Second class particles just walk sometimes

Joint with
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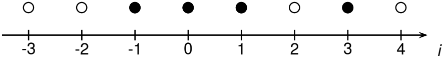
Asymmetric simple exclusion



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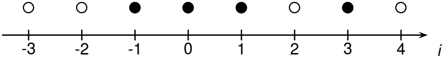

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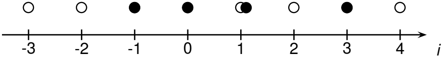

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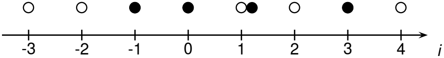

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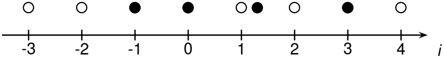

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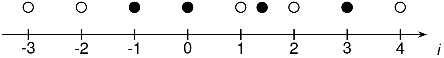

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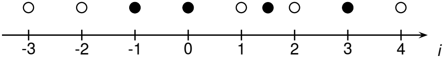

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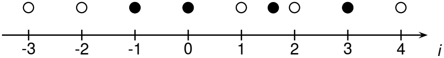

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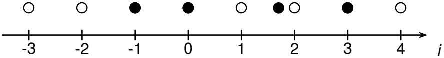

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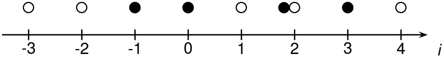

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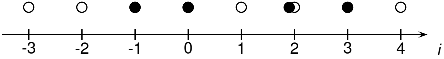

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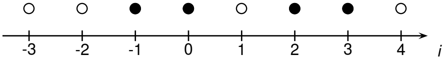

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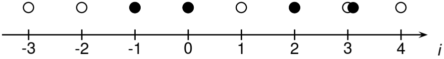

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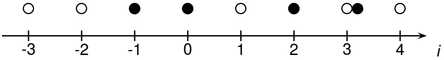

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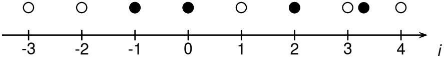

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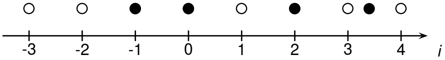

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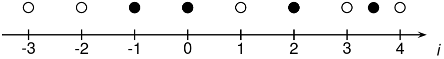

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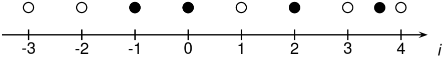

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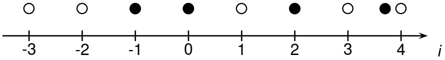

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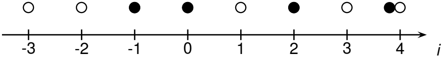

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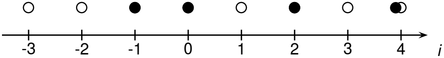

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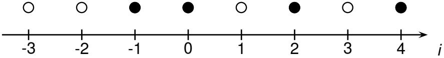

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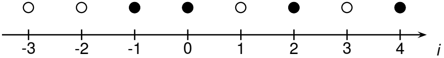

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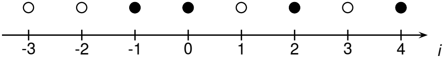

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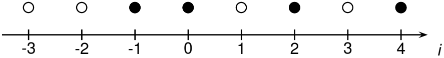

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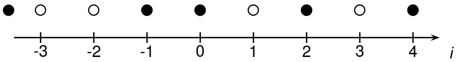

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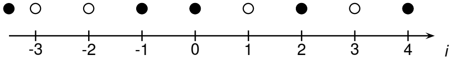

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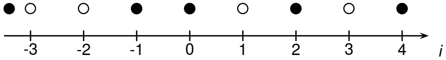

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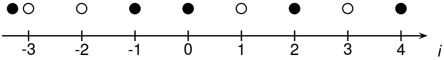

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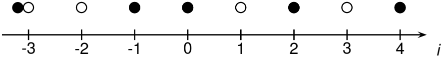

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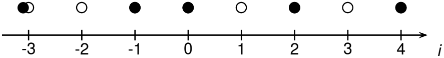

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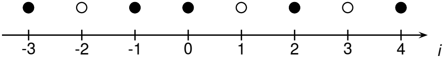

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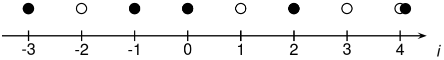

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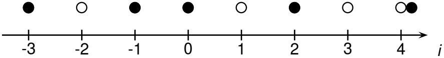

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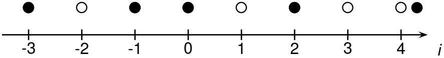

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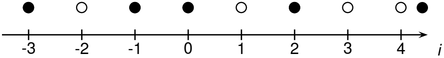

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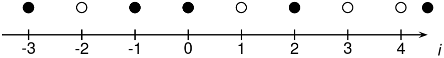

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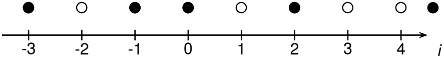

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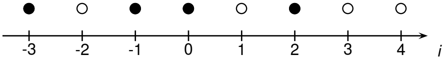

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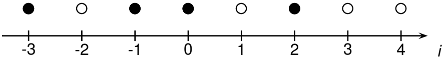

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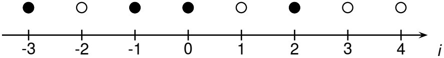

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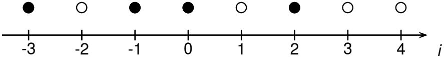

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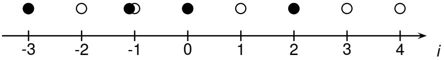

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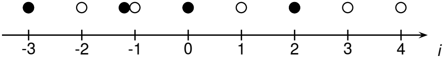

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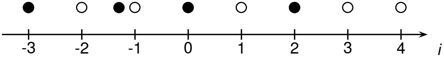

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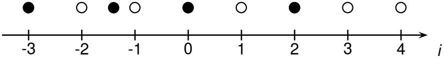

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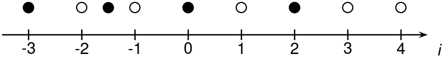

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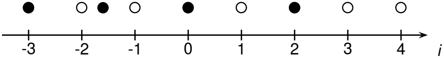

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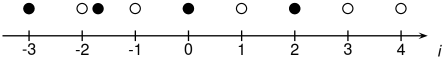

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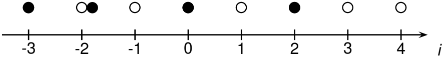

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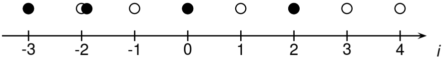

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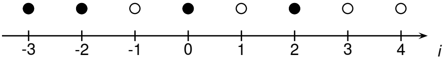
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



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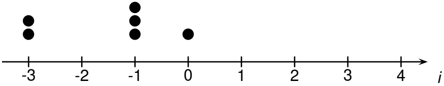
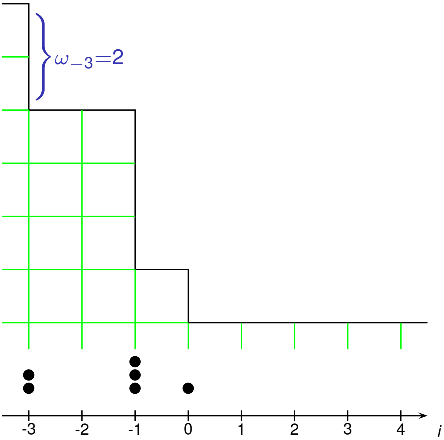
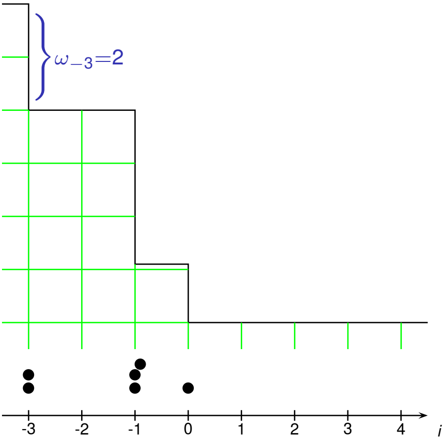
The totally asymmetric zero range process



The totally asymmetric zero range process



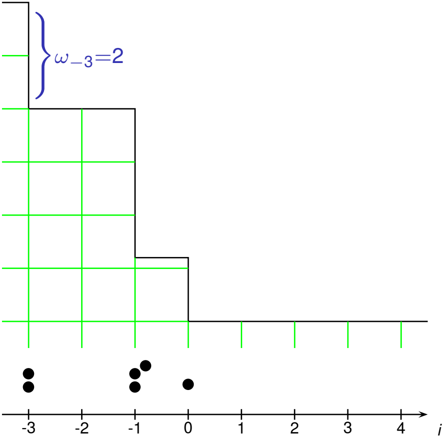
The totally asymmetric zero range process



Particles jump to the right from site i with rate $r(\omega_i)$

(r non-decreasing).

The totally asymmetric zero range process



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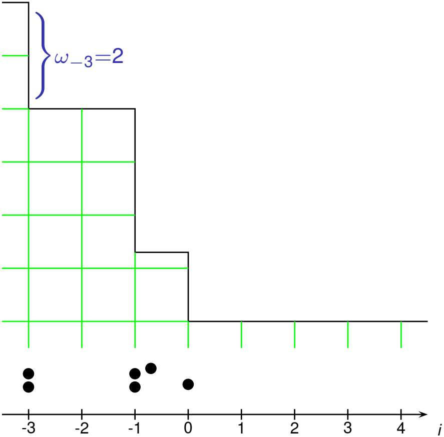

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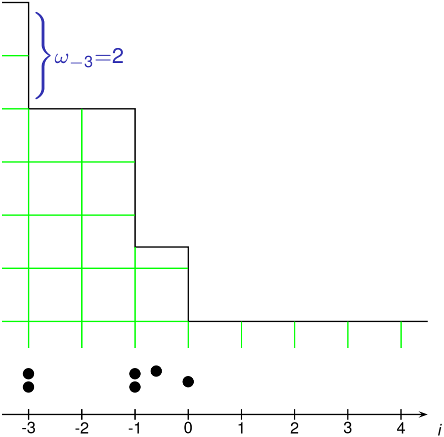

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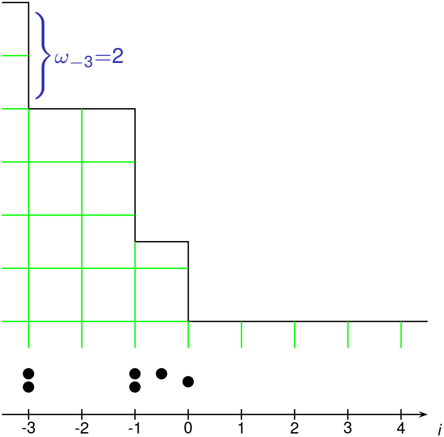

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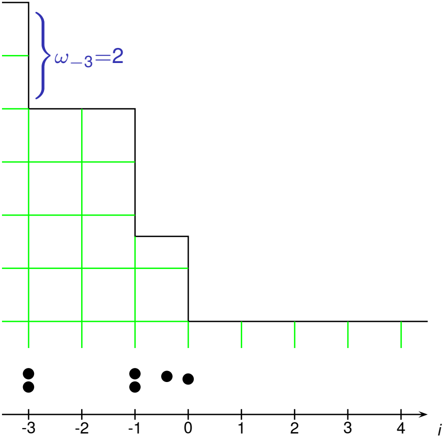

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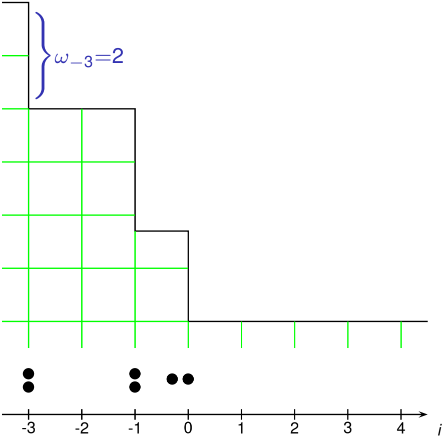

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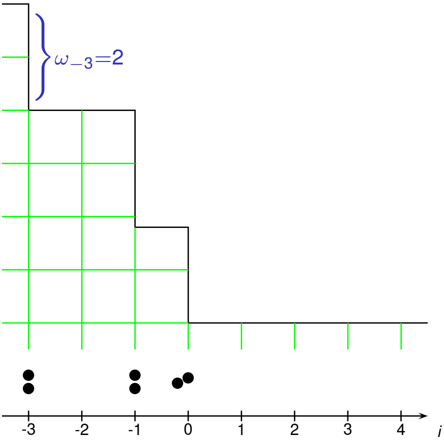

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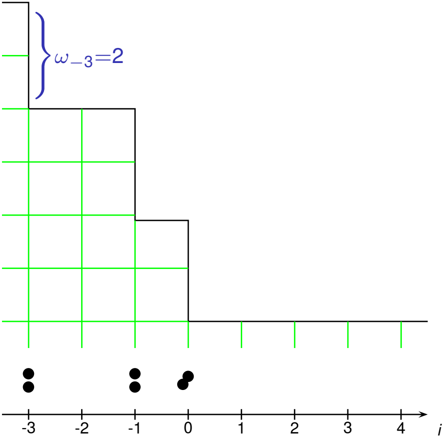

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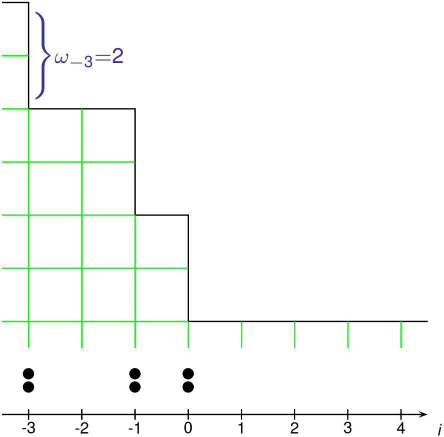

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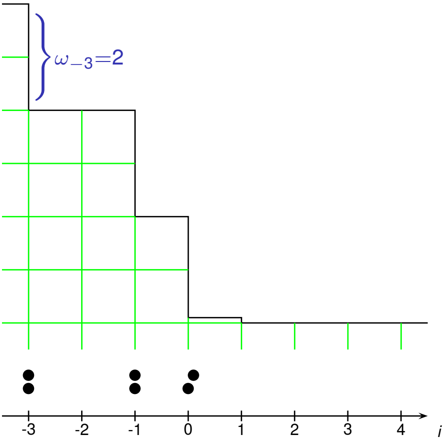

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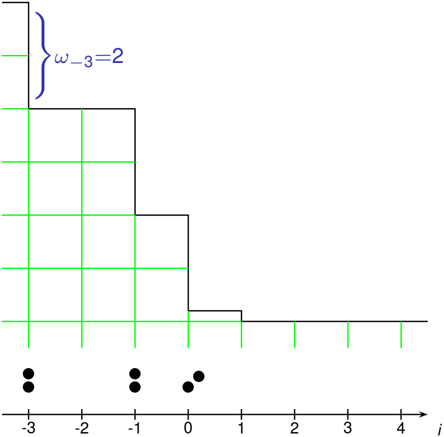

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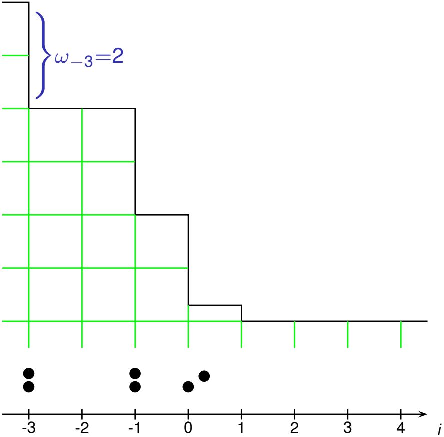

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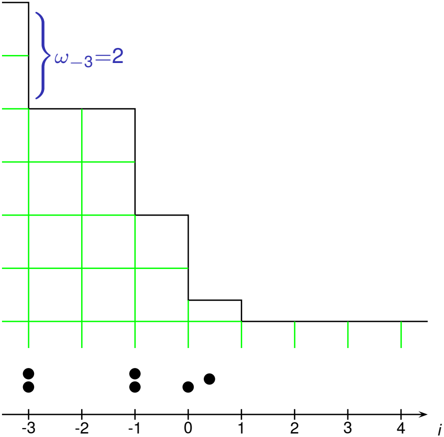

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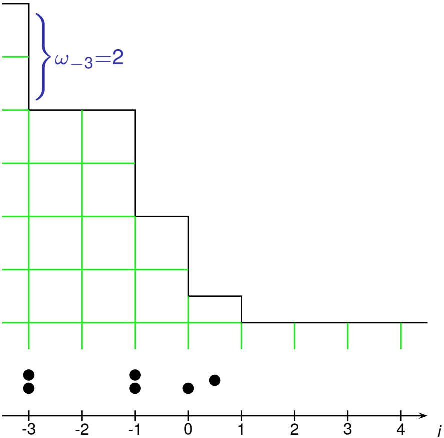

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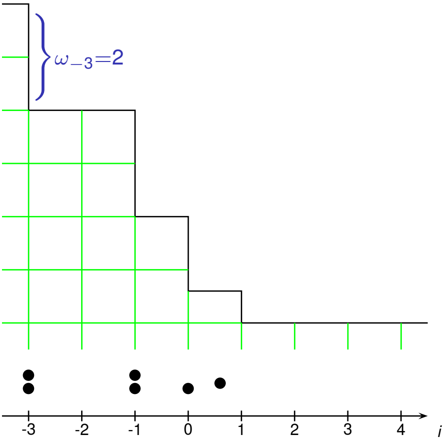

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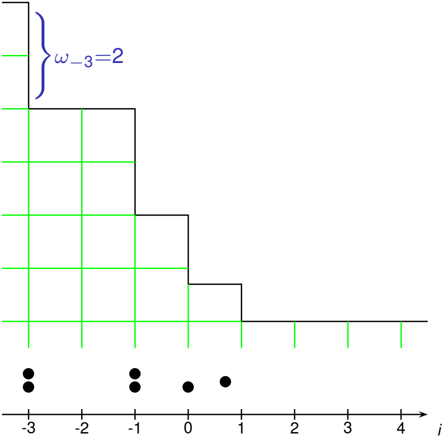

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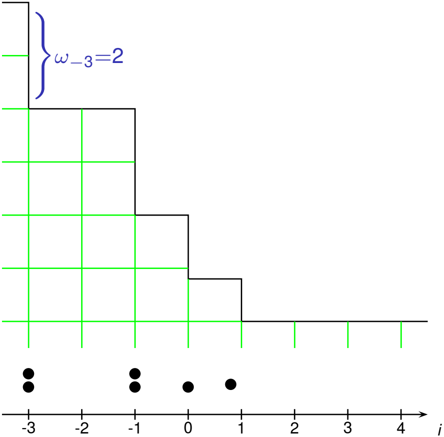

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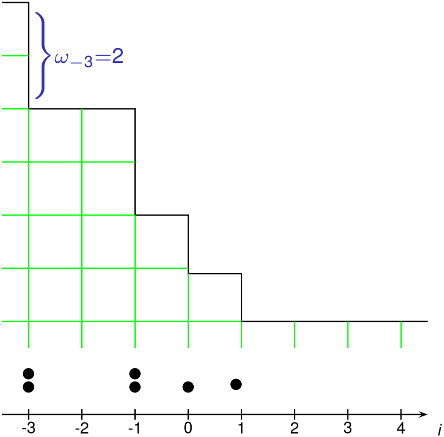

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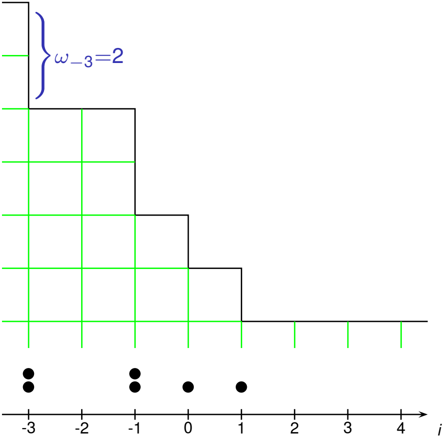

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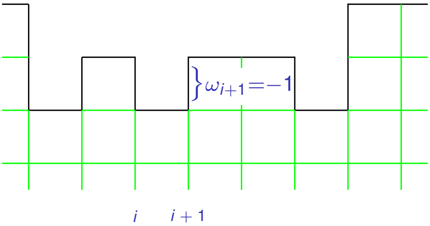
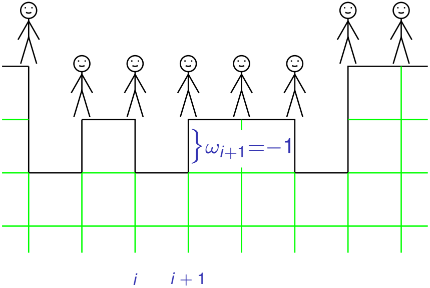
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \\ \| \\ \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ || \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $r(\omega_i)$.

