How to initialise a second class particle?

Joint with
Attila László Nagy

Márton Balázs

University of Bristol

Advances in Last Passage Percolation
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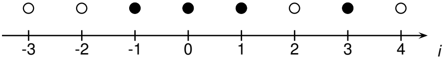
Totally asymmetric simple exclusion



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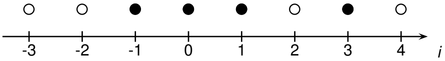

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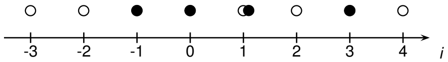

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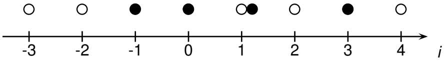

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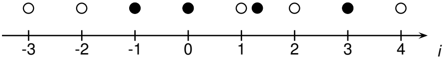

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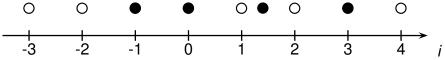

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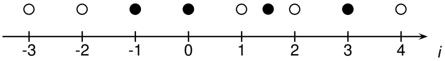

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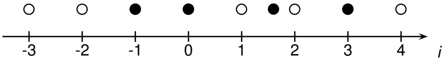

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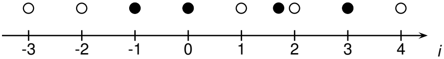

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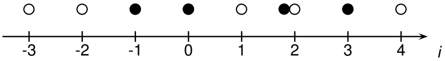

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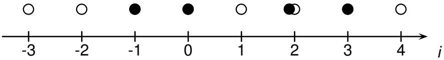

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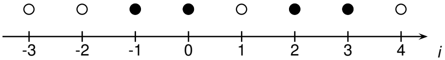

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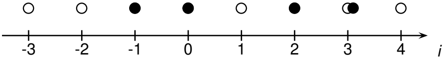

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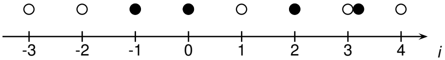

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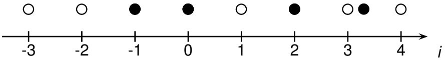

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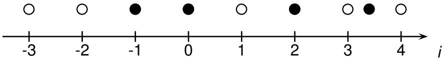

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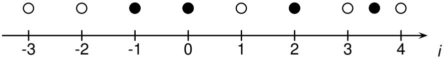

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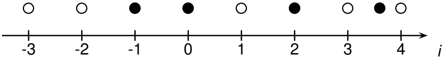

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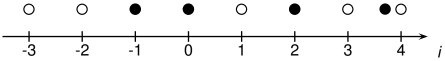

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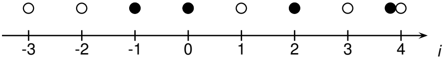

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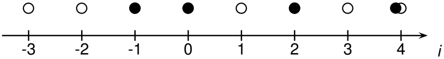

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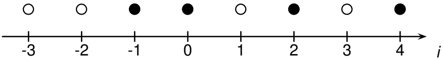

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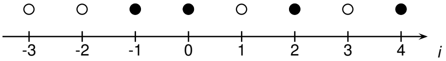

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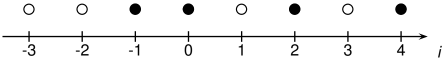

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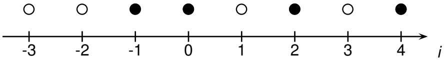

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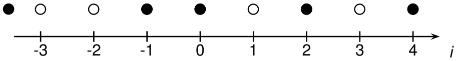

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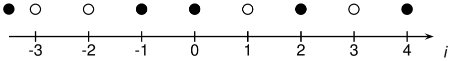

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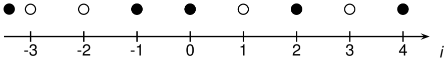

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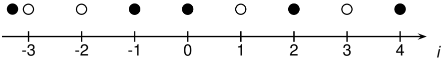

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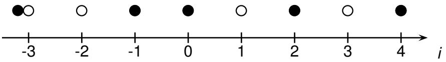

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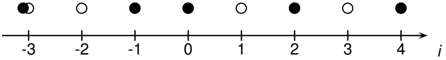

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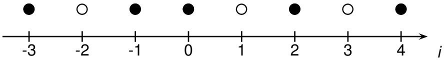

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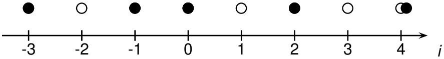

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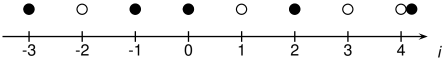

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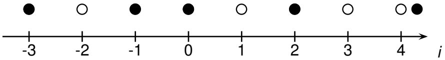

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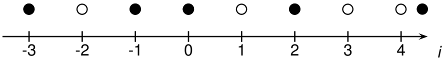

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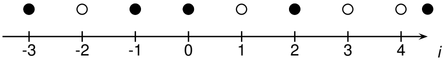

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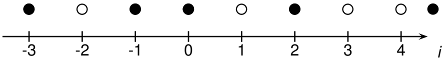

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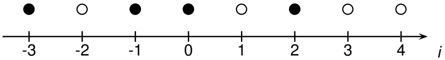

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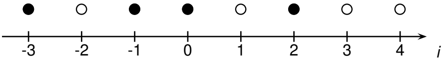

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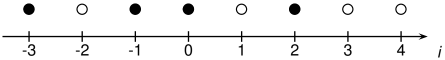

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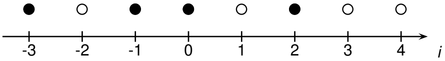

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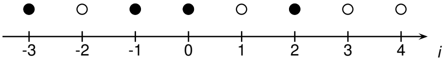

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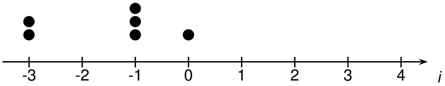
The Bernoulli(ϱ) distribution is time-stationary for any ($0 \leq \varrho \leq 1$). Any translation-invariant stationary distribution is a mixture of Bernoullis.

Totally asymmetric zero range process

$\omega_i \in \mathbb{Z}^+$



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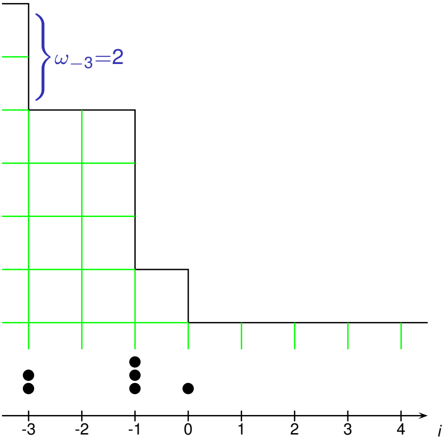

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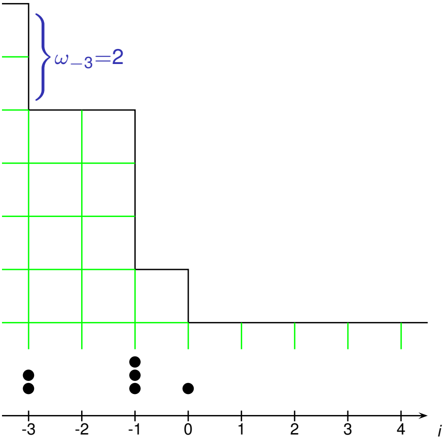

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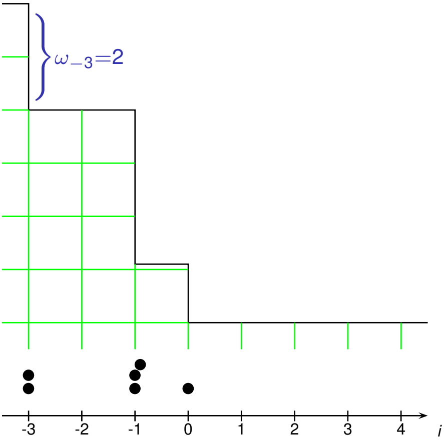

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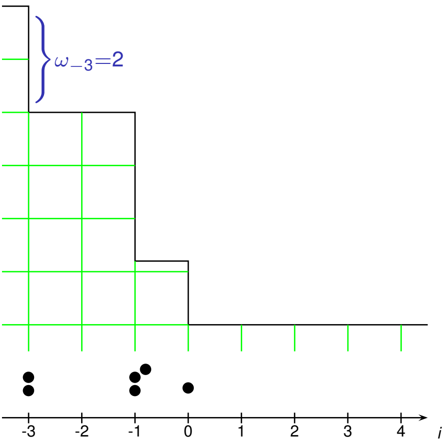

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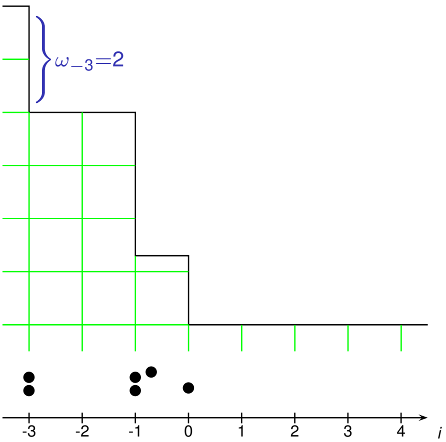

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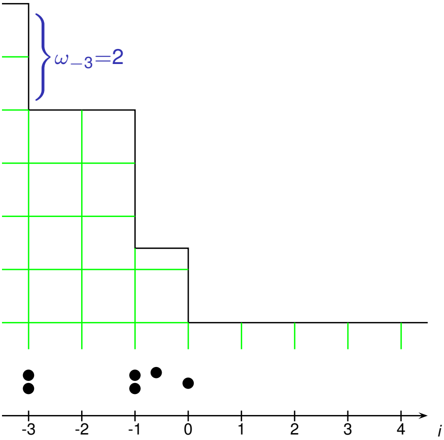

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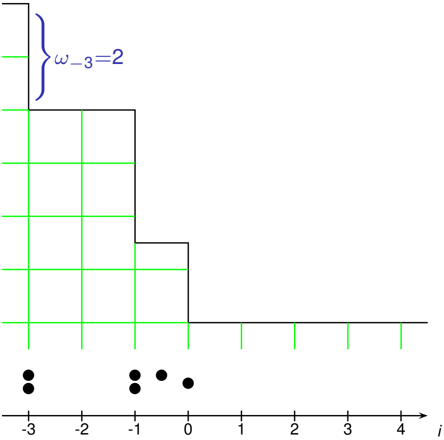

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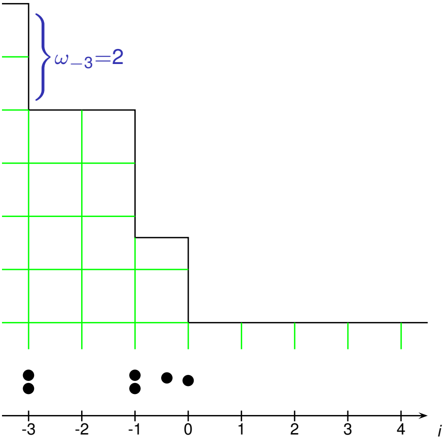
Particles jump to the right with rate $r(\omega_i)$ (r non-decreasing).

Totally asymmetric zero range process



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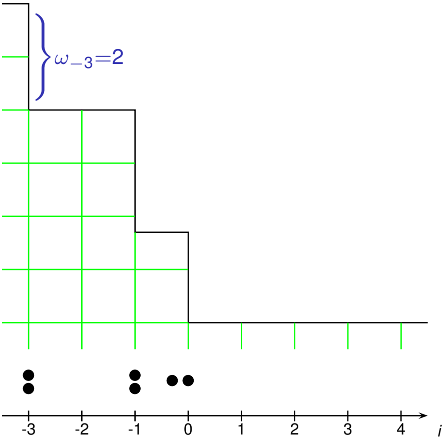

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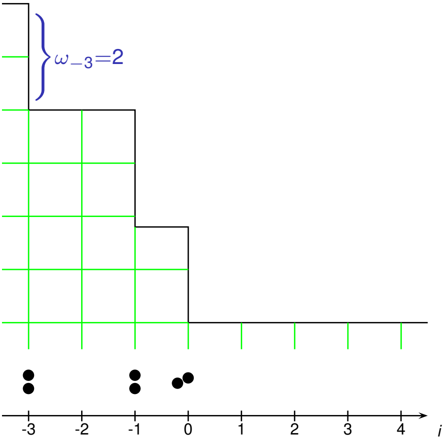

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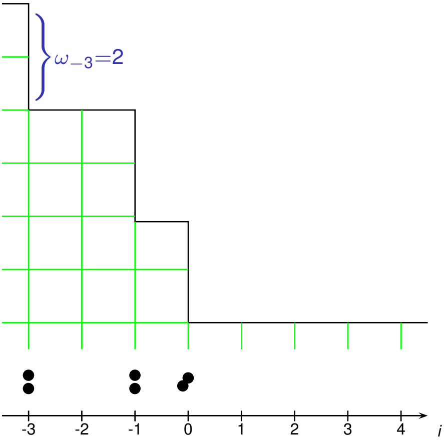

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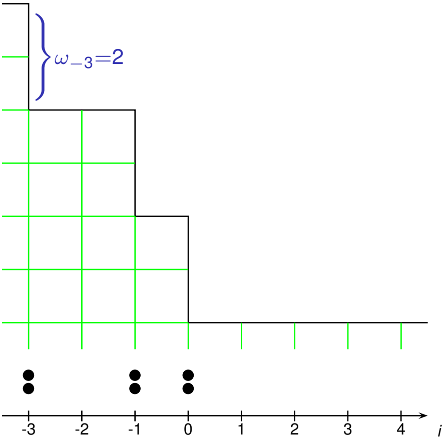

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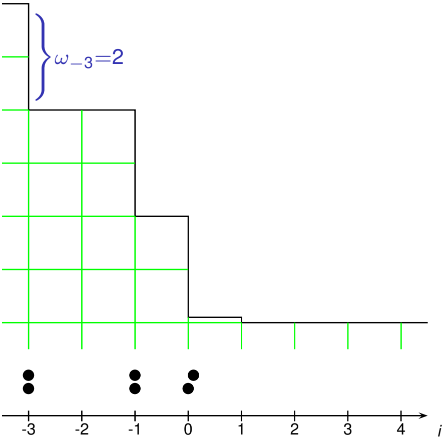

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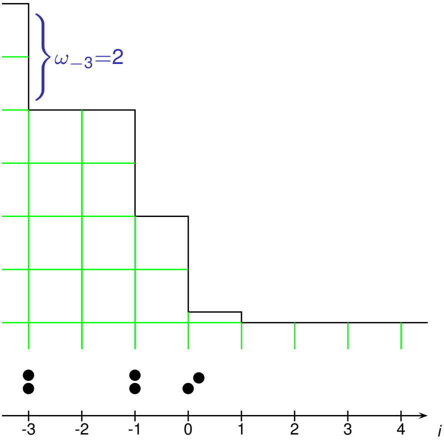

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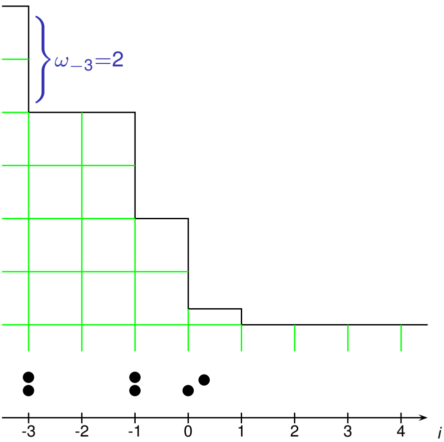

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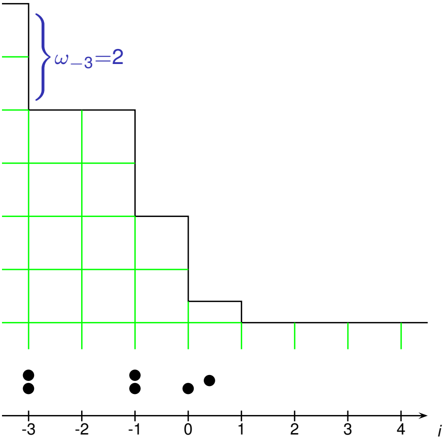

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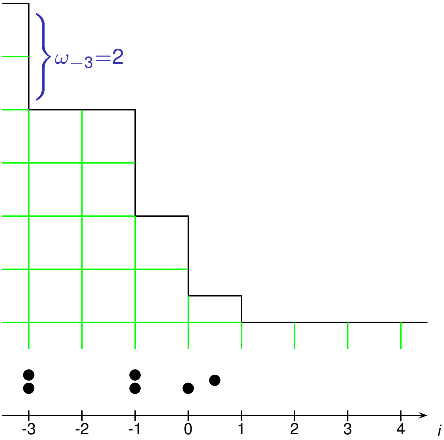

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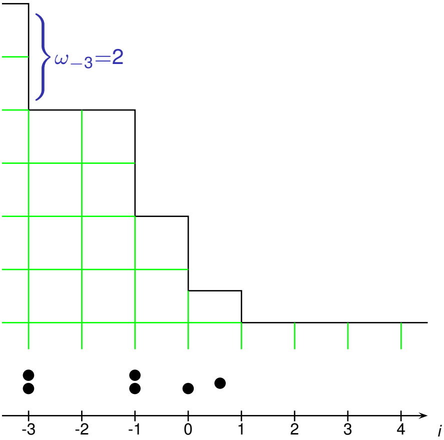

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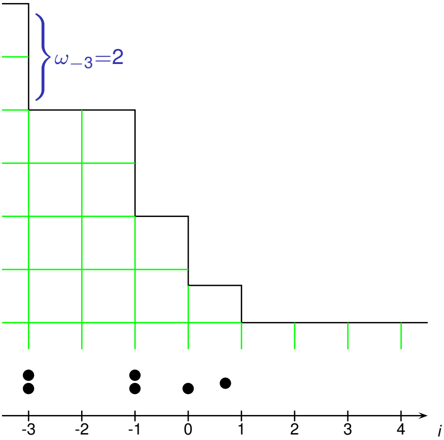

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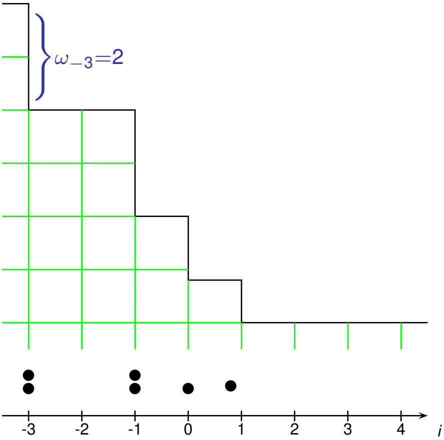

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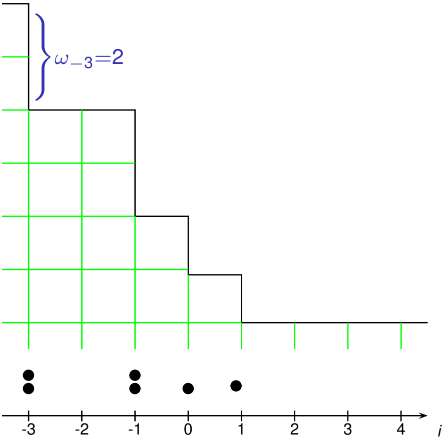

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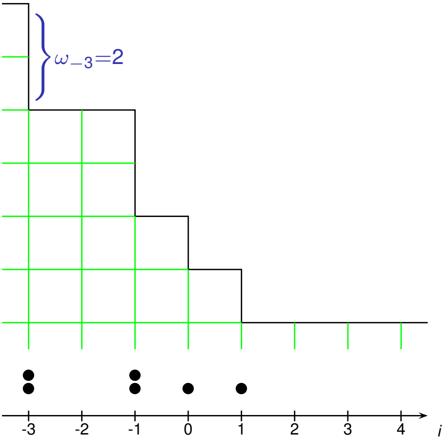

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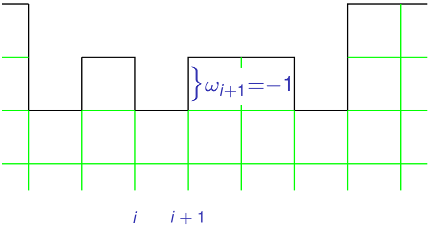
Totally asymmetric zero range process

Extremal translation-invariant stationary distributions are still product, and rather explicit in terms of $r(\cdot)$.

Two special cases:

- $r(\omega_i) = \mathbf{1}\{\omega_i > 0\}$: classical zero range; $\omega_i \sim \mathrm{Geom}(\theta)$.
- $r(\omega_i) = \omega_i$: independent walkers; $\omega_i \sim \mathrm{Poi}(\theta)$.

