mean field model

A model we can say something about:



A mean field model

A model we can say something about:



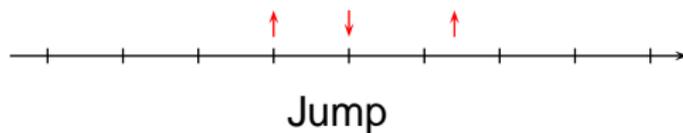
A mean field model

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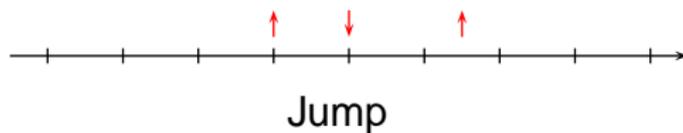
A mean field model

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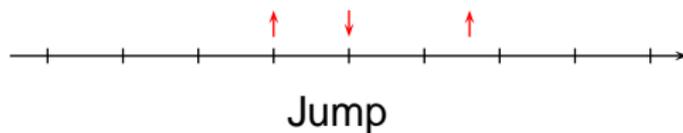
A mean field model

A model we can say something about:



A mean field model

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A mean field model

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A mean field model

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A mean field model

A model we can say something about:



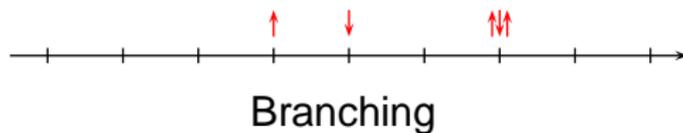
A mean field model

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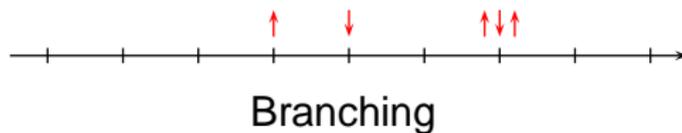
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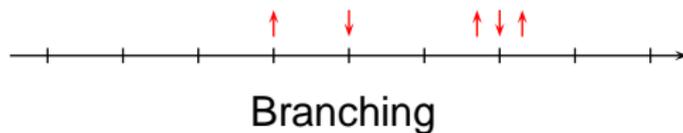
A mean field model

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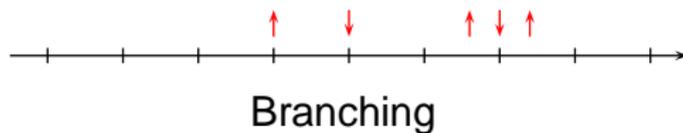
A mean field model

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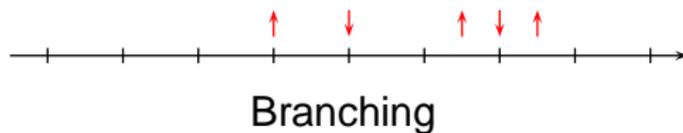
A mean field model

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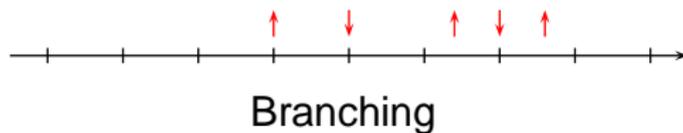
A mean field model

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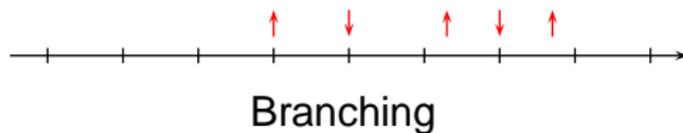
A mean field model

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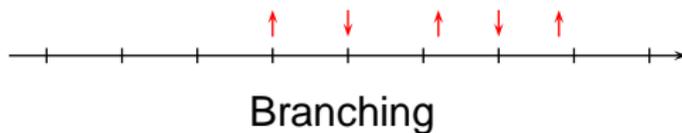
A mean field model

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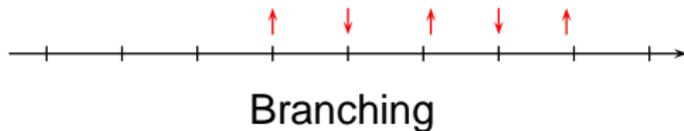
A mean field model

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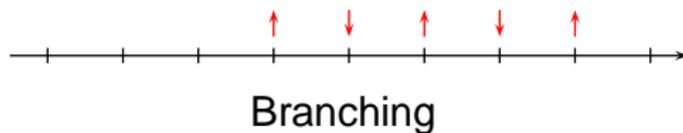
A mean field model

A model we can say something about:



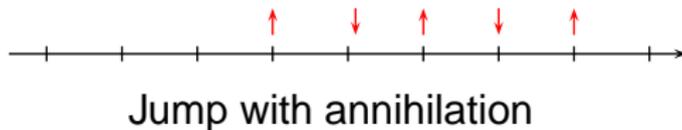
A mean field model

A model we can say something about:



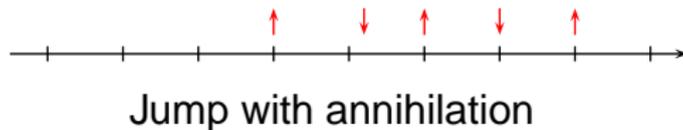
A mean field model

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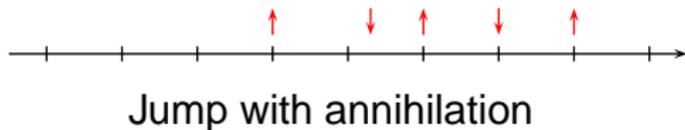
A mean field model

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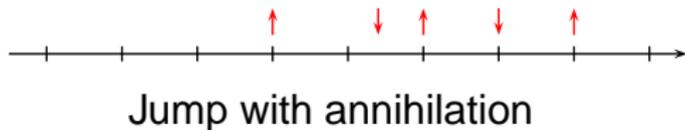
A mean field model

A model we can say something about:



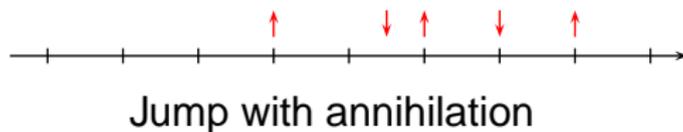
A mean field model

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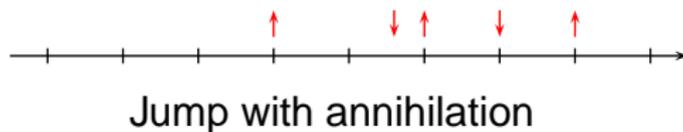
A mean field model

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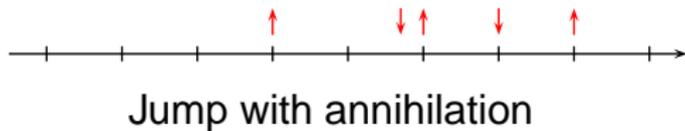
A mean field model

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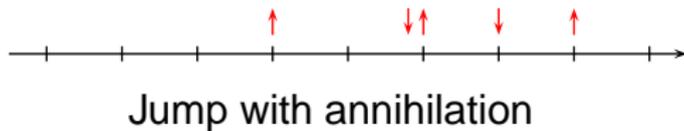
A mean field model

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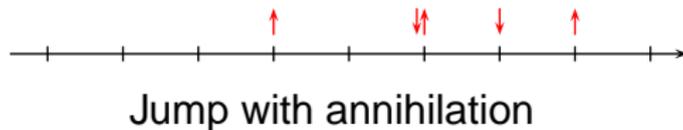
A mean field model

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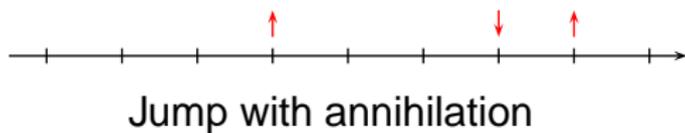
A mean field model

A model we can say something about:



A mean field model

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A mean field model

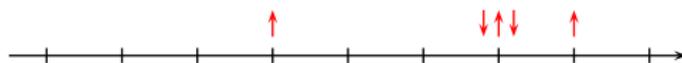
A model we can say something about:



Branching with annihilation

A mean field model

A model we can say something about:



Branching with annihilation

A mean field model

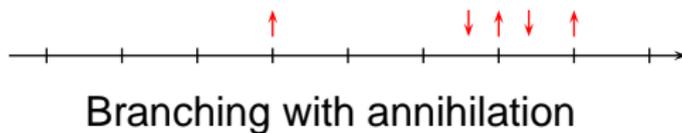
A model we can say something about:



Branching with annihilation

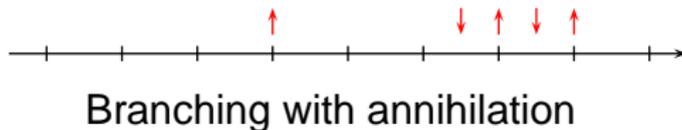
A mean field model

A model we can say something about:



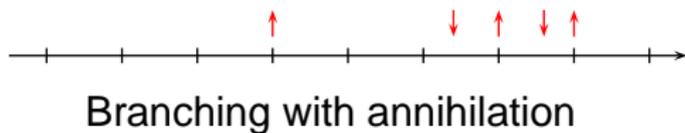
A mean field model

A model we can say something about:



A mean field model

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A mean field model

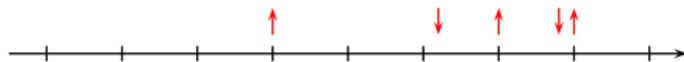
A model we can say something about:



Branching with annihilation

A mean field model

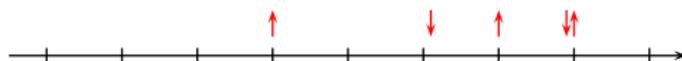
A model we can say something about:



Branching with annihilation

A mean field model

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Branching with annihilation

A mean field model

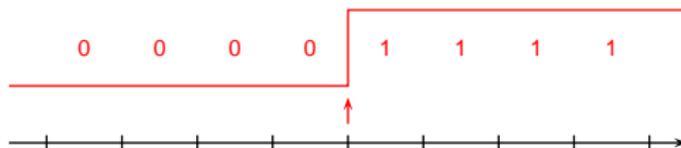
A model we can say something about:



Branching with annihilation

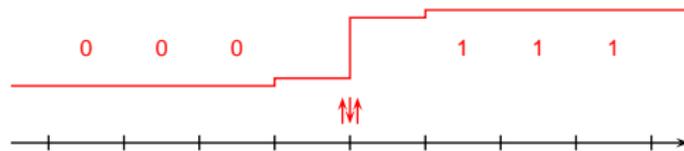
A mean field model

A model we can say something about:



A mean field model

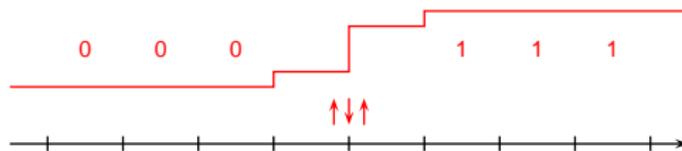
A model we can say something about:



Branching: **exclusion**

A mean field model

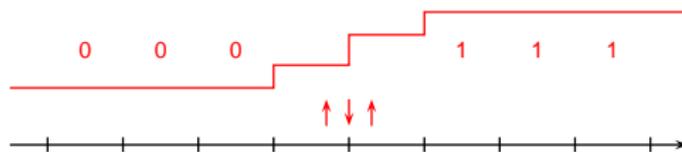
A model we can say something about:



Branching: **exclusion**

A mean field model

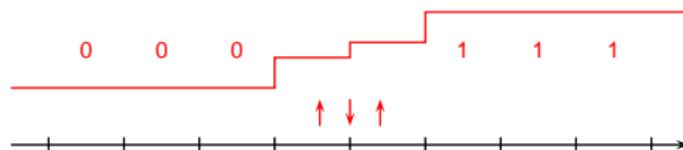
A model we can say something about:



Branching: **exclusion**

A mean field model

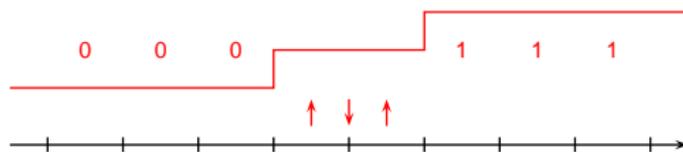
A model we can say something about:



Branching: **exclusion**

A mean field model

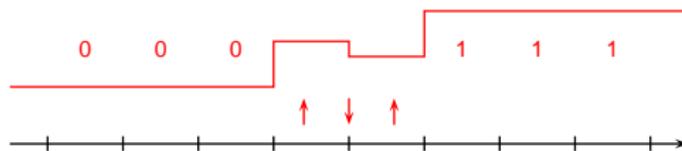
A model we can say something about:



Branching: **exclusion**

A mean field model

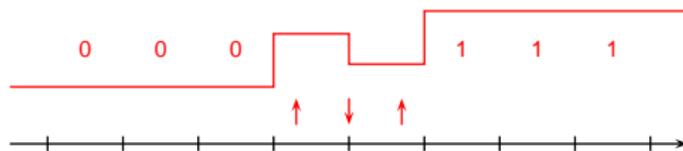
A model we can say something about:



Branching: **exclusion**

A mean field model

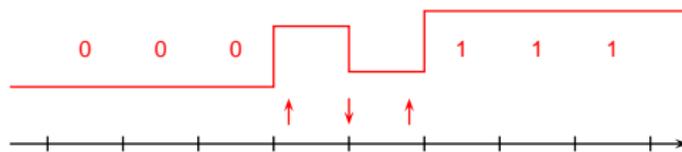
A model we can say something about:



Branching: **exclusion**

A mean field model

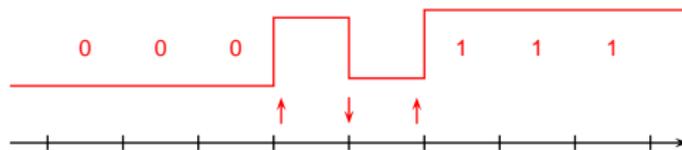
A model we can say something about:



