$t^{1/3}$ scaling of fluctuations in asymmetric interacting systems

Joint with
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MTA-BME Stochastics Research Group

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[1]Bolyai Scholarship of the HAS; OTKA K100473;TÁMOP422

The models

- Asymmetric simple exclusion process
- Zero range
- Bricklayers

Hydrodynamics

- Characteristics

Tool: the second class particle

- Single
- Many second class particles

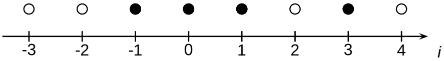
Results

- Normal fluctuations
- Abnormal fluctuations

Proof

- Upper bound
- Lower bound
- Microscopic concavity/convexity

Asymmetric simple exclusion



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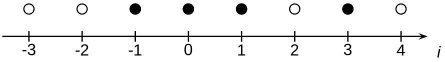

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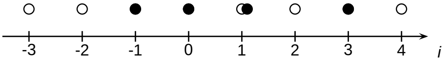

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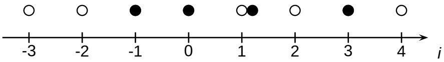

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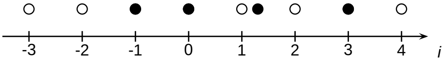

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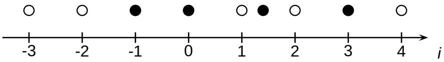

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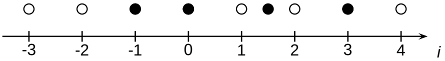

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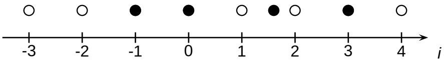

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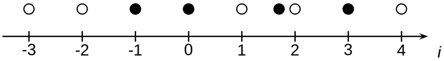

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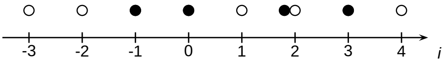

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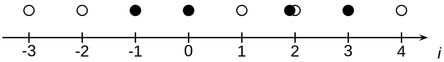

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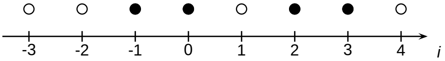

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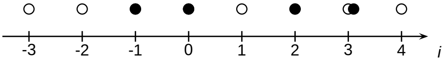

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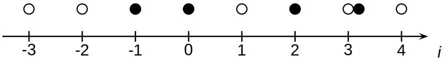

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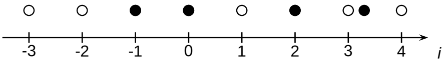

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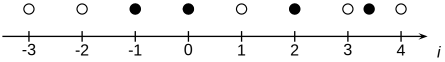

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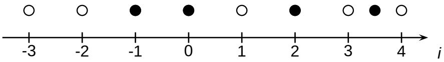

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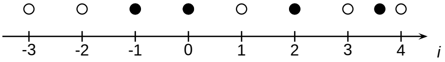

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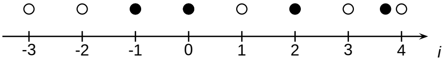

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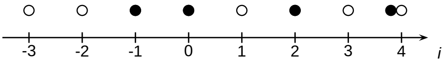

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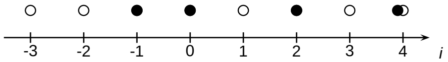

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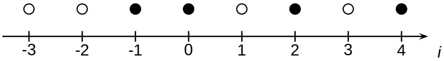

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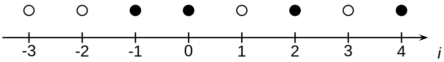

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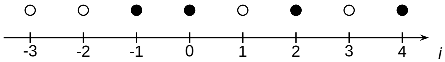

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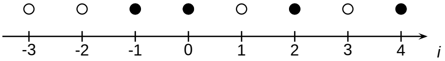

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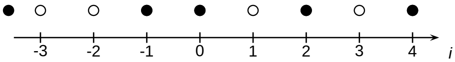

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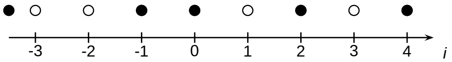

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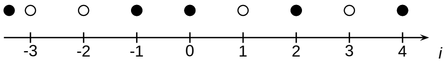

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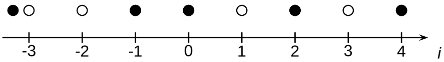

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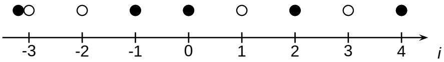

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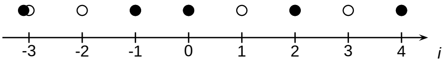

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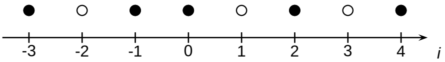

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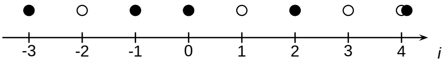

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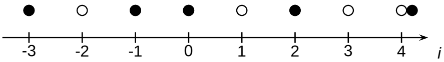

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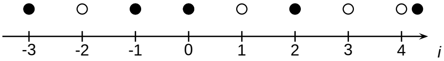

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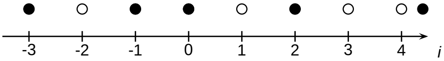

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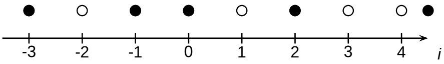

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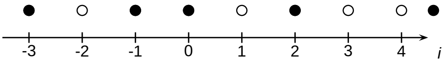

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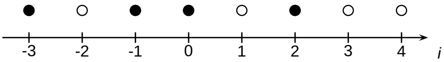

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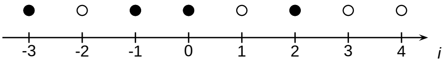

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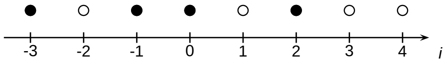

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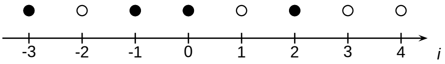

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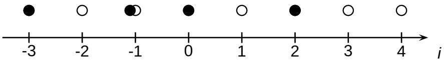

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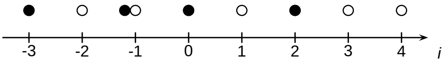

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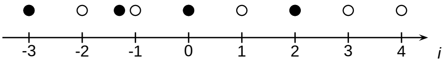

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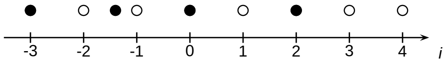
The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



Bernoulli(ϱ) distribution; $\omega_i = 0$ or 1.

Particles try to jump

to the right with rate p,
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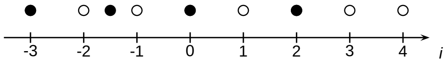
Asymmetric simple exclusion



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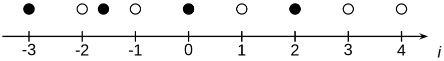

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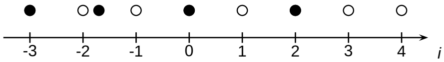

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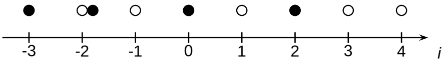

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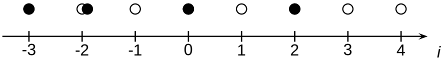

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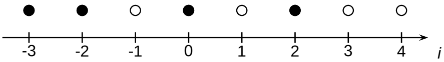

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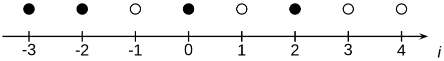

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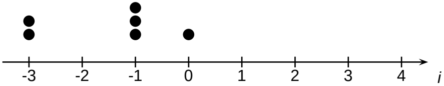
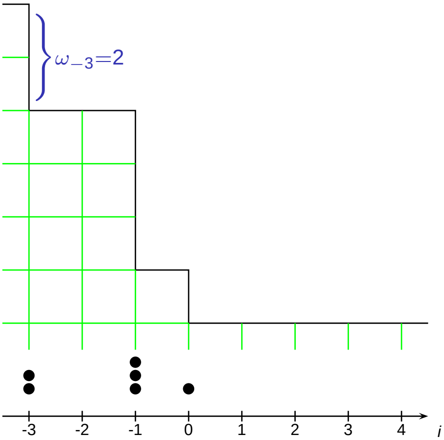
The Bernoulli(ϱ) distribution is time-stationary for any $(0 \le \varrho \le 1)$. Any translation-invariant stationary distribution is a mixture of Bernoullis.

The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

The asymmetric zero range process



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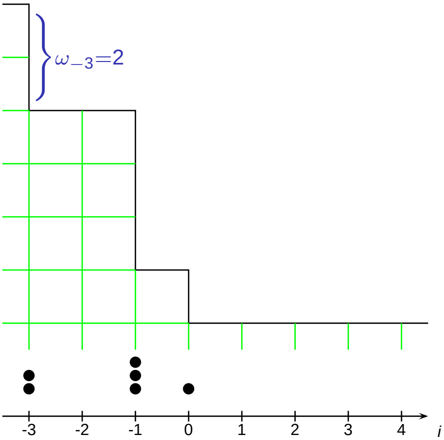
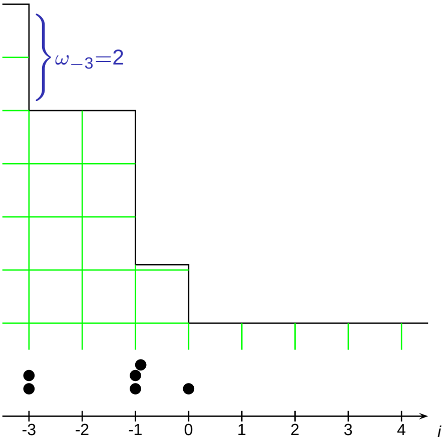


Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
to the right with rate $p \cdot r(\omega_i)$ (r non-decreasing)
to the left with rate $q \cdot r(\omega_i)$ ($q = 1 - p < p$).

The asymmetric zero range process



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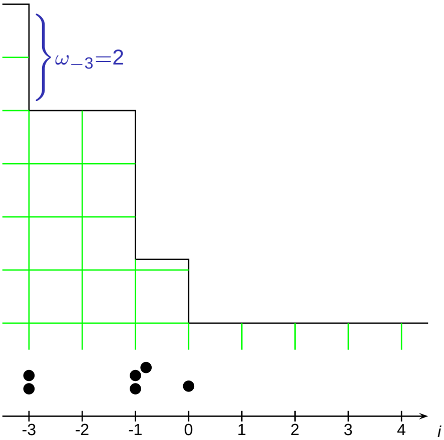

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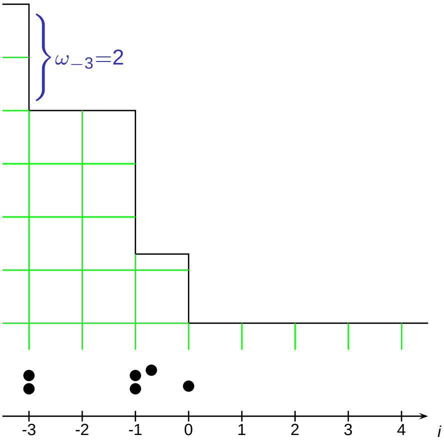

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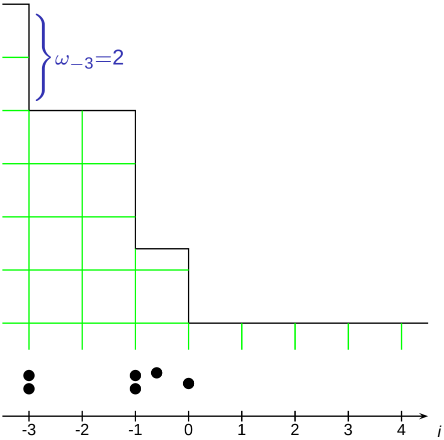

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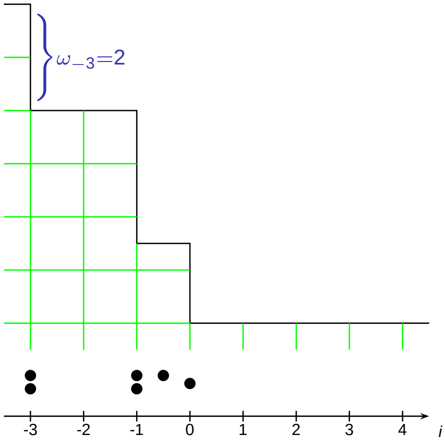

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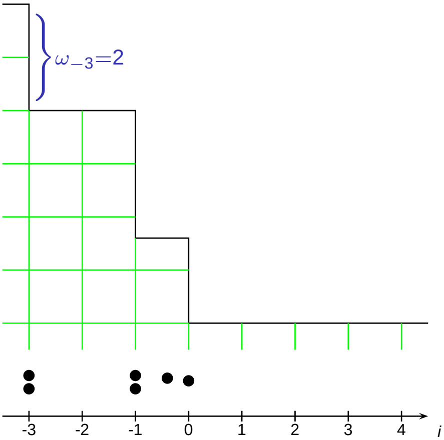

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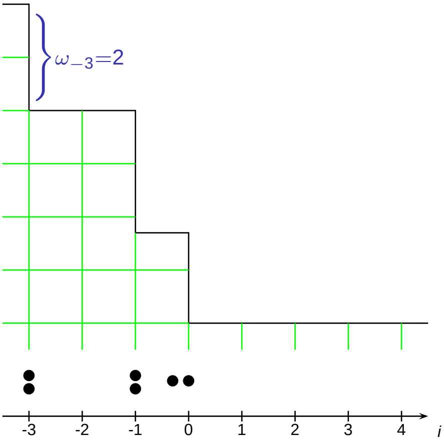

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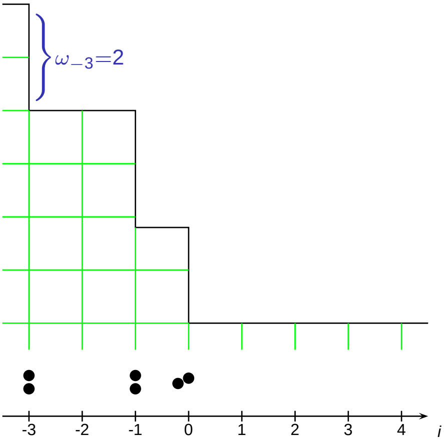

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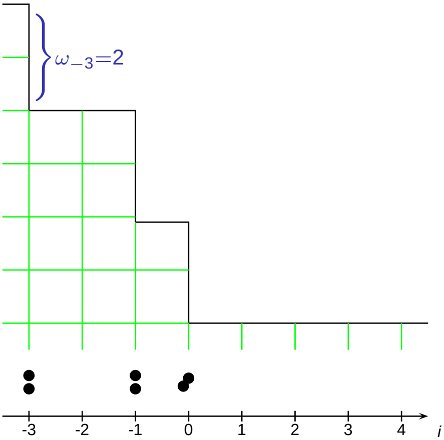

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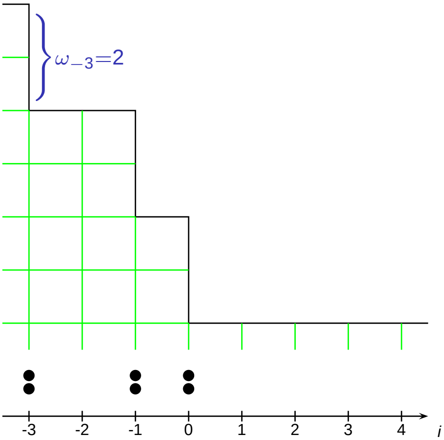

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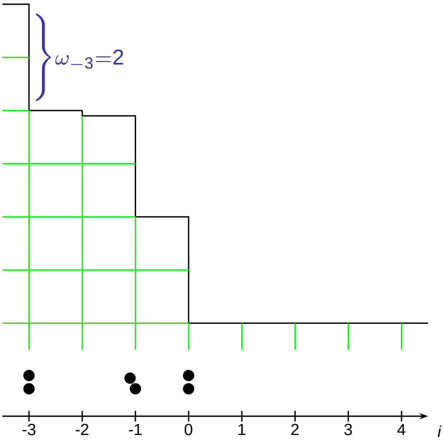

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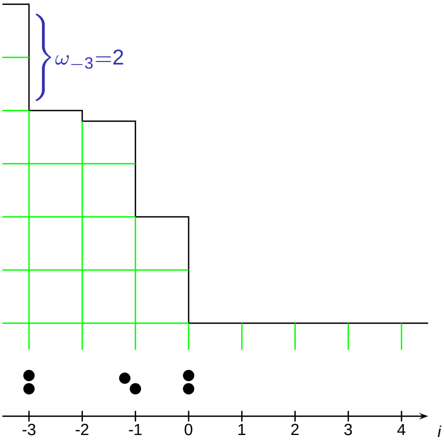

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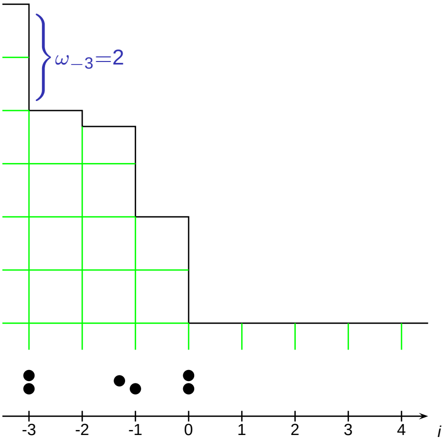

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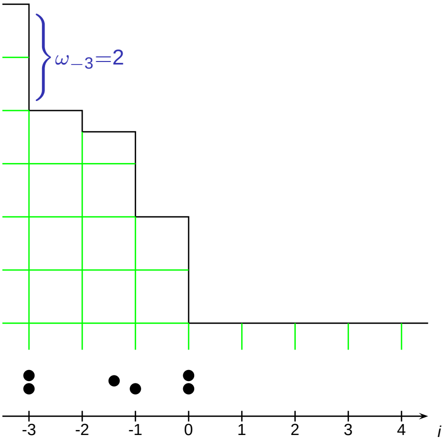

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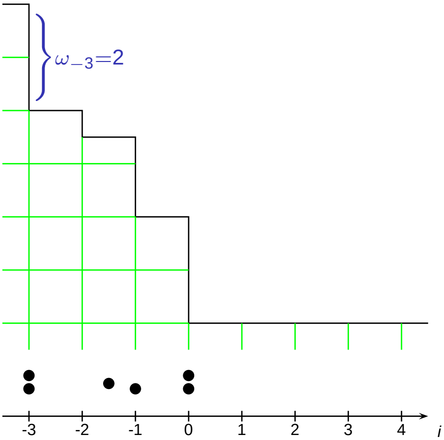

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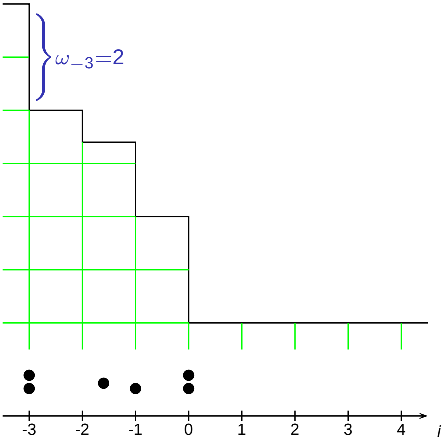

Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
to the right with rate $p \cdot r(\omega_i)$ $\quad$ (r non-decreasing)
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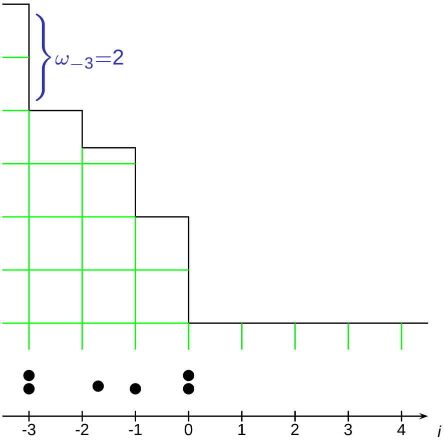
The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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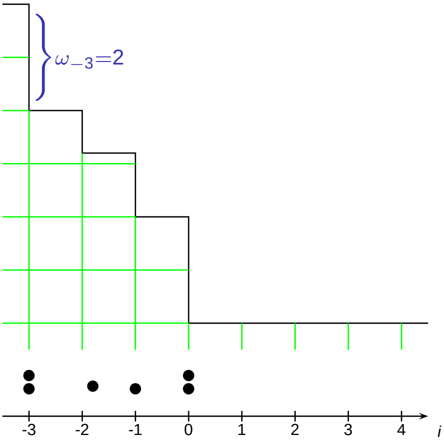

Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
to the right with rate $p \cdot r(\omega_i)$ (r non-decreasing)
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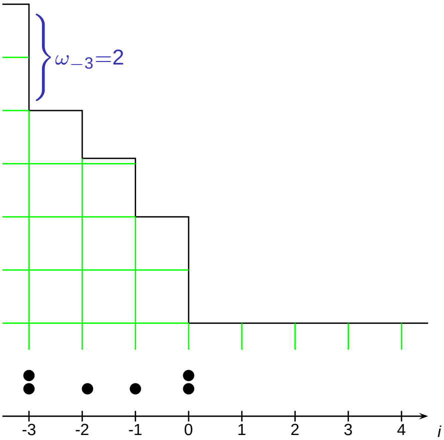
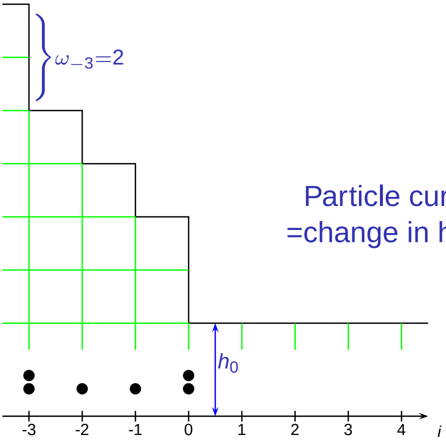
The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
 to the right with rate $p \cdot r(\omega_i)$ (*r* non-decreasing)
 to the left with rate $q \cdot r(\omega_i)$ ($q = 1 - p < p$).

The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
to the right with rate $p \cdot r(\omega_i)$ (r non-decreasing)
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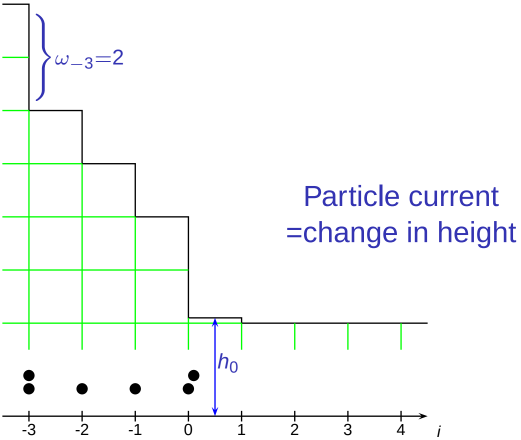
The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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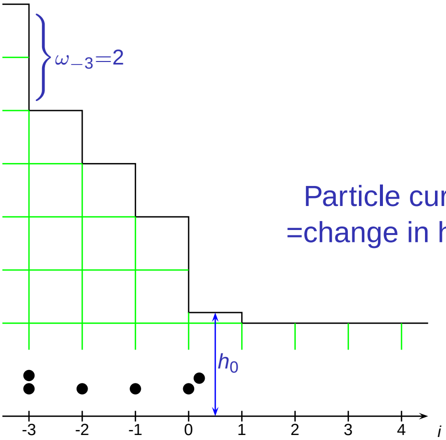
The asymmetric zero range process



Particle current
=change in height

Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
 to the right with rate $p \cdot r(\omega_i)$ (r non-decreasing)
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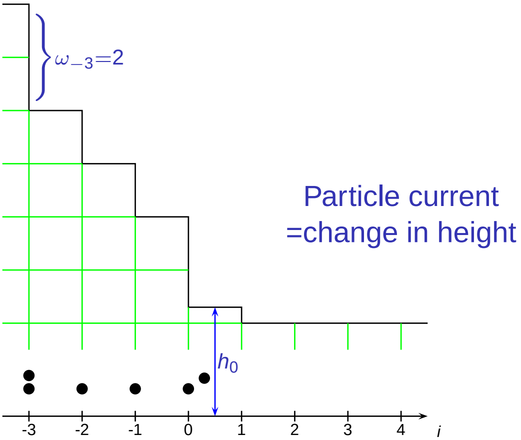
The asymmetric zero range process



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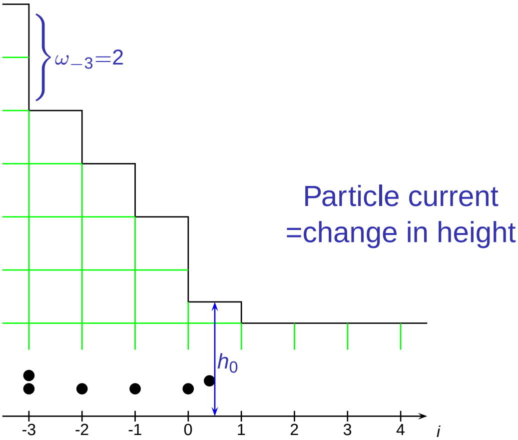
The asymmetric zero range process



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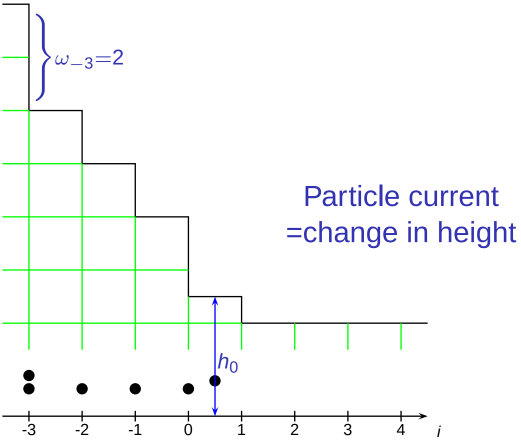

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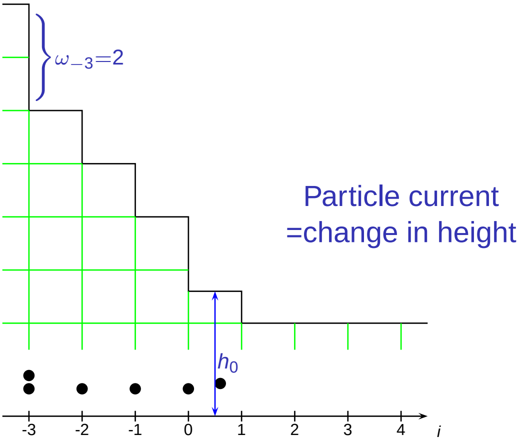

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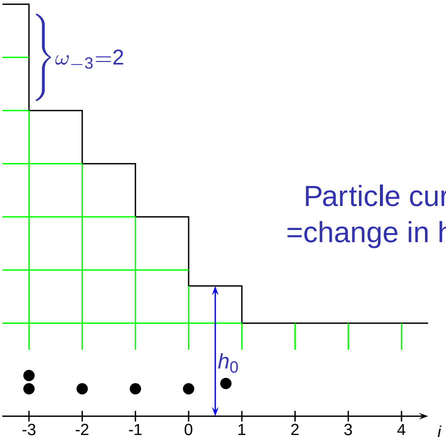

Particle current
=change in height

Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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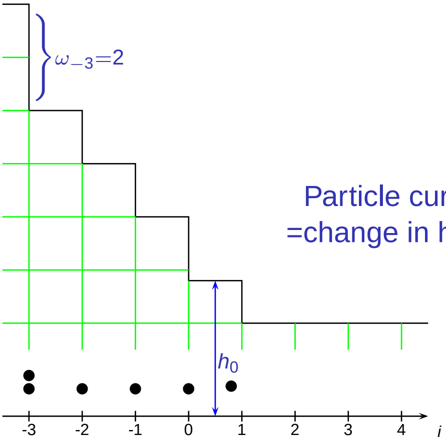

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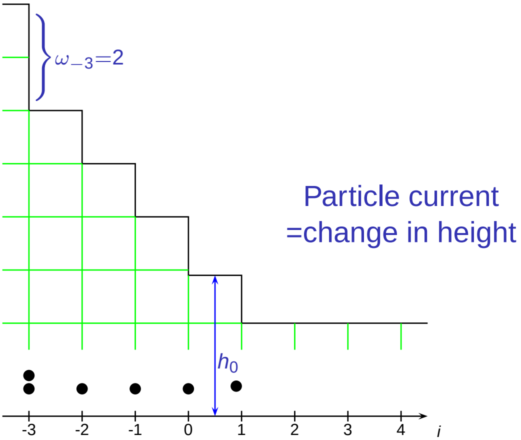

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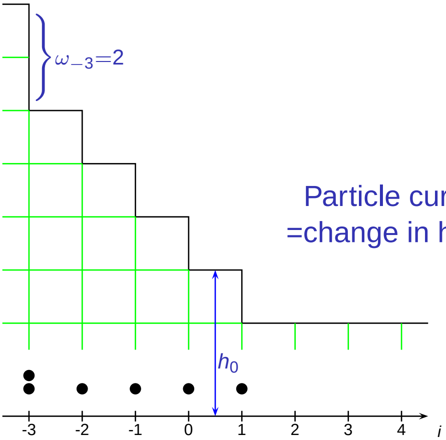

Particle current
=change in height

Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

Particles jump
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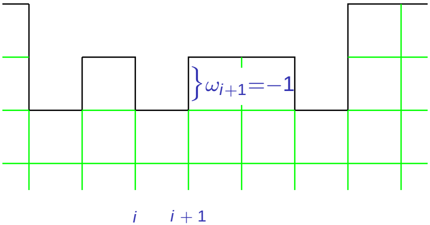
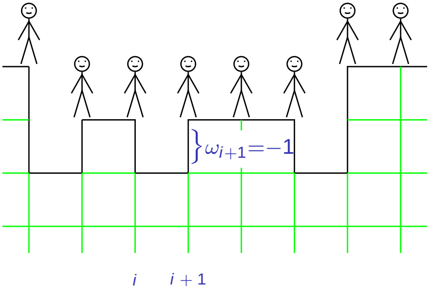
The asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \,\Big\| \, \begin{pmatrix}1\\ -1\end{pmatrix}$$

Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

The asymmetric bricklayers process



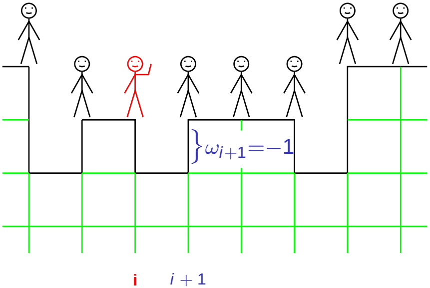
$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

a brick is added with rate $p \cdot [r(\omega_i) + r(-\omega_{i+1})]$
a brick is removed with rate $q \cdot [r(-\omega_i) + r(\omega_{i+1})]$.

$(r(\omega) \cdot r(1-\omega) = 1; \quad r \text{ non-decreasing}; \quad q = 1-p < p)$.

The asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ || \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

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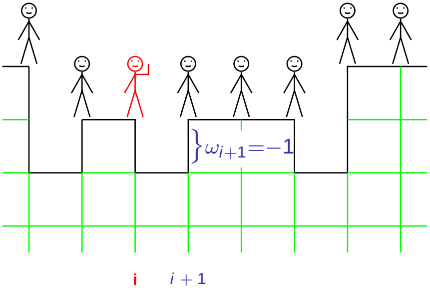

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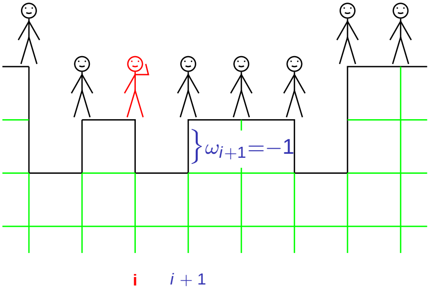

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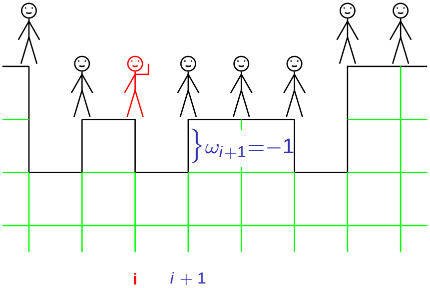

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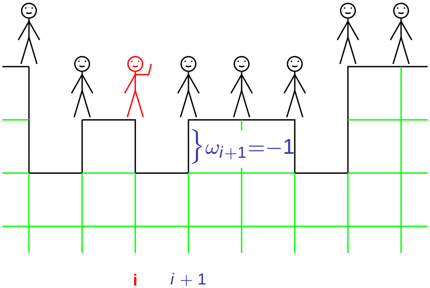

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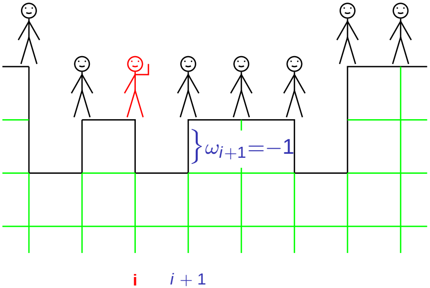

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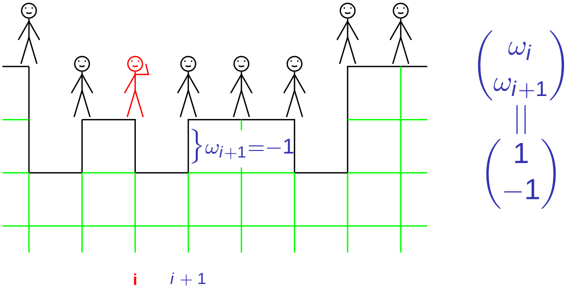

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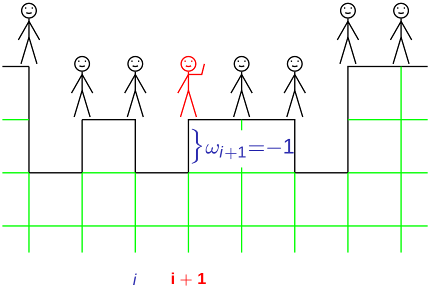

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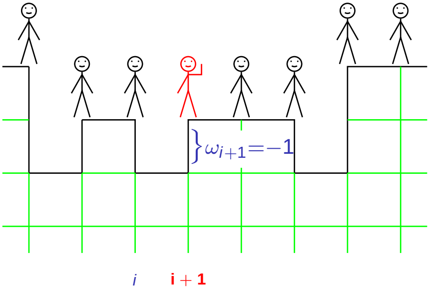

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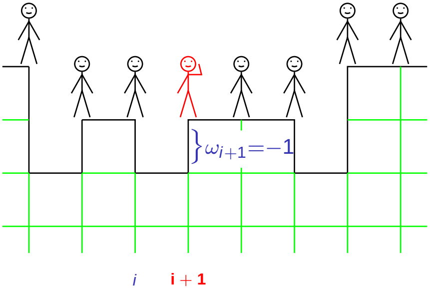

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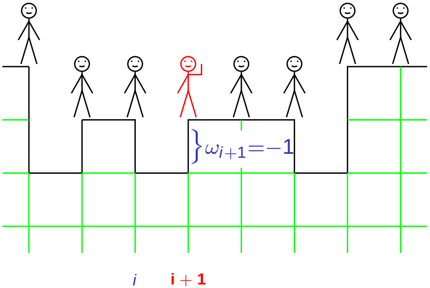

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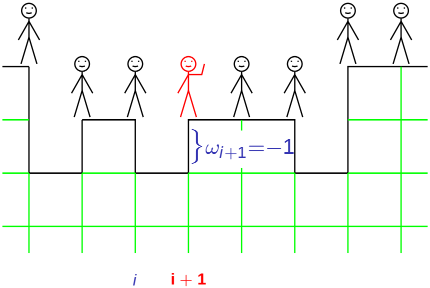

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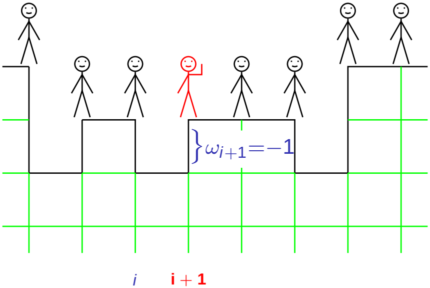

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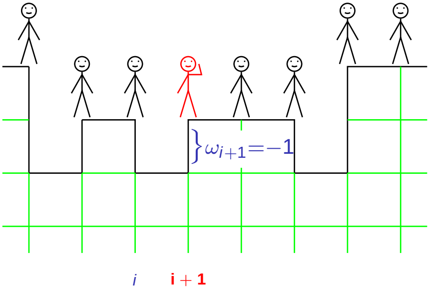

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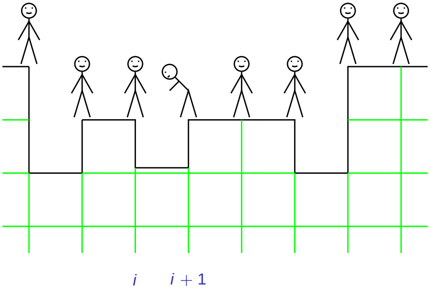

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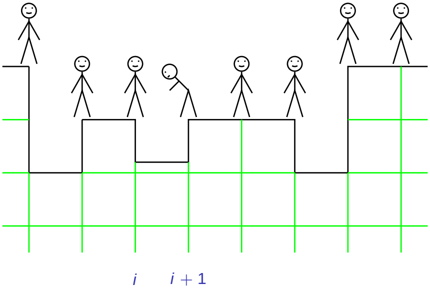

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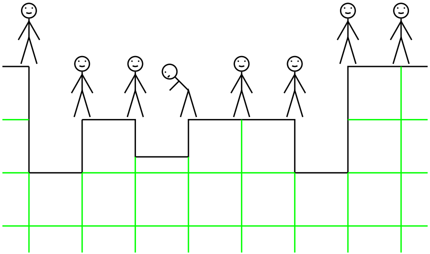

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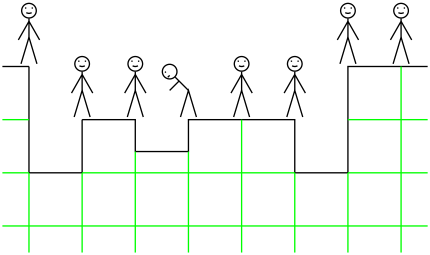

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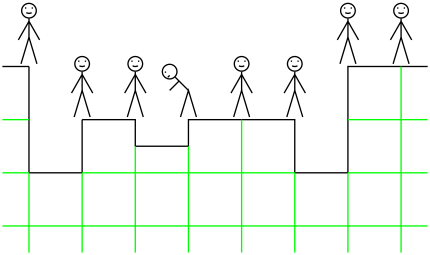

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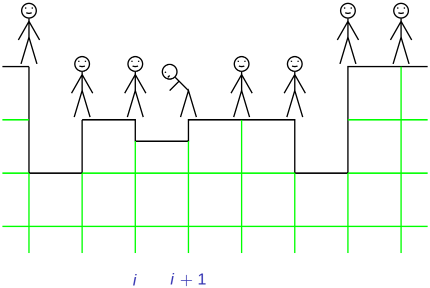

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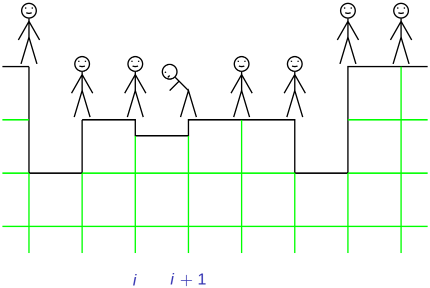

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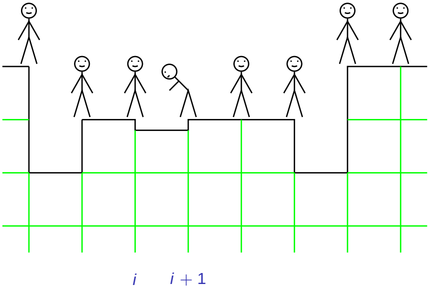

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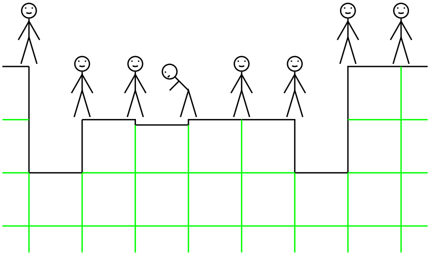

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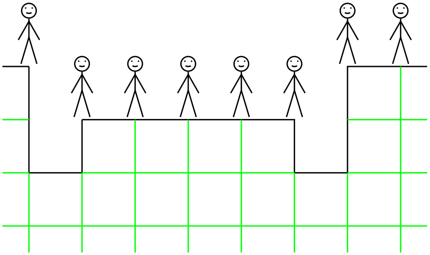

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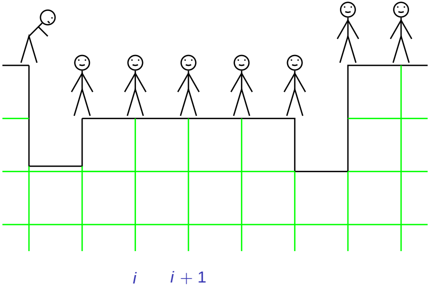

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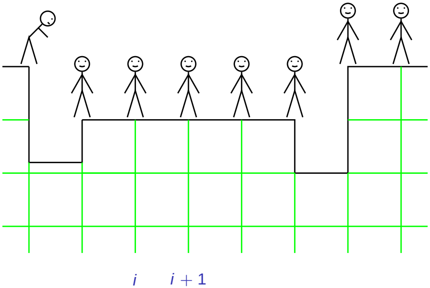