Totally asymmetric bricklayers process



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$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

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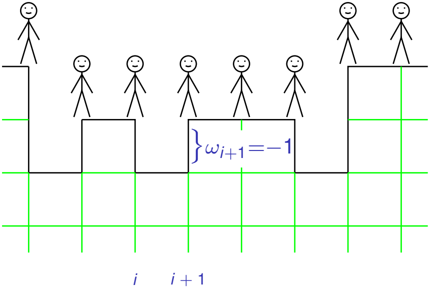

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$(r(\omega) \cdot r(1-\omega) = 1; \quad r$ non-decreasing$)$.

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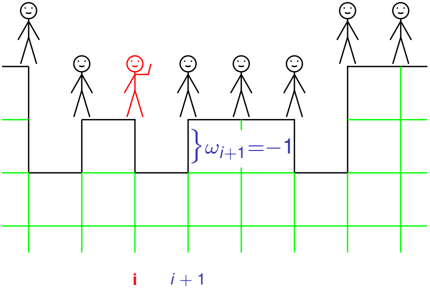

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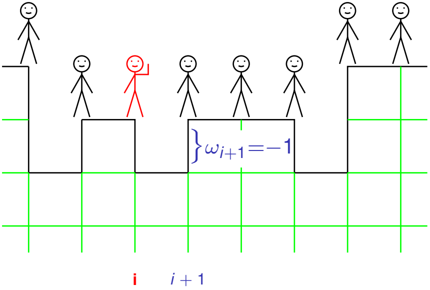

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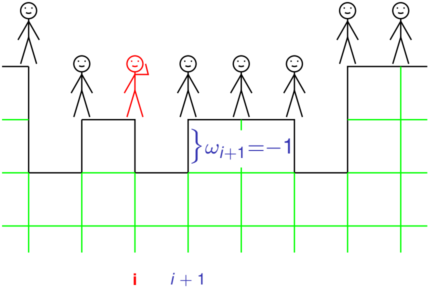

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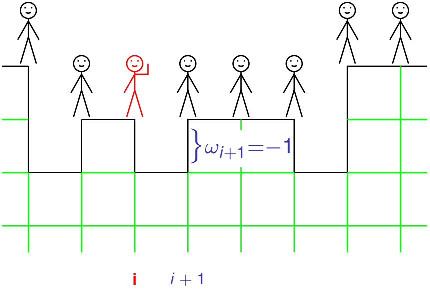

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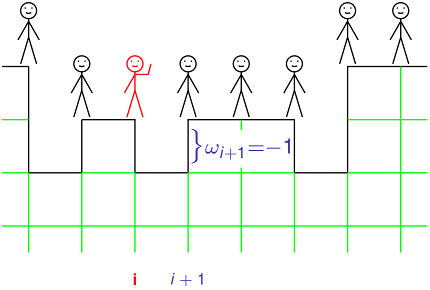

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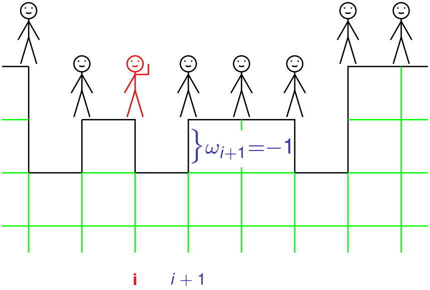

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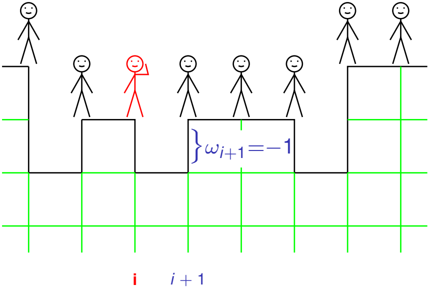

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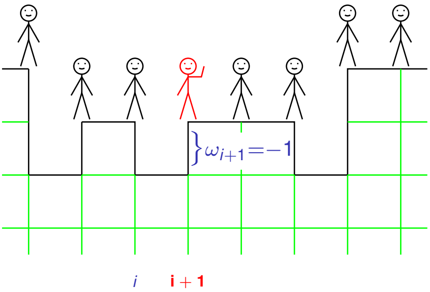

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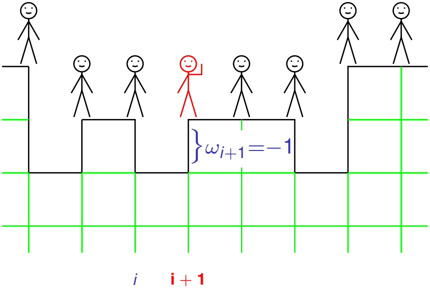

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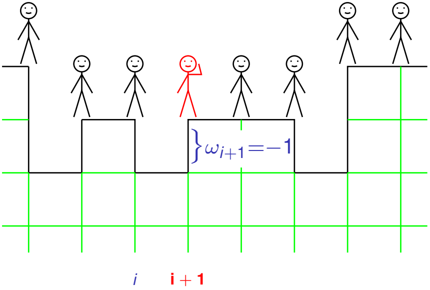

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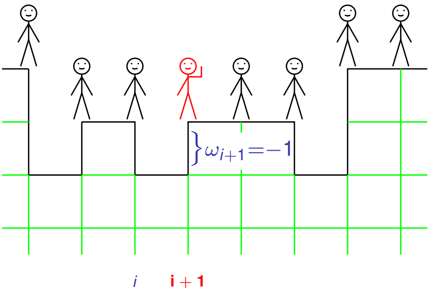

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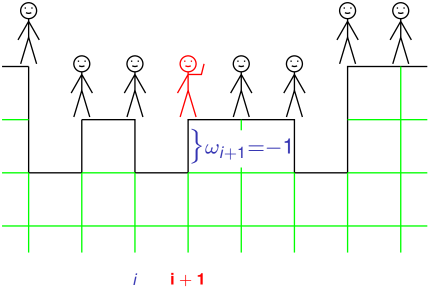

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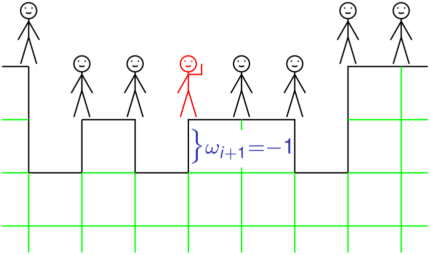

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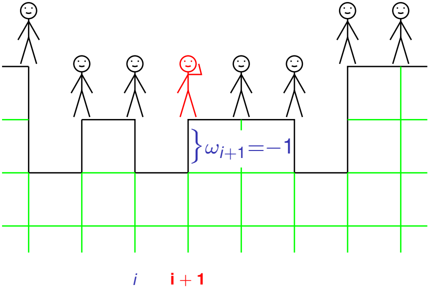

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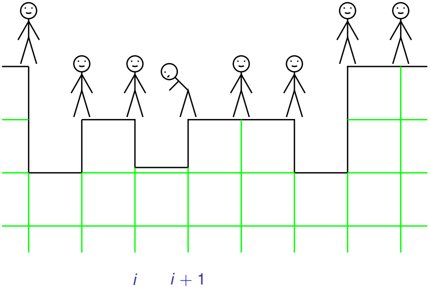

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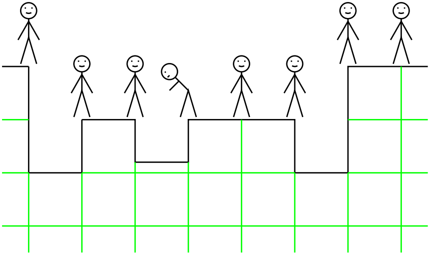

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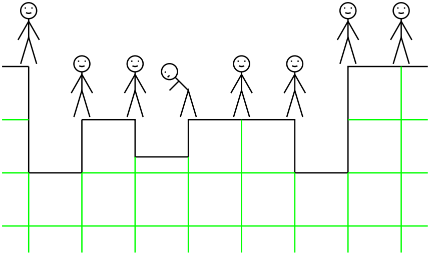

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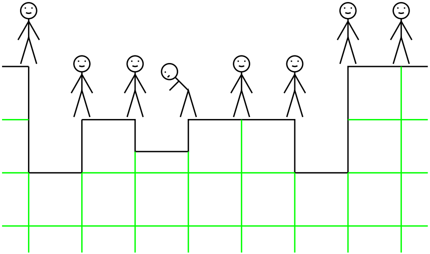

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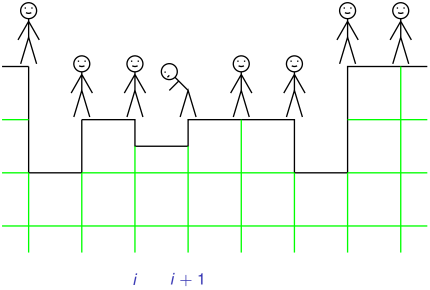

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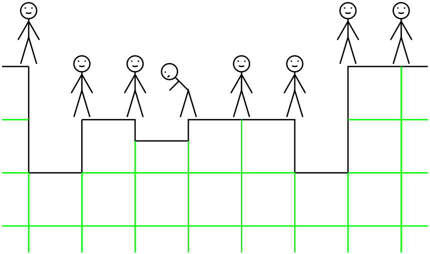

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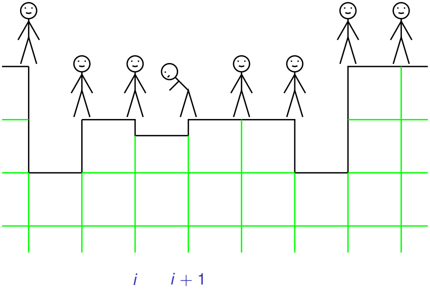

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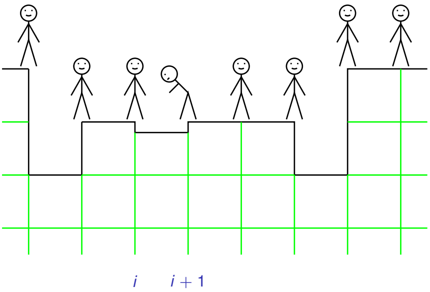

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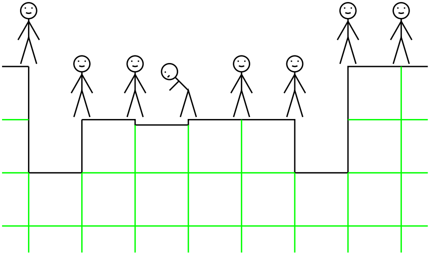

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$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \to \begin{pmatrix} \omega_i - 1 \\ \omega_{i+1} + 1 \end{pmatrix}$$

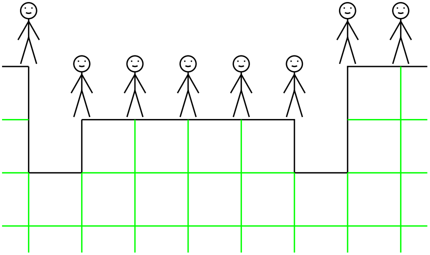
$$\| \qquad \|$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

a brick is added with rate $[r(\omega_i) + r(-\omega_{i+1})]$

$(r(\omega) \cdot r(1 - \omega) = 1; \quad r$ non-decreasing).

Totally asymmetric bricklayers process



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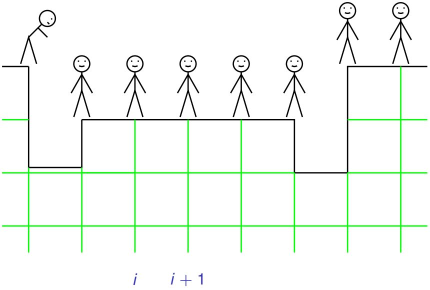

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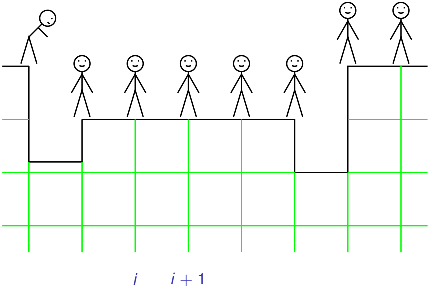

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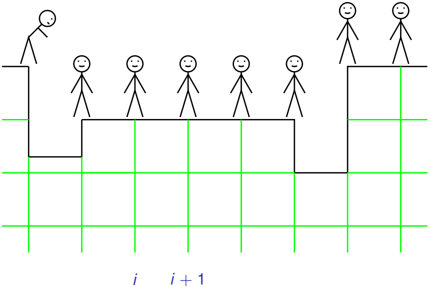

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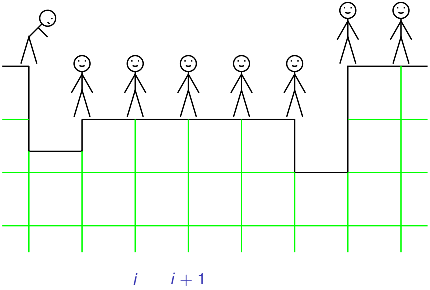

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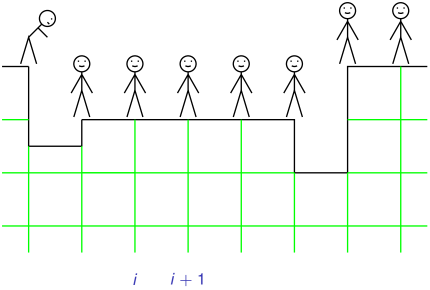

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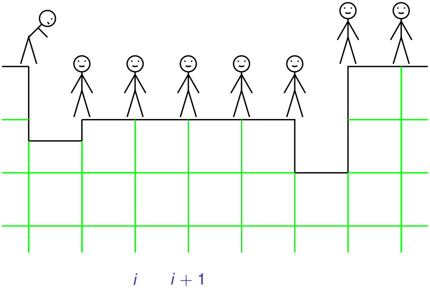

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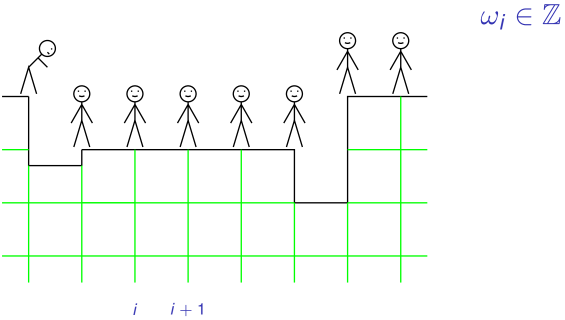

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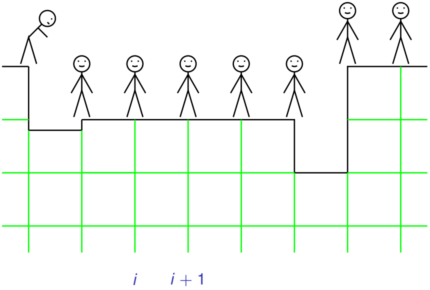

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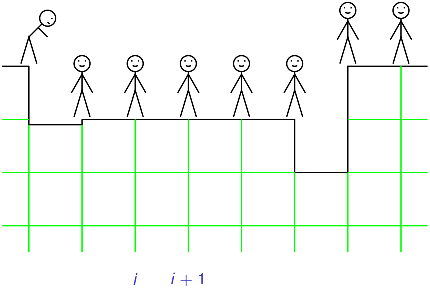

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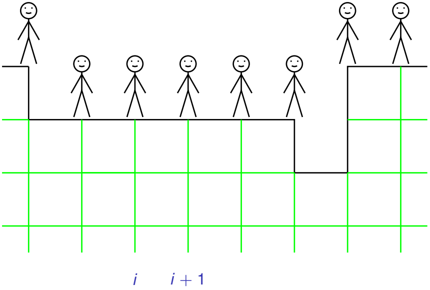

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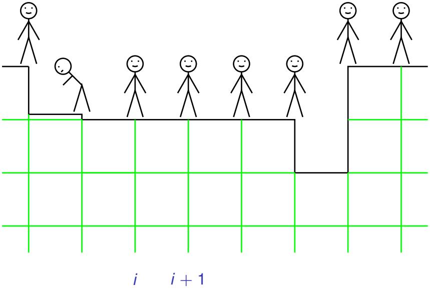

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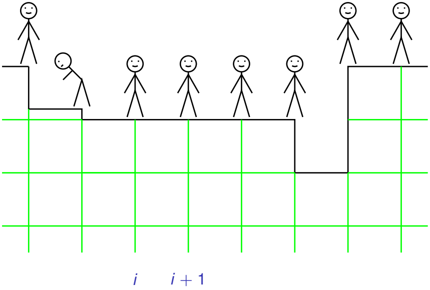

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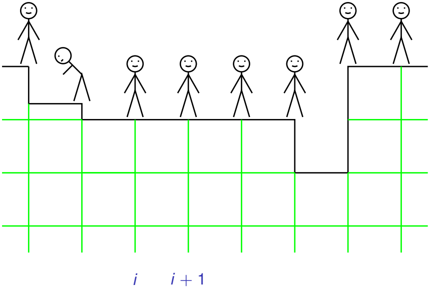

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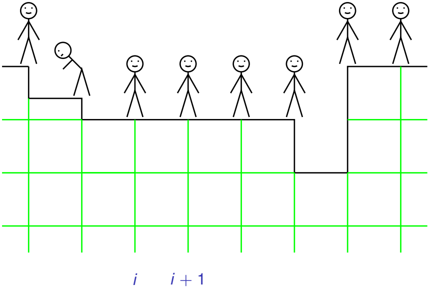

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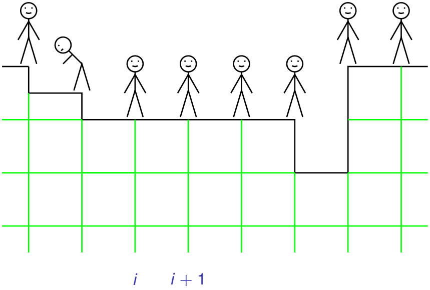

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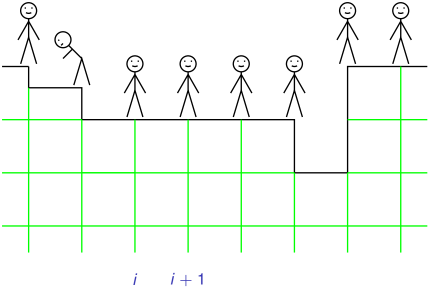

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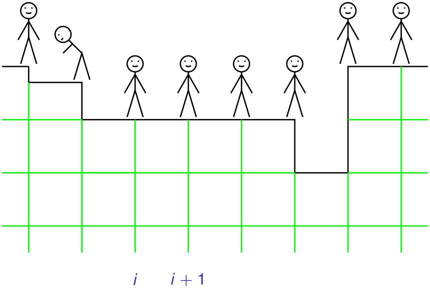

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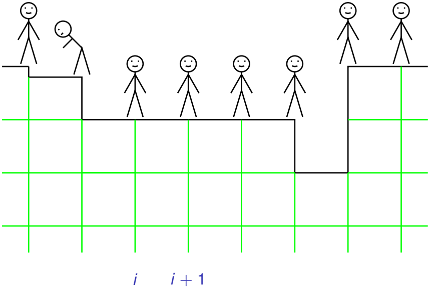

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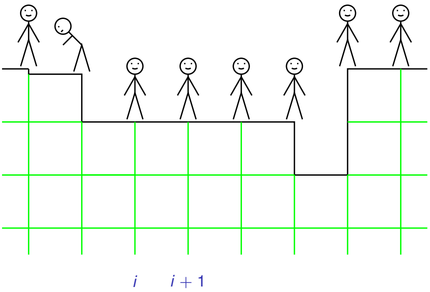

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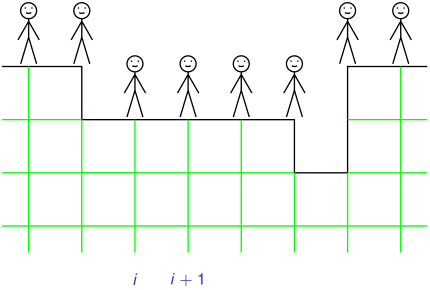

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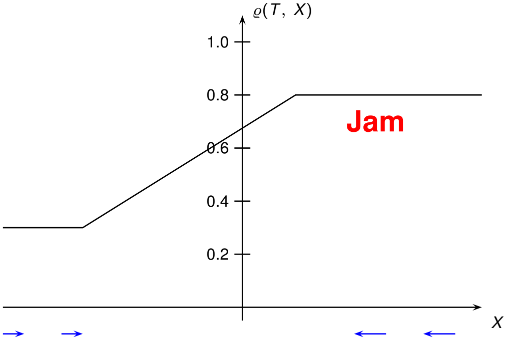
Totally asymmetric bricklayers process

Extremal translation-invariant stationary distributions are still product, and rather explicit in terms of $r(\cdot)$.

