Dependent Double Branching Annihilating Random Walk

Joint with
Attila László Nagy

Márton Balázs

University of Bristol

New Perspectives in Analysis and Probability
University of Sussex
6th March, 2015.

Attractive and non-attractive models

Totally asymmetric simple exclusion process

A $\oplus$ $\ominus$ 0 model

Totally asymmetric zero range process

On large scales

Shocks

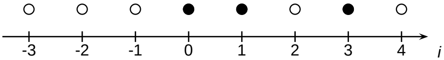
Rarefaction waves

A mean field version

Positive recurrence

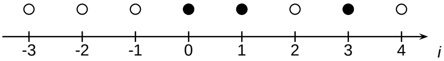
Two words on the proof

The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

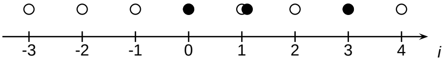
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

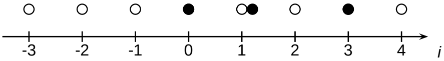
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

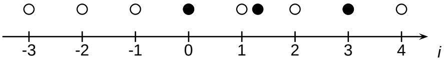
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

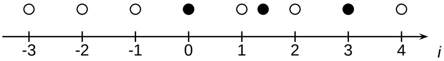
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

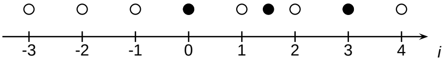
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

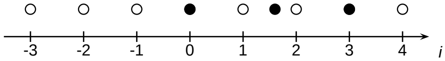
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

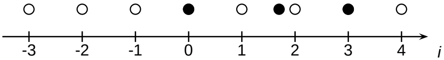
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

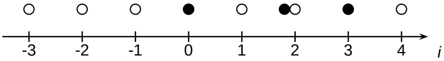
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

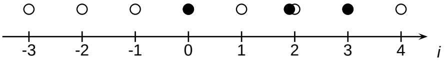
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

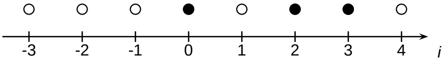
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

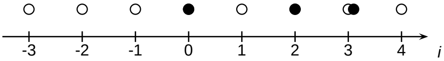
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

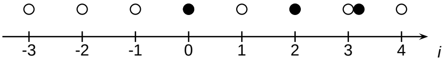
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

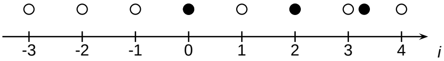
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

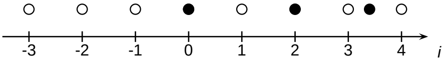
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

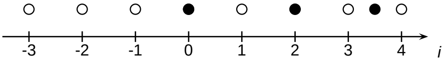
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

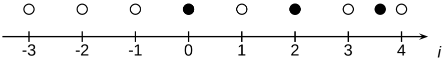
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

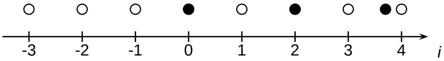
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

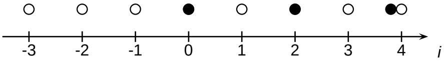
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

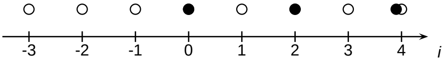
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

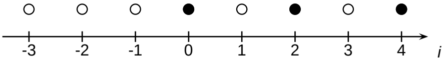
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

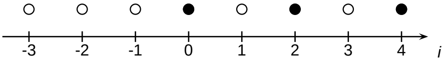
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

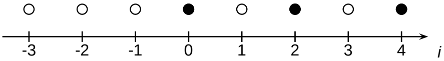
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

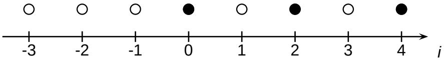
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

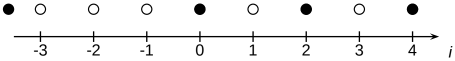
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

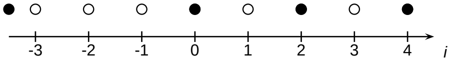
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

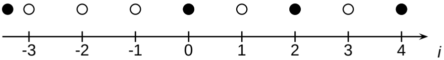
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

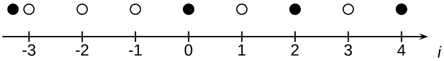
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

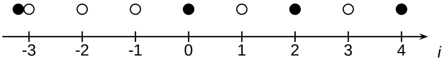
The totally asymmetric simple exclusion process



$$\omega_i(0) \sim \text{Bernoulli}(\varrho) \text{ product distribution.}$$

Particles step to the right with rate 1,
unless the destination site is occupied.

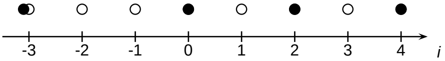
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

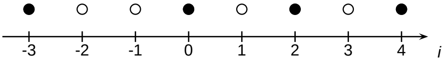
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

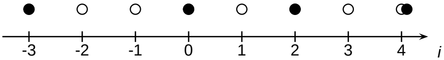
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

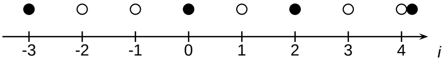
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

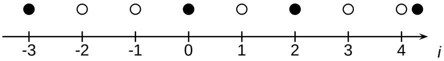
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

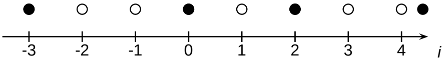
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

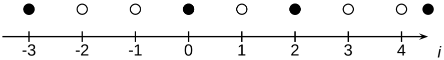
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

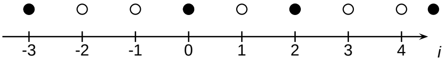
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

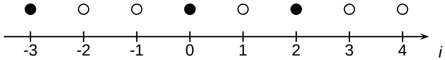
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

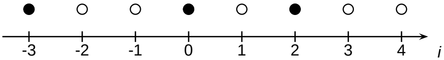
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

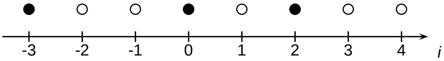
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

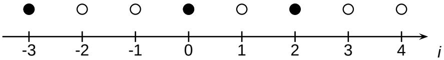
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

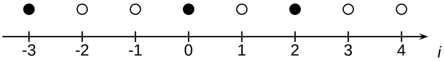
The totally asymmetric simple exclusion process



$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

The totally asymmetric simple exclusion process



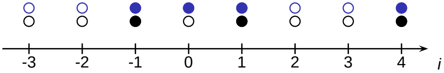
$\omega_i(0) \sim$ Bernoulli(ϱ) product distribution.

Particles step to the right with rate 1,
unless the destination site is occupied.

The Bernoulli(ϱ) product distribution is stationary (and non-reversible) for all $0 \leq \varrho \leq 1$: $\omega_i(t) \sim$ Bernoulli(ϱ).

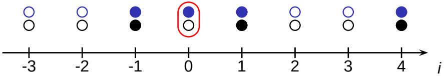
These are the important ($=$ ergodic) stationary distributions.

The second class particle



Stochastic coupling: evolution as close as possible

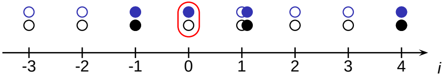
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

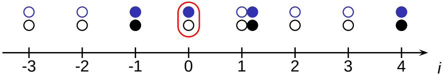
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

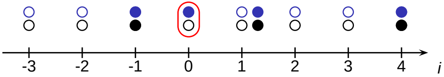
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

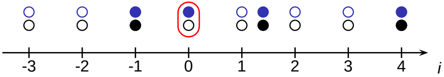
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

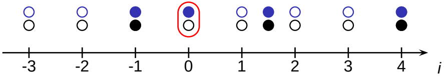
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

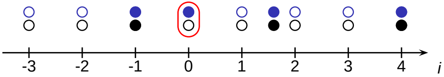
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

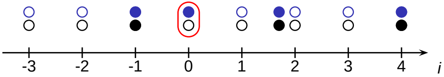
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

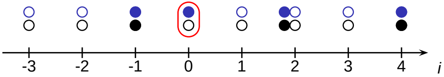
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

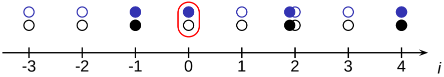
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

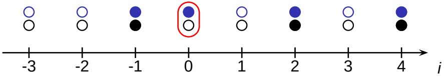
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

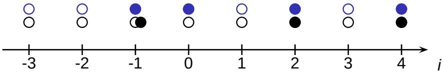
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

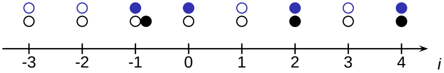
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

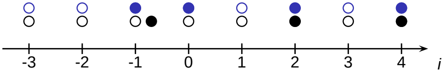
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

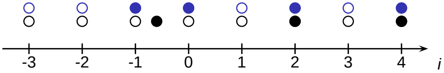
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

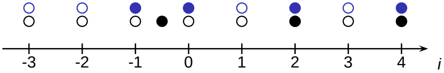
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

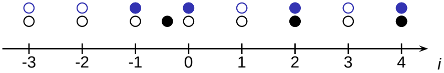
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

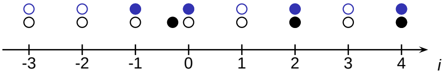
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

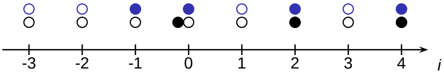
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

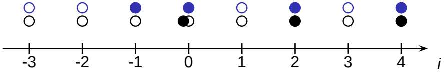
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

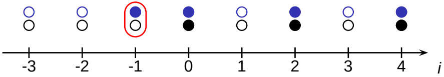
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

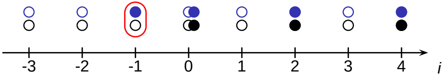
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

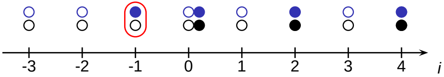
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

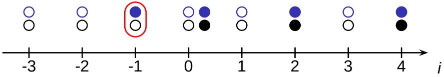
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

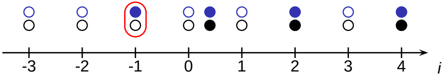
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

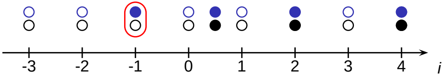
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

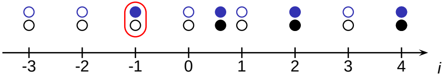
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

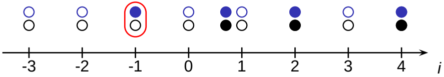
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

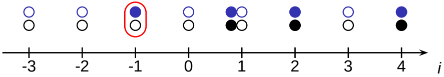
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

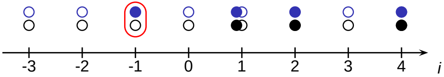
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

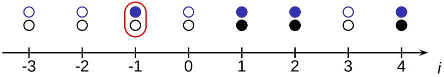
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

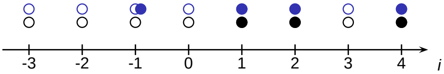
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

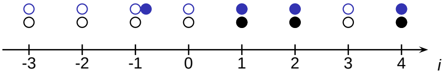
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

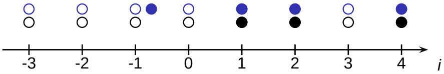
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

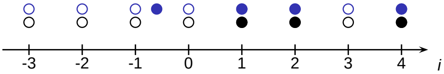
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

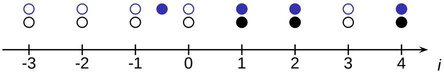
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

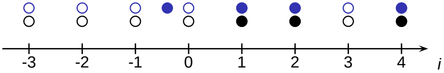
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

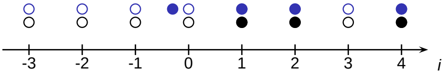
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

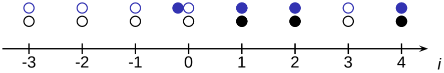
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

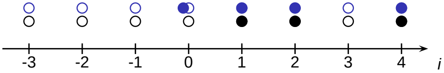
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

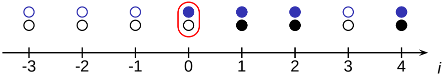
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle.

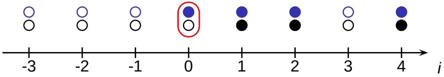
The second class particle



Stochastic coupling: evolution as close as possible

Second class particle. Its position at time t: $Q(t)$.

The second class particle

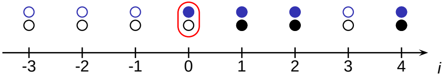


Stochastic coupling: evolution as close as possible

Second class particle. Its position at time t: $Q(t)$.

$$\begin{aligned}\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) &= \mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=0]\cdot(1-\varrho)\\ &\quad+\mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=1]\cdot\varrho-\varrho^2\\ &=\mathbf{E}[\omega_i(t)]\cdot\varrho-\varrho^2.\end{aligned}$$

The second class particle



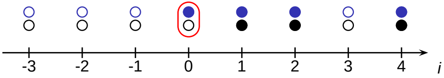
Stochastic coupling: evolution as close as possible

Second class particle. Its position at time t: $Q(t)$.

$$\begin{aligned}\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) &= \mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=0]\cdot(1-\varrho)\\ &\quad+\mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=1]\cdot\varrho-\varrho^2\\ &=\mathbf{E}[\omega_i(t)]\cdot\varrho-\varrho^2.\end{aligned}$$

$$\mathbf{P}\{Q(t)=i\}=\mathbf{E}[\omega_i(t)-\omega_i(t)]=\mathbf{E}[\omega_i(t)]-\mathbf{E}[\omega_i(t)].$$

The second class particle



Stochastic coupling: evolution as close as possible

Second class particle. Its position at time t: $Q(t)$.

$$\mathbf{Cov}(\omega_i(t),\, \omega_0(0)) = \mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=0]\cdot(1-\varrho)$$
$$+\mathbf{E}[\omega_i(t)\omega_0(0)\,|\,\omega_0(0)=1]\cdot\varrho-\varrho^2$$
$$=\mathbf{E}[\omega_i(t)]\cdot\varrho-\varrho^2.$$

$$\mathbf{P}\{Q(t)=i\}=\mathbf{E}[\omega_i(t)-\omega_i(t)]=\mathbf{E}[\omega_i(t)]-\mathbf{E}[\omega_i(t)].$$

$$\varrho=\mathbf{E}[\omega_i(t)]=\mathbf{E}[\omega_i(t)\,|\,\omega_0(0)=0]\cdot(1-\varrho)$$
$$+\mathbf{E}[\omega_i(t)\,|\,\omega_0(0)=1]\cdot\varrho$$
$$=\mathbf{E}[\omega_i(t)]\cdot(1-\varrho)+\mathbf{E}[\omega_i(t)]\cdot\varrho$$

The second class particle

$$\mathbf{Cov}(\omega_i(t),\, \omega_0(0)) = \mathbf{E}[\omega_i(t)] \cdot \varrho - \varrho^2. \tag{1}$$

$$\mathbf{P}\{Q(t) = i\} = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]. \tag{2}$$

$$\varrho = \mathbf{E}[\omega_i(t)] \cdot (1 - \varrho) + \mathbf{E}[\omega_i(t)] \cdot \varrho. \tag{3}$$

The second class particle

$$\mathbf{Cov}(\omega_i(t),\, \omega_0(0)) = \mathbf{E}[\omega_i(t)] \cdot \varrho - \varrho^2. \tag{1}$$

$$\mathbf{P}\{Q(t) = i\} = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]. \tag{2}$$

$$\varrho = \mathbf{E}[\omega_i(t)] \cdot (1 - \varrho) + \mathbf{E}[\omega_i(t)] \cdot \varrho. \tag{3}$$

So,

$$\mathbf{Cov}(\omega_i(t),\, \omega_0(0)) \stackrel{(1)}{=} \varrho \cdot \big(\mathbf{E}[\omega_i(t)] - \varrho\big)$$

The second class particle

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) = \mathbf{E}[\omega_i(t)]\cdot\varrho - \varrho^2. \tag{1}$$

$$\mathbf{P}\{Q(t)=i\} = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]. \tag{2}$$

$$\varrho = \mathbf{E}[\omega_i(t)]\cdot(1-\varrho) + \mathbf{E}[\omega_i(t)]\cdot\varrho. \tag{3}$$

So,

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) \overset{(1)}{=} \varrho\cdot\big(\mathbf{E}[\omega_i(t)] - \varrho\big)$$
$$\overset{(3)}{=} \varrho(1-\varrho)\cdot\big(\mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]\big)$$

The second class particle

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) = \mathbf{E}[\omega_i(t)]\cdot\varrho - \varrho^2. \tag{1}$$

$$\mathbf{P}\{Q(t)=i\} = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]. \tag{2}$$

$$\varrho = \mathbf{E}[\omega_i(t)]\cdot(1-\varrho) + \mathbf{E}[\omega_i(t)]\cdot\varrho. \tag{3}$$

So,

$$\begin{aligned}
\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) &\overset{(1)}{=} \varrho\cdot\big(\mathbf{E}[\omega_i(t)] - \varrho\big)\\
&\overset{(3)}{=} \varrho(1-\varrho)\cdot\big(\mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]\big)\\
&\overset{(2)}{=} \varrho(1-\varrho)\cdot\mathbf{P}\{Q(t)=i\}.
\end{aligned}$$

The second class particle

$$\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) = \mathbf{E}[\omega_i(t)]\cdot\varrho - \varrho^2. \tag{1}$$

$$\mathbf{P}\{Q(t)=i\} = \mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]. \tag{2}$$

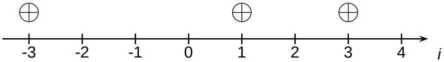
$$\varrho = \mathbf{E}[\omega_i(t)]\cdot(1-\varrho) + \mathbf{E}[\omega_i(t)]\cdot\varrho. \tag{3}$$

So,

$$\begin{aligned}\mathbf{Cov}(\omega_i(t),\,\omega_0(0)) &\overset{(1)}{=} \varrho\cdot\big(\mathbf{E}[\omega_i(t)] - \varrho\big)\\ &\overset{(3)}{=} \varrho(1-\varrho)\cdot\big(\mathbf{E}[\omega_i(t)] - \mathbf{E}[\omega_i(t)]\big)\\ &\overset{(2)}{=} \varrho(1-\varrho)\cdot\mathbf{P}\{Q(t)=i\}.\end{aligned}$$

The second class particle traces information propagation.

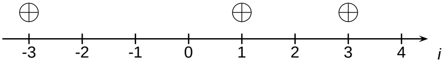
A $\oplus$ $\ominus$ 0 model



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

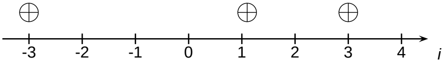
$\oplus$ to the right: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

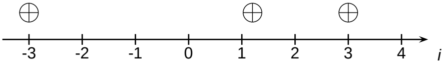
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

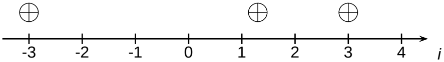
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

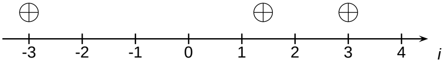
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

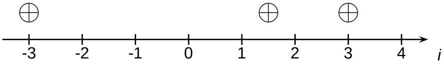
$\oplus$ to the right: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

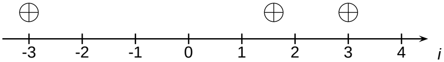
$\oplus$ to the right: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

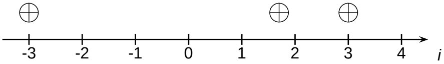
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

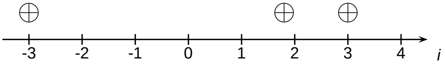
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

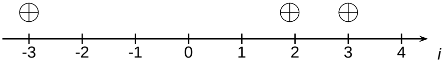
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

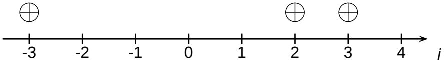
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

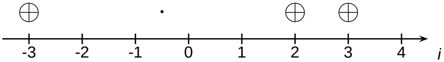
$\oplus$ to the right: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

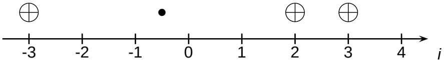
pair creation from vacuum: rate c



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A ⊕ ⊖ 0 model

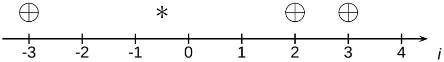
pair creation from vacuum: rate c



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

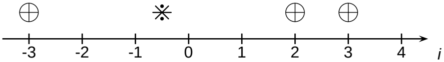
pair creation from vacuum: rate c



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

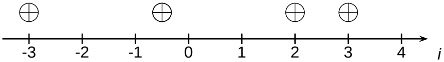
pair creation from vacuum: rate c



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

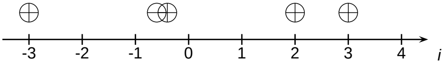
pair creation from vacuum: rate c



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

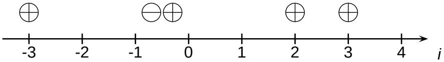
pair creation from vacuum: rate c



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

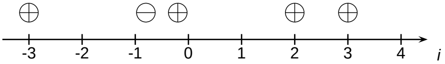
pair creation from vacuum: rate c



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

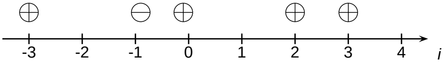
pair creation from vacuum: rate c



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

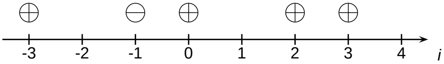
pair creation from vacuum: rate c



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

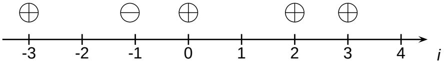
pair creation from vacuum: rate c



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

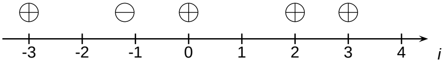
$\ominus$ to the left: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

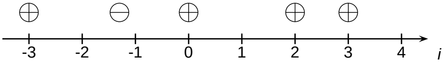
$\ominus$ to the left: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

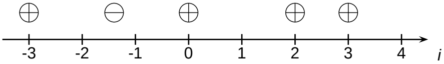
$\ominus$ to the left: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

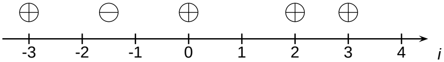
$\ominus$ to the left: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

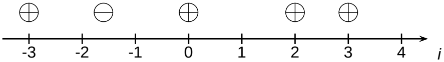
$\ominus$ to the left: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

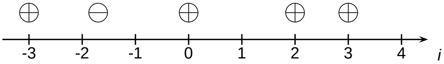
$\ominus$ to the left: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

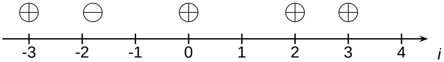
$\ominus$ to the left: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

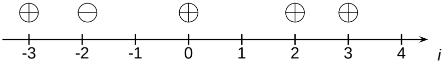
$\ominus$ to the left: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

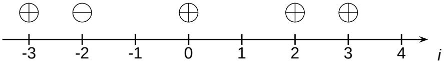
$\ominus$ to the left: rate 1



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A ⊕ ⊖ 0 model

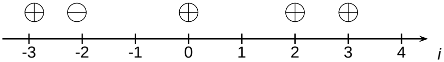
⊖ to the left: rate 1



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

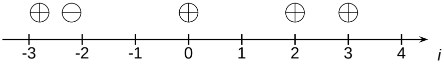
annihilation: rate 2



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

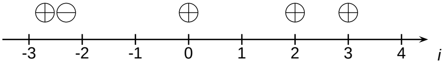
annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

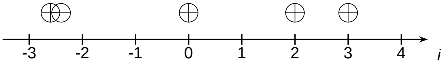
annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

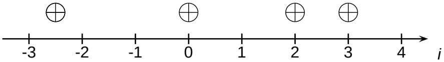
annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

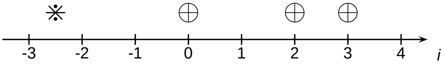
annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

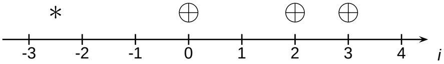
annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

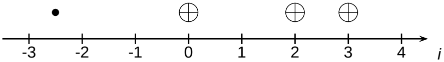
annihilation: rate 2



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A ⊕ ⊖ 0 model

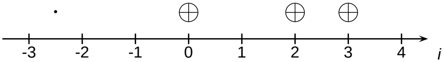
annihilation: rate 2



$\omega_i = -1$, 0, 1: a family of product initial distribution.