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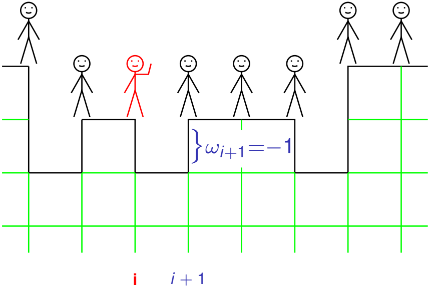
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $\mathbf{r}(\omega_\mathbf{i})$.

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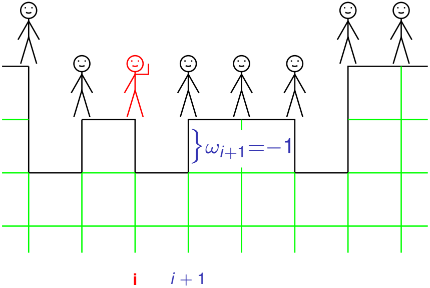
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



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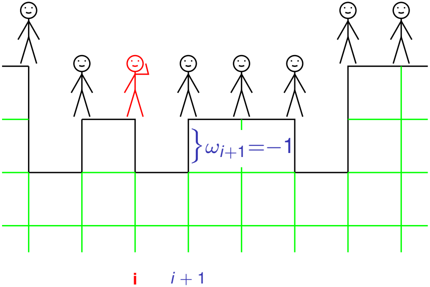

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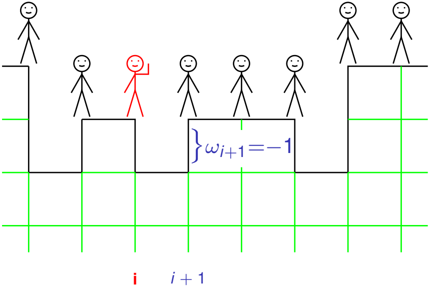

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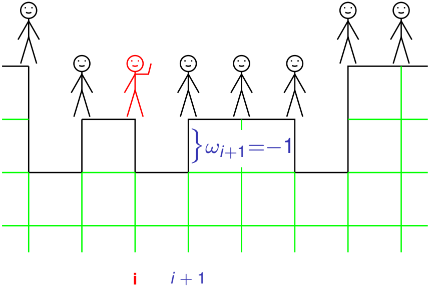

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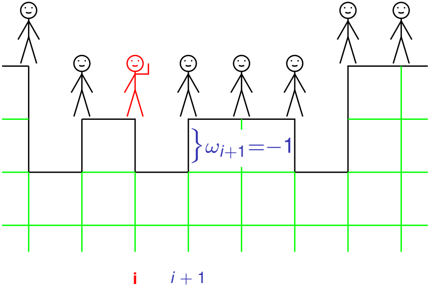

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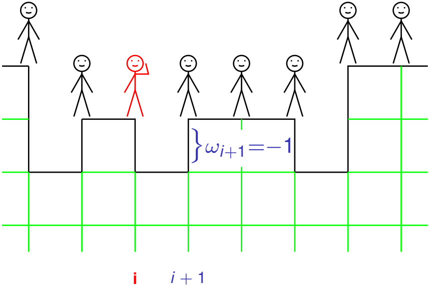

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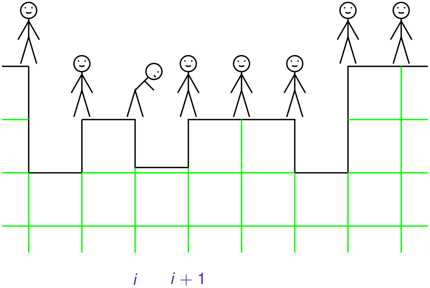

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$
$$\| \qquad\qquad \|$$
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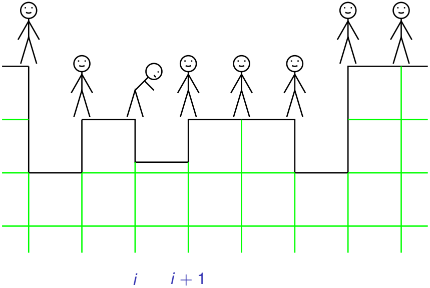

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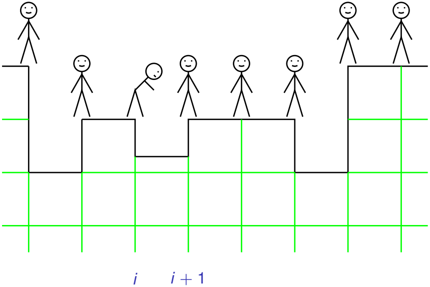

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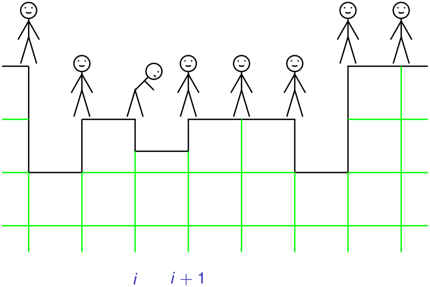

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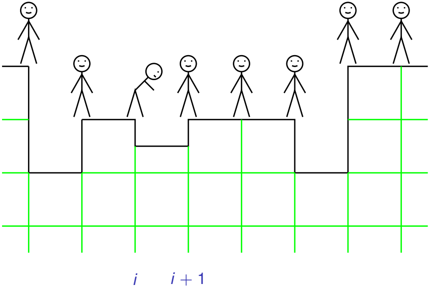

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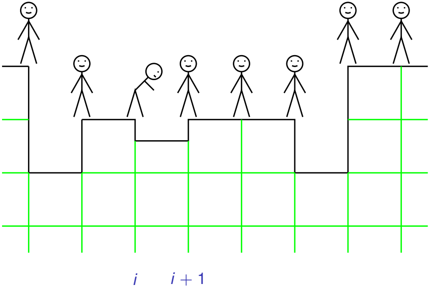

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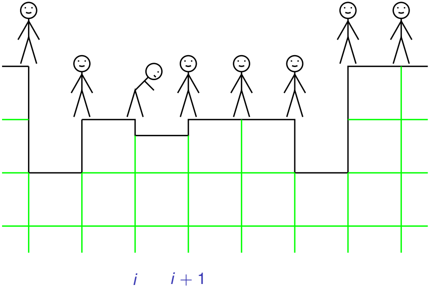

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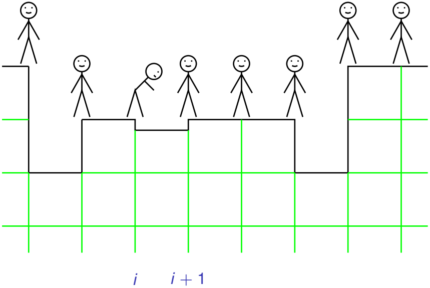

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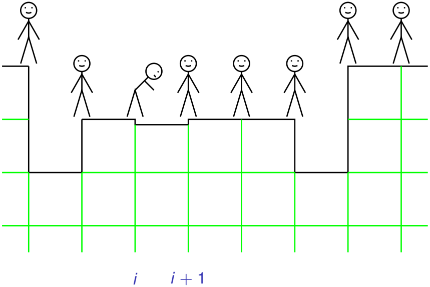

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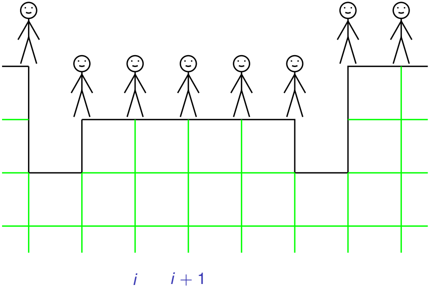

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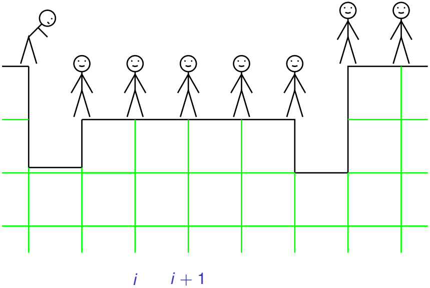

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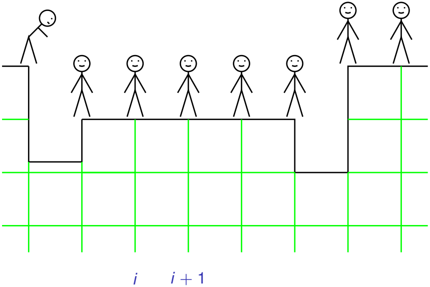

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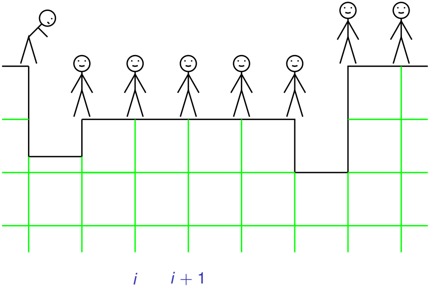

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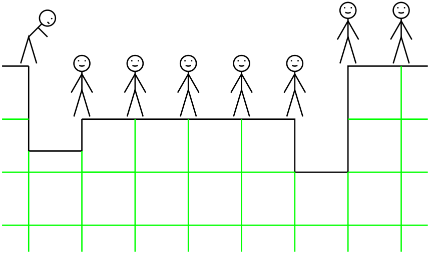

i $\quad$ $i+1$

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\| \qquad\qquad \|$$

$$\begin{pmatrix}1\\ -1\end{pmatrix} \to \begin{pmatrix}0\\ 0\end{pmatrix}$$

a brick is added with rate $r(\omega_i)$.

(r non-decreasing).

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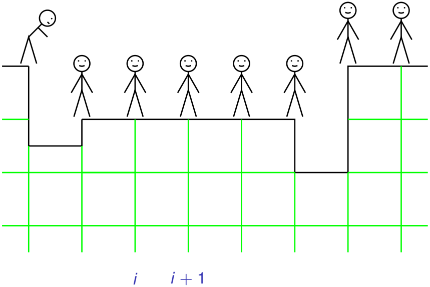

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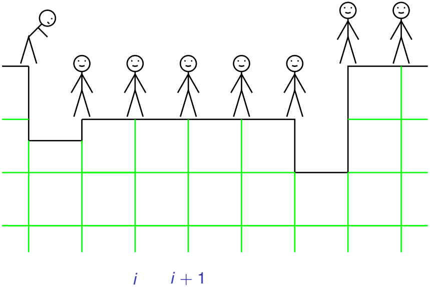

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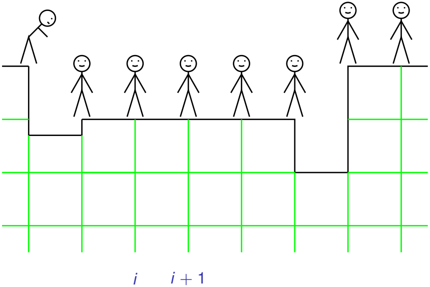

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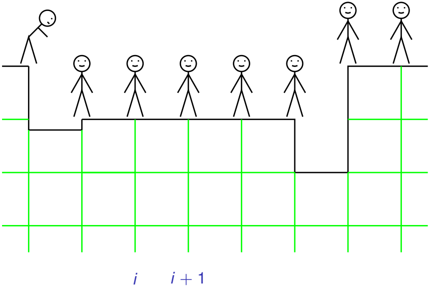

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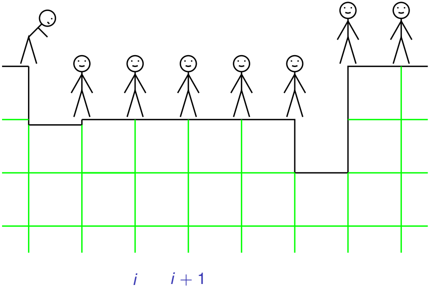

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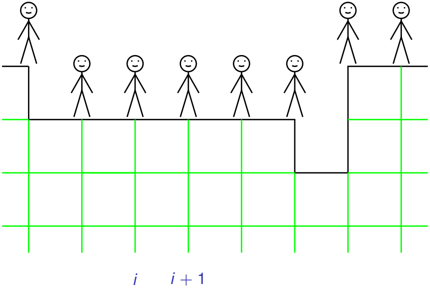

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1} + 1\end{pmatrix}$$

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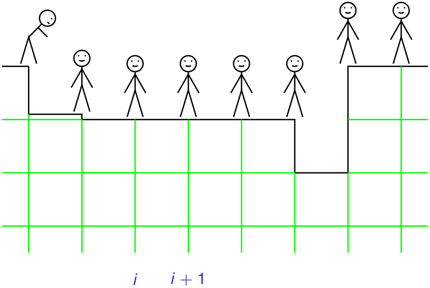

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$
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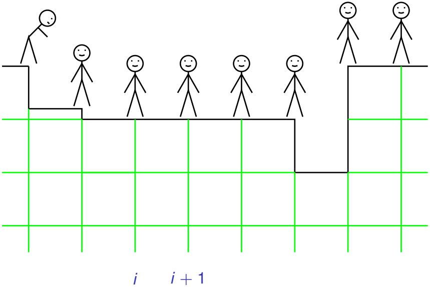

$$\begin{tikzcd} \begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \arrow[r] \arrow[d, equal] & \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix} \arrow[d, equal] \\ \begin{pmatrix}1\\ -1\end{pmatrix} \arrow[r] & \begin{pmatrix}0\\ 0\end{pmatrix} \end{tikzcd}$$

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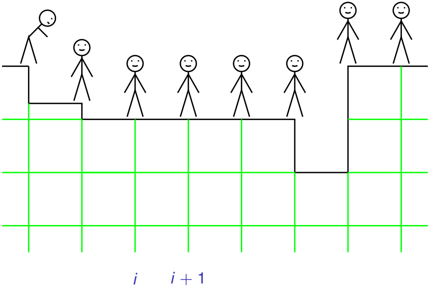

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \rightarrow \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\| \qquad\qquad \|$$

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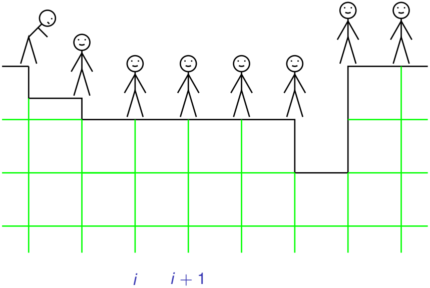

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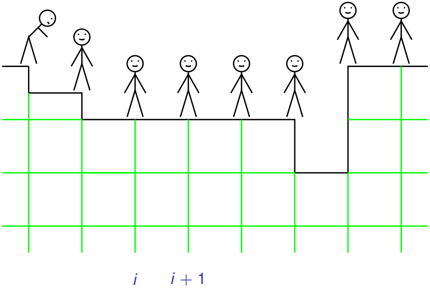

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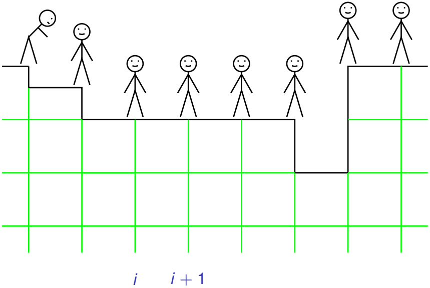

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1} + 1\end{pmatrix}$$

$$\Vert \qquad\qquad \Vert$$

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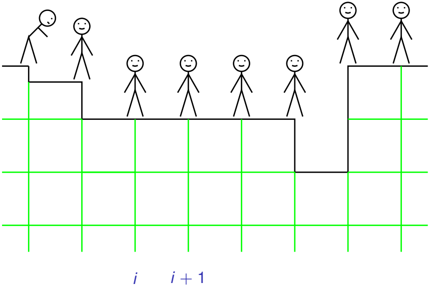

$$\begin{array}{ccc} \begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} & \to & \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}\\ \| & & \| \\ \begin{pmatrix}1\\ -1\end{pmatrix} & \to & \begin{pmatrix}0\\ 0\end{pmatrix}\end{array}$$

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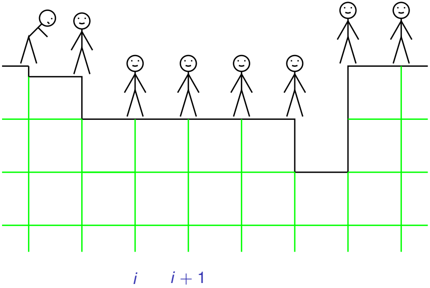

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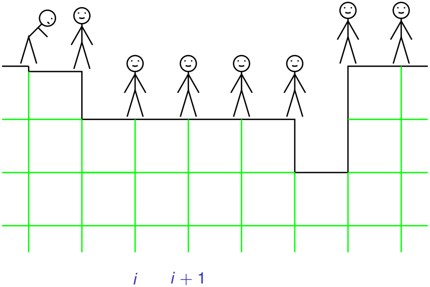

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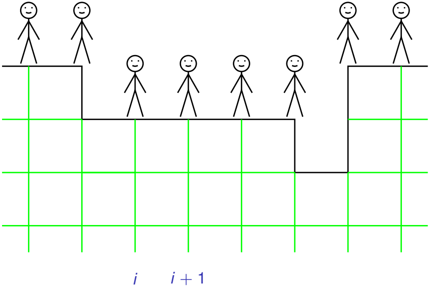

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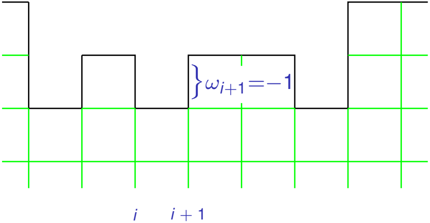
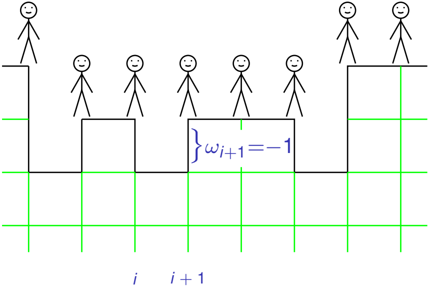
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ || \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

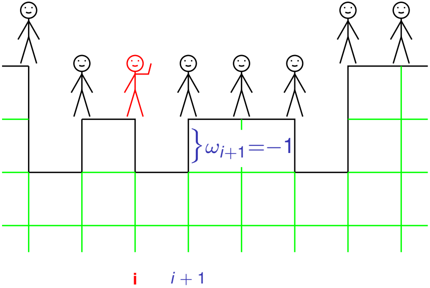
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $r(\omega_i)+r(-\omega_{i+1})$.