Branching: **exclusion**

A mean field model

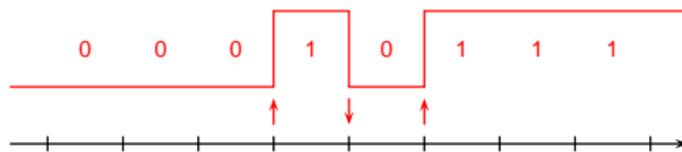
A model we can say something about:



Branching: **exclusion**

A mean field model

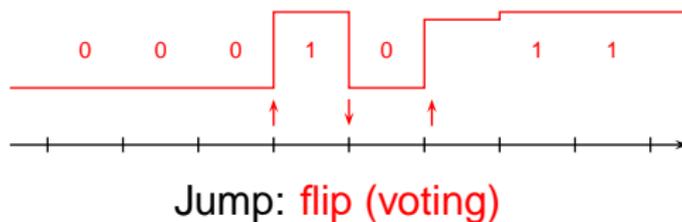
A model we can say something about:



Branching: **exclusion**

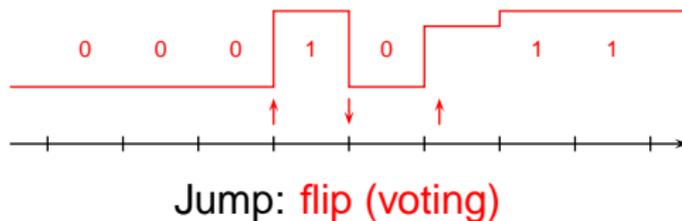
A mean field model

A model we can say something about:



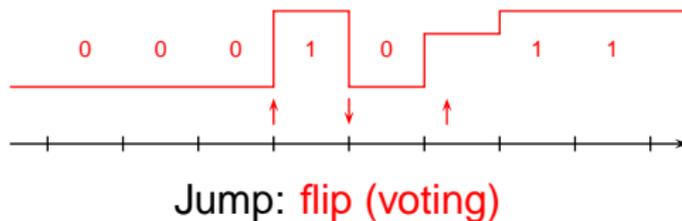
A mean field model

A model we can say something about:



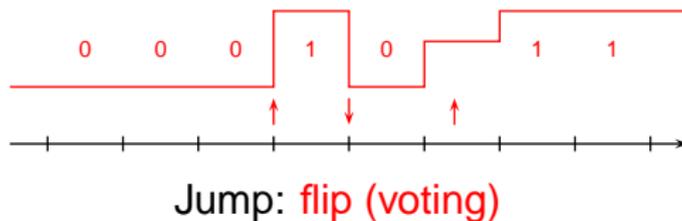
A mean field model

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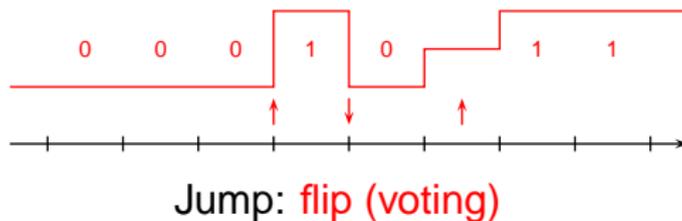
A mean field model

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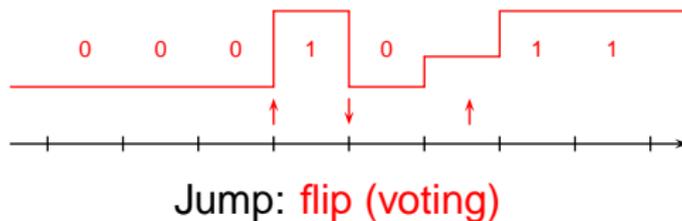
A mean field model

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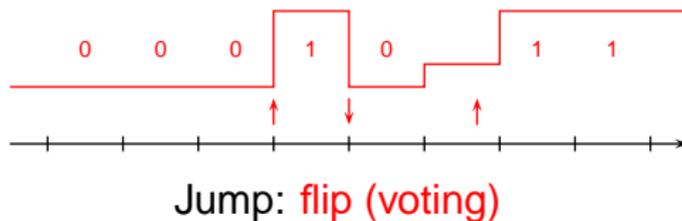
A mean field model

A model we can say something about:



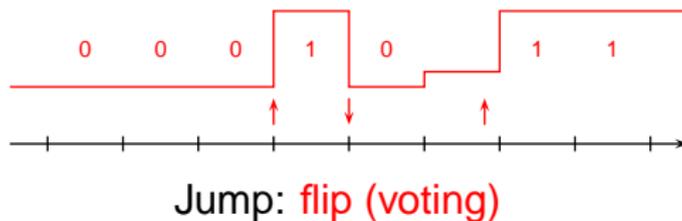
A mean field model

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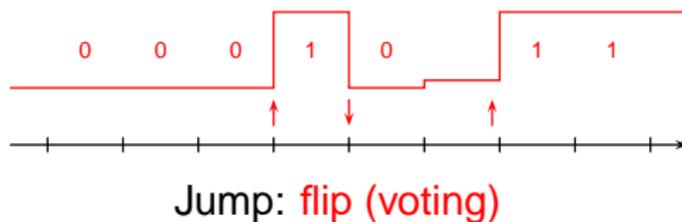
A mean field model

A model we can say something about:



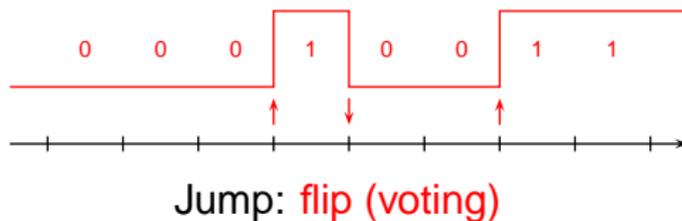
A mean field model

A model we can say something about:



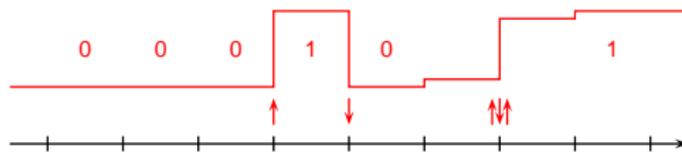
A mean field model

A model we can say something about:



A mean field model

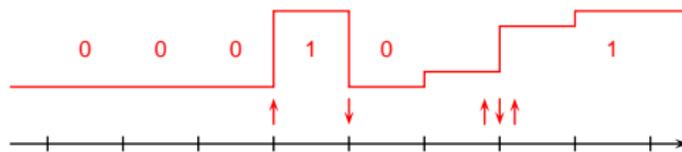
A model we can say something about:



Branching: **exclusion**

A mean field model

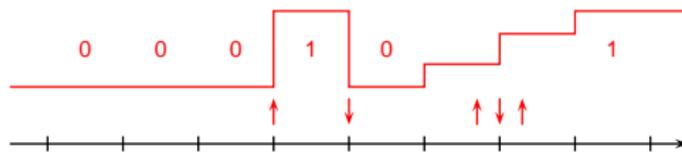
A model we can say something about:



Branching: **exclusion**

A mean field model

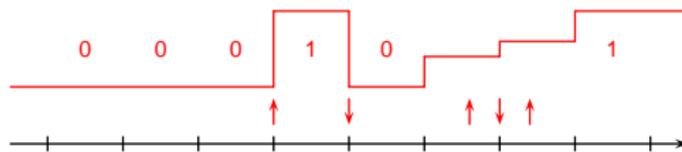
A model we can say something about:



Branching: **exclusion**

A mean field model

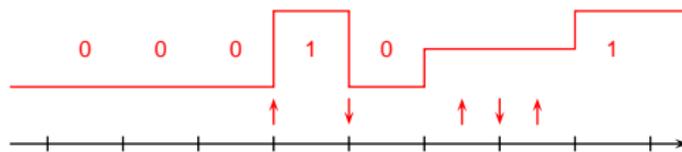
A model we can say something about:



Branching: **exclusion**

A mean field model

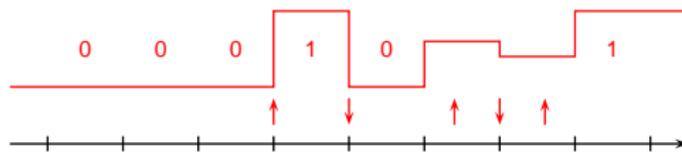
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Branching: **exclusion**

A mean field model

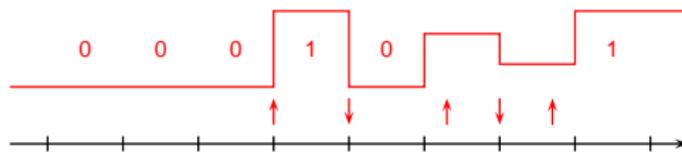
A model we can say something about:



Branching: **exclusion**

A mean field model

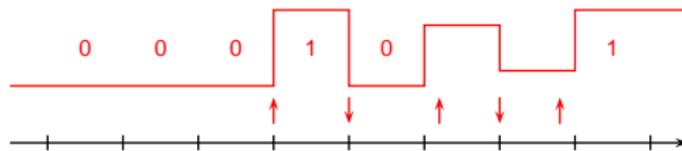
A model we can say something about:



Branching: **exclusion**

A mean field model

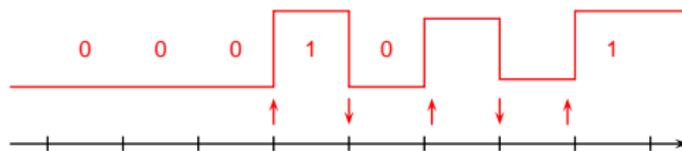
A model we can say something about:



Branching: **exclusion**

A mean field model

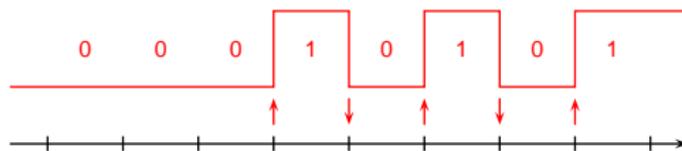
A model we can say something about:



Branching: **exclusion**

A mean field model

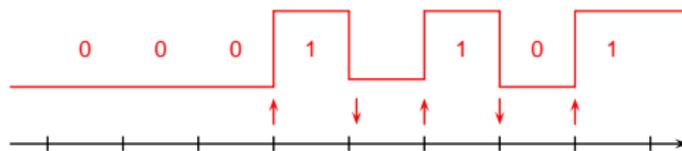
A model we can say something about:



Branching: **exclusion**

A mean field model

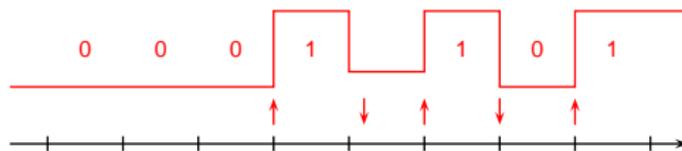
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

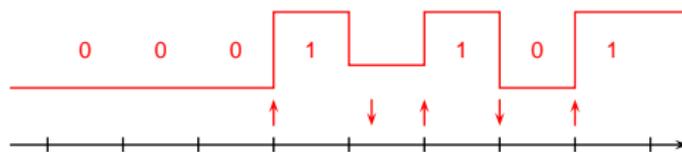
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

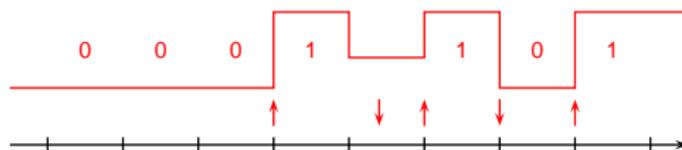
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

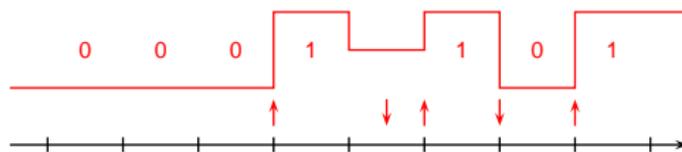
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

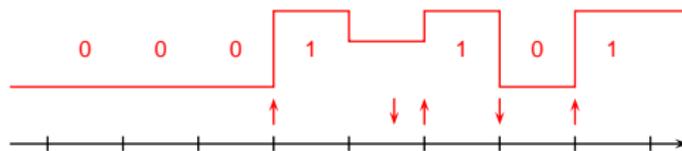
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

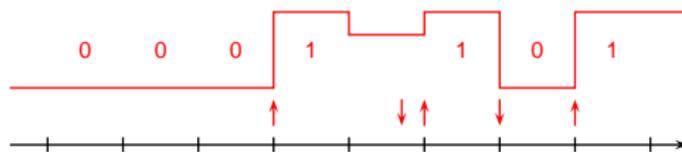
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

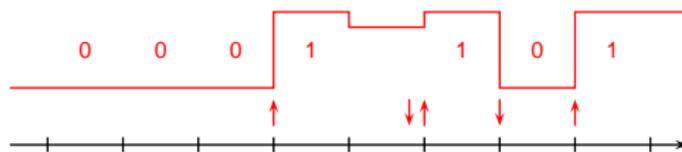
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

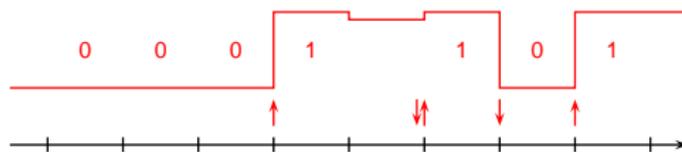
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

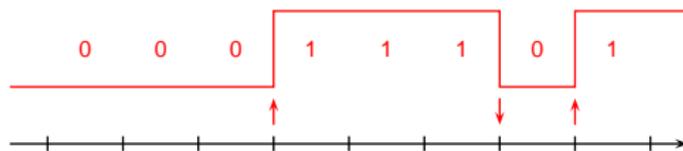
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