A ⊕ ⊖ 0 model

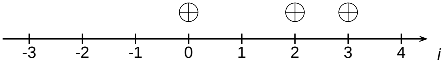
annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

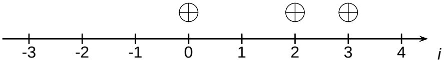
A $\oplus$ $\ominus$ 0 model

annihilation: rate 2



$\omega_i = -1, 0, 1$: a family of product initial distribution.

A $\oplus$ $\ominus$ 0 model

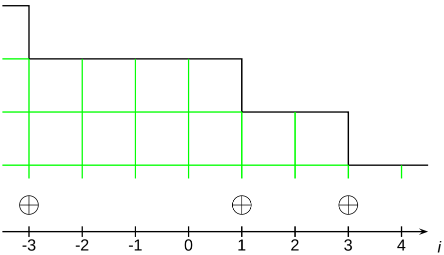


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A ⊕ ⊖ 0 model

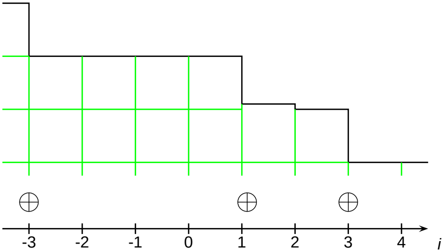


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

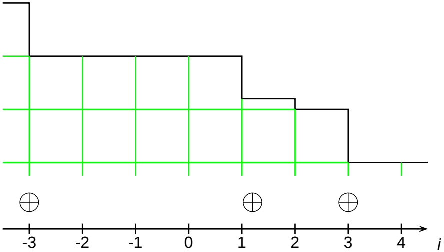


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

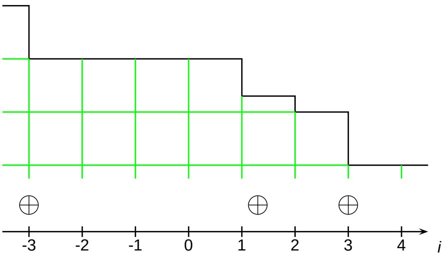


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

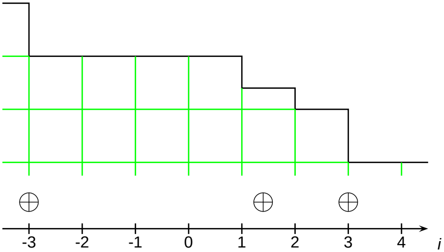


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A ⊕ ⊖ 0 model

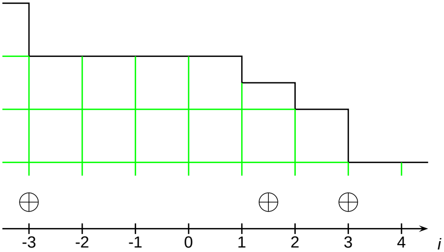


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

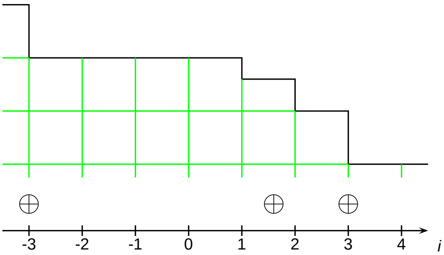


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

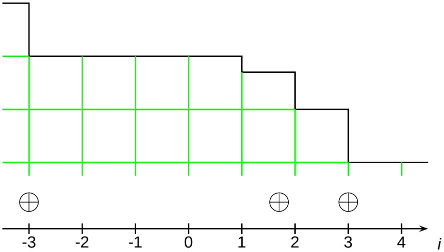


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

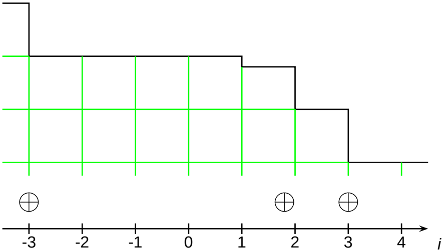


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

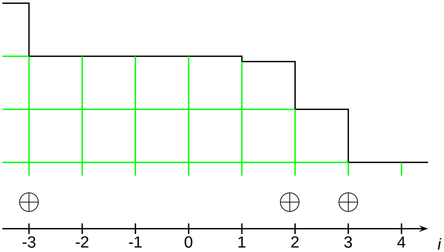


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

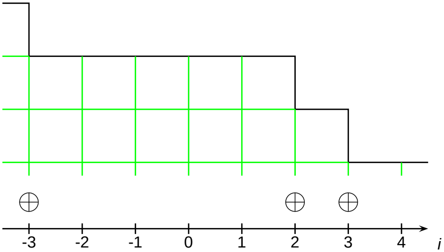


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

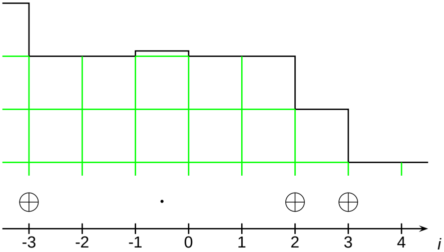


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

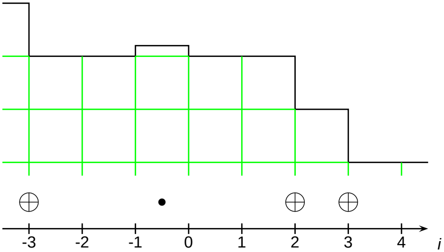


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

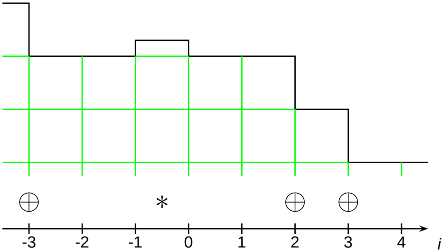


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

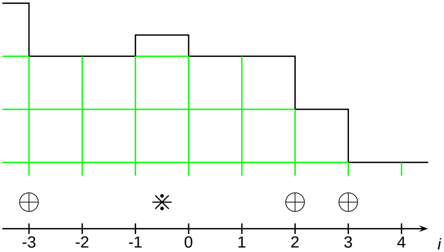


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

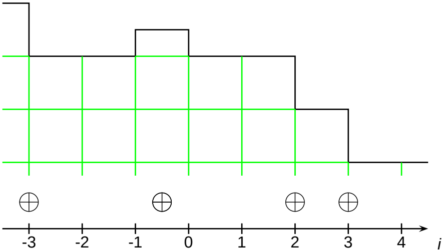


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A ⊕ ⊖ 0 model

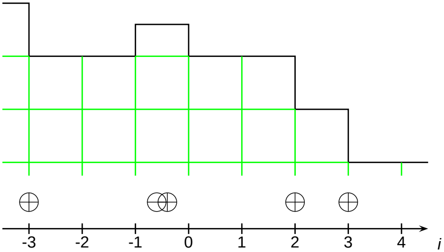


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

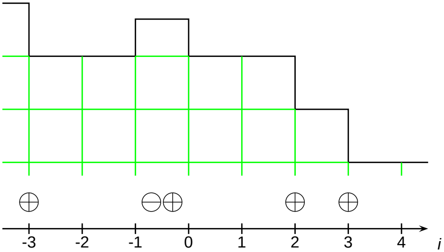


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

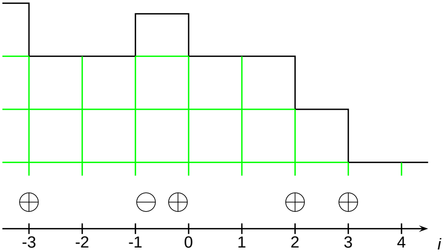


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

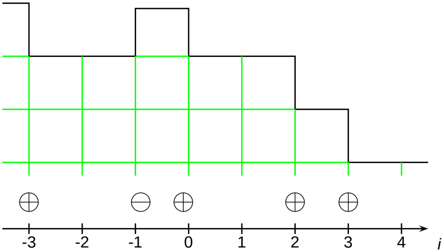


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

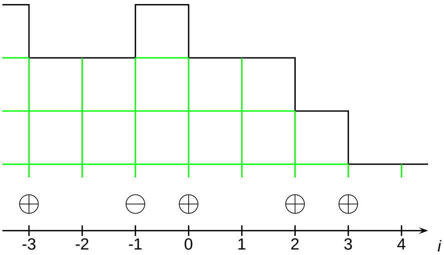


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A ⊕ ⊖ 0 model

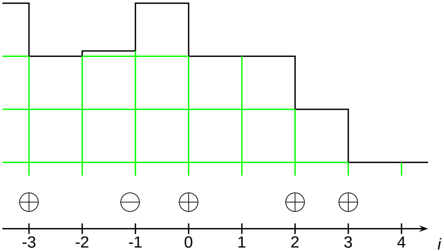


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

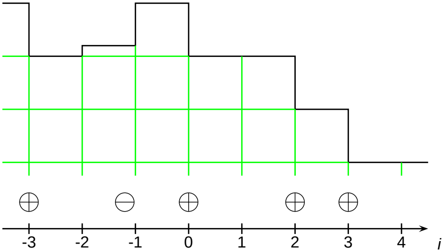


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

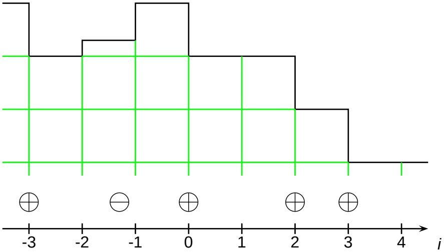


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

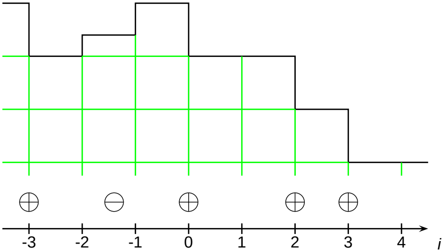


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

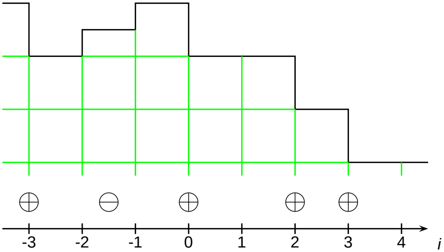


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

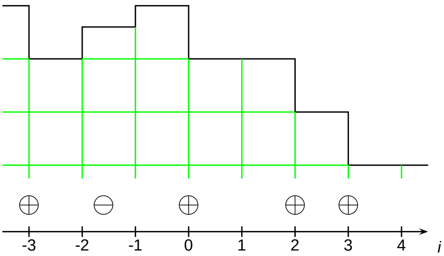


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

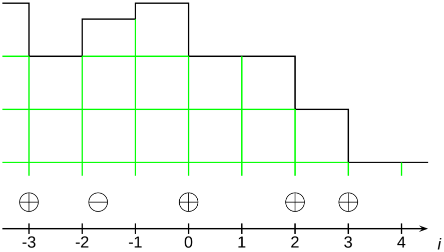


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

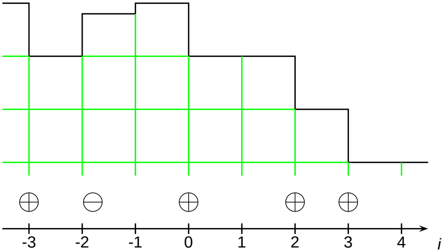


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

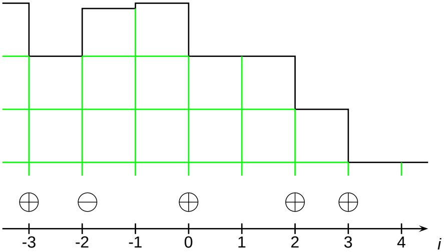


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A ⊕ ⊖ 0 model

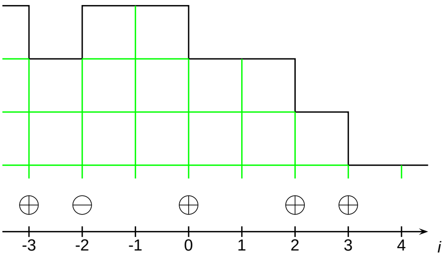


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

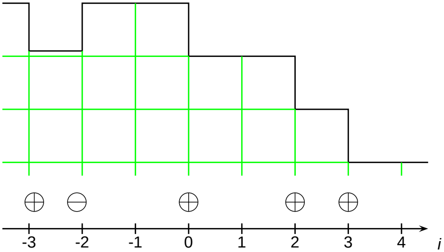


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

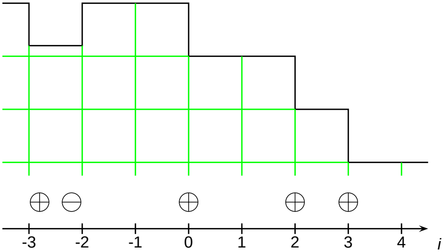


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

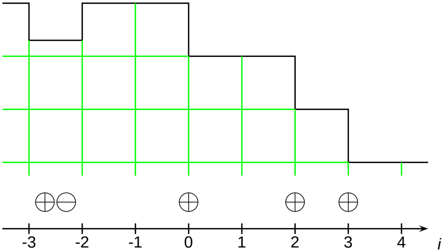


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

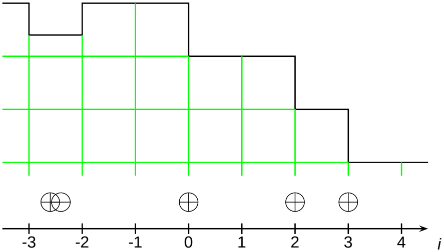


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

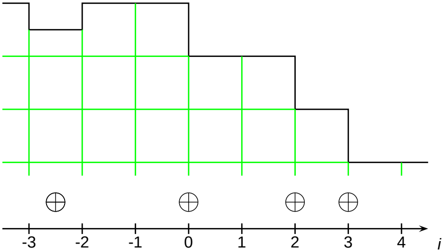


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

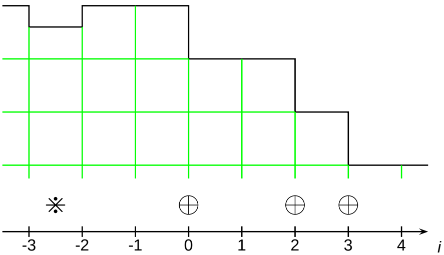


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

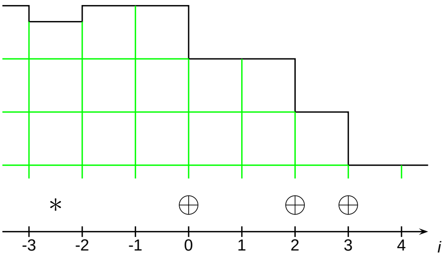


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

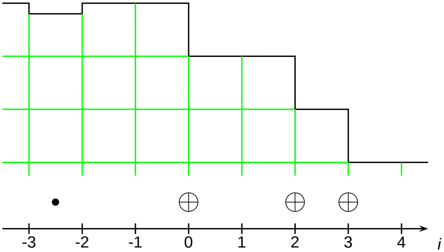


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

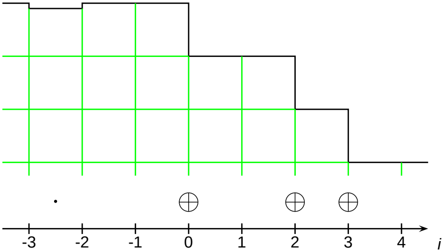


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model

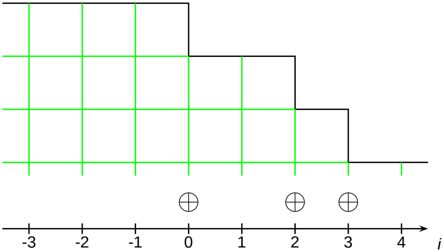


$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important (= ergodic) stationary distributions.

A $\oplus$ $\ominus$ 0 model



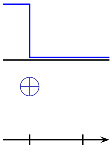
$\omega_i = -1, 0, 1$: a family of product initial distribution.

Those product distributions are stationary (and non-reversible).

These are the important ($=$ ergodic) stationary distributions.

The second class particle

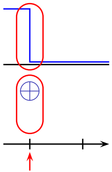
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

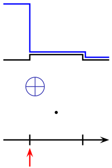
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

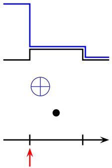
... works much like in TASEP for $c \le 1$. The interesting case:



With rate c

The second class particle

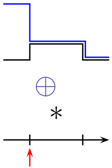
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

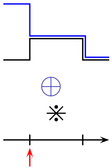
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

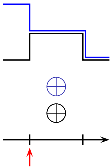
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

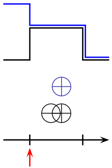
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

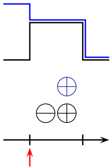
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

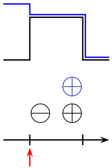
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

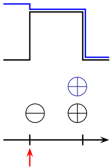
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

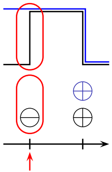
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

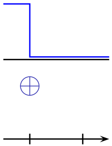
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate c

The second class particle

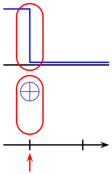
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

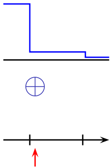
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

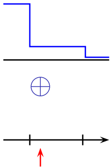
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

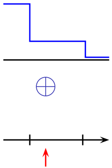
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

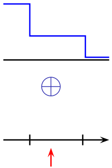
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

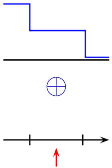
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

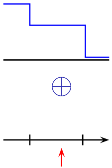
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

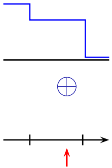
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

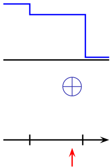
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

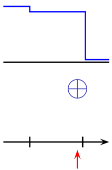
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

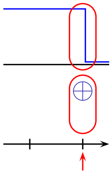
... works much like in TASEP for $c \le 1$. The interesting case:



With rate $1 - c$

The second class particle

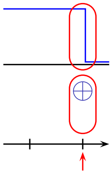
... works much like in TASEP for $c \leq 1$. The interesting case:



With rate $1 - c$

The second class particle

... works much like in TASEP for $c \leq 1$. The interesting case:

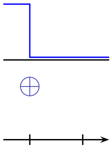


With rate $1 - c$

Attractivity

The second class particle

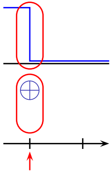
But, for $c > 1$:



With rate 1

The second class particle

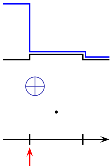
But, for $c > 1$:



With rate 1

The second class particle

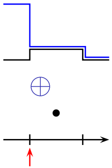
But, for $c > 1$:



With rate 1

The second class particle

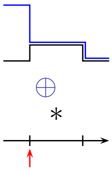
But, for $c > 1$:



With rate 1

The second class particle

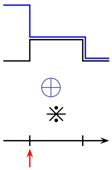
But, for $c > 1$:



With rate 1

The second class particle

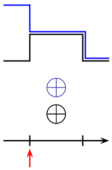
But, for $c > 1$:



With rate 1

The second class particle

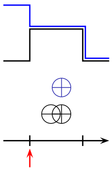
But, for $c > 1$:



With rate 1

The second class particle

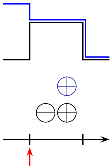
But, for $c > 1$:



With rate 1

The second class particle

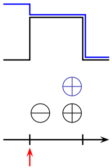
But, for $c > 1$:



With rate 1

The second class particle

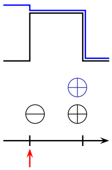
But, for $c > 1$:



With rate 1

The second class particle

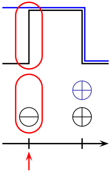
But, for $c > 1$:



With rate 1

The second class particle

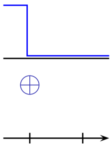
But, for $c > 1$:



With rate 1

The second class particle

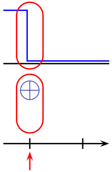
But, for $c > 1$:



With rate $c - 1$

The second class particle

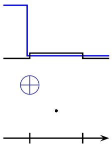
But, for $c > 1$:



With rate $c - 1$

The second class particle

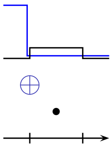
But, for $c > 1$:



With rate $c - 1$

The second class particle

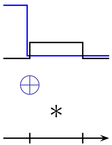
But, for $c > 1$:



With rate $c - 1$

The second class particle

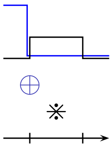
But, for $c > 1$:



With rate $c - 1$

The second class particle

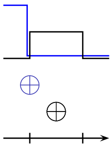
But, for $c > 1$:



With rate $c - 1$

The second class particle

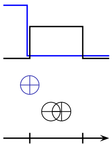
But, for $c > 1$:



With rate $c - 1$

The second class particle

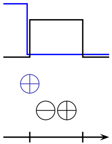
But, for $c > 1$:



With rate $c - 1$

The second class particle

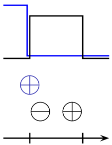
But, for $c > 1$:



With rate $c - 1$

The second class particle

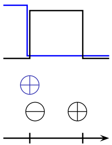
But, for $c > 1$:



With rate $c - 1$

The second class particle

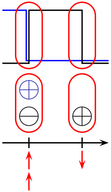
But, for $c > 1$:



With rate $c - 1$

The second class particle

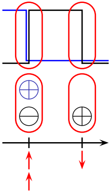
But, for $c > 1$:



With rate $c - 1$

The second class particle

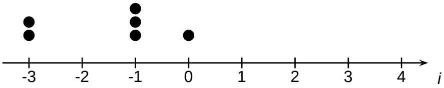
But, for $c > 1$:



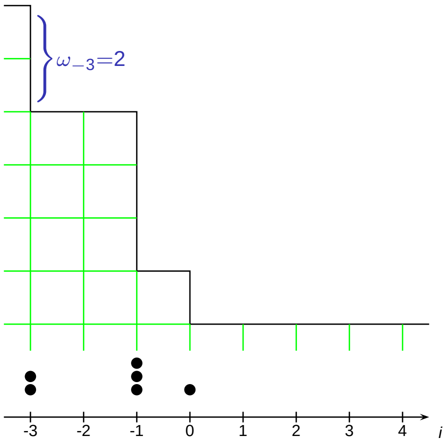
With rate $c - 1$

Non-attractivity

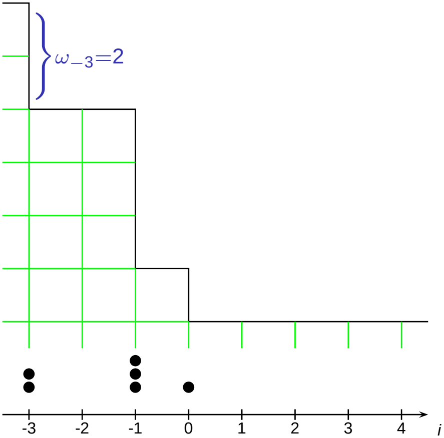
Totally asymmetric zero range process



Totally asymmetric zero range process

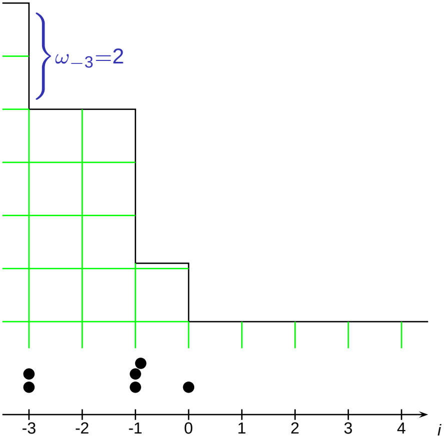


Totally asymmetric zero range process



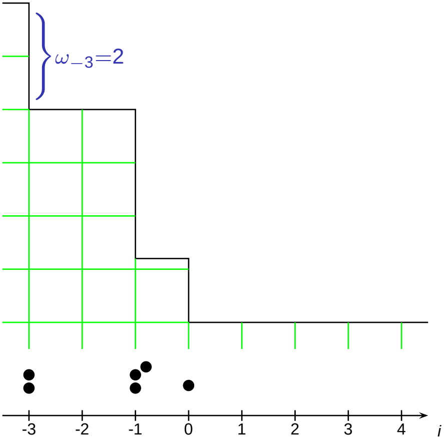
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



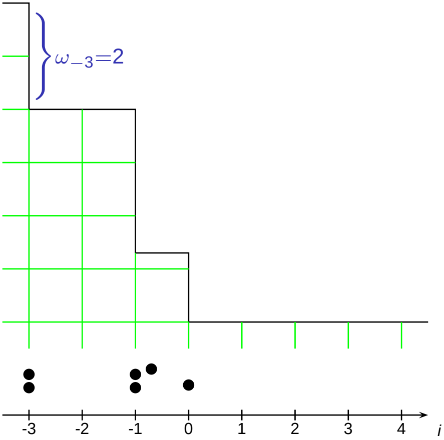
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



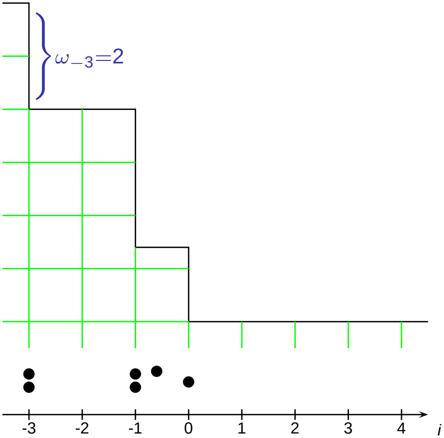
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



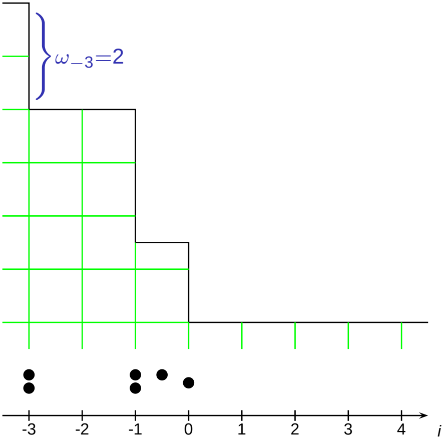
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



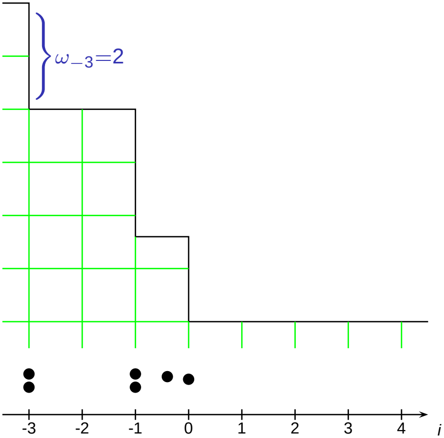
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



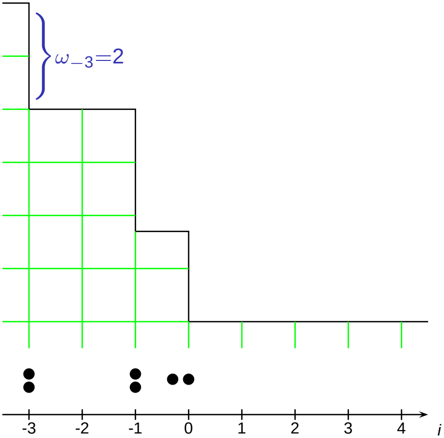
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



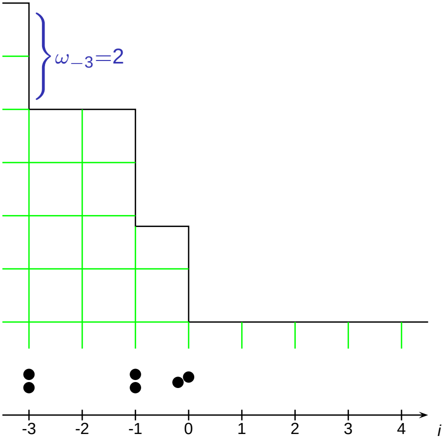
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



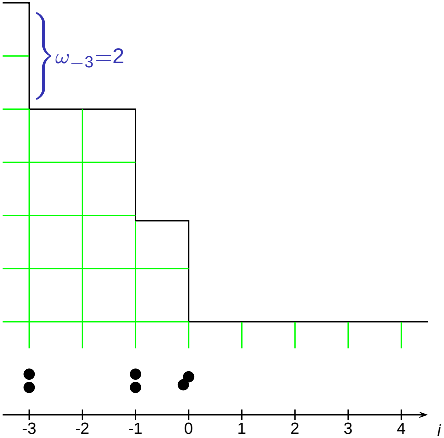
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



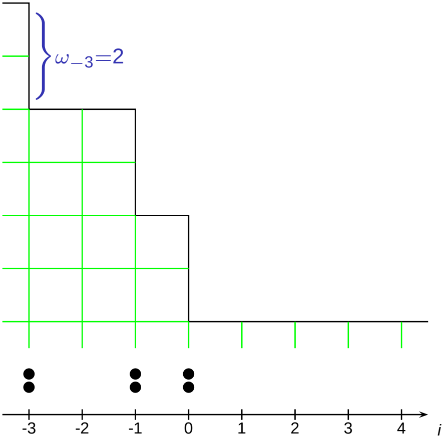
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



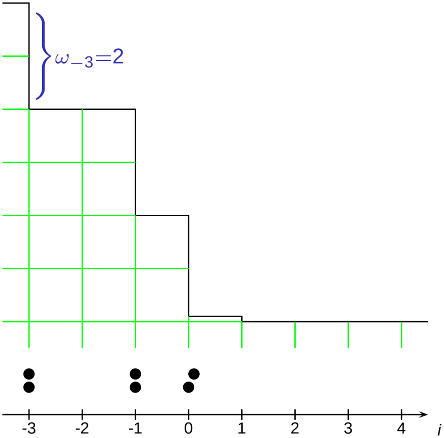
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



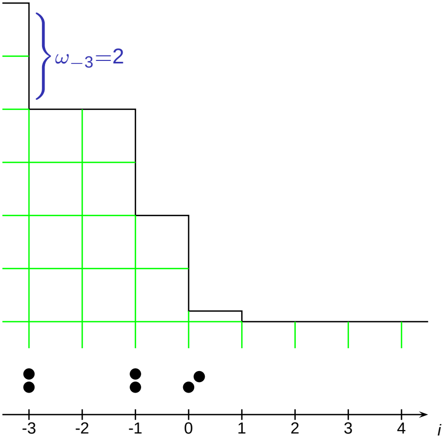
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



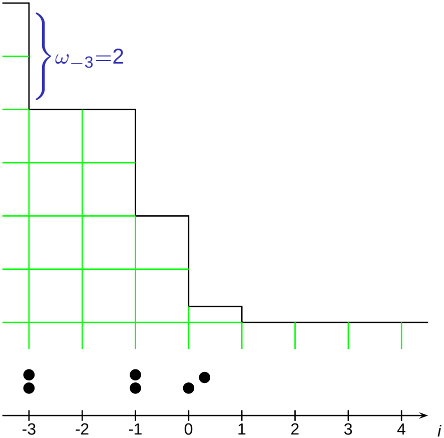
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



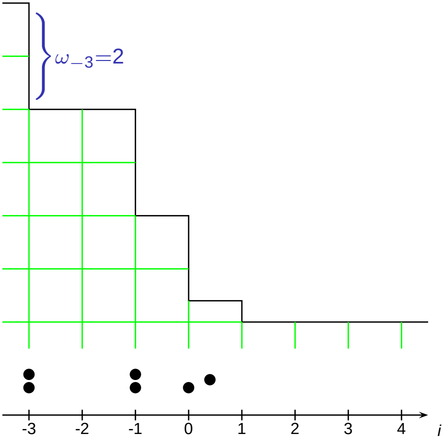
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



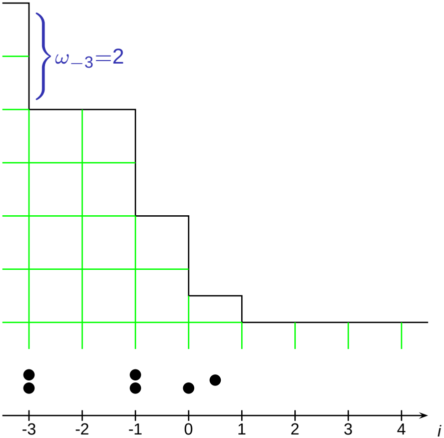
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



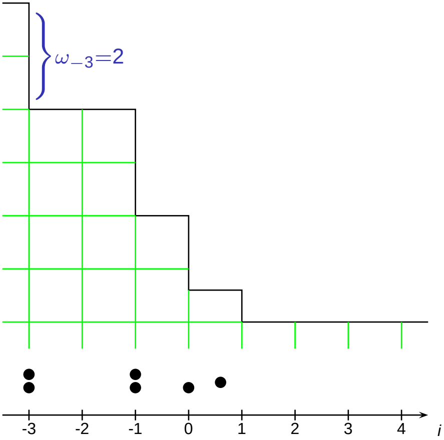
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



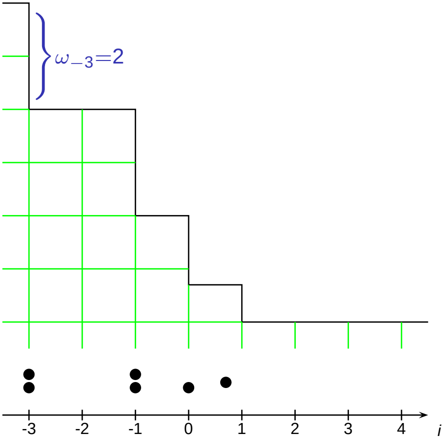
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



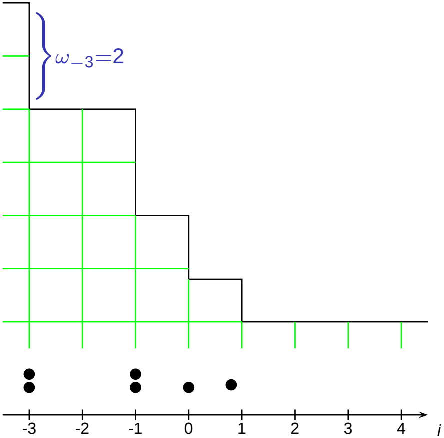
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



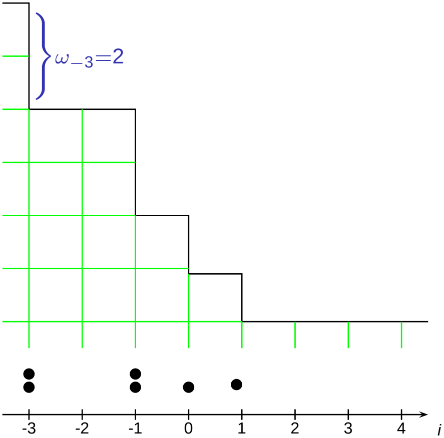
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



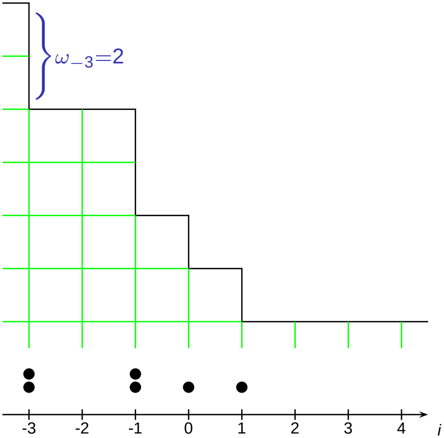
Particles jump to the right with rate $r(\omega_i)$.

Totally asymmetric zero range process



Particles jump to the right with rate $r(\omega_i)$.

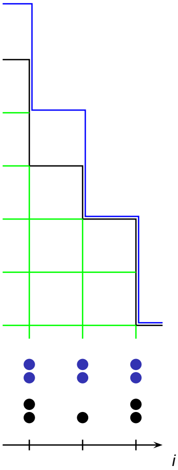
Totally asymmetric zero range process



Particles jump to the right with rate $r(\omega_i)$.

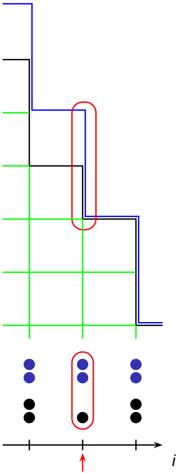
The second class particle: attractive case

States ω and ω only differ at one site.



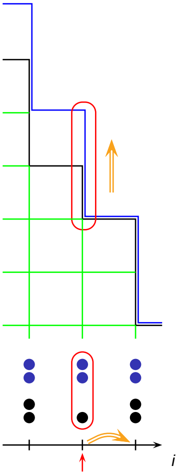
The second class particle: attractive case

States ω and ω only differ at one site.



The second class particle: attractive case

States ω and ω only differ at one site.

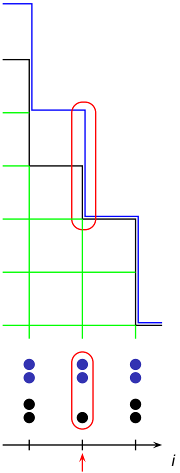


Growth on the right:

rate$\leq$rate

The second class particle: attractive case

States ω and ω only differ at one site.



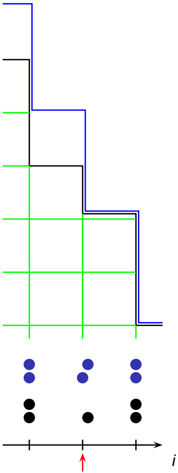
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



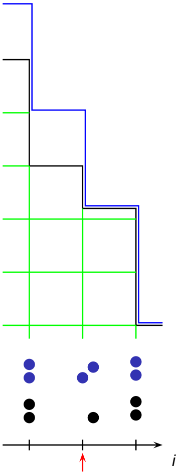
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

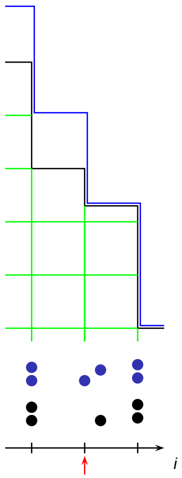
States ω and ω only differ at one site.



Growth on the right:
rate$\le$rate
with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



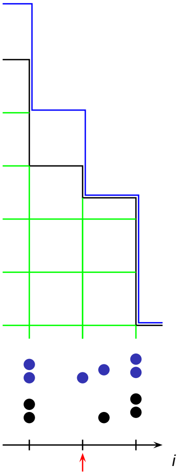
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



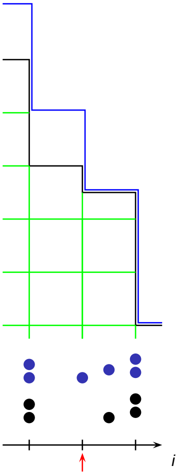
Growth on the right:

rate$\le$rate

with rate:

The second class particle: attractive case

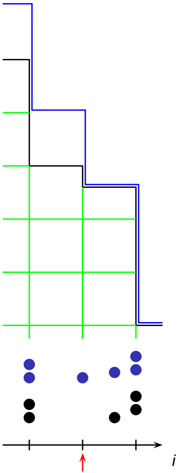
States ω and ω only differ at one site.



Growth on the right:
rate≤rate
with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



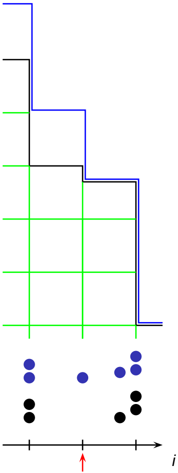
Growth on the right:

rate≤rate

with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



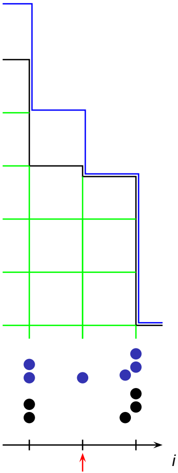
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



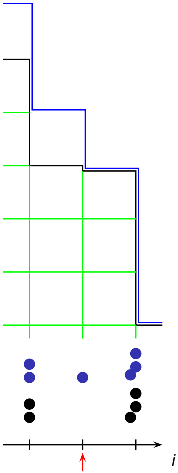
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



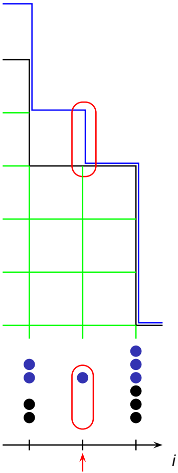
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

States ω and ω only differ at one site.



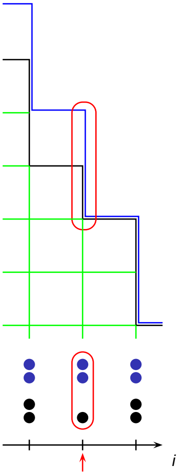
Growth on the right:

rate$\leq$rate

with rate:

The second class particle: attractive case

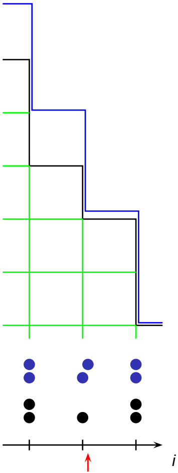
States ω and ω only differ at one site.



Growth on the right:
rate$\leq$rate
with rate-rate:

The second class particle: attractive case

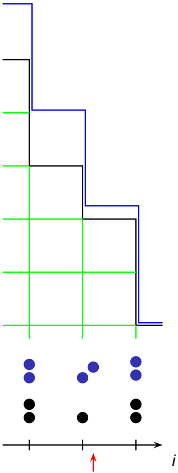
States ω and ω only differ at one site.



Growth on the right:
rate≤rate
with rate-rate:

The second class particle: attractive case

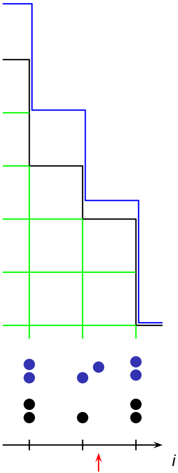
States ω and ω only differ at one site.



Growth on the right:
rate≤rate
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



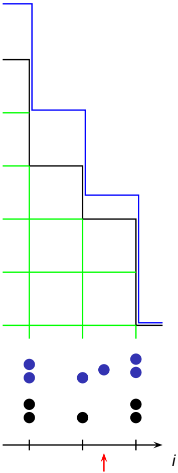
Growth on the right:

rate$\leq$rate

with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



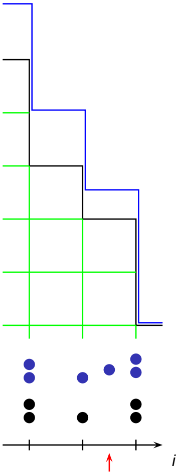
Growth on the right:

rate≤rate

with rate-rate:

The second class particle: attractive case

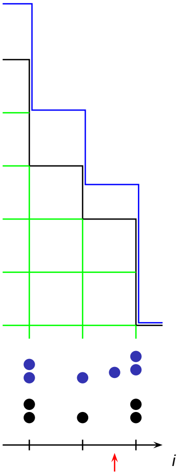
States ω and ω only differ at one site.



Growth on the right:
rate≤rate
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



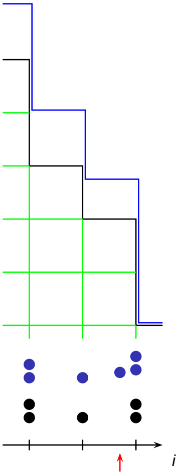
Growth on the right:

rate$\leq$rate

with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



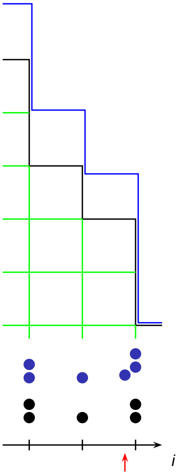
Growth on the right:

rate≤rate

with rate-rate:

The second class particle: attractive case

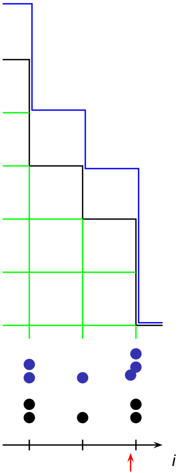
States ω and ω only differ at one site.