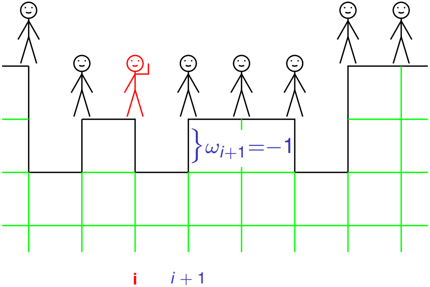
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ \| \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $\mathbf{r}(\omega_\mathbf{i}) + r(-\omega_{i+1})$.

The totally asymmetric bricklayers process



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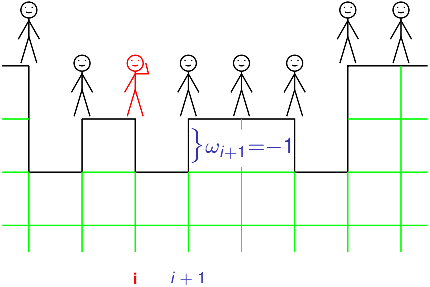

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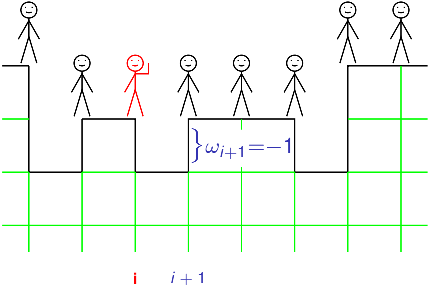

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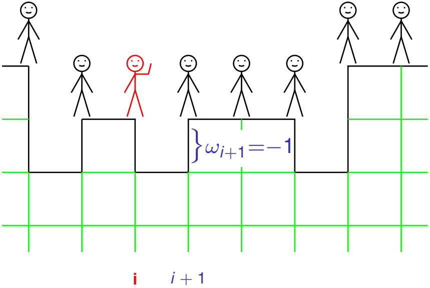

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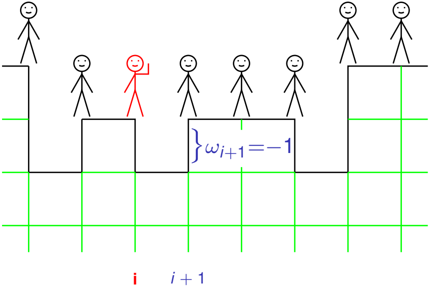

$$\begin{array}{c}\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix}\\ \| \\ \begin{pmatrix}1\\ -1\end{pmatrix}\end{array}$$

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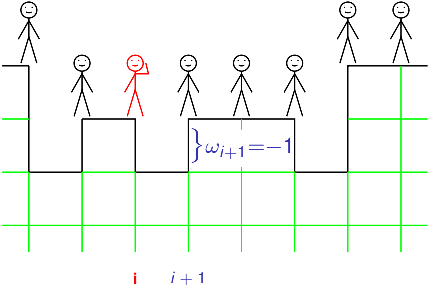

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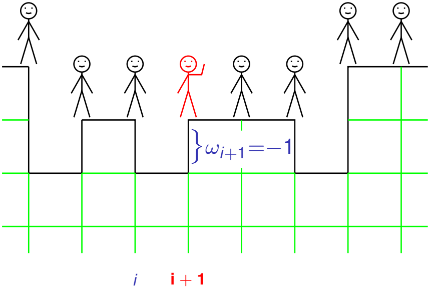

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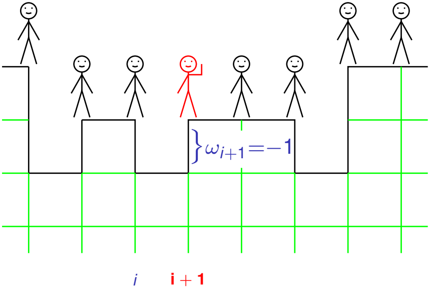

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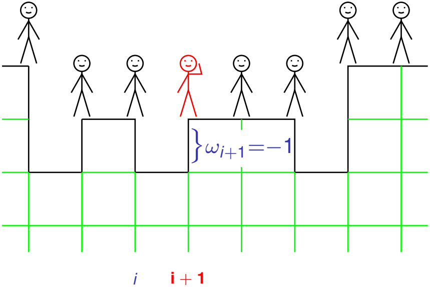

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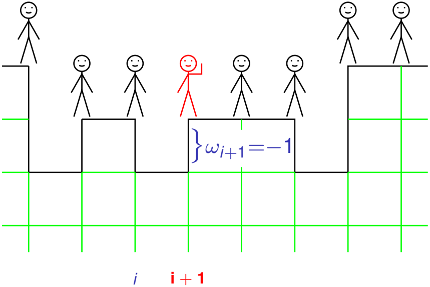

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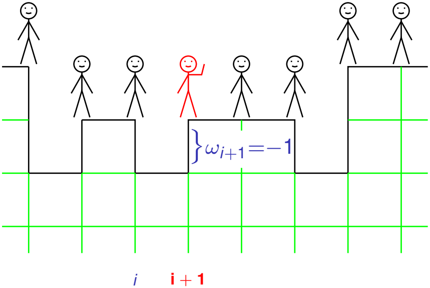

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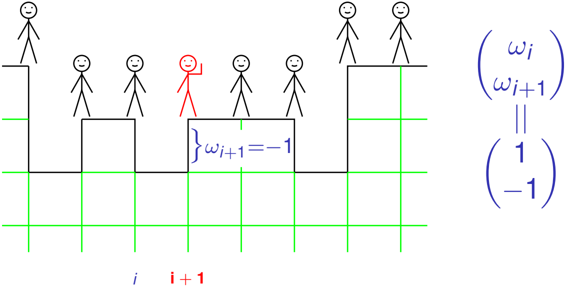

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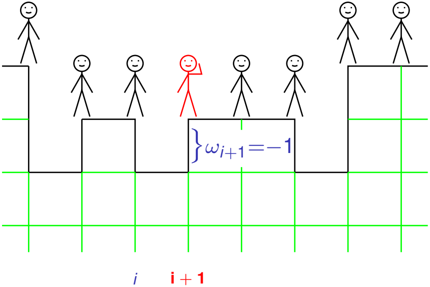

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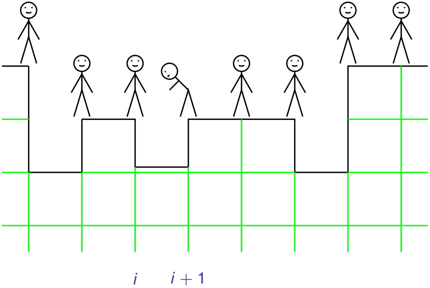

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\begin{tikzcd}\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \arrow[r] \arrow[d, phantom, "\|"] & \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix} \arrow[d, phantom, "\|"] \\ \begin{pmatrix}1\\ -1\end{pmatrix} \arrow[r] & \begin{pmatrix}0\\ 0\end{pmatrix}\end{tikzcd}$$

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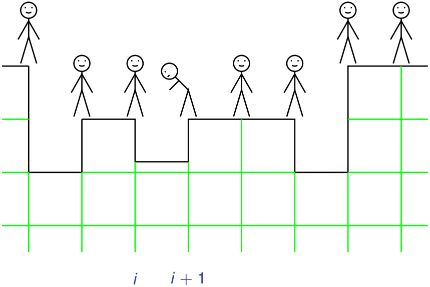
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}$$

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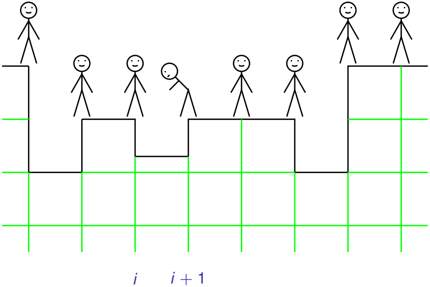

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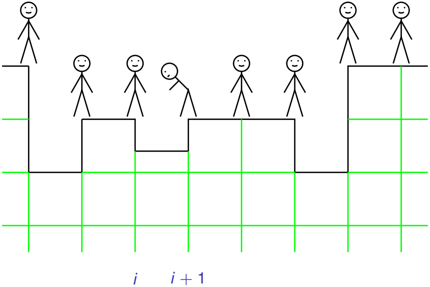

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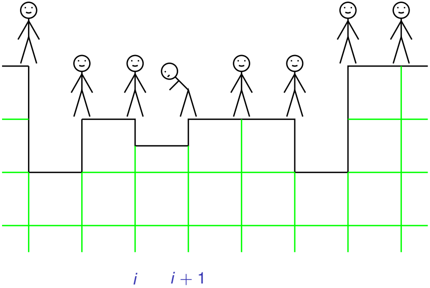

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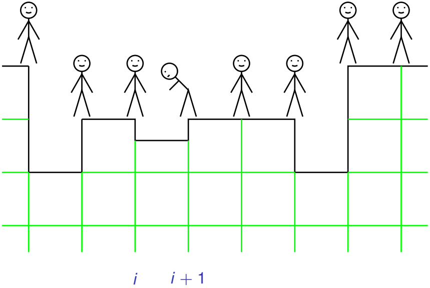

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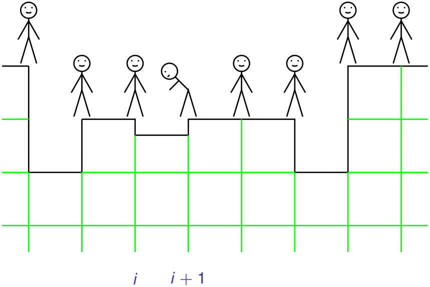

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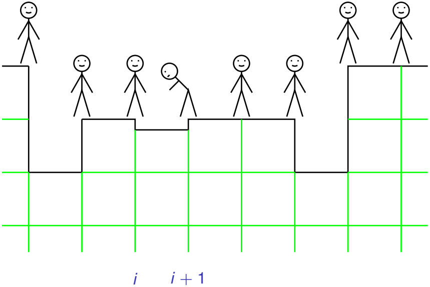

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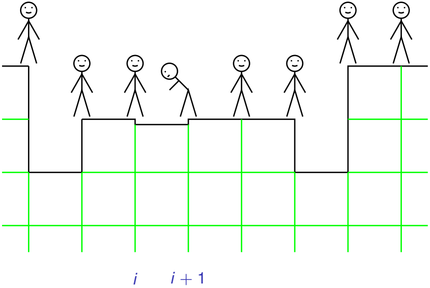

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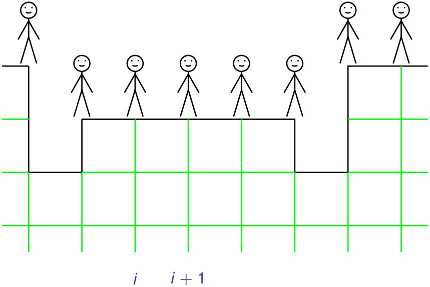

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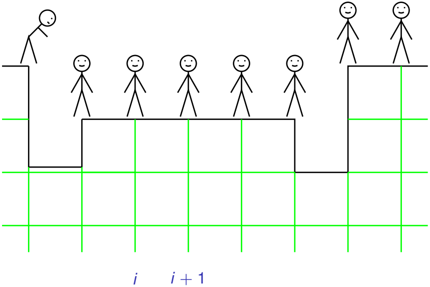

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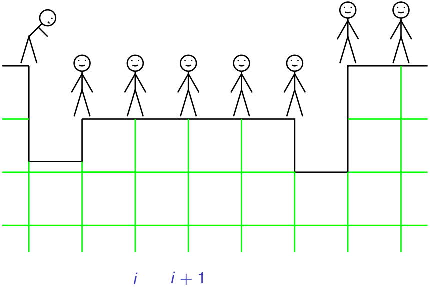

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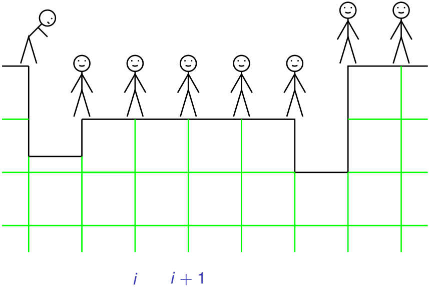

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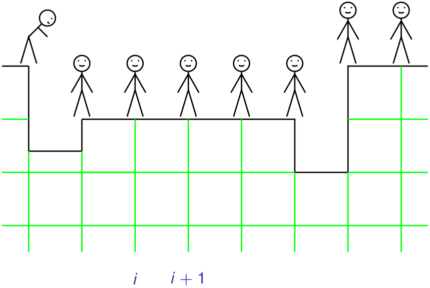

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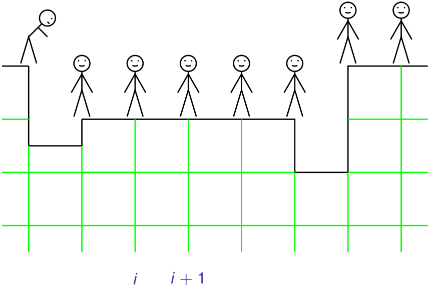

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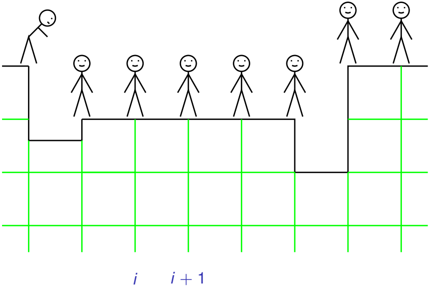

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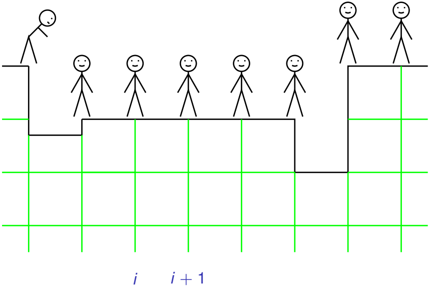

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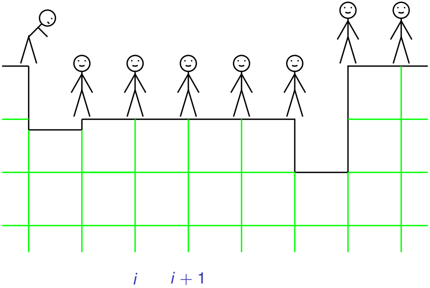

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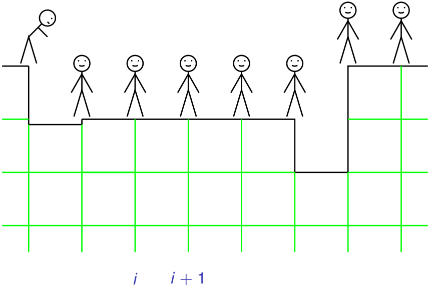

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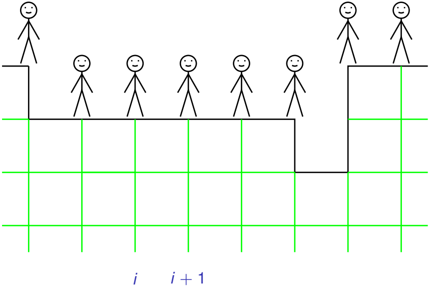

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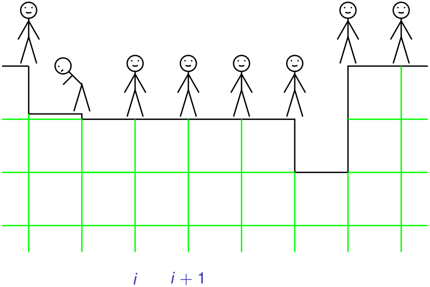

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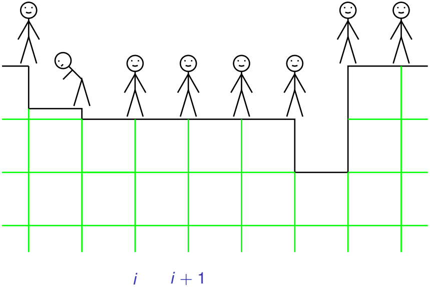

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(with $\|$ between the upper and lower matrices in each column)

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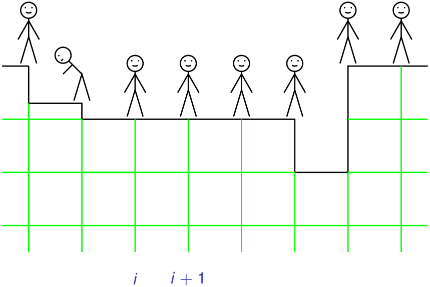

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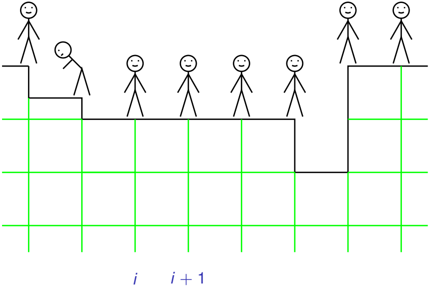

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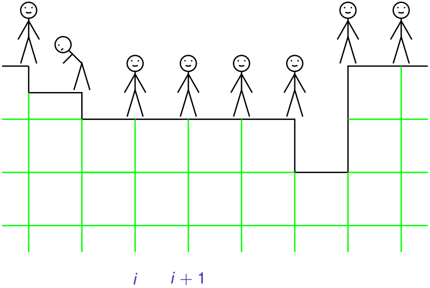

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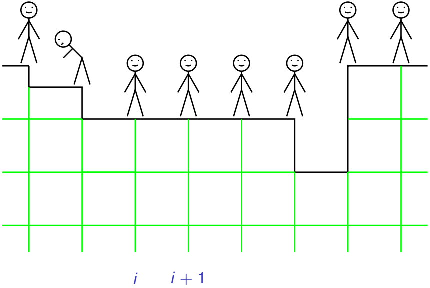

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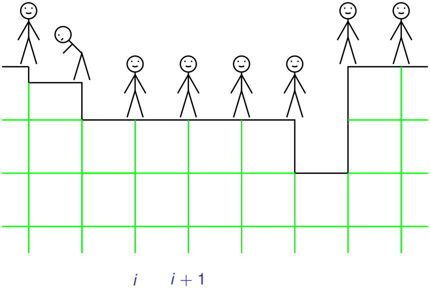

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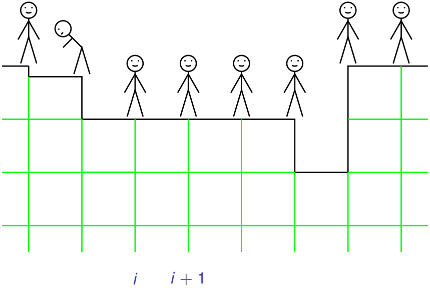

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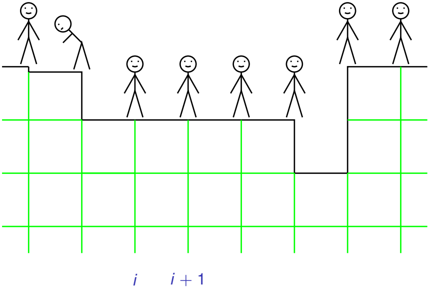

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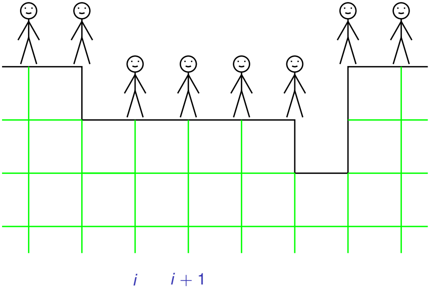

$$\begin{tikzcd}
\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \arrow[r] \arrow[d, equal] & \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix} \arrow[d, equal] \\
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A mirror-symmetrized version of the extended zero range. Left and right jumps of the dynamics cooperate, if $(r(\omega) \cdot r(1-\omega) = 1;\quad r$ non-decreasing).

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Here $r(0)! := 1$, and $r(z+1)! = r(z)! \cdot r(z+1)$ for all $z \in \mathbb{Z}$.