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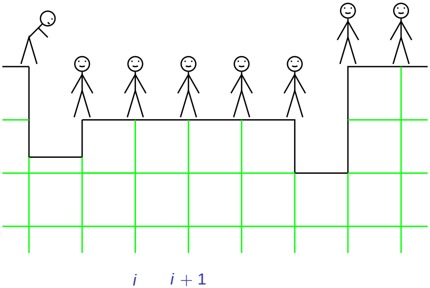

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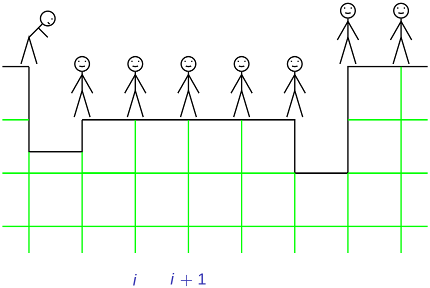

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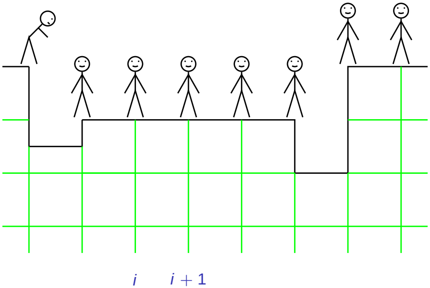

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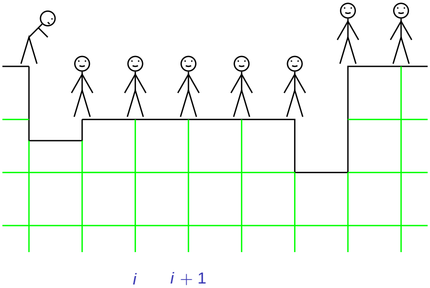

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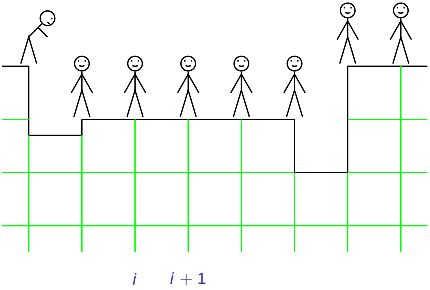

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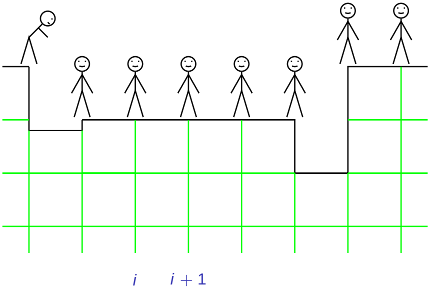

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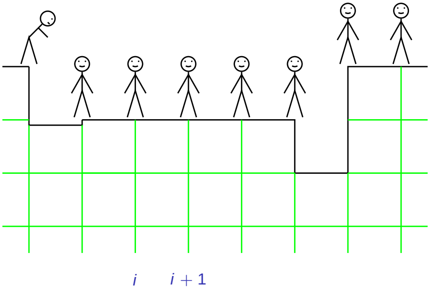

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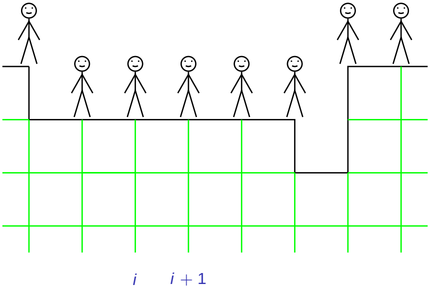

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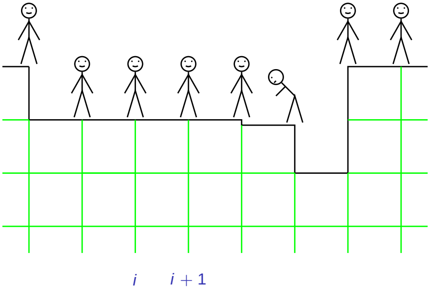

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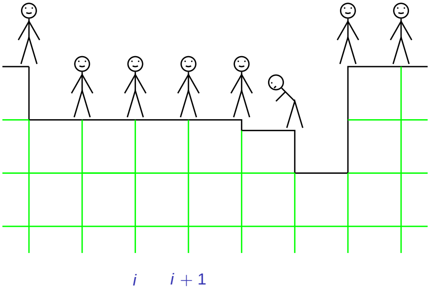

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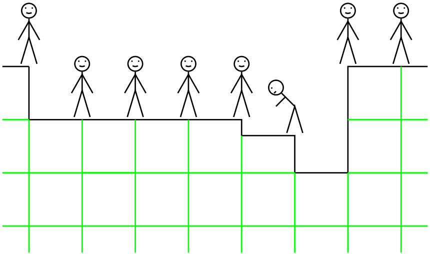

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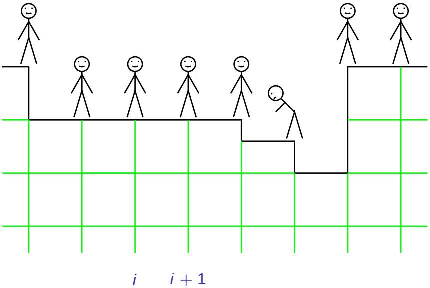

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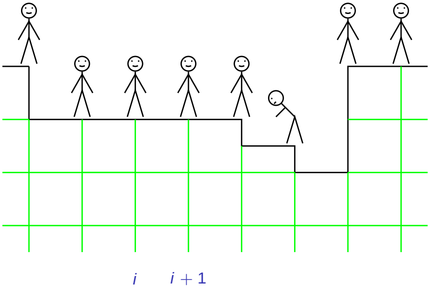

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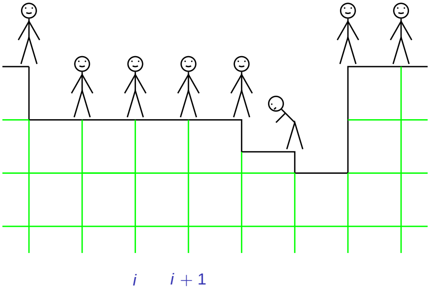

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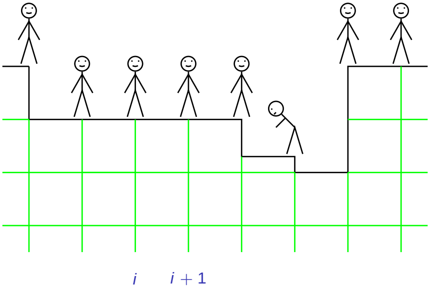

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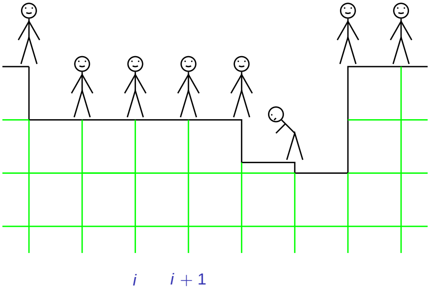

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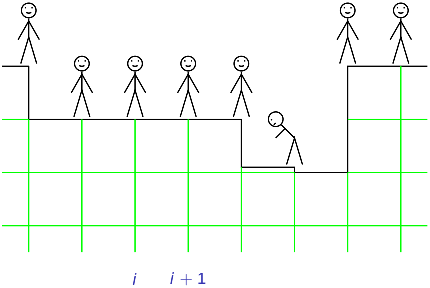

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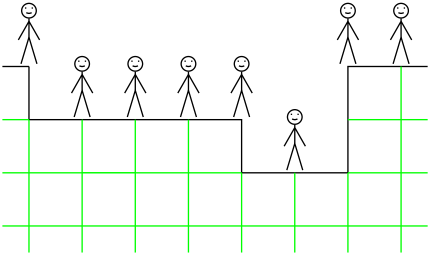

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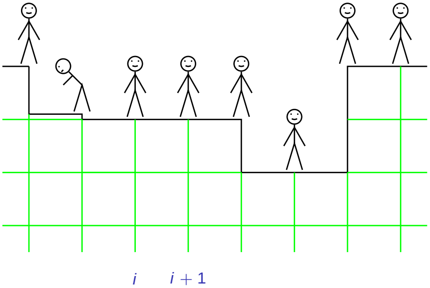

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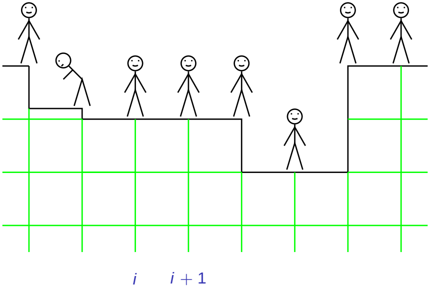

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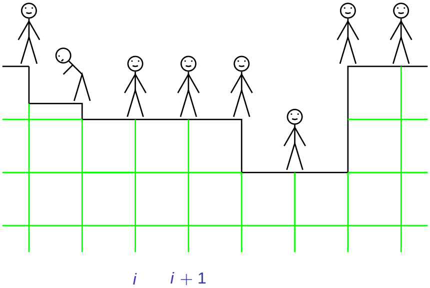

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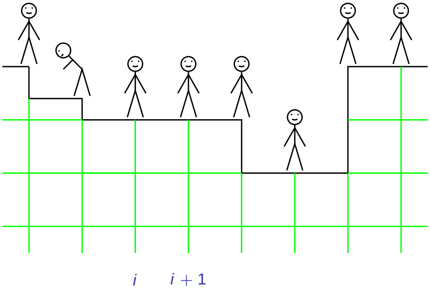

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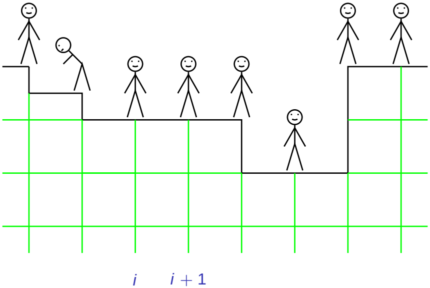

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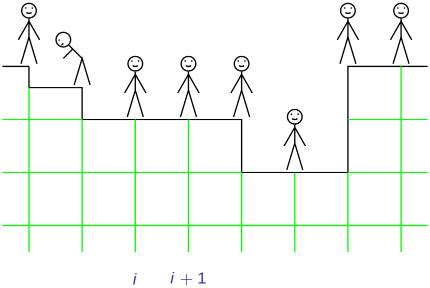

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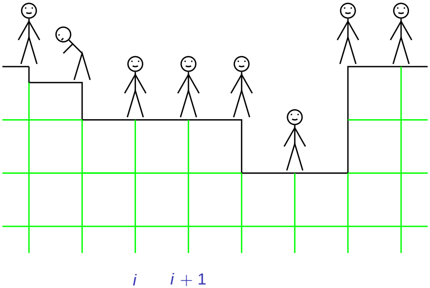

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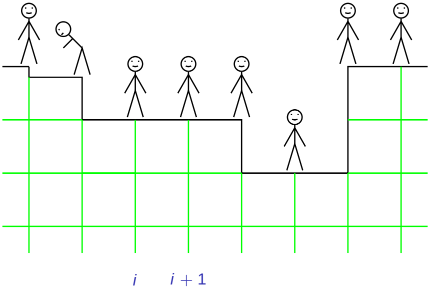

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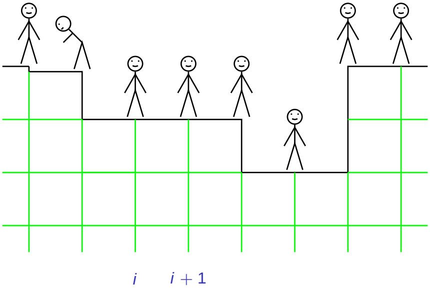

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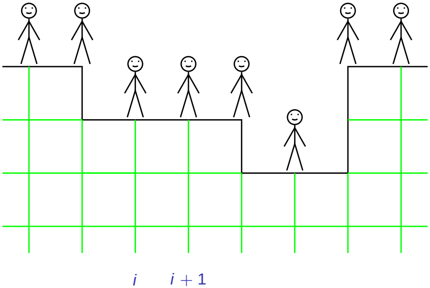

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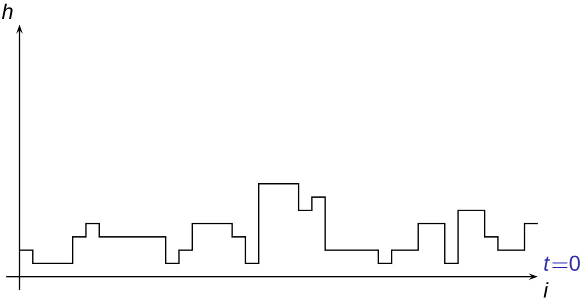

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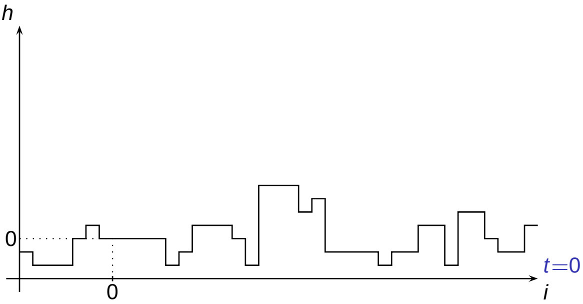

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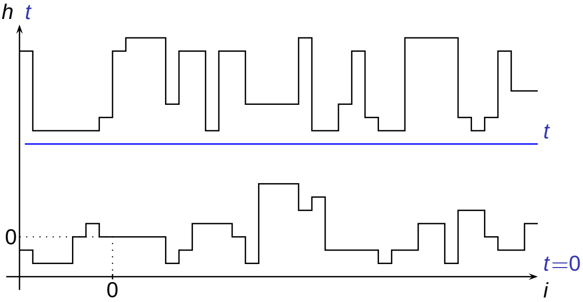

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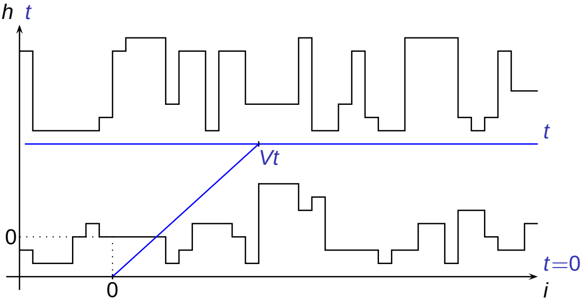

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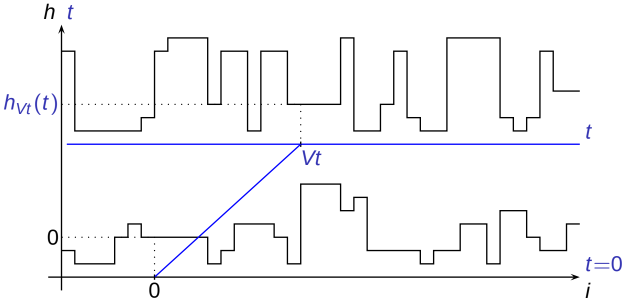

$h_{Vt}(t)$ = height as seen by a moving observer of velocity V.

= net number of particles passing the window $s \mapsto Vs$.

(Remember: particle current=change in height.)

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- The *characteristics* is a path $X(T)$ where $\varrho(T, X(T))$ is constant.

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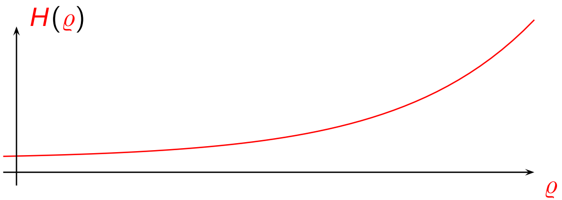

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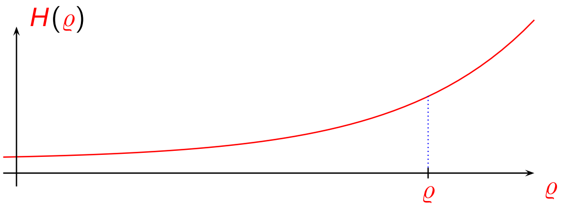

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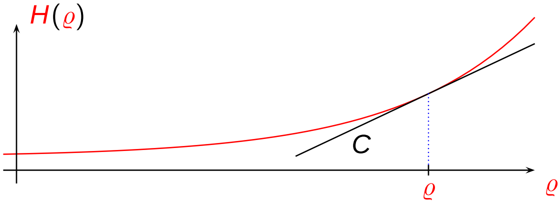

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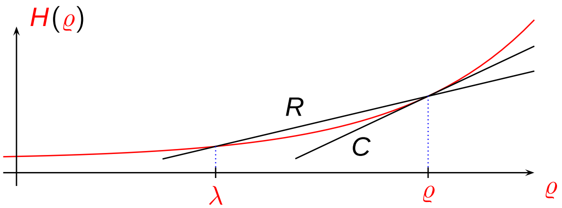

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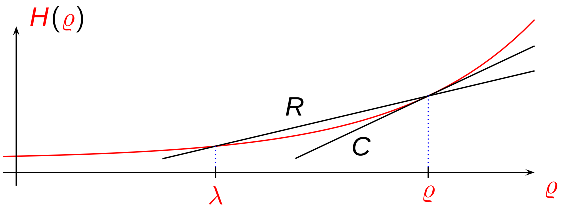

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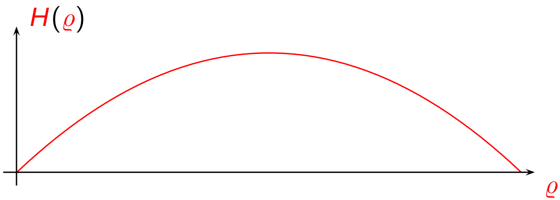

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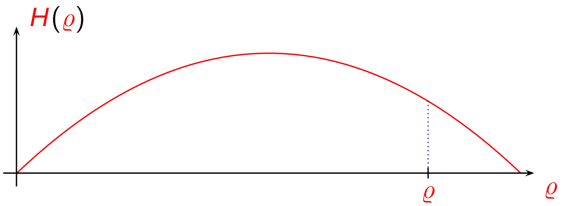

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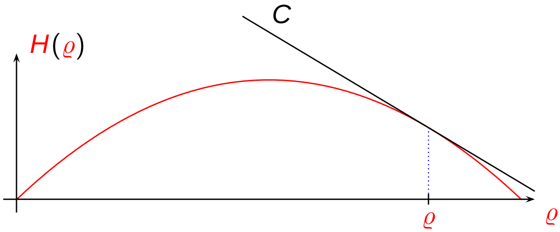

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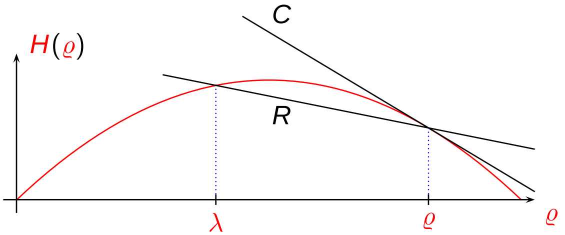

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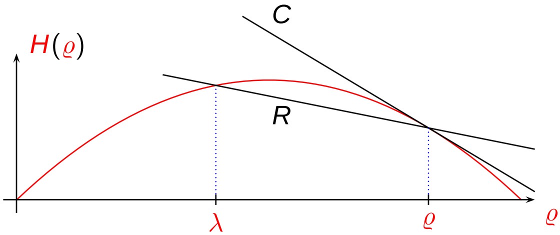

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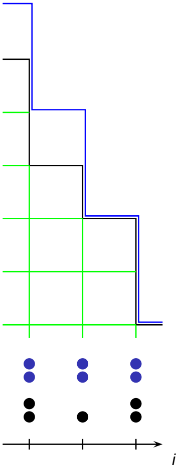
Tool: the second class particle

States ω and ω only differ at one site.



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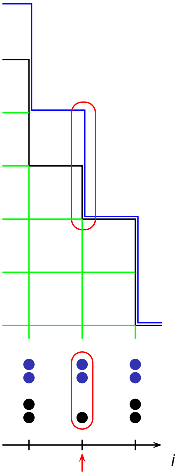

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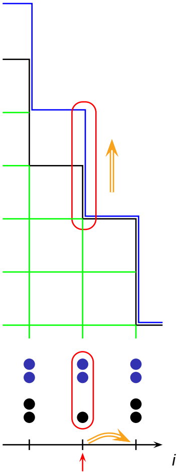

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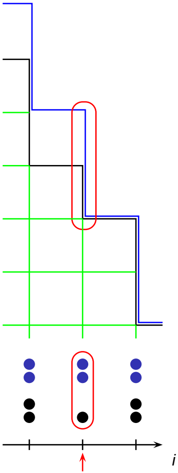

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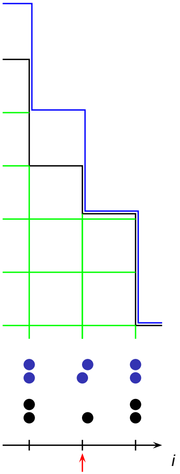

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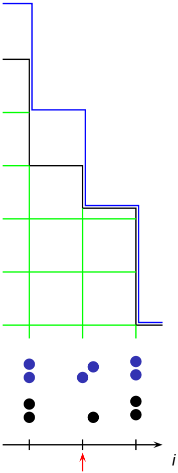

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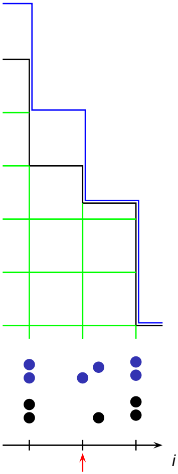

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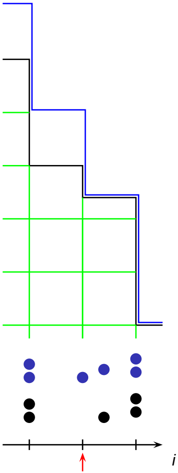

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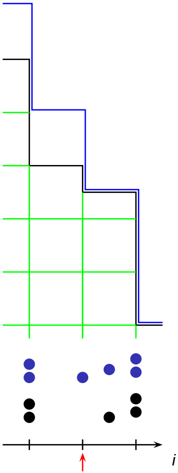

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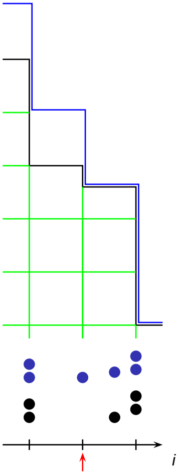

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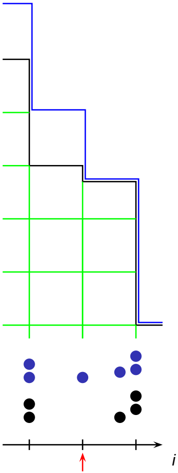

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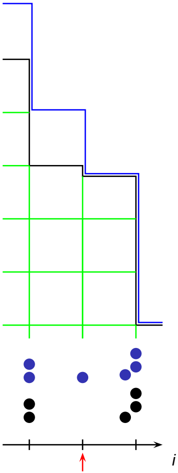

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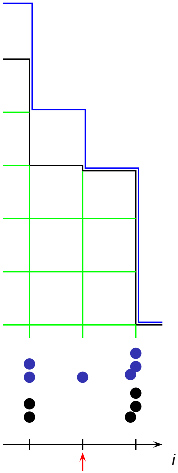

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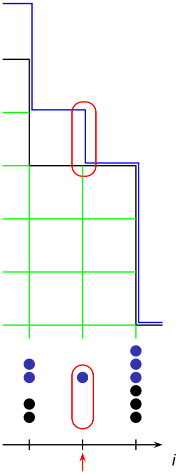

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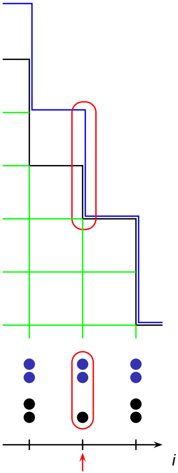

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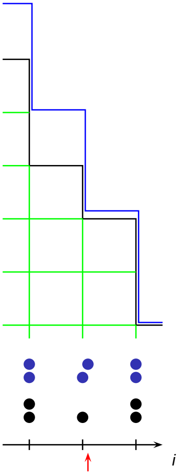

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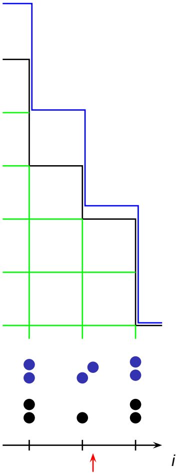

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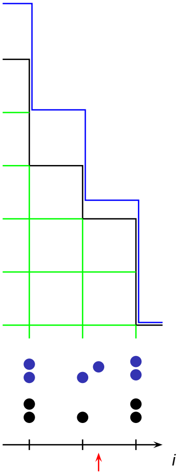

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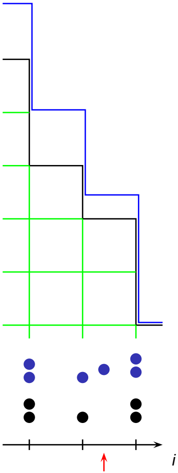

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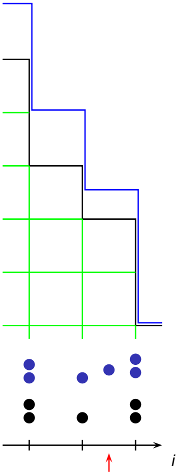

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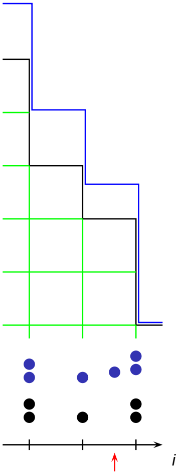

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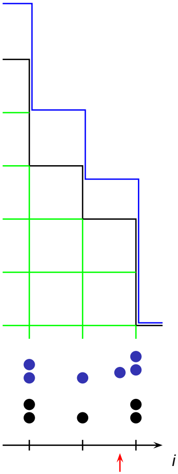

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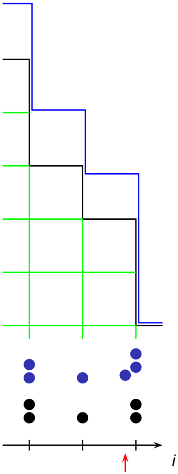

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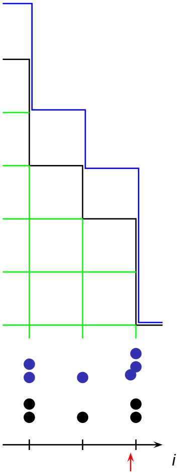

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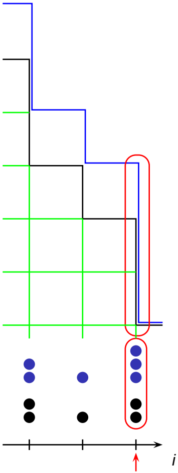

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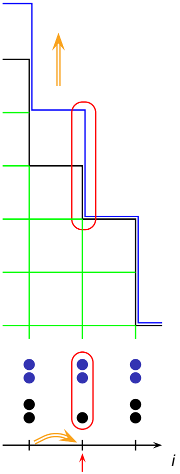

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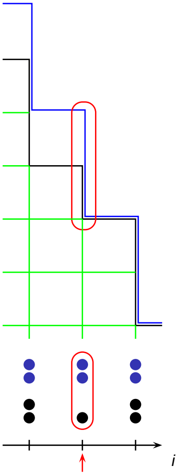

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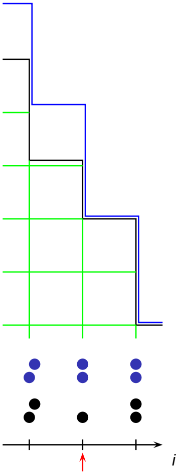

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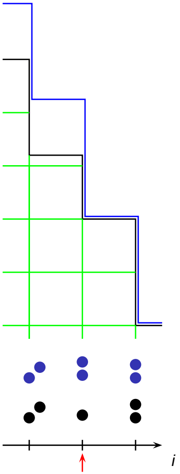

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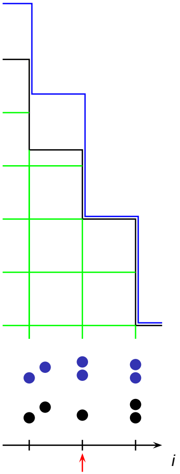

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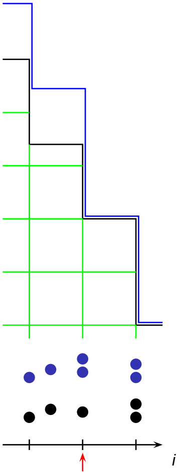

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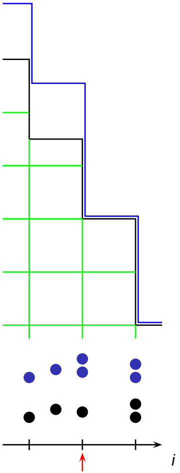

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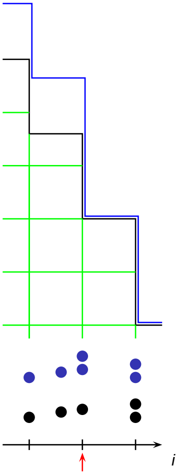

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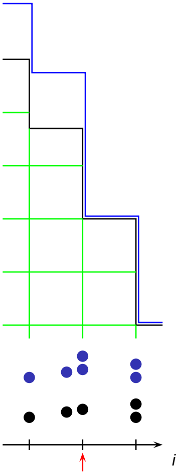

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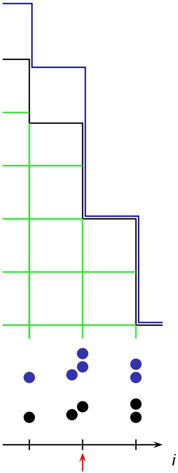

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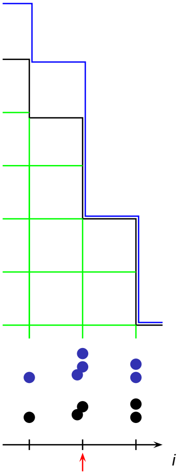

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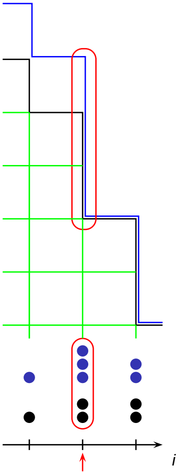

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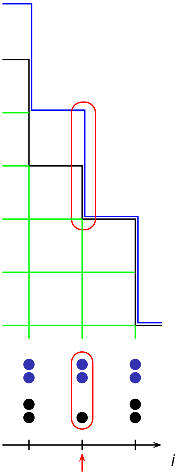

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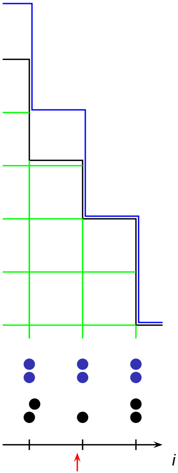
States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

with rate-rate:



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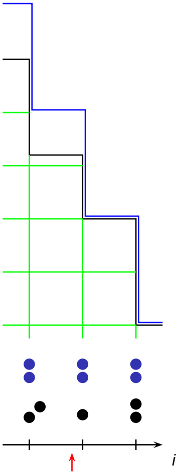

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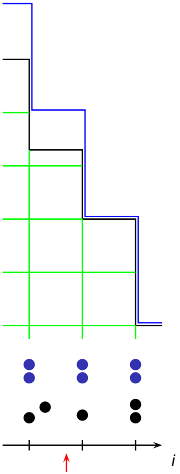

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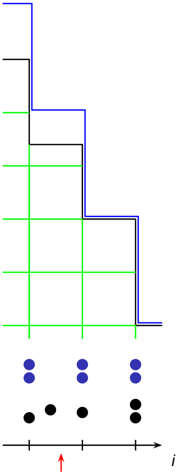

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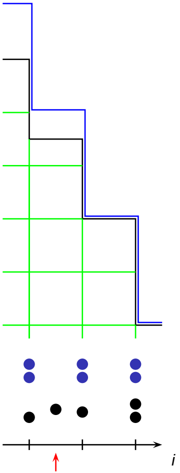

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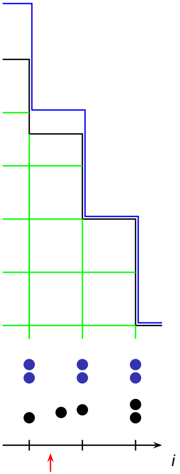

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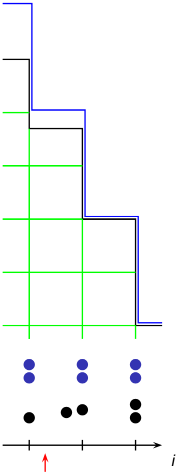

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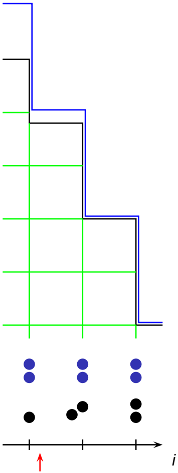

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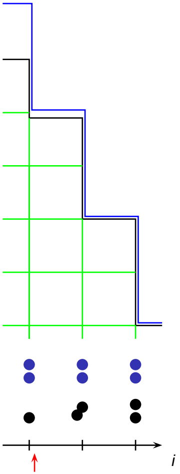

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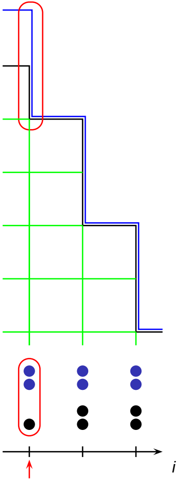

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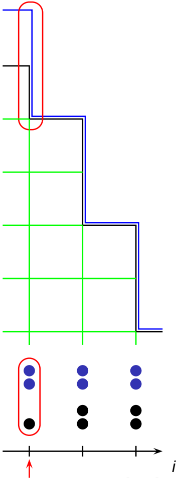

A single discrepancy$\uparrow$, the *second class particle*, is conserved.
Its position at time t is $Q(t)$.

Tool: the second class particle

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{E}(Q(t)) = C \cdot t$$

in the whole family of processes.

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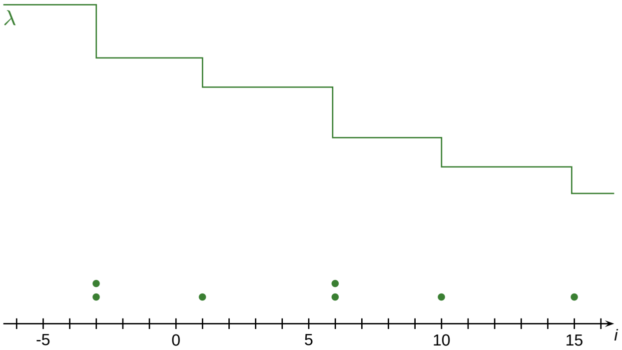

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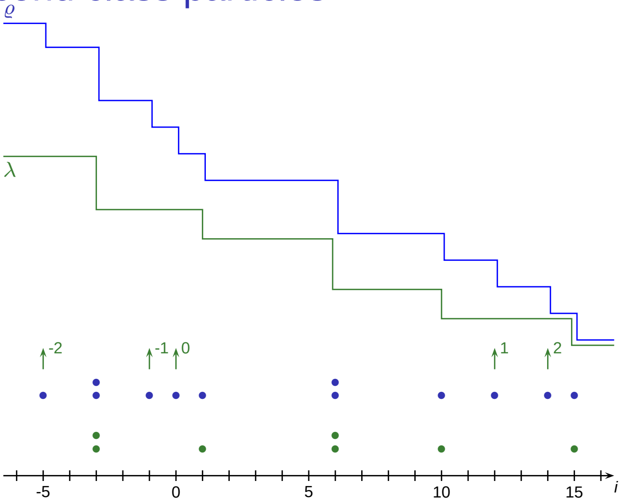

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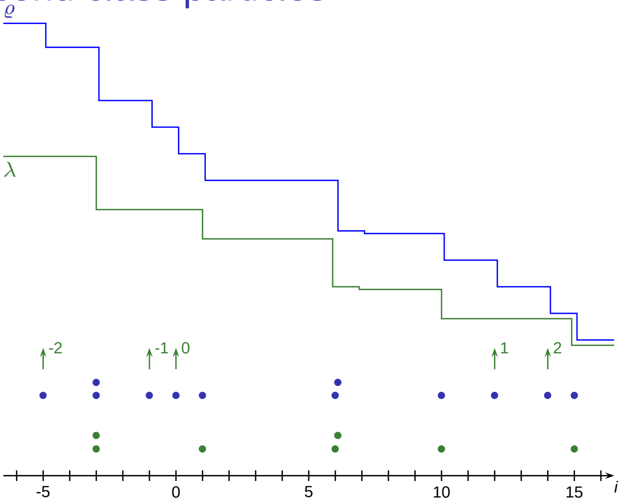

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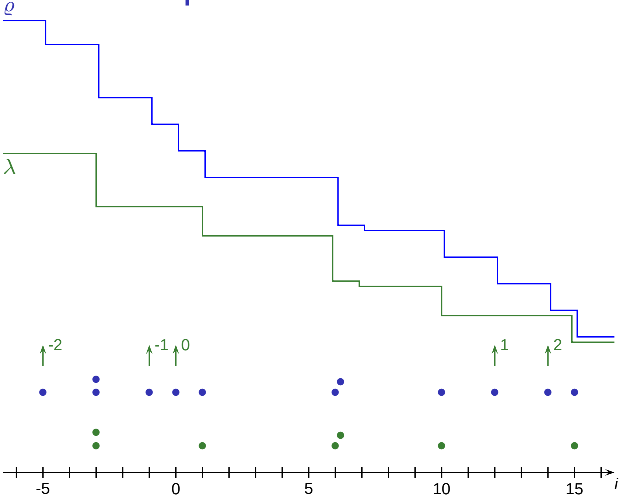

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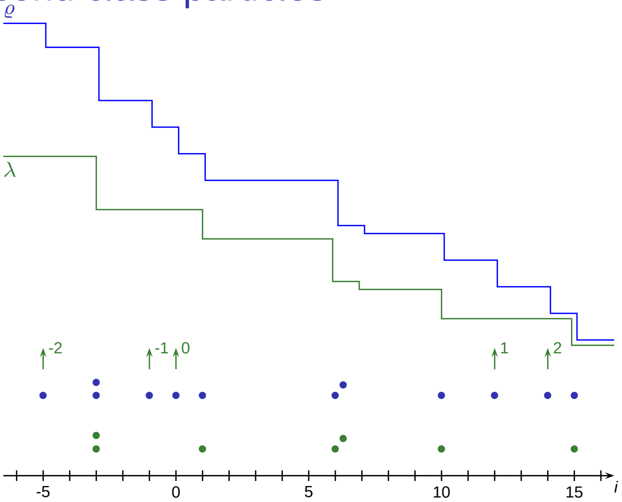

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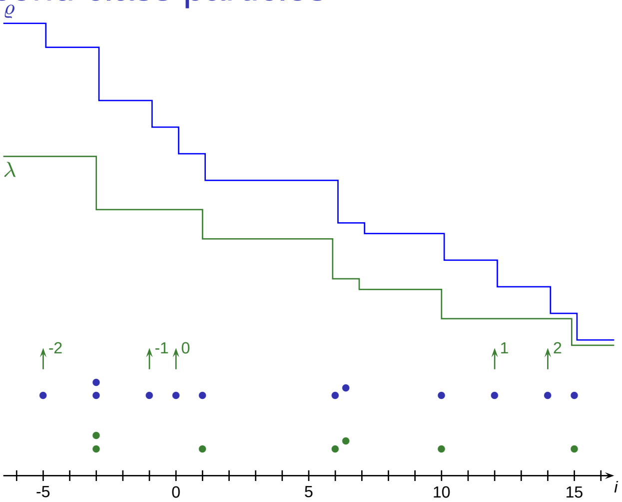

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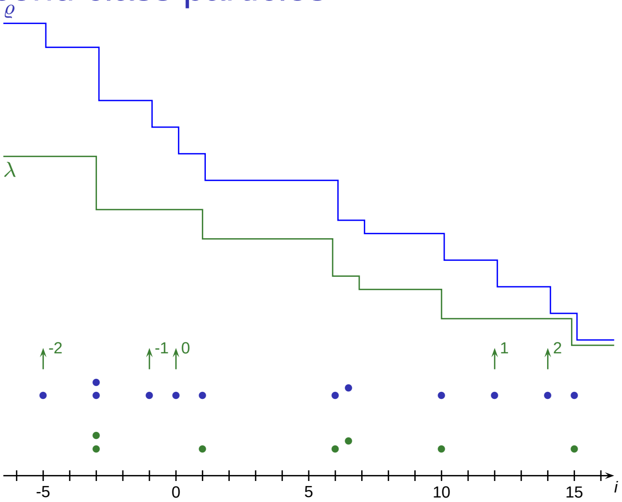

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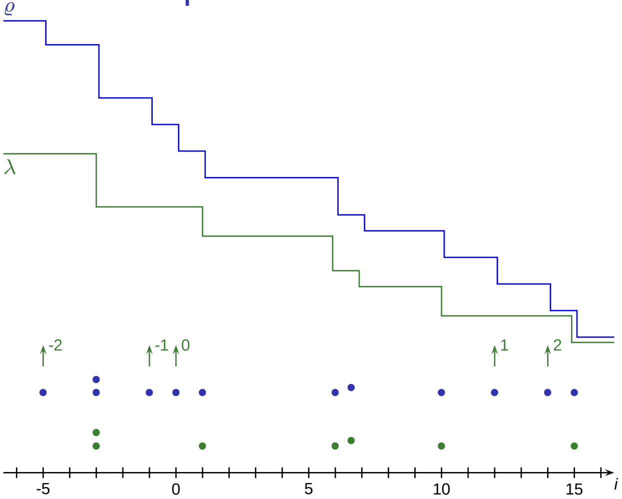