Growth on the right:
rate≤rate
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



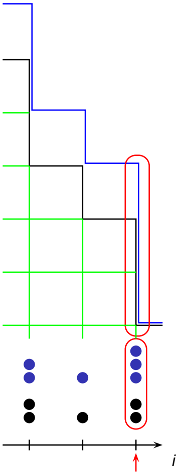
Growth on the right:

rate$\leq$rate

with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



Growth on the right:

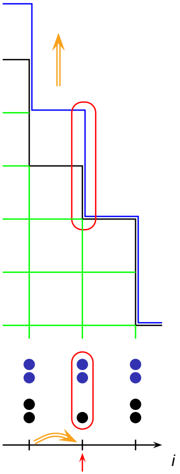
rate≤rate

with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.

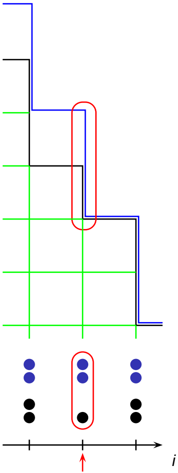
Growth on the left:
rate $\geq$ rate



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate
with rate:



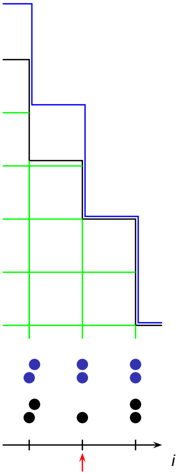
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

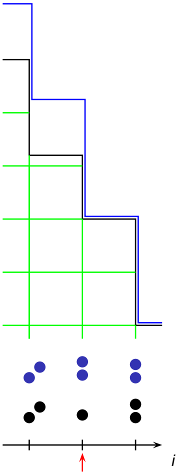
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

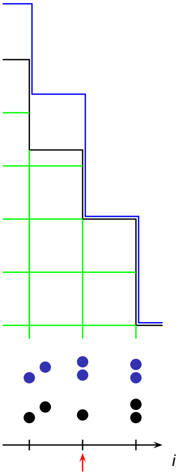
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

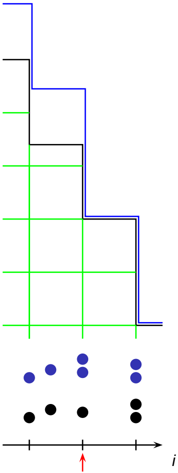
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

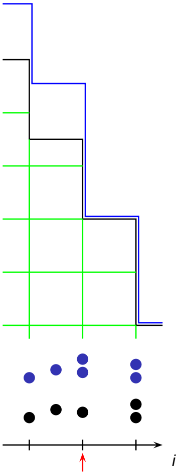
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

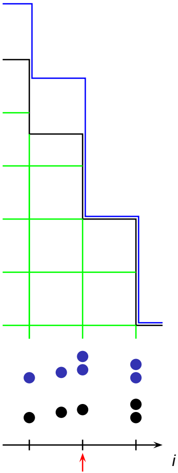
Growth on the left:
rate≥rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

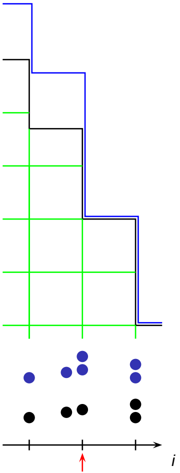
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

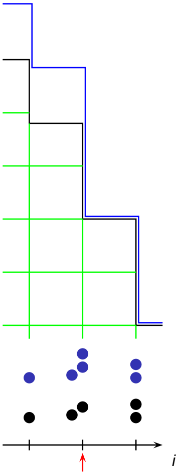
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate≥rate
with rate:



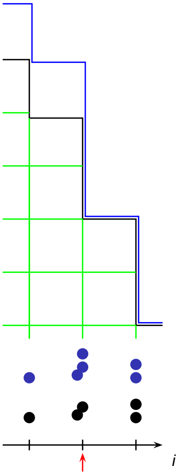
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

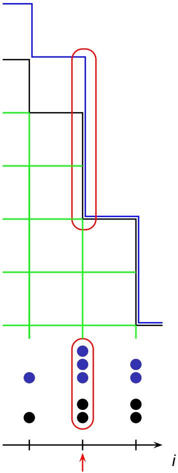
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

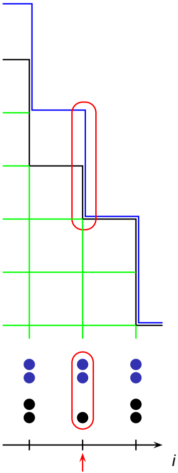
Growth on the left:
rate$\geq$rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

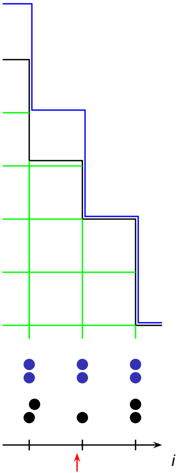
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

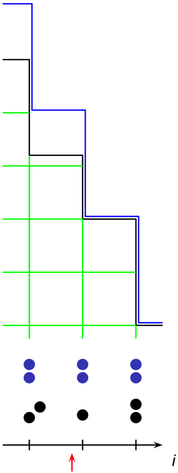
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate
with rate-rate:



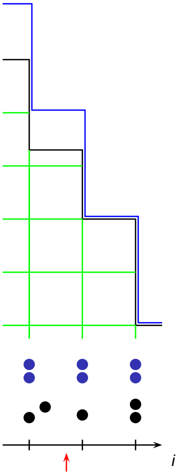
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

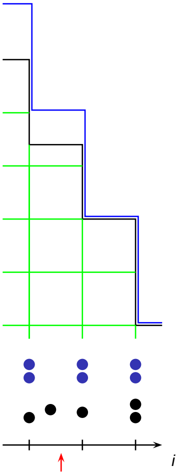
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

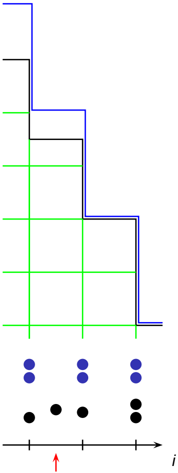
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate
with rate-rate:



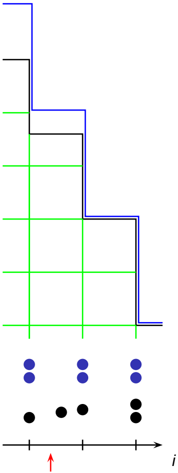
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

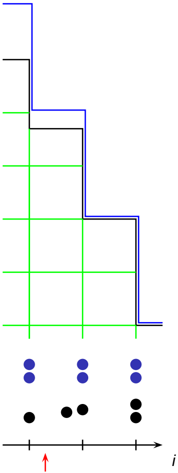
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

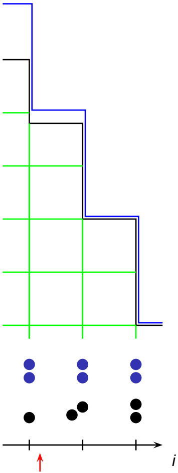
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

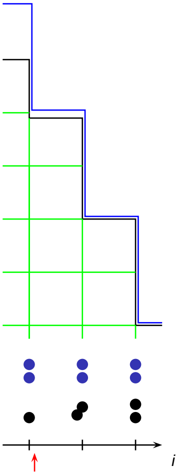
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

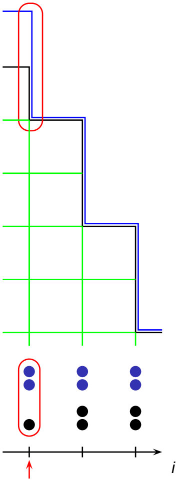
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

States ω and ω only differ at one site.

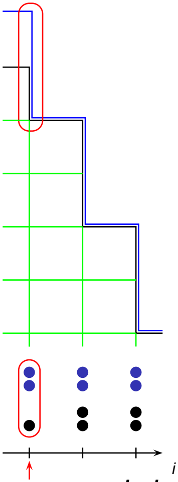
Growth on the left:
rate$\geq$rate
with rate-rate:



The second class particle: attractive case

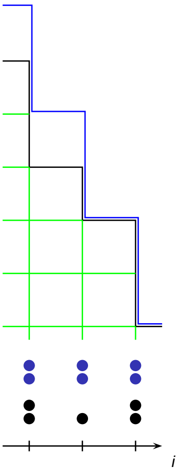
States ω and ω only differ at one site.

Growth on the left:
rate$\geq$rate
with rate-rate:

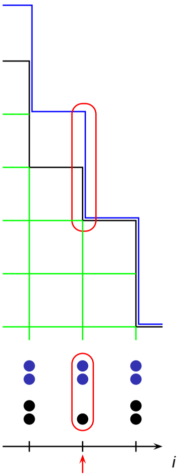


A single discrepancy↑, the *second class particle*, is conserved.

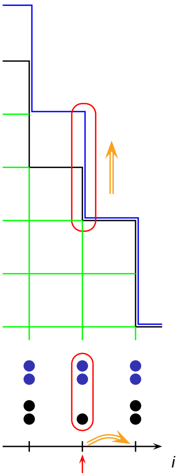
The second class particle: non-attractive case



The second class particle: non-attractive case

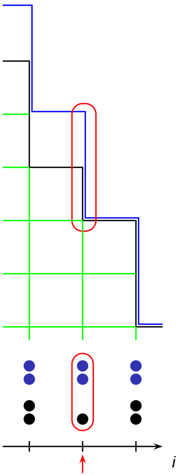


The second class particle: non-attractive case



Growth on the right:
rate > rate

The second class particle: non-attractive case

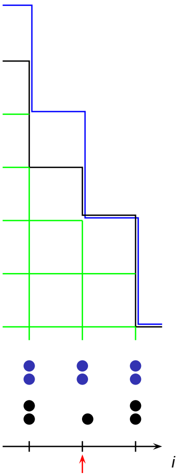


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

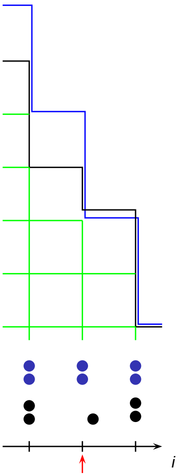


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

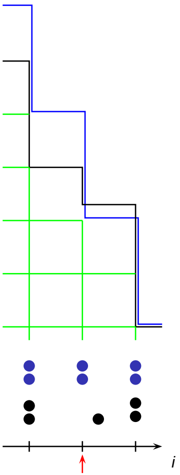


Growth on the right:

rate$>$rate

rate-rate:

The second class particle: non-attractive case

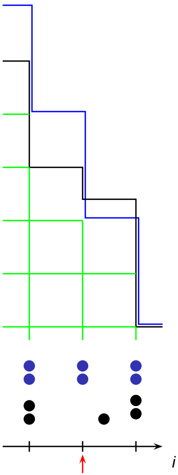


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

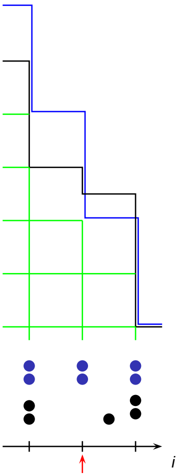


Growth on the right:

rate $>$ rate

rate-rate:

The second class particle: non-attractive case

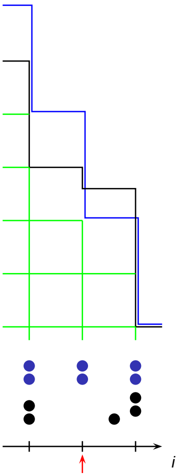


Growth on the right:

rate$>$rate

rate-rate:

The second class particle: non-attractive case

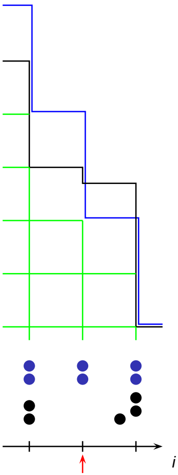


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

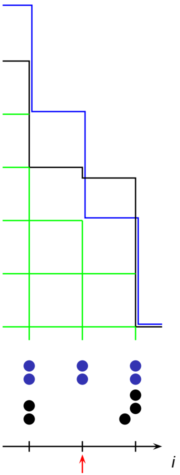


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

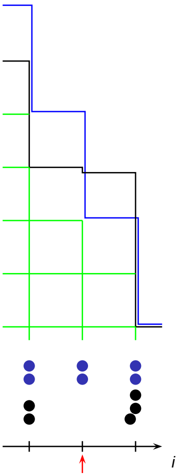


Growth on the right:

rate$>$rate

rate-rate:

The second class particle: non-attractive case

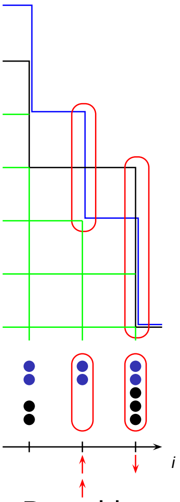


Growth on the right:

rate$>$rate

rate-rate:

The second class particle: non-attractive case

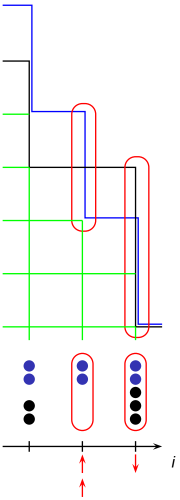


Growth on the right:

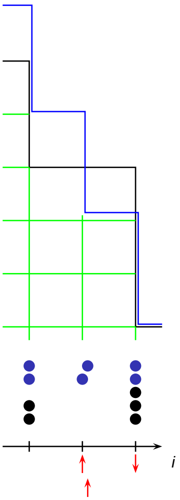
rate $>$ rate

rate-rate:

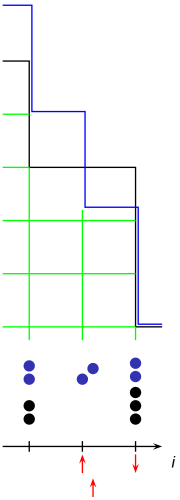
The second class particle: non-attractive case



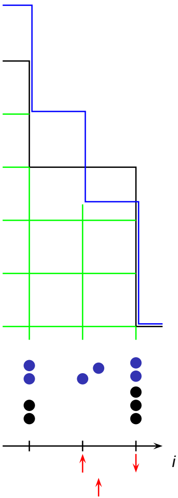
The second class particle: non-attractive case



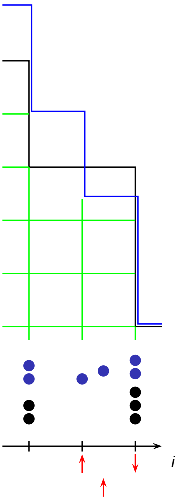
The second class particle: non-attractive case



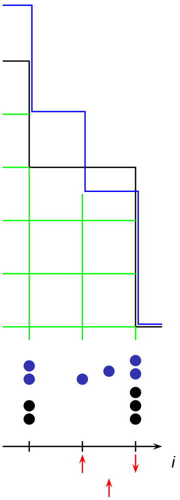
The second class particle: non-attractive case



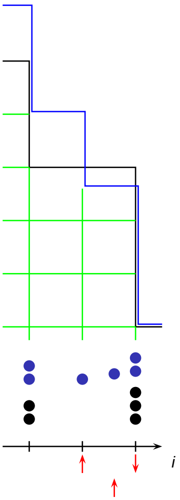
The second class particle: non-attractive case



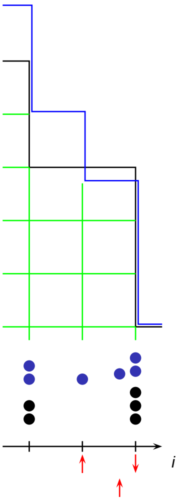
The second class particle: non-attractive case



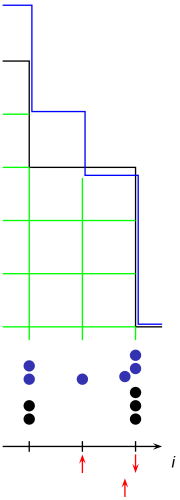
The second class particle: non-attractive case



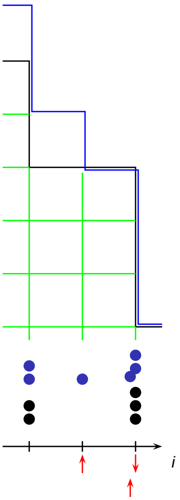
The second class particle: non-attractive case



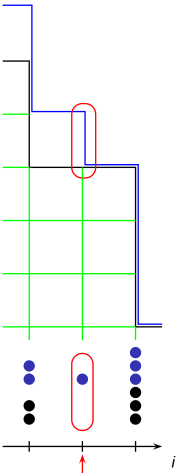
The second class particle: non-attractive case



The second class particle: non-attractive case



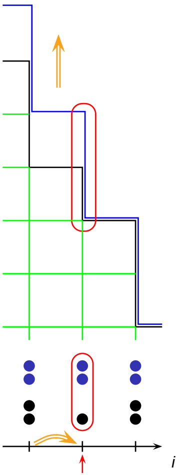
The second class particle: non-attractive case



Annihilating

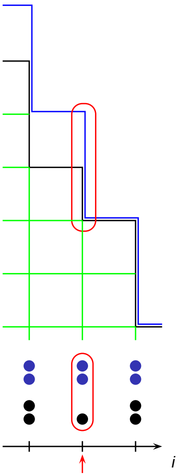
The second class particle: non-attractive case

Growth on the left:
rate<rate



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

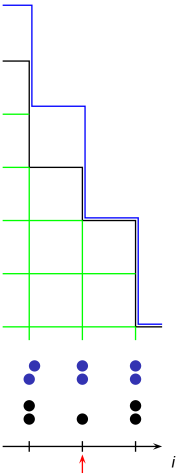


The second class particle: non-attractive case

Growth on the left:

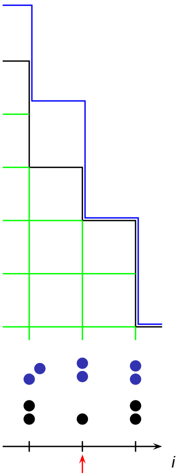
rate$<$rate

with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate$<$rate
with rate-rate:

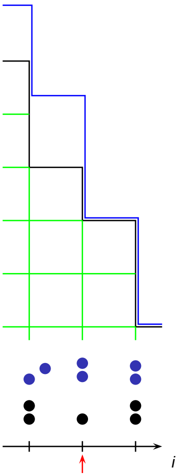


The second class particle: non-attractive case

Growth on the left:

rate<rate

with rate-rate:

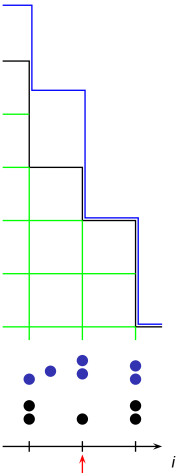


The second class particle: non-attractive case

Growth on the left:

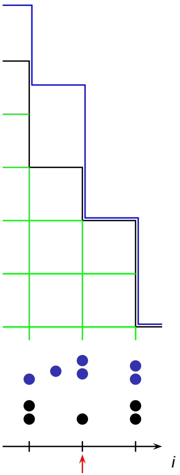
rate<rate

with rate-rate:



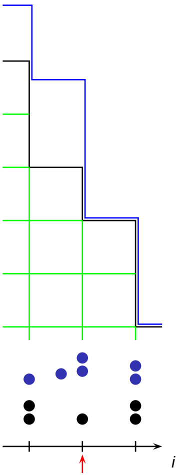
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



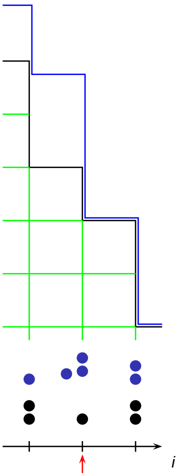
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

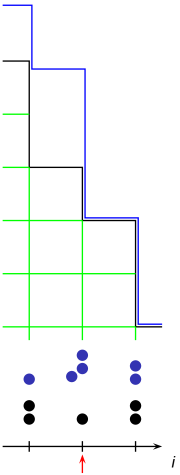


The second class particle: non-attractive case

Growth on the left:

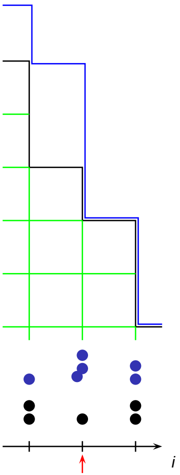
rate<rate

with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

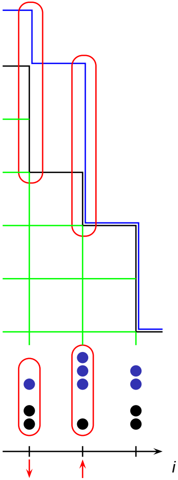


The second class particle: non-attractive case

Growth on the left:

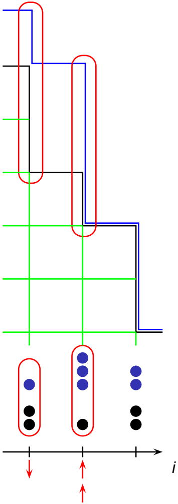
rate<rate

with rate-rate:

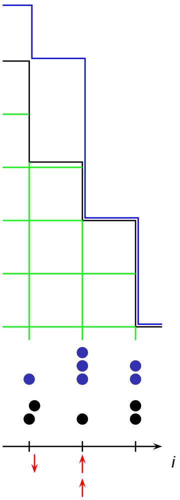


Branching

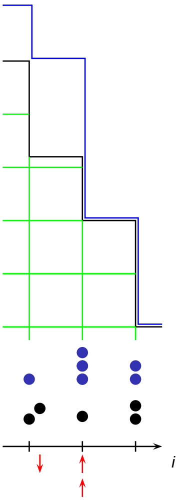
The second class particle: non-attractive case



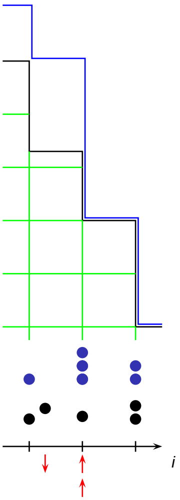
The second class particle: non-attractive case



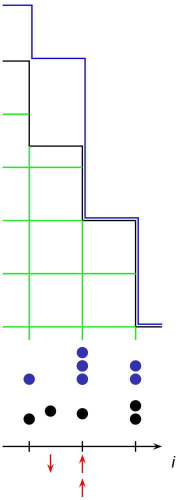
The second class particle: non-attractive case



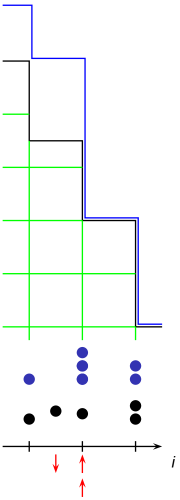
The second class particle: non-attractive case



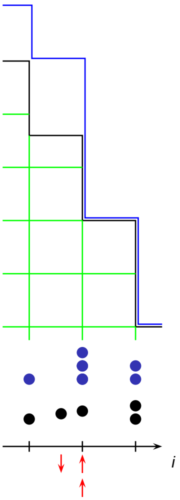
The second class particle: non-attractive case



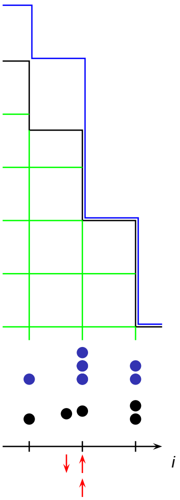
The second class particle: non-attractive case



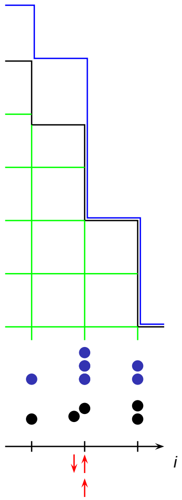
The second class particle: non-attractive case



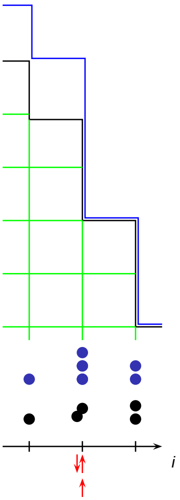
The second class particle: non-attractive case



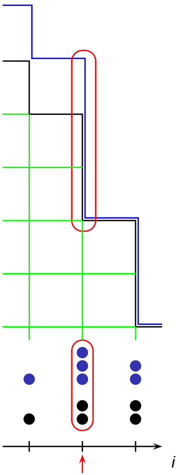
The second class particle: non-attractive case



The second class particle: non-attractive case



The second class particle: non-attractive case



Annihilating

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i$$

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i\\ &= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t - \mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t + \mathfrak{o}(t)}{t}\end{aligned}$$

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i \\ &= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t - \mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t + \mathfrak{o}(t)}{t} \\ &= \varrho_{i-1}[1-\varrho_i] - \varrho_i[1-\varrho_{i+1}].\end{aligned}$$

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i \\ &= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t-\mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t+\mathfrak{o}(t)}{t} \\ &= \varrho_{i-1}[1-\varrho_i]-\varrho_i[1-\varrho_{i+1}].\end{aligned}$$

Define now $T=t\cdot\varepsilon$, $X=i\cdot\varepsilon$, $\hat{\varrho}(T,X)=\varrho_i(t)=\varrho_{\frac{X}{\varepsilon}}\left(\frac{T}{\varepsilon}\right)$.