Hydrodynamics (very briefly)

The *density* $\varrho = \varrho(\theta) := \mathbf{E}^{\theta}(\omega)$ and the *hydrodynamic flux* $H = H^{\theta} := \mathbf{E}^{\theta}$[growth rate] both depend on a parameter ϱ or θ of the stationary distribution.

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For the ASEP, $H(\varrho) = (p - q) \cdot \varrho(1 - \varrho)$, concave.

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$\rightsquigarrow$ TAGEZRP, TAEBLP.

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$\rightsquigarrow$ Either convex or concave, discontinuous shock solutions exist.

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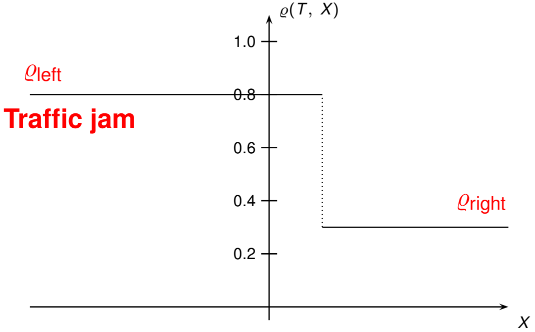
Rarefaction wave



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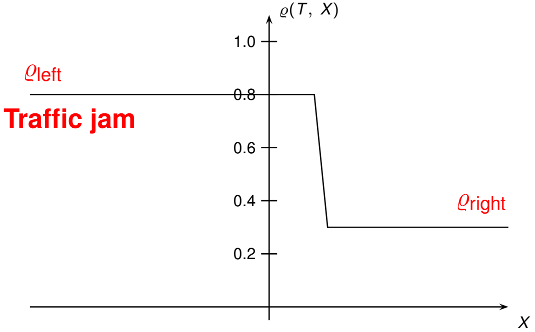

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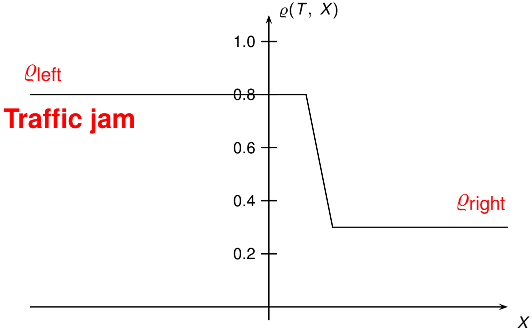

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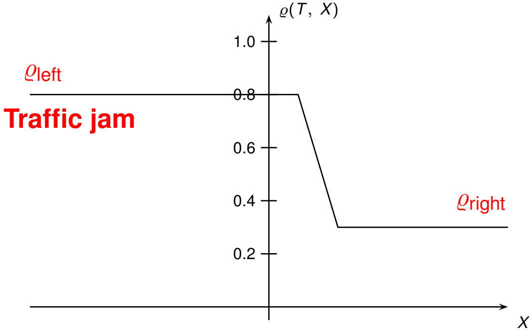

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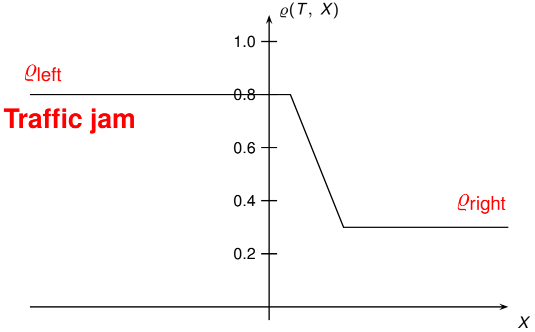
Rarefaction wave



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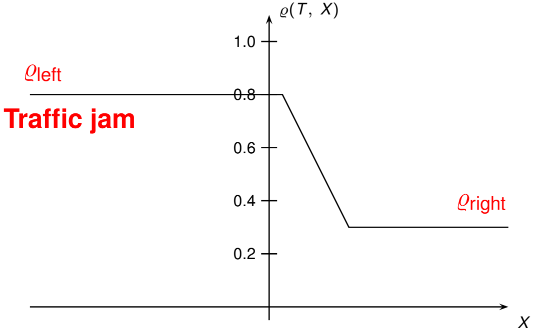
Rarefaction wave



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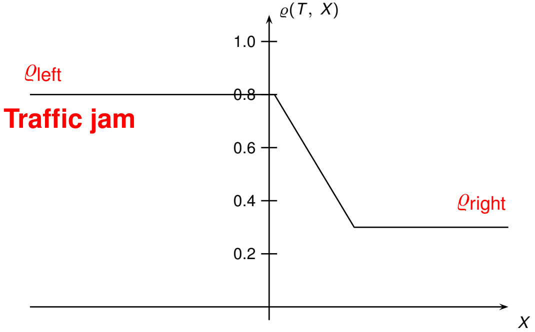
Rarefaction wave



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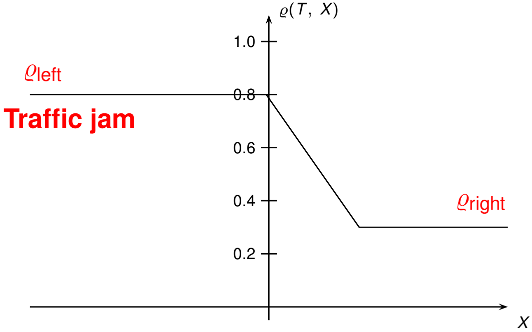
Rarefaction wave



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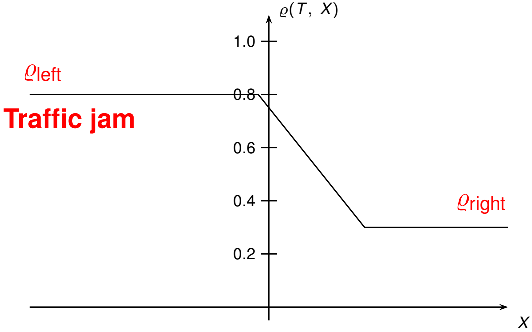
Rarefaction wave



Traffic jam

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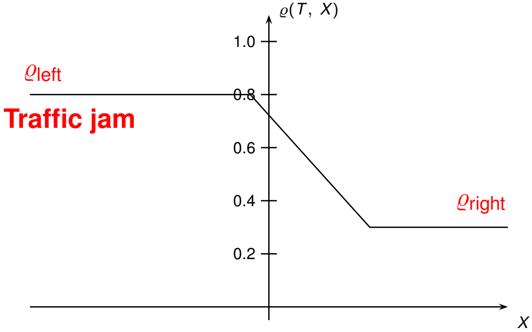
Rarefaction wave



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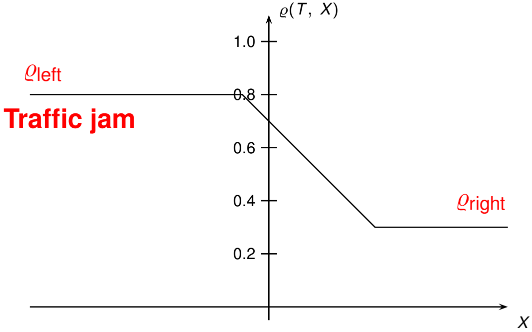
Rarefaction wave



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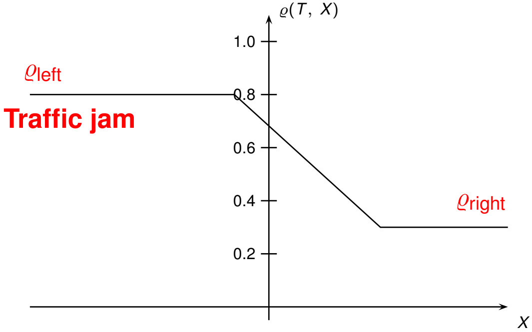

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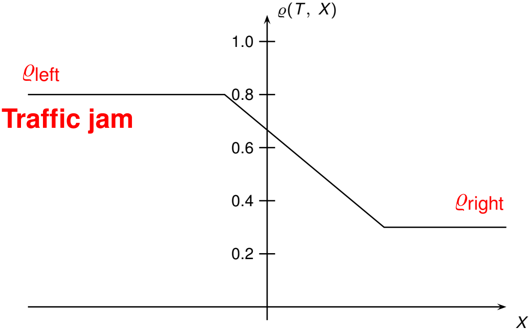
Rarefaction wave



Traffic jam

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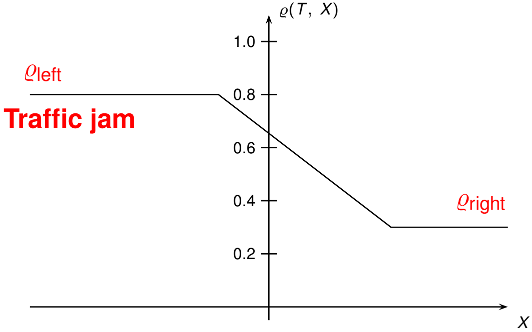
Rarefaction wave



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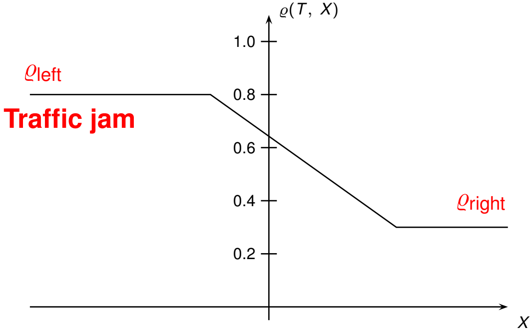
Rarefaction wave



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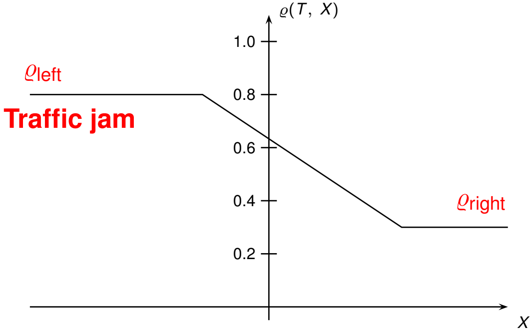
Rarefaction wave



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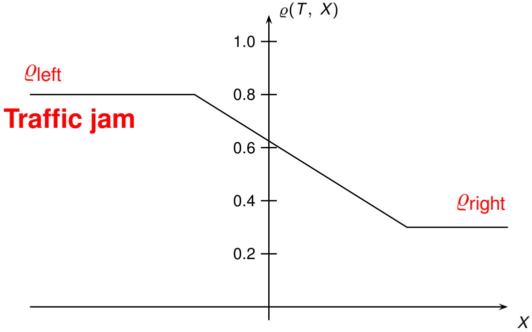
Rarefaction wave



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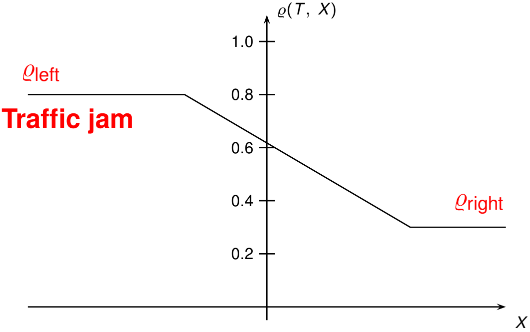

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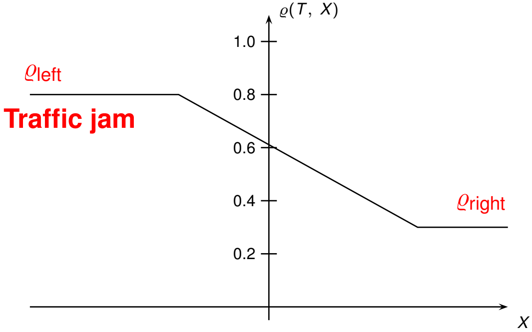
Rarefaction wave



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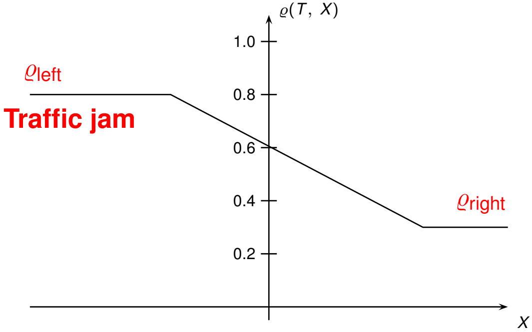

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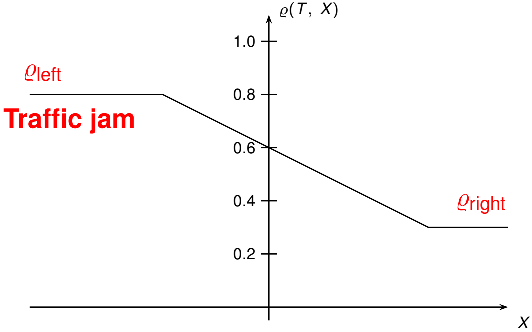

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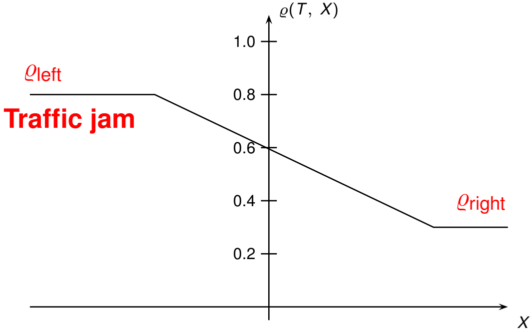
Rarefaction wave



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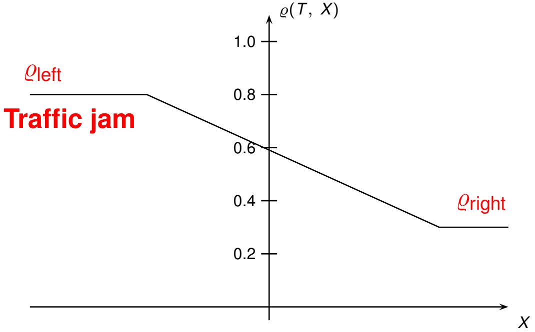
Rarefaction wave



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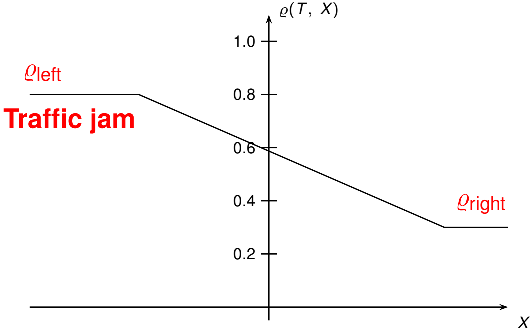
Rarefaction wave



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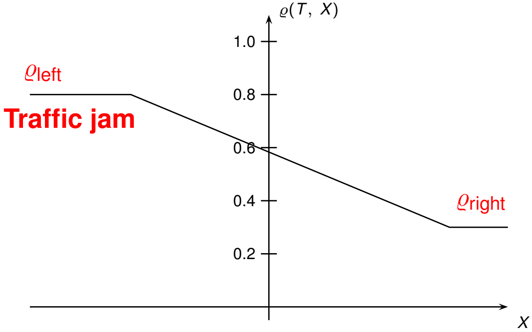
Rarefaction wave



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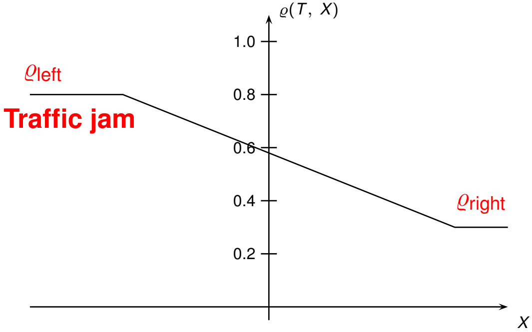
Rarefaction wave



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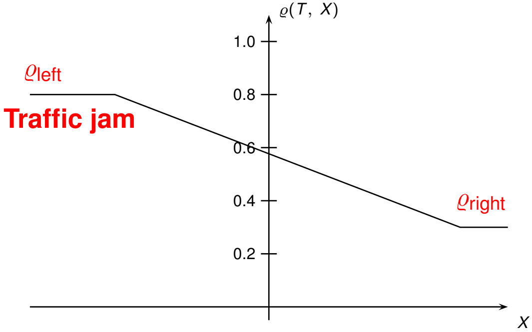
Rarefaction wave



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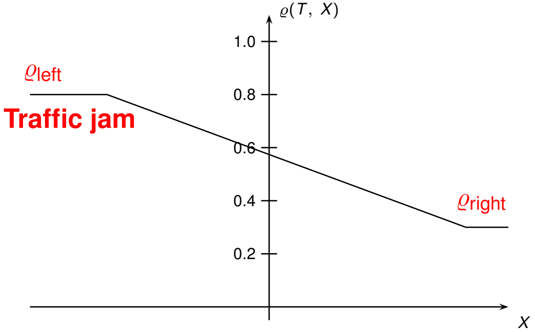
Rarefaction wave



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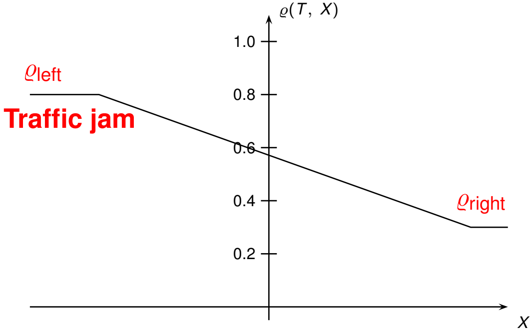
Rarefaction wave



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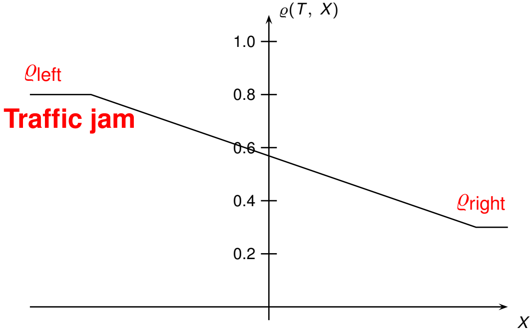
Rarefaction wave



Traffic jam

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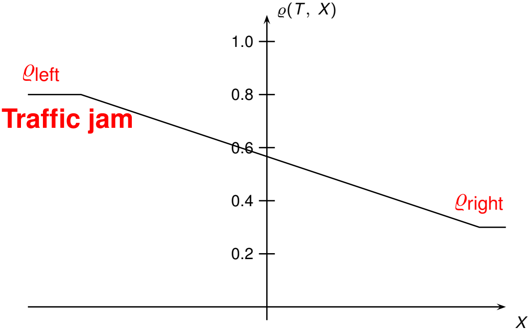
Rarefaction wave



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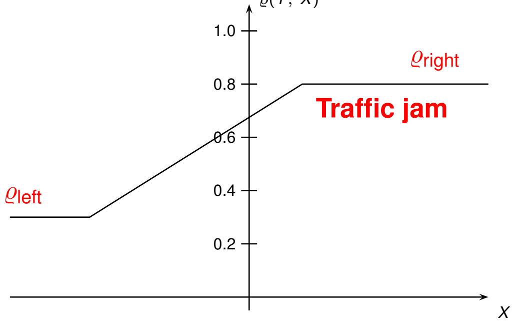
Shock wave



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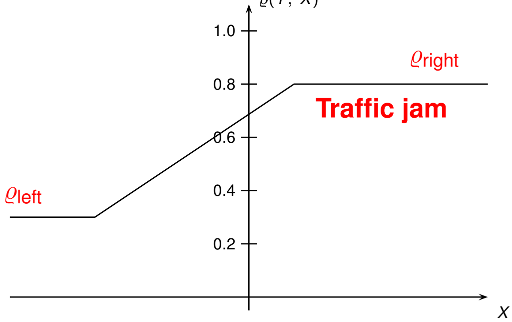
Shock wave