A special case: $r(\omega_i) = \mathrm{e}^{\beta\omega_i}$: $\omega_i \sim$ discrete Gaussian$(\frac{\theta}{\beta}, \frac{1}{\sqrt{\beta}})$.

Rescaled version: shock



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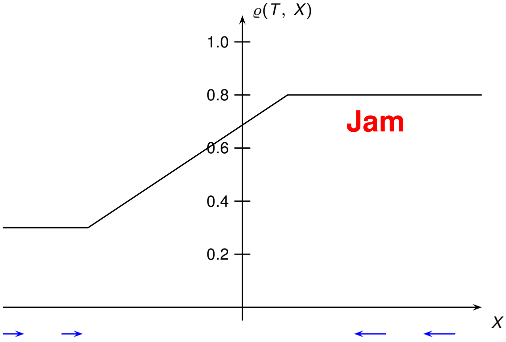

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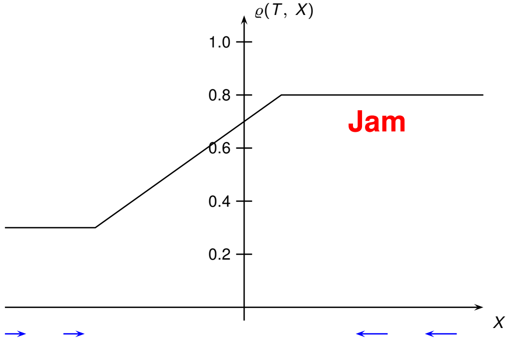

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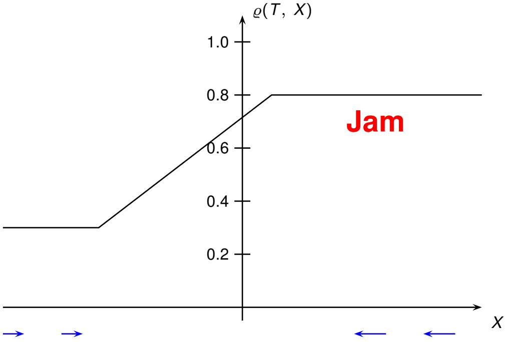

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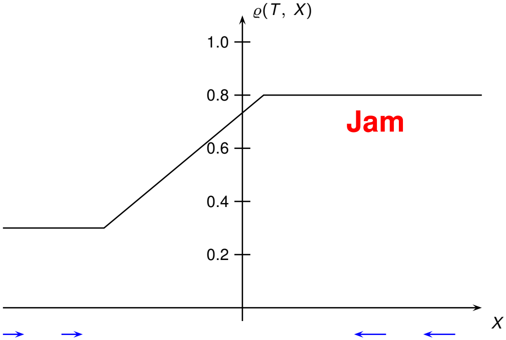

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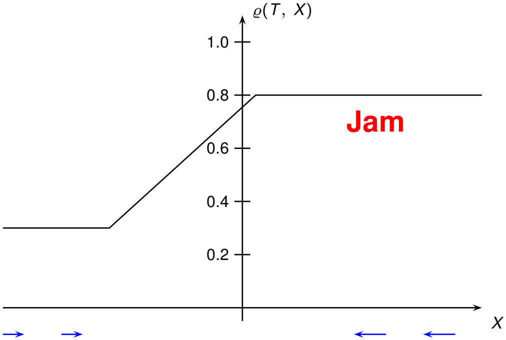

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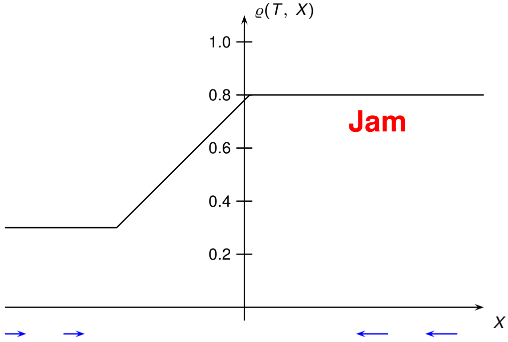

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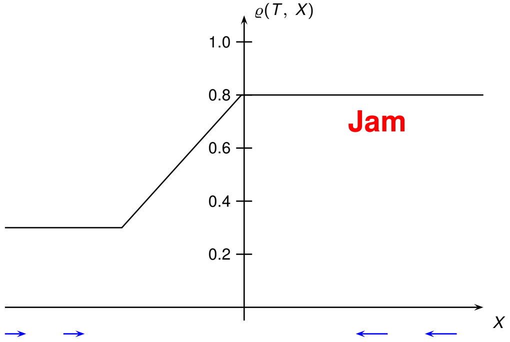

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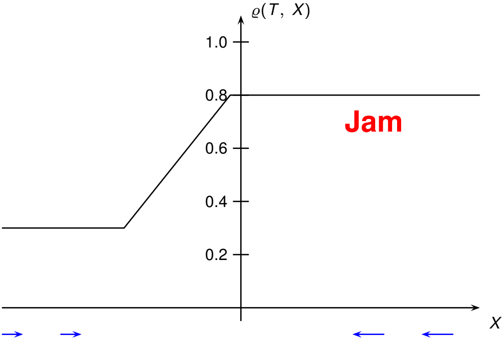

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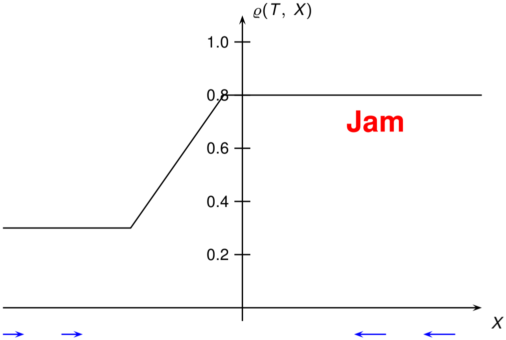

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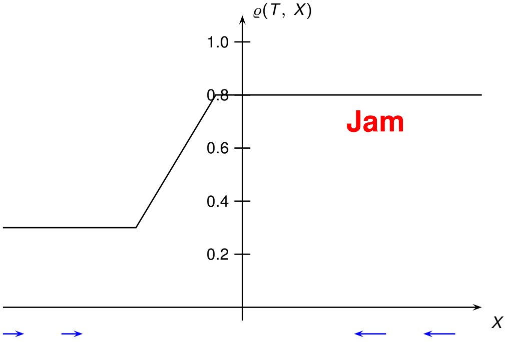

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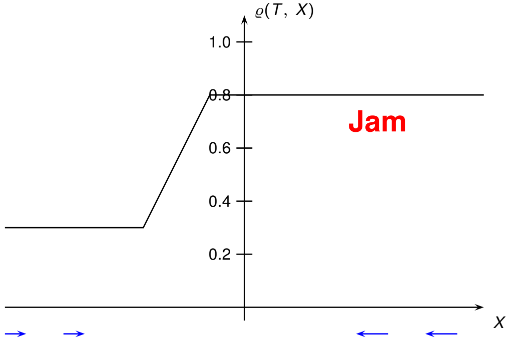

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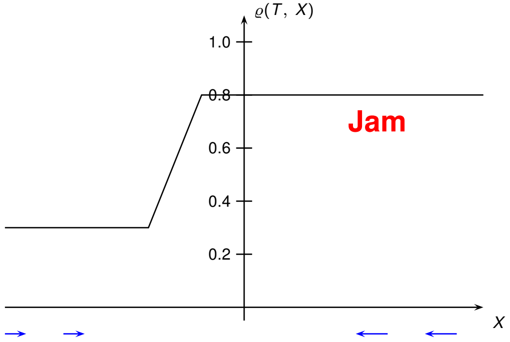

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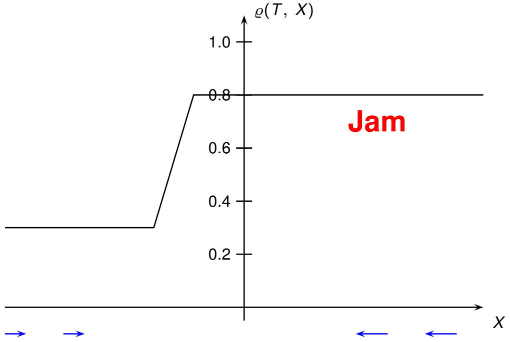

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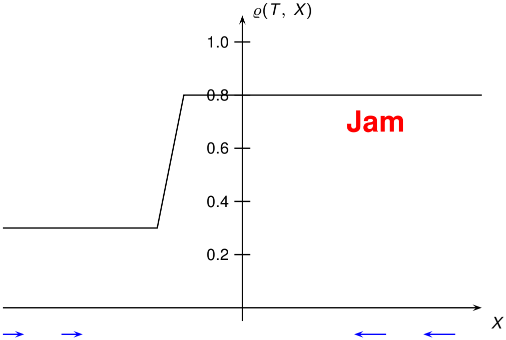

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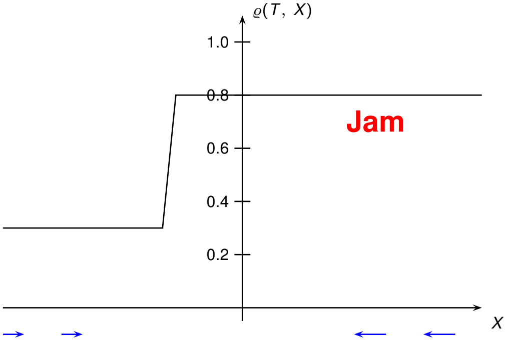

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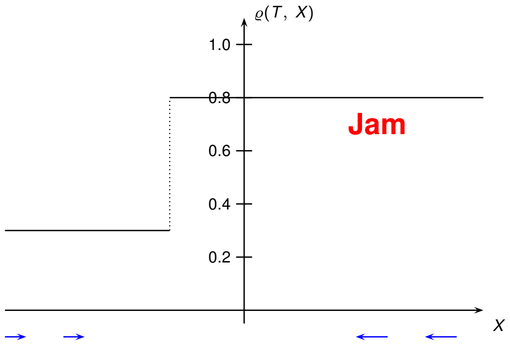

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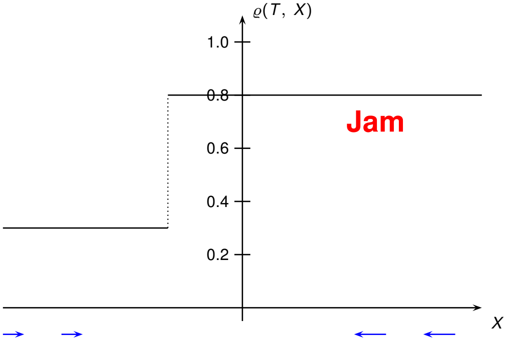

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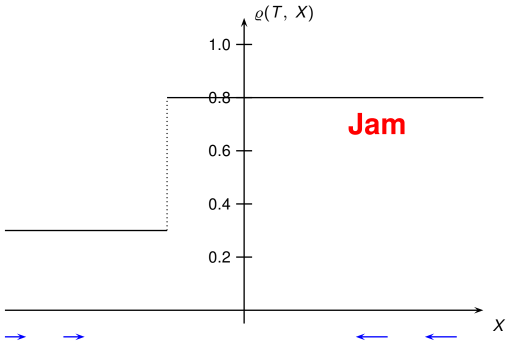

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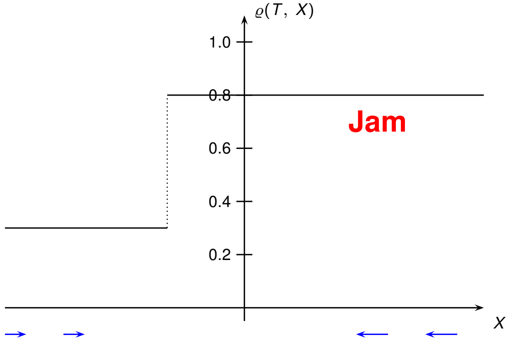

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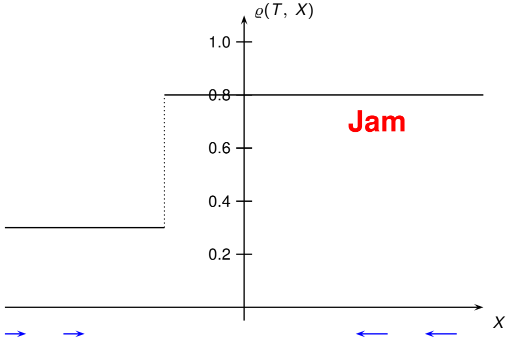

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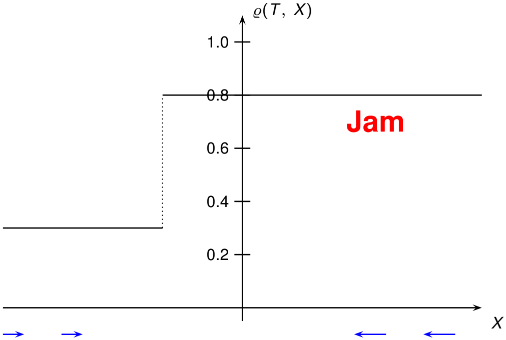

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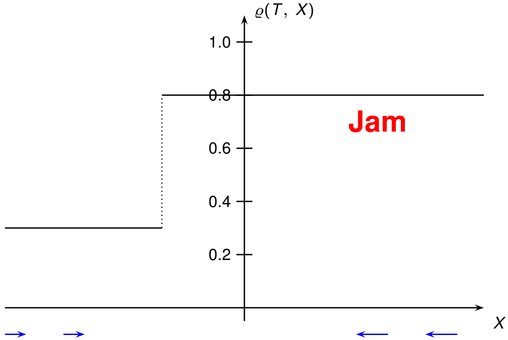

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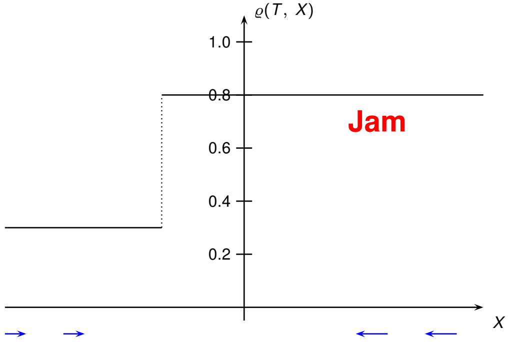

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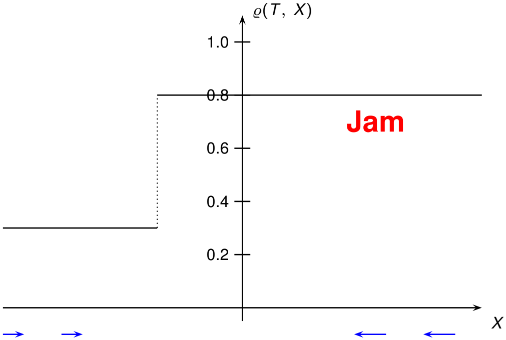

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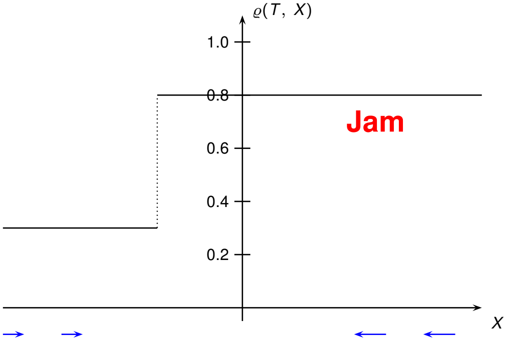

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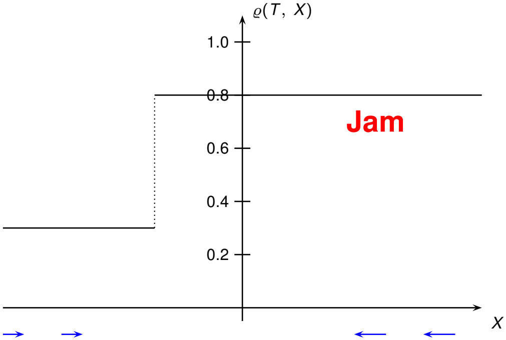

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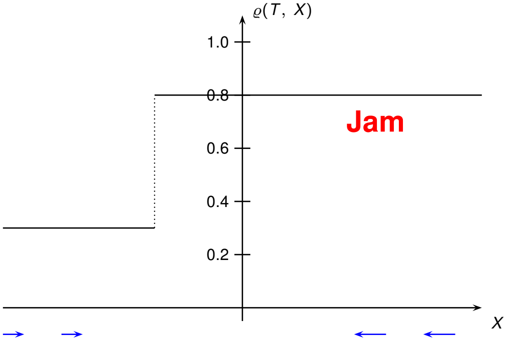

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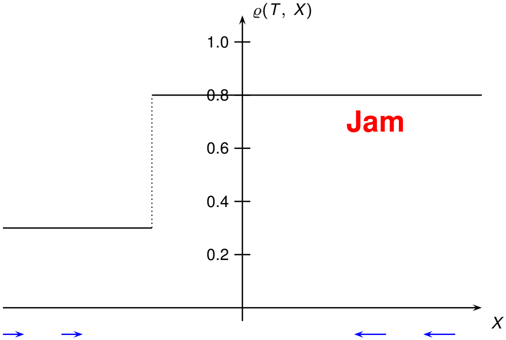

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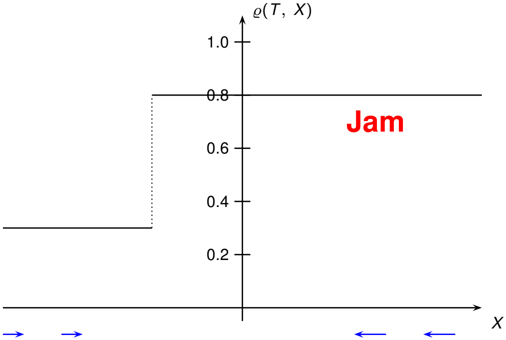

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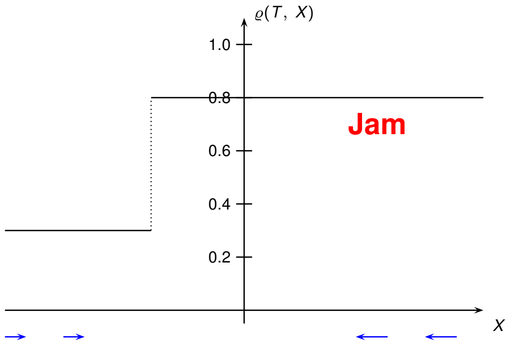

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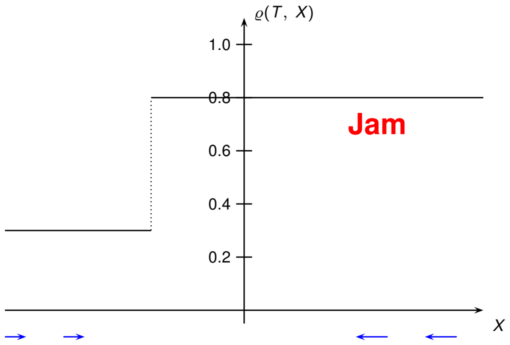

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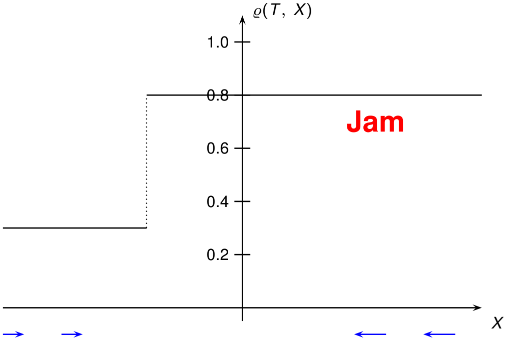

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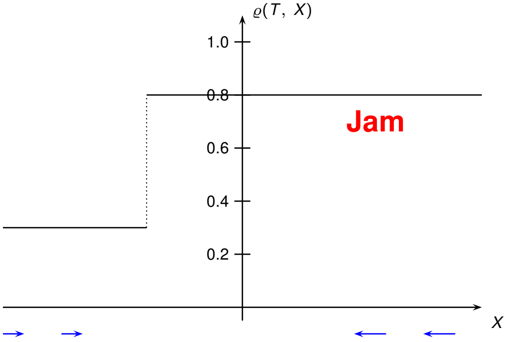