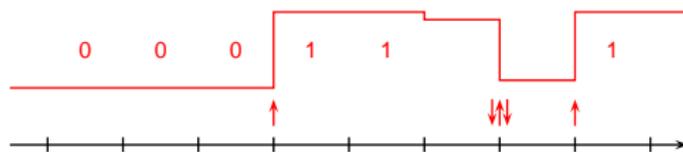
A model we can say something about:



Jump with annihilation: **flip (voting)**

A mean field model

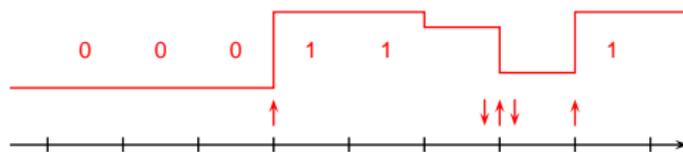
A model we can say something about:



Branching with annihilation: **exclusion**

A mean field model

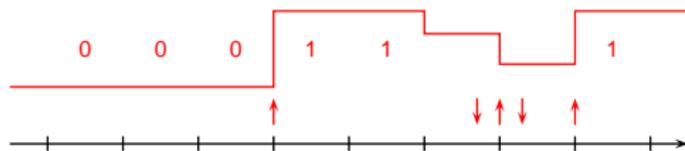
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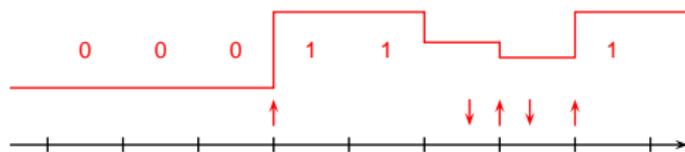
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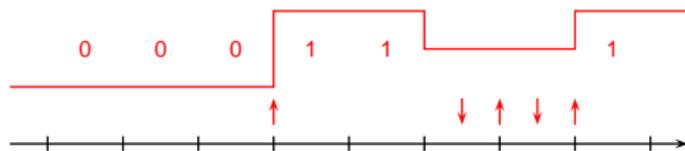
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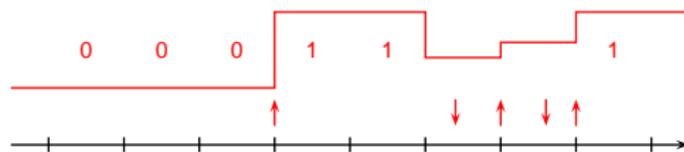
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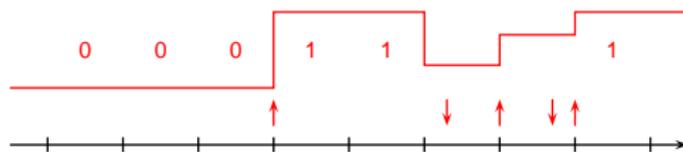
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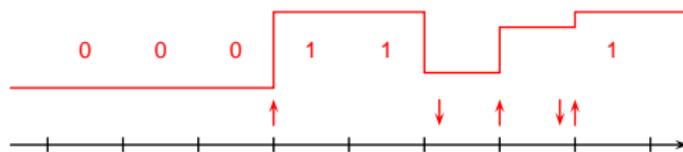
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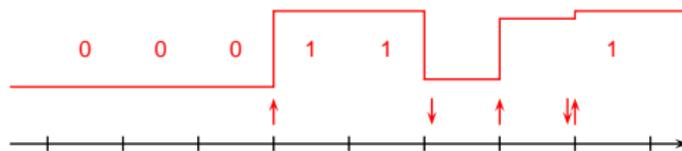
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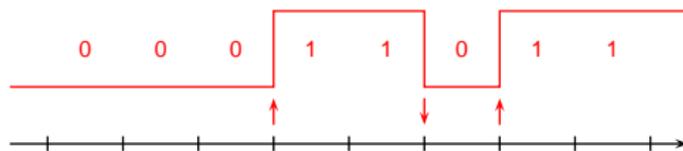
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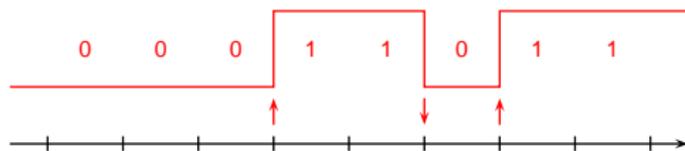
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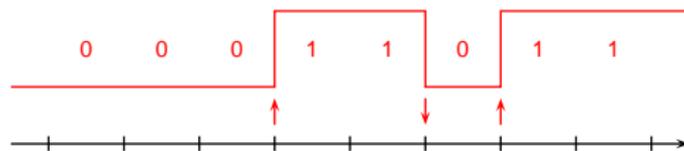