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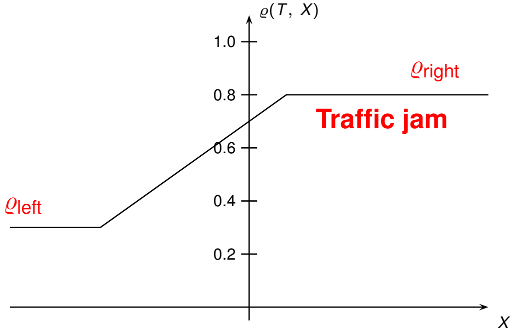
Shock wave



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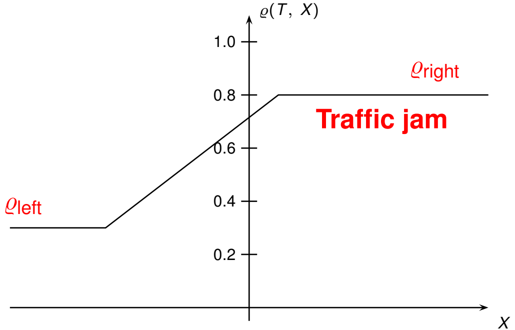
Shock wave



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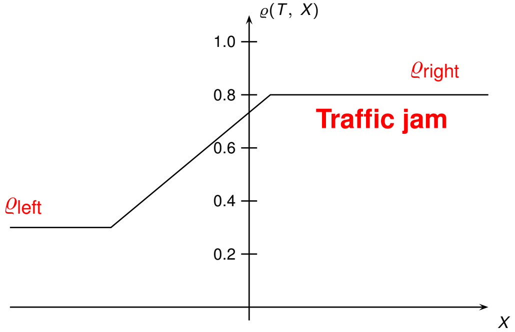
Shock wave



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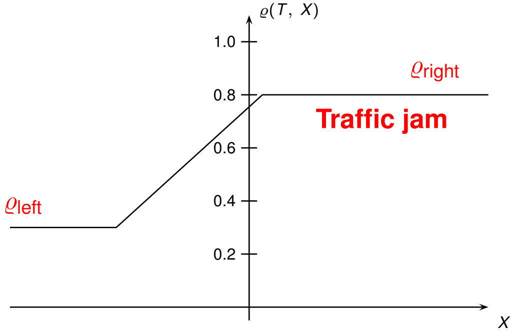

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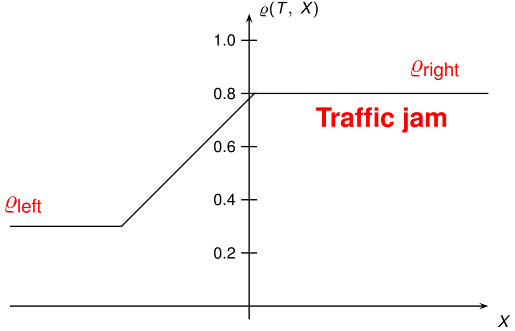

Traffic jam

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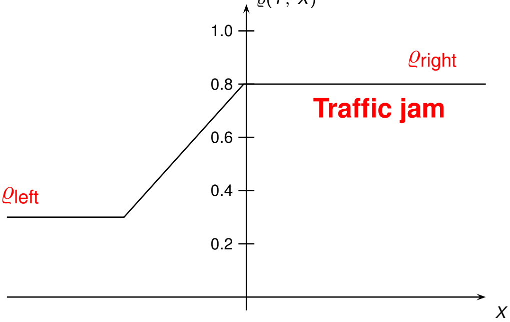
Shock wave



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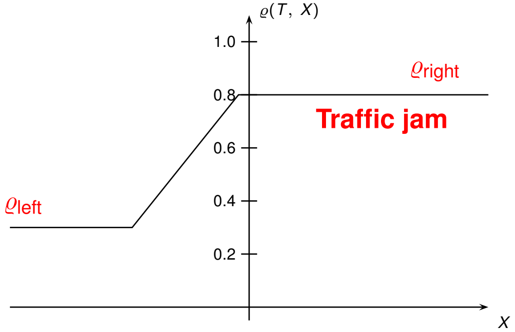
Shock wave



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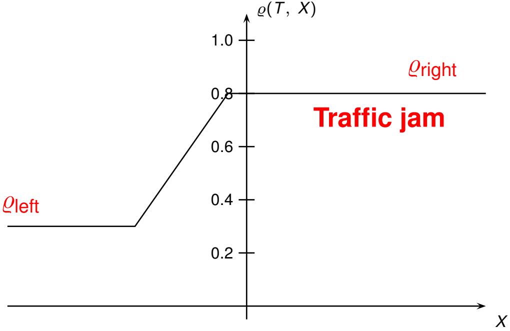

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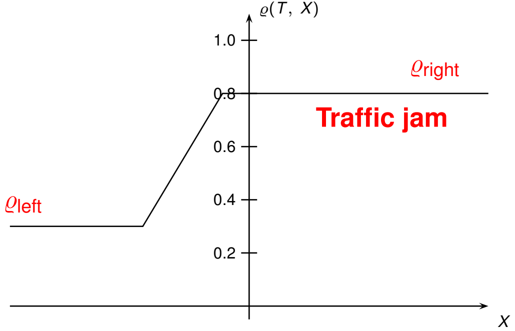

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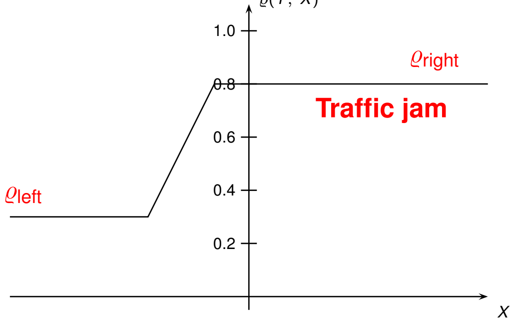
Shock wave



Traffic jam

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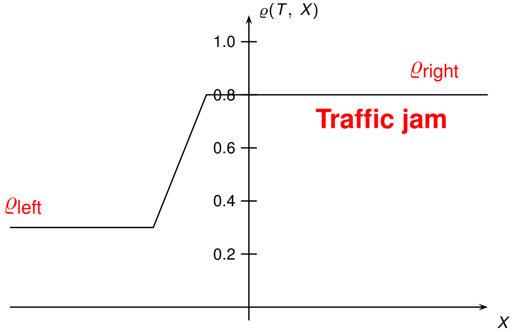
Shock wave



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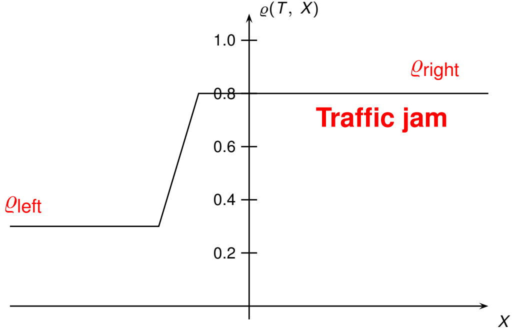
Shock wave



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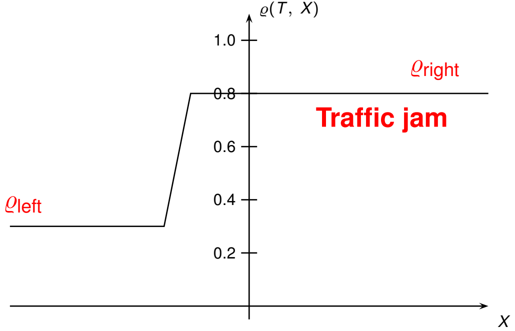
Shock wave



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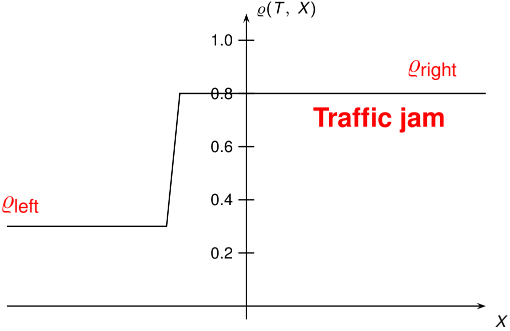
Shock wave



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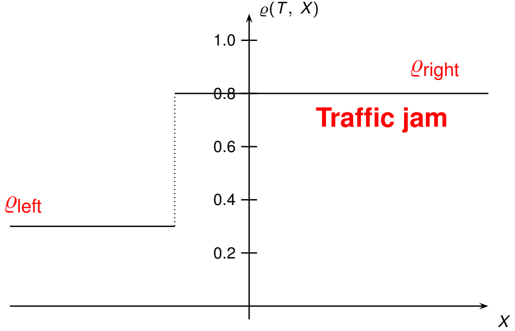
Shock wave



Traffic jam

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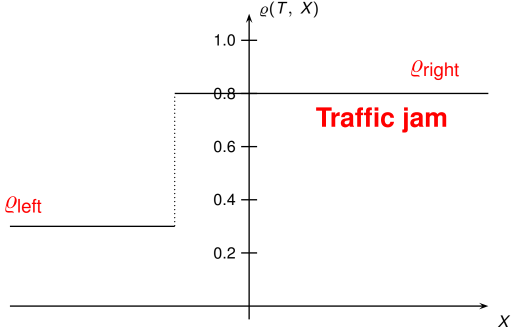
Shock wave



Traffic jam

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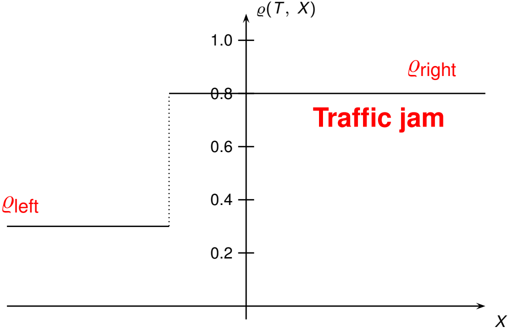
Shock wave



Traffic jam

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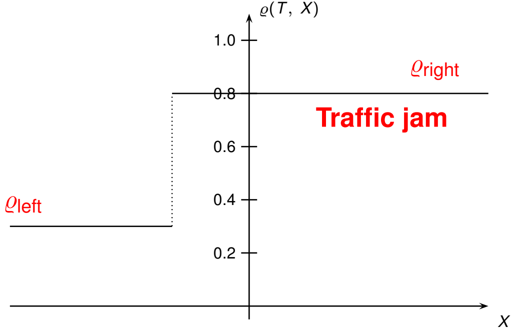
Shock wave



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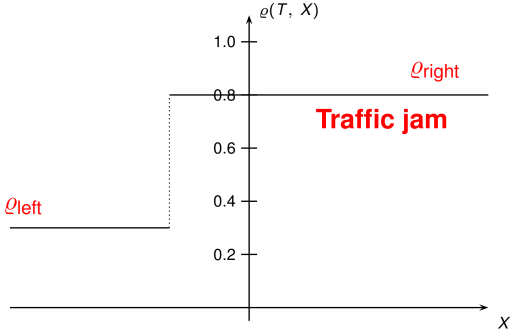
Shock wave



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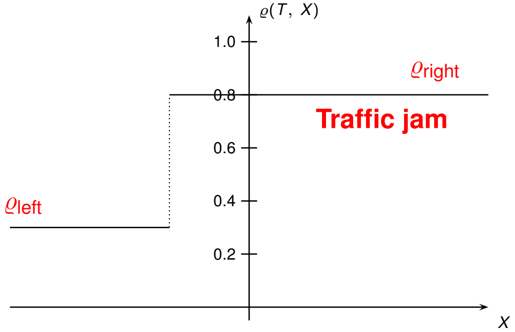
Shock wave



Traffic jam

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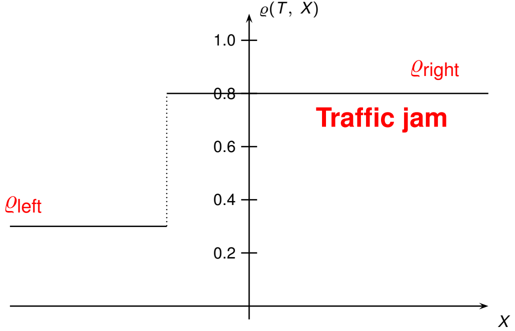

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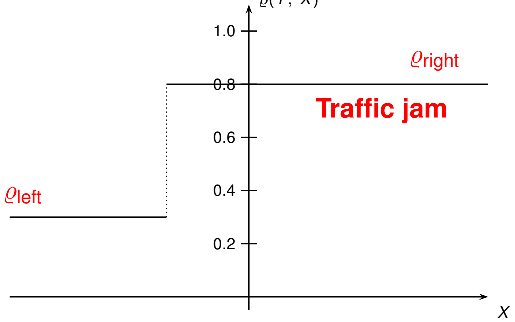

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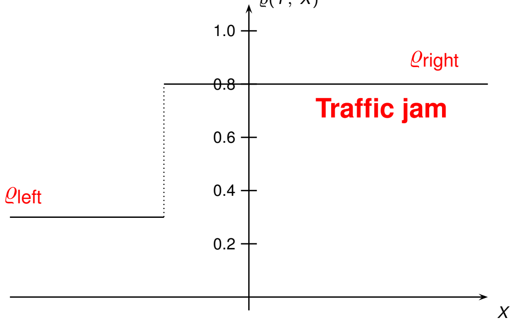

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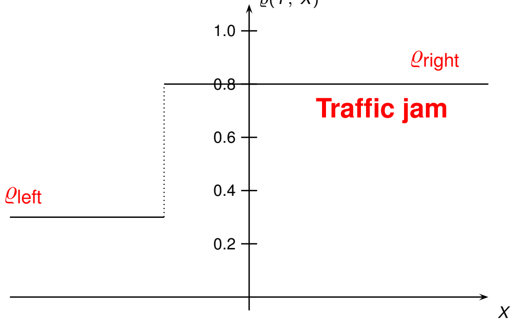

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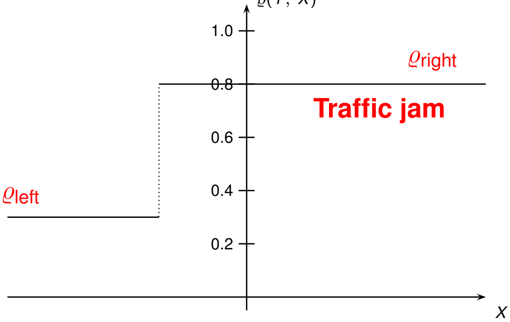

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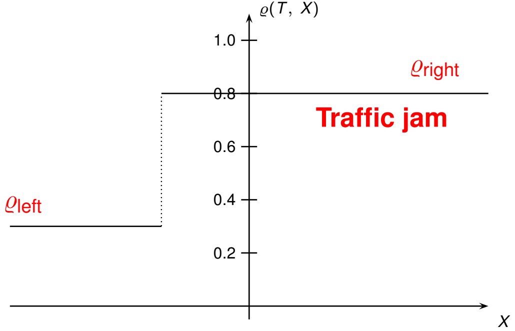

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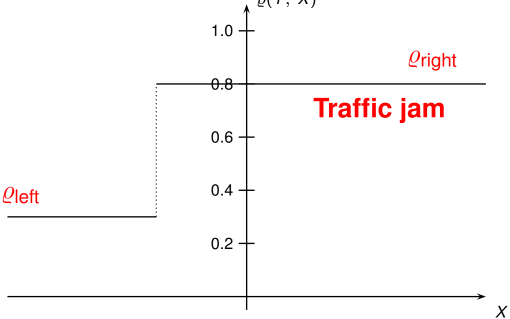

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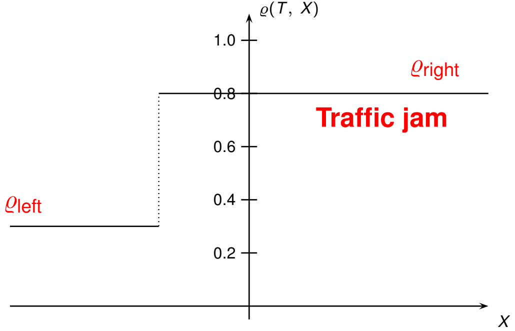

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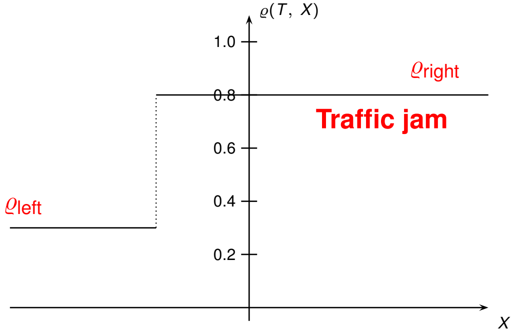

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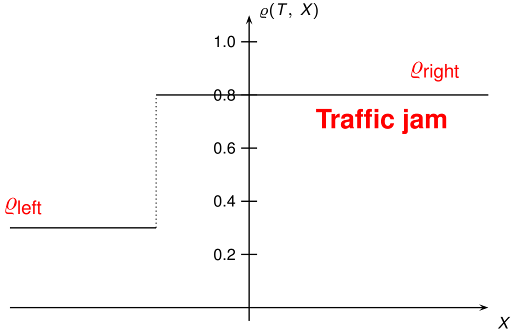

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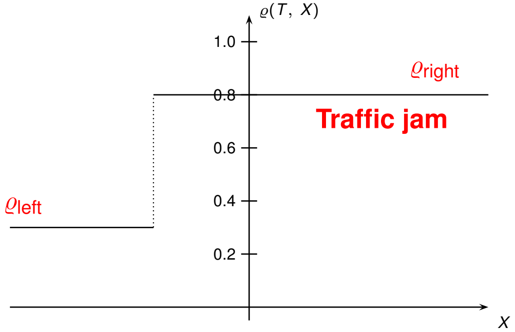

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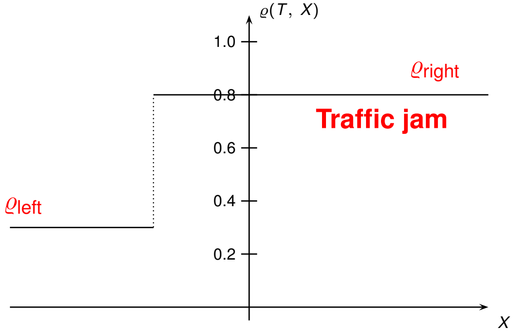

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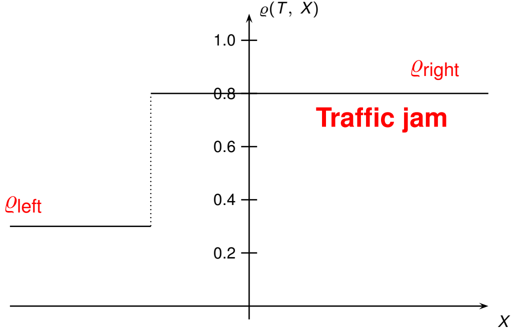

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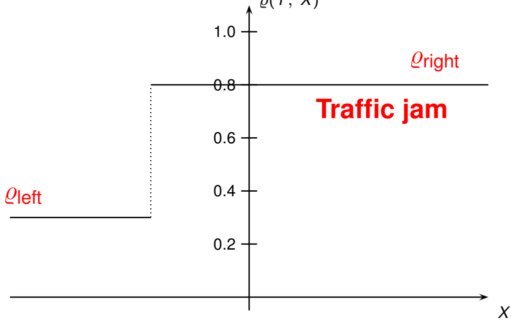

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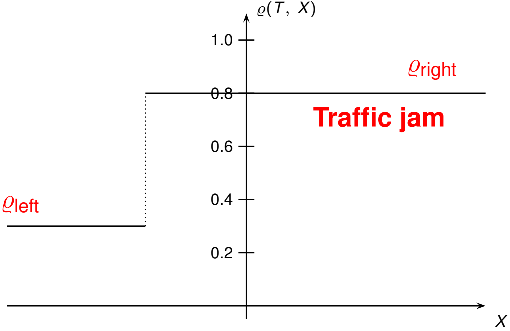