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i \\ &= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t-\mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t+\mathfrak{o}(t)}{t} \\ &= \varrho_{i-1}[1-\varrho_i]-\varrho_i[1-\varrho_{i+1}].\end{aligned}$$

Define now $T=t\cdot\varepsilon$, $X=i\cdot\varepsilon$, $\hat{\varrho}(T,X)=\varrho_i(t)=\varrho_{\frac{X}{\varepsilon}}\left(\frac{T}{\varepsilon}\right)$.

$$\varepsilon\frac{\partial}{\partial T}\hat{\varrho}=\hat{\varrho}(T,X-\varepsilon)\big[1-\hat{\varrho}(T,X)\big]-\hat{\varrho}(T,X)\big[1-\hat{\varrho}(T,X+\varepsilon)\big]$$

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i \\ &= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t - \mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t + \mathfrak{o}(t)}{t} \\ &= \varrho_{i-1}[1-\varrho_i] - \varrho_i[1-\varrho_{i+1}].\end{aligned}$$

Define now $T = t\cdot\varepsilon$, $X = i\cdot\varepsilon$, $\hat{\varrho}(T,X) = \varrho_i(t) = \varrho_{\frac{X}{\varepsilon}}\left(\frac{T}{\varepsilon}\right)$.

$$\varepsilon\frac{\partial}{\partial T}\hat{\varrho} = \hat{\varrho}(T,X-\varepsilon)\big[1-\hat{\varrho}(T,X)\big] - \hat{\varrho}(T,X)\big[1-\hat{\varrho}(T,X+\varepsilon)\big]$$

$$\frac{\partial}{\partial T}\hat{\varrho} = \frac{\hat{\varrho}(T,X-\varepsilon)\big[1-\hat{\varrho}(T,X)\big] - \hat{\varrho}(T,X)\big[1-\hat{\varrho}(T,X+\varepsilon)\big]}{\varepsilon}$$

On large scales

Let us now allow the density to change slowly in space. The change of density at position i:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varrho_i &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}\omega_i \\ &= \lim_{t\to 0}\frac{\mathbf{P}\{\omega_{i-1}=1,\ \omega_i=0\}t-\mathbf{P}\{\omega_i=1,\ \omega_{i+1}=0\}t+\mathfrak{o}(t)}{t} \\ &= \varrho_{i-1}[1-\varrho_i]-\varrho_i[1-\varrho_{i+1}].\end{aligned}$$

Define now $T=t\cdot\varepsilon$, $X=i\cdot\varepsilon$, $\hat{\varrho}(T,X)=\varrho_i(t)=\varrho_{\frac{X}{\varepsilon}}\left(\frac{T}{\varepsilon}\right)$.

$$\varepsilon\frac{\partial}{\partial T}\hat{\varrho}=\hat{\varrho}(T,X-\varepsilon)\big[1-\hat{\varrho}(T,X)\big]-\hat{\varrho}(T,X)\big[1-\hat{\varrho}(T,X+\varepsilon)\big]$$

$$\frac{\partial}{\partial T}\hat{\varrho}=\frac{\hat{\varrho}(T,X-\varepsilon)\big[1-\hat{\varrho}(T,X)\big]-\hat{\varrho}(T,X)\big[1-\hat{\varrho}(T,X+\varepsilon)\big]}{\varepsilon}$$

$$\frac{\partial}{\partial T}\hat{\varrho}+\frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})]=0 \qquad (\textit{Burgers eq.}).$$

Burgers eq.: characteristics

$$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1 - \hat{\varrho})] = 0$$

Burgers eq.: nonlinear PDE.

Burgers eq.: characteristics

$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})] = 0$ Burgers eq.: nonlinear PDE.

Characteristics: find a path $X(T)$ where $\hat{\varrho}\big(T, X(T)\big)$ is a constant:

$$\frac{\mathrm{d}}{\mathrm{d}T}\hat{\varrho}\big(T, X(T)\big) = 0$$

Burgers eq.: characteristics

$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})] = 0$ Burgers eq.: nonlinear PDE.

Characteristics: find a path $X(T)$ where $\hat{\varrho}\big(T, X(T)\big)$ is a constant:

$$\frac{\mathrm{d}}{\mathrm{d}T}\hat{\varrho}\big(T, X(T)\big) = 0$$

$$\frac{\partial}{\partial T}\hat{\varrho} + \dot{X}(T) \cdot \frac{\partial}{\partial X}\hat{\varrho} = 0$$

Burgers eq.: characteristics

$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})] = 0$ Burgers eq.: nonlinear PDE.

Characteristics: find a path $X(T)$ where $\hat{\varrho}\big(T, X(T)\big)$ is a constant:

$$\frac{\mathrm{d}}{\mathrm{d}T}\hat{\varrho}\big(T, X(T)\big) = 0$$

$$\frac{\partial}{\partial T}\hat{\varrho} + \dot{X}(T)\cdot\frac{\partial}{\partial X}\hat{\varrho} = 0$$

$$\frac{\partial}{\partial T}\hat{\varrho} + (1-2\hat{\varrho})\cdot\frac{\partial}{\partial X}\hat{\varrho} = 0$$

Burgers eq.: characteristics

$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})] = 0$ Burgers eq.: nonlinear PDE.

Characteristics: find a path $X(T)$ where $\hat{\varrho}\big(T, X(T)\big)$ is a constant:

$$\frac{\mathrm{d}}{\mathrm{d}T}\hat{\varrho}\big(T, X(T)\big) = 0$$
$$\frac{\partial}{\partial T}\hat{\varrho} + \dot{X}(T)\cdot\frac{\partial}{\partial X}\hat{\varrho} = 0$$
$$\frac{\partial}{\partial T}\hat{\varrho} + (1-2\hat{\varrho})\cdot\frac{\partial}{\partial X}\hat{\varrho} = 0$$

The characteristic velocity: $\dot{X}(T) = 1 - 2\hat{\varrho}$.

Burgers eq.: characteristics

$\frac{\partial}{\partial T}\hat{\varrho} + \frac{\partial}{\partial X}[\hat{\varrho}(1-\hat{\varrho})] = 0$ Burgers eq.: nonlinear PDE.

Characteristics: find a path $X(T)$ where $\hat{\varrho}\big(T, X(T)\big)$ is a constant:

$$\frac{\mathrm{d}}{\mathrm{d}T}\hat{\varrho}\big(T, X(T)\big) = 0$$

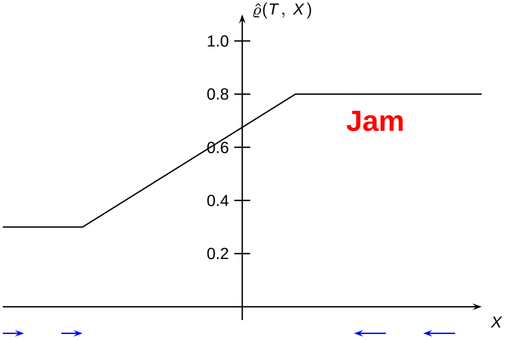
$$\frac{\partial}{\partial T}\hat{\varrho} + \dot{X}(T)\cdot\frac{\partial}{\partial X}\hat{\varrho} = 0$$

$$\frac{\partial}{\partial T}\hat{\varrho} + (1-2\hat{\varrho})\cdot\frac{\partial}{\partial X}\hat{\varrho} = 0$$

The characteristic velocity: $\dot{X}(T) = 1 - 2\hat{\varrho}$.

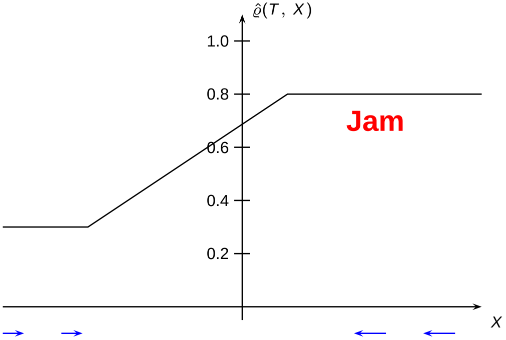
Second class particles are known to follow the characteristics.

On large scales



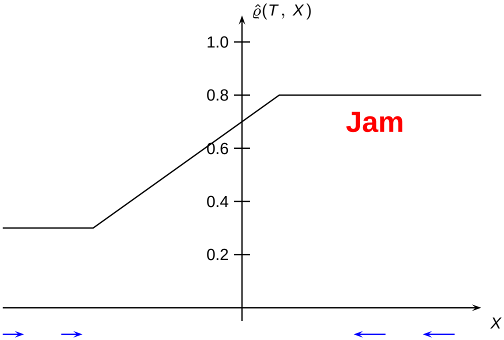
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



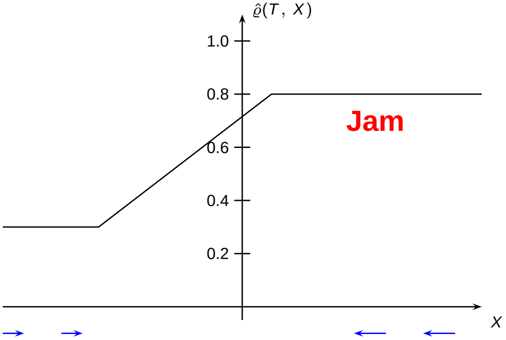
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



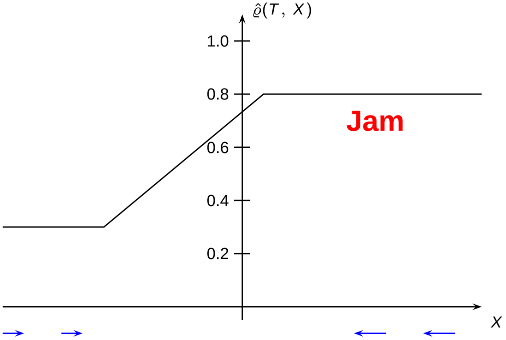
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



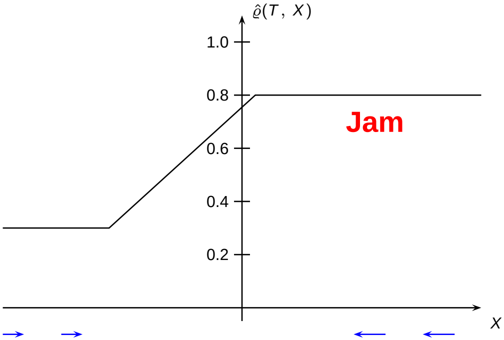
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



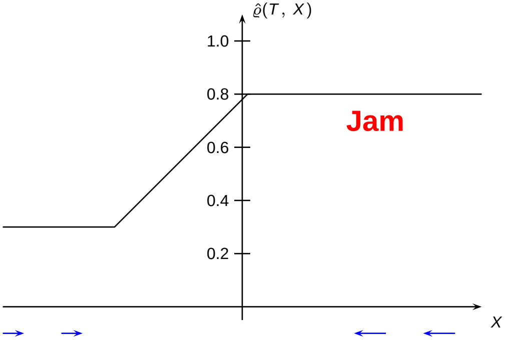
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



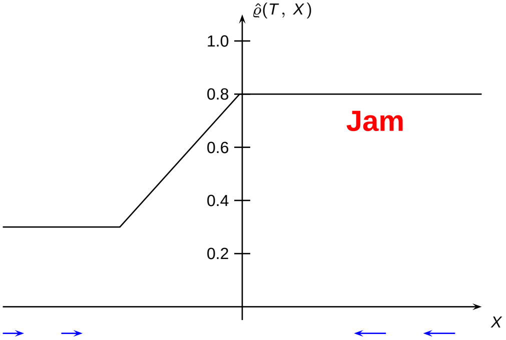
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



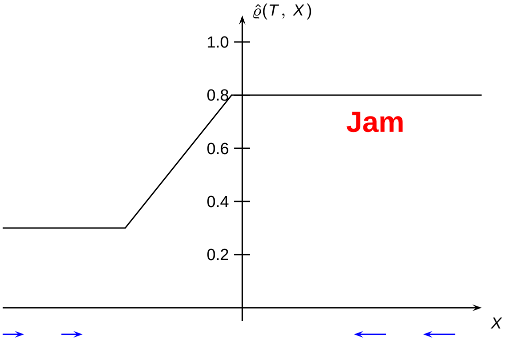
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



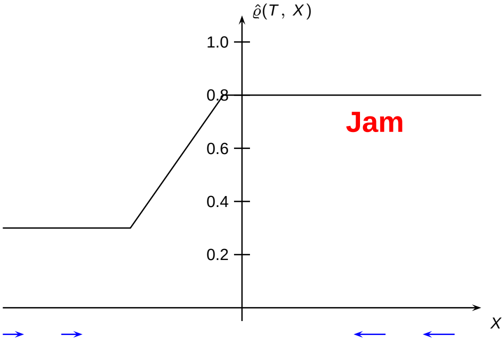
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



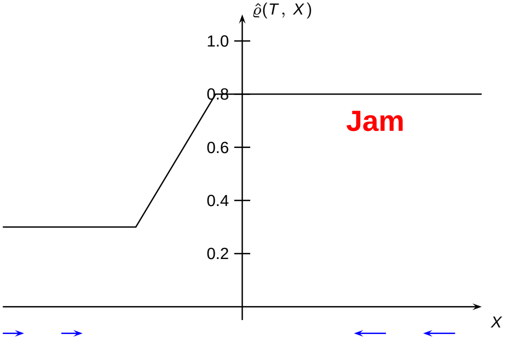
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



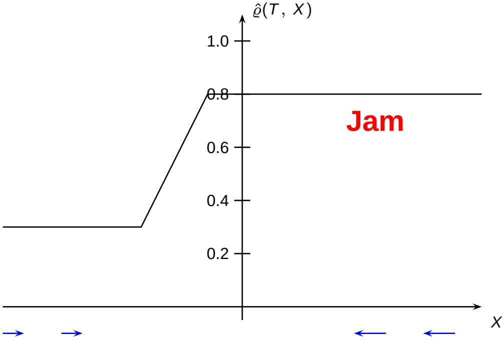
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



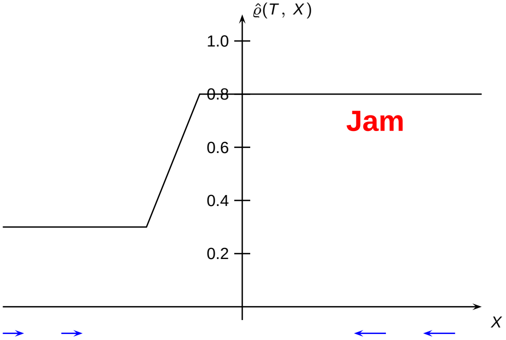
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



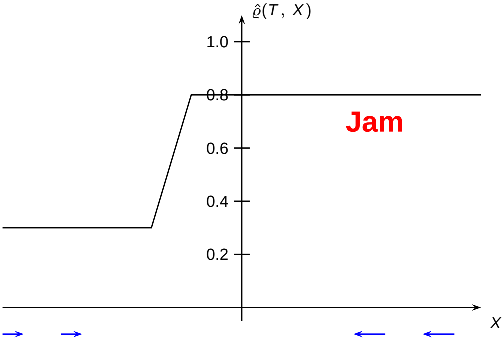
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



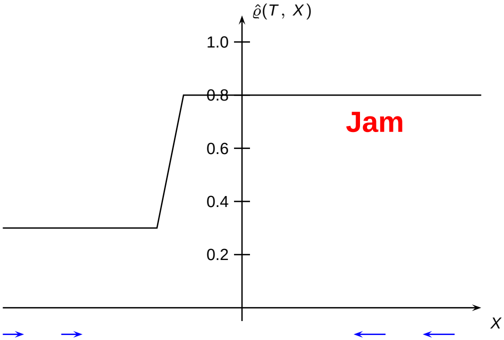
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



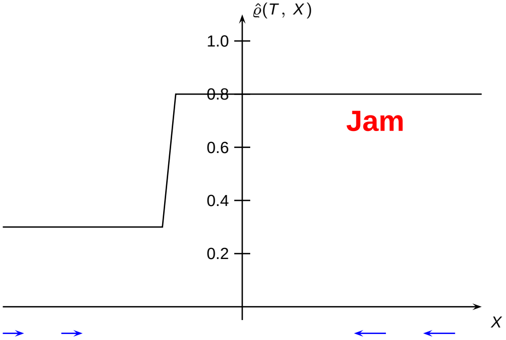
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



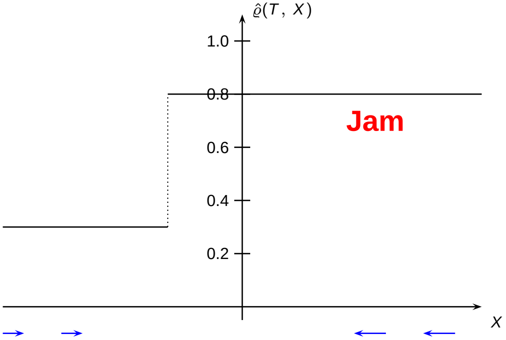
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



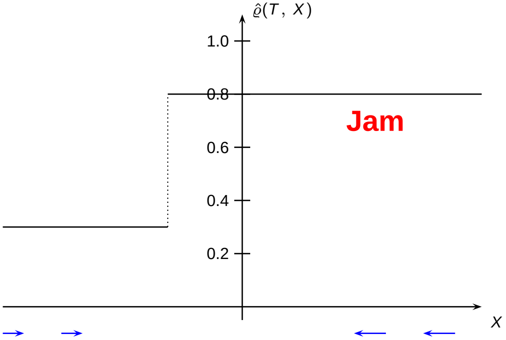
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



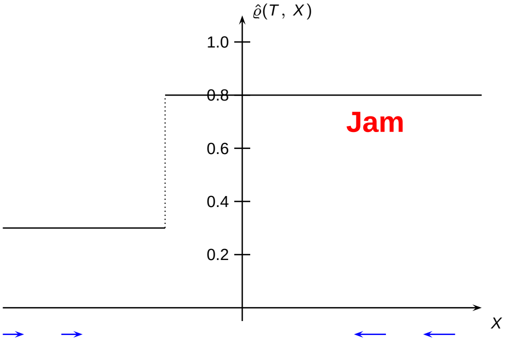
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



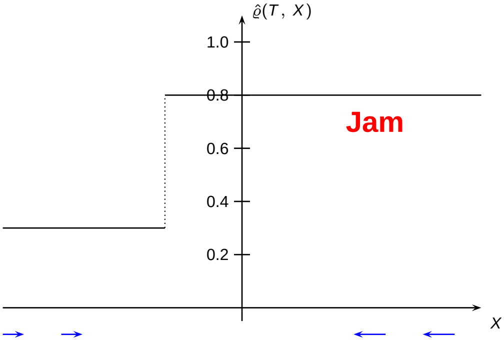
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



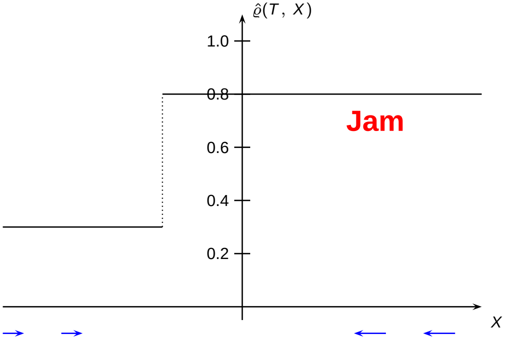
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



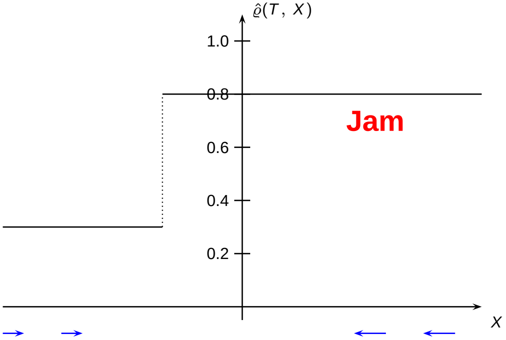
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



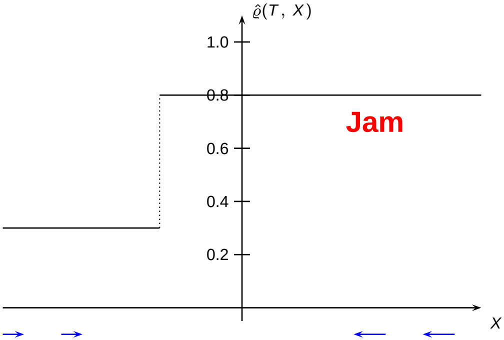
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