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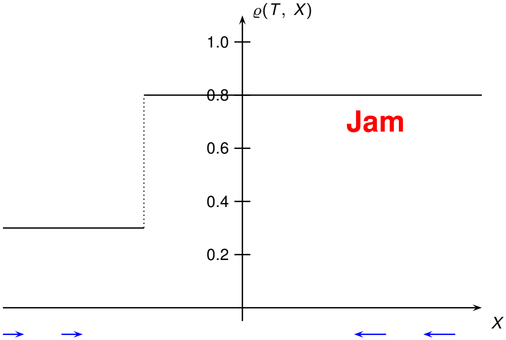

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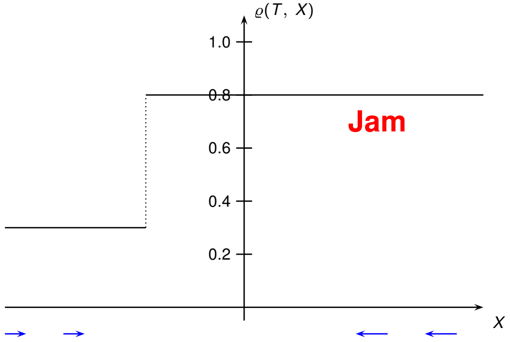

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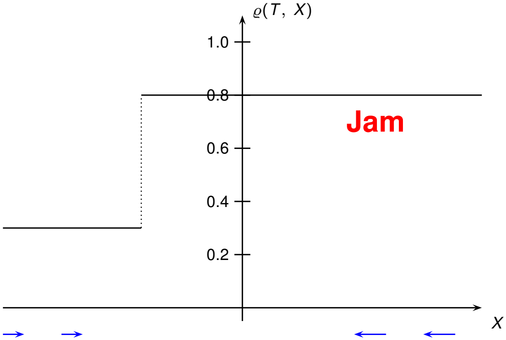

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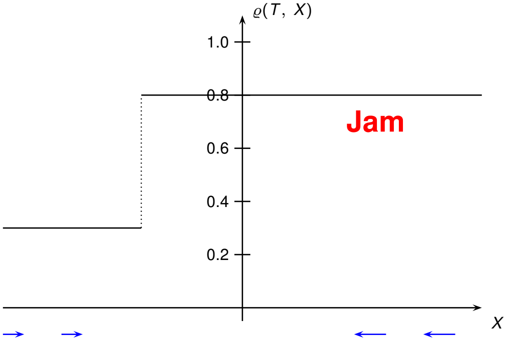

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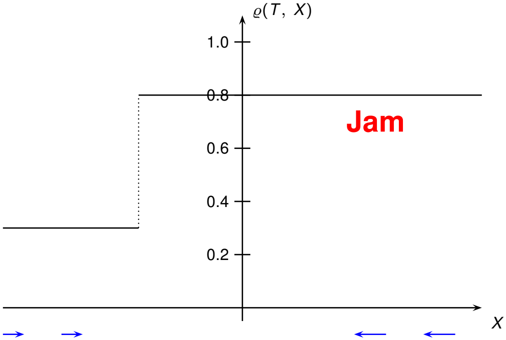

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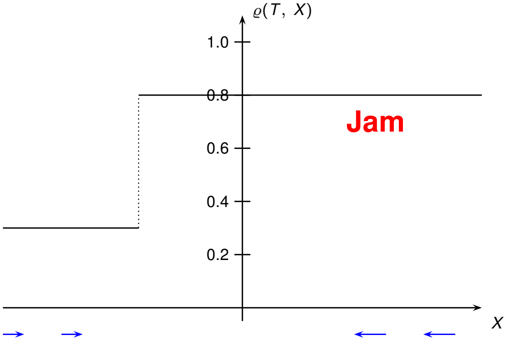

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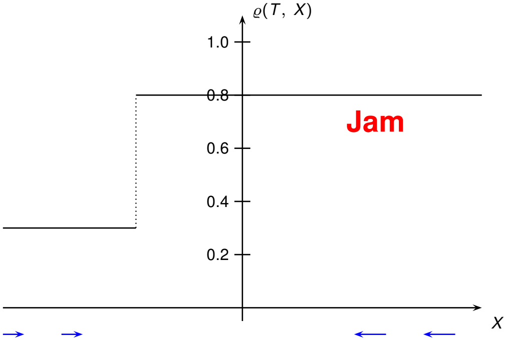

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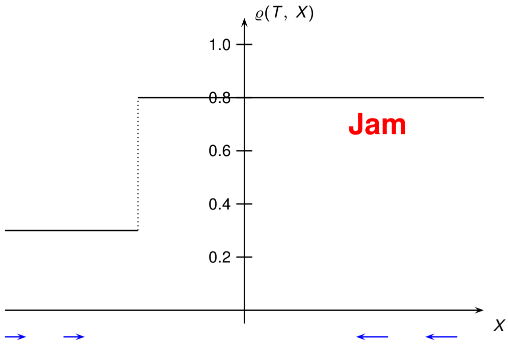

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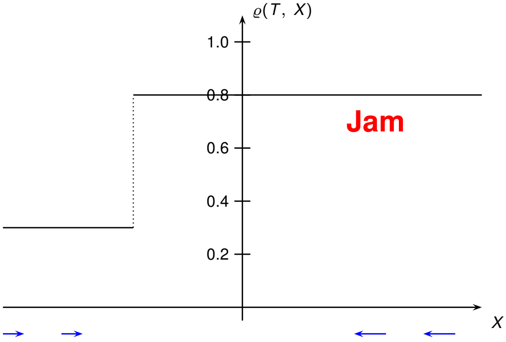

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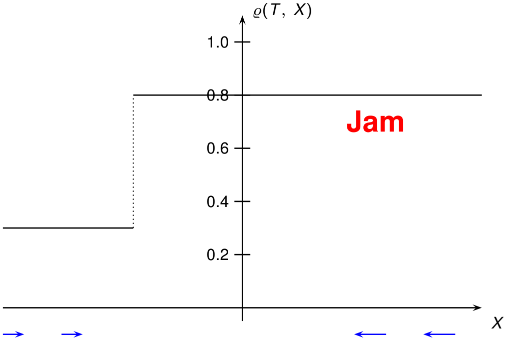

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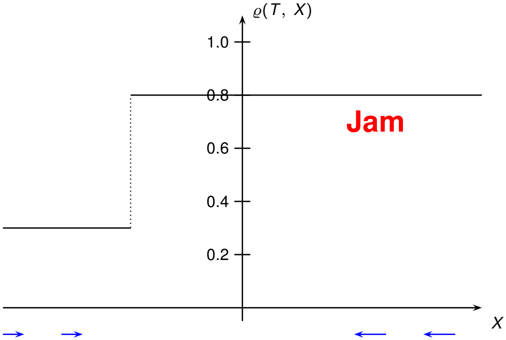

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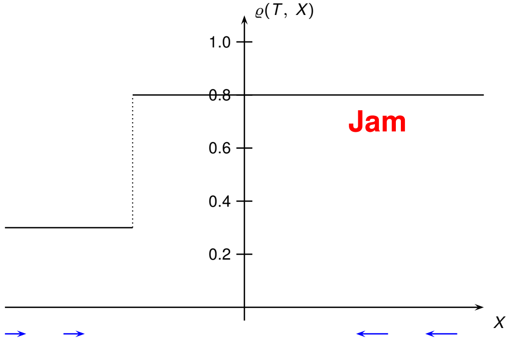

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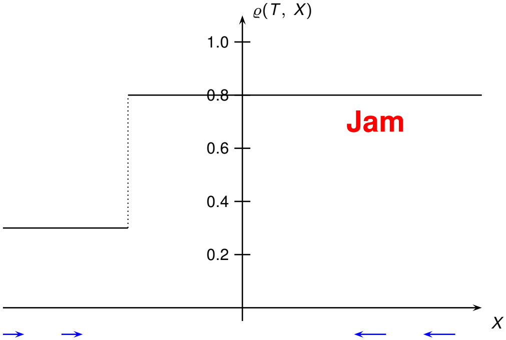

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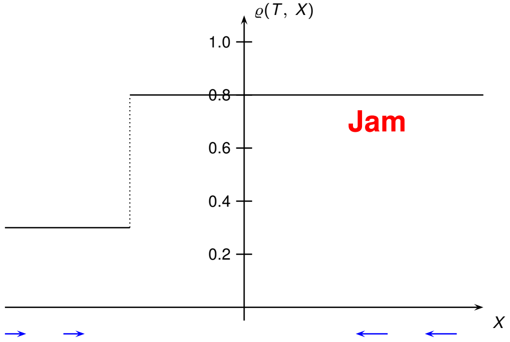

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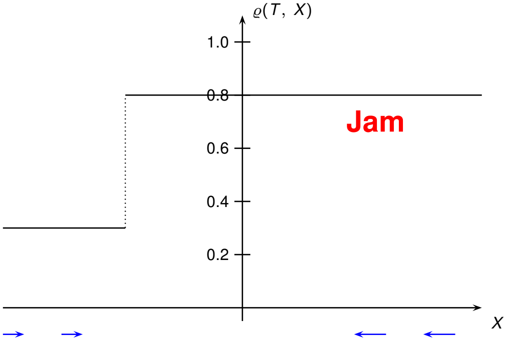

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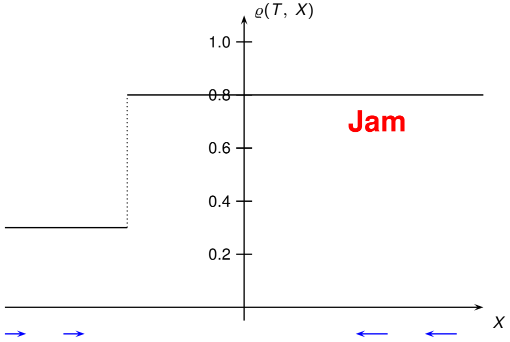

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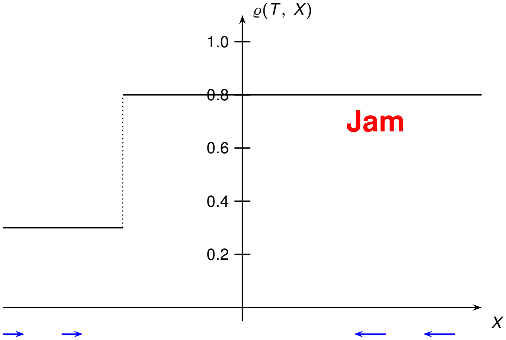

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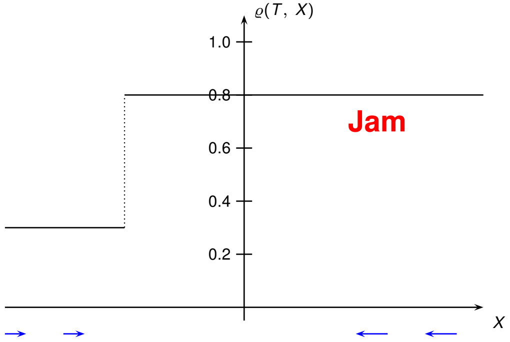

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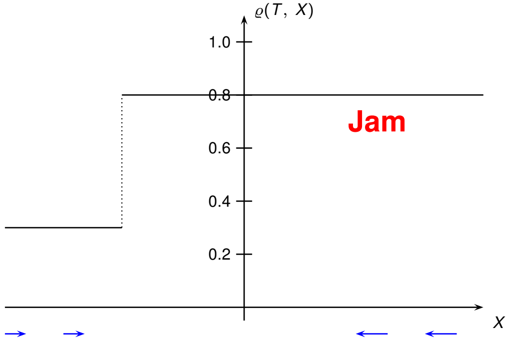

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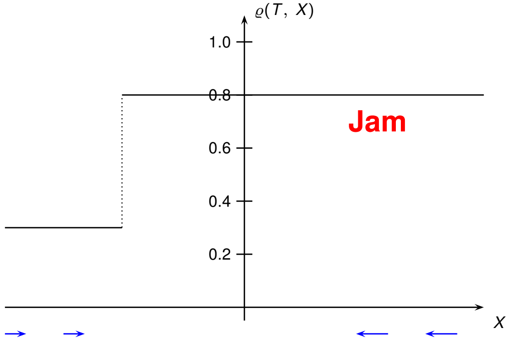

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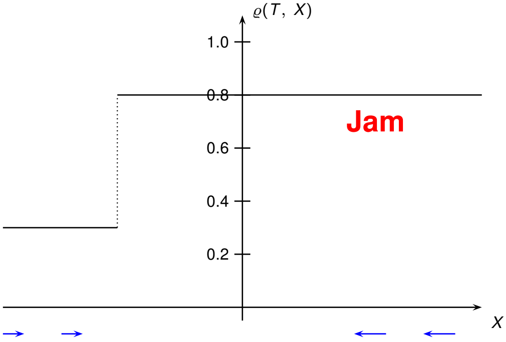

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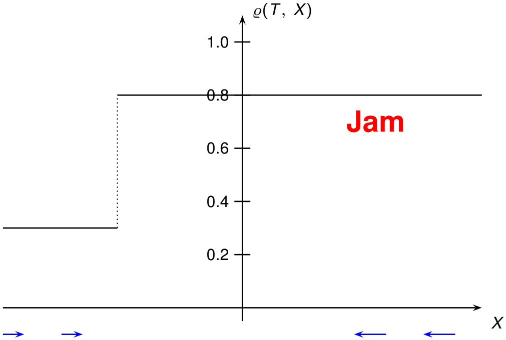

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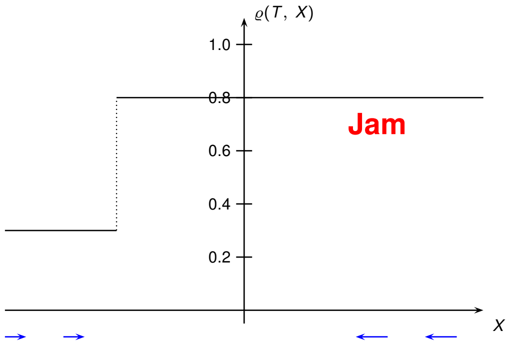

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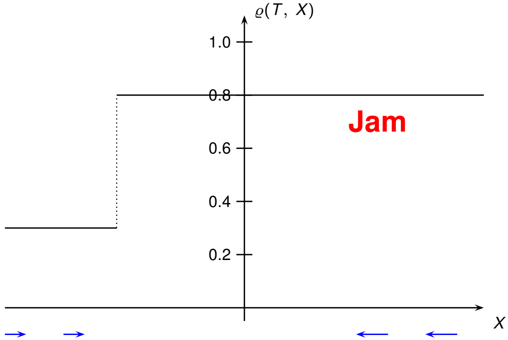

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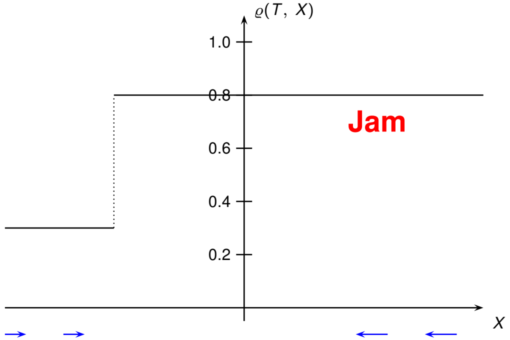

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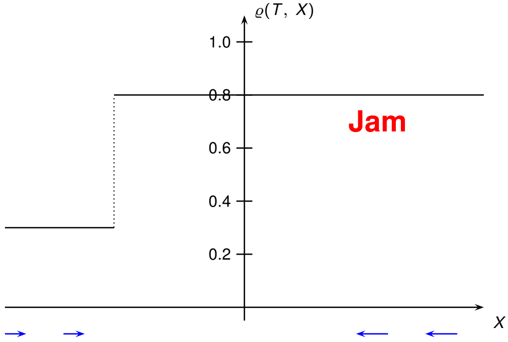

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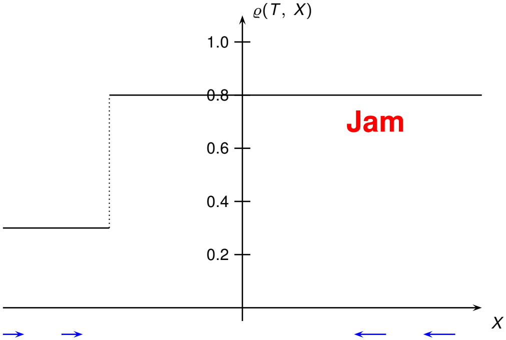
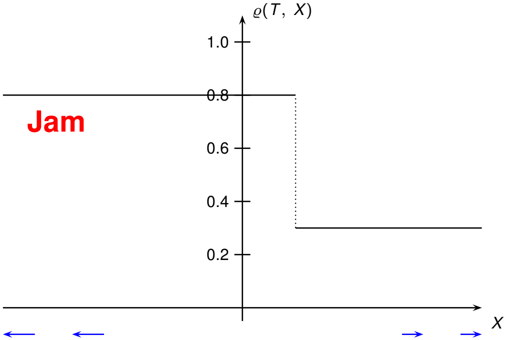


Rescaled version: rarefaction fan



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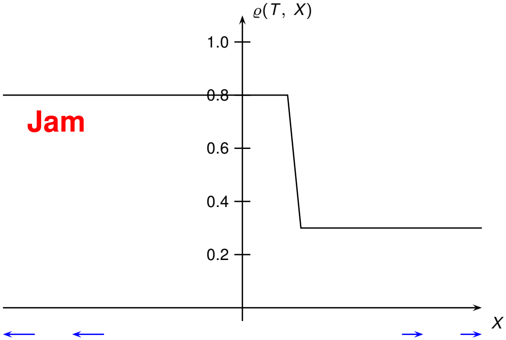

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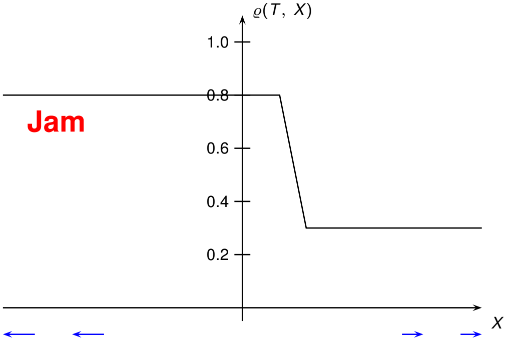

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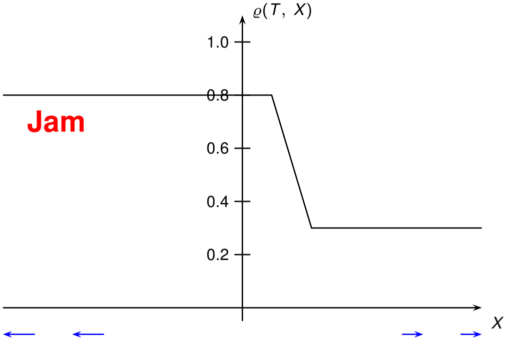

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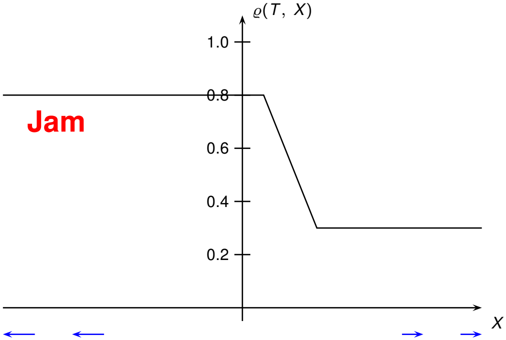

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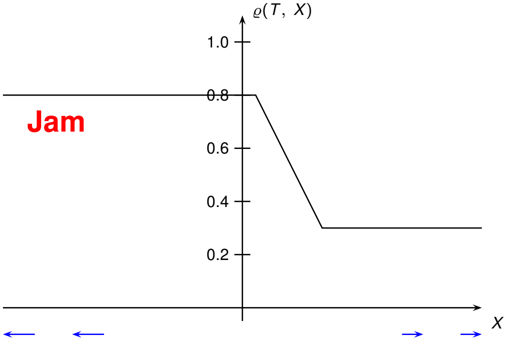

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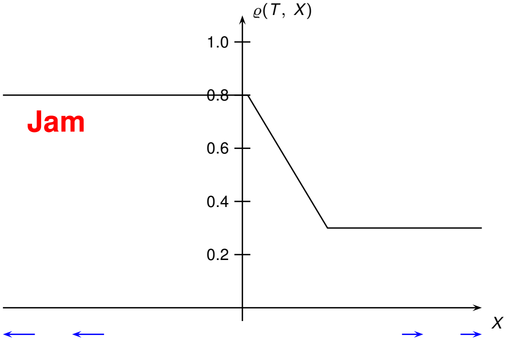