Branching with annihilation: **exclusion**

- ▶ Double branching-annihilating random walks (DBARW)

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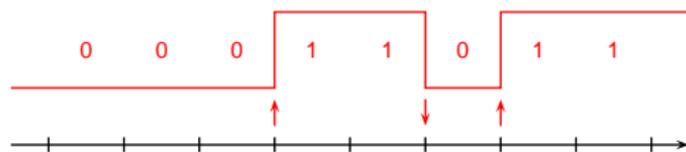


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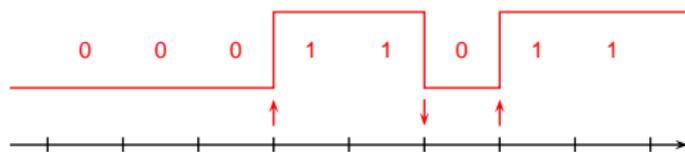


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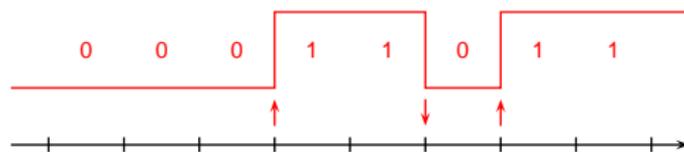


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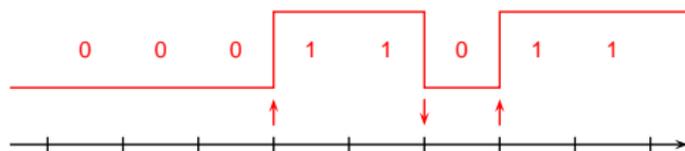


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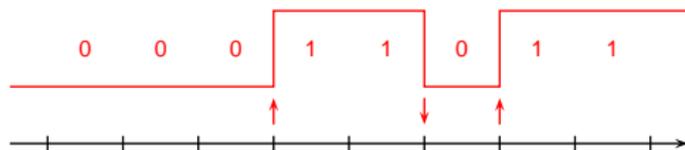
Question: Is the process, as seen by the leftmost \uparrow , recurrent?

DBARW



First instance of DBARW we could find in the literature: [A. Sudbury '90](#). Positive recurrence: [V. Belitsky, P.A. Ferrari, M.V. Menshikov and S.Y. Popov '01](#); [A. Sturm and J.M. Swart '08](#).
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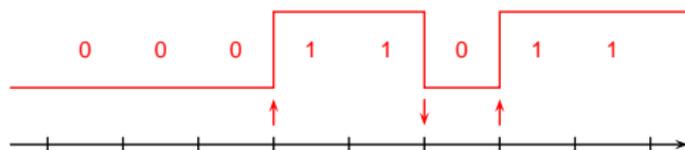
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But: true second class particles interact (*common background of first class particles*).

↪ Repeat the Sturm-Swart proof with configuration dependent jump rates. **Jump rates can depend on the whole configuration.**

DBARW

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- ▶ Weak dependence on particles far away.
- ▶ No repulsion in the jumping rates between particles. (*A bit of repulsion locally is still OK.*)

Positive recurrence

Theorem

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- ▶ *(Extension of all this to non nearest neighbour symmetric branching.)*

An example

- ▶ Branching rates: constant.
- ▶ Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{particle on right}} \frac{1}{\text{distance}^{\alpha}},$$

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Unfortunately we do **not** seem to be there yet... This is **not** covered at the moment. **But a small modification that respects parity in a peculiar way seems to work.**

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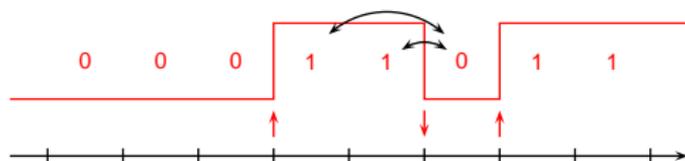
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This one is fine.

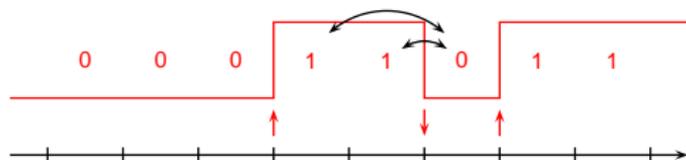
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Main tool 1: the number of inversions, i.e., wrongly ordered 1-0 pairs.

If there are too many of them, the generator is negative on the number of these pairs.

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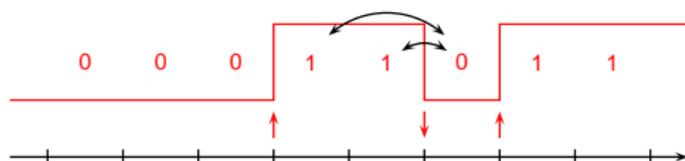
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Thank you.