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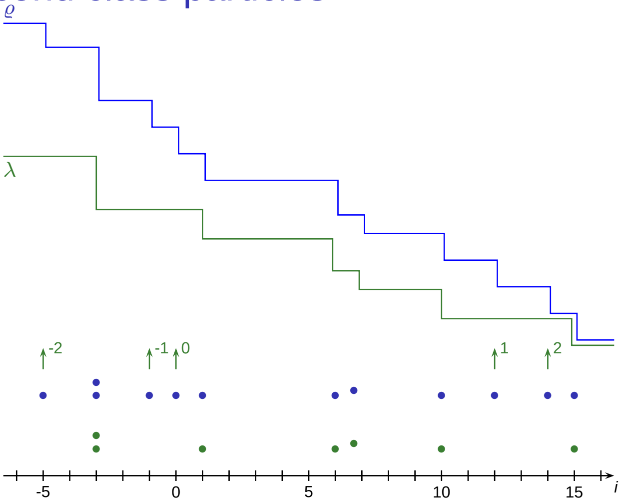

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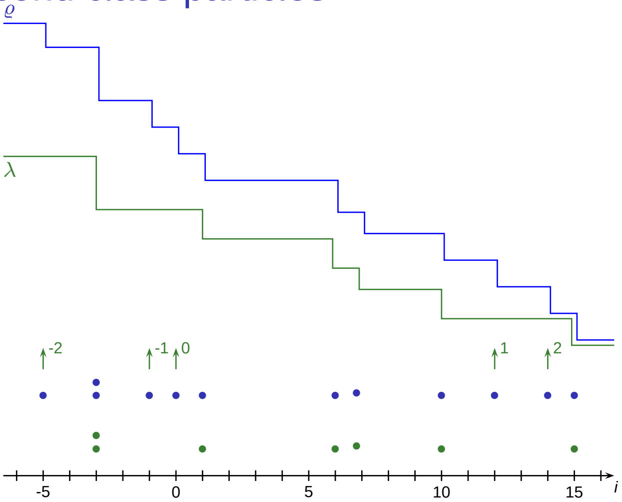

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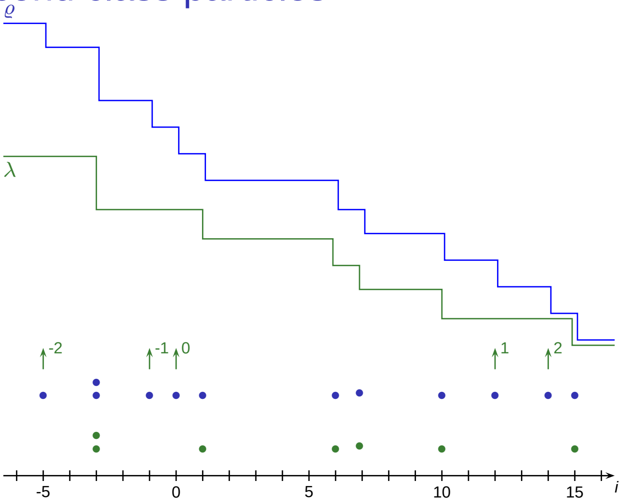

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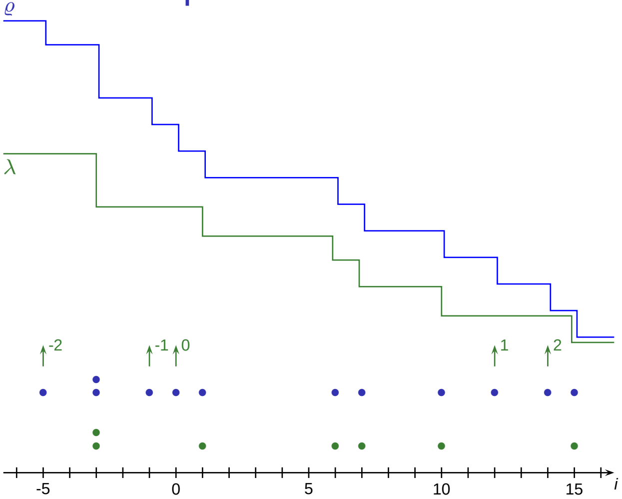

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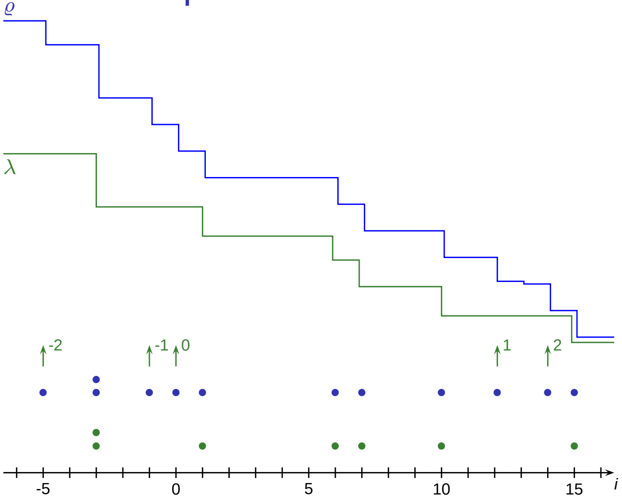

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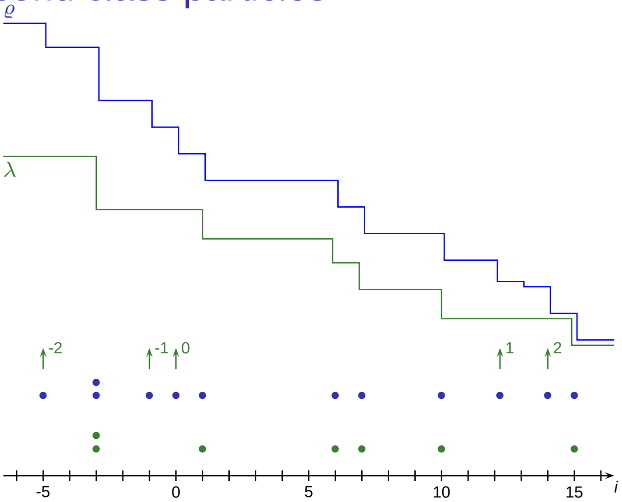

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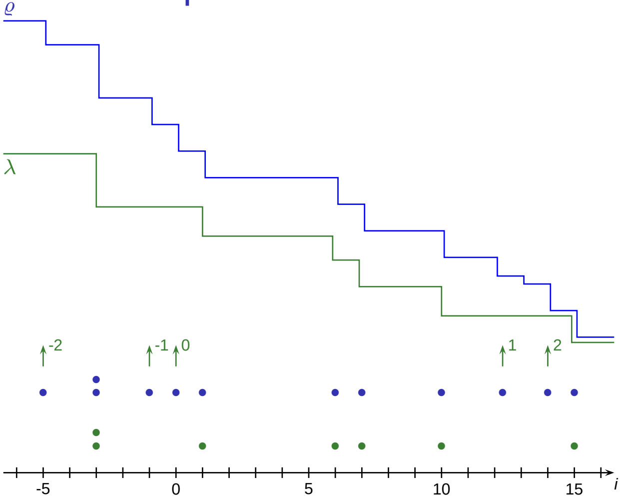

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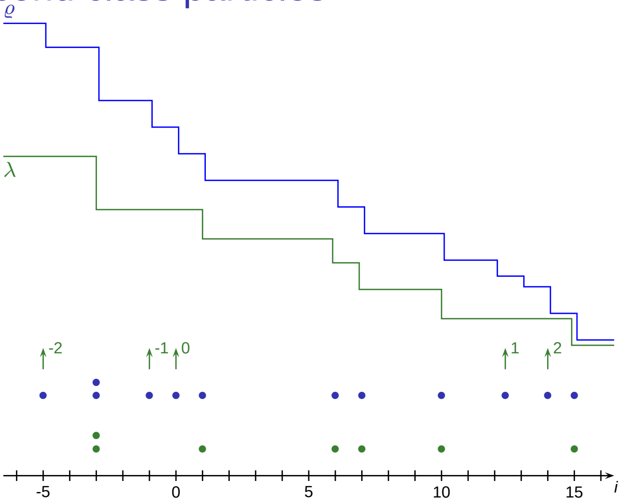

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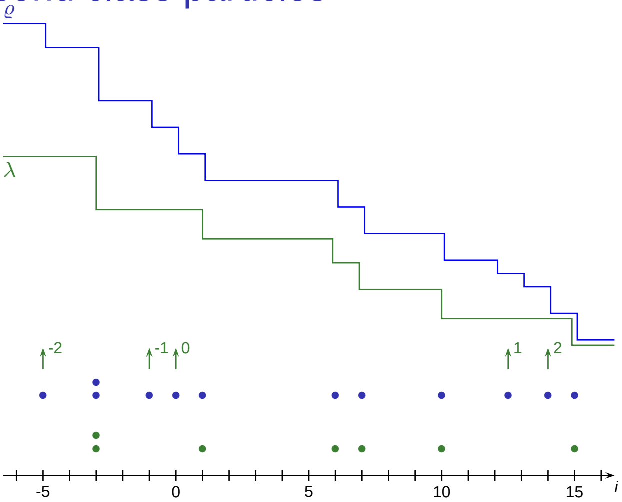

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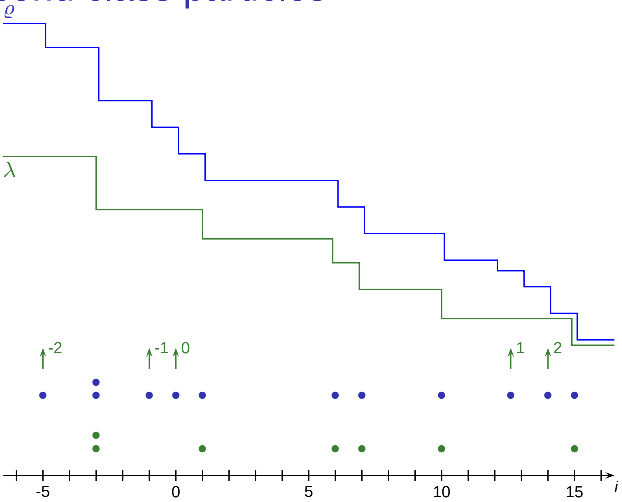

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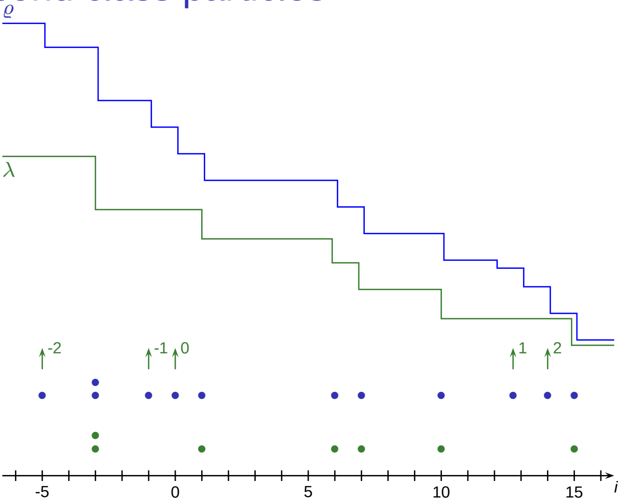

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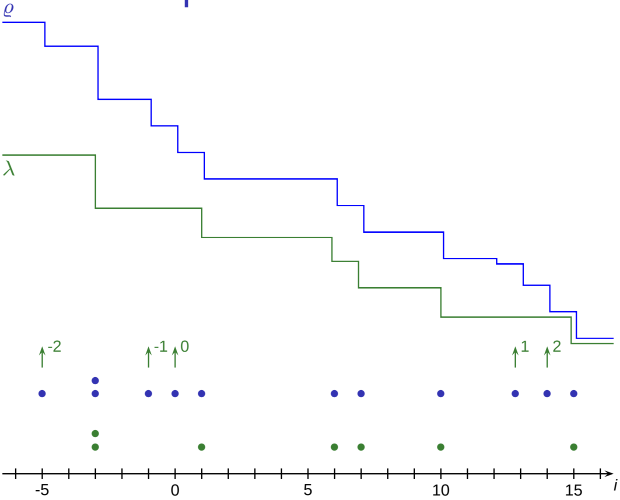

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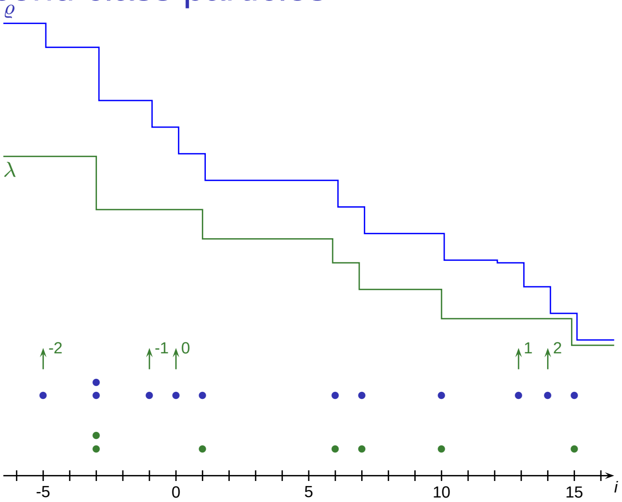

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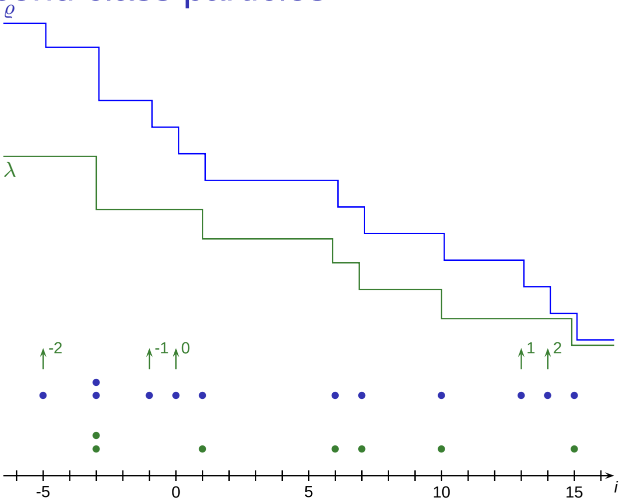

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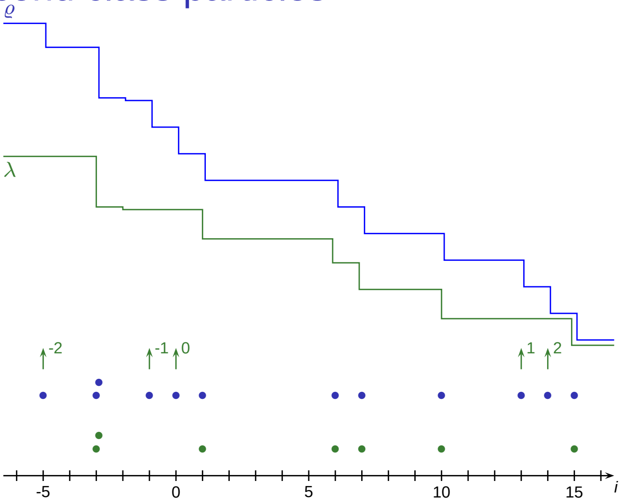

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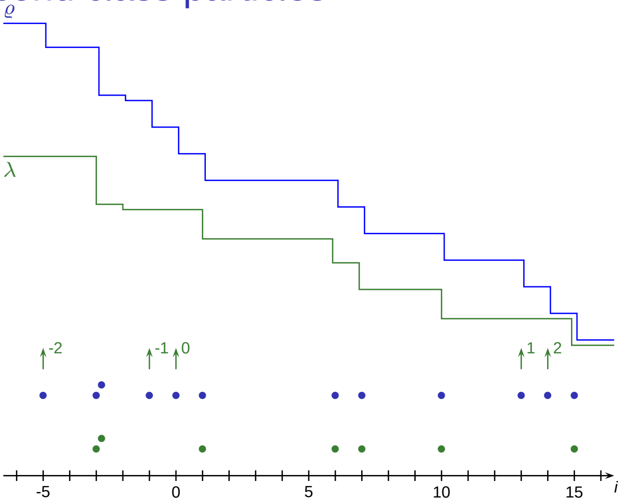

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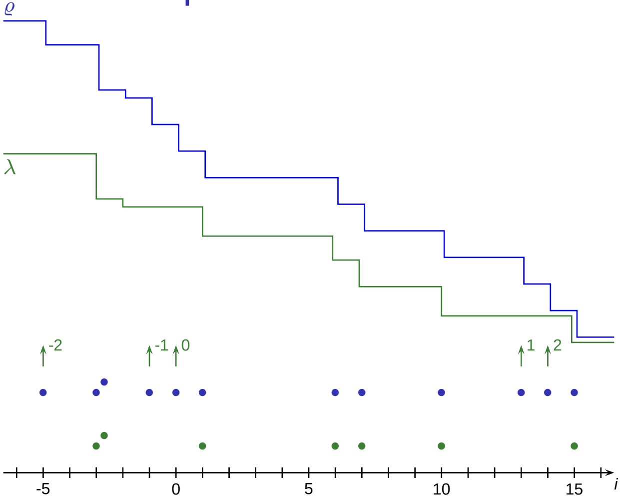

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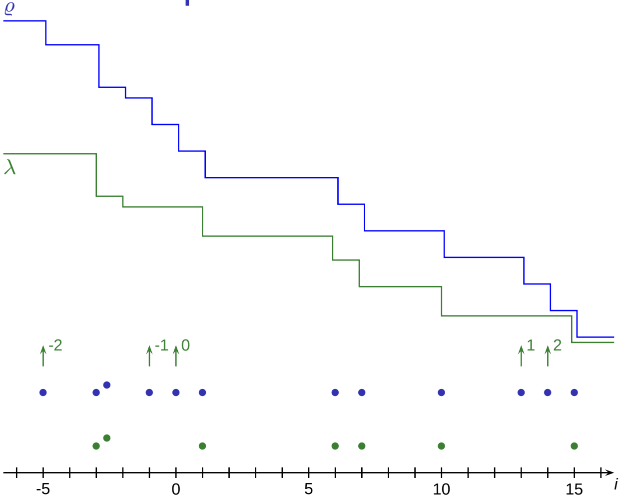

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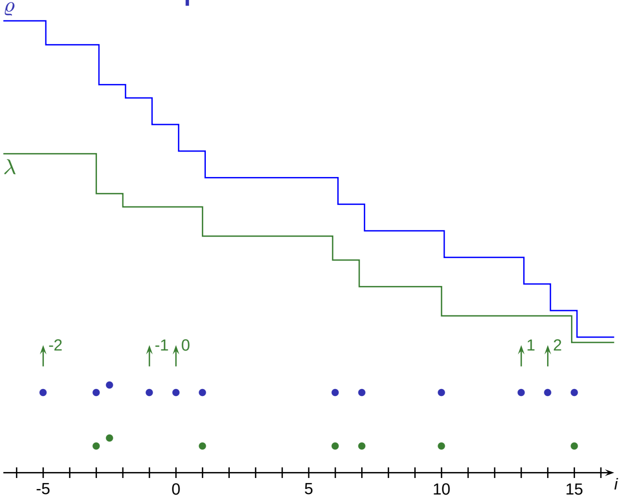

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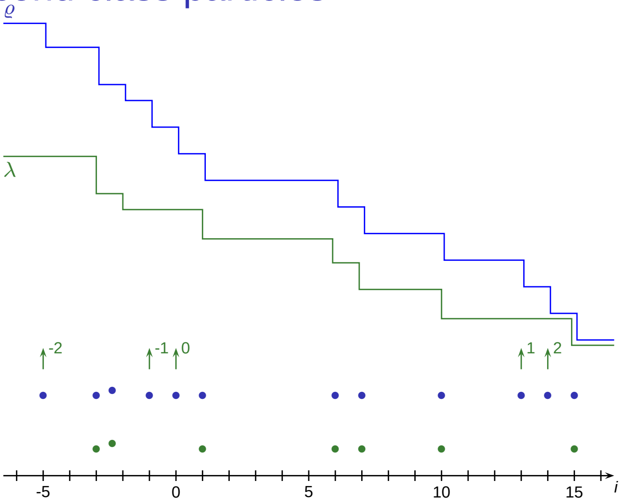

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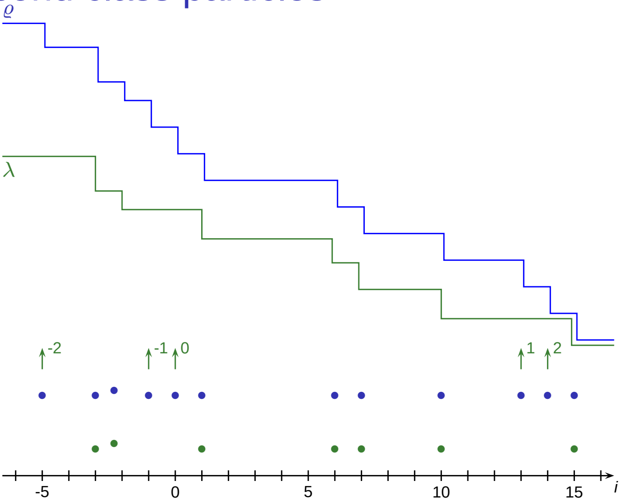

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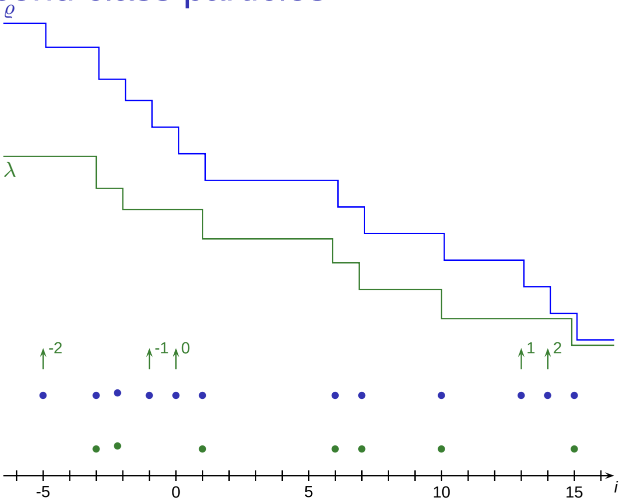

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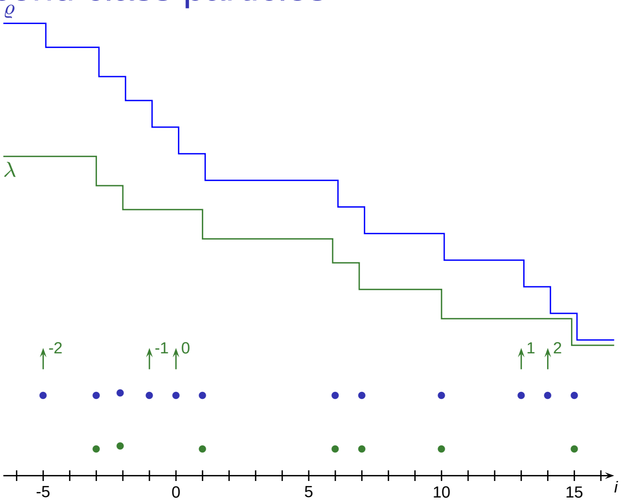

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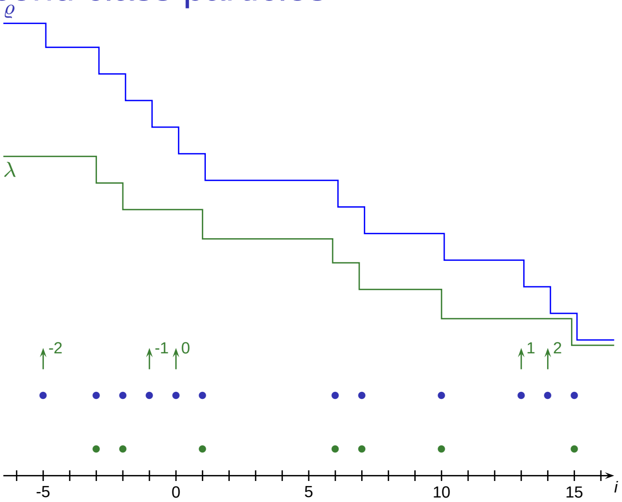

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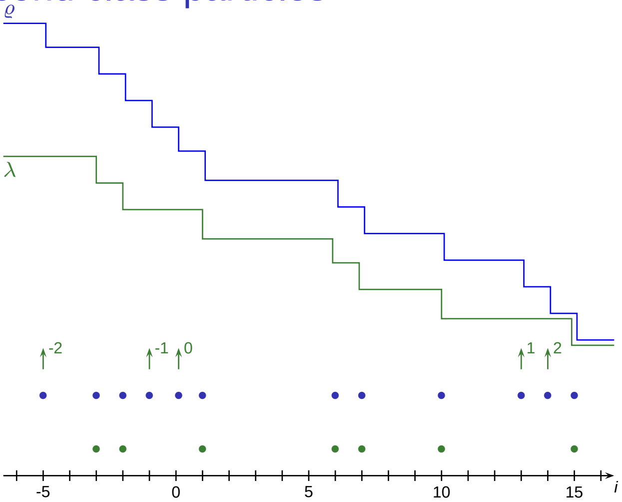

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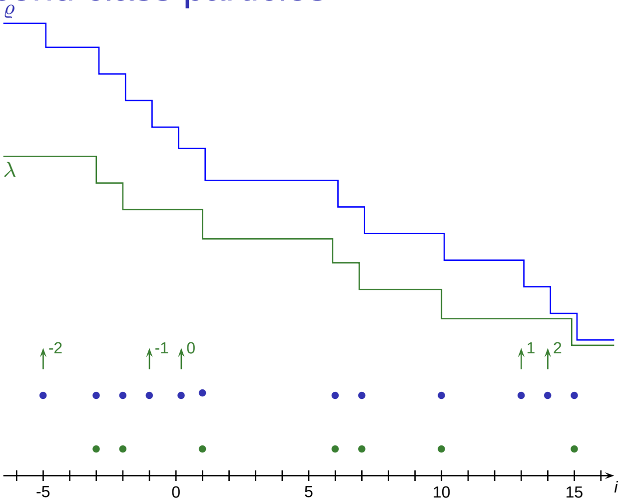

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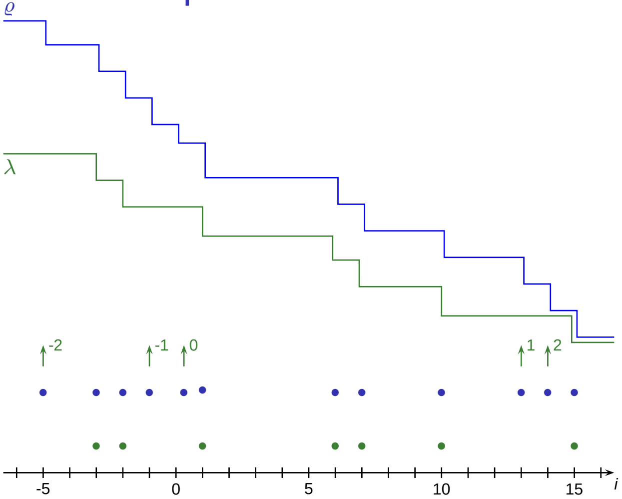

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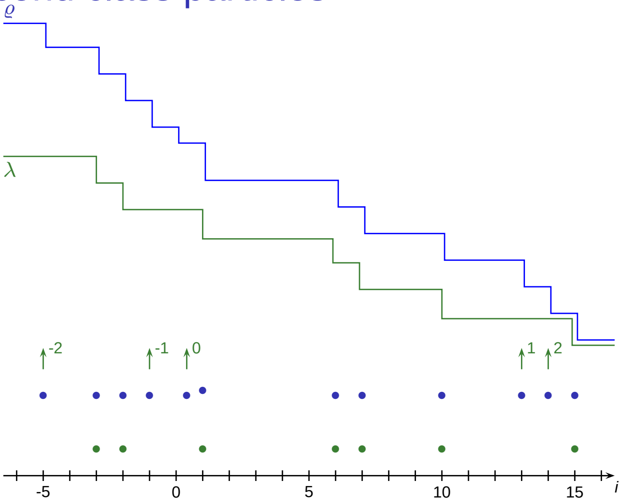

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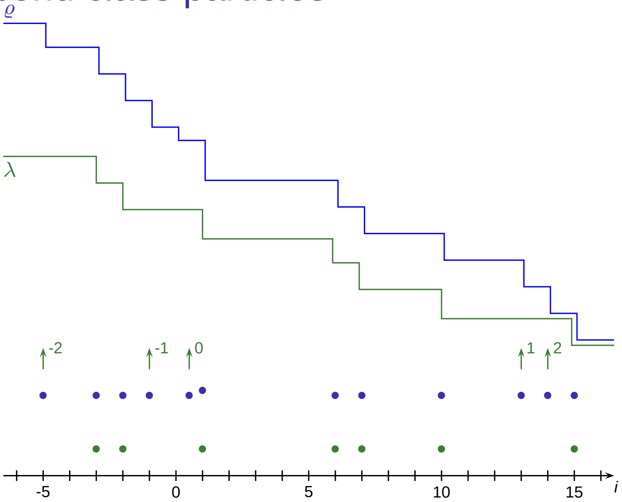

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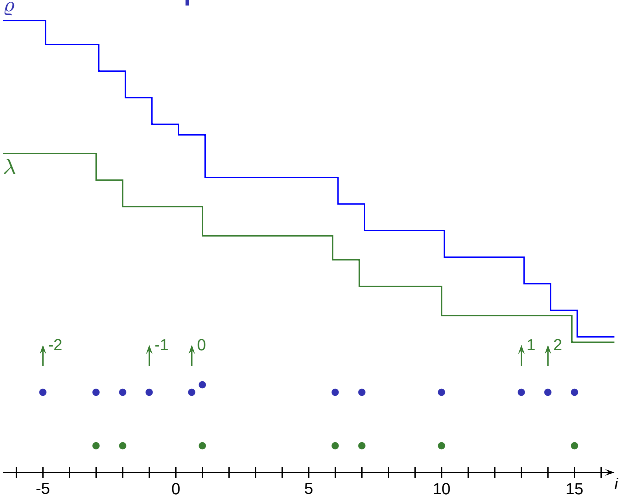

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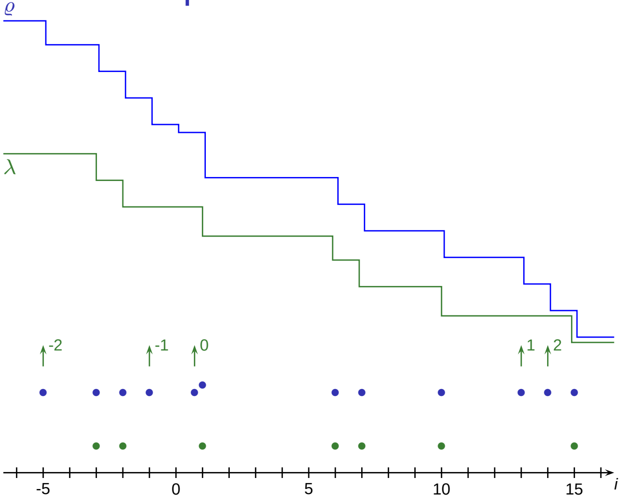

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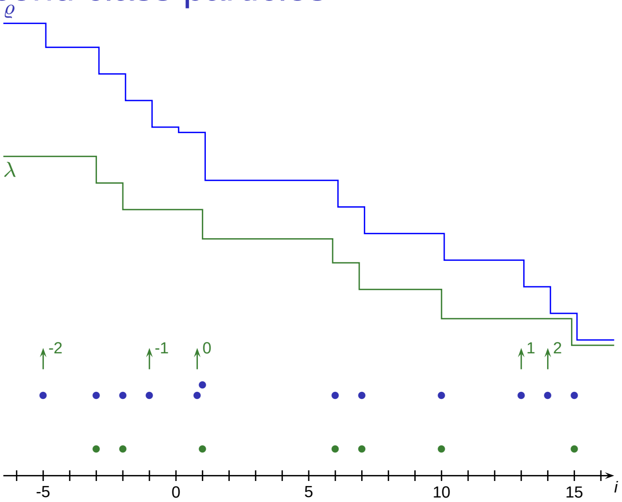

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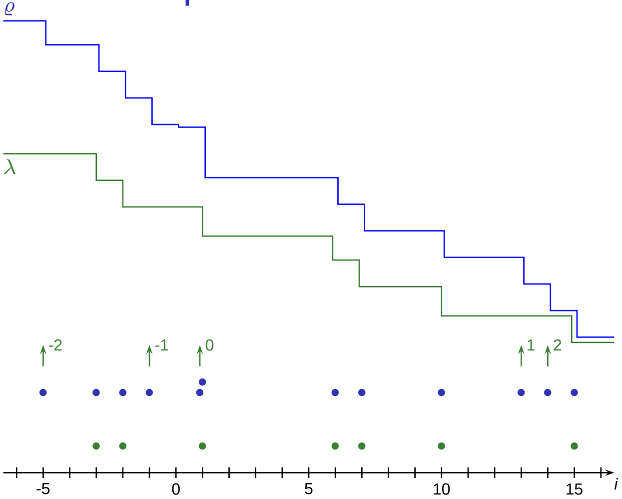

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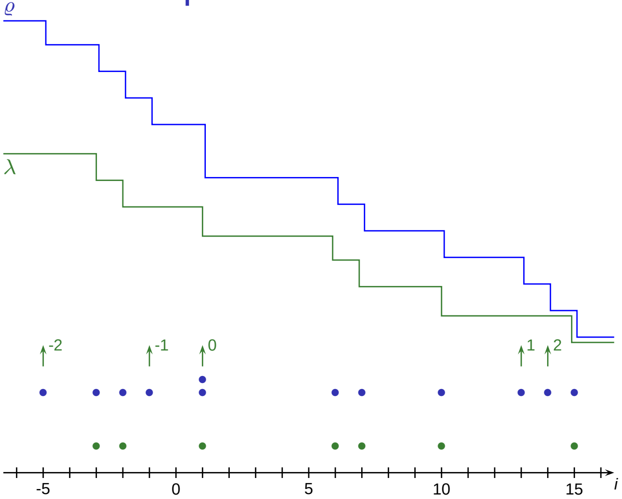

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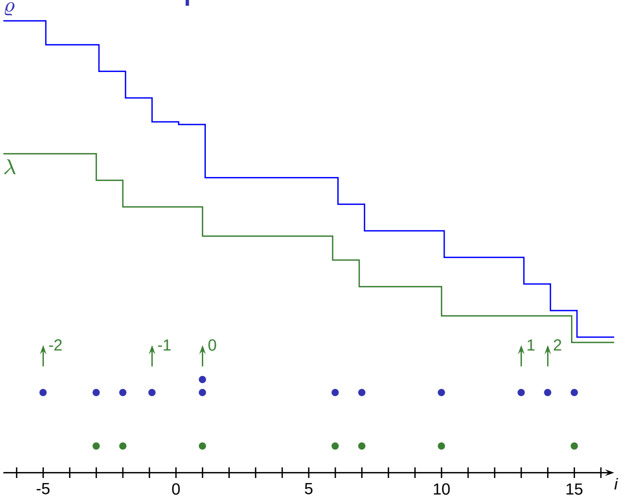

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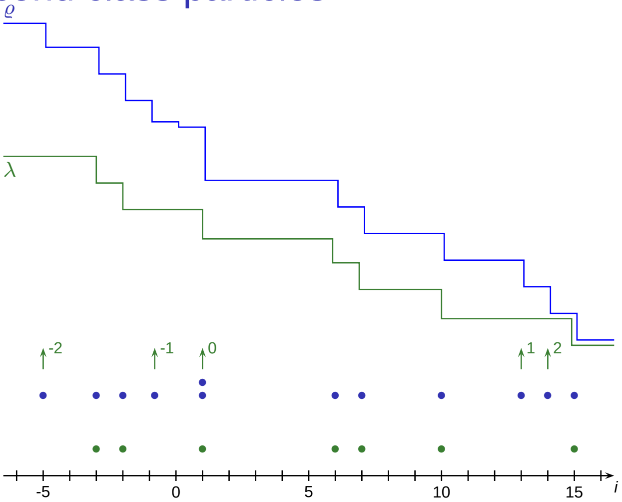

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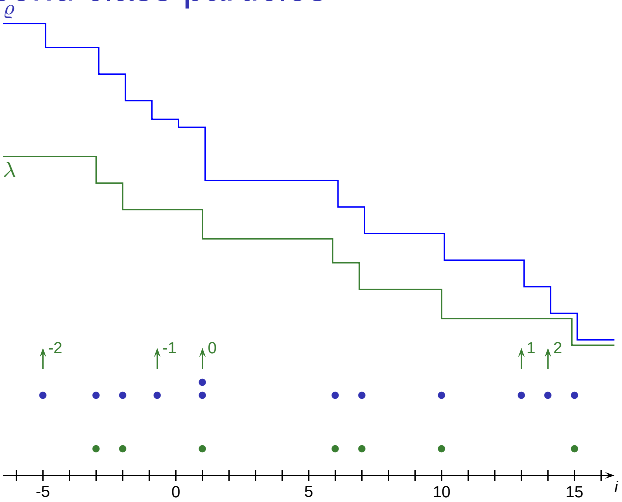

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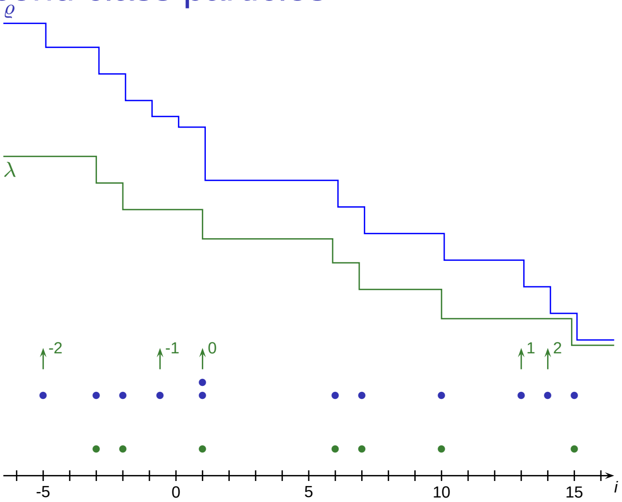

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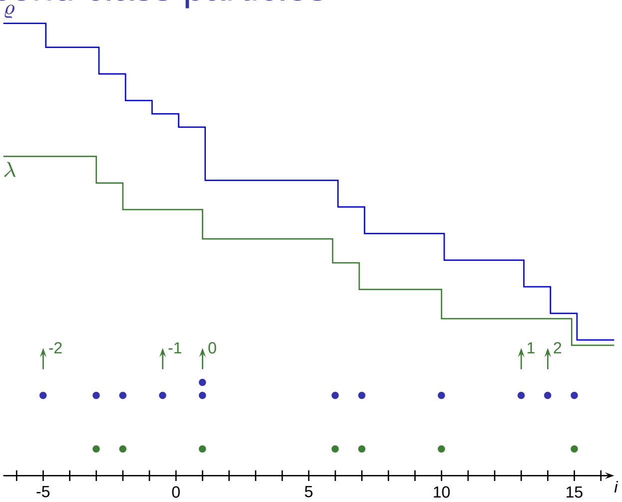

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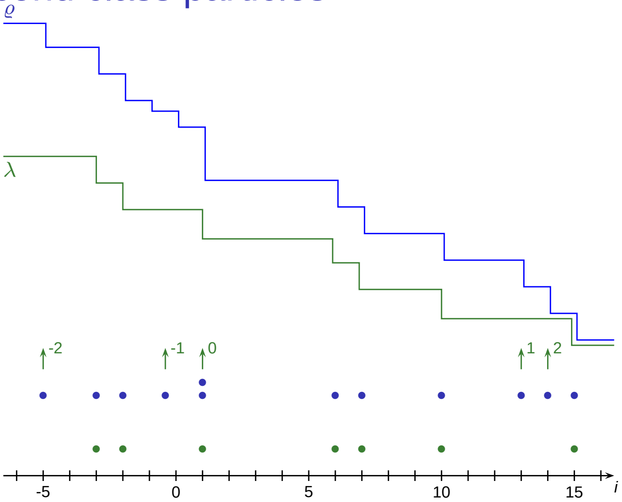

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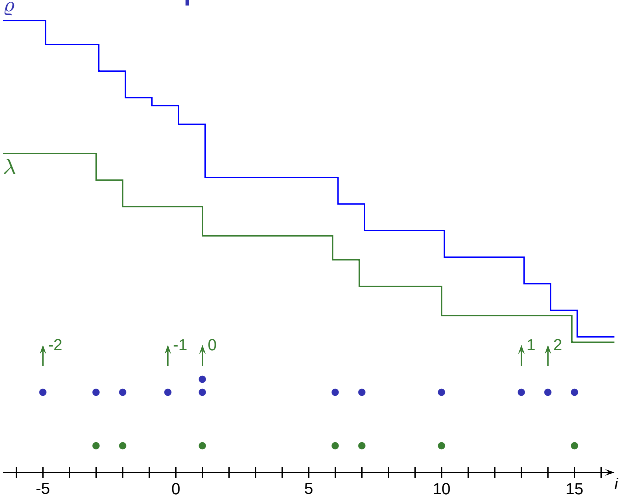

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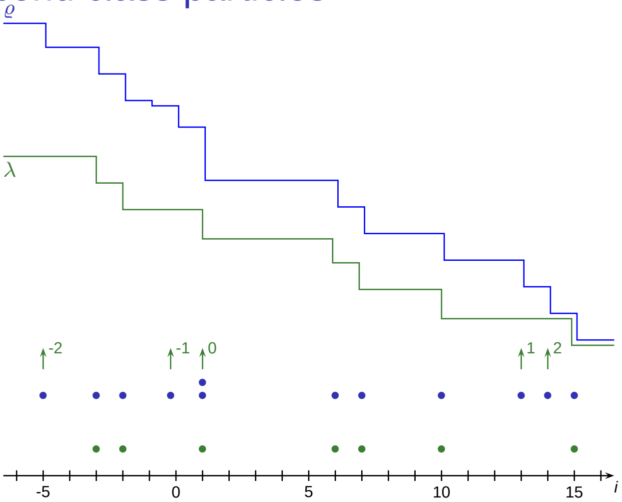