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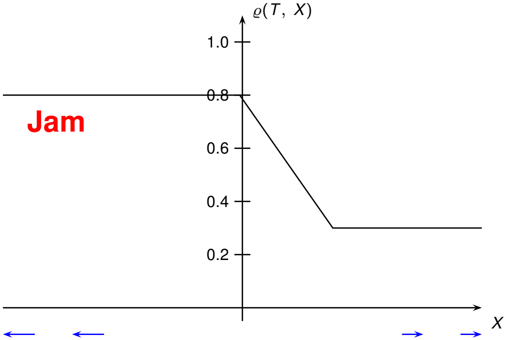

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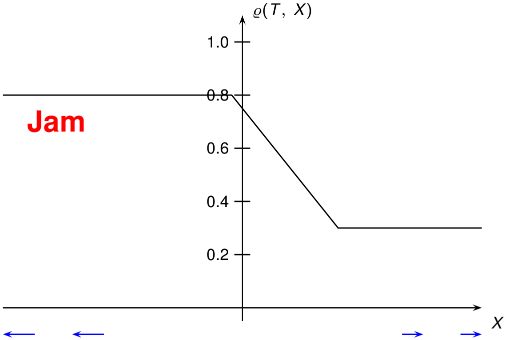

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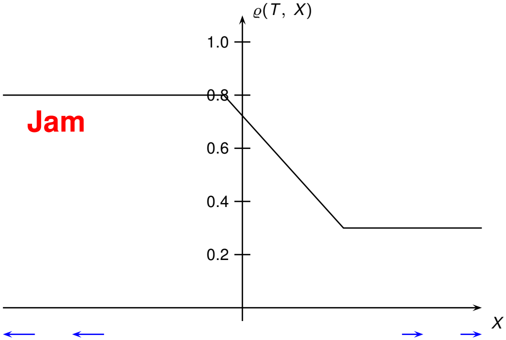

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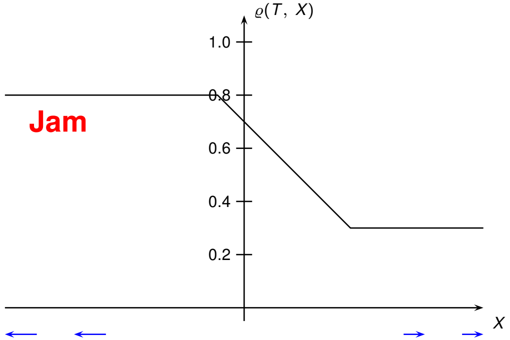

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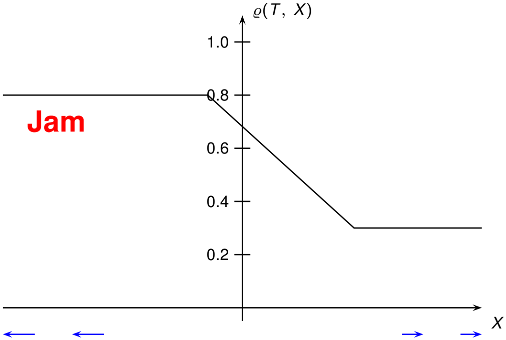

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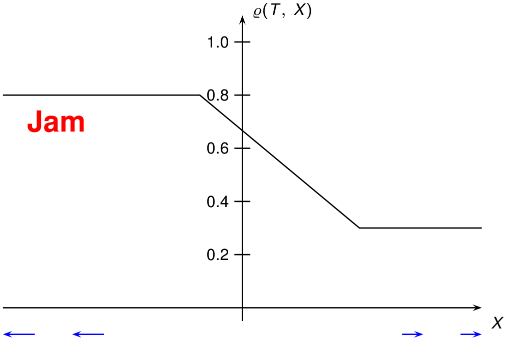

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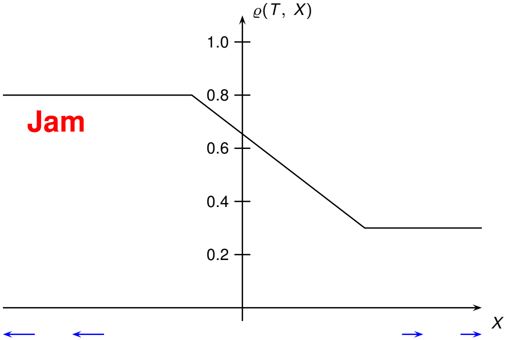

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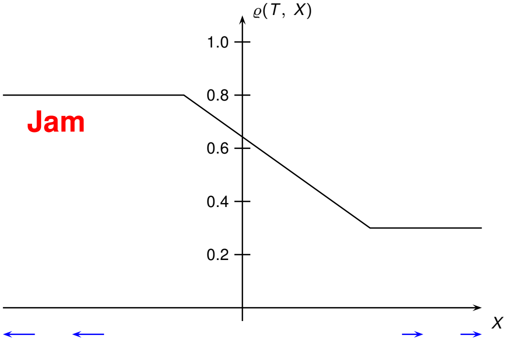

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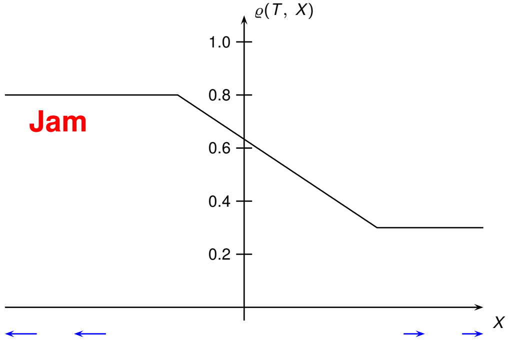

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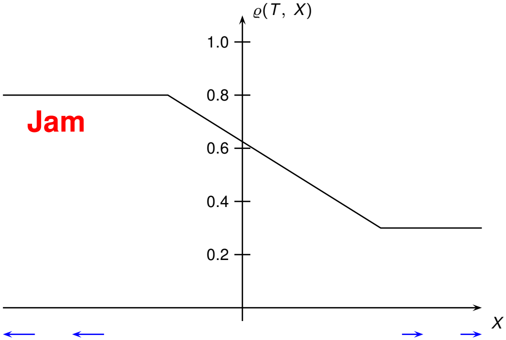

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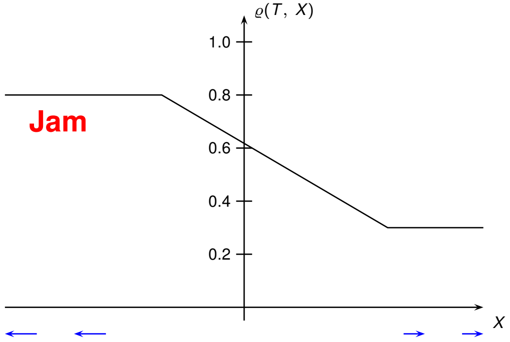

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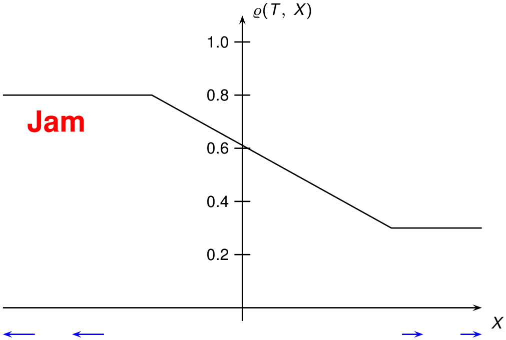

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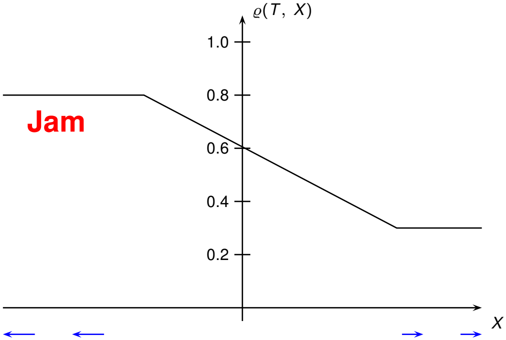

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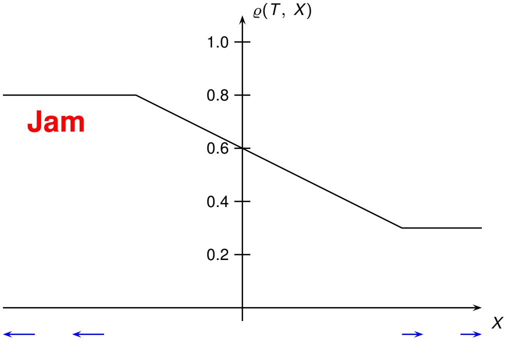

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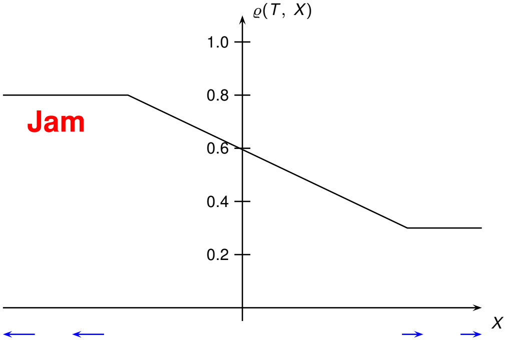

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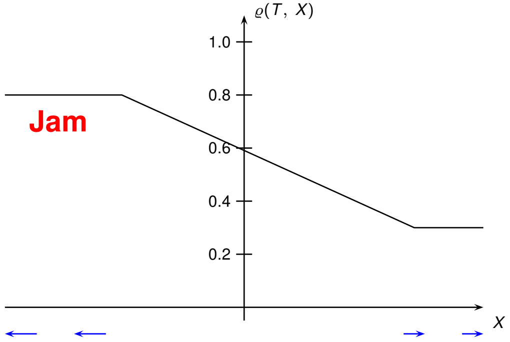

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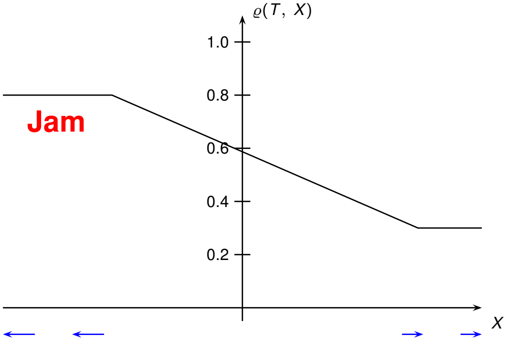

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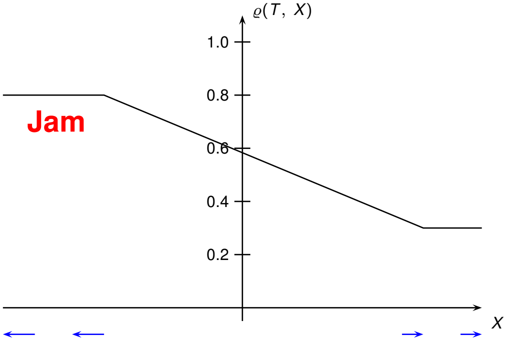

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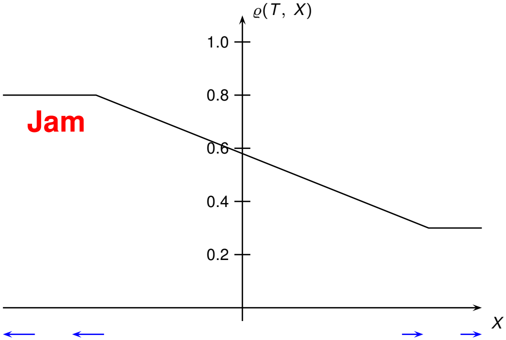

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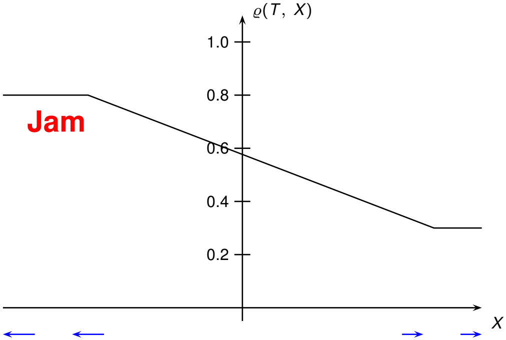

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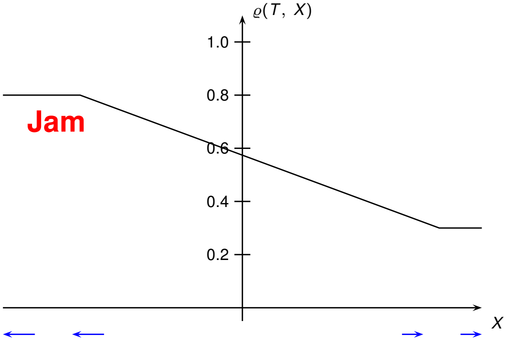

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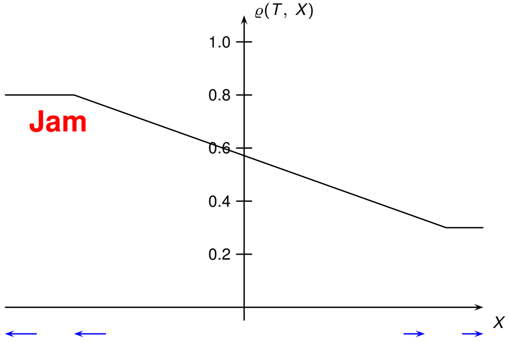

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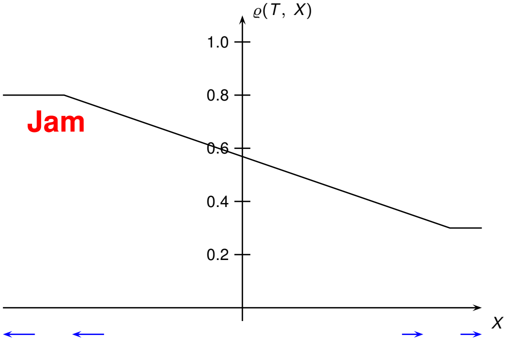

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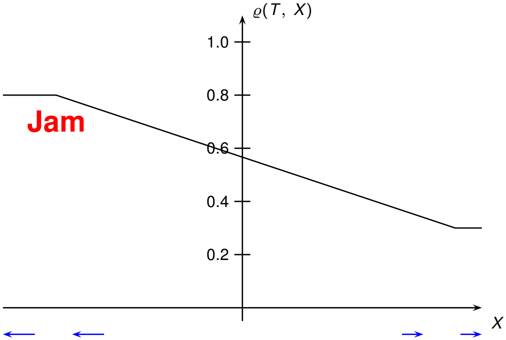
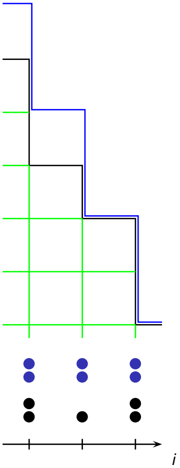


The second class particle

States ω and η only differ at one site.



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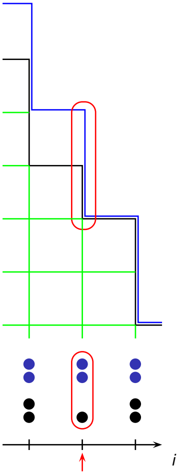

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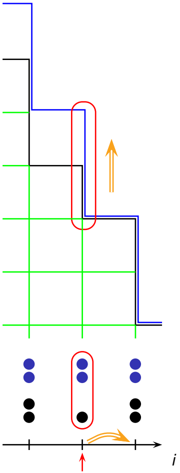

Growth on the right:
rate≤rate

The second class particle

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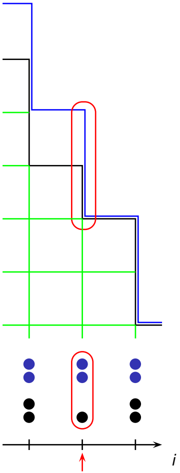

Growth on the right:
rate≤rate
with rate:

The second class particle

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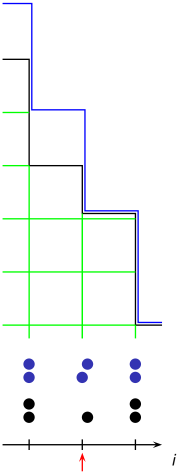

Growth on the right:
rate≤rate
with rate:

The second class particle

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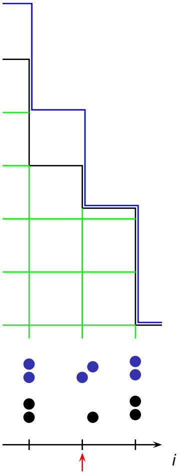

Growth on the right:
rate$\leq$rate
with rate:

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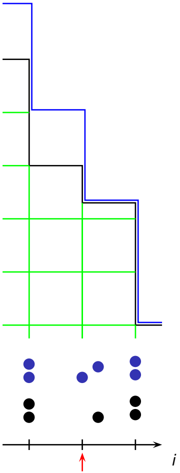

Growth on the right:
rate≤rate
with rate:

The second class particle

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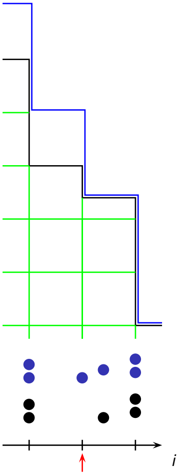

Growth on the right:
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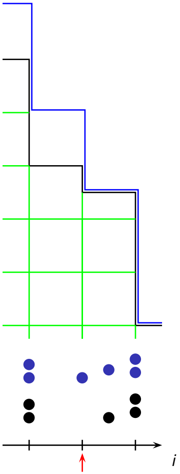

Growth on the right:

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with rate:

The second class particle

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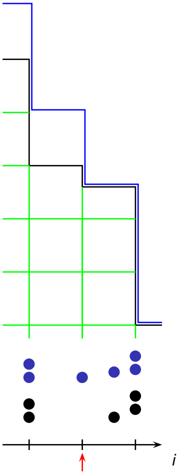

Growth on the right:
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with rate:

The second class particle

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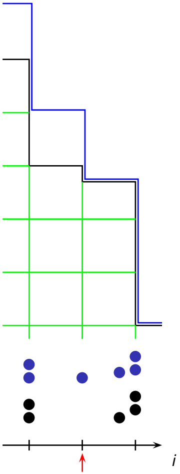

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with rate:

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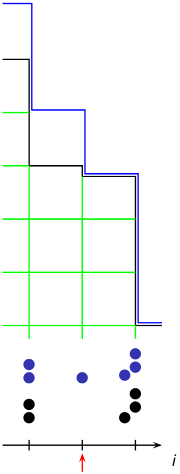

Growth on the right:
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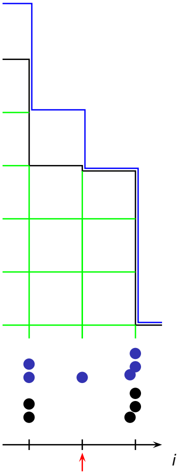
with rate:

The second class particle

States ω and η only differ at one site.



Growth on the right:
rate≤rate
with rate:

The second class particle

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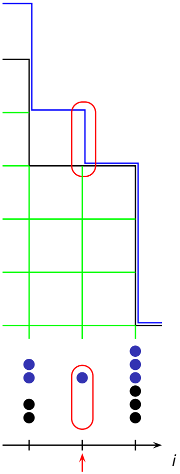
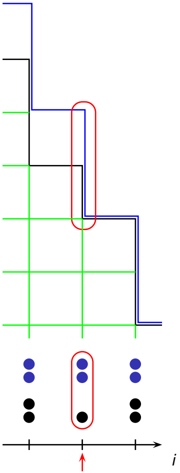


Growth on the right:
rate$\leq$rate
with rate:

The second class particle

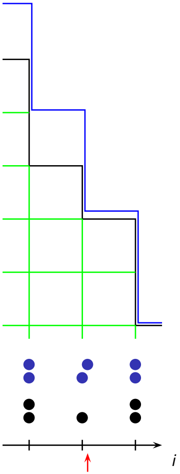
States ω and η only differ at one site.



Growth on the right:
rate≤rate
with rate-rate:

The second class particle

States ω and η only differ at one site.