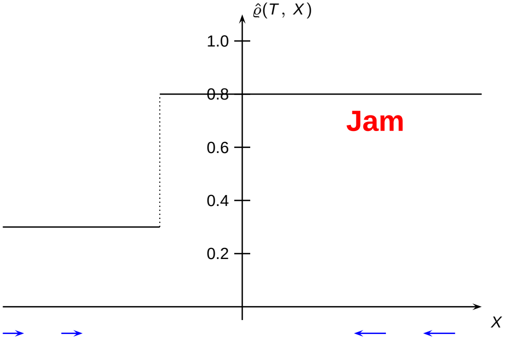
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



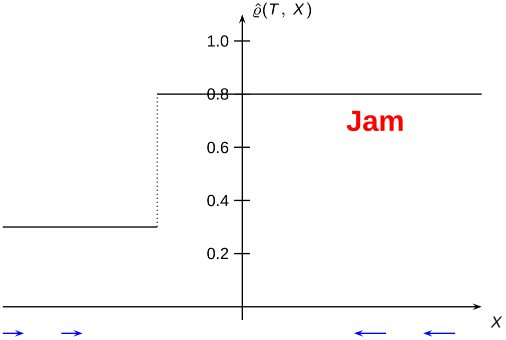
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



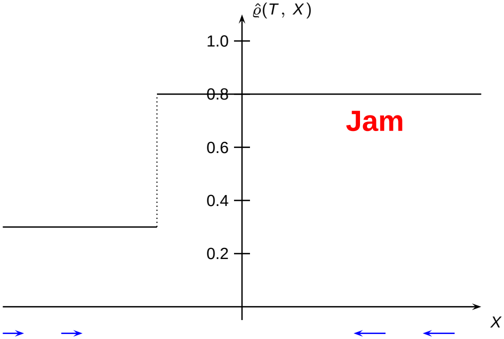
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



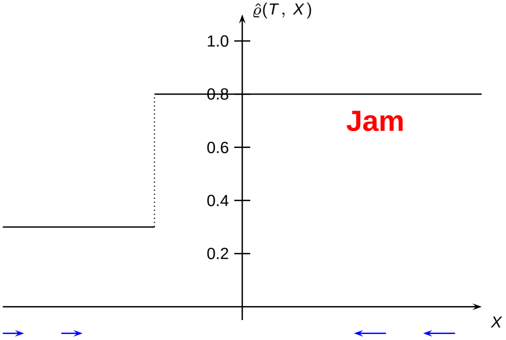
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



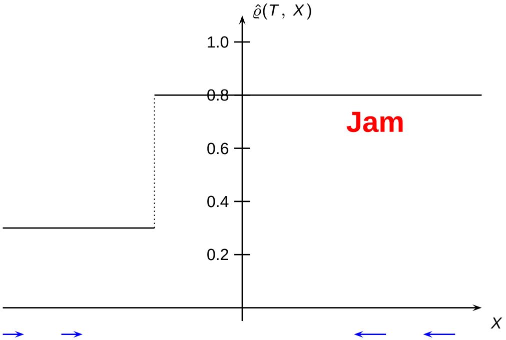
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



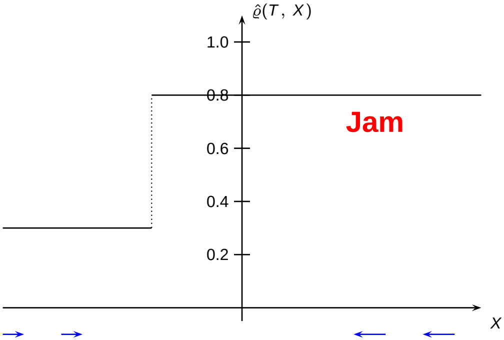
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



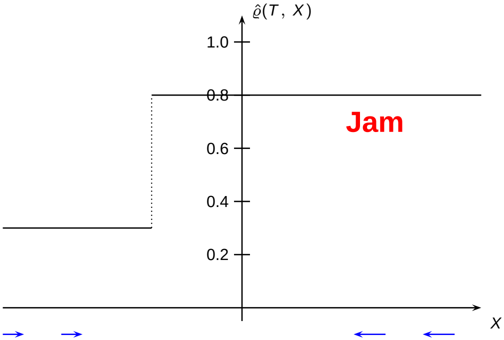
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



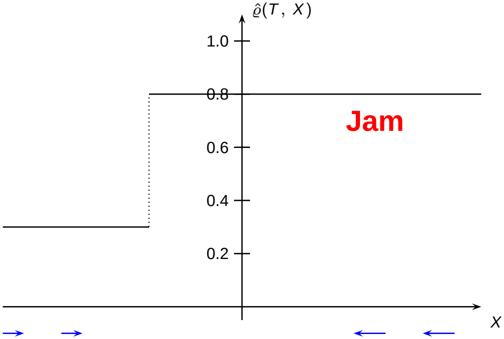
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



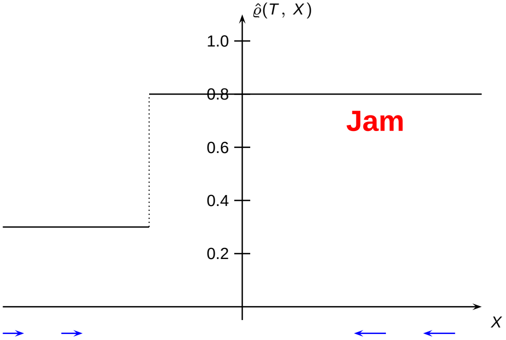
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



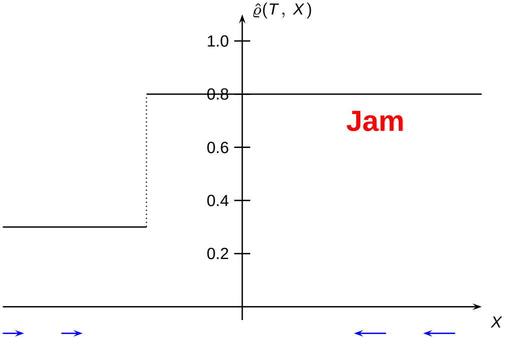
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



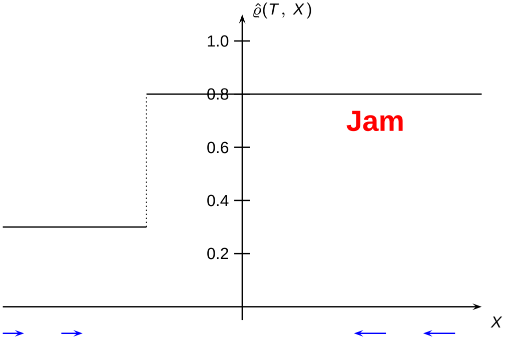
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



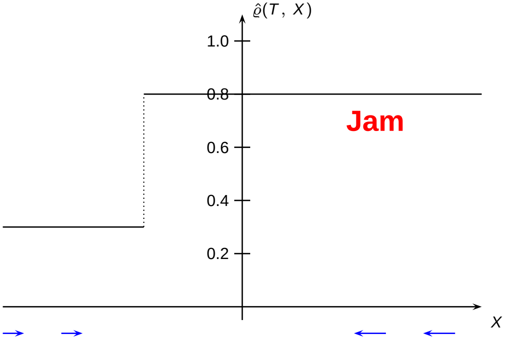
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



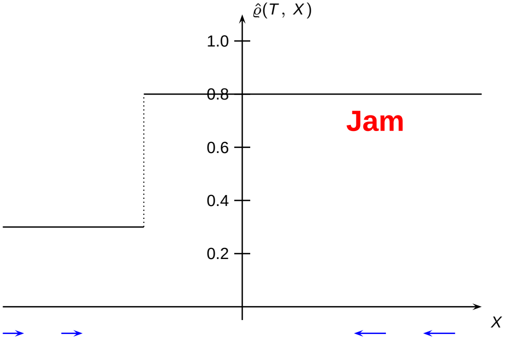
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



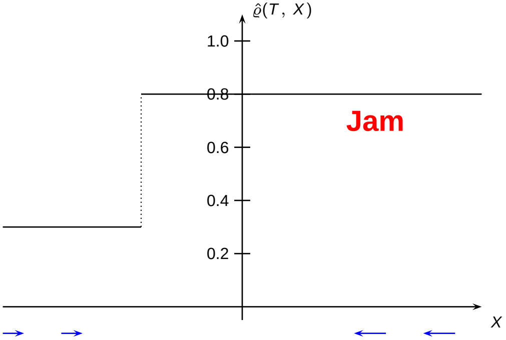
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



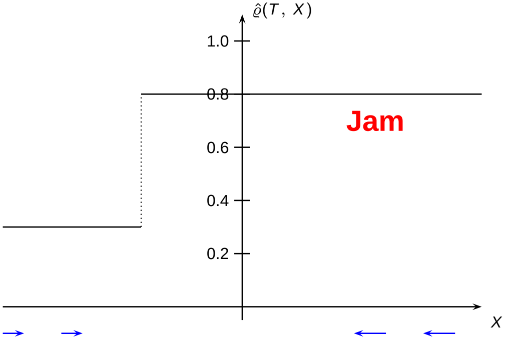
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



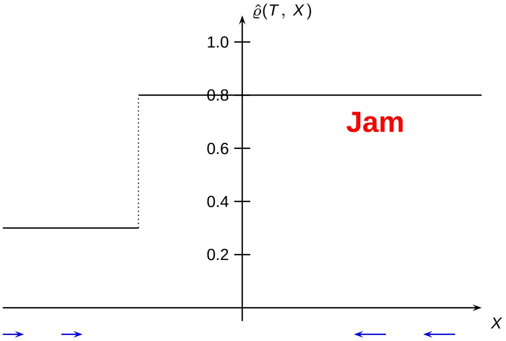
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



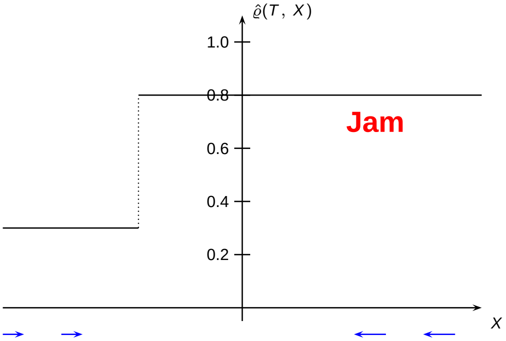
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



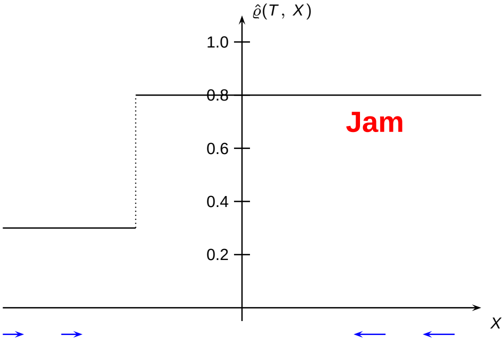
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



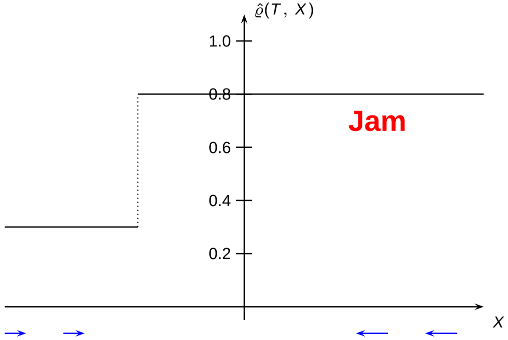
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



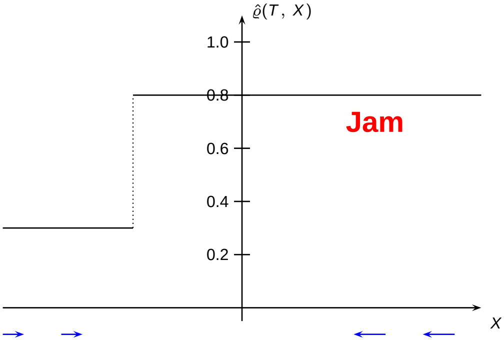
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



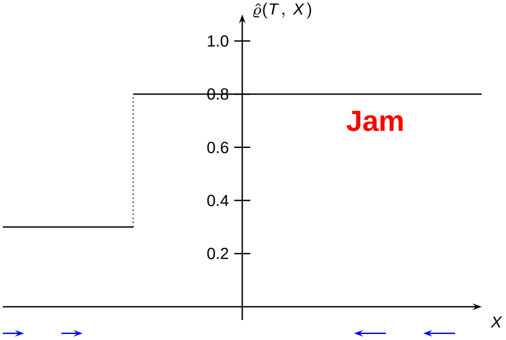
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



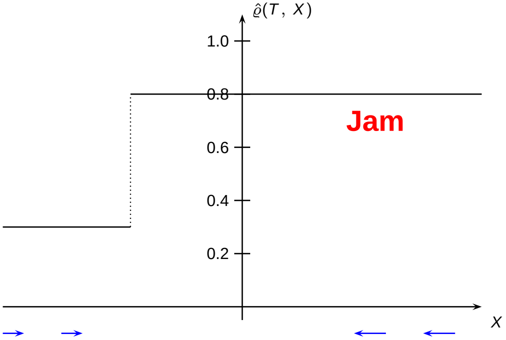
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



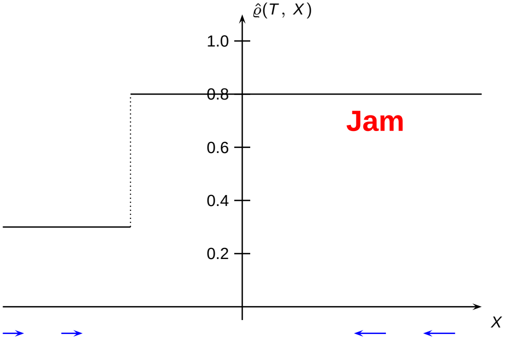
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



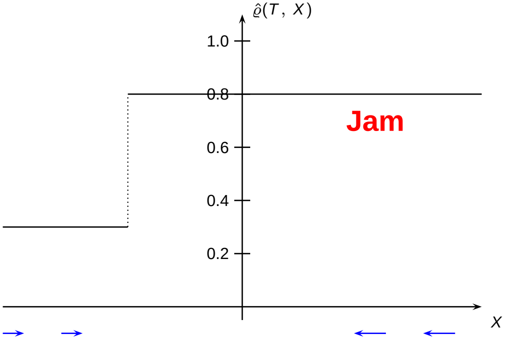
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



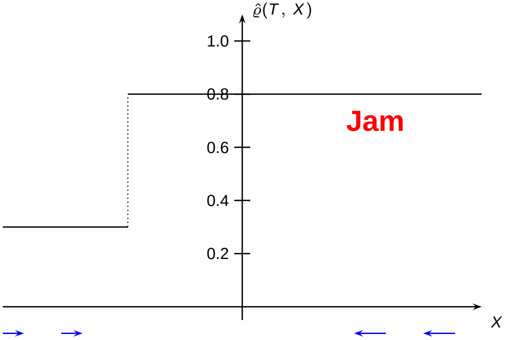
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



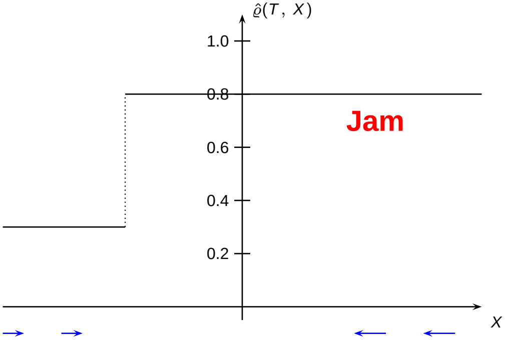
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



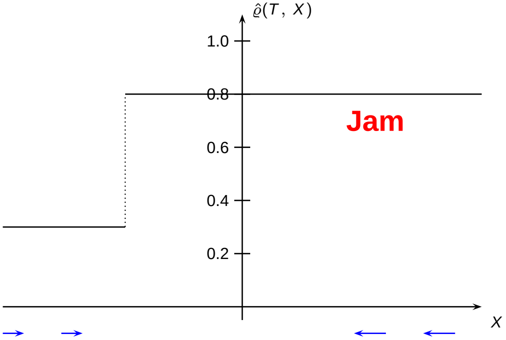
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



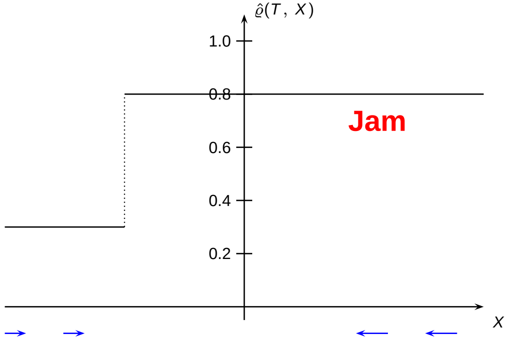
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



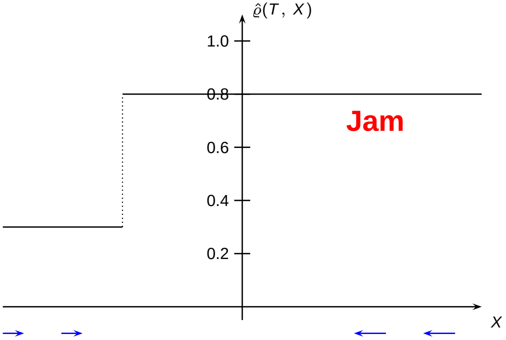
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



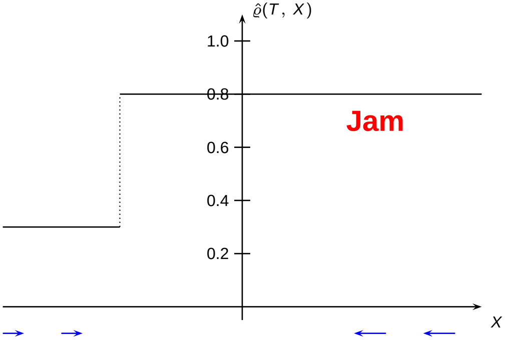
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



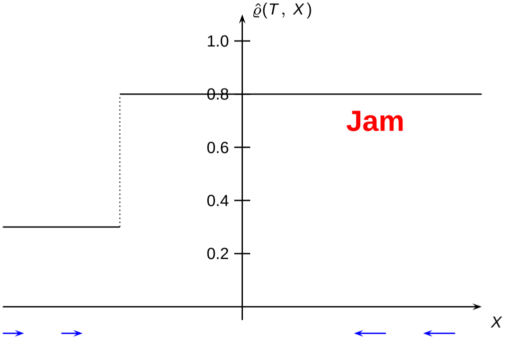
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



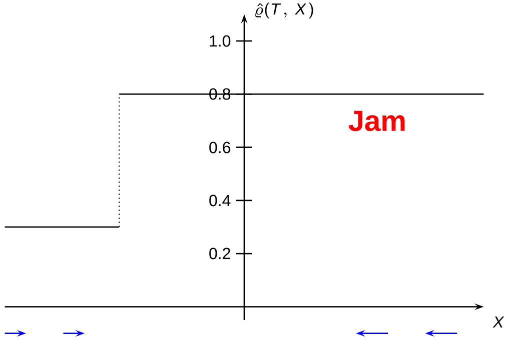
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



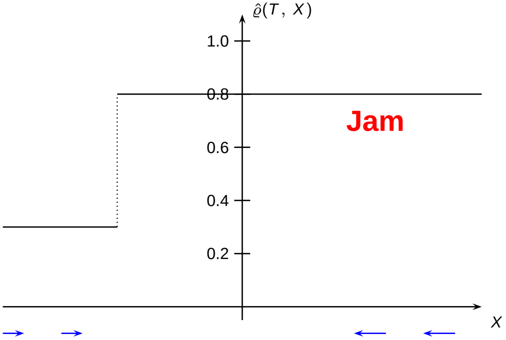
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



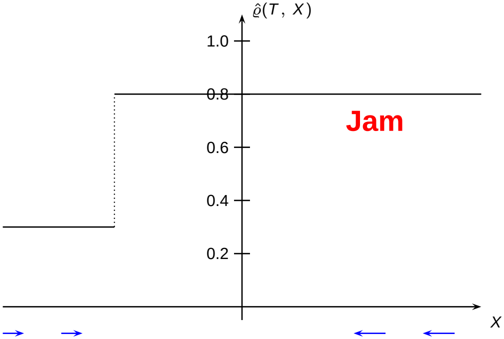
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



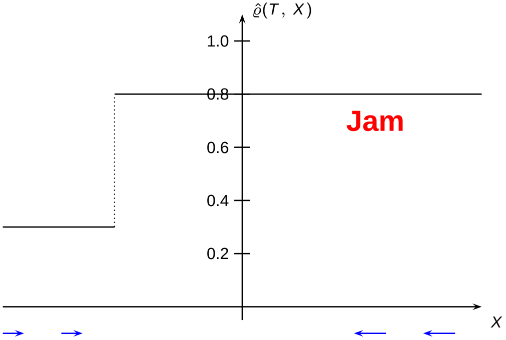
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



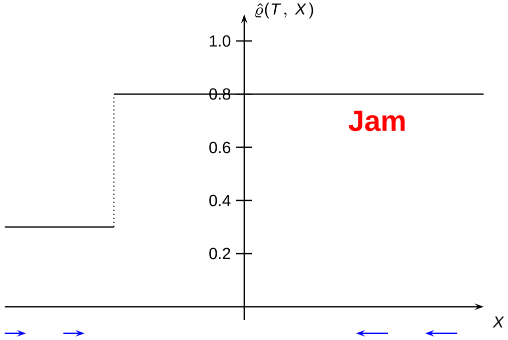
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



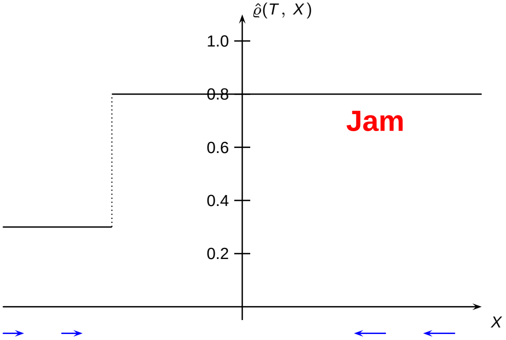
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



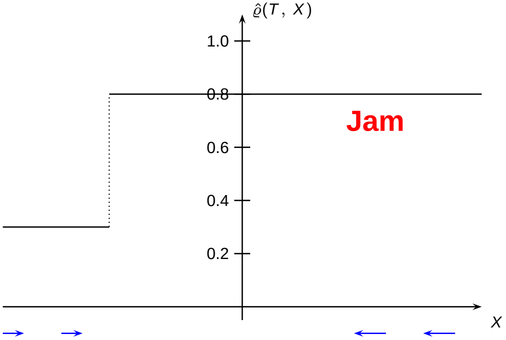
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