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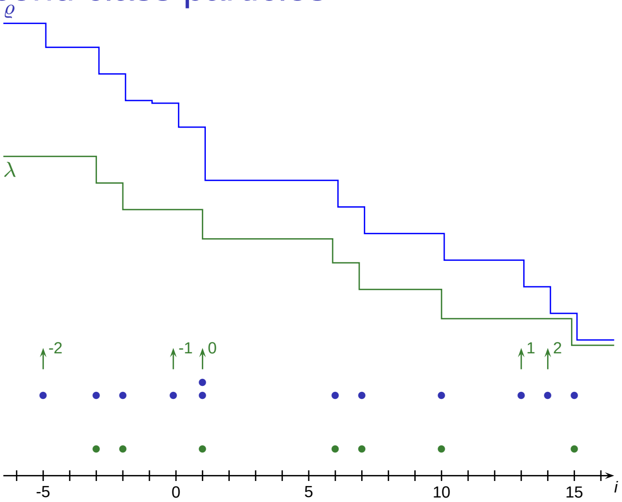

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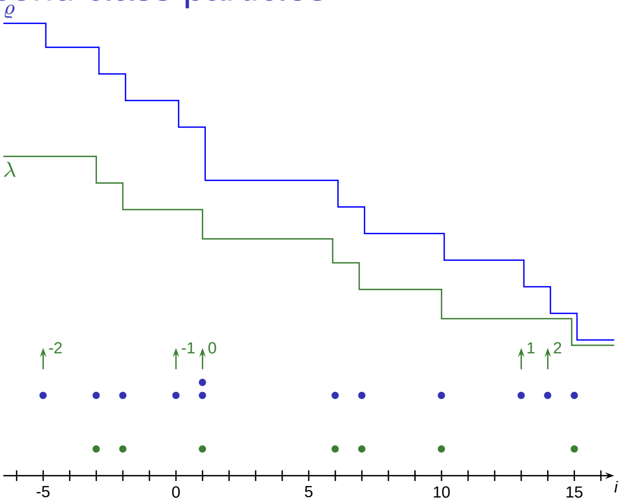

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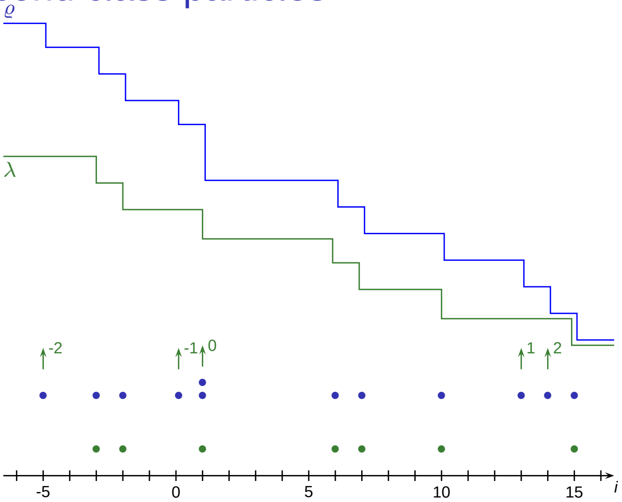

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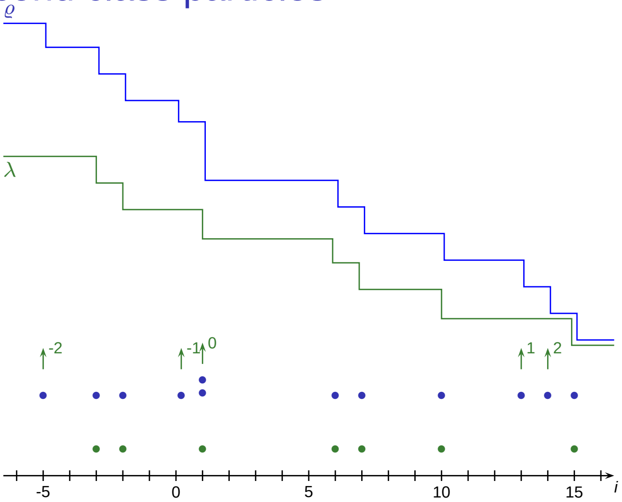

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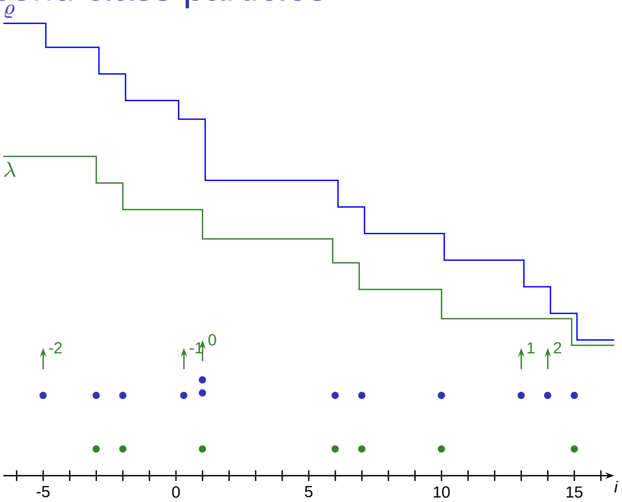

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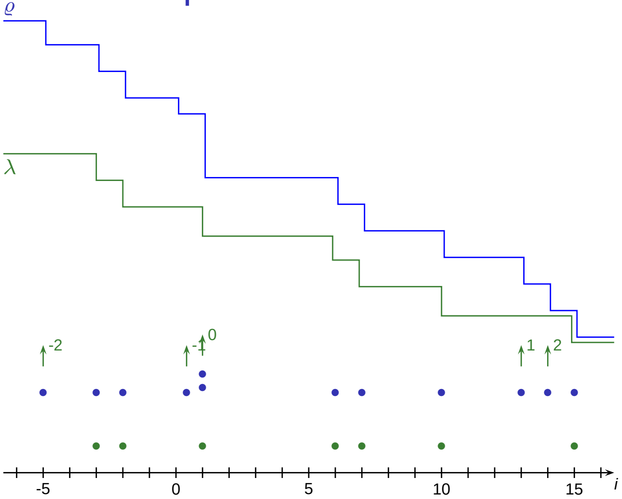

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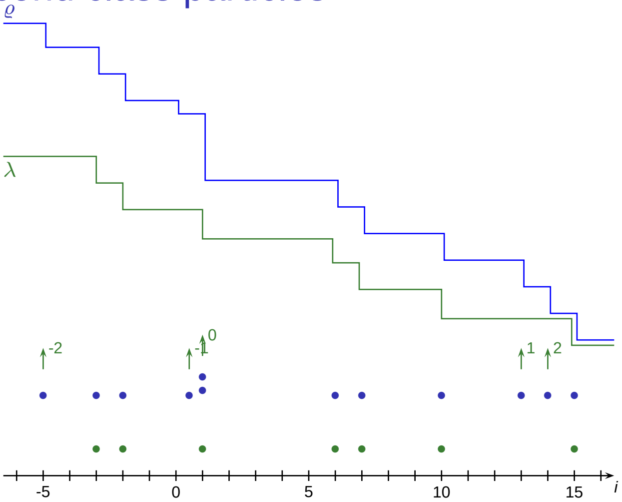

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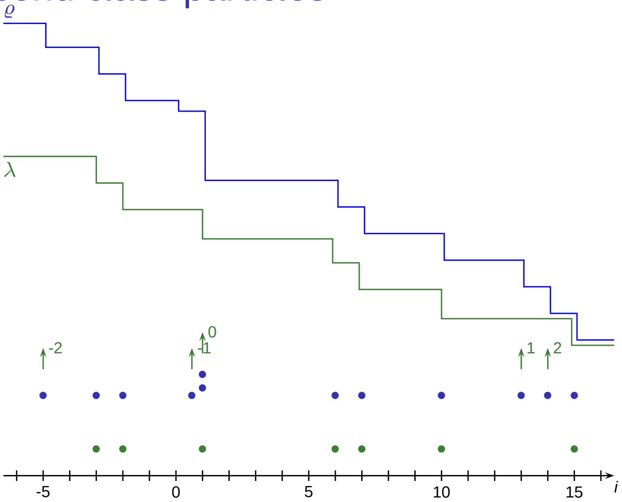

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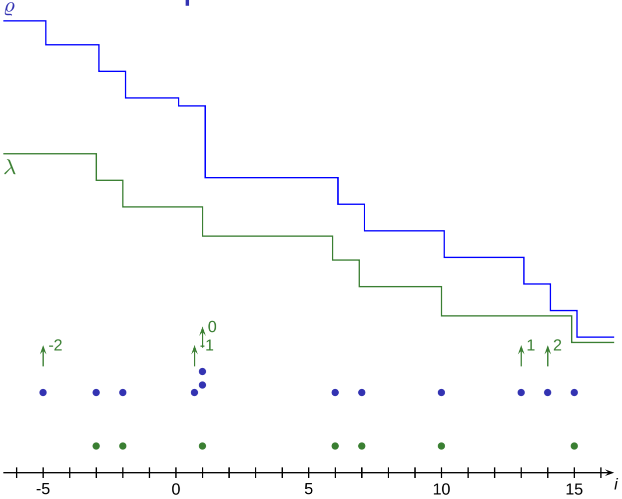

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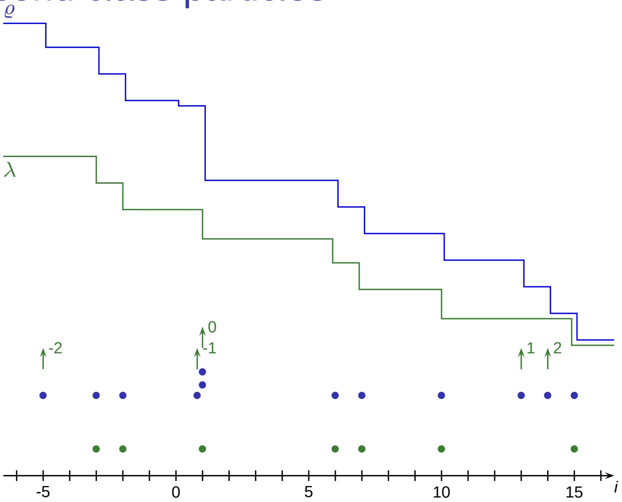

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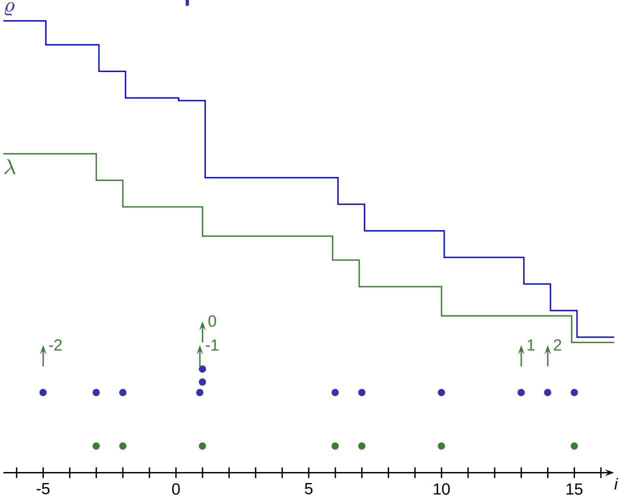

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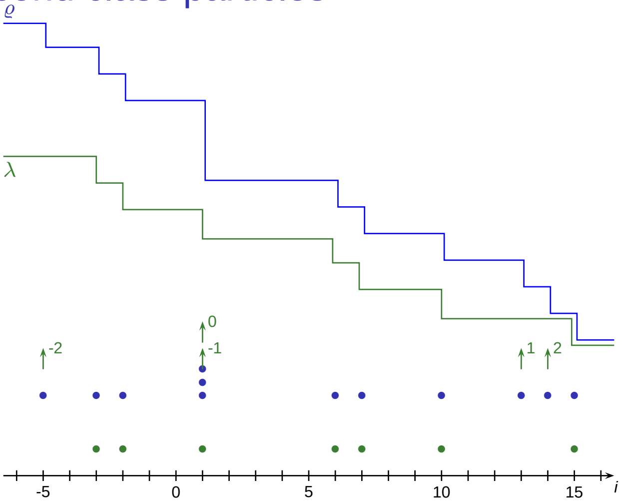

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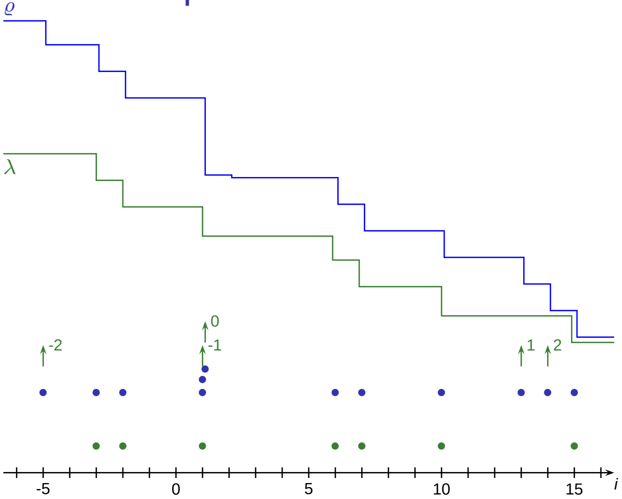

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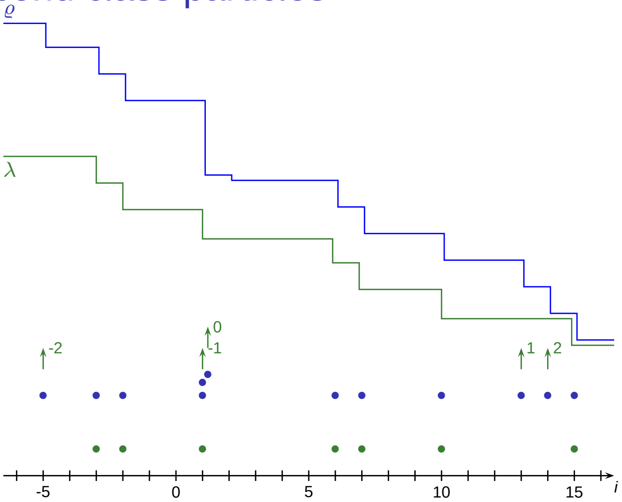

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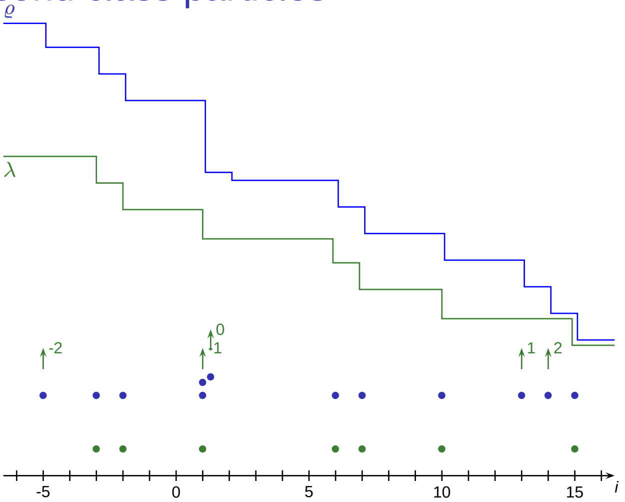

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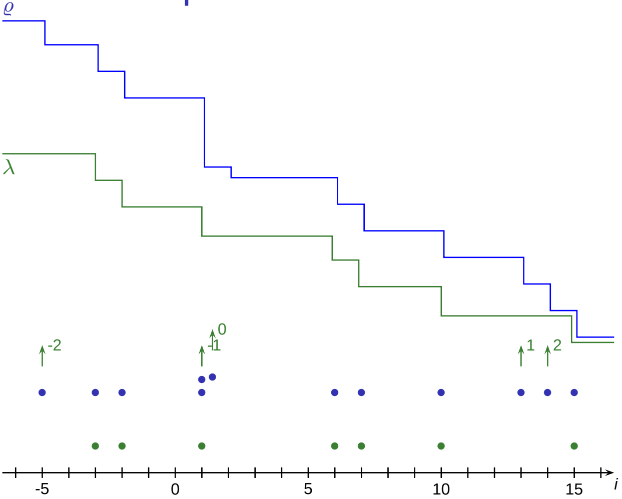

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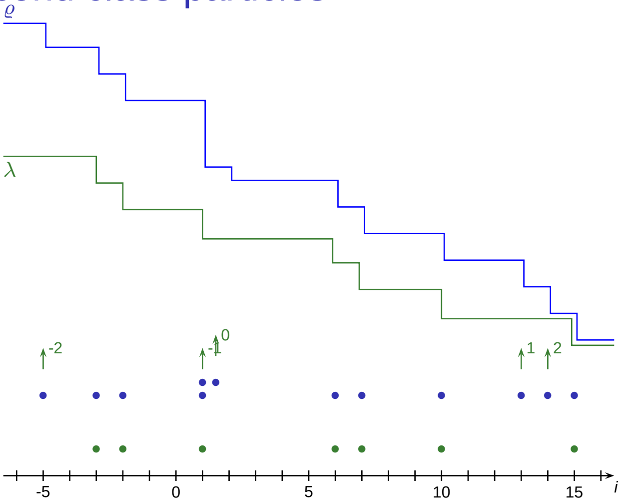

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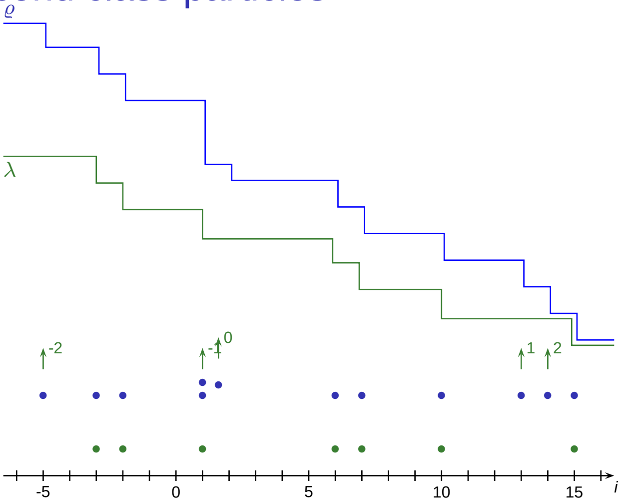

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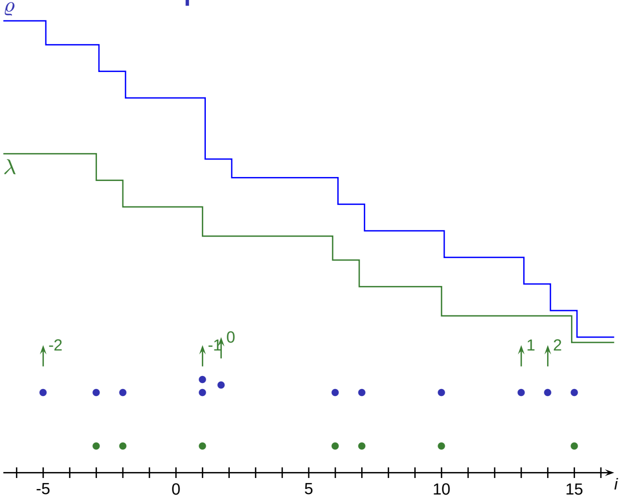

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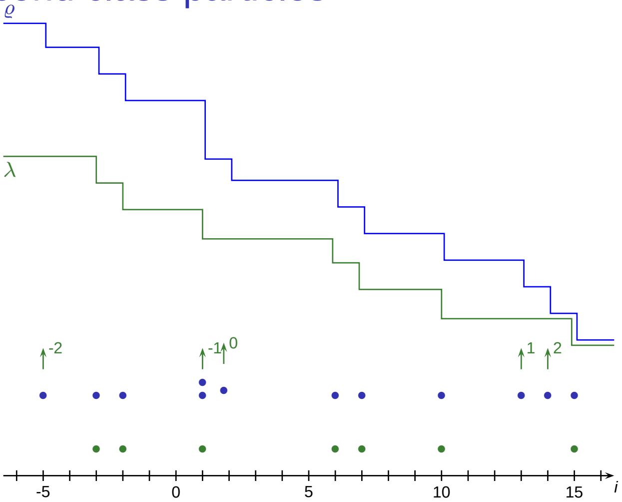

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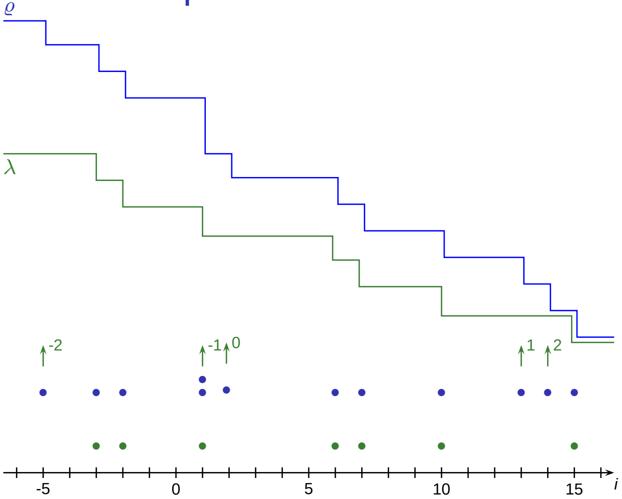

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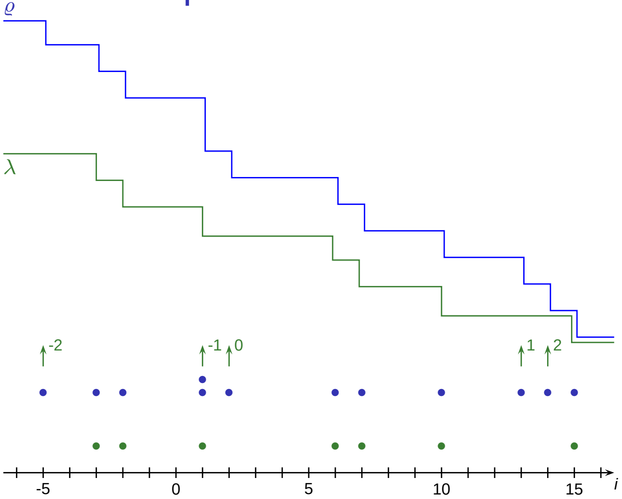

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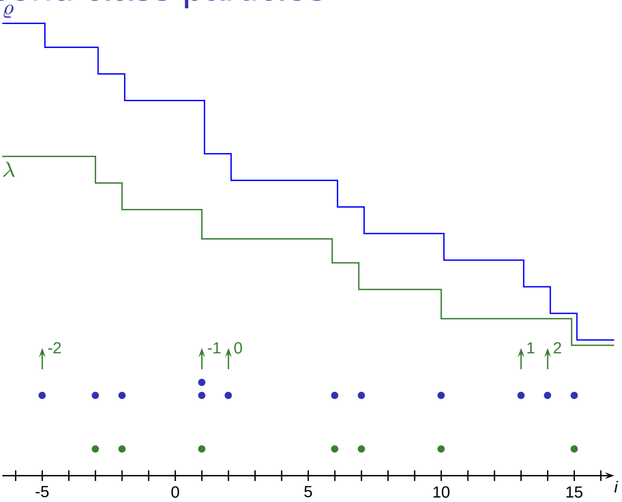

Picture:

The position $X(t)$ of $\uparrow^0$ follows the Rankine-Hugoniot speed R.

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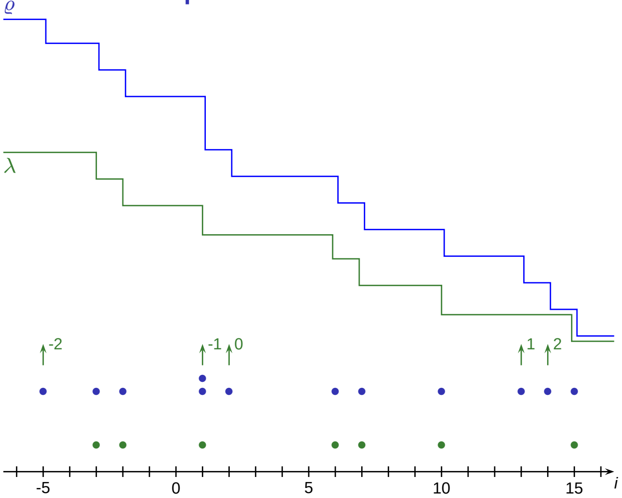

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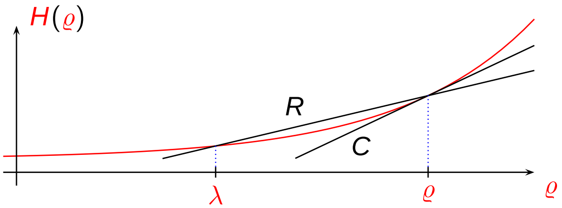
Characteristics (very briefly)

Convex flux (some cases of AZRP, ABLP):



Recall $C = H'(\varrho) > R = \dfrac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$

$C = H'(\varrho) = \mathbf{E}Q/t$ $\mathbf{E}X/t = R = [H(\varrho) - H(\lambda)]/(\varrho - \lambda)$

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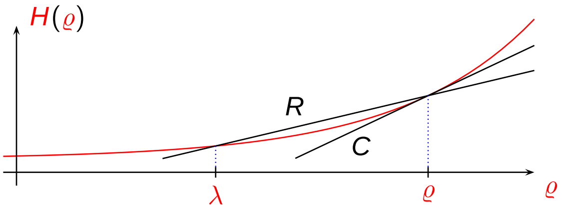

Recall $$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Do we have $$Q(t) \overset{?}{\geq} X(t)$$

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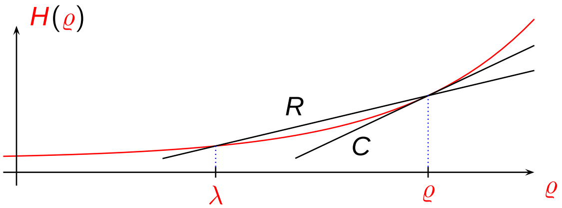

Recall $$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Do we have $$Q(t) \overset{?}{\geq} X(t) - \text{tight error}$$

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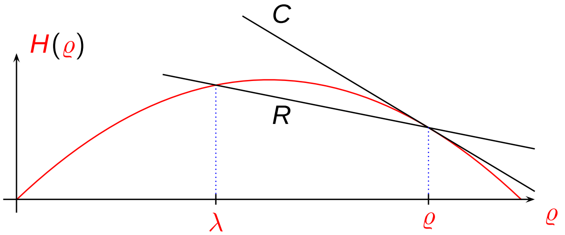
Characteristics (very briefly)

Concave flux (ASEP, AZRP):



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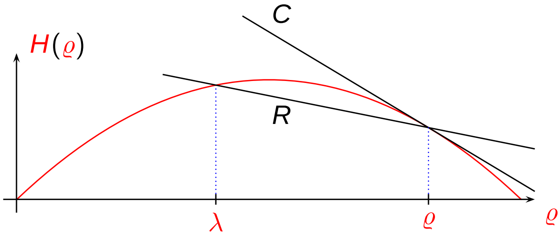

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Do we have $\quad Q(t) \overset{?}{\leq} X(t)$

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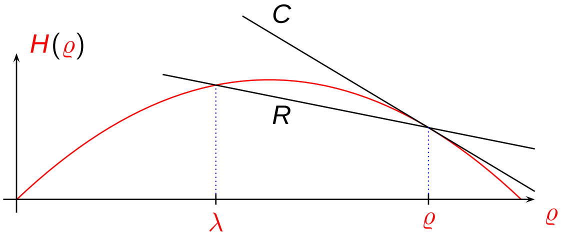

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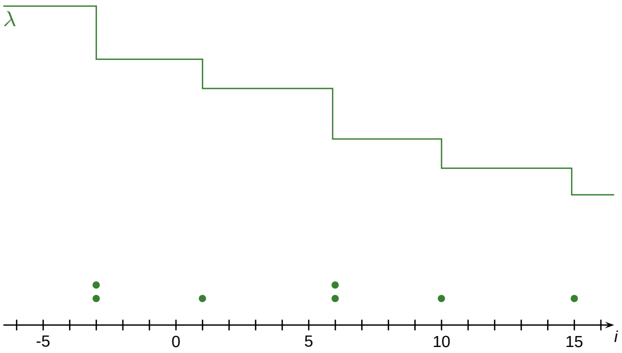
Many second class particles



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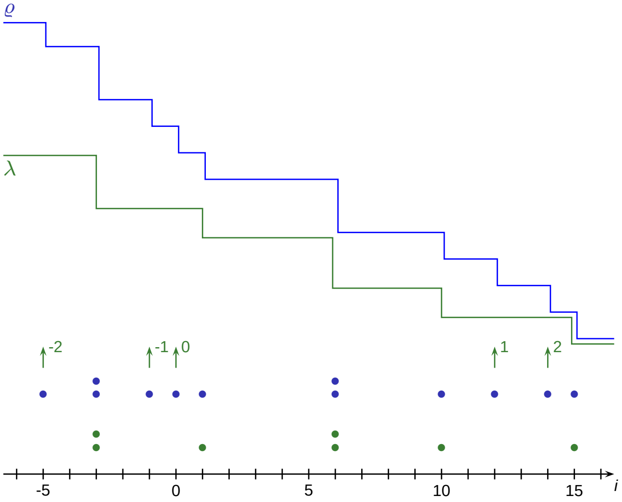

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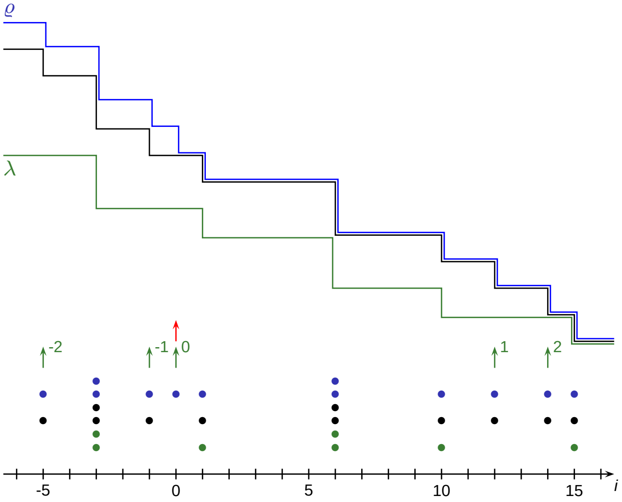
Many second class particles plus one



Couple three processes, and $X(t)$ to $Q(t)$.

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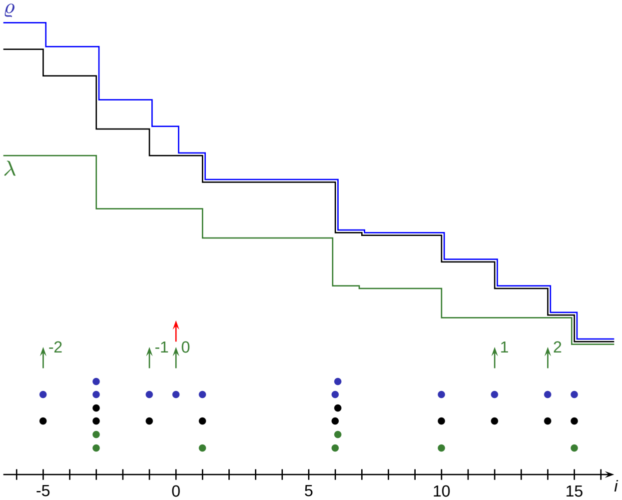

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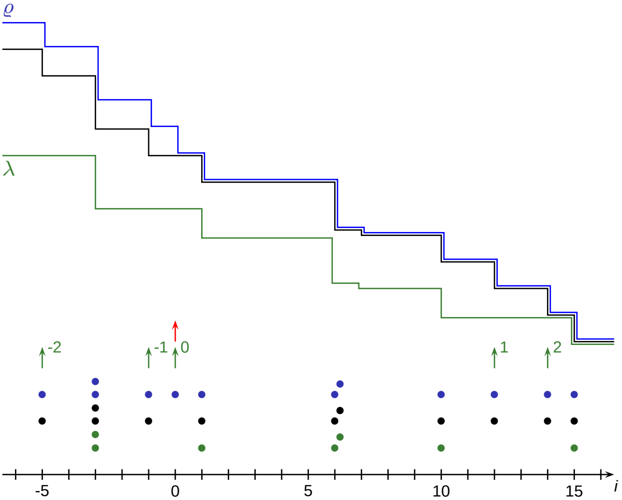

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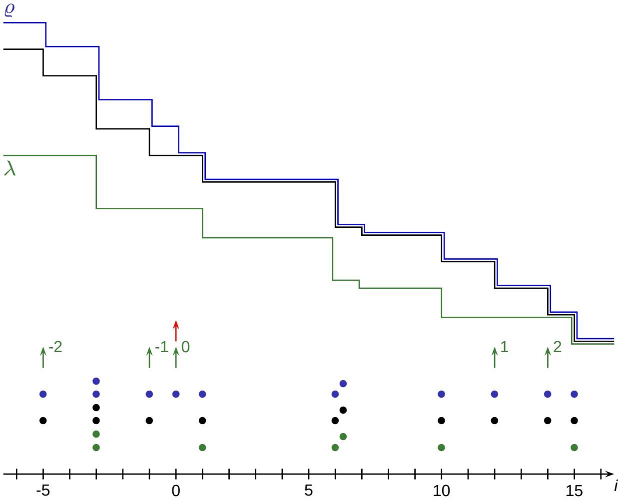

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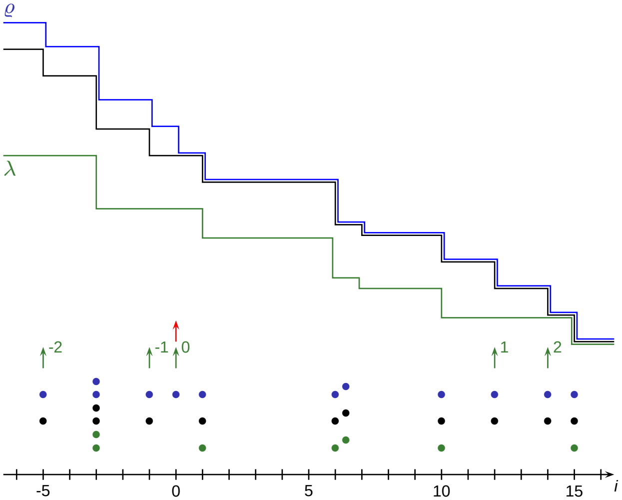

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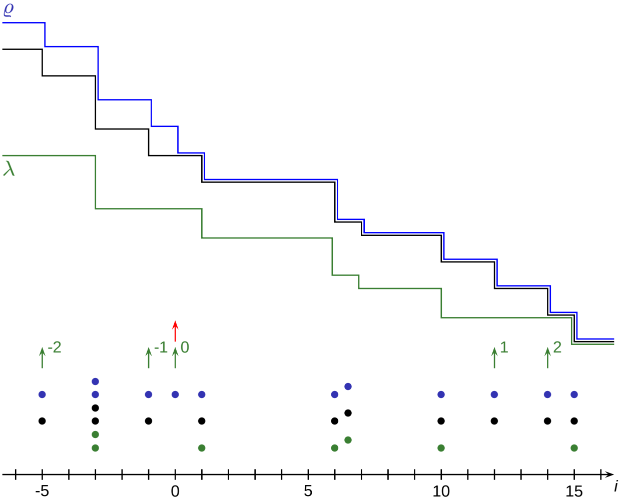

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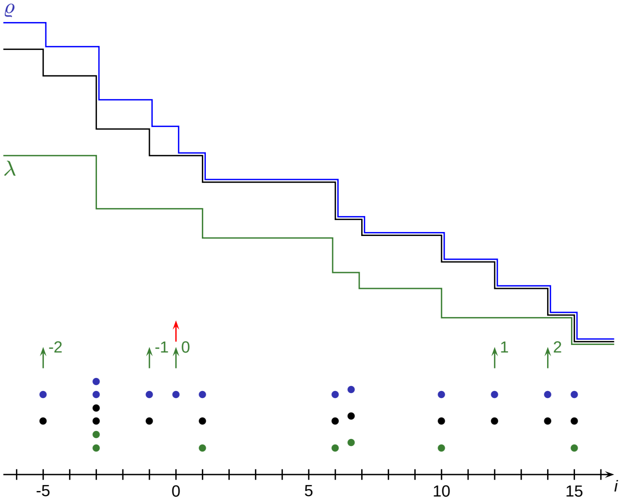

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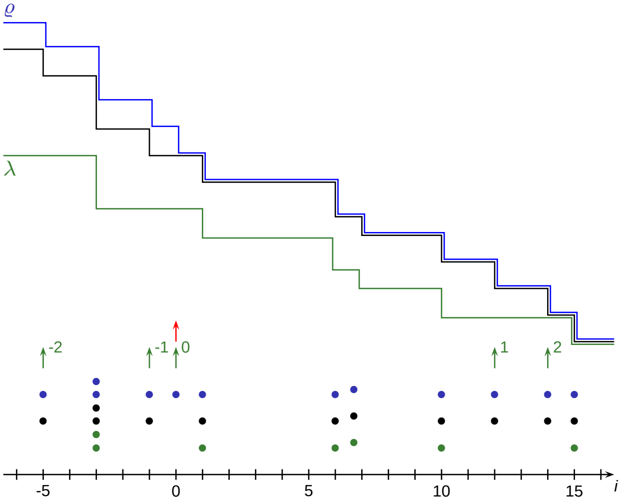

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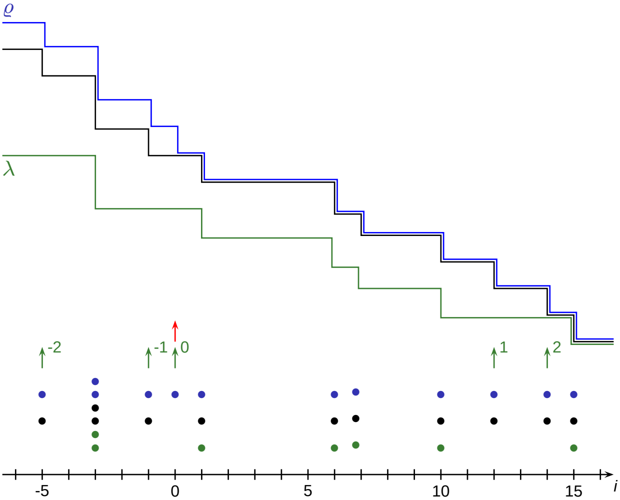

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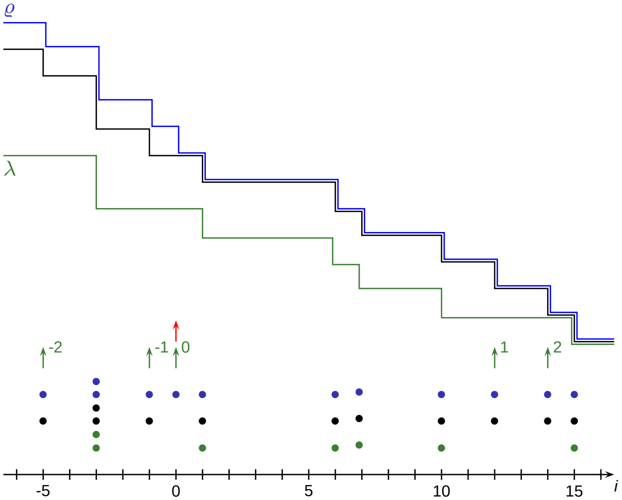

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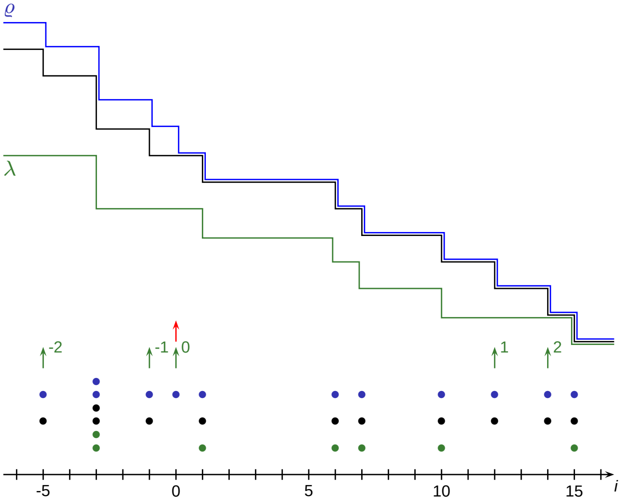

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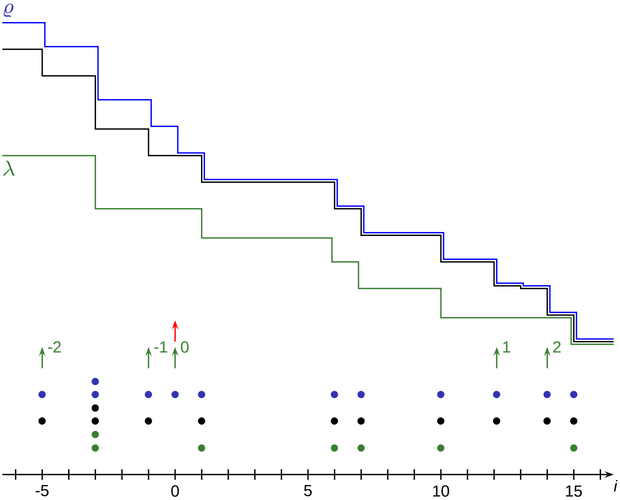