Growth on the right:
rate≤rate
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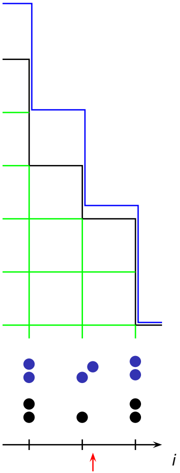

Growth on the right:
rate≤rate
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The second class particle

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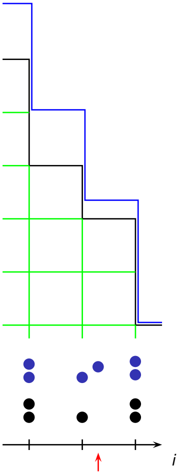

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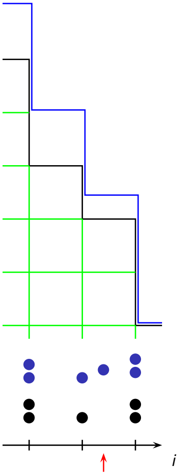

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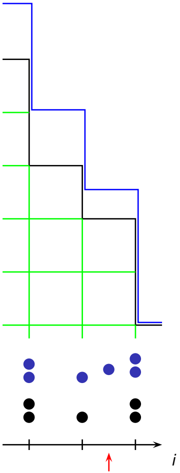

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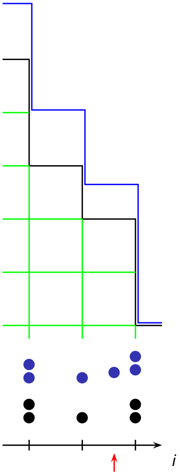

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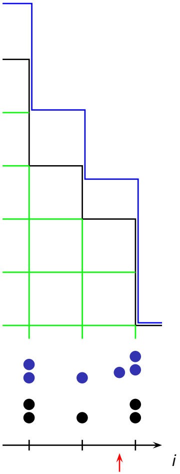

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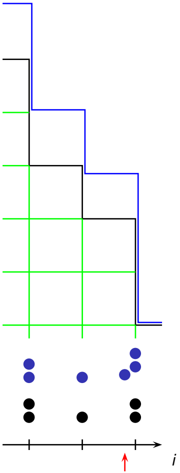

Growth on the right:
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The second class particle

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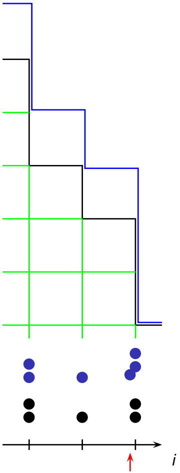

Growth on the right:
rate≤rate
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The second class particle

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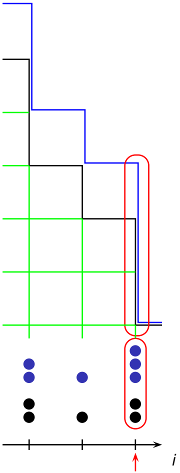


Growth on the right:

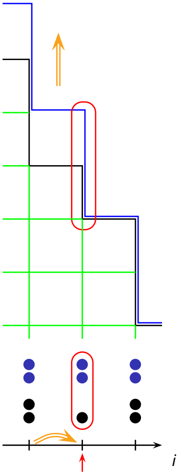
rate$\leq$rate

with rate-rate:

The second class particle

States ω and η only differ at one site.

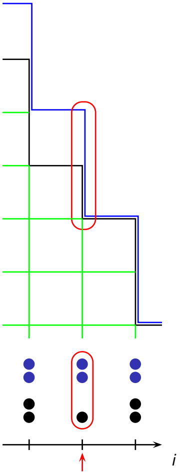
Growth on the left:
rate$\geq$rate



The second class particle

States ω and η only differ at one site.

Growth on the left:
rate≥rate
with rate:



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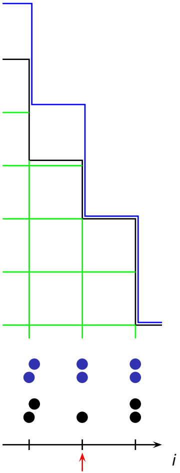

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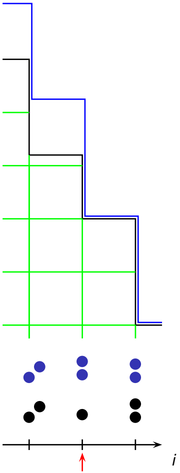

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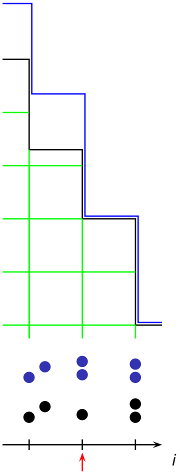

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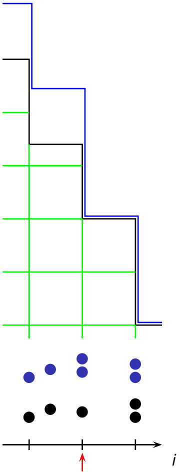

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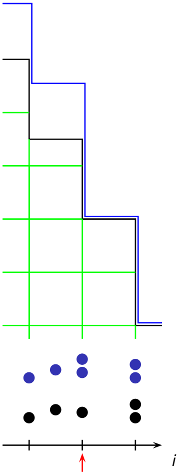

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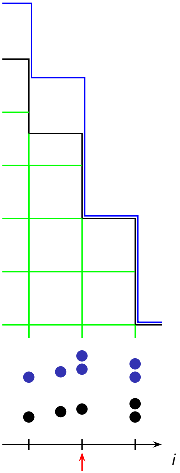

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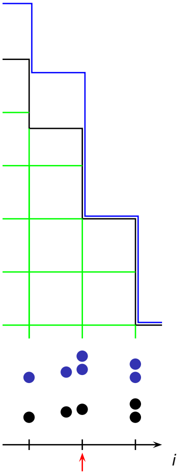

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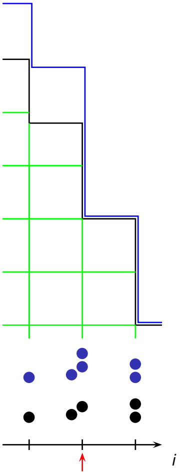

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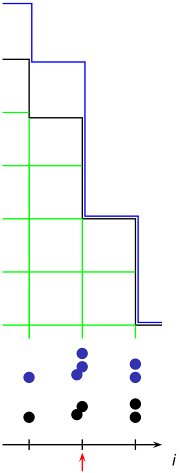

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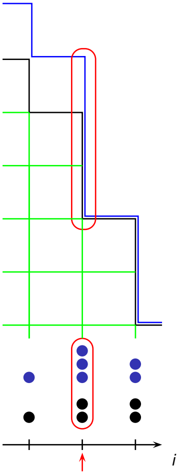

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Growth on the left:

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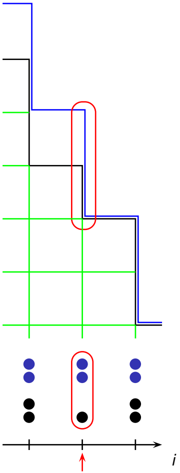

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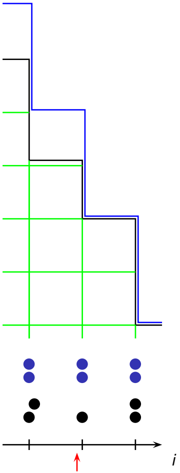

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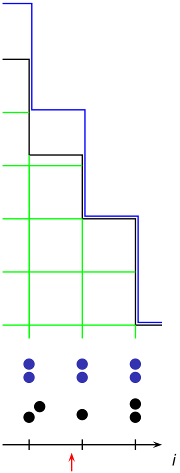

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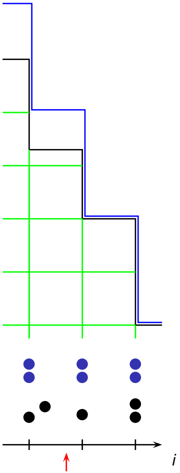

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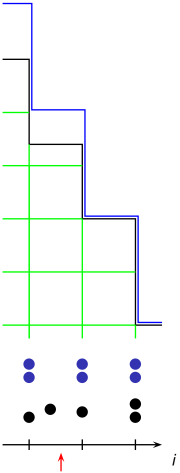

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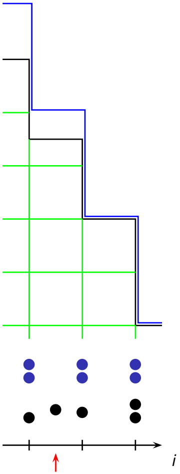

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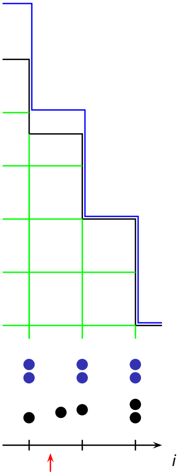

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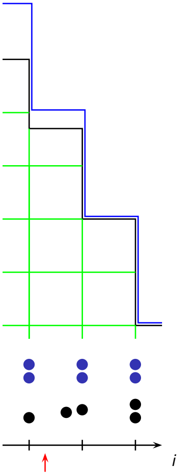

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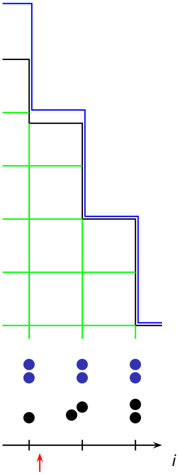

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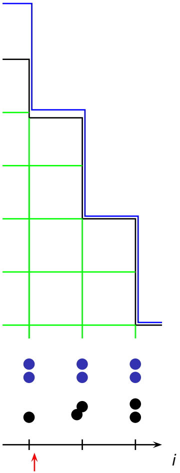

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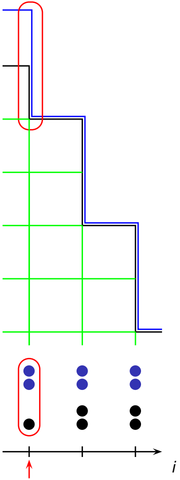

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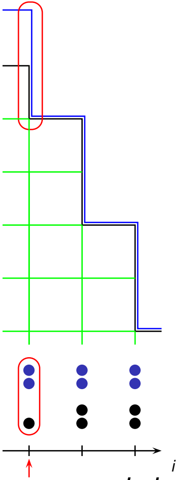
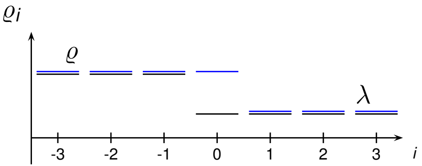


A single discrepancy ↑, the *second class particle*, is conserved.
Its position at time t is $Q(t)$.

Ferrari-Kipnis '95 for TASEP

Blue TASEP ω:
Bernoulli(ϱ) for sites $\{\dots, -2, -1, 0\}$,
Bernoulli(λ) for sites $\{1, 2, 3, \dots\}$.

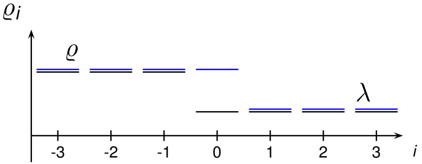
Black TASEP η:
Bernoulli(ϱ) for sites $\{\dots, -3, -2, -1\}$,
Bernoulli(λ) for sites $\{0, 1, 2, \dots\}$.



$h_i(t)$, $g_i(t)$ are the respective numbers of particles jumping over the edge $(i, i+1)$ by time t ($i > 0$).

Ferrari-Kipnis '95 for TASEP, Part 1

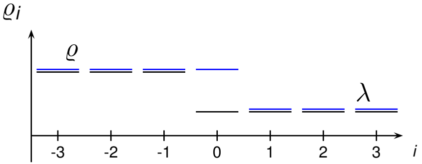
First realization:



Ferrari-Kipnis '95 for TASEP, Part 1

First realization:

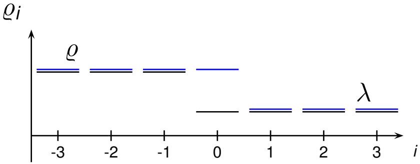
- $\omega_i(0) = \eta_i(0) \sim \text{Bernoulli}(\varrho)$ for $i < 0$



Ferrari-Kipnis '95 for TASEP, Part 1

First realization:

- $\omega_i(0) = \eta_i(0) \sim \text{Bernoulli}(\varrho)$ for $i < 0$
- $(\omega_0(0), \eta_0(0)) = (0, 0)$ w. prob. $1 - \varrho$
  $(\omega_0(0), \eta_0(0)) = (1, 0)$ w. prob. $\varrho - \lambda$
  $(\omega_0(0), \eta_0(0)) = (1, 1)$ w. prob. λ



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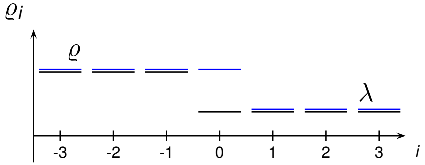
- $\omega_i(0) = \eta_i(0) \sim \text{Bernoulli}(\lambda)$ for $i > 0$



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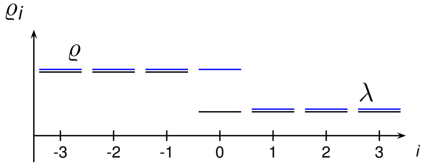
- $(\omega_0(0), \eta_0(0)) = (0, 0)$ w. prob. $1 - \varrho$
  $(\omega_0(0), \eta_0(0)) = (1, 0)$ w. prob. $\varrho - \lambda$ — 2nd class particle
  $(\omega_0(0), \eta_0(0)) = (1, 1)$ w. prob. λ
- $\omega_i(0) = \eta_i(0) \sim \text{Bernoulli}(\lambda)$ for $i > 0$



Ferrari-Kipnis '95 for TASEP, Part 1

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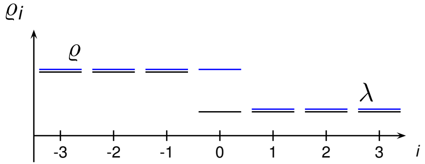
- $(\omega_0(0), \eta_0(0)) = (0, 0)$ w. prob. $1 - \varrho$
  $(\omega_0(0), \eta_0(0)) = (1, 0)$ w. prob. $\varrho - \lambda$ — 2^{nd} class particle
  $(\omega_0(0), \eta_0(0)) = (1, 1)$ w. prob. λ
- $\omega_i(0) = \eta_i(0) \sim \text{Bernoulli}(\lambda)$ for $i > 0$

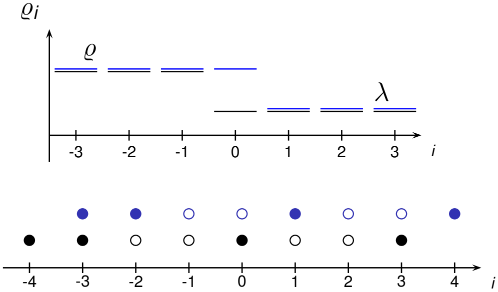


$$\mathbf{E}h_i(t) - \mathbf{E}g_i(t) = \mathbf{E}(h_i(t) - g_i(t)) = (\varrho - \lambda) \cdot \mathbf{P}\{Q(t) > i\}.$$

Ferrari-Kipnis ’95 for TASEP, Part 2

Second realization:

$$\omega_i(t) \equiv \eta_{i-1}(t) \qquad \forall i,\ \forall t.$$



Ferrari-Kipnis '95 for TASEP, Part 2

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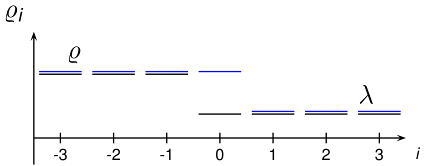

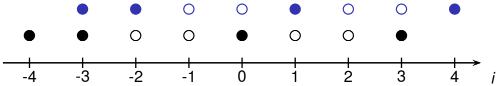

$$\mathbf{E}h_i(t) - \mathbf{E}g_i(t) = \mathbf{E}(h_i(t) - g_i(t)) = \mathbf{E}(\eta_i(t) - \eta_i(0)) = \mathbf{E}\eta_i(t) - \mathbf{E}\eta_i(0).$$

Ferrari-Kipnis '95 for TASEP

Thus,

$$\mathbf{E}h_i(t) - \mathbf{E}g_i(t) = \mathbf{E}(h_i(t) - g_i(t)) = (\varrho - \lambda) \cdot \mathbf{P}\{Q(t) > i\},$$
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Ferrari-Kipnis '95 for TASEP

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$$\mathbf{P}\{Q(t) > i\} = \frac{\mathbf{E}\eta_i(t) - \mathbf{E}\eta_i(0)}{\varrho - \lambda}.$$

Ferrari-Kipnis ’95 for TASEP

Thus,

$$\mathbf{E}h_i(t) - \mathbf{E}g_i(t) = \mathbf{E}(h_i(t) - g_i(t)) = (\varrho - \lambda) \cdot \mathbf{P}\{Q(t) > i\},$$
$$\mathbf{E}h_i(t) - \mathbf{E}g_i(t) = \mathbf{E}(h_i(t) - g_i(t)) = \mathbf{E}\eta_i(t) - \mathbf{E}\eta_i(0),$$

$$\mathbf{P}\{Q(t) > i\} = \frac{\mathbf{E}\eta_i(t) - \mathbf{E}\eta_i(0)}{\varrho - \lambda}.$$

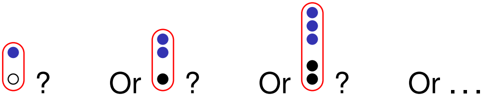
Combine with hydrodynamics to conclude

$$\frac{Q(t)}{t} \Rightarrow \begin{cases} \text{shock velocity} & \text{in a shock,} \\ \mathrm{U}\big(H'(\varrho),\, H'(\lambda)\big) & \text{in a rarefaction wave.} \end{cases}$$

Let's generalise

Other models have more than 0 or 1 particles per site. How do we start the second class particle?

- Shall we do



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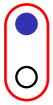
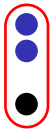
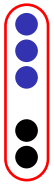
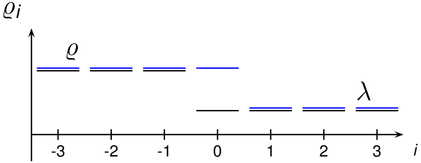
   ? Or  ? Or  ? Or …

- Recall for TASEP we increased λ to ϱ by adding or not adding a 2^{nd} class particle.

  $(\omega_0(0), \eta_0(0)) = (0, 0)$ w. prob. $1 - \varrho$

  $(\omega_0(0), \eta_0(0)) = (1, 0)$ w. prob. $\varrho - \lambda$

  $(\omega_0(0), \eta_0(0)) = (1, 1)$ w. prob. λ



Let's generalise: problems with coupling

Fix $\lambda < \varrho \leq \lambda + 1$. Is there a joint distribution of (ω_0, η_0) such that

- the first marginal is $\omega_0 \sim$ stati. μ^ϱ;
- the second marginal is $\eta_0 \sim$ stati. μ^λ;
- $\eta_0 \leq \omega_0 \leq \eta_0 + 1$?

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- *No for Geometric (classical TAZRP with $r(\omega_i) = \mathbf{1}\{\omega_i > 0\}$).*
- *No for Poisson (indep. walkers with $r(\omega_i) = \omega_i$).*
- *Yes for discrete Gaussian (bricklayers with $r(\omega_i) = \mathrm{e}^{\beta\omega_i}$).*

Let's generalise

Keep calm and couple anyway.

Find a coupling measure ν with

- first marginal $\omega_0 \sim$ stati. μ^ϱ;
- second marginal $\eta_0 \sim$ stati. μ^λ;
- zero weight whenever $\omega_0 \notin \{\eta_0, \eta_0 + 1\}$.

Not many choices:

$$\begin{aligned}
\nu(x, x) &= \mu^\varrho\{-\infty \dots x\} - \mu^\lambda\{-\infty \dots x - 1\},\\
\nu(x + 1, x) &= \mu^\lambda\{-\infty \dots x\} - \mu^\varrho\{-\infty \dots x\},\\
\nu &= \text{zero elsewhere.}
\end{aligned}$$

Let's generalise

$$\nu(x,\,x)=\mu^{\varrho}\{-\infty\dots x\}-\mu^{\lambda}\{-\infty\dots x-1\},$$
$$\nu(x+1,\,x)=\mu^{\lambda}\{-\infty\dots x\}-\mu^{\varrho}\{-\infty\dots x\},$$
$$\nu=\text{zero elsewhere.}$$

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- Bad news: $\nu(x,\,x)$ can be negative (e.g., Geom., Poi).

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Let's generalise

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- Good news: *Who cares?* No 2nd class particle there.
- Good news: $\nu(x+1,\, x) \geq 0$ (attractivity).