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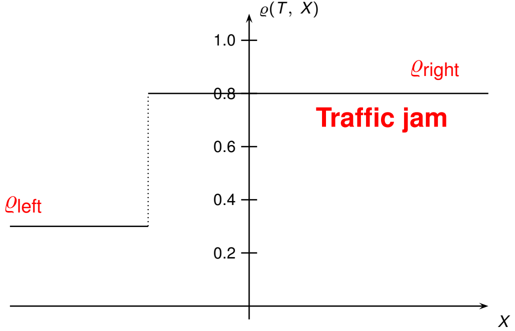

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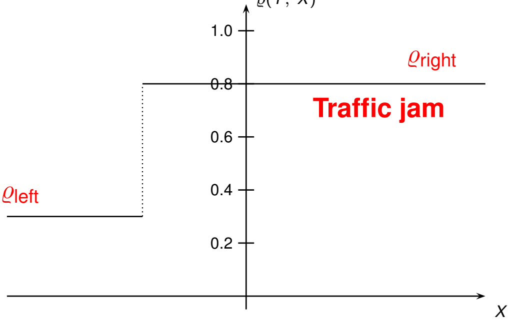

Traffic jam

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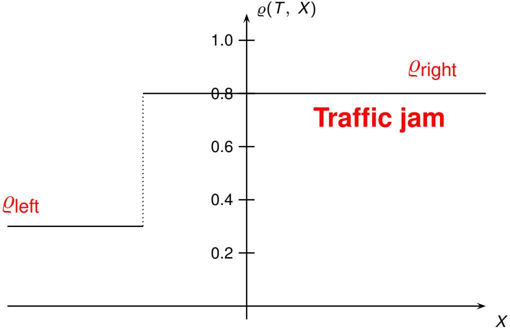

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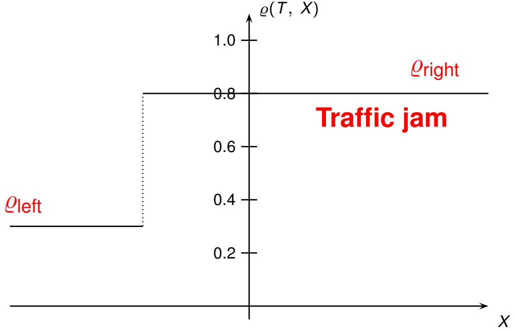

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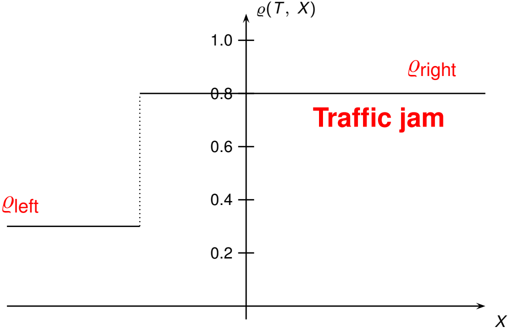

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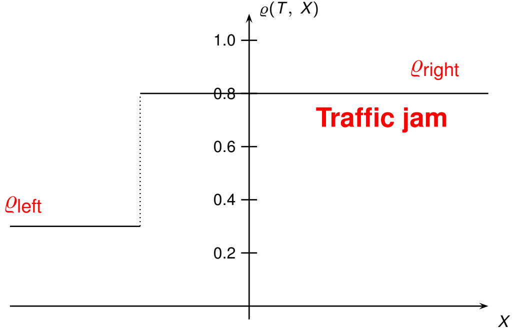

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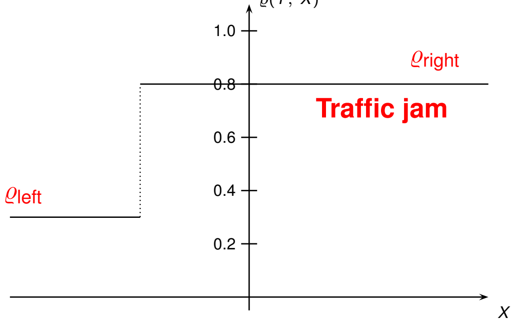

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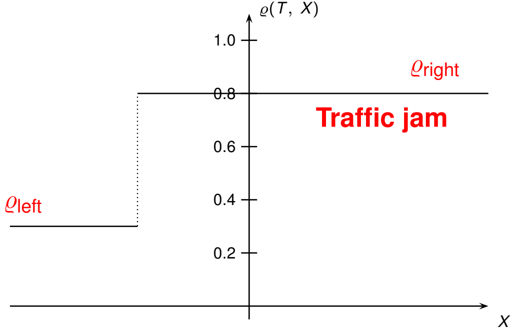

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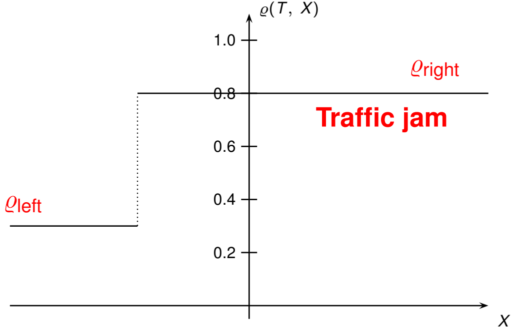

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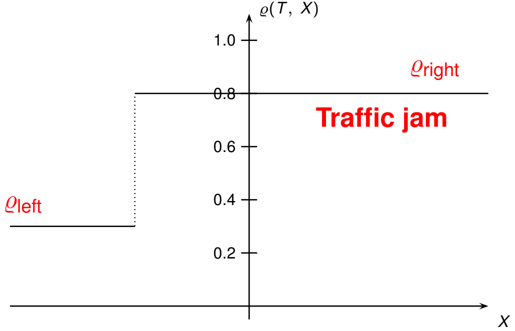

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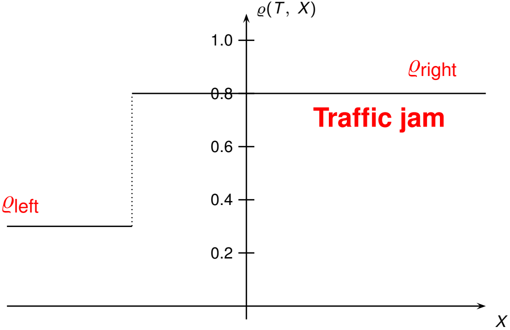

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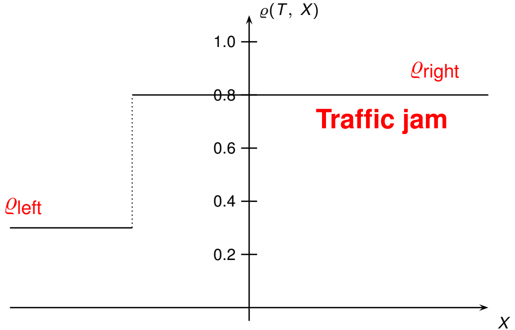

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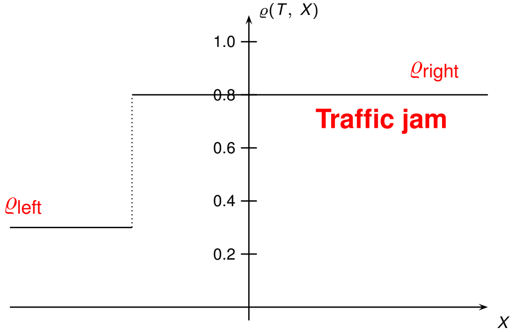

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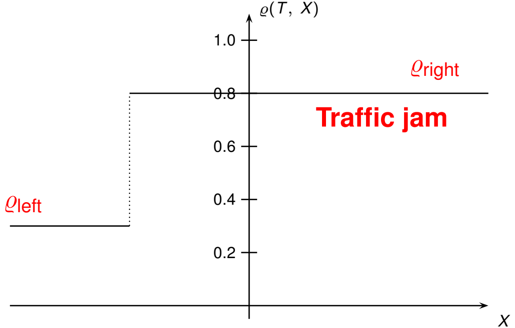

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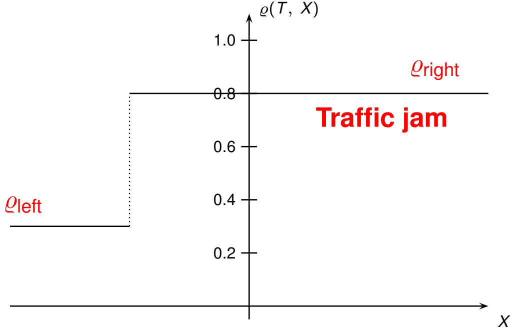

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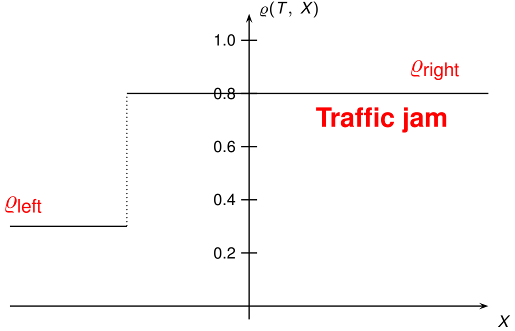

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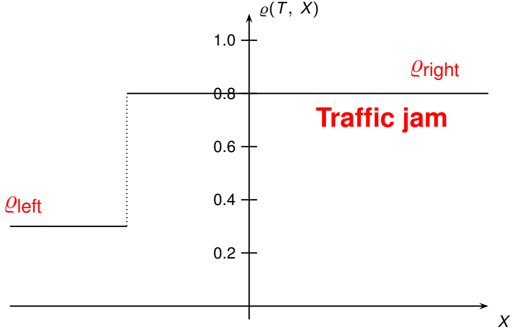

Traffic jam

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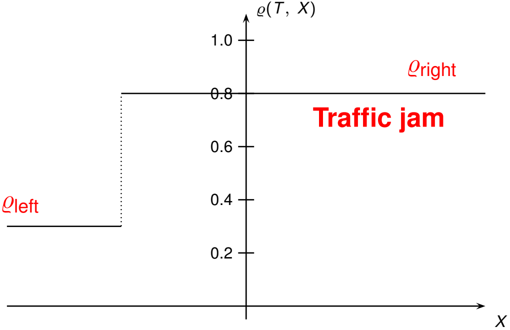

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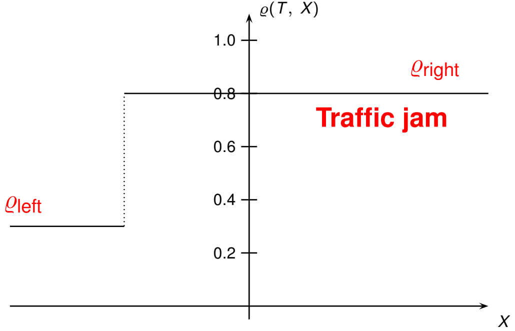

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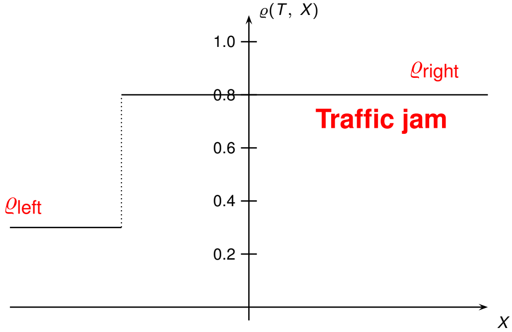

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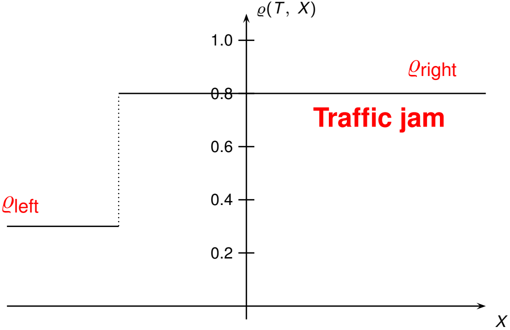

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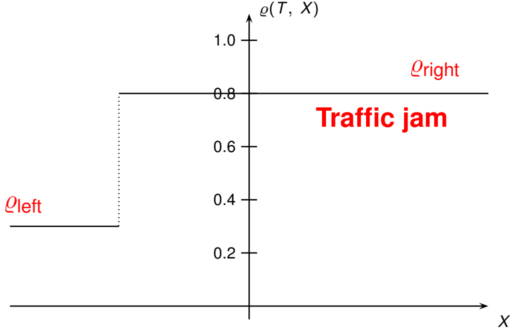

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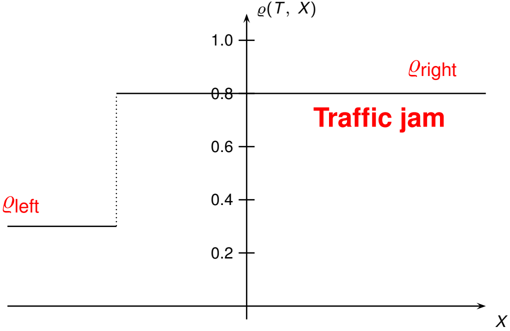

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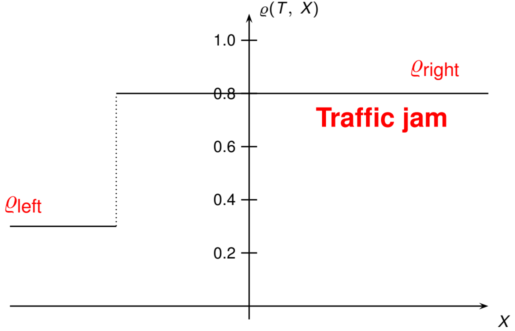

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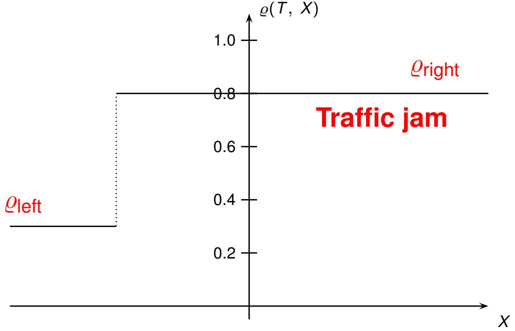

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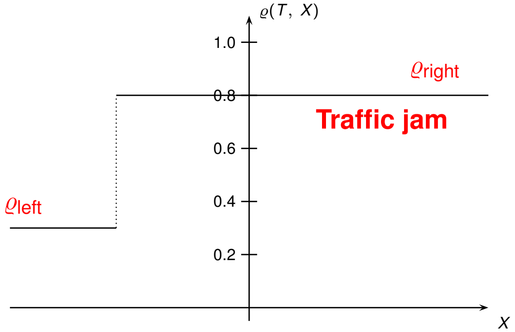
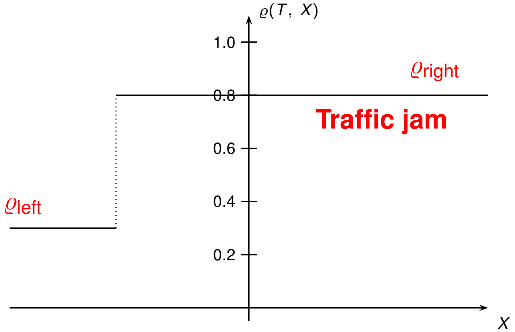


Discontinuous shock appears. Its velocity is given by *the Rankine-Hugoniot speed for densities* ϱ_{left} *and* ϱ_{right}

$$R = \frac{H(\varrho_{\text{left}}) - H(\varrho_{\text{right}})}{\varrho_{\text{left}} - \varrho_{\text{right}}}.$$

Shock wave



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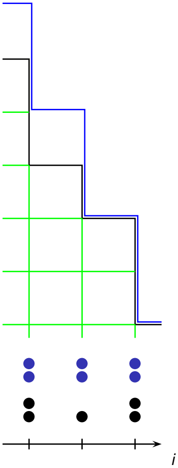
Let's look for the corresponding microscopic structure.

The second class particle

States ω and ω only differ at one site.



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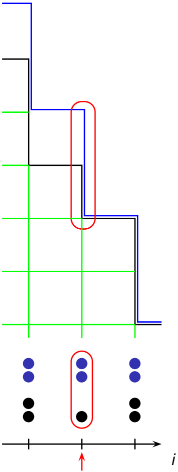

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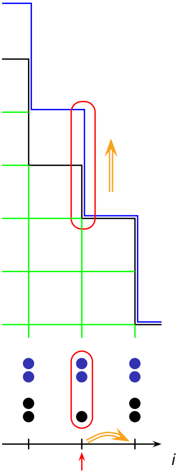

Growth on the right:

rate$\leq$rate

The second class particle

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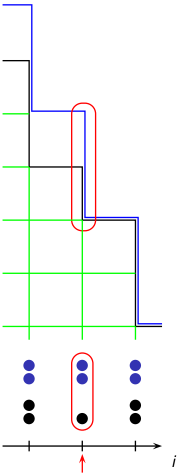

Growth on the right:
rate$\leq$rate
with rate:

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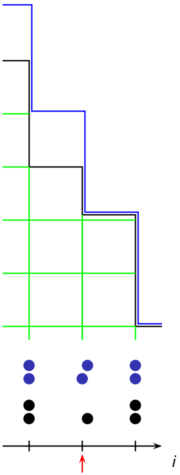

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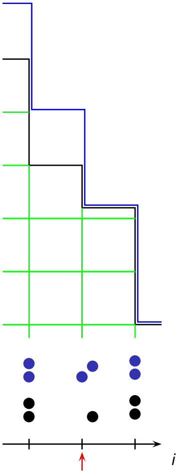

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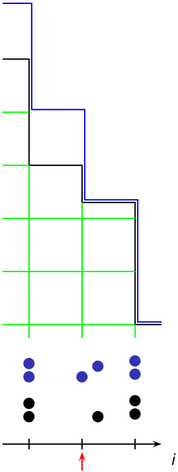

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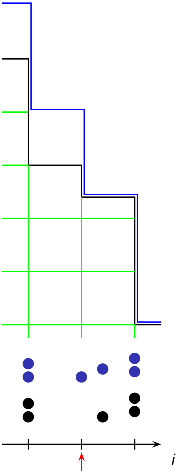
with rate:

The second class particle

States ω and ω only differ at one site.