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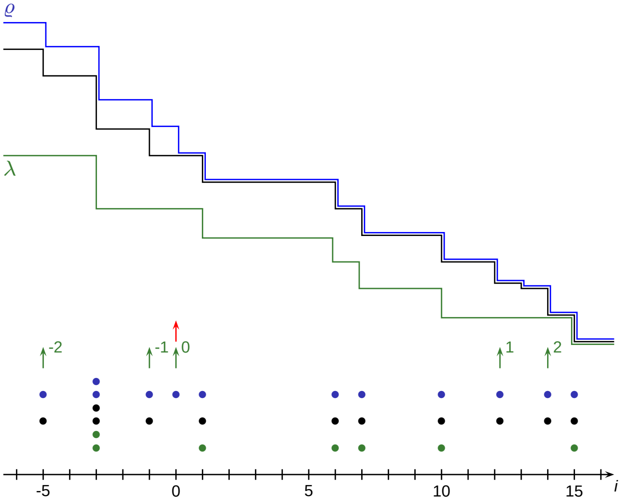

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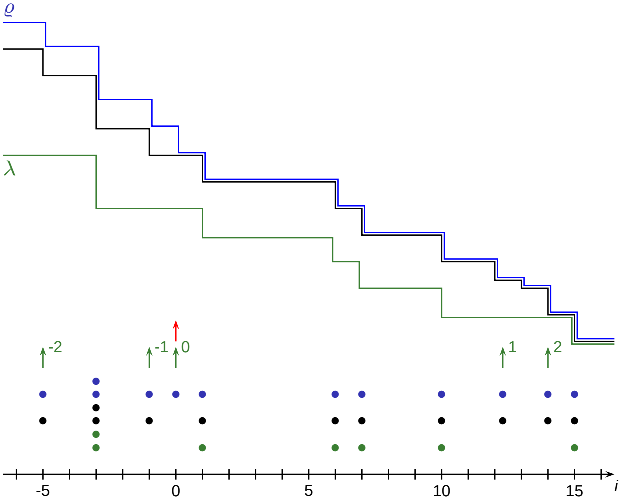

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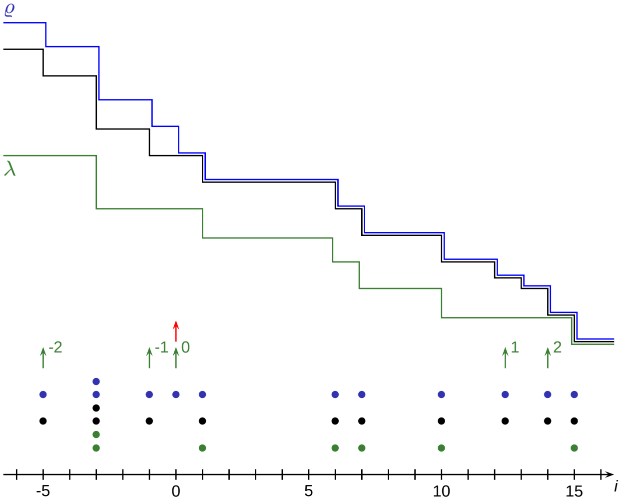

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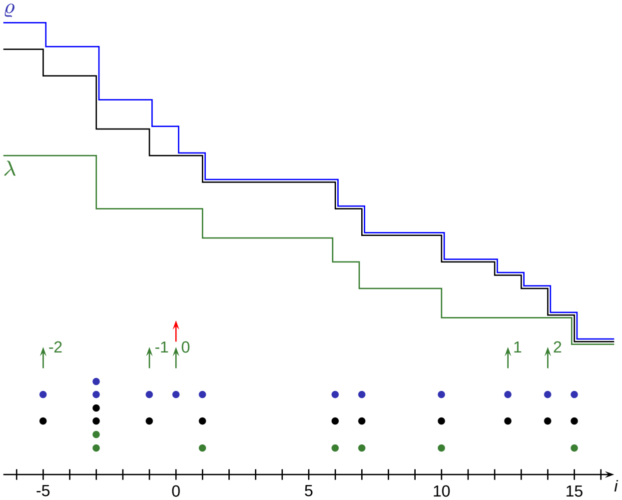

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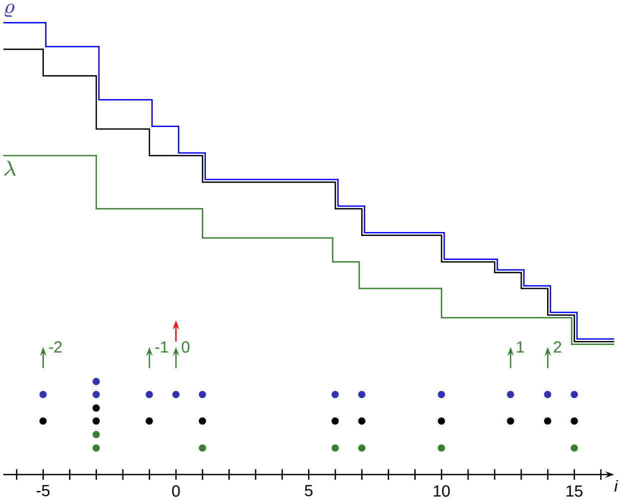

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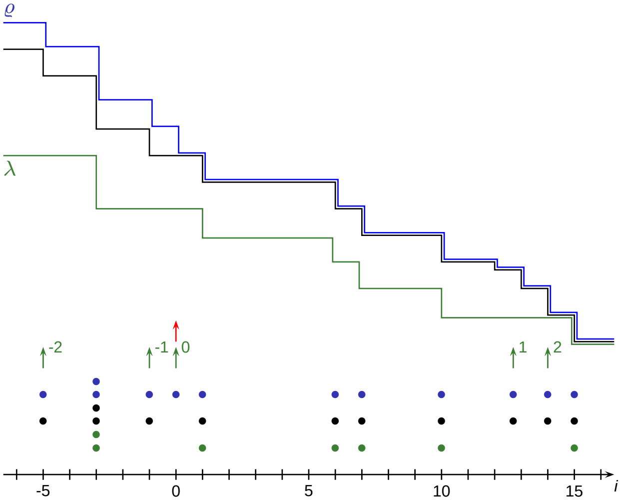

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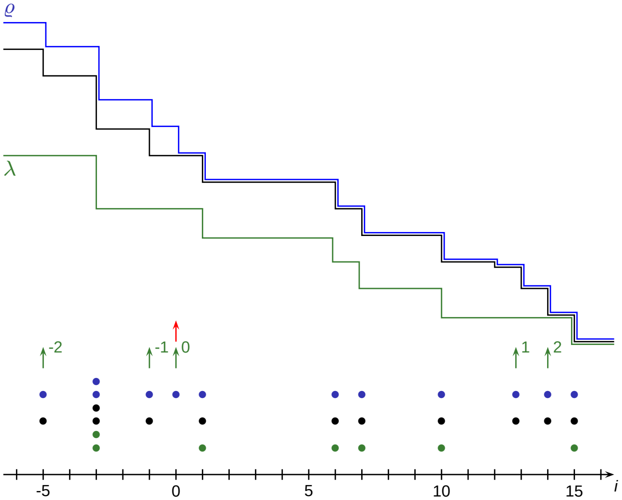

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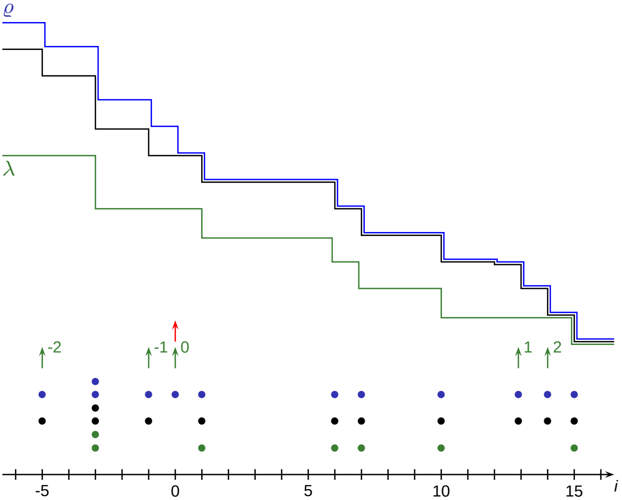

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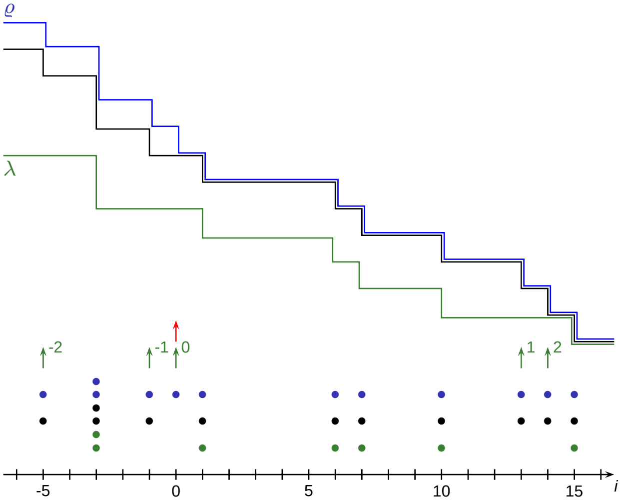

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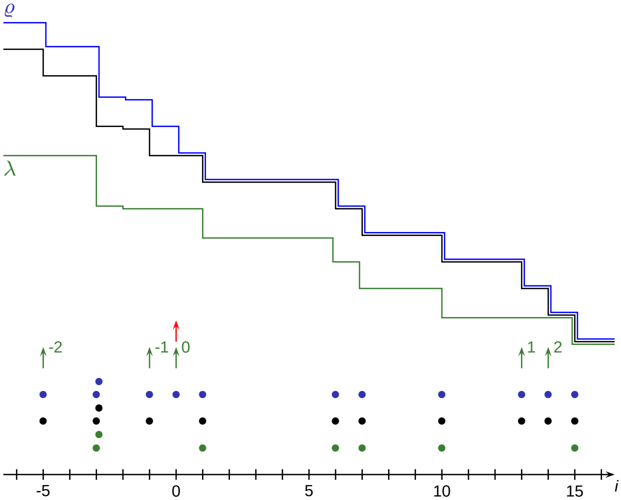

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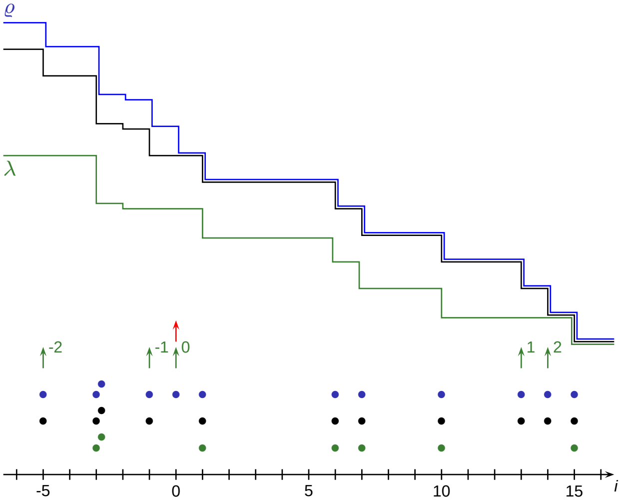

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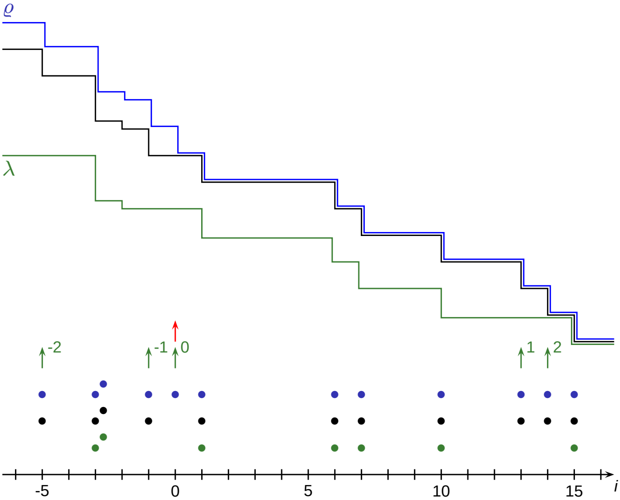

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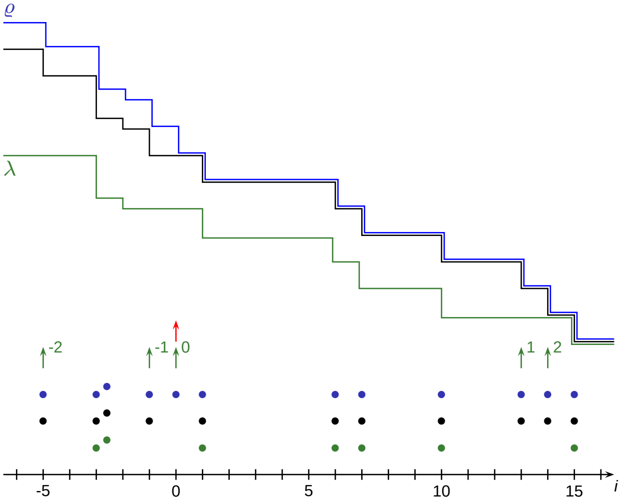

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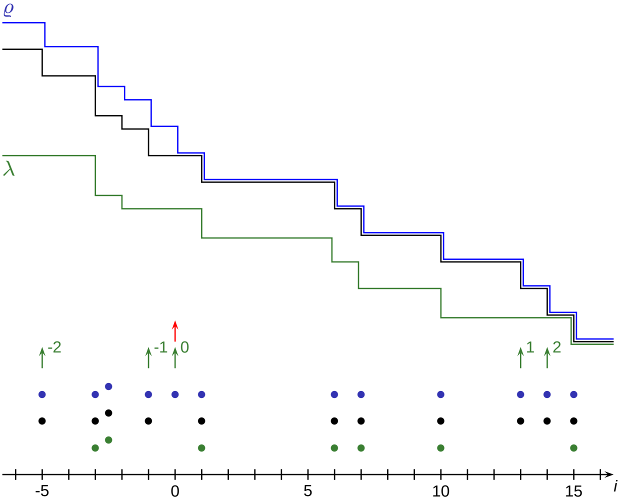

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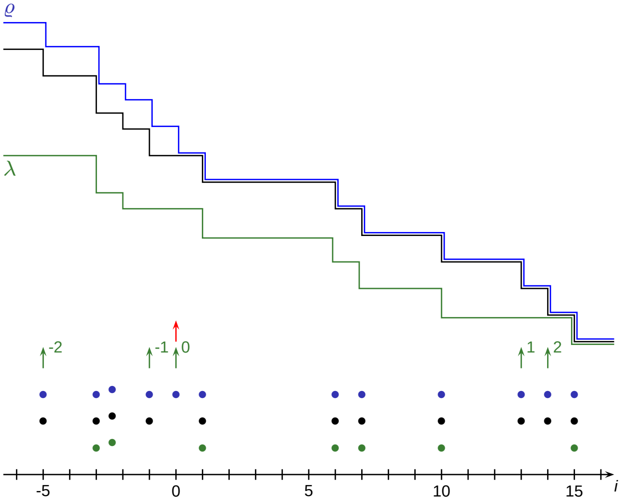

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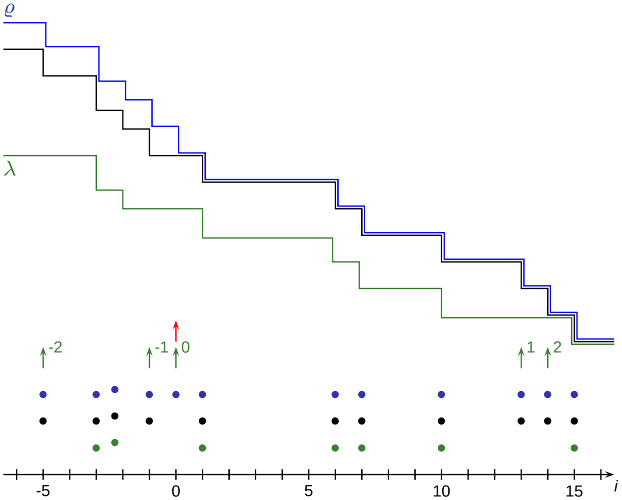

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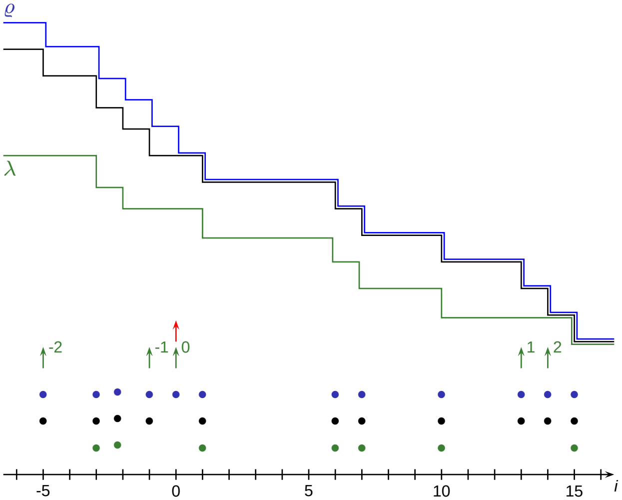

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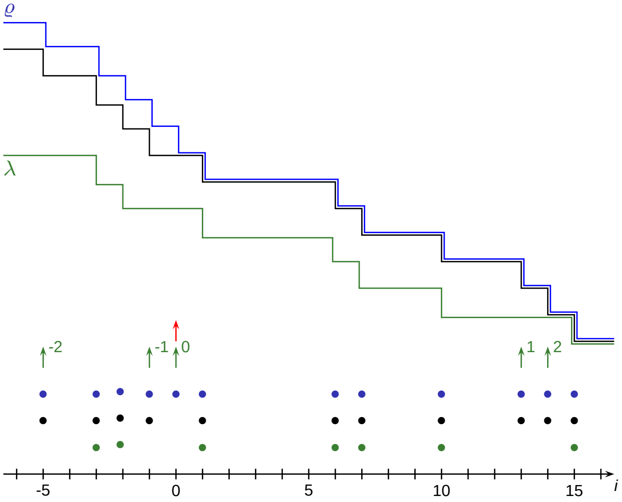

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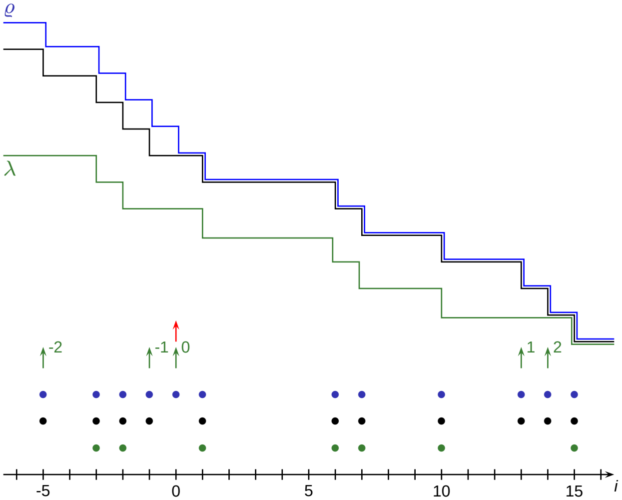

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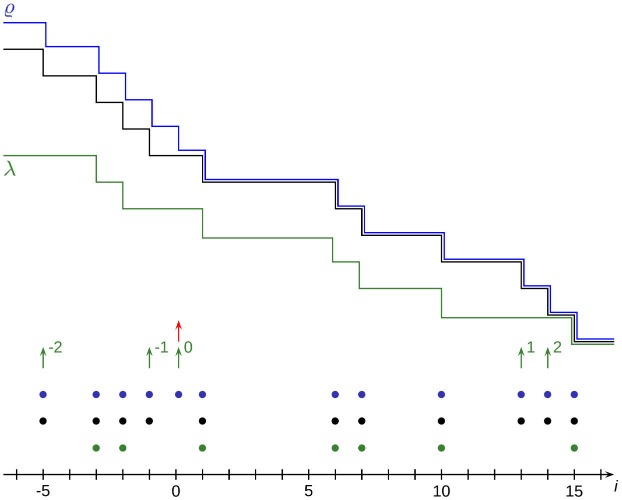

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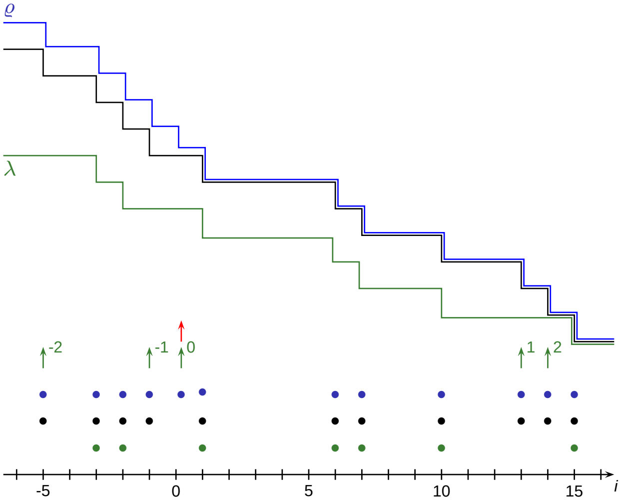

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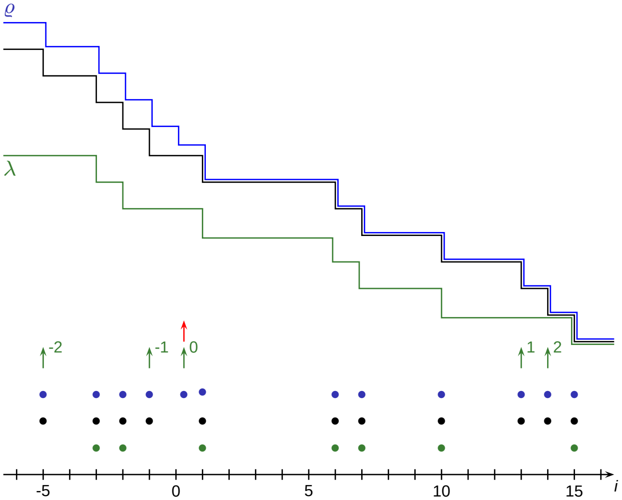

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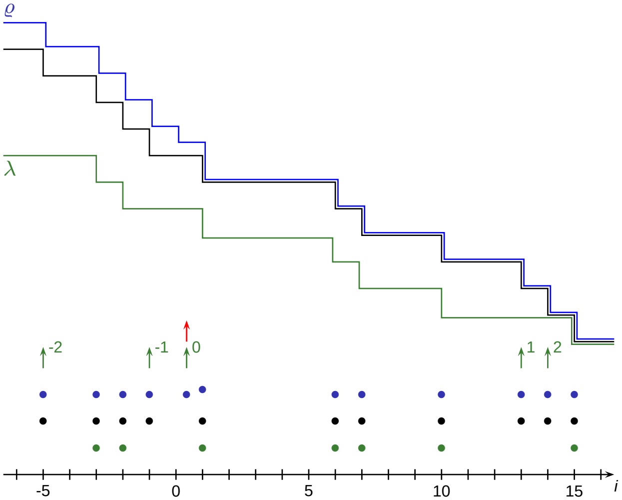

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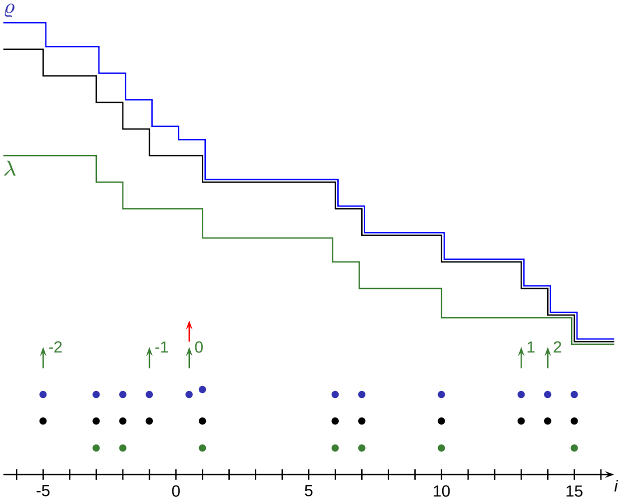

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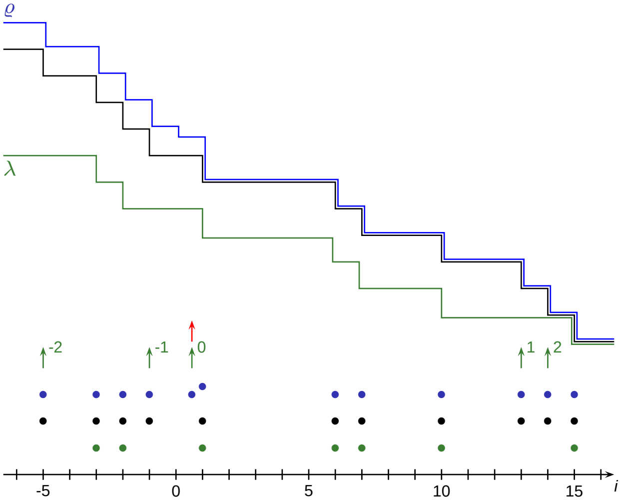

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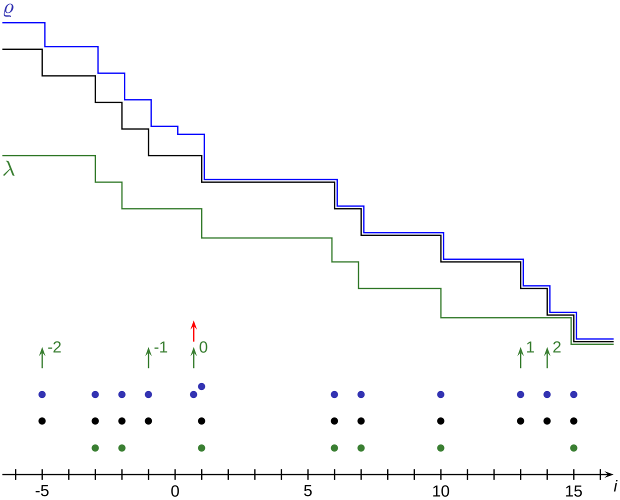

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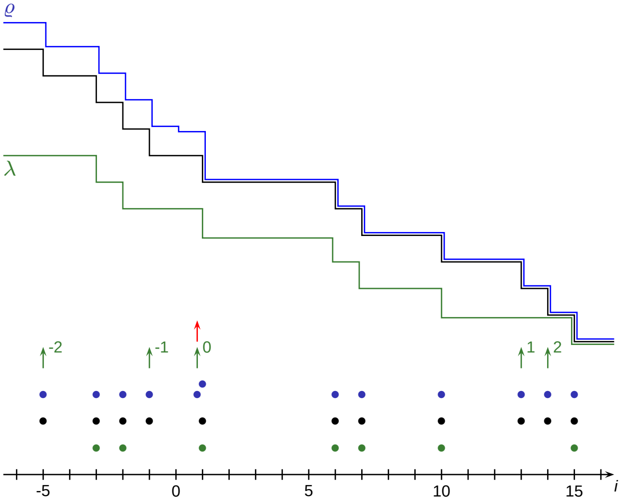

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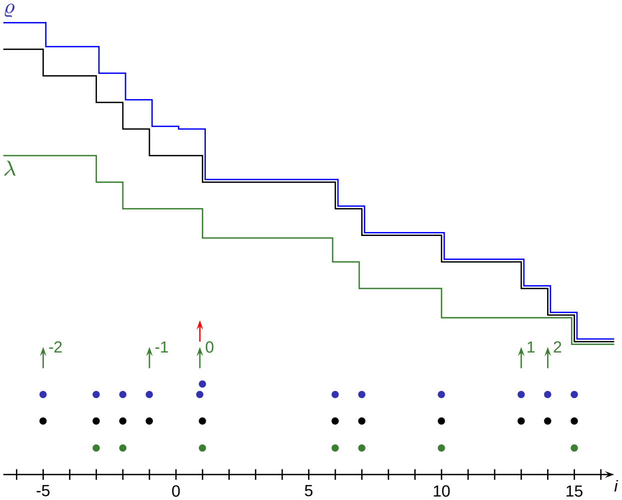

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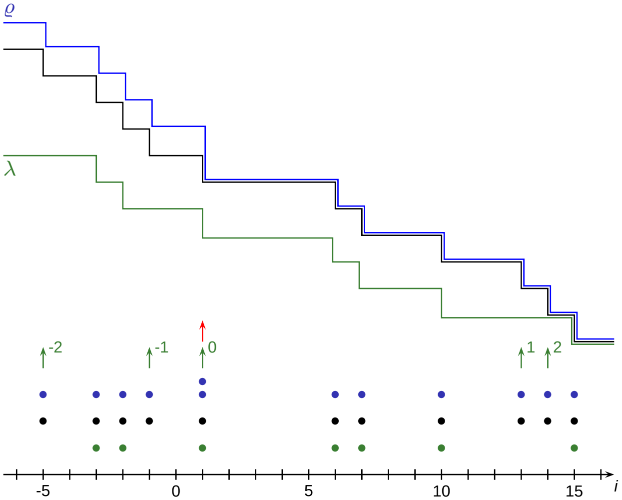

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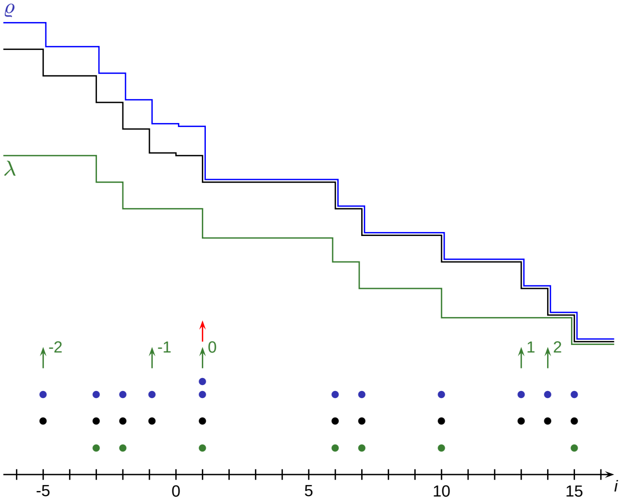

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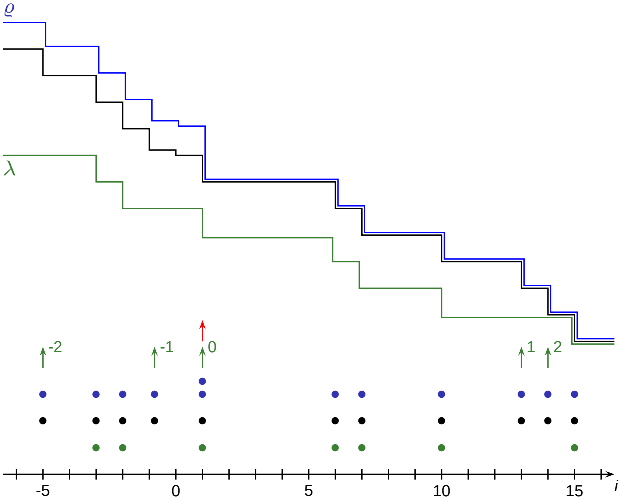

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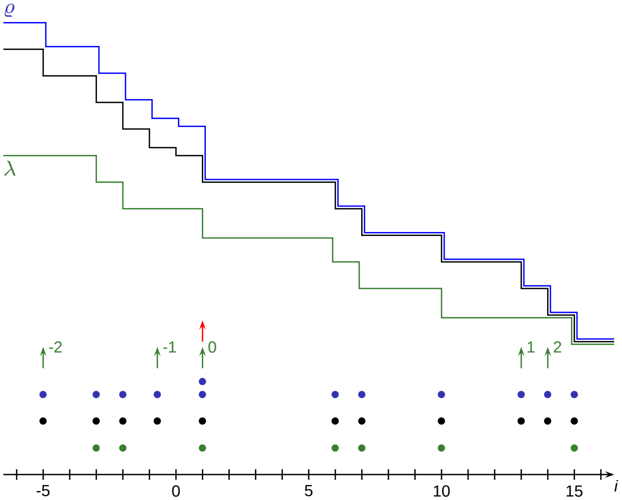

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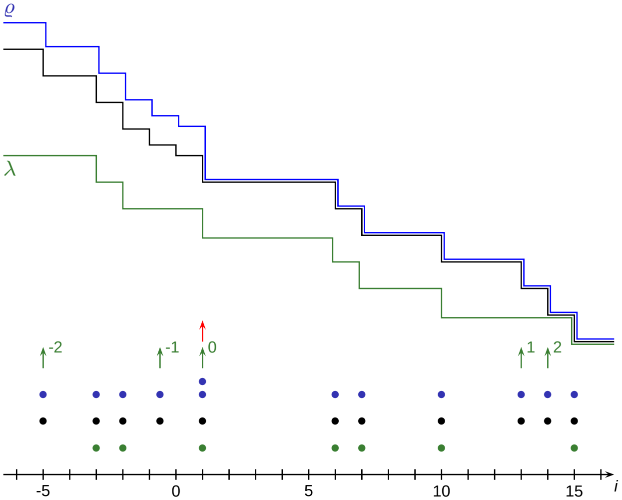

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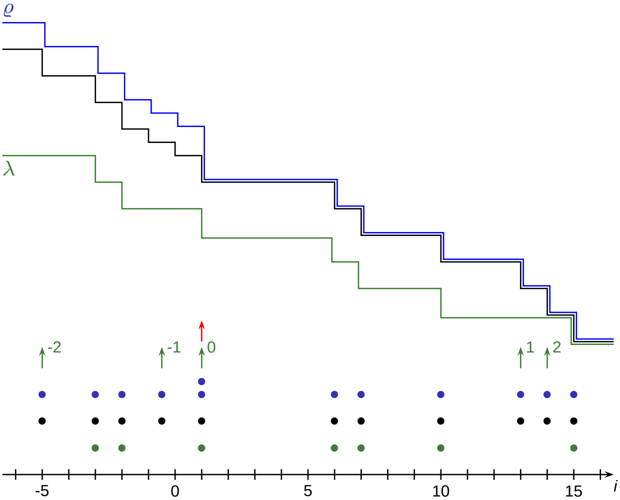

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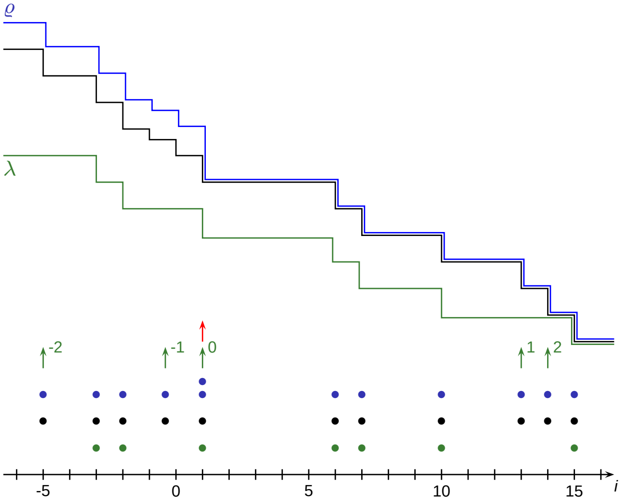

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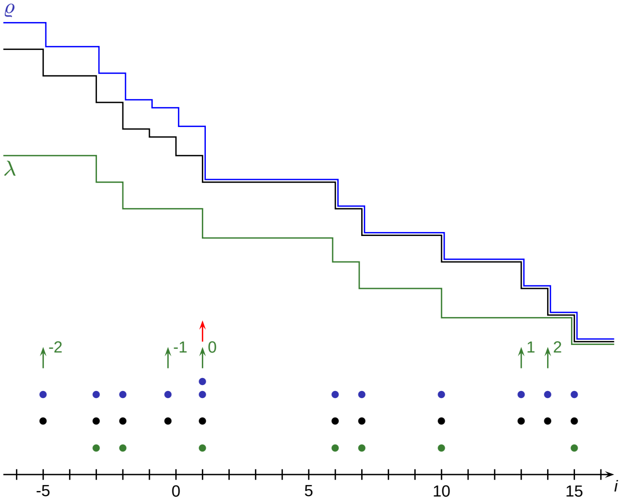

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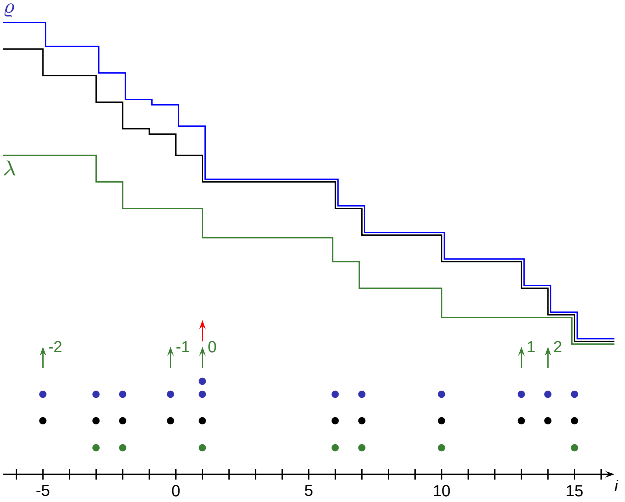

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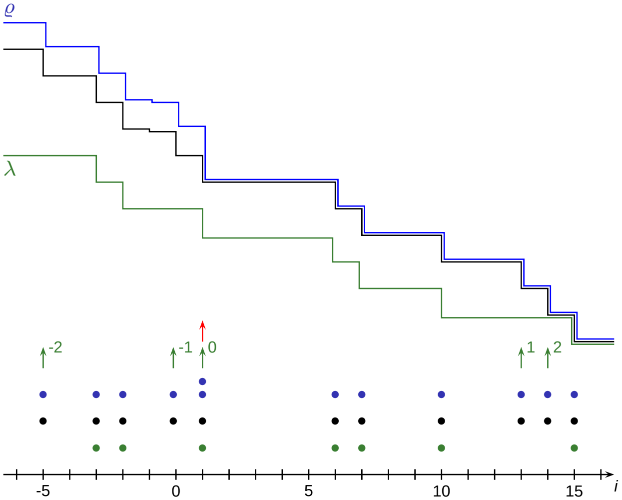

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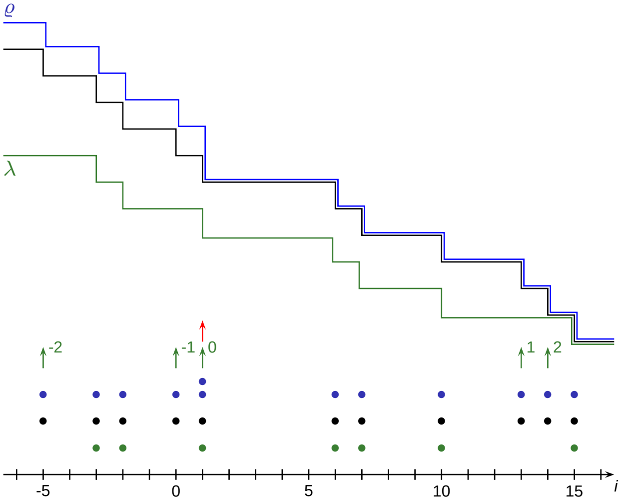

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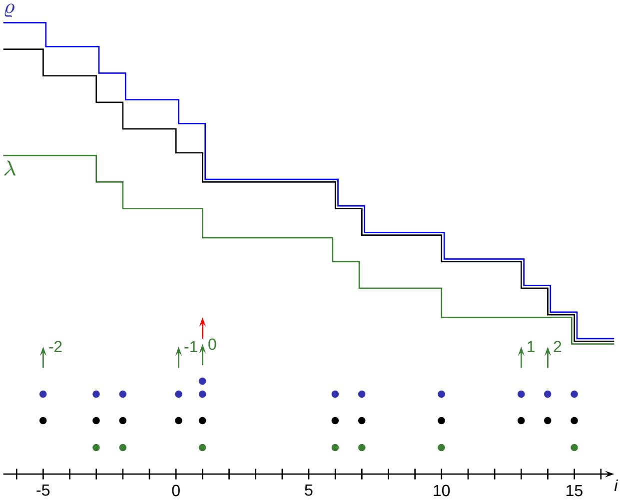

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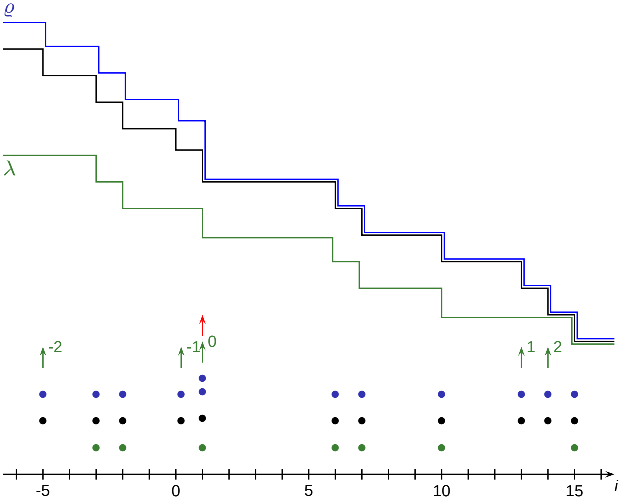

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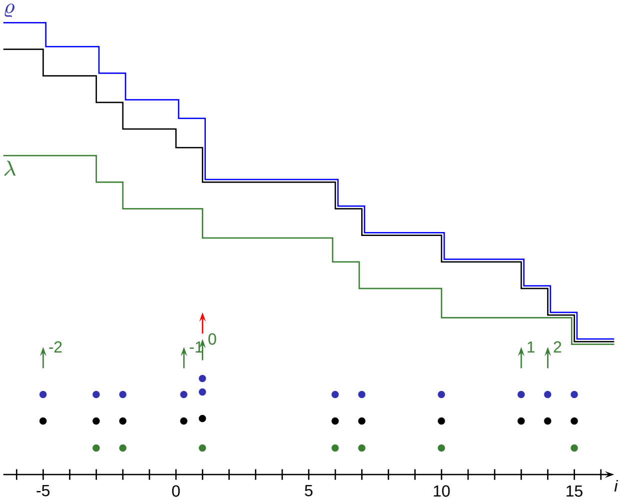