We can still use the *signed measure* ν formally, as we only care about $\nu(x+1,\, x)$. Scale this up to get the initial distribution at the site of the second class particle:

$$\mu(\omega_0,\, \eta_0) = \mu(\eta_0+1,\, \eta_0) = \frac{\nu(\eta_0+1,\, \eta_0)}{\sum_x \nu(x+1,\, x)} = \frac{\nu(\eta_0+1,\, \eta_0)}{\varrho-\lambda}.$$

Let's generalise

$$\mu(\omega_0,\, \eta_0) = \frac{\nu(\eta_0 + 1,\, \eta_0)}{\varrho - \lambda}$$

- is a proper probability distribution;
- actually agrees with the coupling measure ν conditioned on a 2nd class particle when ν behaves nicely (Bernoulli, discr.Gaussian);

Let's generalise

Theorem

Starting in

$$\bigotimes_{i<0} \mu_i^{\varrho} \otimes \mu_0 \otimes \bigotimes_{i>0} \mu_i^{\lambda},$$

$$\lim_{N\to\infty} \mathbf{P}\Big\{\frac{Q(NT)}{N} > X\Big\} = \frac{\varrho(X,\, T) - \lambda}{\varrho - \lambda}$$

where $\varrho(X,\, T)$ is the entropy solution of the hydrodynamic equation with initial data

$$\begin{array}{ll} \varrho & \textit{on the left} \\ \lambda & \textit{on the right.} \end{array}$$

What do we have?

$$\lim_{N\to\infty} \mathbf{P}\left\{\frac{Q(NT)}{N} > X\right\} = \frac{\varrho(X,\, T) - \lambda}{\varrho - \lambda}$$

$\rightsquigarrow$ The solution $\varrho(X,\, T)$ is the distribution of the velocity for Q.

- Shock: distribution is step function, velocity is deterministic (LLN).
- Rarefaction wave: distribution is continuous, velocity is random (e.g., Uniform for TASEP).

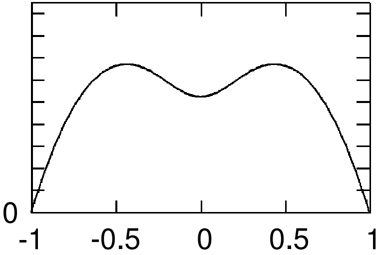
A fun model (B., A.L. Nagy, I. Tóth, B. Tóth)

$\omega_i = -1, 0, 1;$

$$
\begin{aligned}
(0, -1) &\to (-1, 0) &\qquad \text{with rate } \frac{1}{2},\\
(1, 0) &\to (0, 1) &\qquad \text{with rate } \frac{1}{2},\\
(1, -1) &\to (0, 0) &\qquad \text{with rate } 1,\\
(0, 0) &\to (-1, 1) &\qquad \text{with rate } c.
\end{aligned}
$$

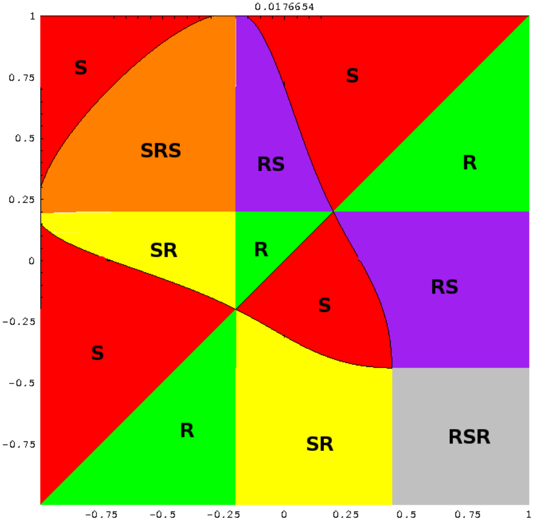
A fun model (B., A.L. Nagy, I. Tóth, B. Tóth)

Hydrodynamic flux $H(\varrho)$, for certain c:



A fun model (B., A.L. Nagy, I. Tóth, B. Tóth)

Here is what can happen (**R**: rarefaction wave, **S**: Shock):



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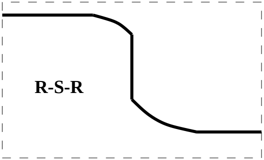
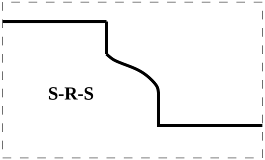
Examples for $\varrho(T, X)$:





$$\lim_{N\to\infty} \mathbf{P}\left\{\frac{Q(NT)}{N} > X\right\} = \frac{\varrho(X,\ T) - \lambda}{\varrho - \lambda}$$

$\rightsquigarrow$ The solution $\varrho(X,\ T)$ is the distribution of the velocity for Q.

I haven't seen a walk with a random velocity of *mixed distribution* before.

Storytelling. . .

$$\mu(\omega_0,\,\eta_0)=\frac{\nu(\eta_0+1,\,\eta_0)}{\varrho-\lambda}$$

In the 1/3-fluctuations papers (B., J. Komjáthy, T. Seppäläinen) we had to start the second class particle in a $\varrho=\lambda$ flat environment. We came up with a measure $\hat\mu$ for this which worked nicely with our formulas. *But at that time we had no idea why.*

As it turns out: $\hat\mu=\lim\limits_{\lambda\nearrow\varrho}\mu$.

Symmetric case

Everything works with partially asymmetric models (allow left jumps too).

In fact everything works for symmetric models as well. The hydrodynamic scaling is diffusive there with the limit being of heat equation type. In this case:

Symmetric case

Theorem (Symmetric version)

Starting in

$$\bigotimes_{i<0} \mu_i^{\varrho} \otimes \mu_0 \otimes \bigotimes_{i>0} \mu_i^{\lambda},$$

$$\lim_{N\to\infty} \mathbf{P}\Big\{\frac{Q(NT)}{\sqrt{N}} > X\Big\} = \frac{\varrho(X,\ T) - \lambda}{\varrho - \lambda}$$

where $\varrho(X,\ T)$ is the entropy solution of the hydrodynamic equation with initial data

$$\begin{array}{ll} \varrho & \textit{on the left} \\ \lambda & \textit{on the right.} \end{array}$$

SSEP: CLT (of course...). Other models: interesting!

One more result

Theorem

If μ^ϱ are the stationary product marginals then, under our initial distribution, $\eta_{Q(t)}(t)$ is stationary in time.

Proof.

Repeat the argument with $\mathbf{E}\Phi\big(\eta_i(t)\big)$ instead of $\mathbf{E}g_i(t)$. □

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This was not even a question with exclusion.

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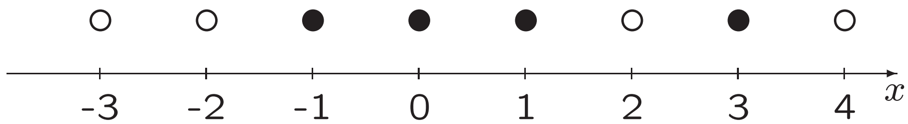
This was not even a question with exclusion.

Only the site $Q(t)$!

TASEP and the corner growth model

TASEP: Interacting particles



Bernoulli(ϱ) distribution

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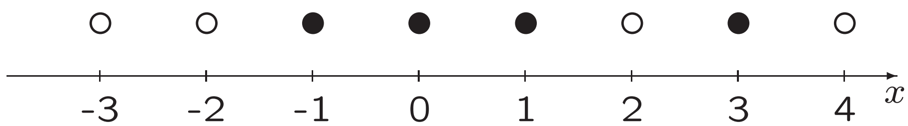
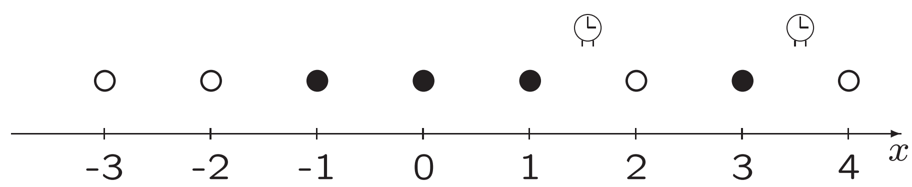


Bernoulli(ϱ) distribution

(particle, hole) pairs become
(hole, particle) pairs with rate 1.

TASEP: Interacting particles



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That is: waiting times ⏲ $\sim$ Exponential(1).

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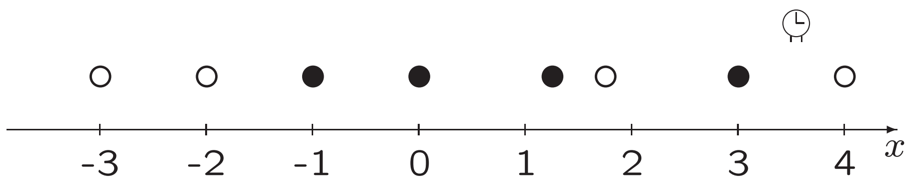

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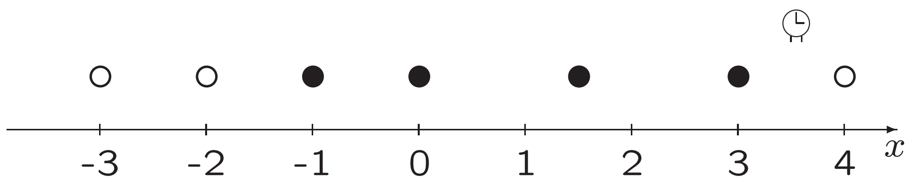

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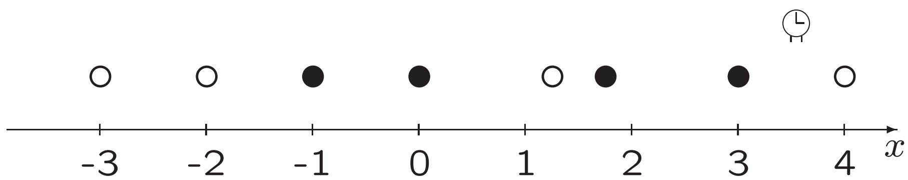

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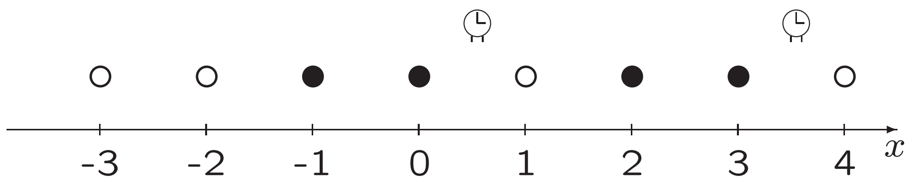

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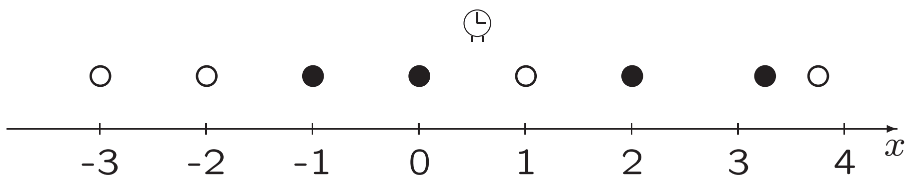

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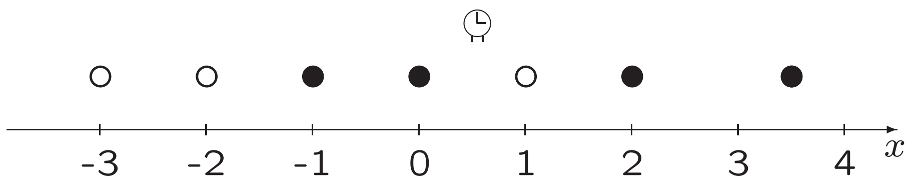

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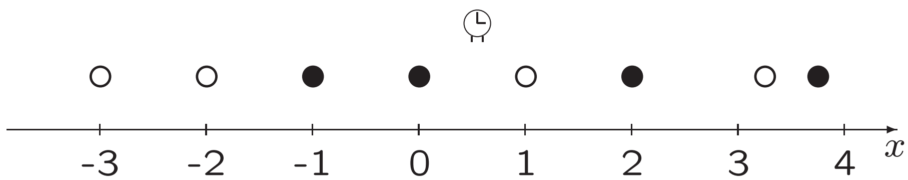

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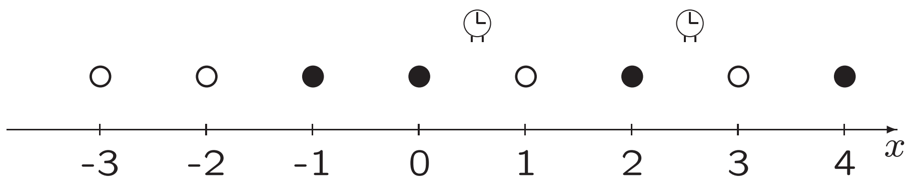

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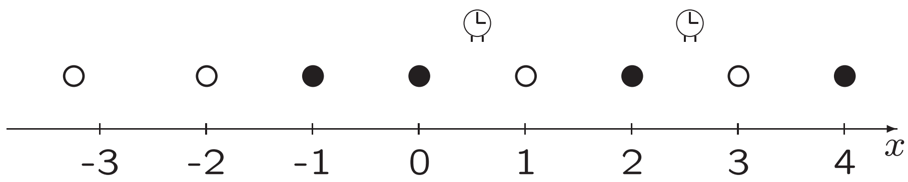

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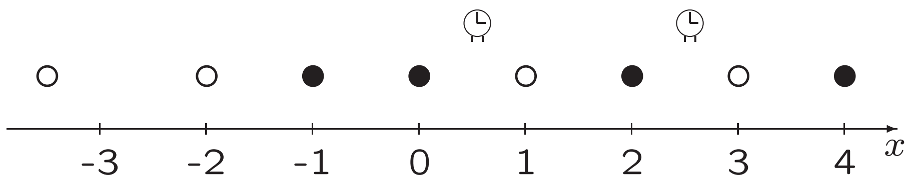

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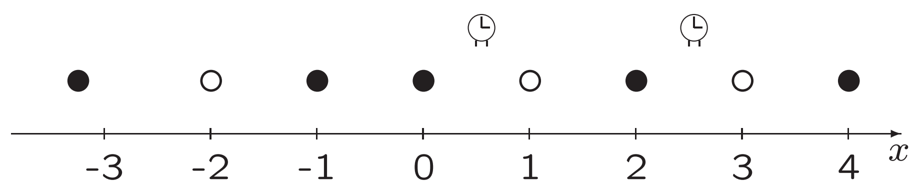

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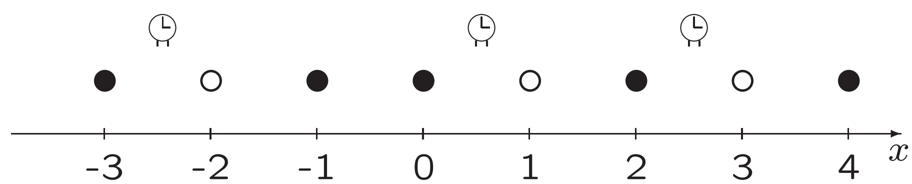

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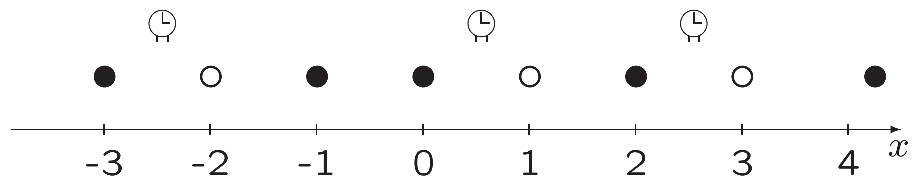

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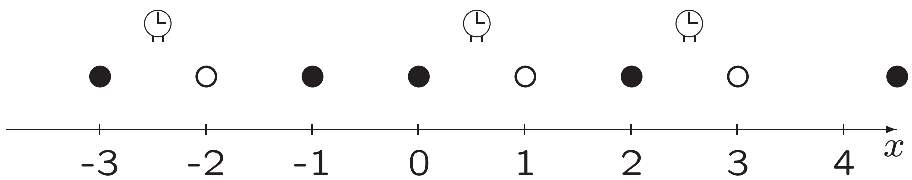

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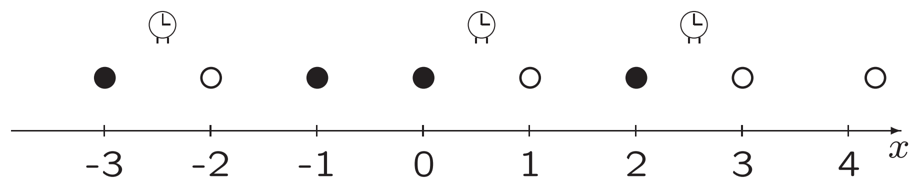

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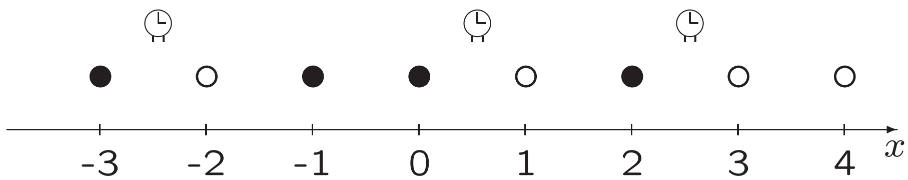

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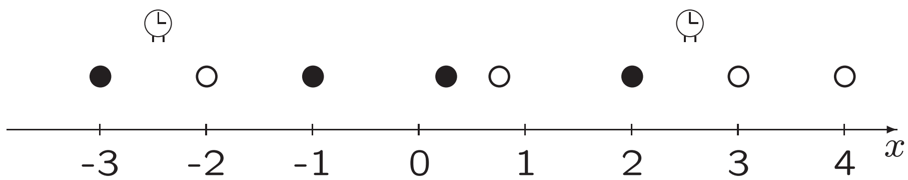

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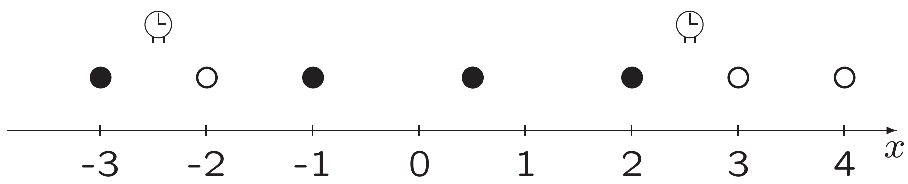

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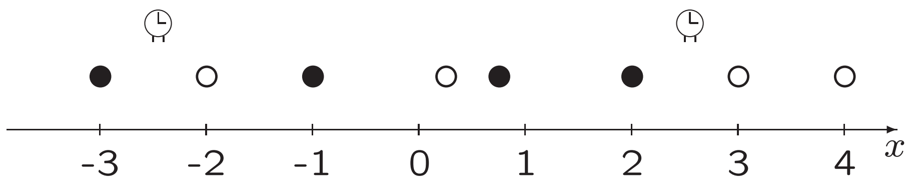

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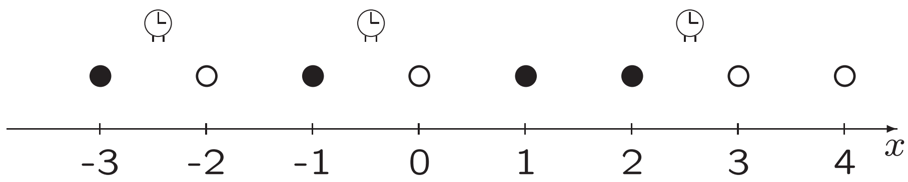
TASEP: Interacting particles



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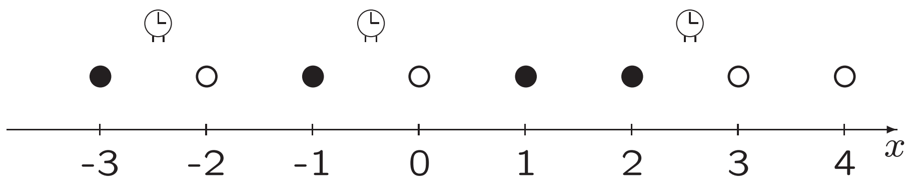
TASEP: Interacting particles

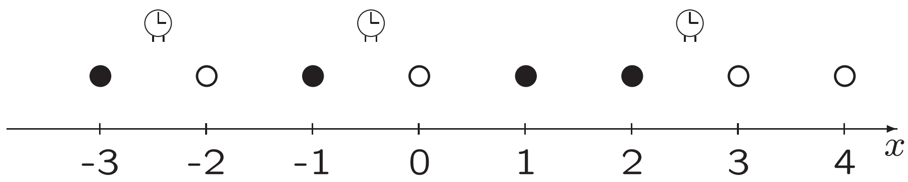


Bernoulli(ϱ) distribution

(particle, hole) pairs become
(hole, particle) pairs with rate 1.
That is: waiting times ⏲ $\sim$ Exponential(1).
$\rightsquigarrow$ Markov process.

Particles try to jump to the right, but block each other.