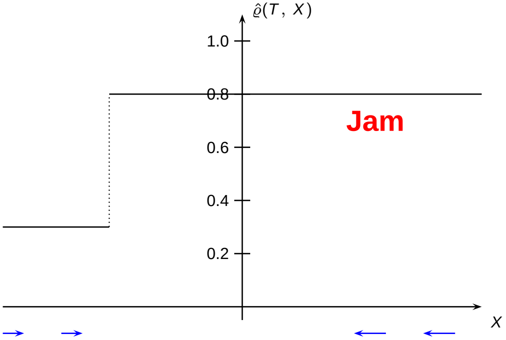
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

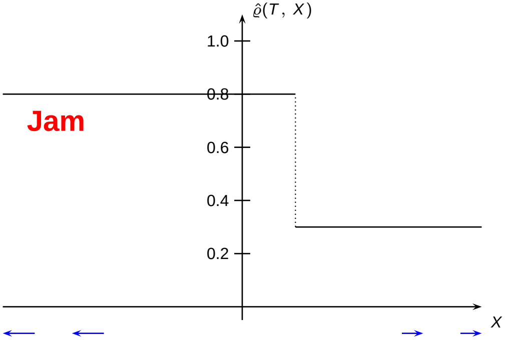
On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

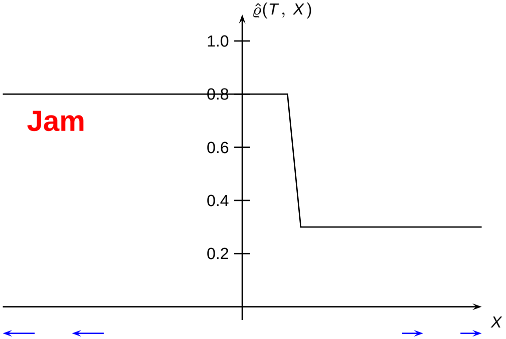
Shock

On large scales



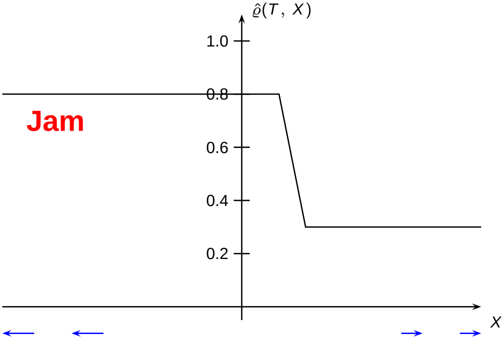
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



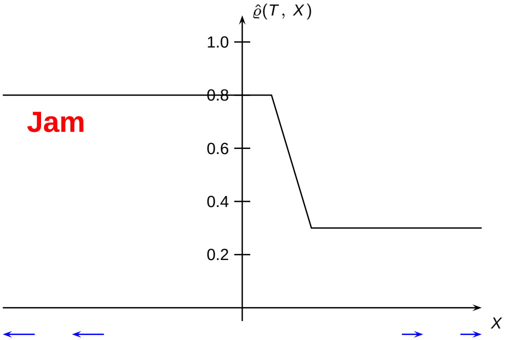
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



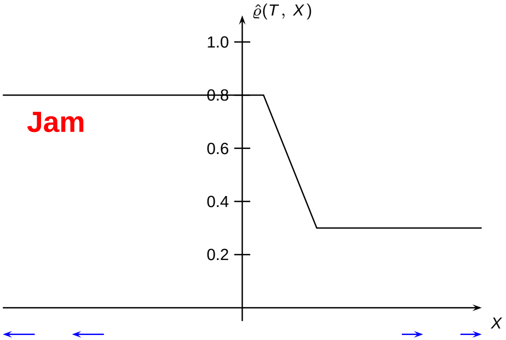
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



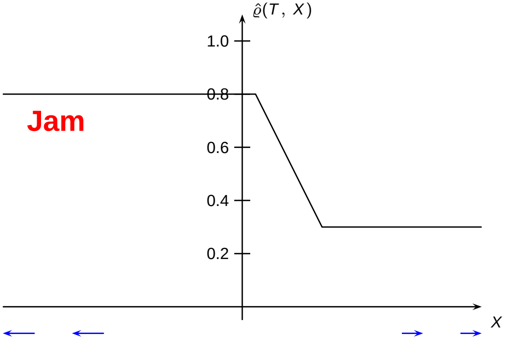
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



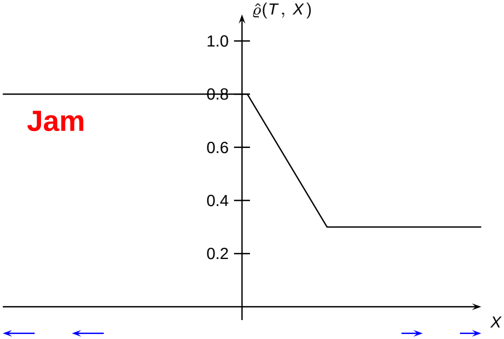
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



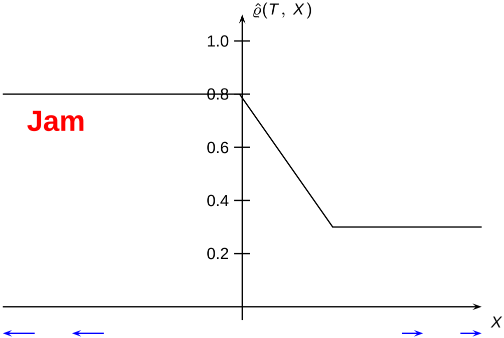
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



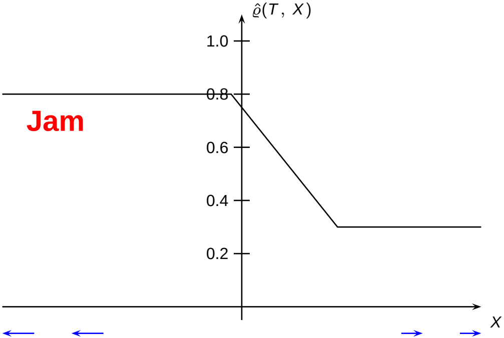
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



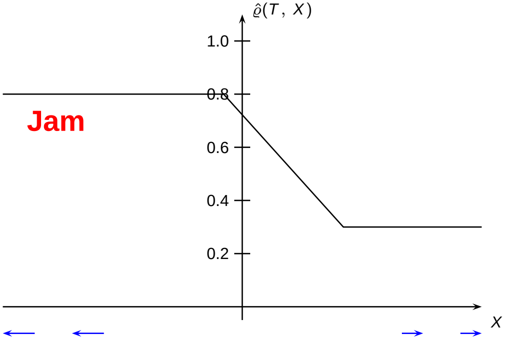
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



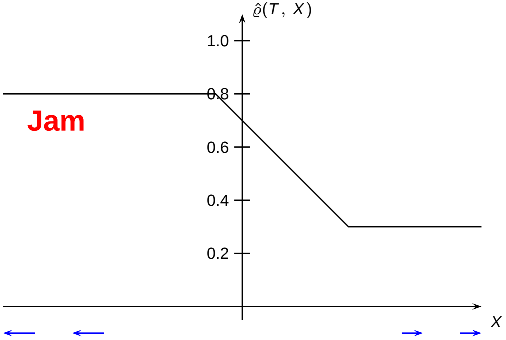
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



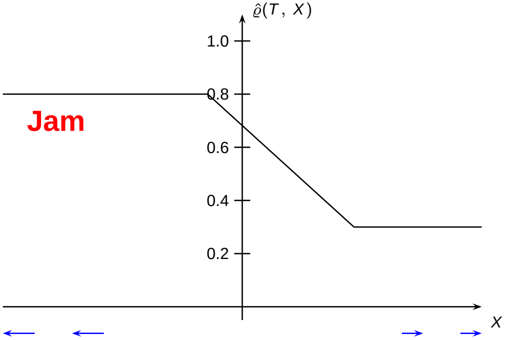
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



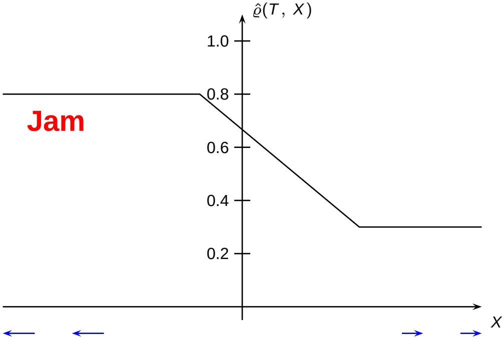
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



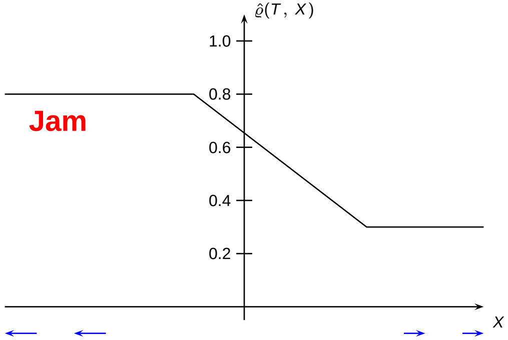
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



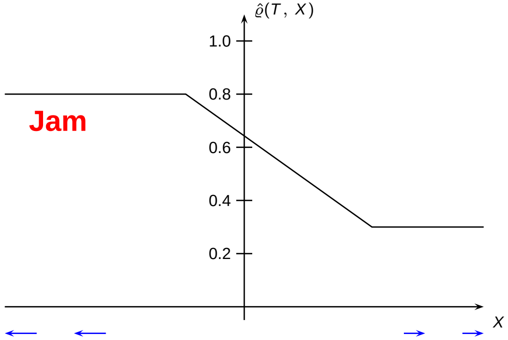
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



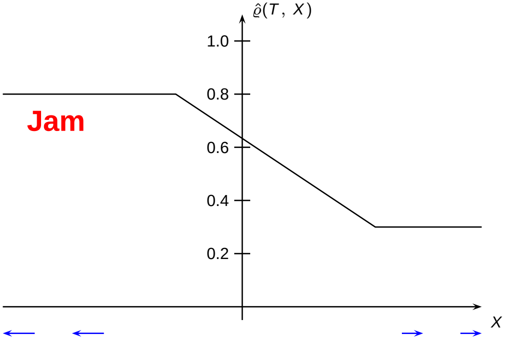
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



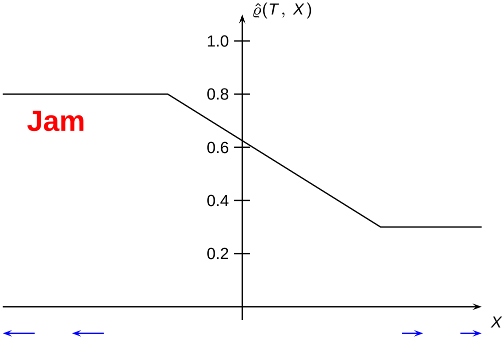
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



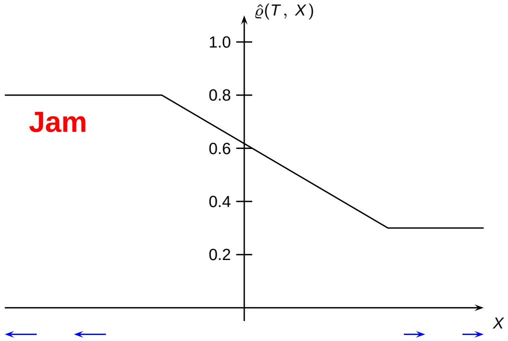
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



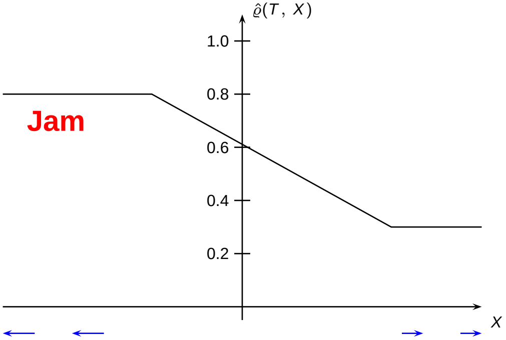
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



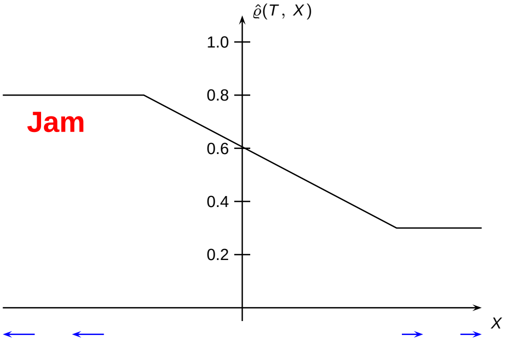
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



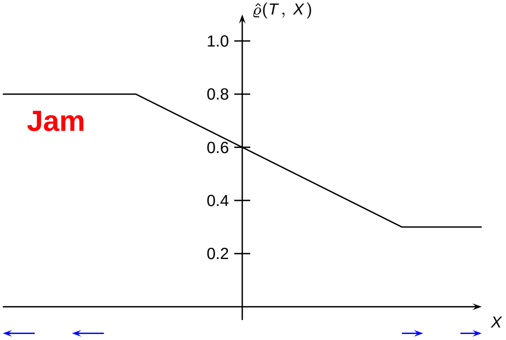
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



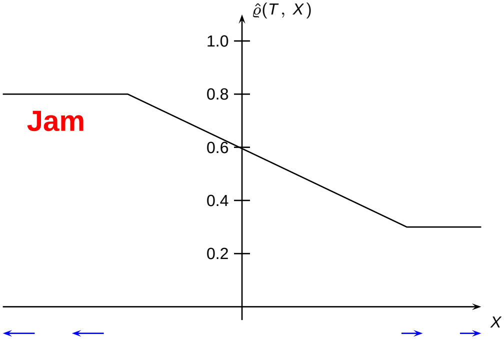
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



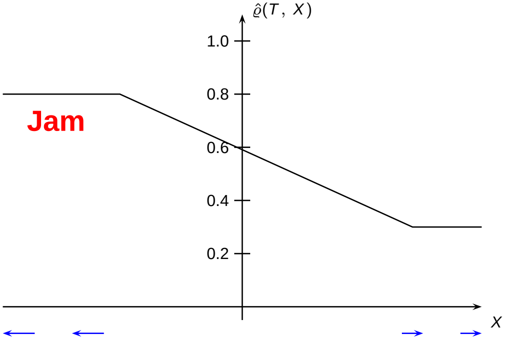
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



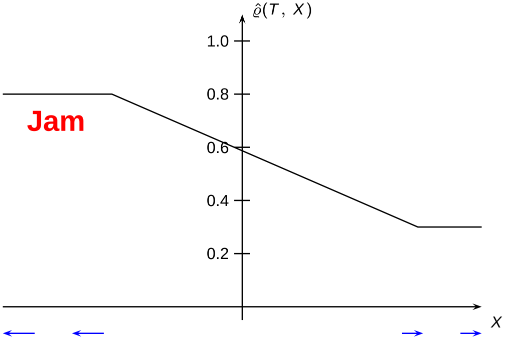
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



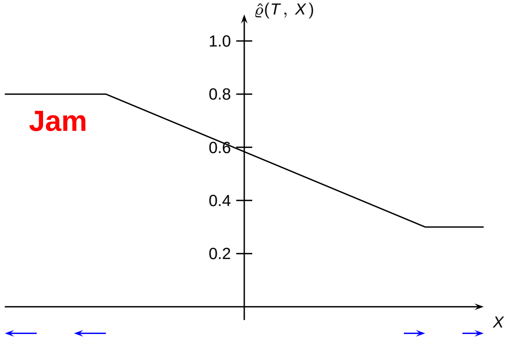
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



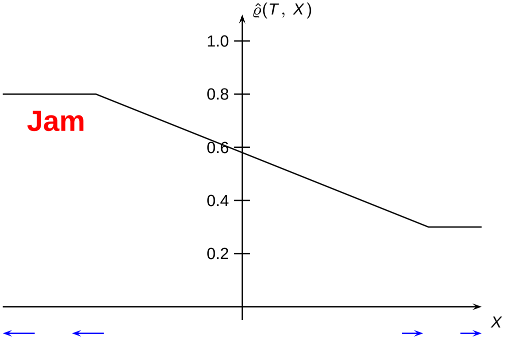
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



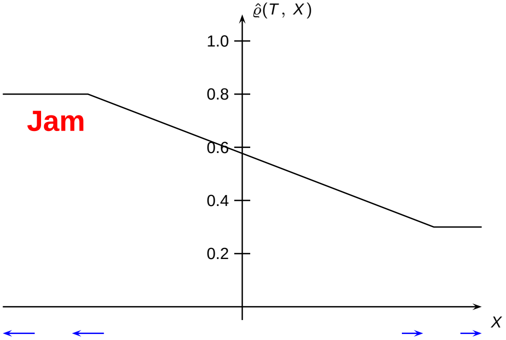
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



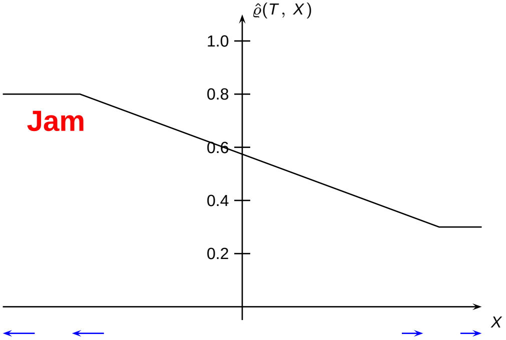
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



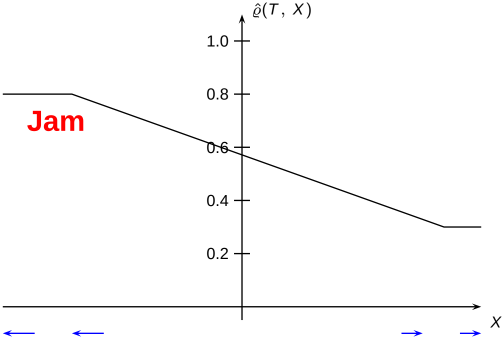
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



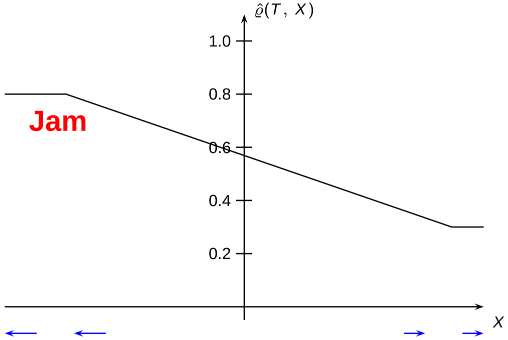
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



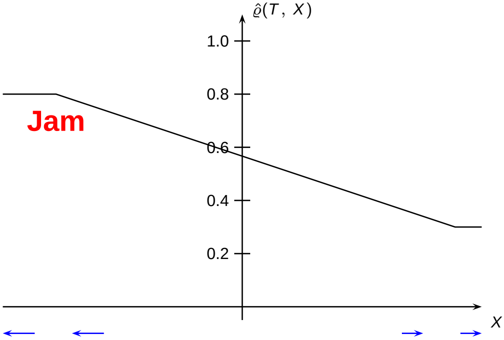
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



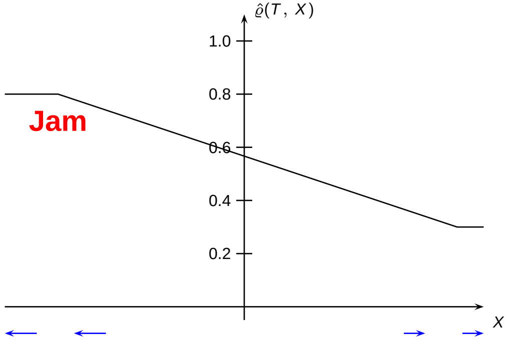
$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

On large scales



$$\dot{X}(T) = 1 - 2\hat{\varrho}$$

Rarefaction wave

The second class particle: non-attractive case

We are facing a

- nearest neighbour
- parity conserving
- branching
- annihilating process
- on the dynamic background of first class particles.

The aim is to control the number of $\uparrow$ and $\downarrow$'s. Idea from Bálint Tóth.