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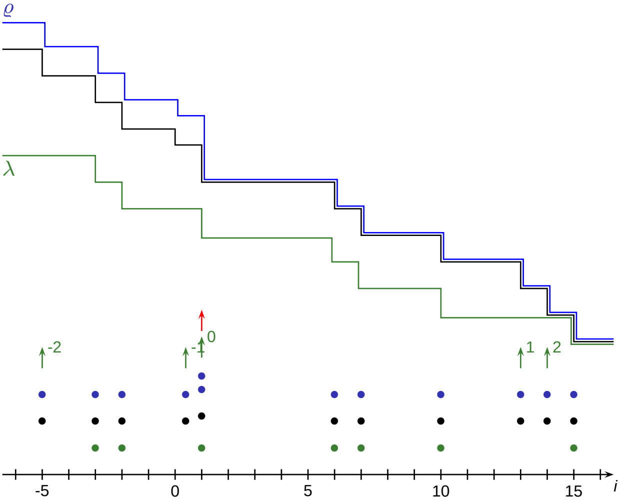

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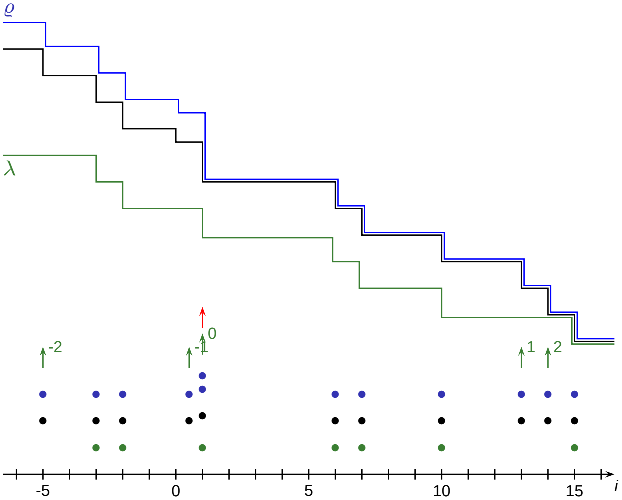

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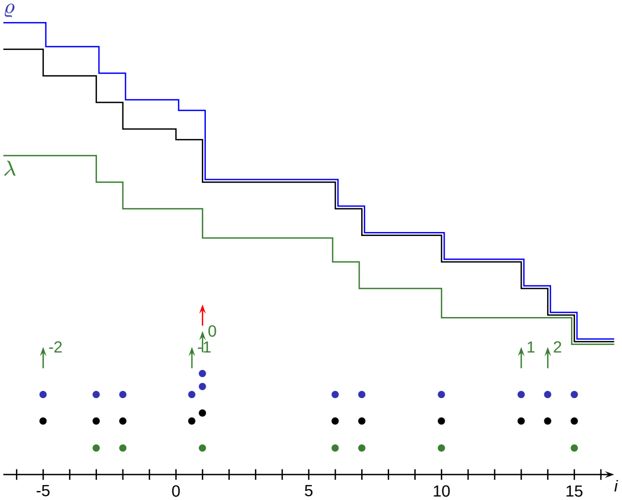

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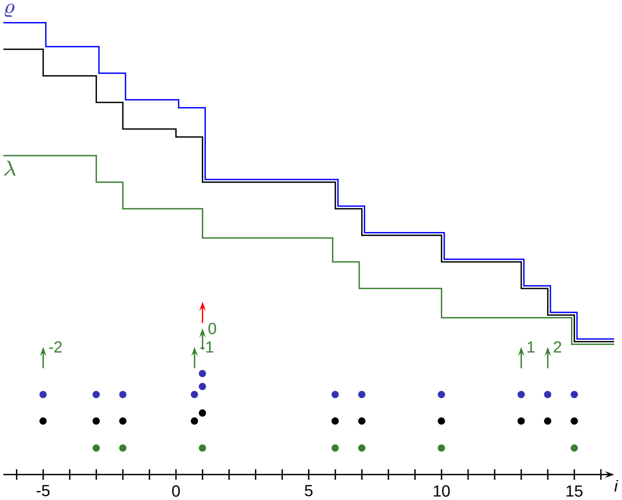

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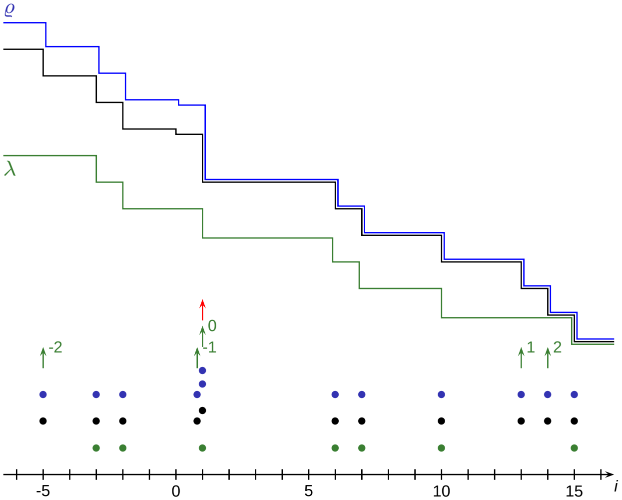

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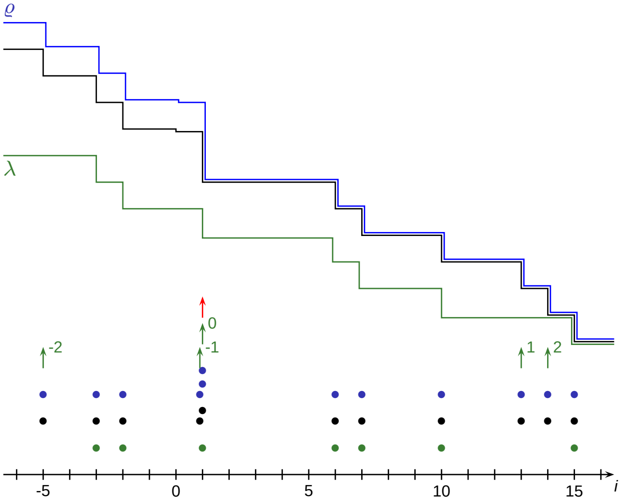

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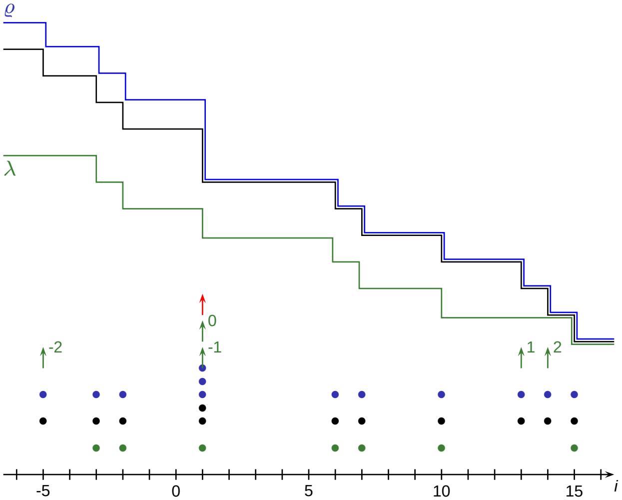

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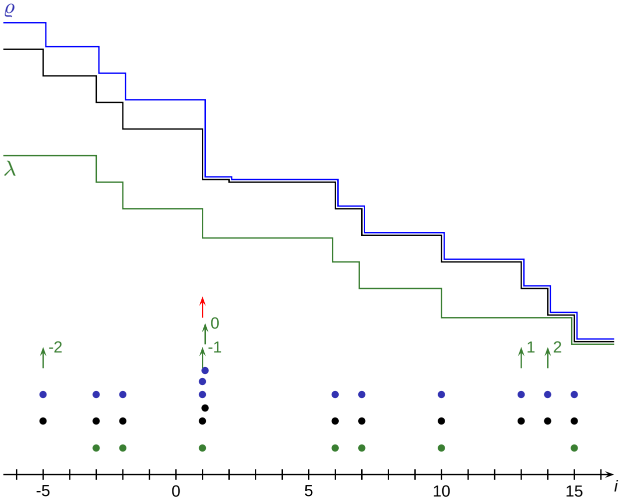

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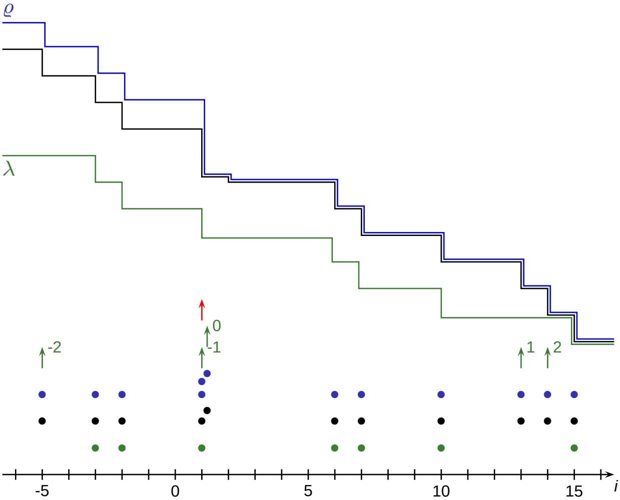

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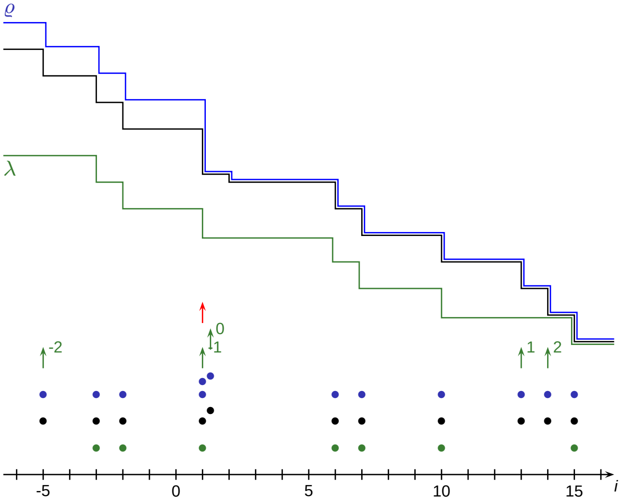

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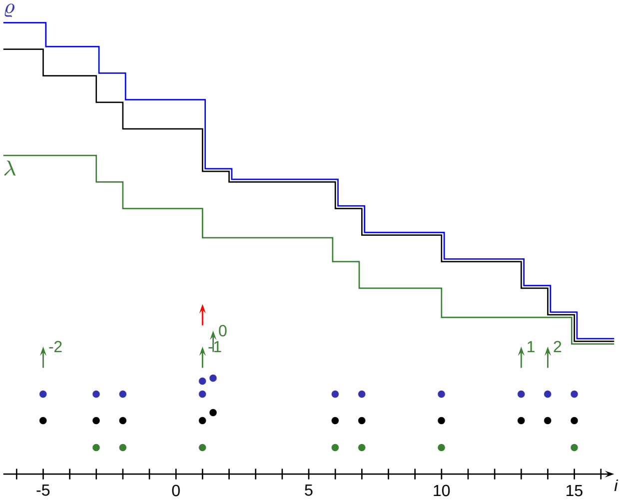

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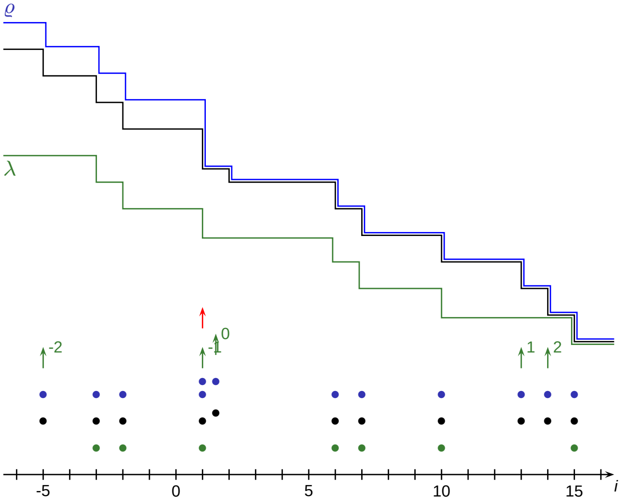

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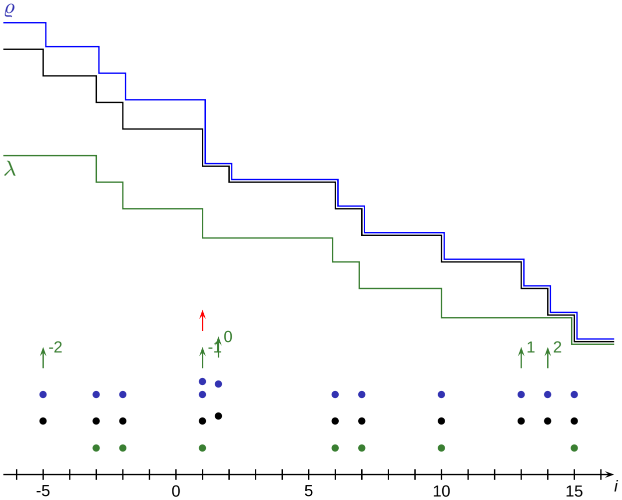

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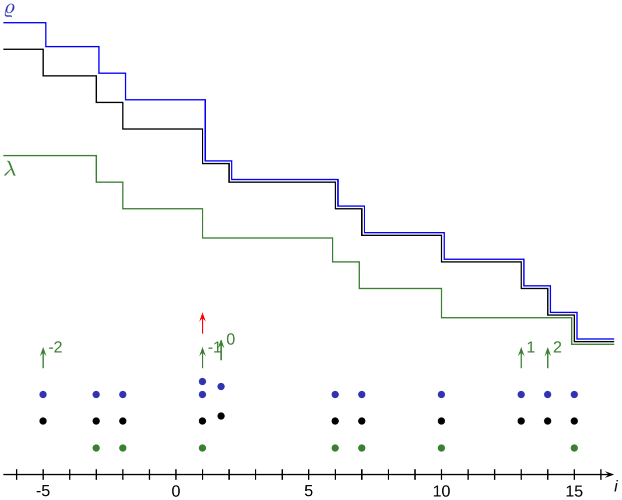

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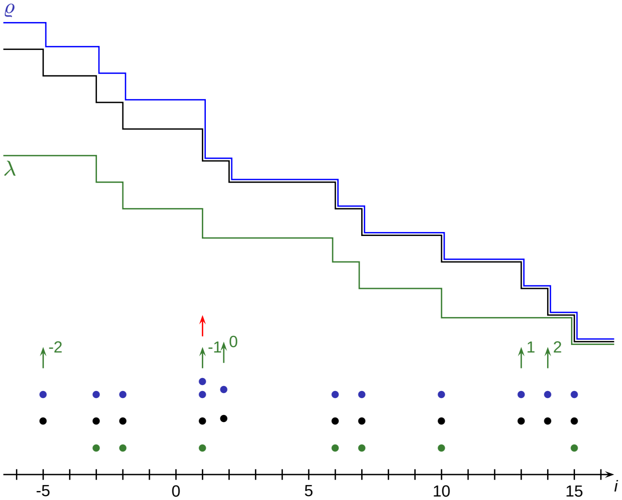

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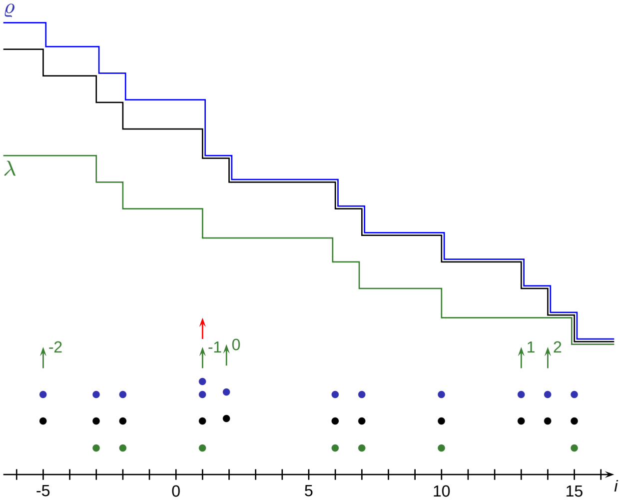

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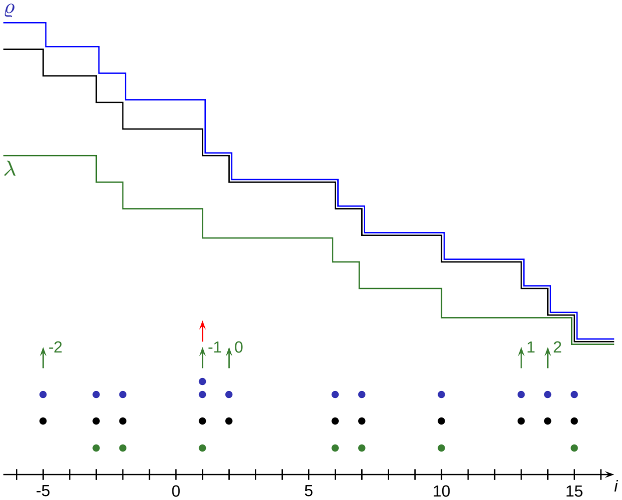

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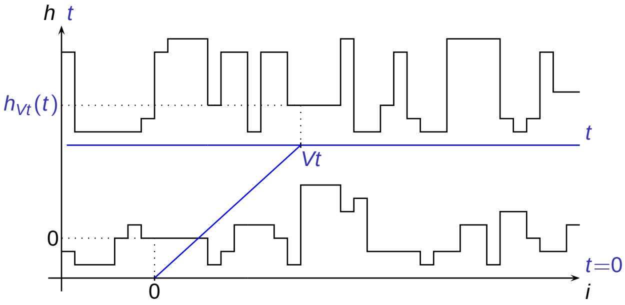

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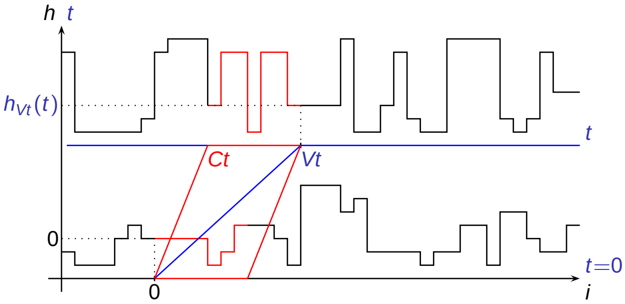

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There is a huge literature now on limit distribution results, using combinatorial and asymptotic analytic tools.

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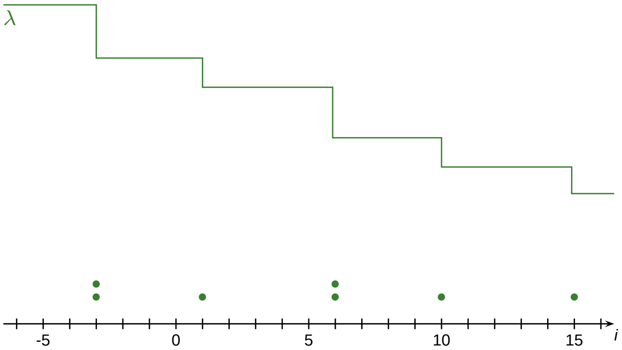

Second class particle current: difference in growth.

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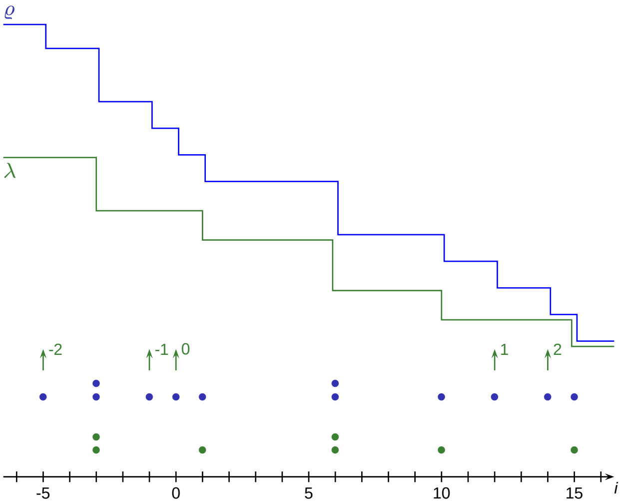

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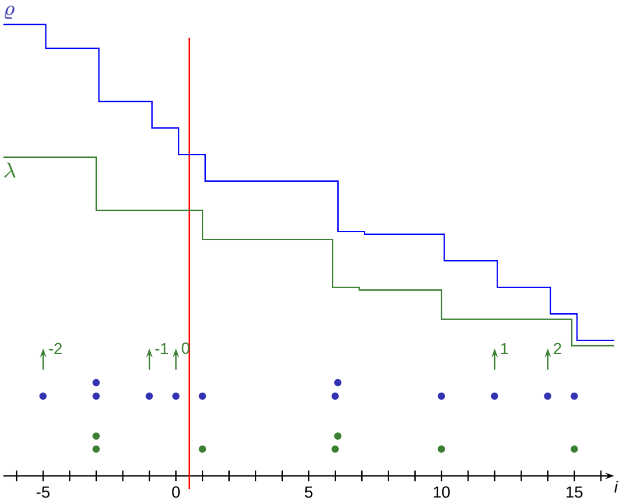

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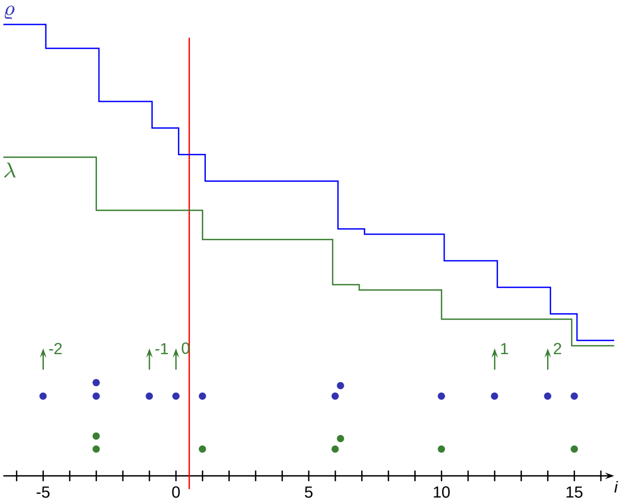

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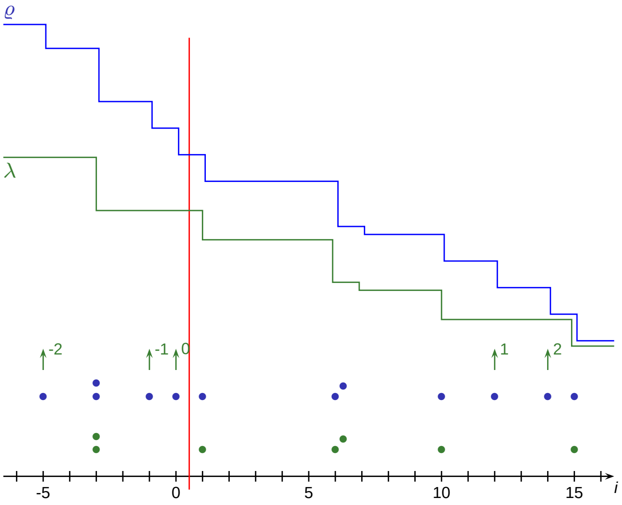

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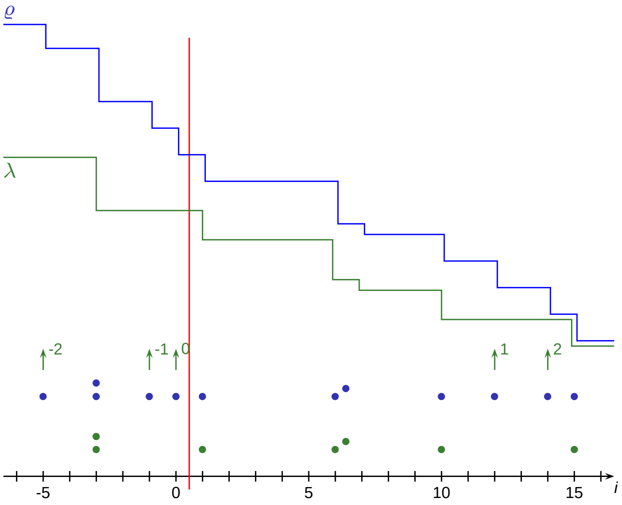

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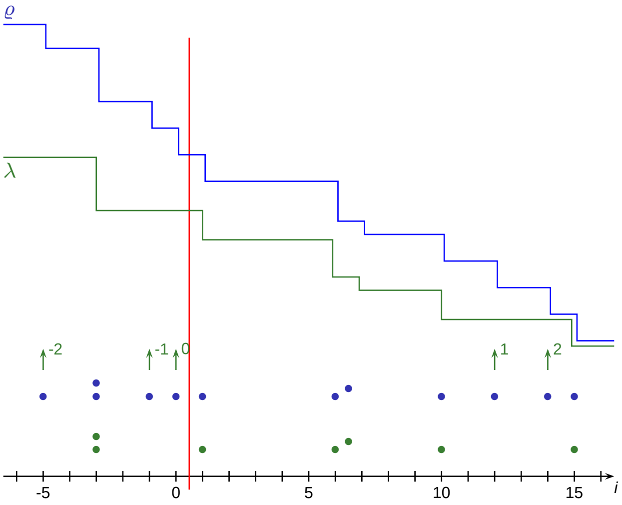

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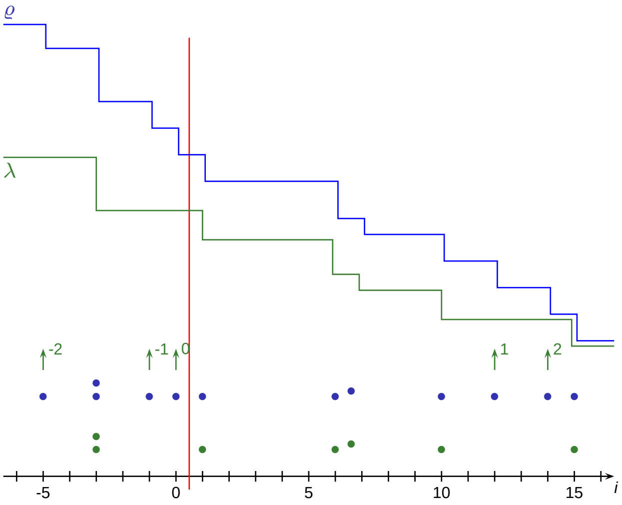

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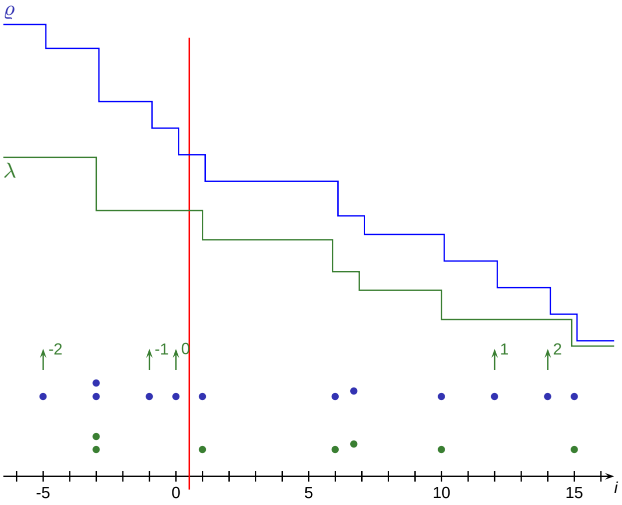

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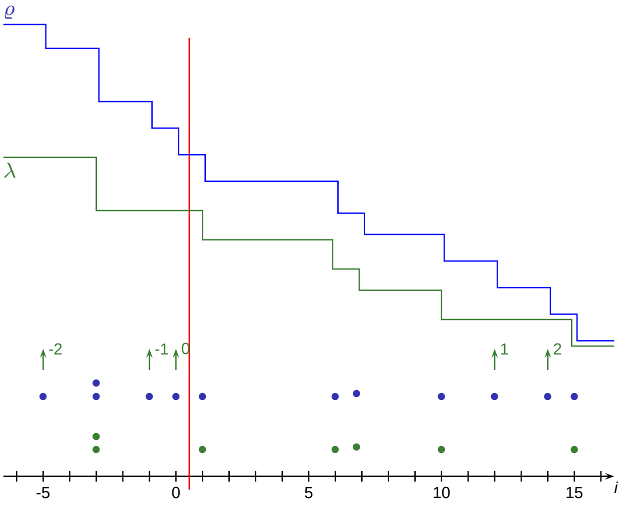

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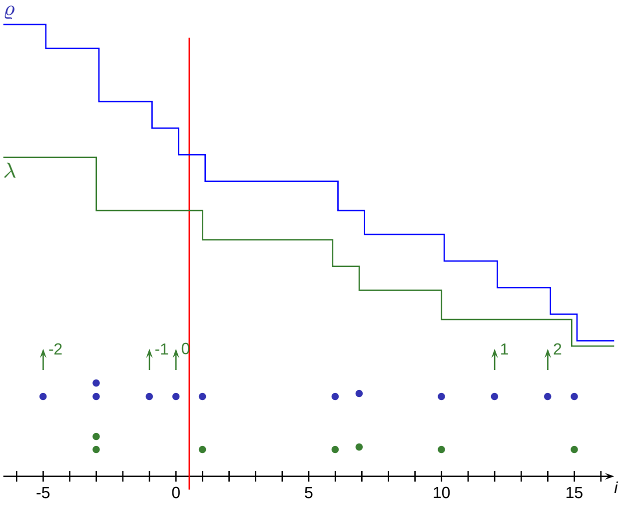

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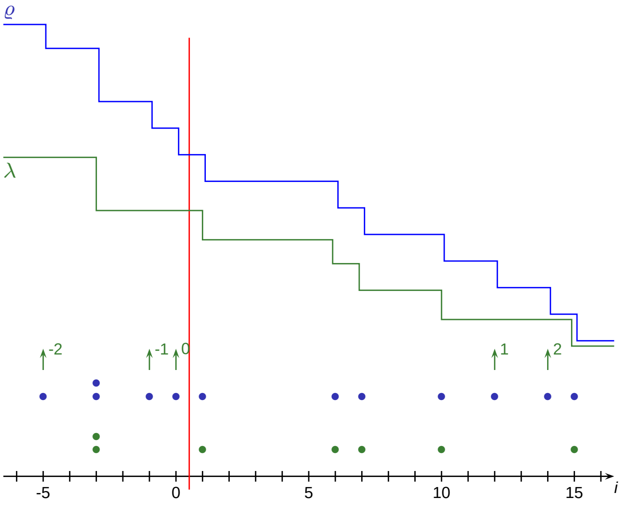

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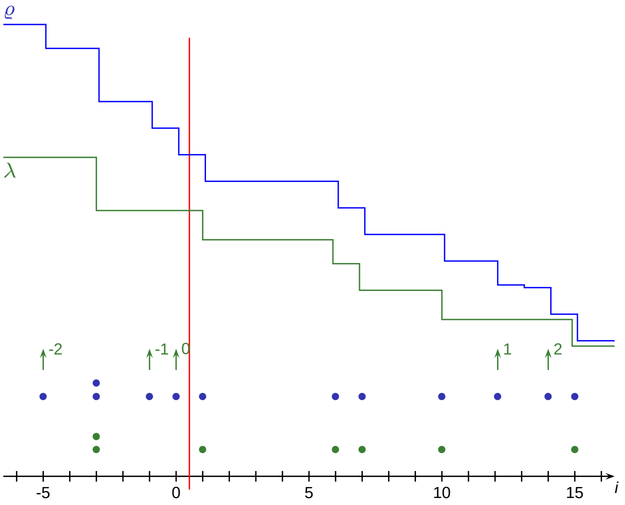

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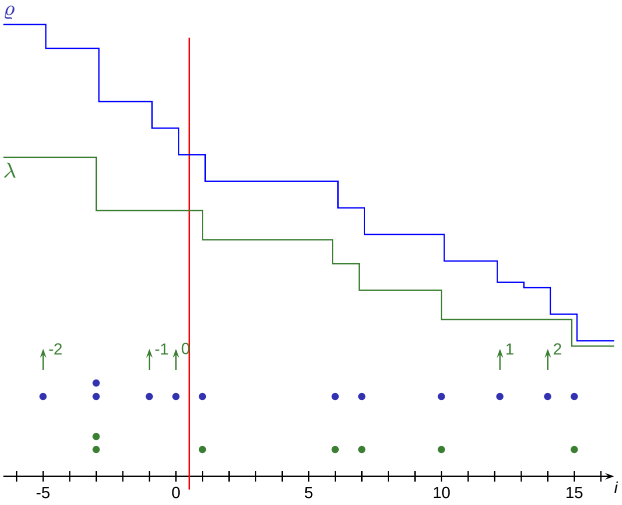

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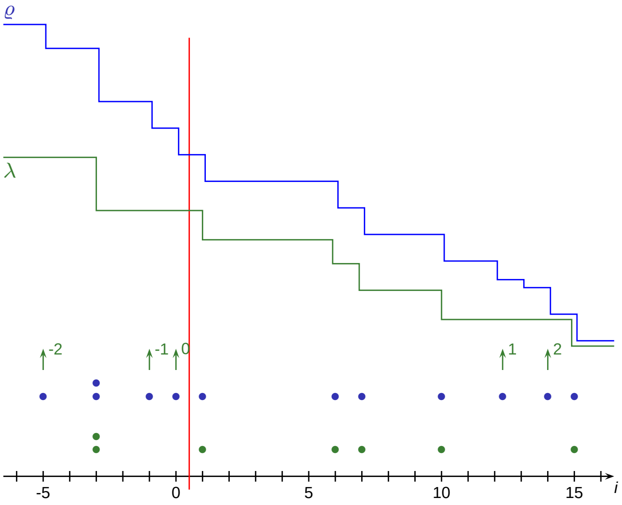

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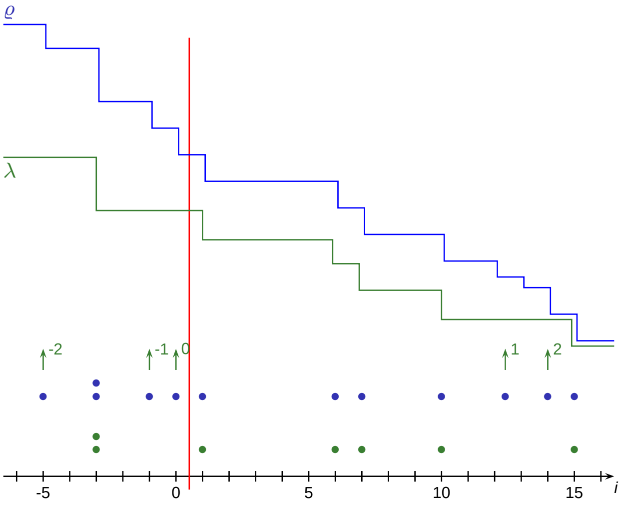

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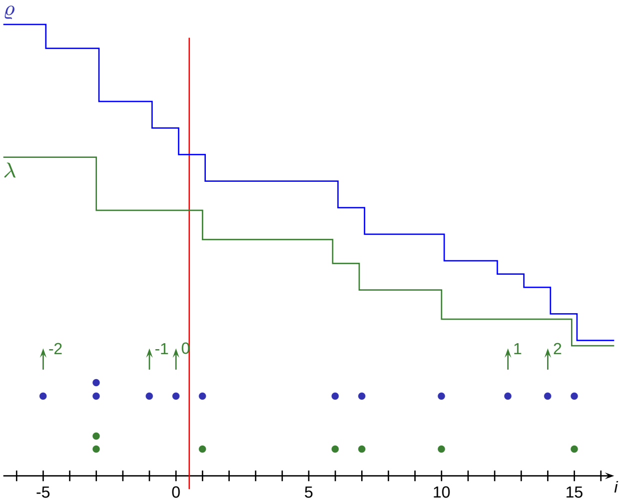

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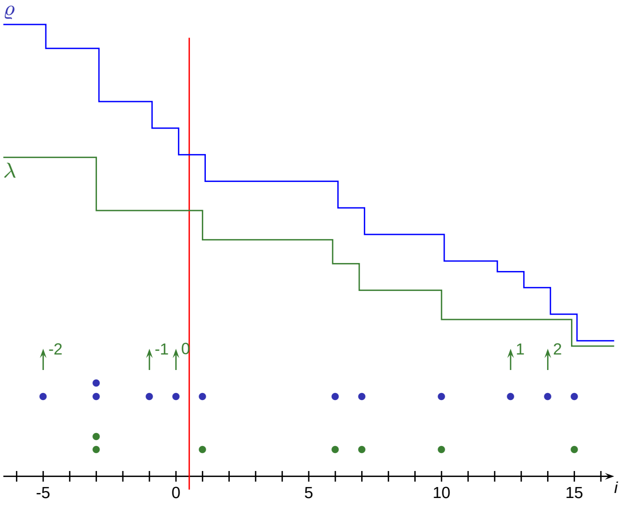

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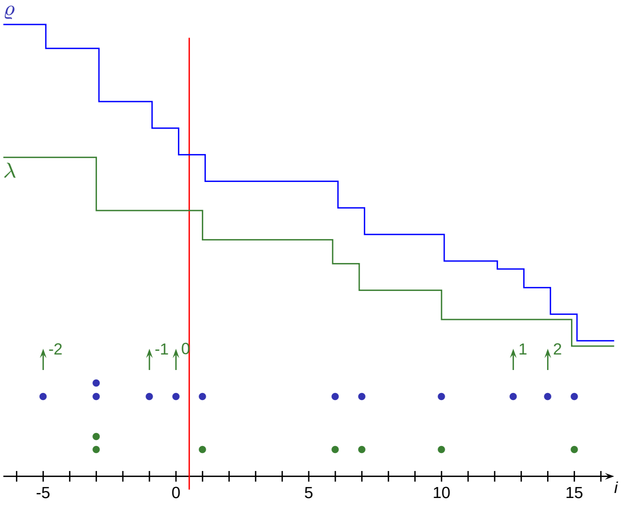

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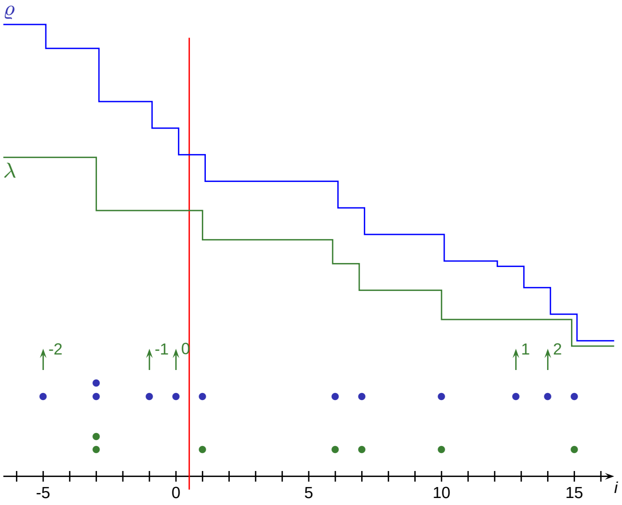

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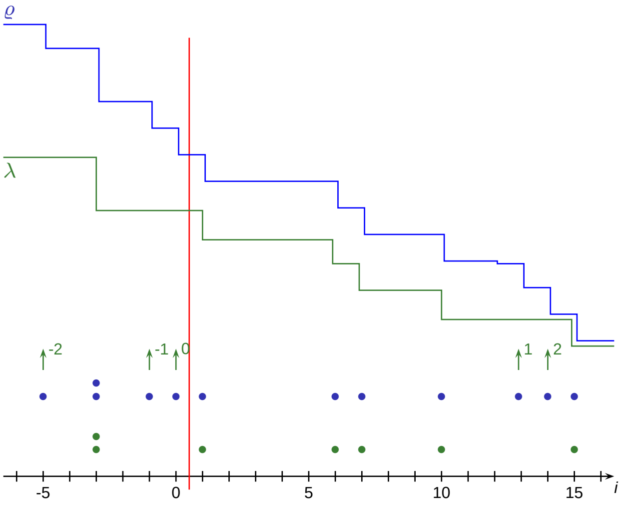

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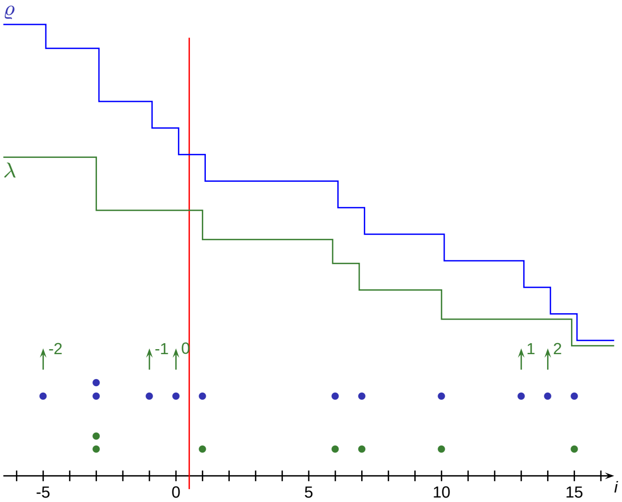

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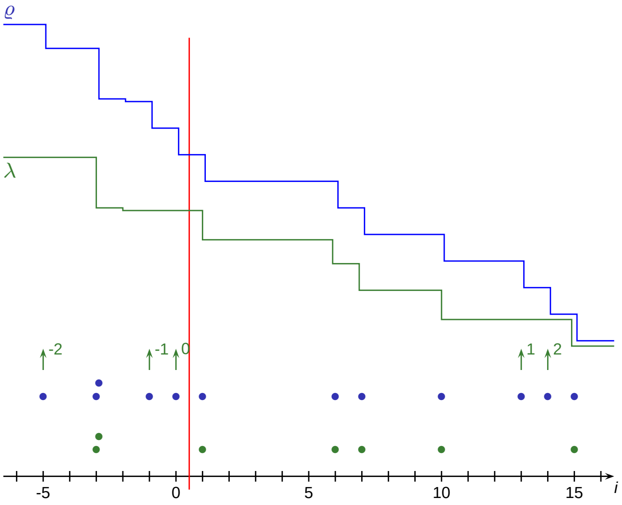