Growth on the right:

rate≤rate

with rate:

The second class particle

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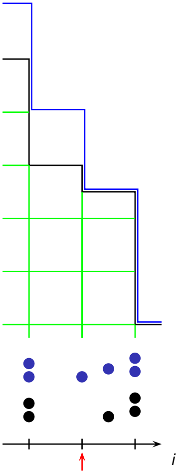

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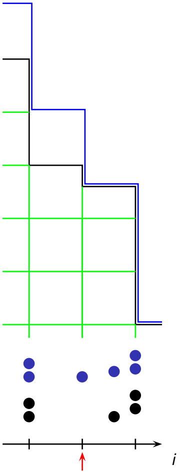

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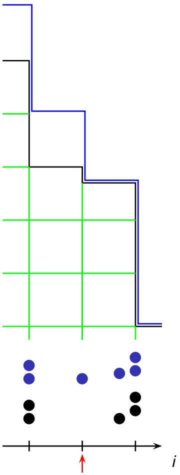

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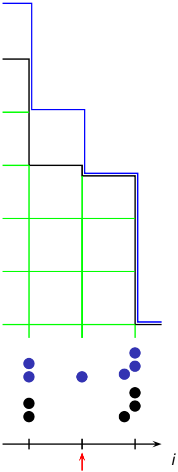

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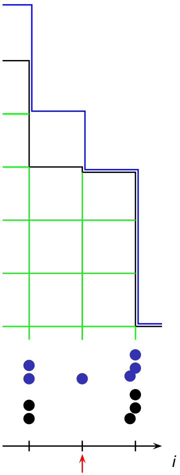

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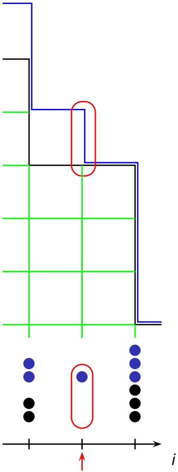

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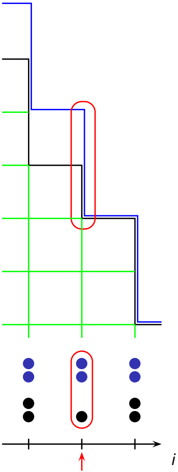

Growth on the right:
rate≤rate
with rate-rate:

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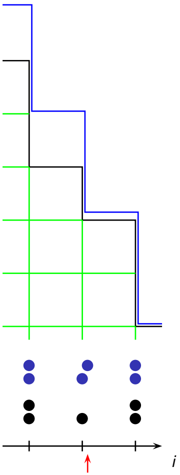

Growth on the right:

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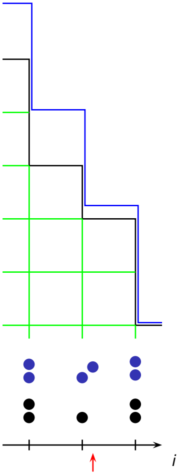

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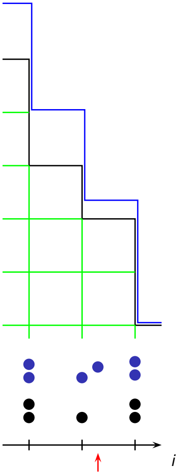

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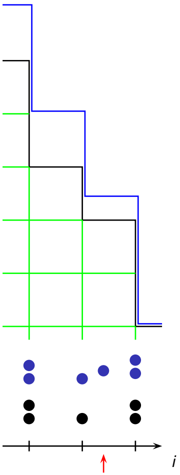

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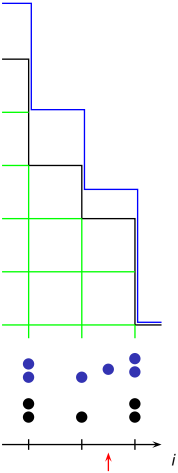

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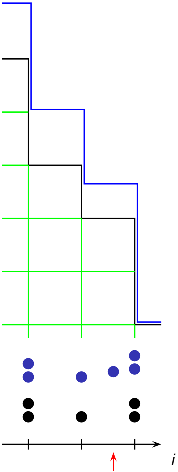

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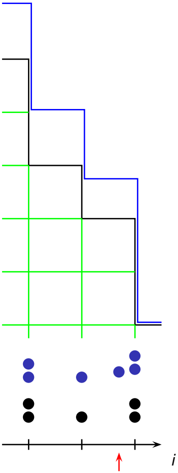

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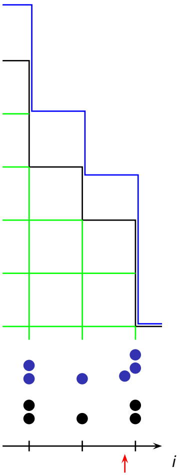

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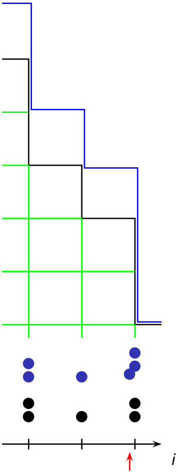

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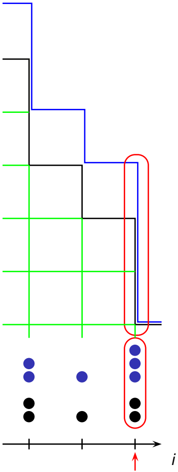

Growth on the right:

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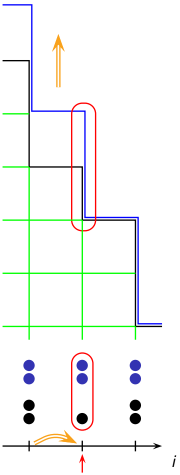

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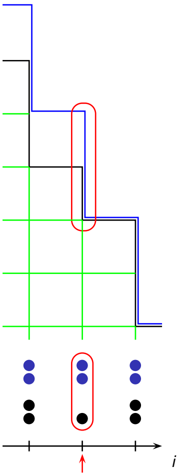
with rate:



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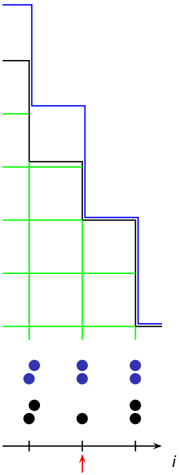

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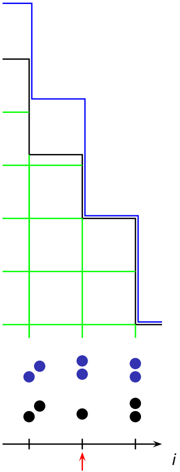

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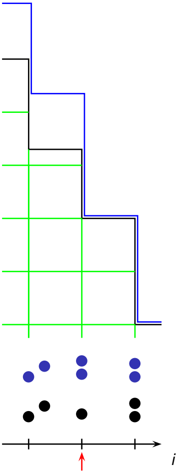

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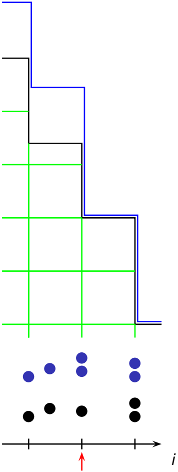

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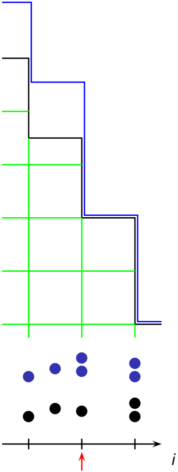

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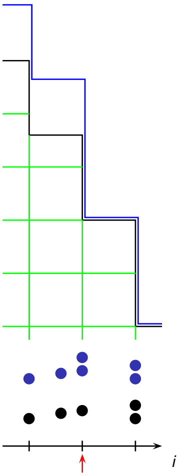

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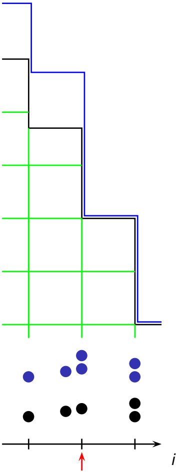

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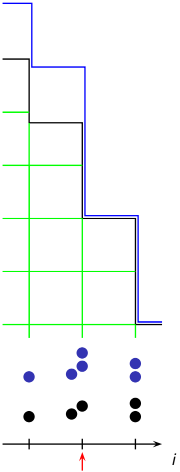

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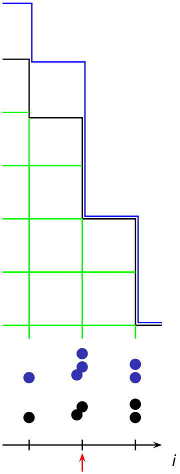

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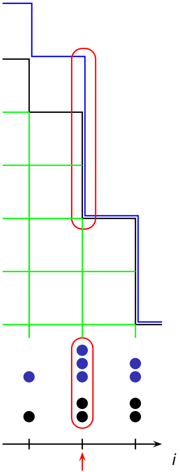

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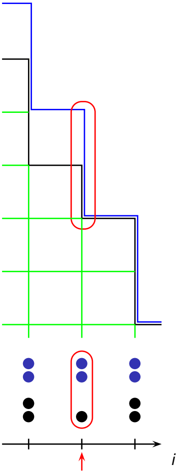

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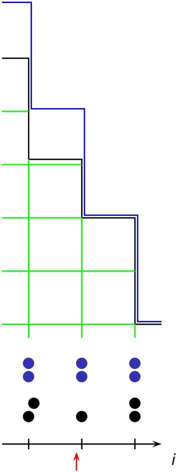

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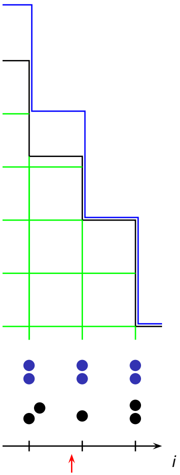

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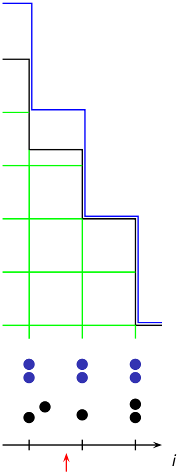

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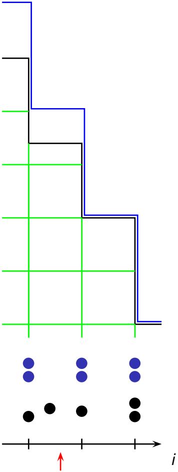

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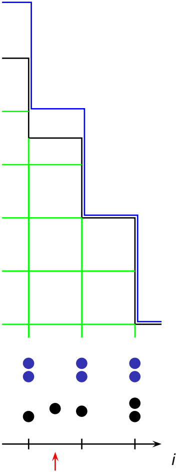

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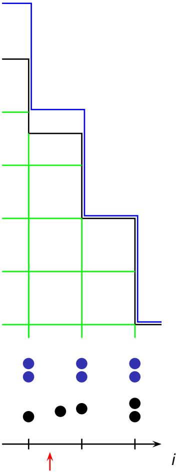

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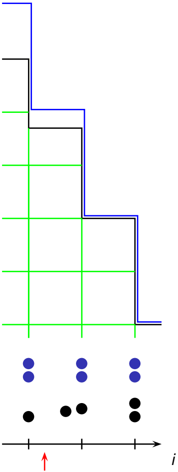

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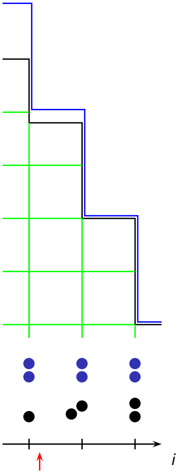

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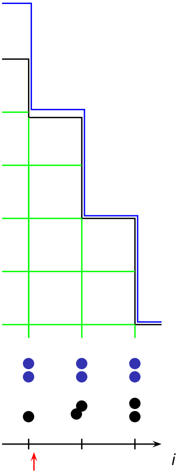

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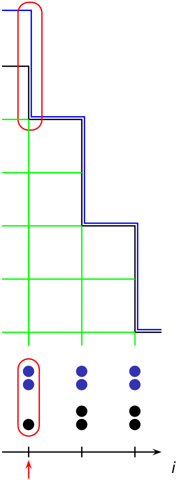

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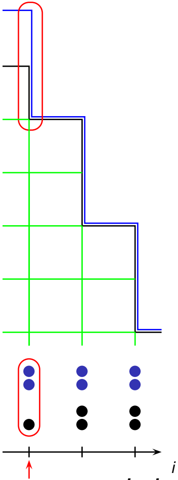
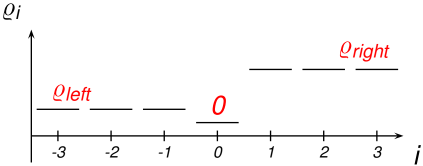


A single discrepancy ↑, the *second class particle*, is conserved.

Earlier results: as seen by the second class particle

Theorem (Derrida, Lebowitz, Speer ’97)

For the ASEP, the Bernoulli product distribution with densities



is stationary for the process, as seen from the second class particle, if

$$\frac{\varrho_{right}\cdot(1-\varrho_{left})}{\varrho_{left}\cdot(1-\varrho_{right})}=\frac{p}{q}.$$

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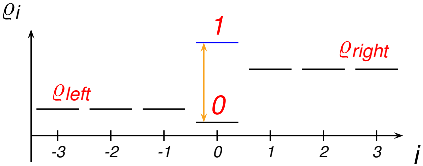

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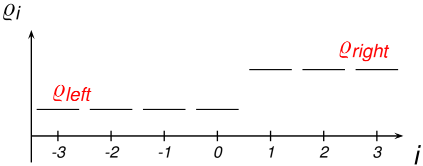
Earlier results: random walking shocks

Theorem (Belitsky and Schütz '02)

For the ASEP with the very same parameters, the Bernoulli product distribution μ_0 *with densities*



evolves according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = p \cdot \frac{\varrho_{left}}{\varrho_{right}} \cdot [\mu_{-1} - \mu_0] + q \cdot \frac{\varrho_{right}}{\varrho_{left}} \cdot [\mu_1 - \mu_0].$$

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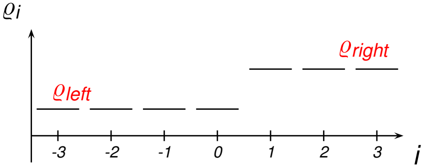

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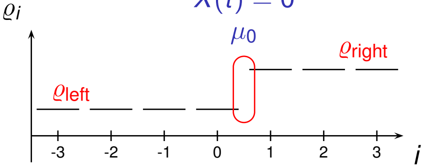
Multiple shocks and their interactions are also handled.

Earlier results: random walking shocks

Interpretation: random walking shock $\mu(t) = \mu_{X(t)}$:

with rate $p \cdot \frac{\varrho_{\text{left}}}{\varrho_{\text{right}}}$:

$X(t) = 0$



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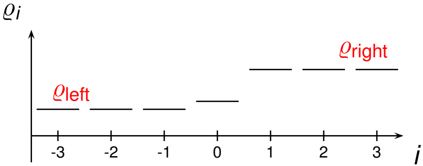

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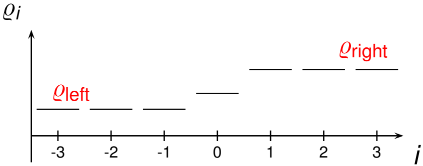

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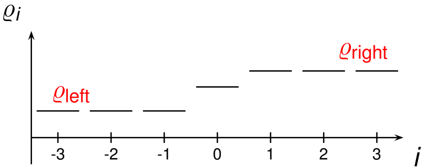

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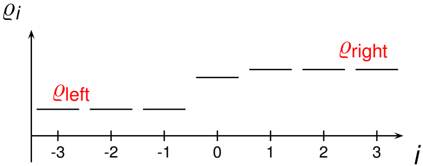

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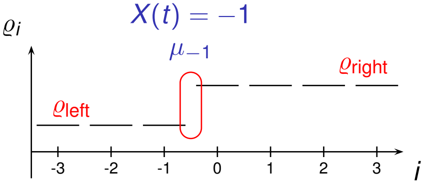

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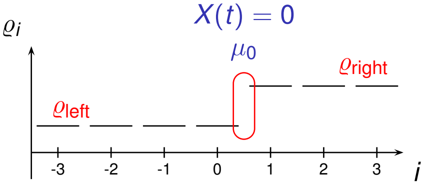

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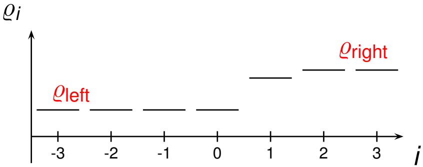

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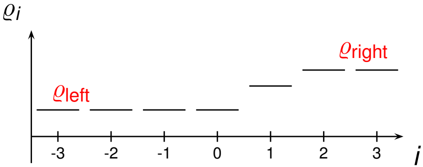

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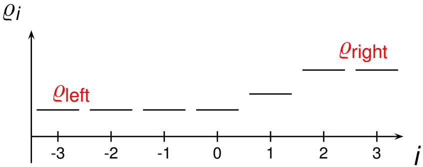

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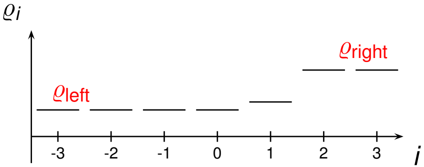

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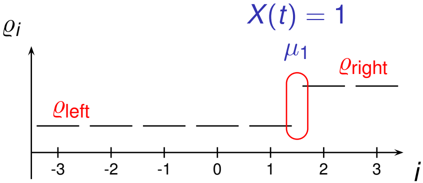

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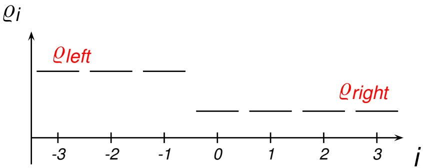
Theorem (B. ’01)

For the TAEBLP, the product distribution of marginals μ^{ϱ_i} with densities



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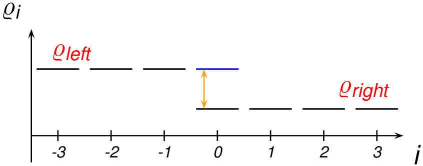

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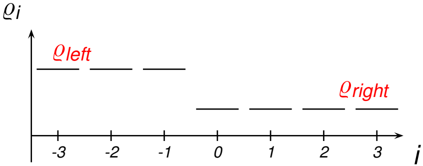

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$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = C_{left} \cdot [\mu_{-1} - \mu_0] + C_{right} \cdot [\mu_1 - \mu_0].$$

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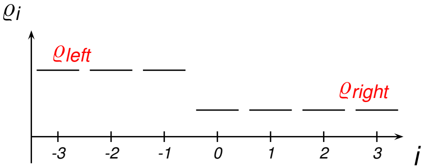

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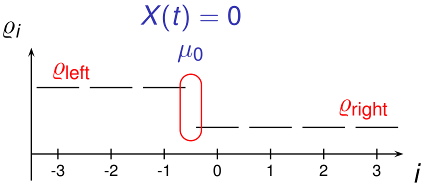

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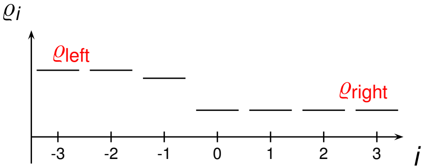

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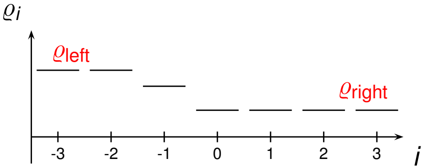

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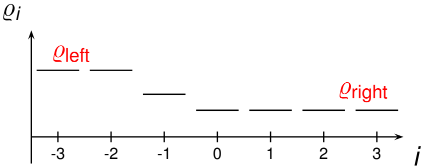

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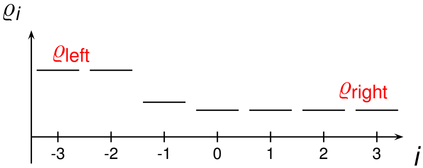

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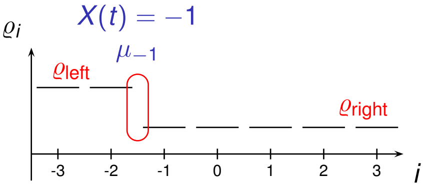

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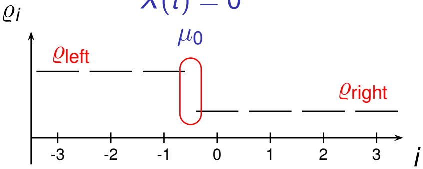

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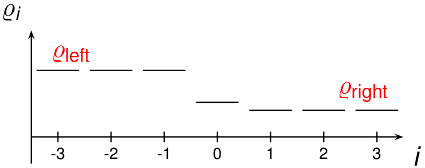

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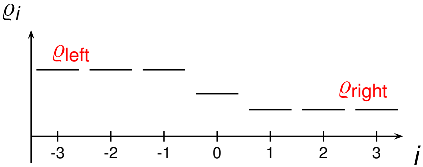

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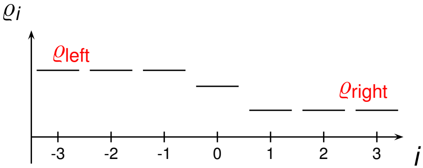

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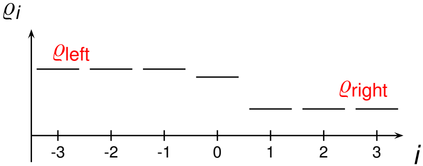

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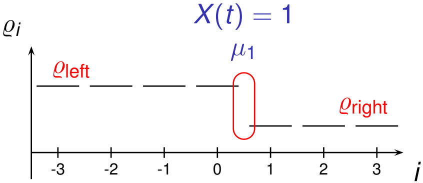

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Is it the second class particle that performs the simple random walk in the middle of a shock?