$\rightsquigarrow$ `homog2.avi`

A mean field model

A model we can say something about:

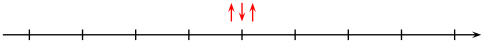
A mean field model

A model we can say something about:

Branching

A mean field model

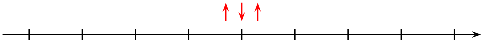
A model we can say something about:



Branching

A mean field model

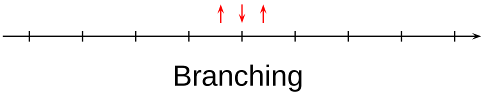
A model we can say something about:



Branching

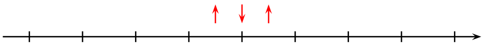
A mean field model

A model we can say something about:



A mean field model

A model we can say something about:



Branching

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

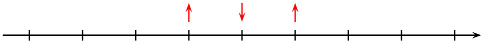
A mean field model

A model we can say something about:

Branching

A mean field model

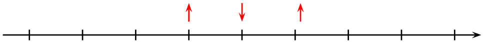
A model we can say something about:



Branching

A mean field model

A model we can say something about:



Jump

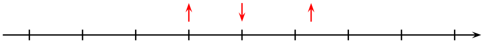
A mean field model

A model we can say something about:

Jump

A mean field model

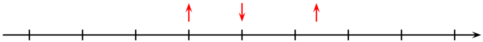
A model we can say something about:



Jump

A mean field model

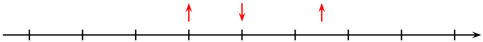
A model we can say something about:



Jump

A mean field model

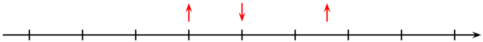
A model we can say something about:



Jump

A mean field model

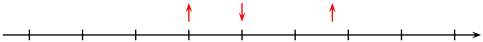
A model we can say something about:



Jump

A mean field model

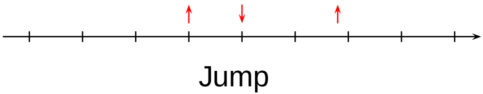
A model we can say something about:



Jump

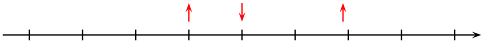
A mean field model

A model we can say something about:



A mean field model

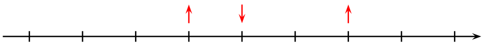
A model we can say something about:



Jump

A mean field model

A model we can say something about:



Jump

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

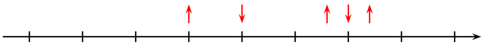
A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:



Branching

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:

Branching

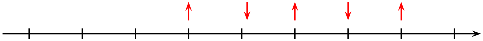
A mean field model

A model we can say something about:

Branching

A mean field model

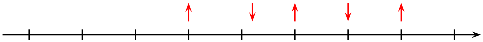
A model we can say something about:



Jump with annihilation

A mean field model

A model we can say something about:



Jump with annihilation

A mean field model

A model we can say something about:

Jump with annihilation

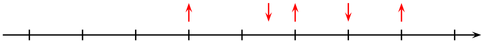
A mean field model

A model we can say something about:

Jump with annihilation

A mean field model

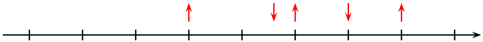
A model we can say something about:



Jump with annihilation

A mean field model

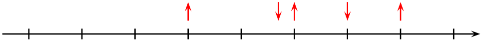
A model we can say something about:



Jump with annihilation

A mean field model

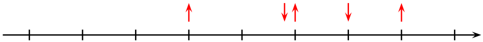
A model we can say something about:



Jump with annihilation

A mean field model

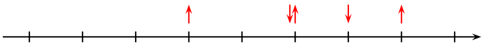
A model we can say something about:



Jump with annihilation

A mean field model

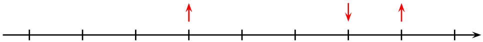
A model we can say something about:



Jump with annihilation

A mean field model

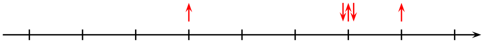
A model we can say something about:



Jump with annihilation

A mean field model

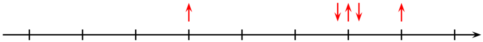
A model we can say something about:



Branching with annihilation

A mean field model

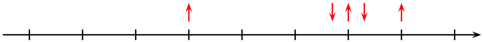
A model we can say something about:



Branching with annihilation

A mean field model

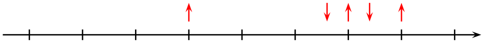
A model we can say something about:



Branching with annihilation

A mean field model

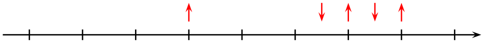
A model we can say something about:



Branching with annihilation

A mean field model

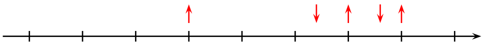
A model we can say something about:



Branching with annihilation

A mean field model

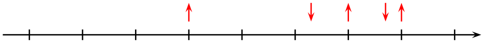
A model we can say something about:



Branching with annihilation

A mean field model

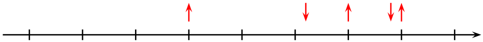
A model we can say something about:



Branching with annihilation

A mean field model

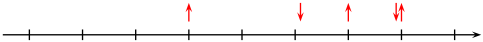
A model we can say something about:



Branching with annihilation

A mean field model

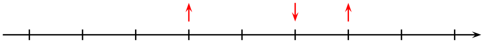
A model we can say something about:



Branching with annihilation

A mean field model

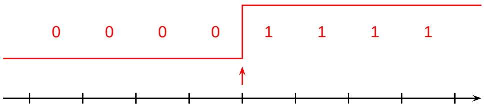
A model we can say something about:



Branching with annihilation

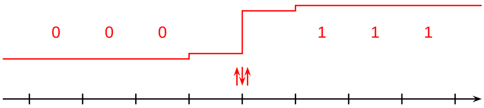
A mean field model

A model we can say something about:



A mean field model

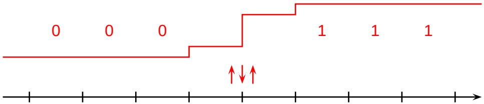
A model we can say something about:



Branching: exclusion

A mean field model

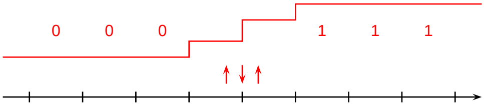
A model we can say something about:



Branching: exclusion

A mean field model

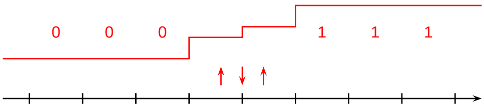
A model we can say something about:



Branching: exclusion

A mean field model

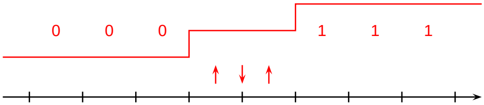
A model we can say something about:



Branching: exclusion

A mean field model

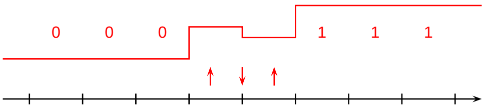
A model we can say something about:



Branching: exclusion

A mean field model

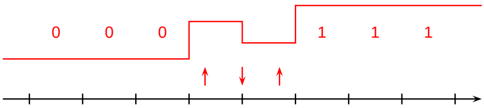
A model we can say something about:



Branching: exclusion

A mean field model

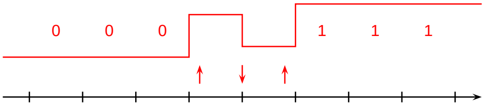
A model we can say something about:



Branching: exclusion

A mean field model

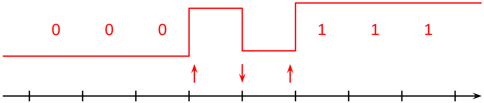
A model we can say something about:



Branching: exclusion

A mean field model

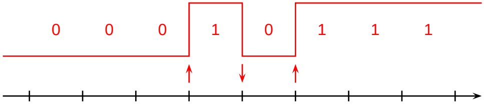
A model we can say something about:



Branching: exclusion

A mean field model

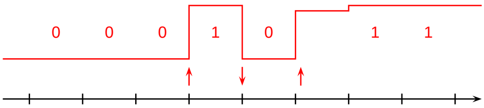
A model we can say something about:



Branching: exclusion

A mean field model

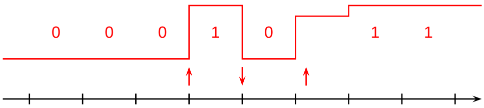
A model we can say something about:



Jump: flip (voting)

A mean field model

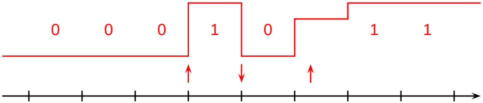
A model we can say something about:



Jump: flip (voting)

A mean field model

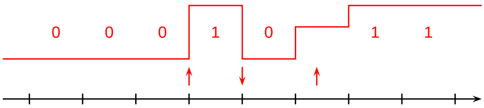
A model we can say something about:



Jump: flip (voting)

A mean field model

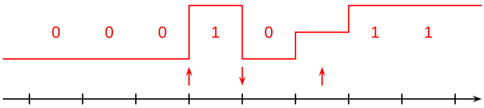
A model we can say something about:



Jump: flip (voting)

A mean field model

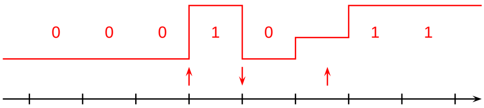
A model we can say something about:



Jump: flip (voting)

A mean field model

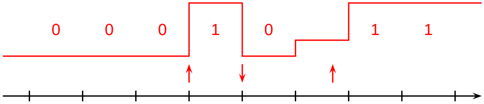
A model we can say something about:



Jump: flip (voting)

A mean field model

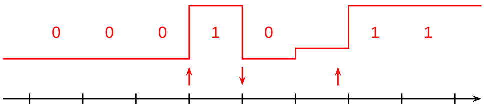
A model we can say something about:



Jump: flip (voting)

A mean field model

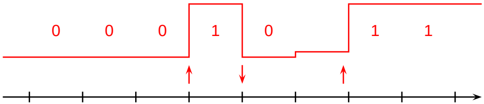
A model we can say something about:



Jump: flip (voting)

A mean field model

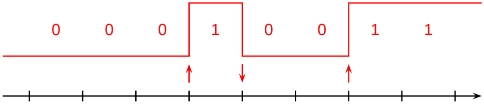
A model we can say something about:



Jump: flip (voting)

A mean field model

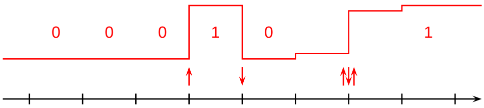
A model we can say something about:



Jump: flip (voting)

A mean field model

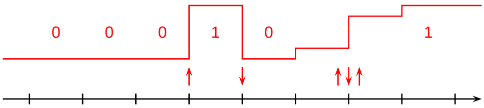
A model we can say something about:



Branching: exclusion

A mean field model

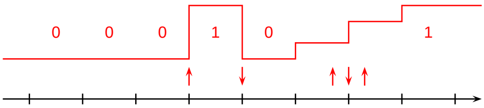
A model we can say something about:



Branching: exclusion

A mean field model

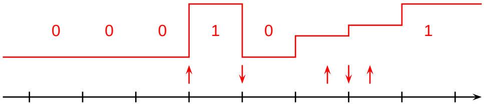
A model we can say something about:



Branching: exclusion

A mean field model

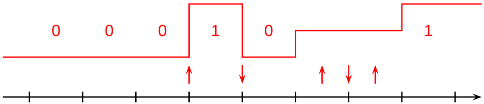
A model we can say something about:



Branching: exclusion

A mean field model

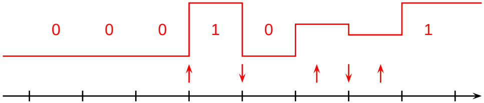
A model we can say something about:



Branching: exclusion

A mean field model

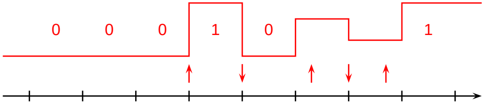
A model we can say something about:



Branching: exclusion

A mean field model

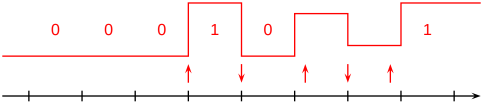
A model we can say something about:



Branching: exclusion

A mean field model

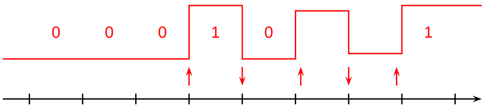
A model we can say something about:



Branching: exclusion

A mean field model

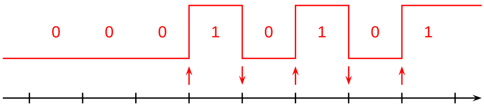
A model we can say something about:



Branching: exclusion

A mean field model

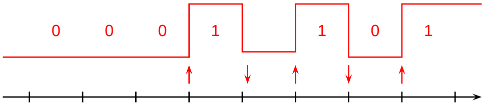
A model we can say something about:



Branching: exclusion

A mean field model

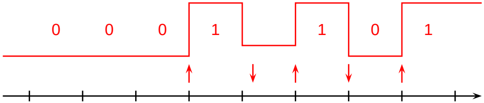
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

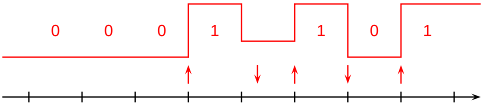
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

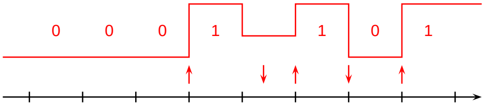
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

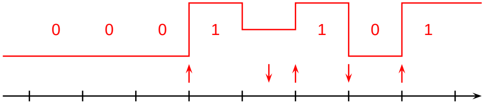
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

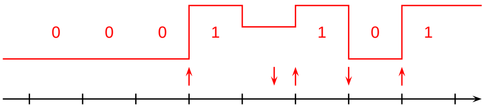
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

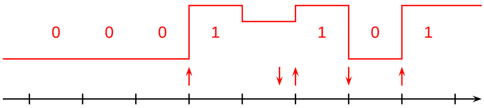
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

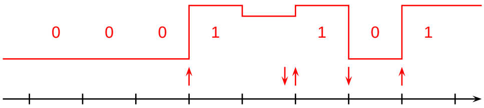
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

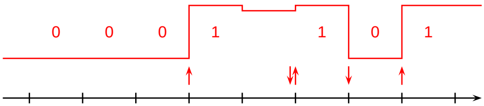
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

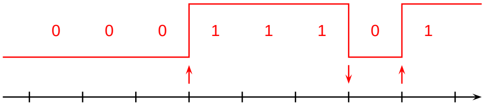
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

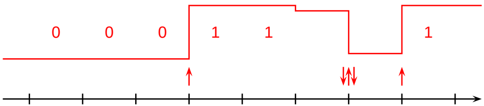
A model we can say something about:



Jump with annihilation: flip (voting)

A mean field model

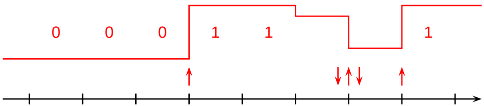
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

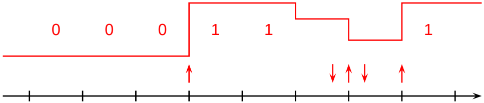
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

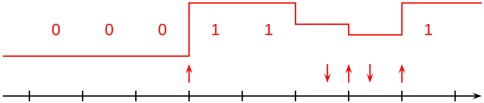
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

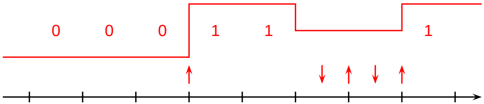
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

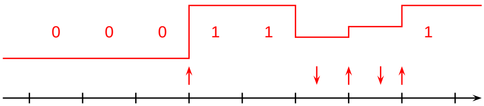
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

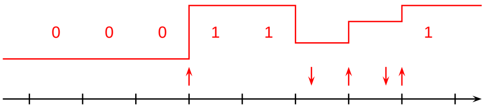
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

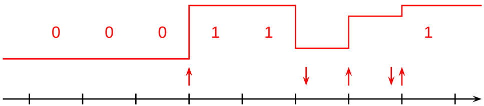
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

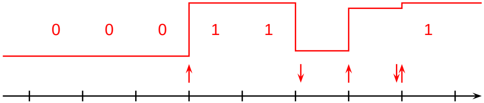
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

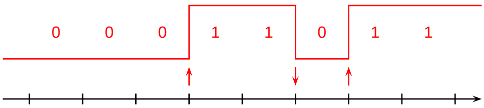
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

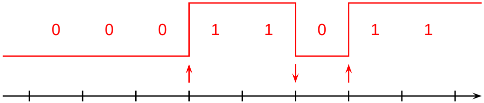
A model we can say something about:



Branching with annihilation: exclusion

A mean field model

A model we can say something about:

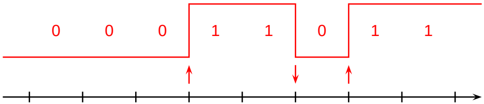


Branching with annihilation: exclusion

- Double branching-annihilating random walks (DBARW)

A mean field model

A model we can say something about:

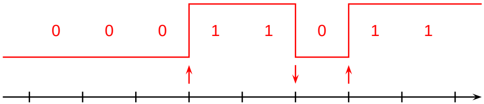


Branching with annihilation: exclusion

- Double branching-annihilating random walks (DBARW)
- $\uparrow$'s and $\downarrow$'s always alternate;

A mean field model

A model we can say something about:

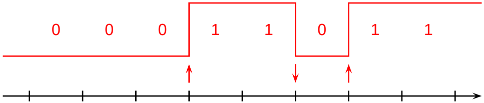


Branching with annihilation: exclusion

- Double branching-annihilating random walks (DBARW)
- ↑'s and ↓'s always alternate;
- their algebraic sum is constant in time.

A mean field model

A model we can say something about:

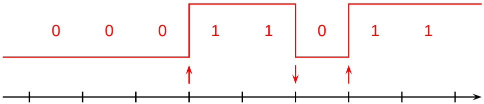


Branching with annihilation: exclusion

- Double branching-annihilating random walks (DBARW)
- ↑'s and ↓'s always alternate;
- their algebraic sum is constant in time.
- Nothing is monotone.

A mean field model

A model we can say something about:

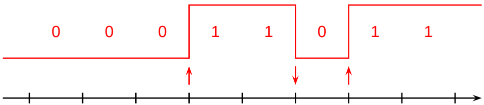


Branching with annihilation: exclusion

- Double branching-annihilating random walks (DBARW)
- ↑'s and ↓'s always alternate;
- their algebraic sum is constant in time.
- Nothing is monotone.

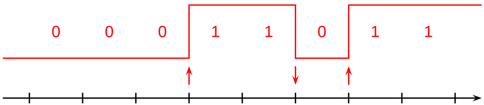
Question: Is the process, as seen by the leftmost ↑, recurrent?

DBARW



First instance of DBARW we could find in the literature: A. Sudbury '90. Positive recurrence: V. Belitsky, P.A. Ferrari, M.V. Menshikov and S.Y. Popov '01; A. Sturm and J.M. Swart '08. *Results are very sensitive to the details of branching.*

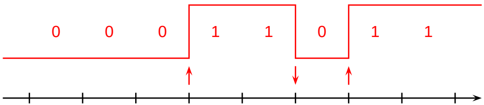
DBARW



First instance of DBARW we could find in the literature: A. Sudbury '90. Positive recurrence: V. Belitsky, P.A. Ferrari, M.V. Menshikov and S.Y. Popov '01; A. Sturm and J.M. Swart '08. *Results are very sensitive to the details of branching.*

But: true second class particles interact (*common background of first class particles*).

DBARW



First instance of DBARW we could find in the literature: A. Sudbury '90. Positive recurrence: V. Belitsky, P.A. Ferrari, M.V. Menshikov and S.Y. Popov '01; A. Sturm and J.M. Swart '08. *Results are very sensitive to the details of branching.*

But: true second class particles interact (*common background of first class particles*).

$\rightsquigarrow$ Repeat the Sturm-Swart proof with configuration dependent jump rates. Jump rates can depend on the whole configuration.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.
- Uniform lower bound on jumping rates: no particles are stuck.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.
- Uniform lower bound on jumping rates: no particles are stuck.
- Bounds on the branching rates.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.
- Uniform lower bound on jumping rates: no particles are stuck.
- Bounds on the branching rates.
- Bounds on the difference for branching rates of $\uparrow$'s and $\downarrow$'s.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.
- Uniform lower bound on jumping rates: no particles are stuck.
- Bounds on the branching rates.
- Bounds on the difference for branching rates of $\uparrow$'s and $\downarrow$'s.
- Weak dependence on particles far away.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.
- Uniform lower bound on jumping rates: no particles are stuck.
- Bounds on the branching rates.
- Bounds on the difference for branching rates of $\uparrow$'s and $\downarrow$'s.
- Weak dependence on particles far away.
- No repulsion in the jumping rates between particles. (*A bit of repulsion locally is still OK.*)

Positive recurrence

Theorem

Then, starting from a single $\uparrow$:

- *The process takes finitely many steps in finite time (construction).*

Positive recurrence

Theorem

Then, starting from a single $\uparrow$:

- *The process takes finitely many steps in finite time (construction).*
- *The width of the process has all moments finite.*

Positive recurrence

Theorem

Then, starting from a single $\uparrow$:

- *The process takes finitely many steps in finite time (construction).*
- *The width of the process has all moments finite.*
- *The process as seen from the leftmost $\uparrow$ is positive recurrent.*

Positive recurrence

Theorem

Then, starting from a single $\uparrow$:

- *The process takes finitely many steps in finite time (construction).*
- *The width of the process has all moments finite.*
- *The process as seen from the leftmost $\uparrow$ is positive recurrent.*
- *The stationary distribution sees a finite expected number of particles.*

Positive recurrence

Theorem

Then, starting from a single $\uparrow$:

- *The process takes finitely many steps in finite time (construction).*
- *The width of the process has all moments finite.*
- *The process as seen from the leftmost $\uparrow$ is positive recurrent.*
- *The stationary distribution sees a finite expected number of particles.*
- *(Extension of all this to non nearest neighbour symmetric branching.)*

An example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{particle on right}} \frac{1}{\text{distance}^\alpha},$$

jump rate to the left:

$$\frac{1}{2} + \sum_{\text{particle on left}} \frac{1}{\text{distance}^\alpha},$$

$\alpha > 1$.

An example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{particle on right}} \frac{1}{\text{distance}^{\alpha}},$$

jump rate to the left:

$$\frac{1}{2} + \sum_{\text{particle on left}} \frac{1}{\text{distance}^{\alpha}},$$

$\alpha > 1$.

Unfortunately we do not seem to be there yet... This is not covered at the moment. But a small modification that respects parity in a peculiar way works.

Another example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{gaps } L_i \text{ on the right}} \frac{1}{L_i^\alpha}$$

jump rate to the left:

$$\frac{1}{2} + \sum_{\text{gaps } L_i \text{ on the left}} \frac{1}{L_i^\alpha}$$

$\alpha > 1$. ($\sim$ like a rank dependent model but decreasing with distance.)

Another example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{gaps } L_i \text{ on the right}} \frac{1}{L_i^\alpha}$$

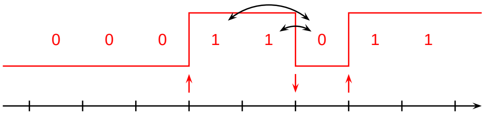
jump rate to the left:

$$\frac{1}{2} + \sum_{\text{gaps } L_i \text{ on the left}} \frac{1}{L_i^\alpha}$$

$\alpha > 1$. ($\sim$ like a rank dependent model but decreasing with distance.)

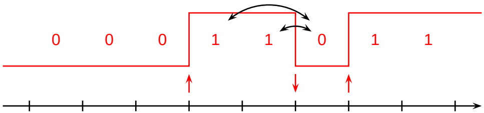
This one is fine.

Two words on the proof



Main tool 1: the number of inversions, i.e., wrongly ordered 1-0 pairs.
If there are too many of them, the generator is negative on the number of these pairs.

Two words on the proof

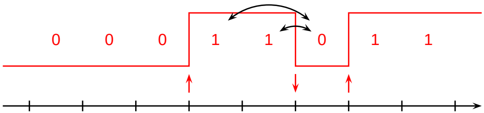


Main tool 1: the number of inversions, i.e., wrongly ordered 1-0 pairs.
If there are too many of them, the generator is negative on the number of these pairs.

Main tool 2: if the process is not tight, then on the long run there cannot be any finite number of particles:

$$\frac{1}{T}\int_0^T \mathbf{P}\{\text{number}(t) < N\}\,\mathrm{d}t \to 0 \qquad (\forall N).$$

Two words on the proof



<u>Main tool 1:</u> the number of inversions, i.e., wrongly ordered 1-0 pairs.
If there are too many of them, the generator is negative on the number of these pairs.

<u>Main tool 2:</u> if the process is not tight, then on the long run there cannot be any finite number of particles:

$$\frac{1}{T}\int_0^T \mathbf{P}\{\text{number}(t) < N\}\,\mathrm{d}t \to 0 \qquad (\forall N).$$

Thank you.