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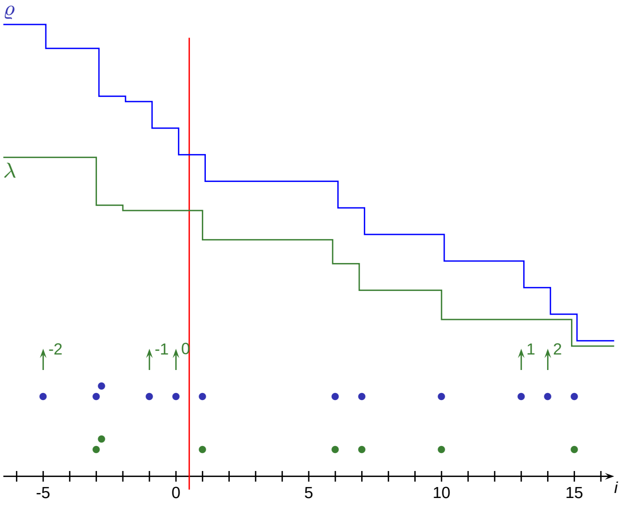

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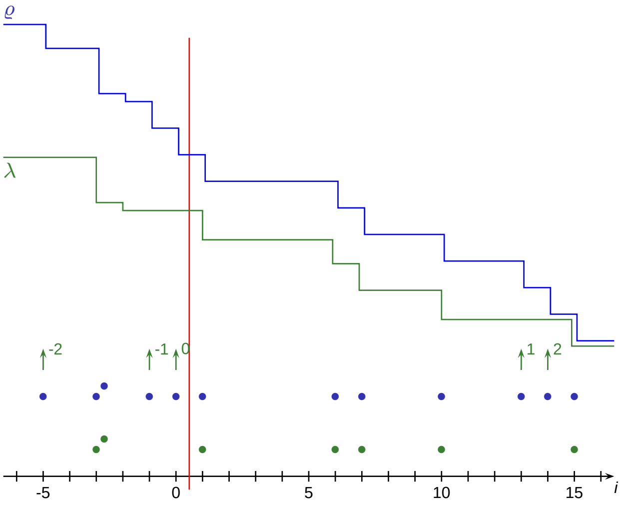

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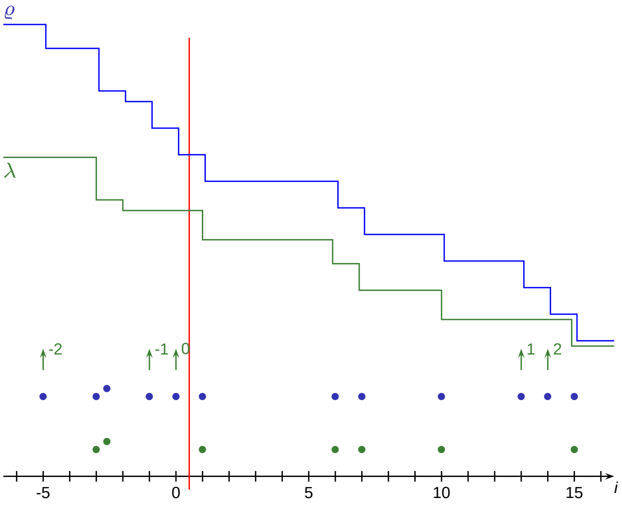

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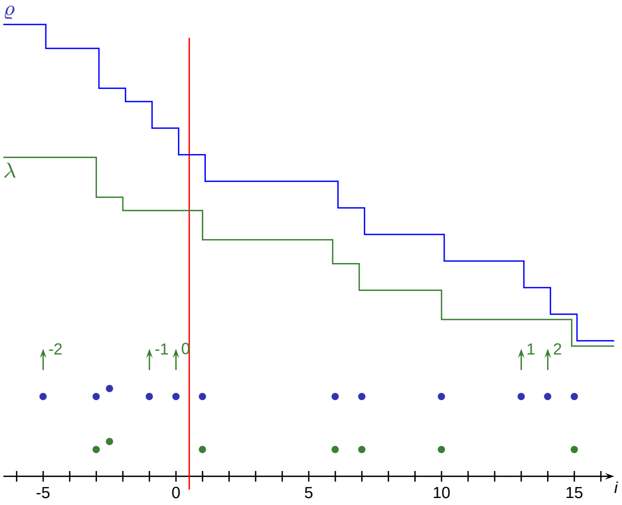

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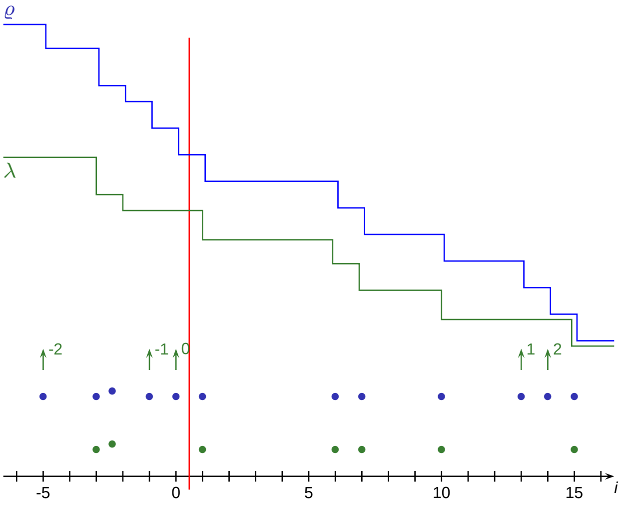

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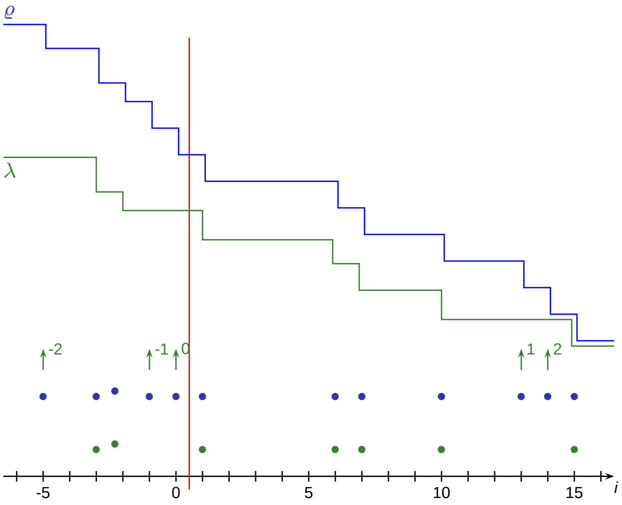

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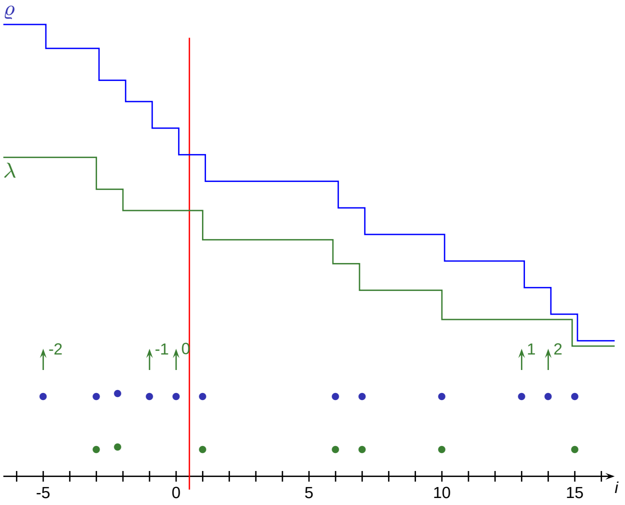

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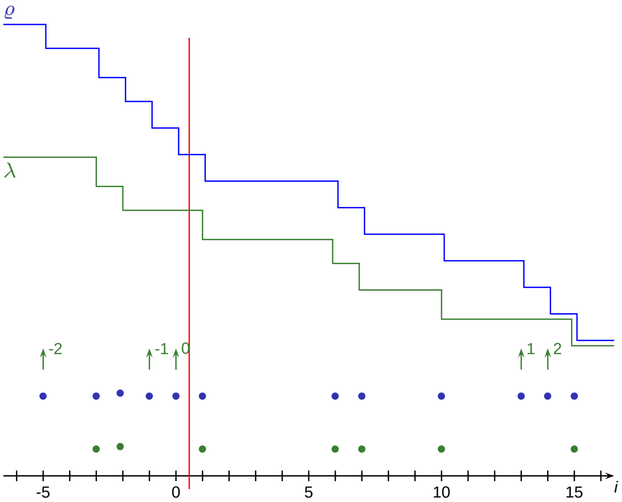

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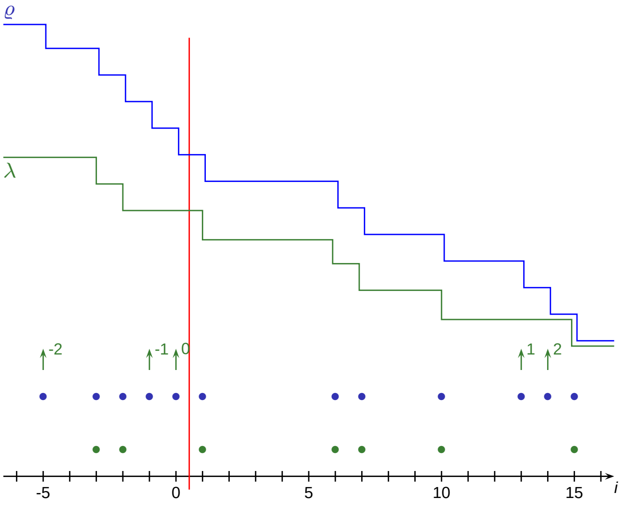

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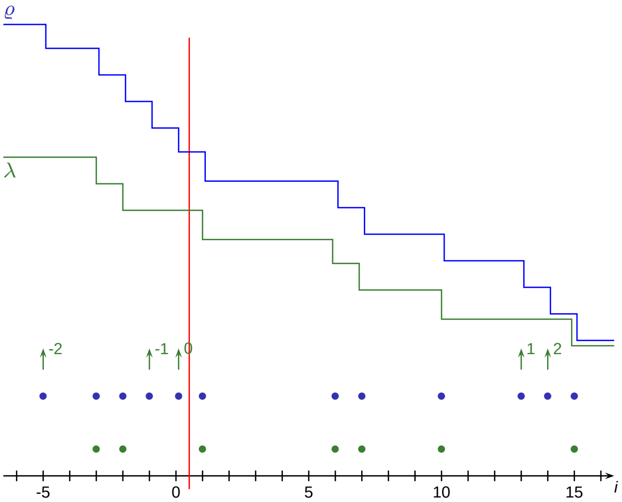

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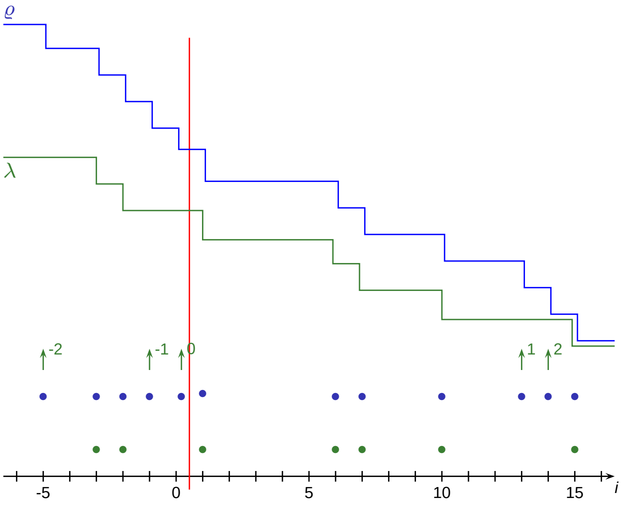

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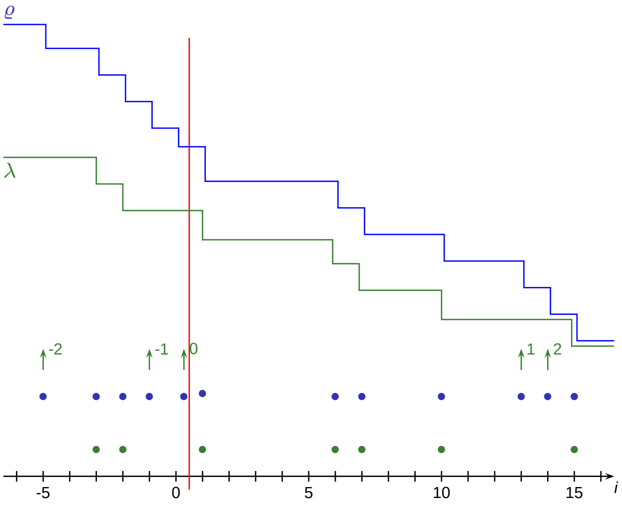

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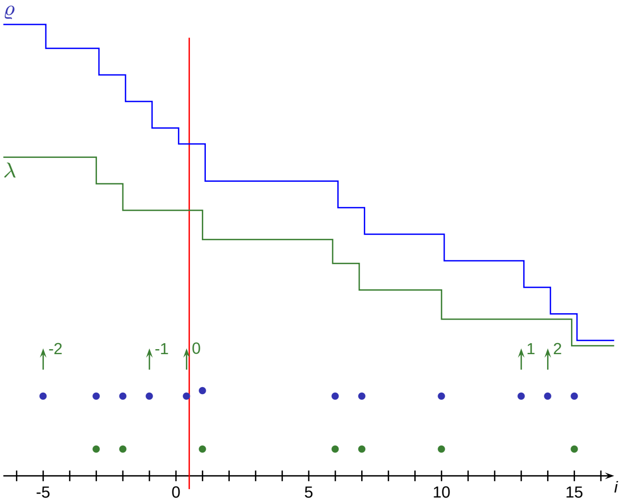

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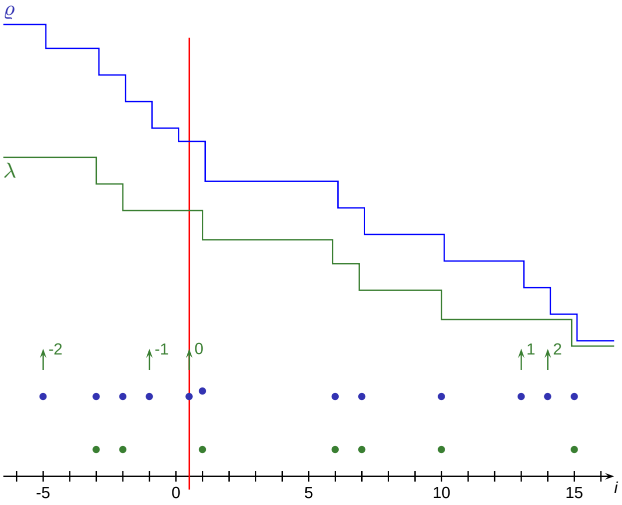

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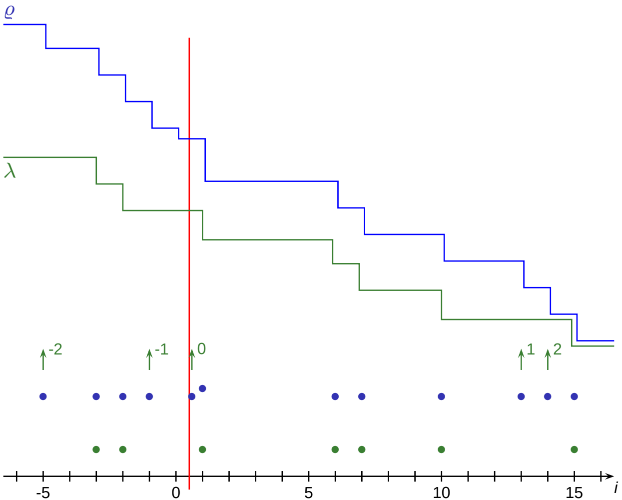

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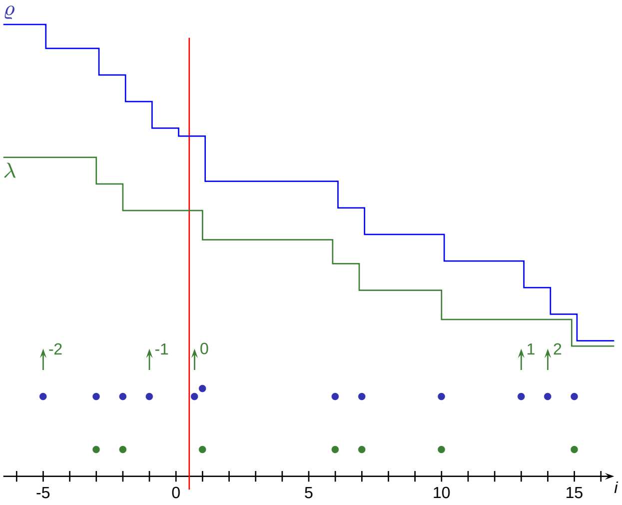

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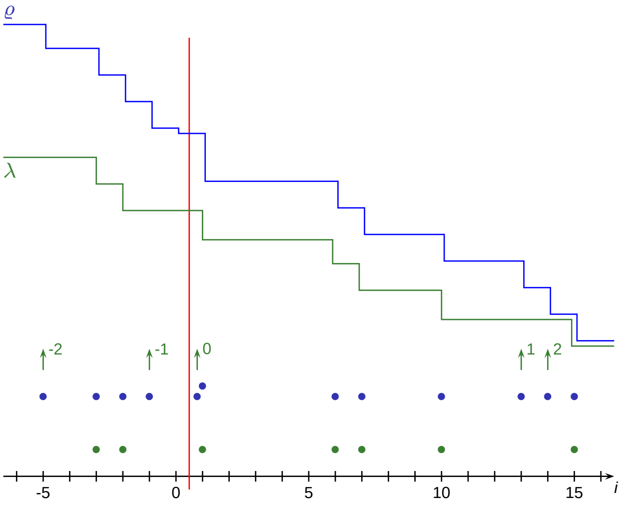

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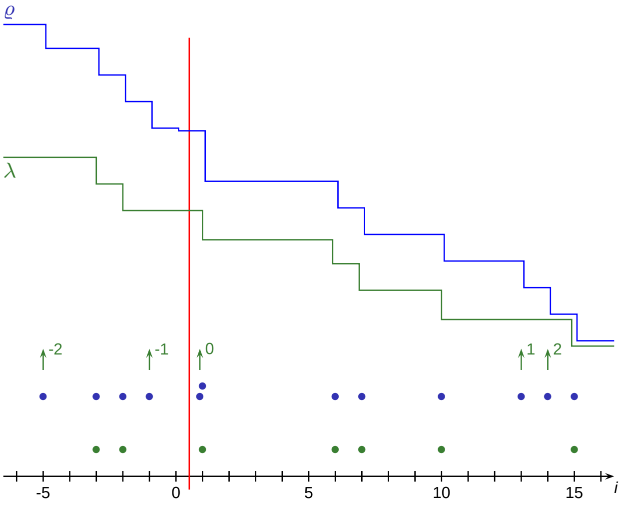

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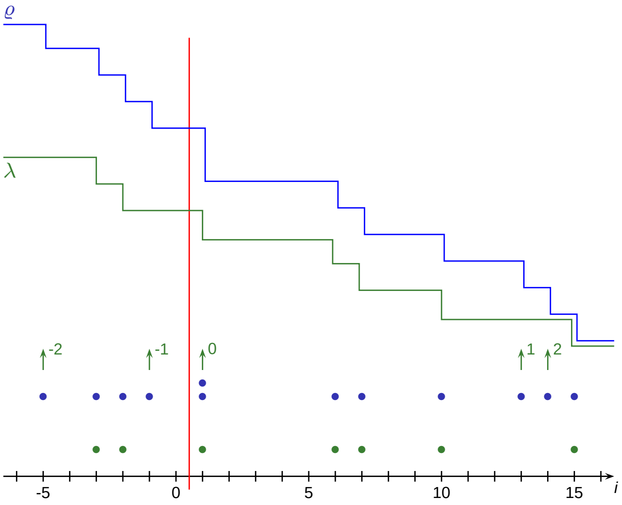

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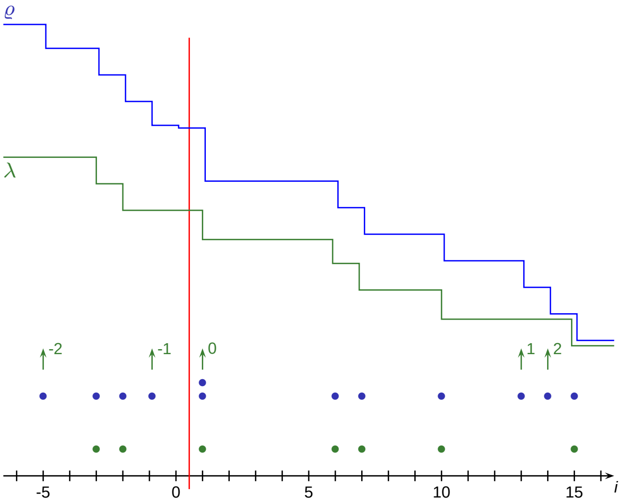

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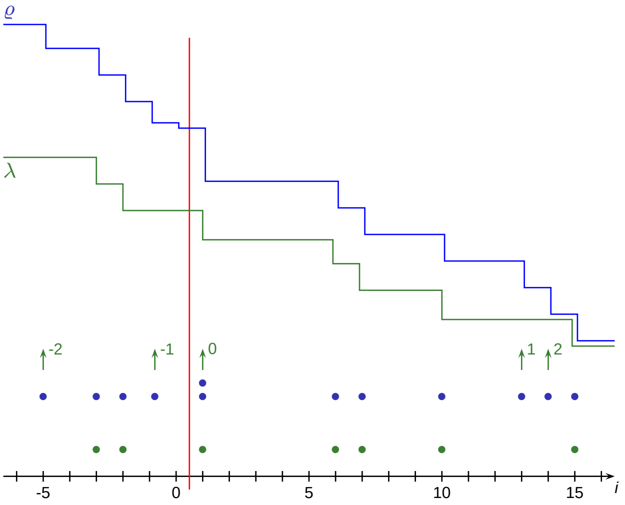

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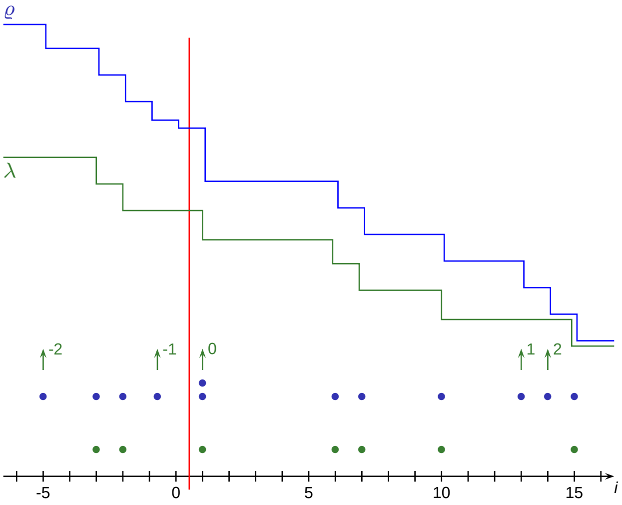

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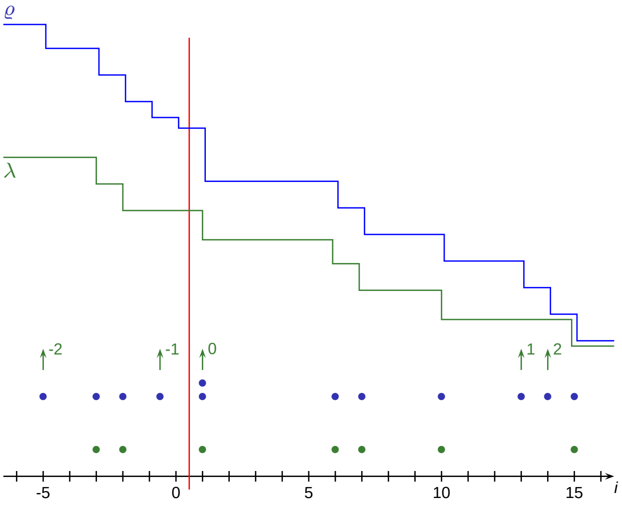

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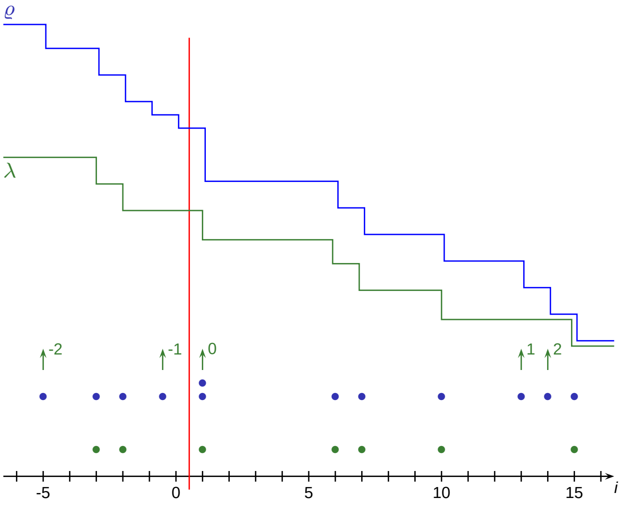

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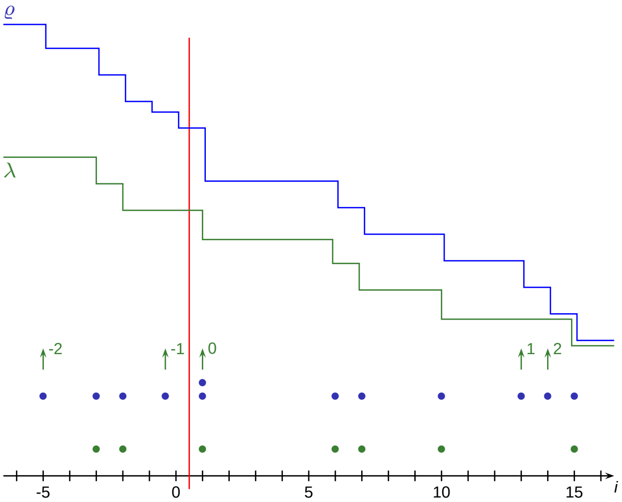

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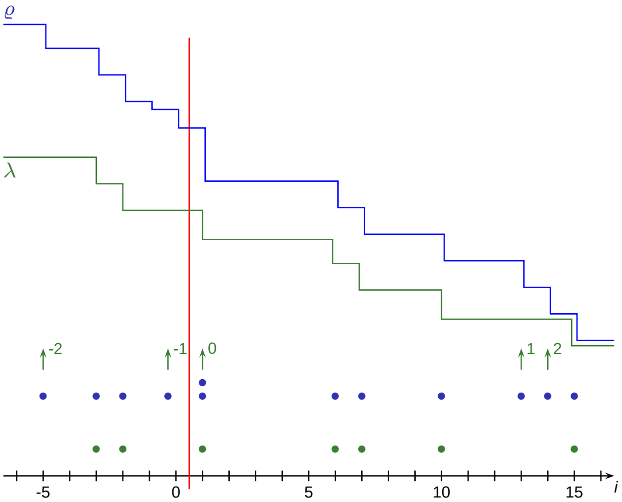

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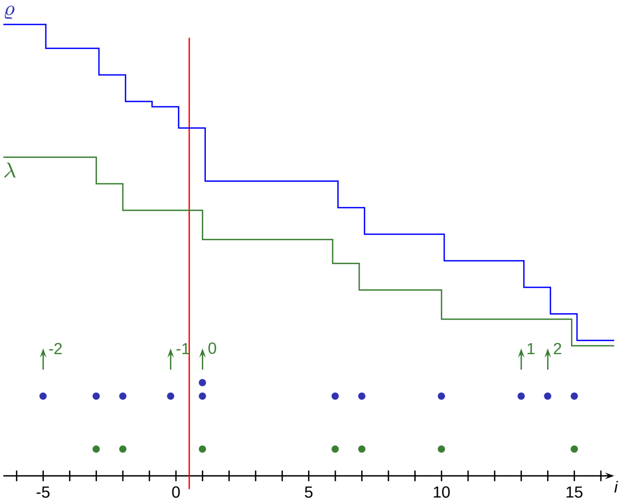

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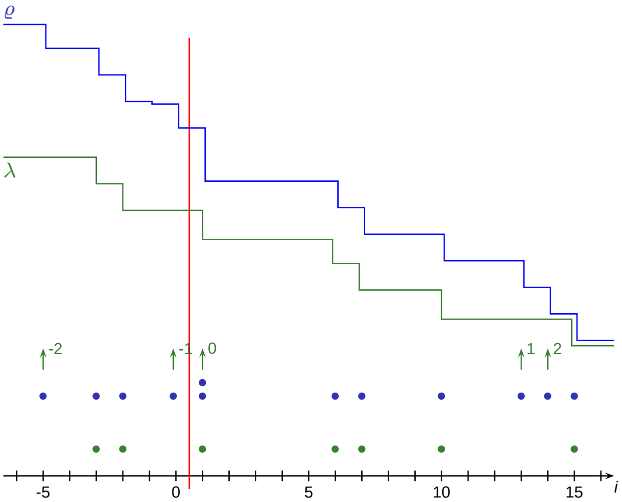

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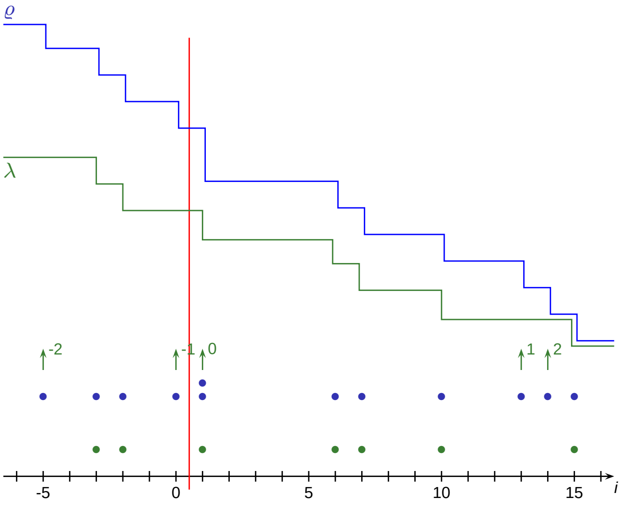

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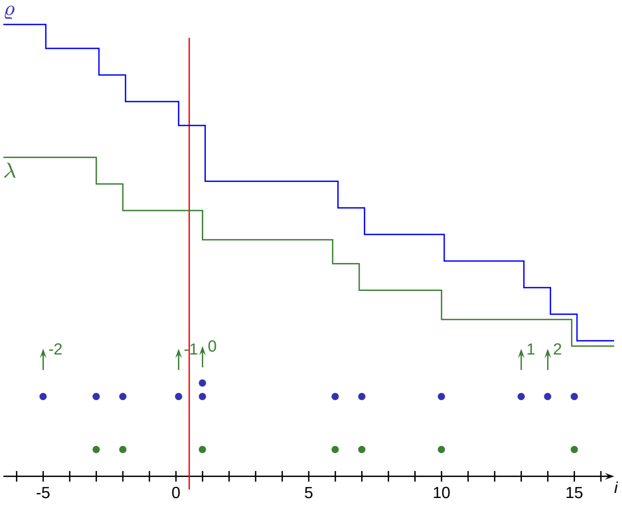

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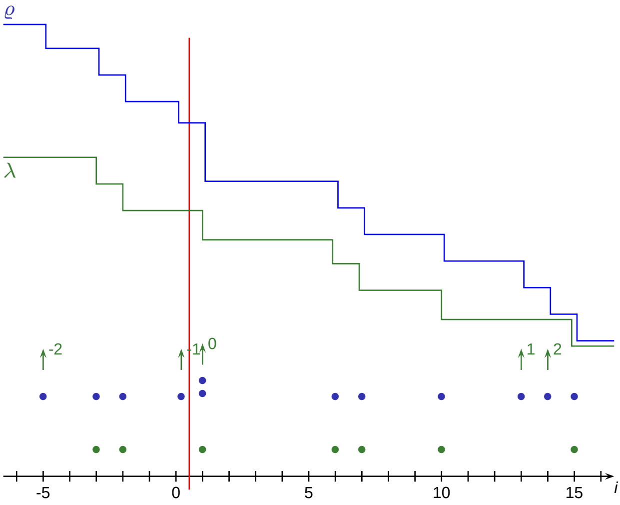

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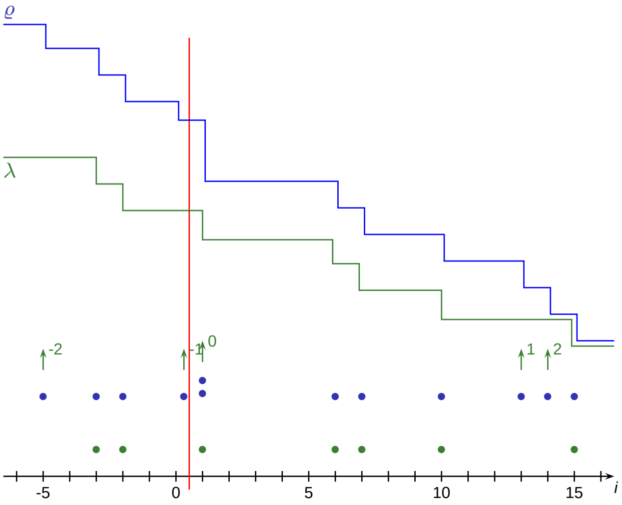

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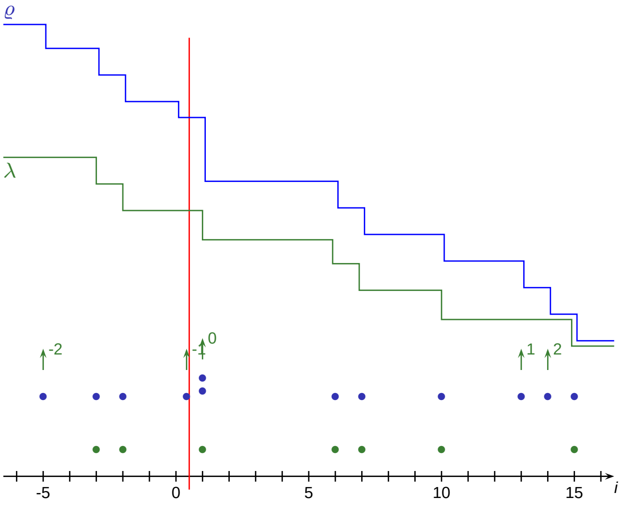

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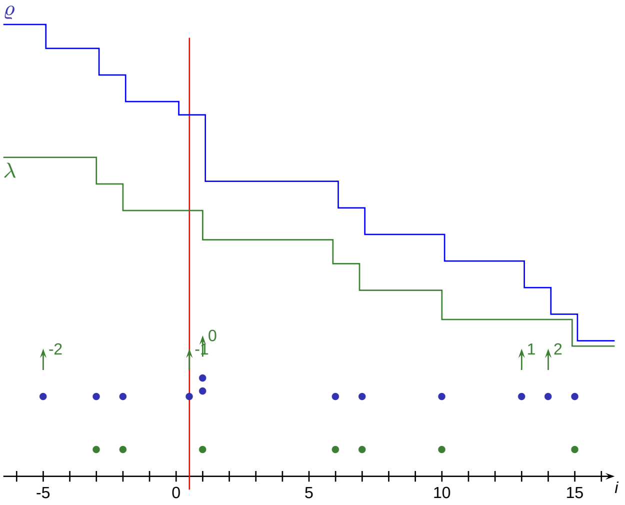

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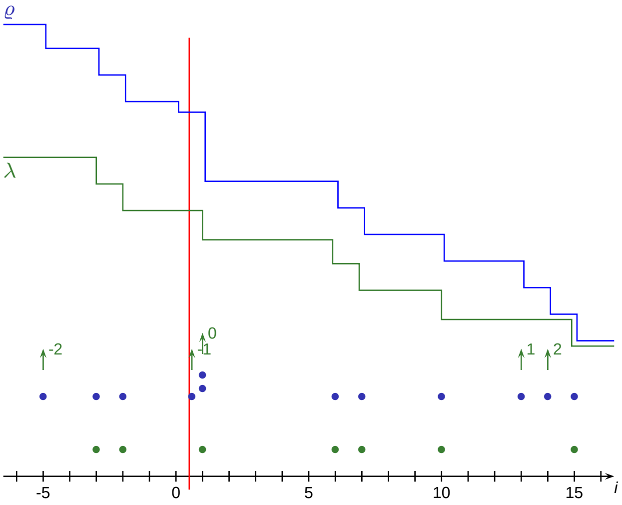

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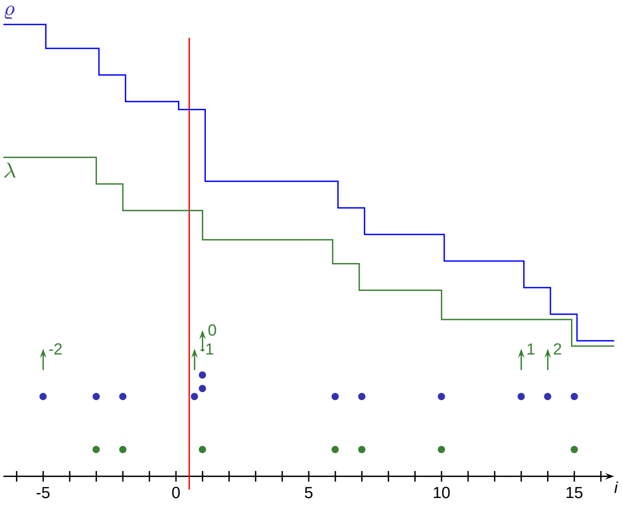

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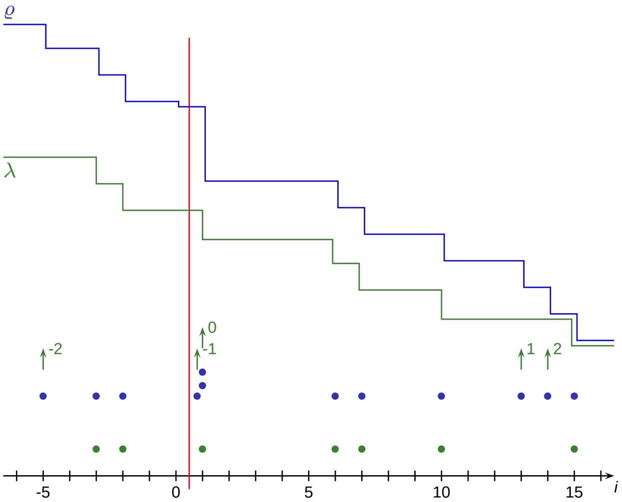

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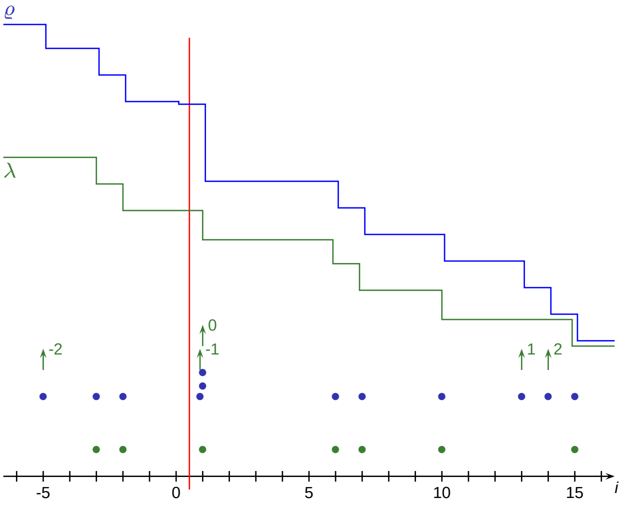

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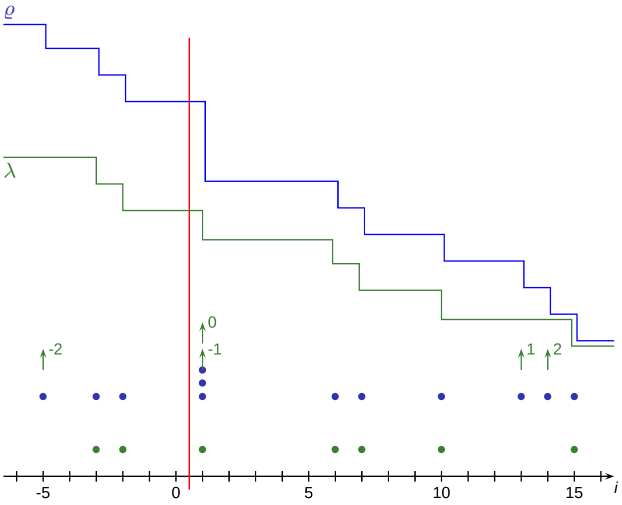

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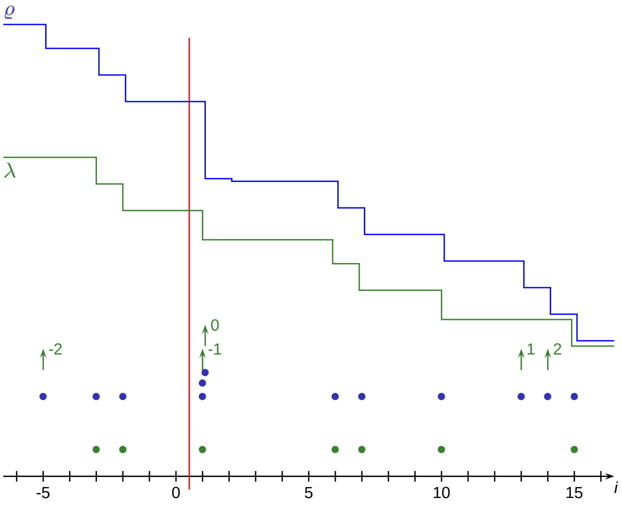