In what sense? Annealed w.r.t. the initial shock distribution... But what does this mean?

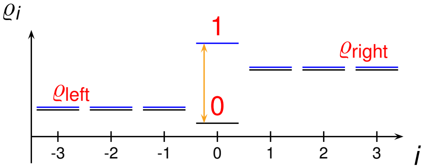
Here is the question:

For the ASEP, let ν_0 be the Bernoulli product distribution

$$\nu_0 = \Bigl(\bigotimes_{i<0} \mu^{\varrho_{\text{left}}}\Bigr) \otimes \bigl(\delta\bigr) \otimes \Bigl(\bigotimes_{i>0} \mu^{\varrho_{\text{right}}}\Bigr),$$

where

$$\mu^{\varrho}(\omega = \omega) = \begin{cases} \varrho, & \text{if } \omega = 1, \\ 1 - \varrho, & \text{if } \omega = 0; \end{cases} \qquad \delta(0,\, 1) = 1.$$



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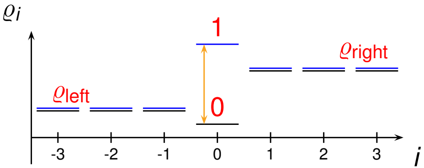

Does it satisfy

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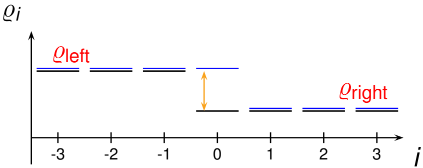

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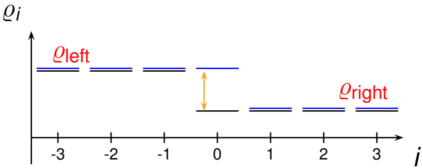

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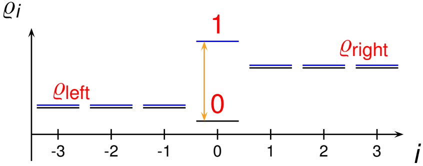
This explains both types of the previous results.

The presence of a second class particle in the measure significantly simplifies the computations. $\rightsquigarrow$ This is how we discovered the TAGEZRP.

Why do we like...

- ASEP: Fundamental example, nice combinatorics, unique second class particle.



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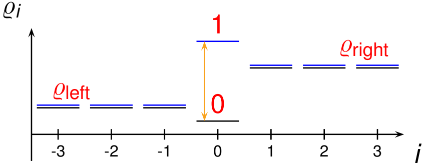
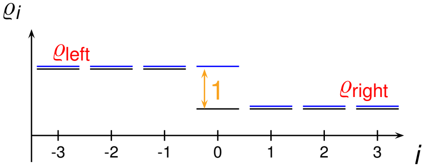


- EBLP, TAGEZRP: Nice convex rates; $\omega \sim \mu^{\varrho} \Rightarrow \omega + 1 \sim \mu^{\varrho+1}$.



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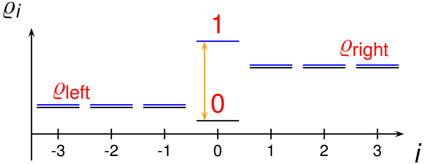

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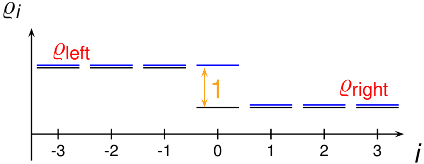

- TA$\mathfrak{q}$-ZRP: Nice concave rates; *Bethe Ansatz, exact solvability. . . ?*

The story of a search

The ZRP family is simple enough that we can feed it into the

Big RandomWalkingShocksMachine.

Here's what comes out (L. Duffy, D. Pantelli, B. '18).

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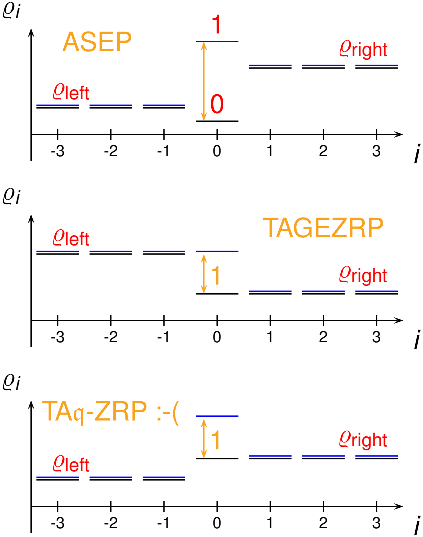
- Surprise: TA$\mathfrak{q}$-ZRP; $r(\omega) = 1 - \mathfrak{q}^{\omega}$.

TAq-ZRP



Impossible to find this without the second class particle.

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We also see that shocks+second class particles

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- It's a mess for TA$\mathfrak{q}$-ZRP. Shocks+second class particles locally interact *and* their jump rates depend on ranks,
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Boundaries for the finite-volume case

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Liggett's condition (Liggett '75):

- Add hypothetical sites of density ϱ at the ends of the volume.
- Apply jump boundary rates as normal, except averaged out at the hypothetical sites.
- $\rightsquigarrow$ the usual product measures of density ϱ are still stationary in finite or half-infinite volume.

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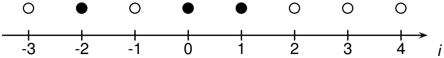
🥂 🎂 *Happy birthday Gunter!* 🎂 🥂

Thank you.

A similar result: branching coalescing random walk



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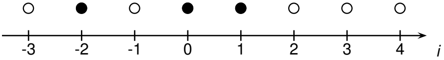

With rate p: jump to the right

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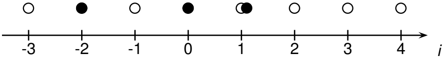

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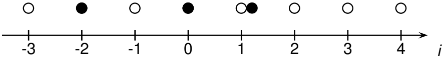

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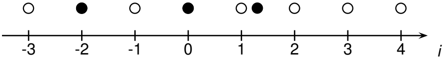

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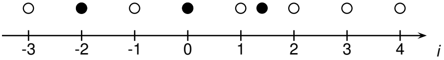

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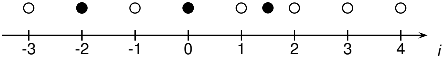

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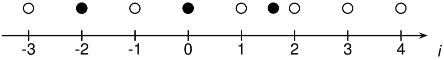

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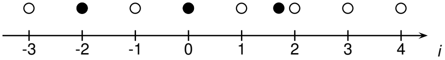

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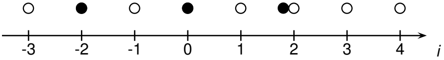

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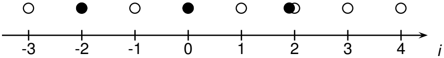

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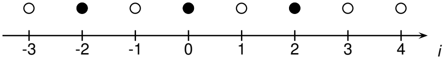

With rate q: jump to the left

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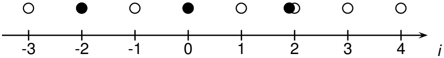

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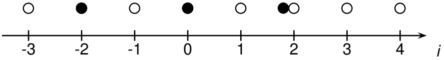

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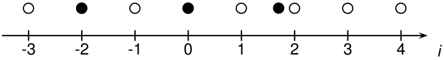

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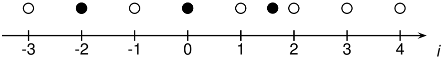

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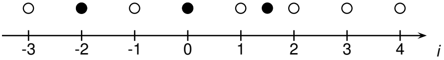

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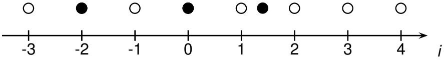

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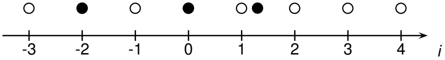

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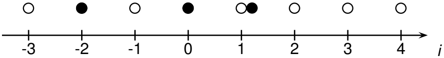

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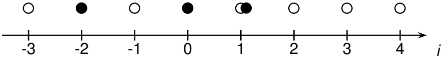

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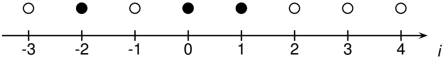

With rate c_r: coalescence to the right

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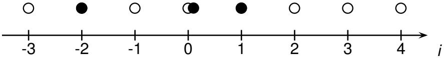

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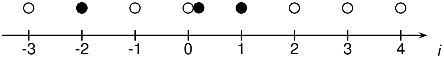

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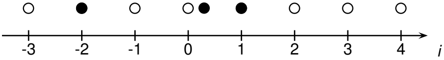

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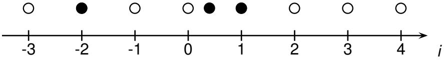

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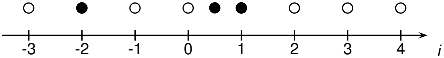

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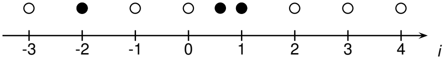

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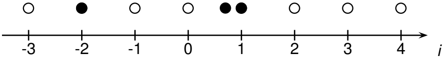

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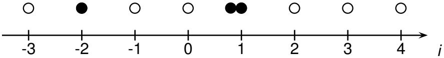

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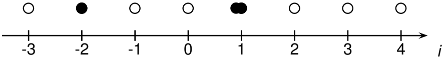

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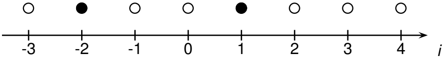

With rate b_l: branching to the left

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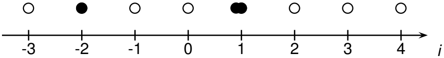

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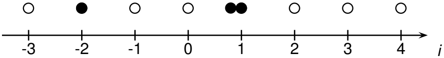

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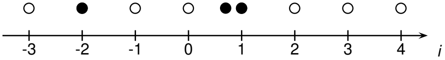

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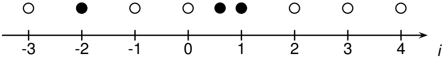

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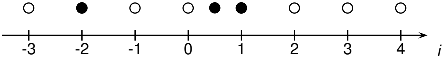

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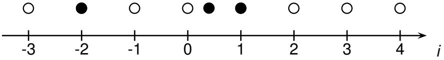

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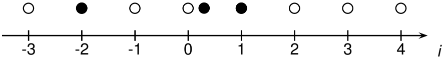

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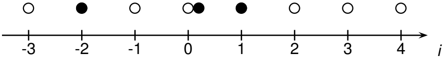

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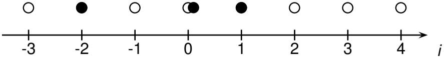

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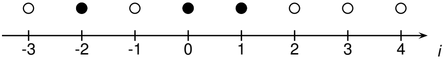

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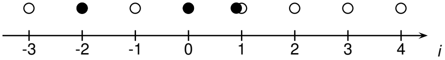

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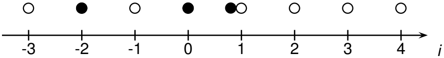

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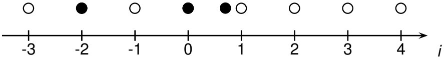

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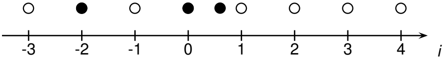

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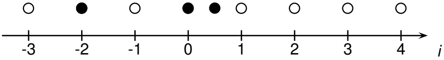

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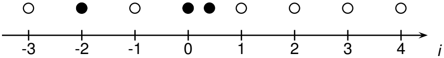

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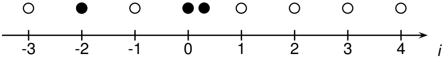

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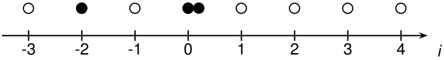

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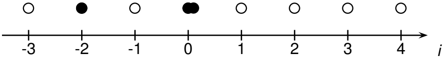

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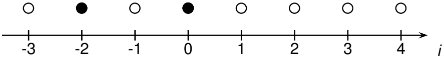

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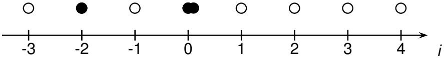

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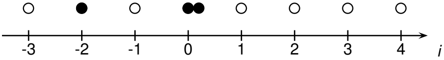

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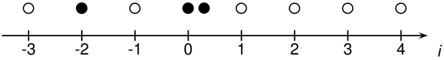
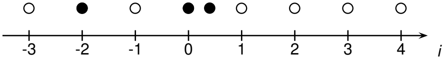


With rate b_r: branching to the right

A similar result: branching coalescing random walk



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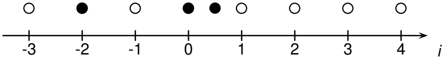

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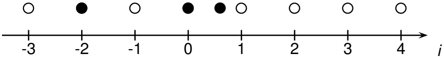

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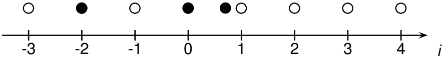

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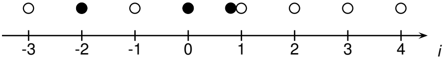

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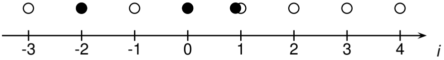

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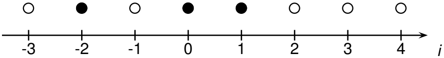

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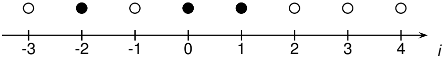


With rate b_r: branching to the right

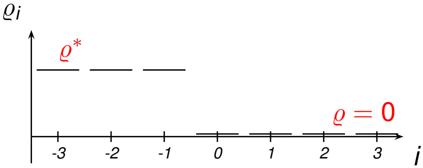
The Bernoulli(ϱ^*) distribution is stationary for

$$\varrho^* = \frac{b_l + b_r}{b_l + b_r + c_l + c_r}.$$

Earlier results: as seen by the rightmost particle

Theorem

For the BCRW, the Bernoulli product distribution with densities

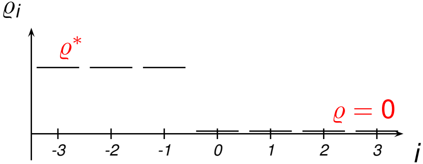


is stationary for the process, as seen from the rightmost particle.

Earlier results: random walking shocks

Theorem (Krebs, Jafarpour and Schütz '03)

For the BCRW with the very same parameters, the Bernoulli product distribution μ_0 *with densities*



evolves according to

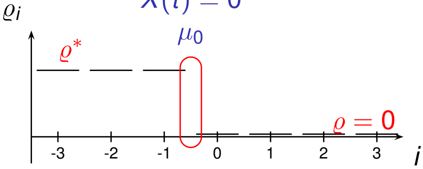
$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\mu_{-1} - \mu_0]$$
$$+ p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\mu_1 - \mu_0].$$

Earlier results: random walking shocks

Interpretation: random walking shock $\mu(t) = \mu_{X(t)}$:

with rate $\frac{c_l\cdot(b_l+b_r)+q\cdot(c_l+c_r)}{b_l+b_r+c_l+c_r}$:

$X(t) = 0$



$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = \frac{c_l\cdot(b_l+b_r)+q\cdot(c_l+c_r)}{b_l+b_r+c_l+c_r}\cdot[\mu_{-1}-\mu_0]$$
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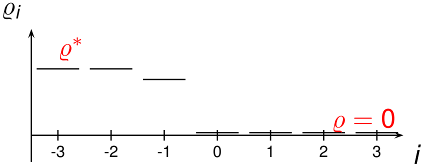

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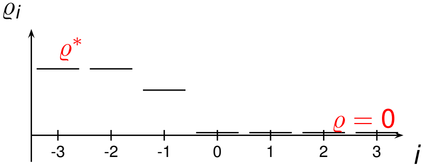

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\mu_{-1} - \mu_0] + p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\mu_1 - \mu_0].$$

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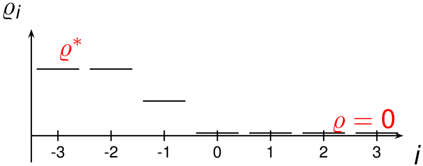

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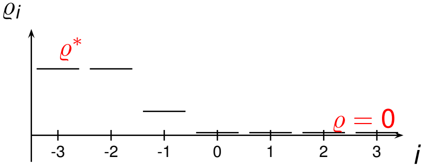

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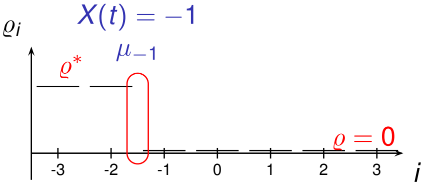

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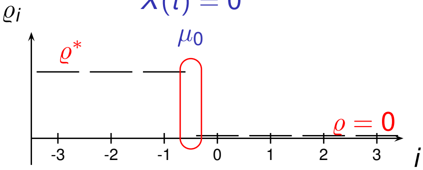

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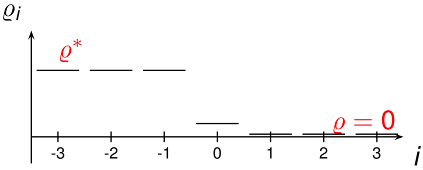

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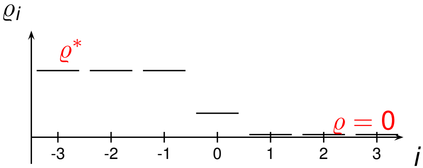

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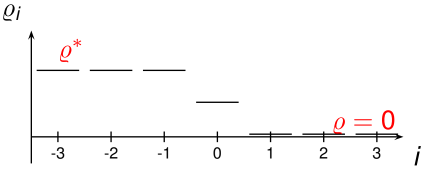

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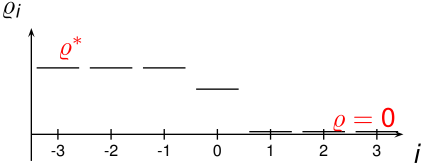

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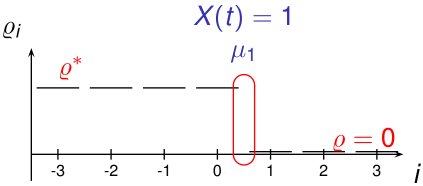

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The question:

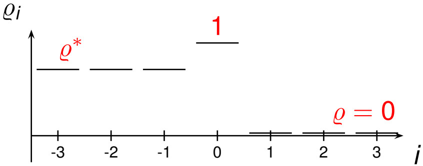
Is it the rightmost particle that performs the random walk?

Here is the question:

For the BCRW, let ν_0 be the Bernoulli product distribution

$$\nu_0 = \Big(\bigotimes_{i<0} \mu^{\varrho^*}\Big) \otimes \big(\delta\big) \otimes \Big(\bigotimes_{i>0} \mu^{0}\Big),$$

where $\delta(0) = 1$.



Does it satisfy

$$\frac{\mathrm{d}}{\mathrm{d}t}\nu_0 = \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\nu_{-1} - \nu_0]$$
$$+ p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\nu_1 - \nu_0]?$$

The answer

- ... is, of course, yes again. [Gy. Farkas, P. Kovács, A. Rákos, B. '10]

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- ... is, of course, yes again. [Gy. Farkas, P. Kovács, A. Rákos, B. '10]
- Fronts of the other direction: $0 - 1 - \varrho^*$ can also be handled.

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Thank you.