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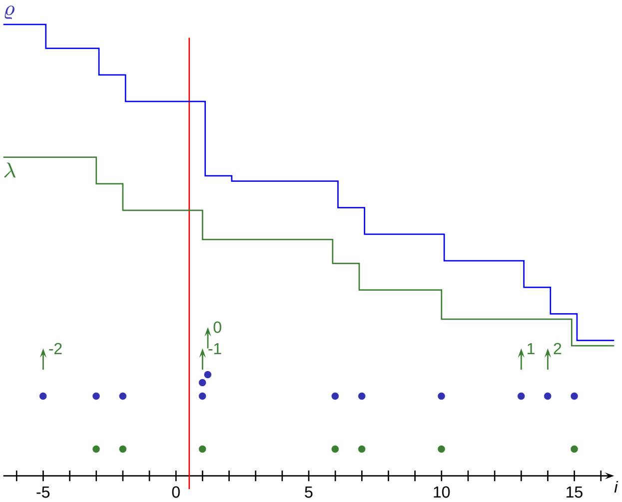

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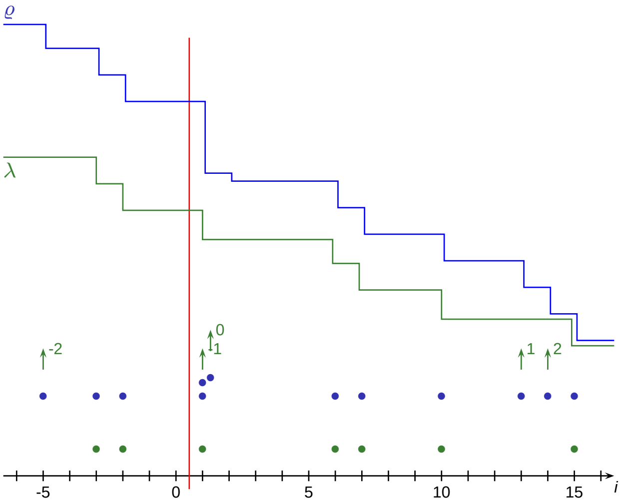

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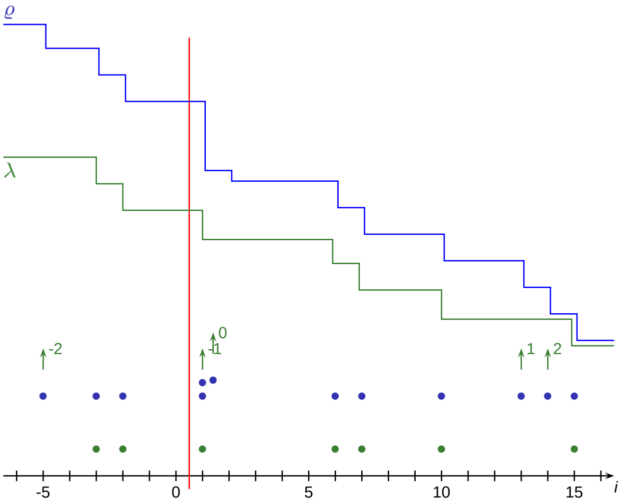

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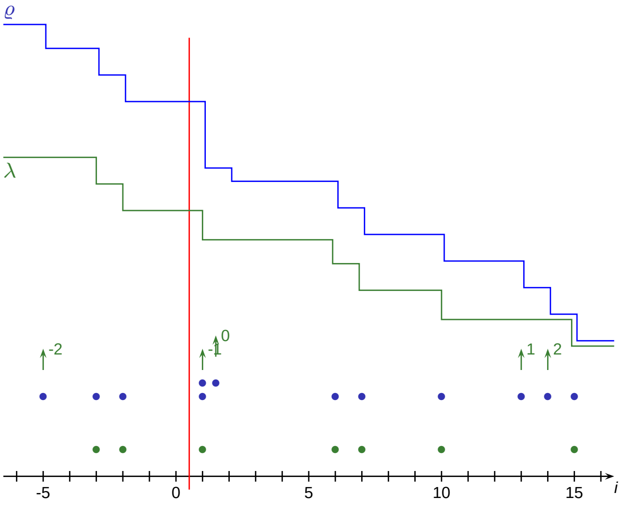

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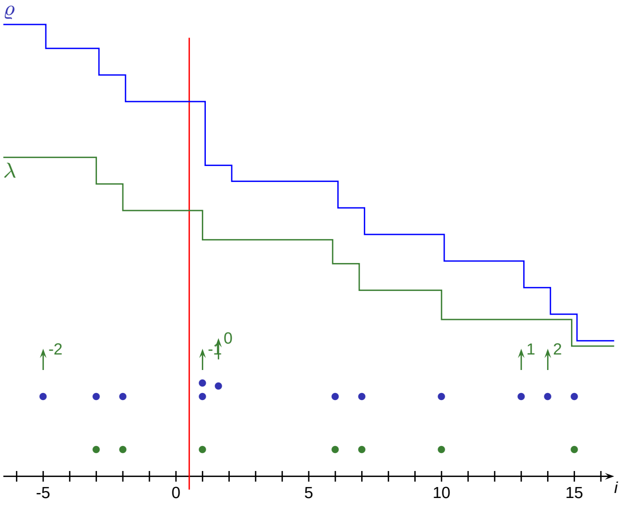

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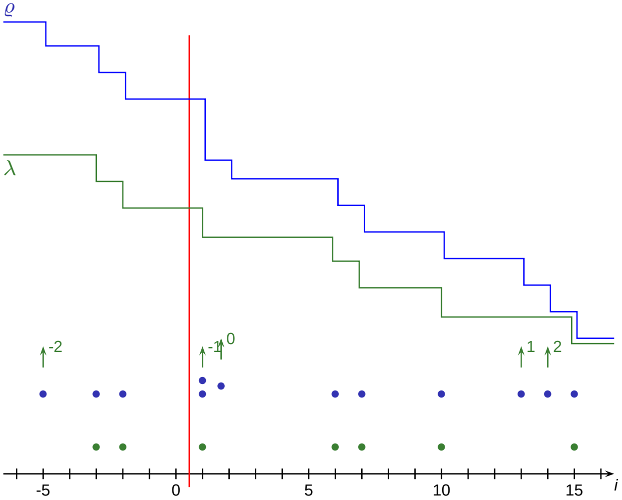

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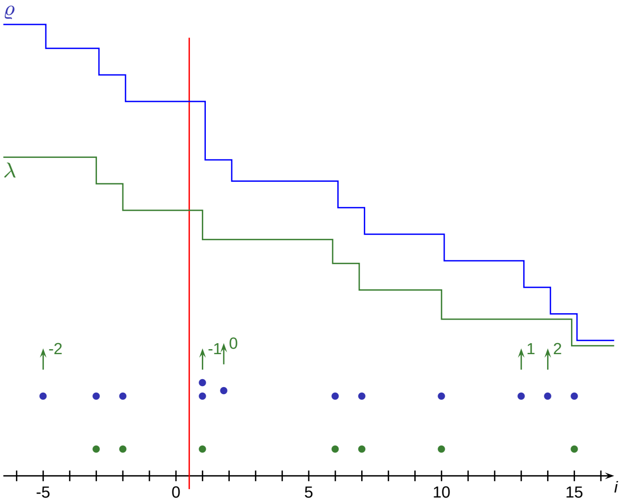

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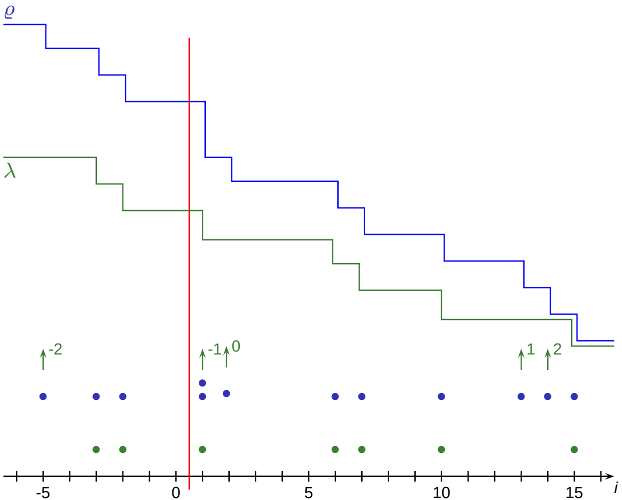

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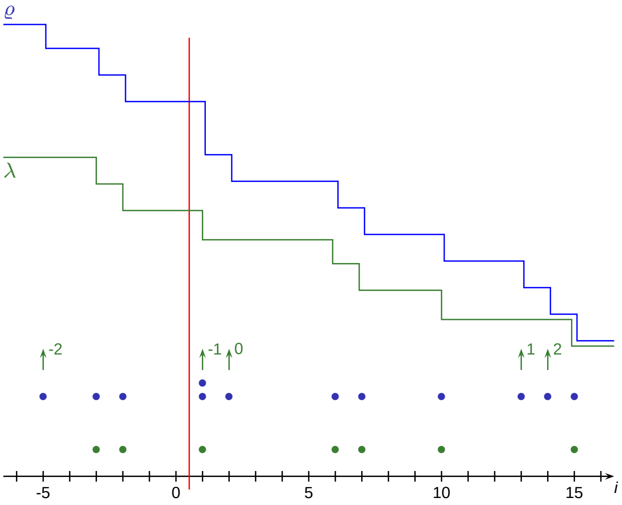

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Upper bound (concave case)

$\mathbf{P}\{Q(t) \text{ is too large}\}$

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Micro conc.: $Q(t) < X(t) +$ tight error

Upper bound (concave case)

$$\begin{aligned}\mathbf{P}\{Q(t)\text{ is too large}\} &\le \mathbf{P}\{X(t)\text{ is too large}\}\\ &\le \mathbf{P}\{\text{too many}\uparrow\text{'s have crossed } Ct\}\end{aligned}$$

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Upper bound (concave case)

$$
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\mathbf{P}\{Q(t) \text{ is too large}\} &\leq \mathbf{P}\{X(t) \text{ is too large}\} \\
&\leq \mathbf{P}\{\text{too many} \uparrow\text{'s have crossed } Ct\} \\
&\leq \mathbf{P}\{h_{Ct}(t) - h_{Ct}(t) \text{ is too large}\}.
\end{aligned}
$$

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Centering $h_{Ct}(t) - h_{Ct}(t)$ brings in a second-order Taylor-expansion of $H(\varrho)$.

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Optimize "too large(λ)" in λ,

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Optimize "too large(λ)" in λ, use Chebyshev's inequality and relate $\mathbf{Var}(h_{Ct}(t))$ to $\mathbf{Var}(h_{Ct}(t))$.

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The computations result in (remember $\mathbf{E}(Q(t)) = Ct$)

$$\mathbf{P}\{Q(t) - Ct \ge u\} \le c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t)).$$

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Upper bound

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{Var}(h_{Ct}(t)) = c \cdot \mathbf{E}|Q(t) - C \cdot t|$$

in the whole family of processes.

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Upper bound

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Hence proceed with

$$\mathbf{P}\{Q(t) - Ct \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{E}|Q(t) - C \cdot t|$$

$\mathbf{P}\{Q(t) - Ct \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t))$ $\mathbf{E}Q(t) = Ct$

Upper bound

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

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with $\widetilde{Q}(t) := Q(t) - Ct$ and $E := \mathbf{E}|\widetilde{Q}(t)|$.

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that is, $E^3 \le c \cdot t^2$.

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\mathbf{Var}(h_{Ct}(t)) &\overset{\text{Thm}}{=} \text{const.} \cdot \mathbf{E}|Q(t) - Ct| \\
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In the upper bound, the relevant orders were

$$u \text{ (deviation of } Q(t)) \sim t^{2/3}, \qquad \varrho - \lambda \sim t^{-1/3}.$$

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The critical feature in both the upper bound and lower bound was microscopic convexity/concavity: $Q(t) \geq X(t)$ (convex) or $Q(t) \leq X(t)$ (concave).

Microscopic convexity/concavity

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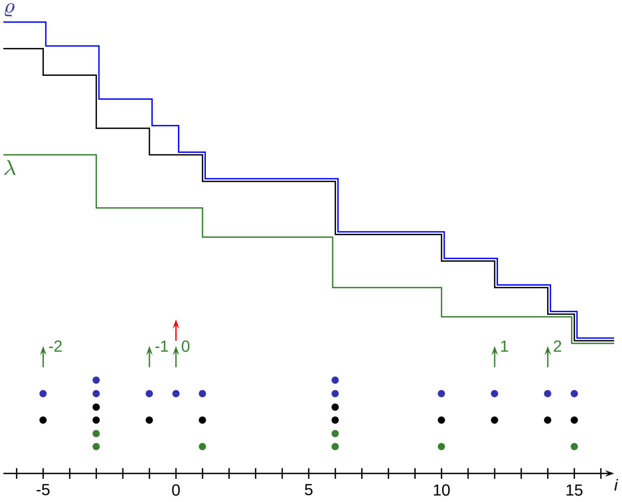
The critical feature: microscopic concavity

$Q(t) \leq X(t) +$ tight error



Goal: to understand $Q(t)$ on the background process of the ↑'s.

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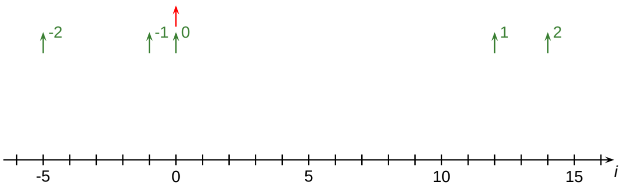

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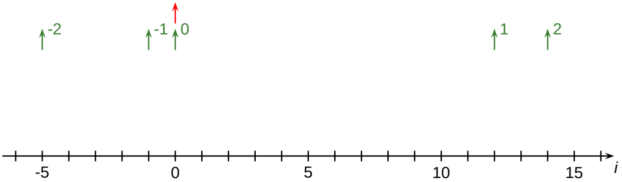

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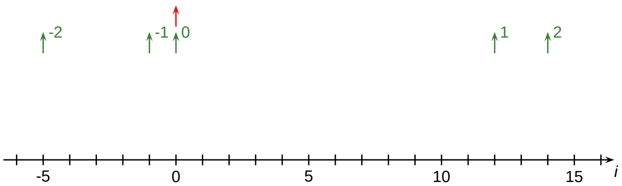

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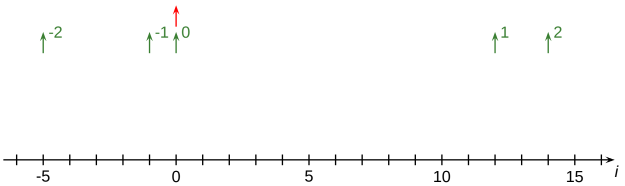

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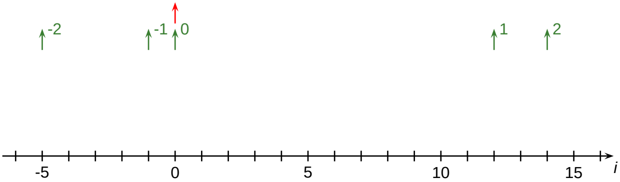

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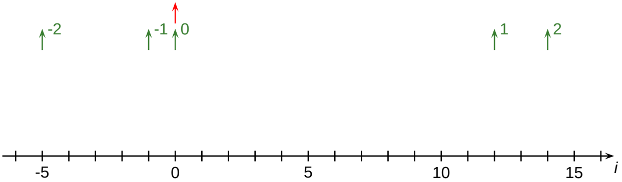

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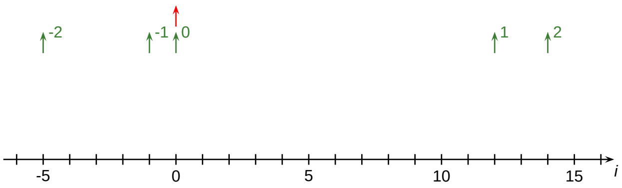

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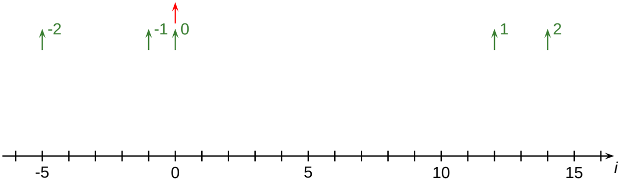

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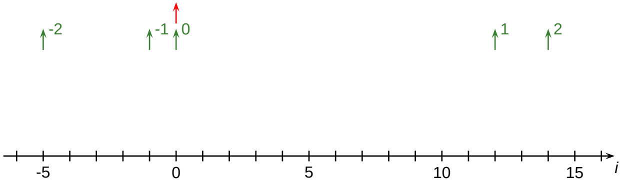

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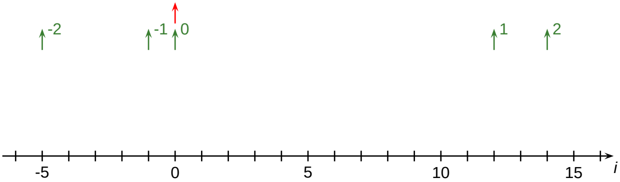

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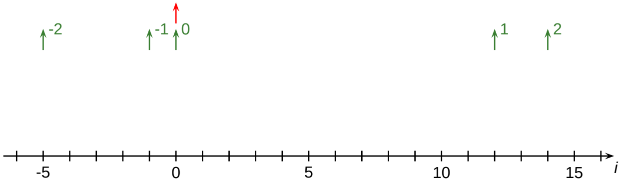

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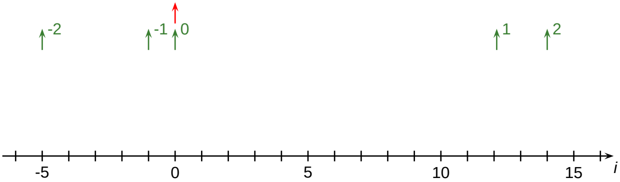

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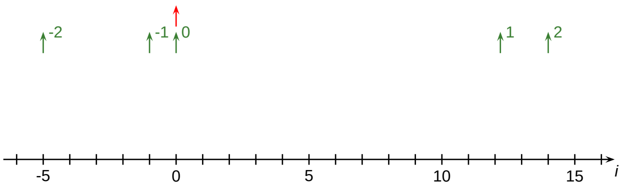

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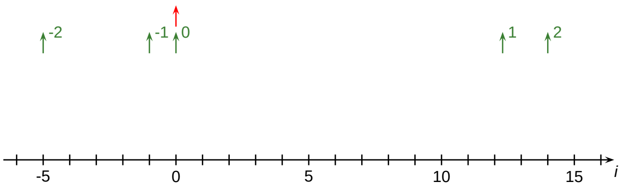

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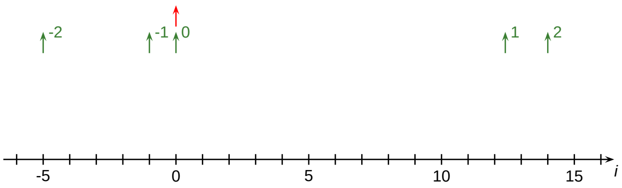

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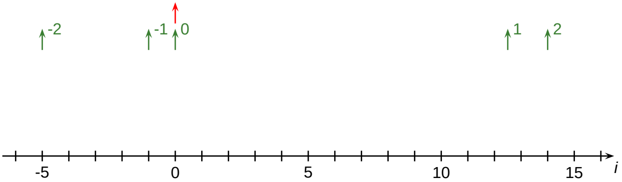

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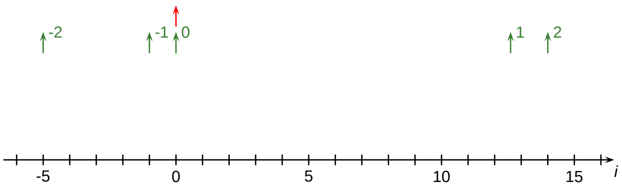

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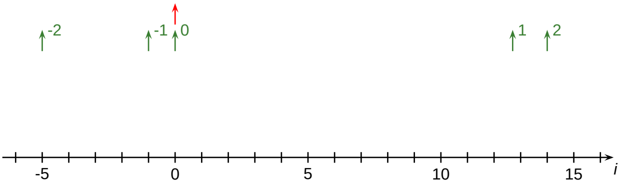

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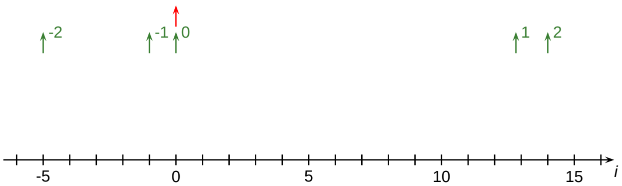

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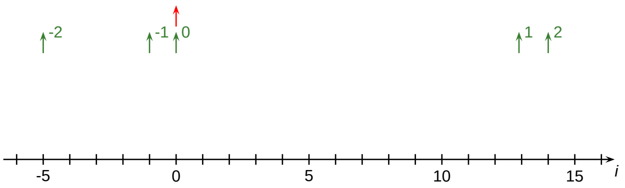

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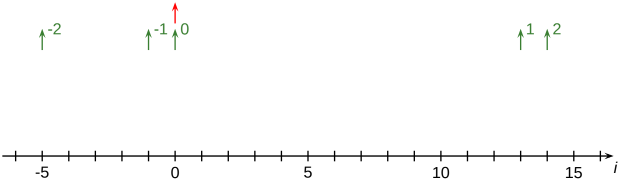

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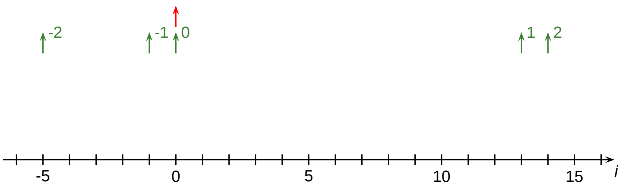

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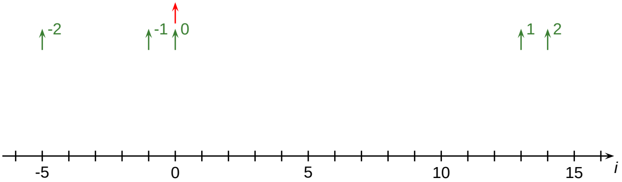

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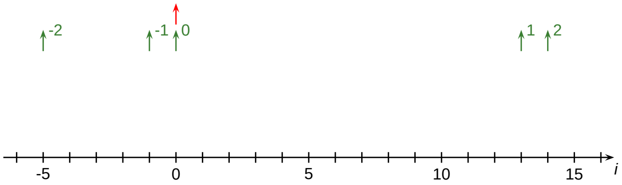

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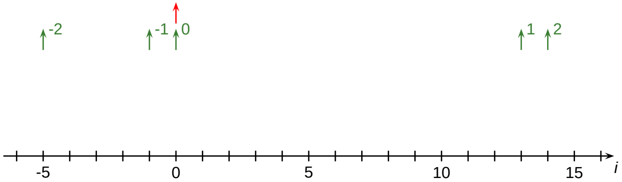

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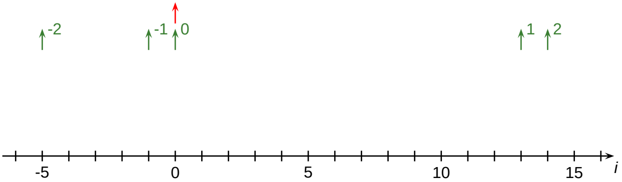

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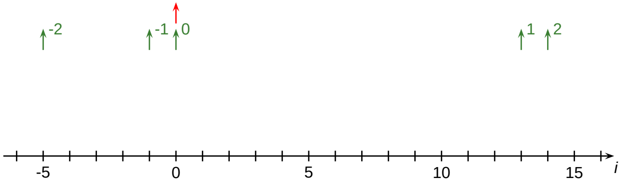

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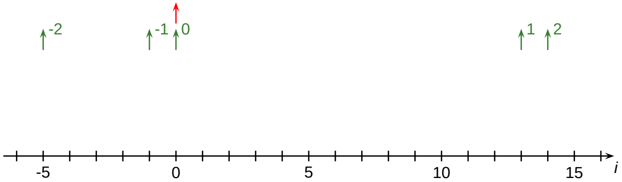

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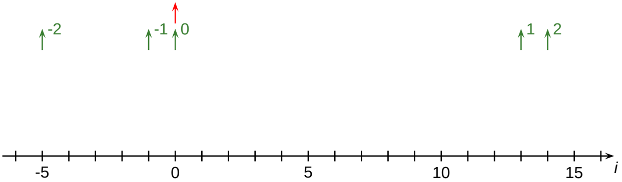

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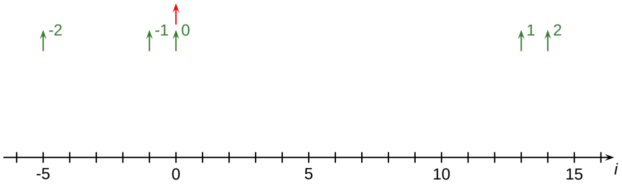

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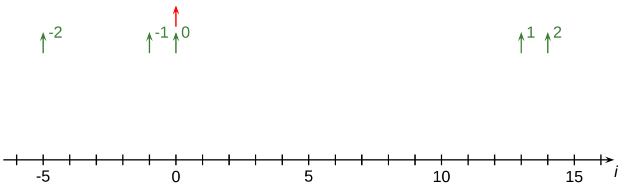

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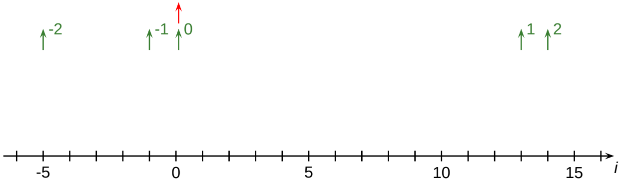

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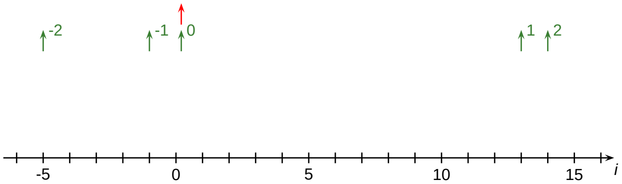

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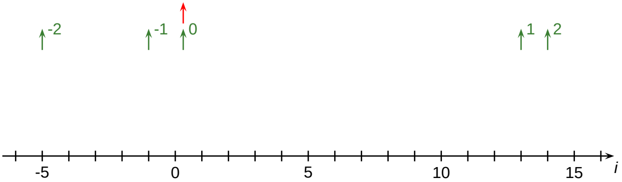

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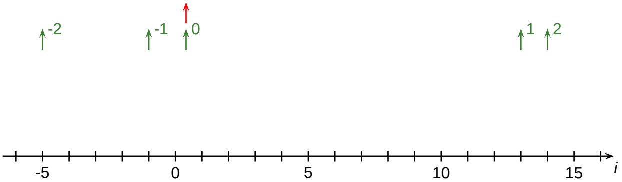

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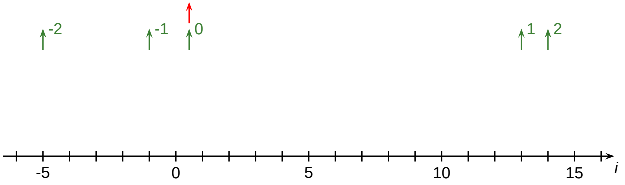

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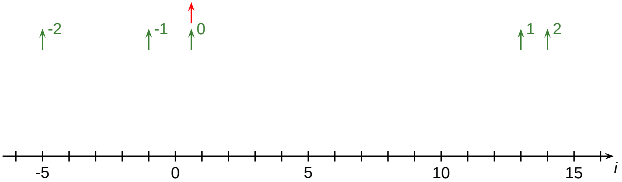

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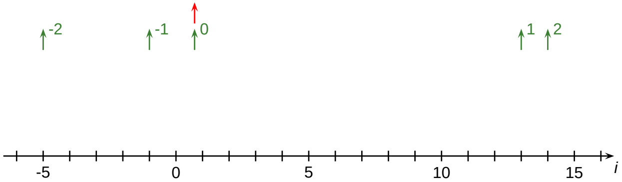

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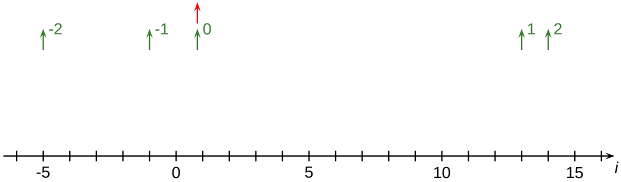

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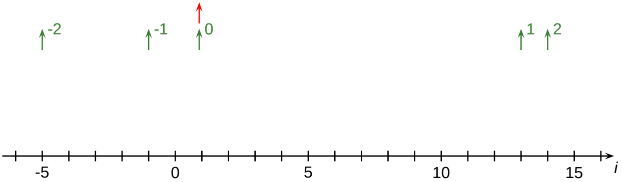

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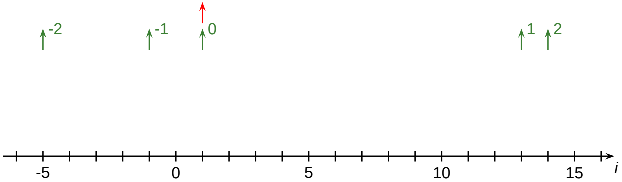

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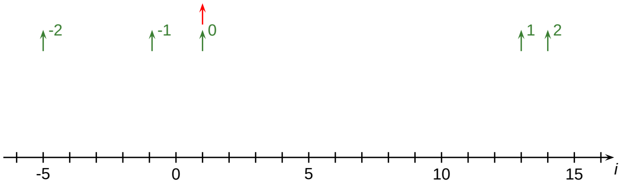

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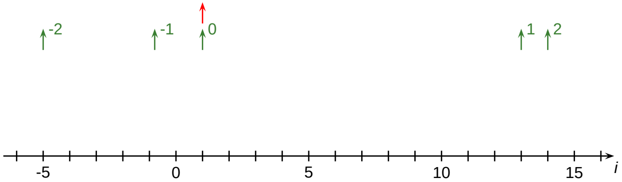

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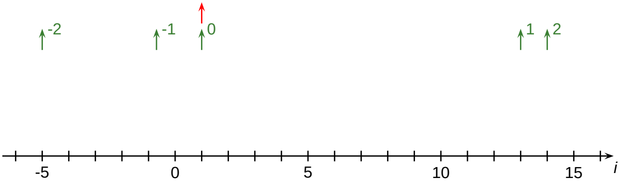

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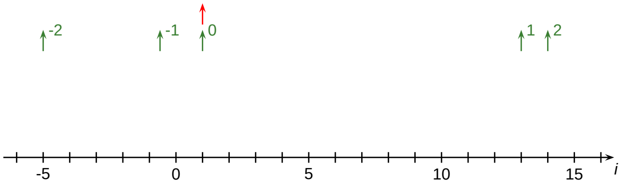

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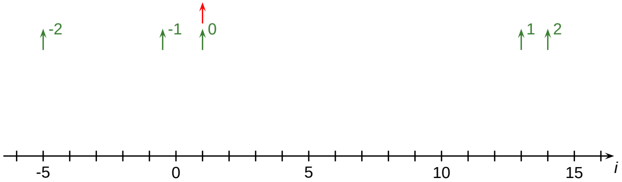

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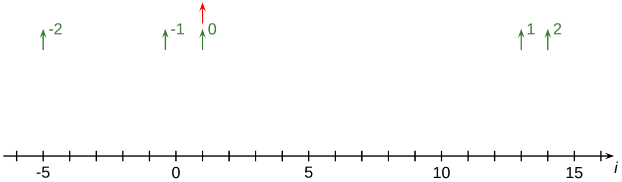

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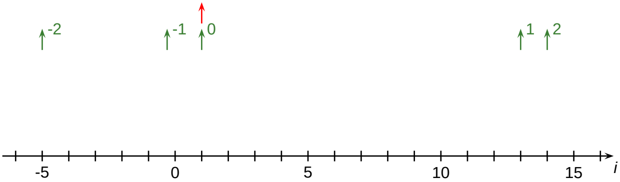

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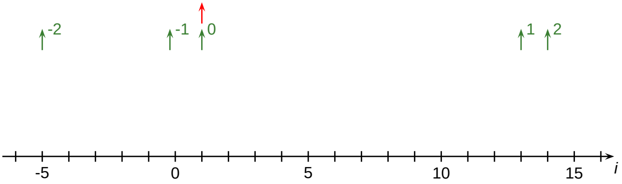

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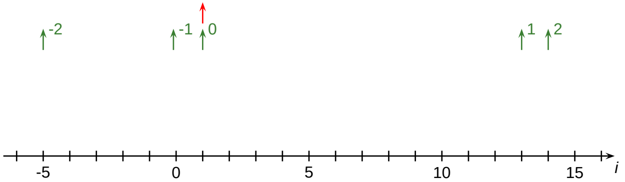

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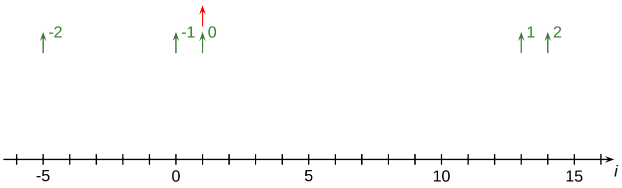

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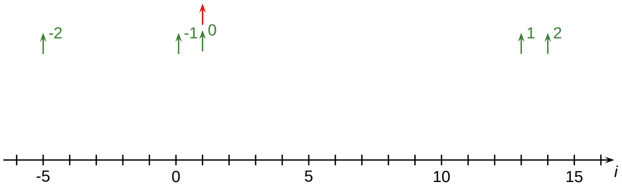

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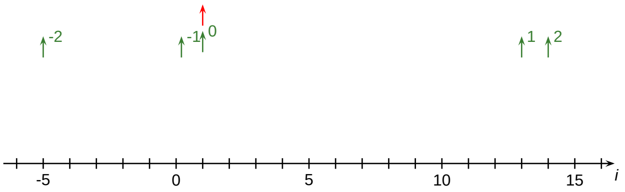

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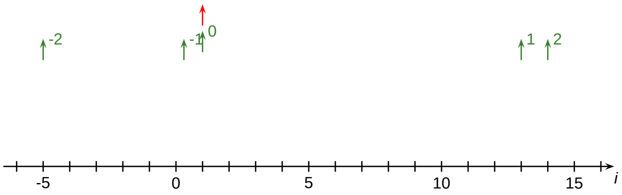

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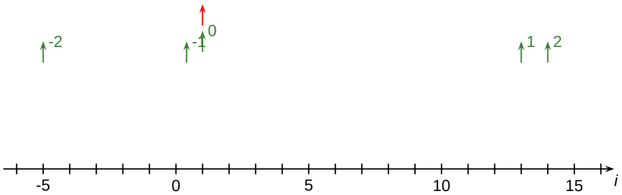

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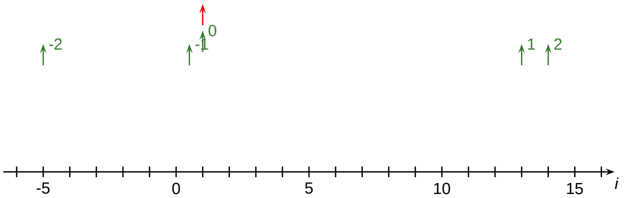

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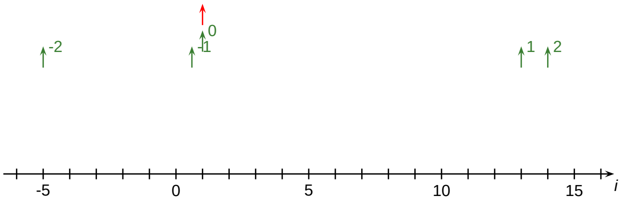

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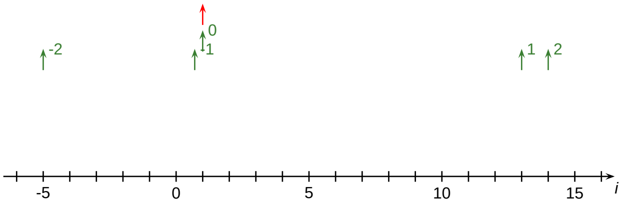

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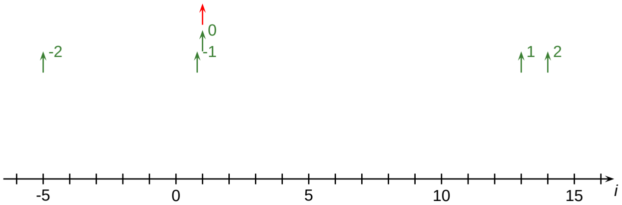

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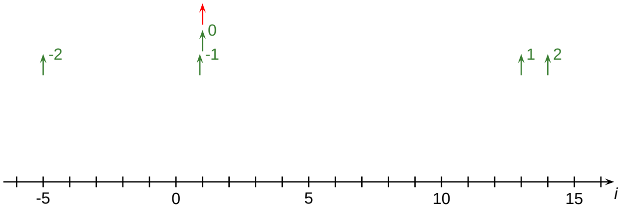

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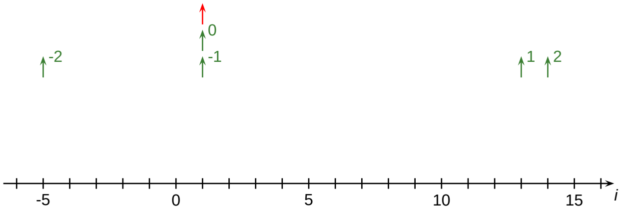

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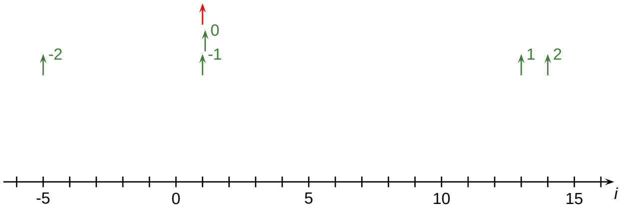

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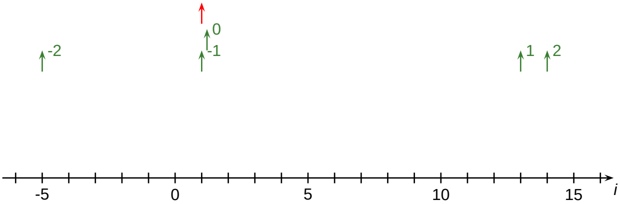

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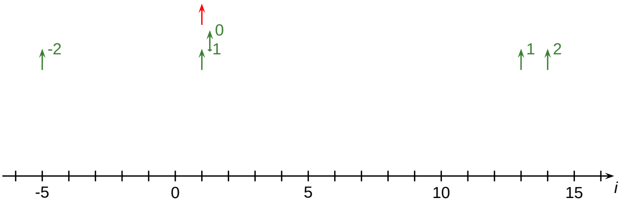

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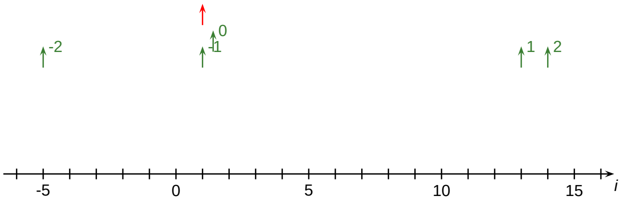

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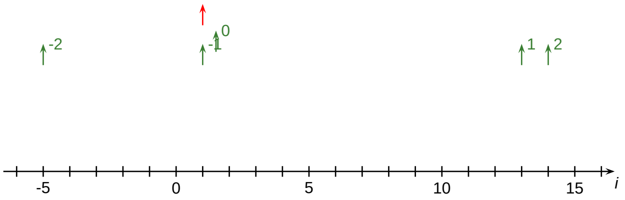

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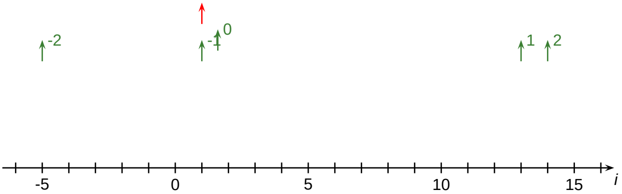

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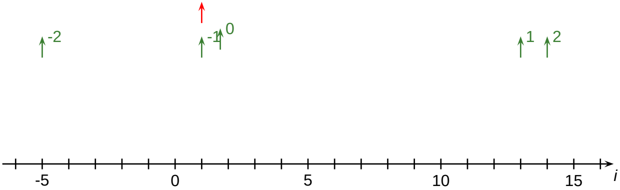

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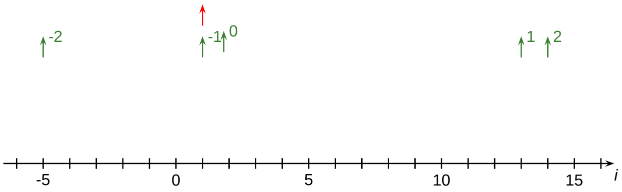

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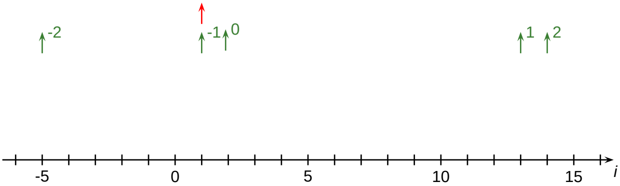

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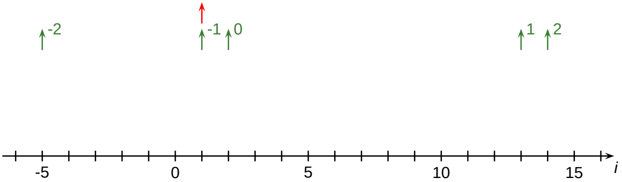

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This is the form of microscopic concavity we currently use: $m_Q(t)$ is dominated by a time-independent distribution with finite variance.

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Once this is done, we could proceed with less and less convex/concave models to see how $t^{1/3}$ scaling turns to $t^{1/4}$ for linear models (random average process, linear rate AZRP)...

Thank you.