TASEP: Interacting particles



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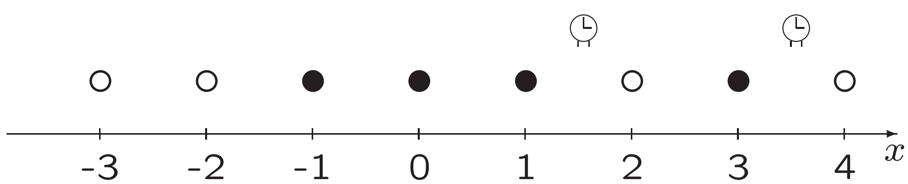
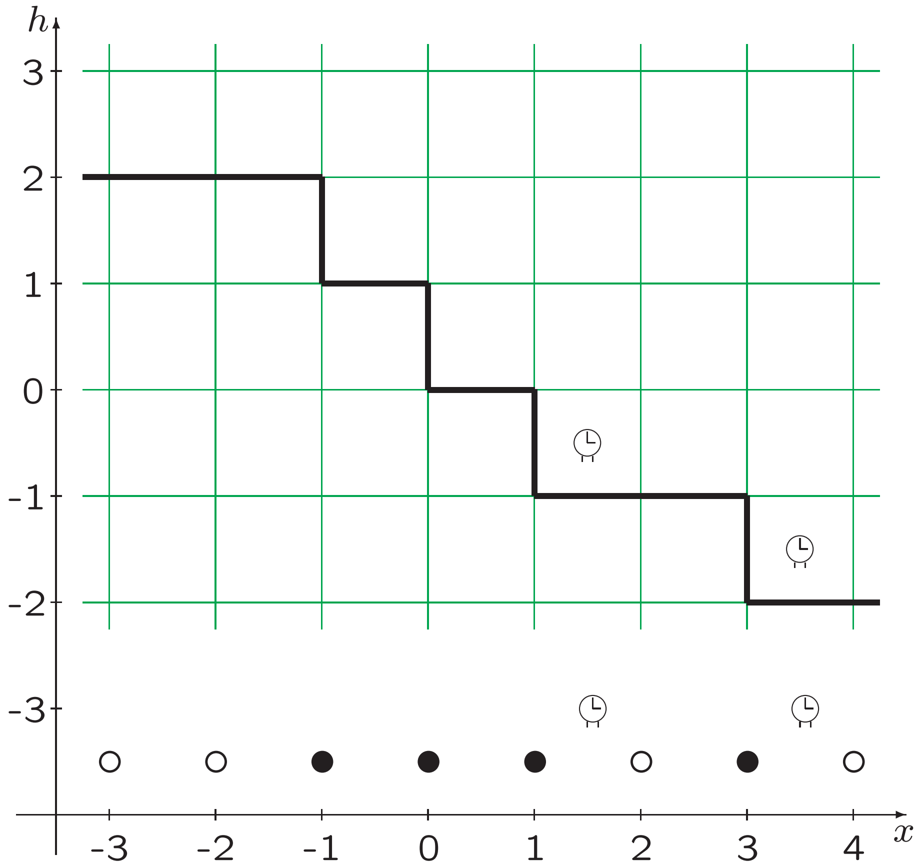
The Bernoulli(ϱ) distribution is time-stationary for any ($0 \le \varrho \le 1$). Any translation-invariant stationary distribution is a mixture of Bernoullis.

TASEP: Surface growth



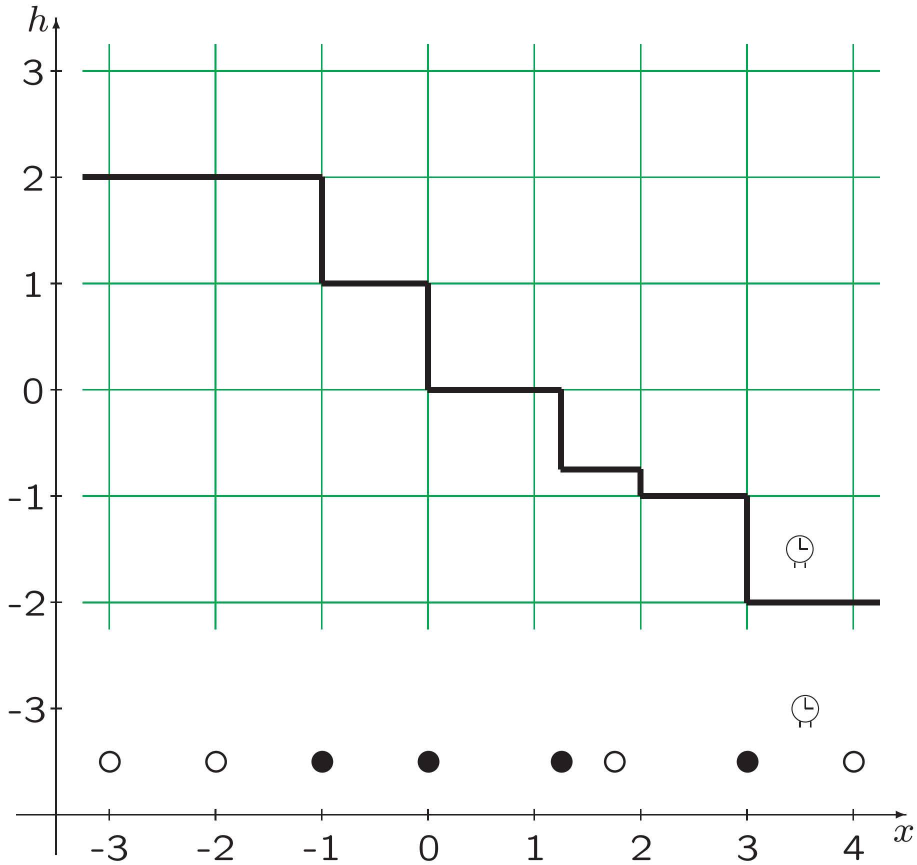
Bernoulli(ϱ) distribution

TASEP: Surface growth



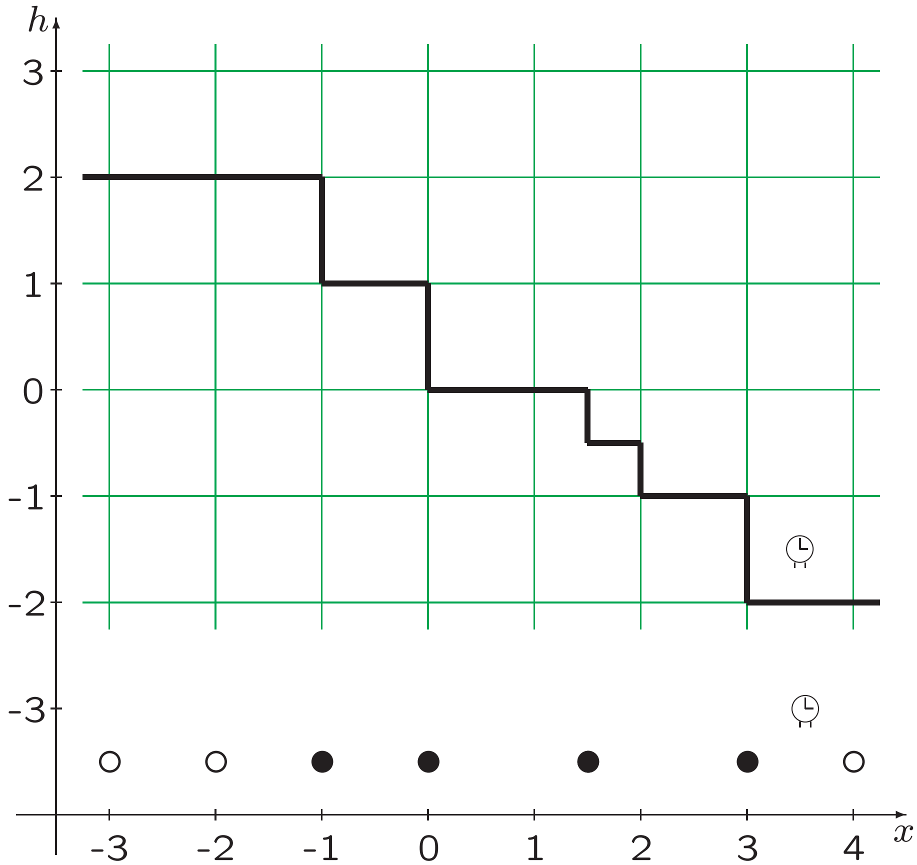
Bernoulli(ϱ) distribution

TASEP: Surface growth



Bernoulli(ϱ) distribution

TASEP: Surface growth



Bernoulli(ϱ) distribution

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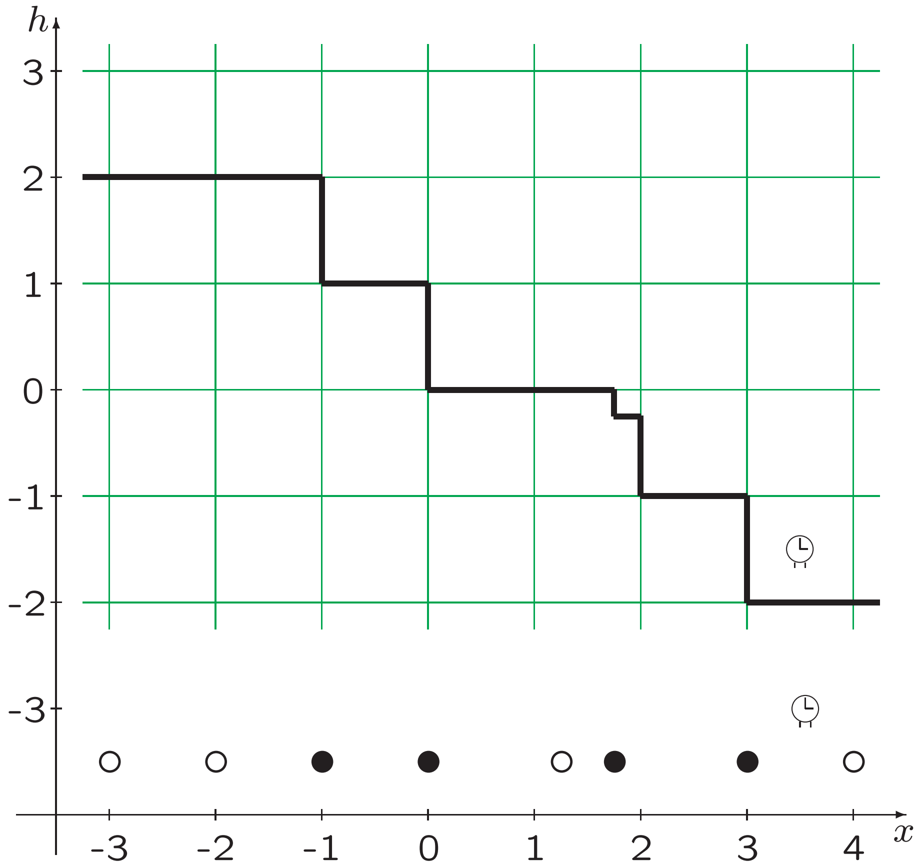

Bernoulli(ϱ) distribution

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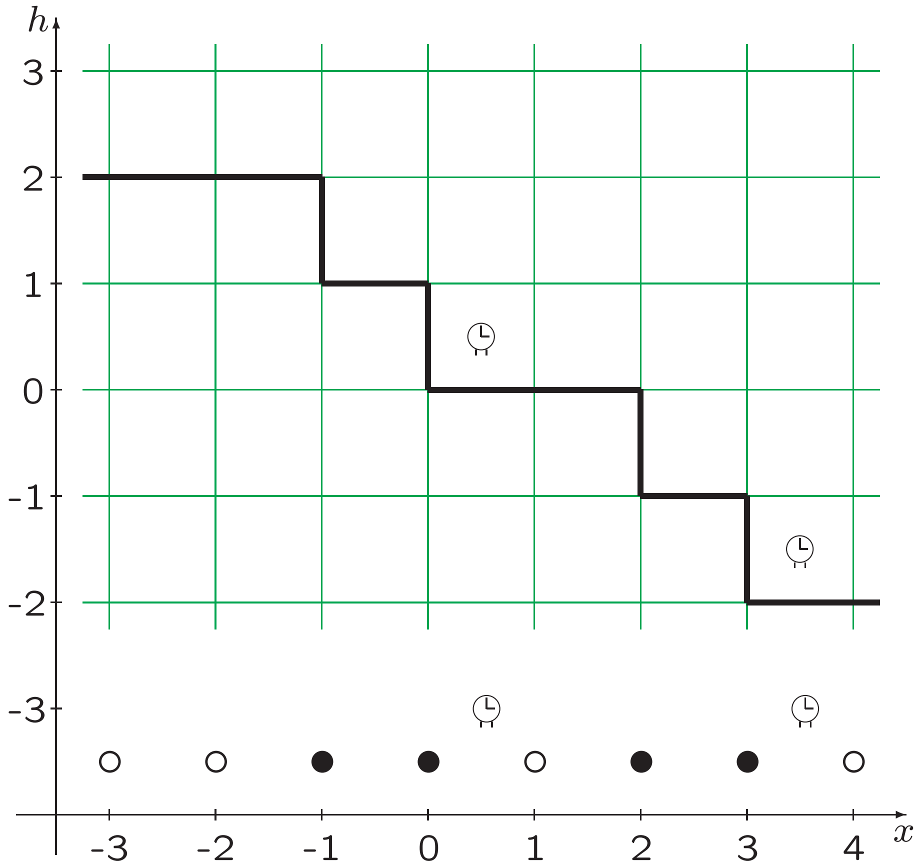

Bernoulli(ϱ) distribution

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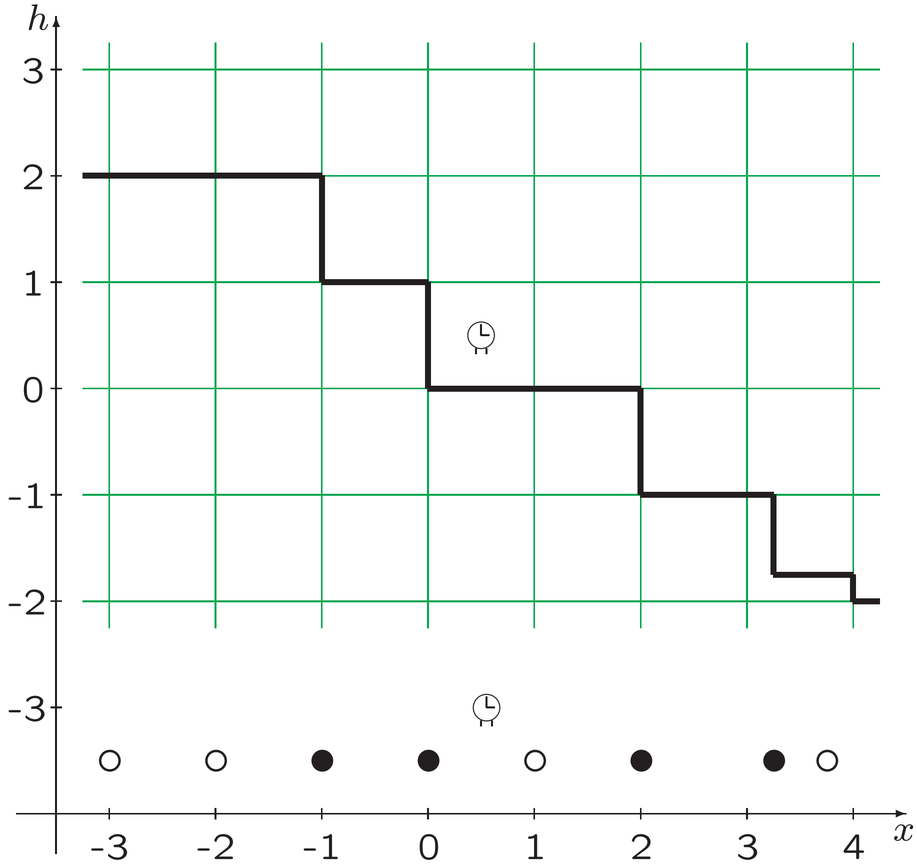

Bernoulli(ϱ) distribution

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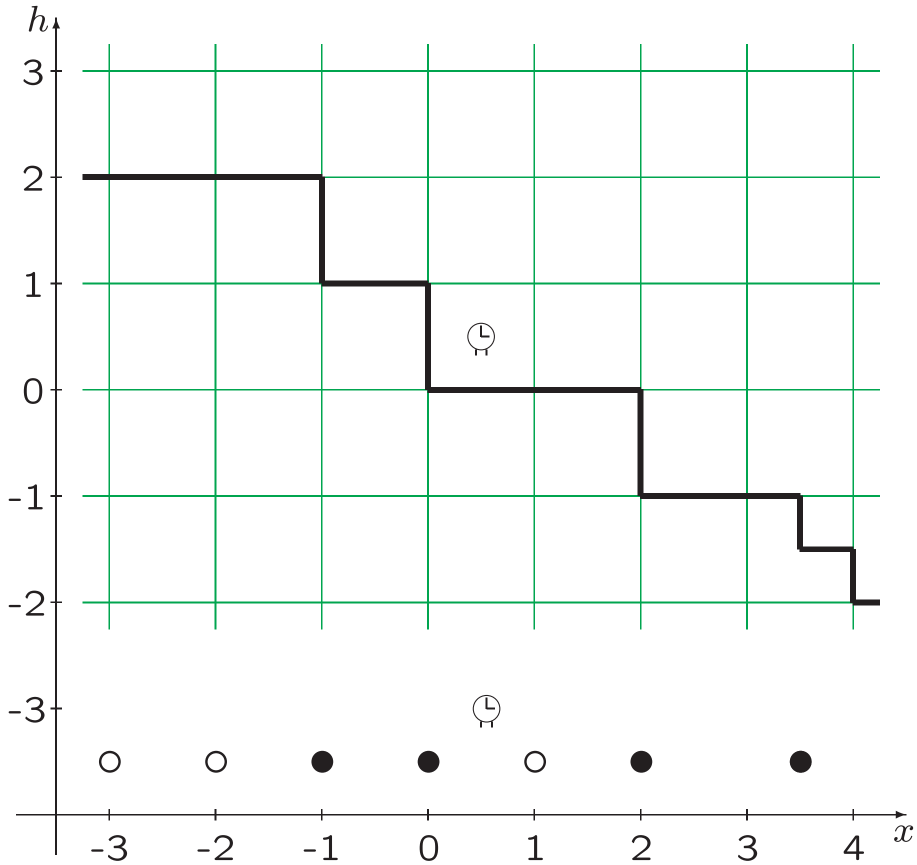

Bernoulli(ϱ) distribution

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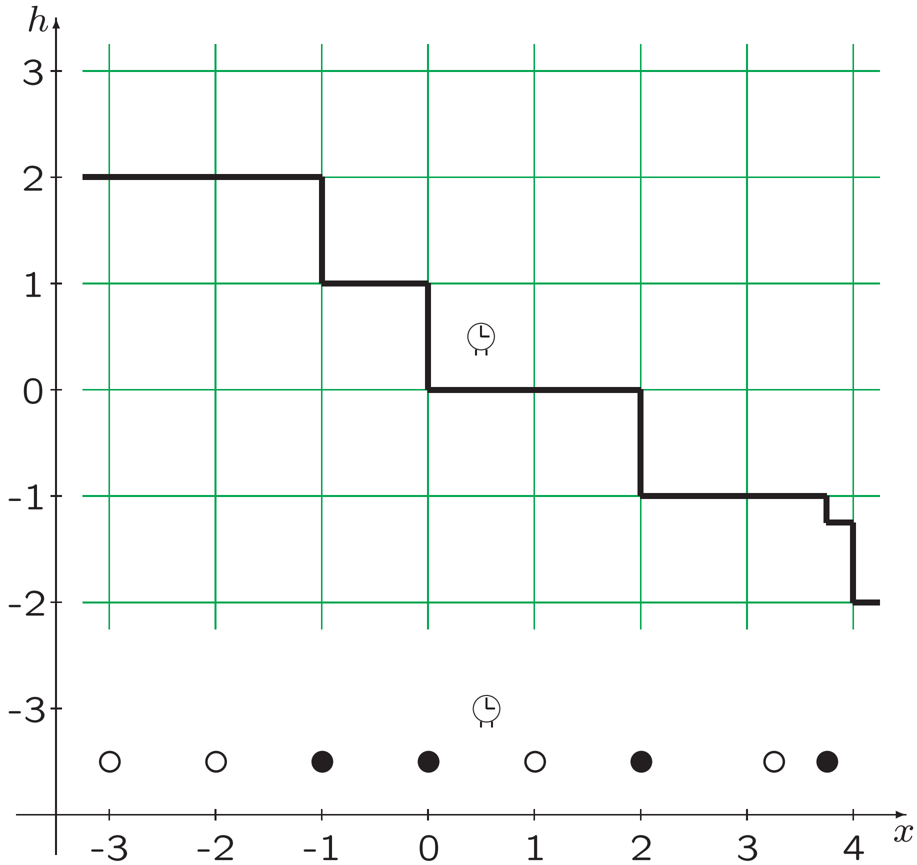

Bernoulli(ϱ) distribution

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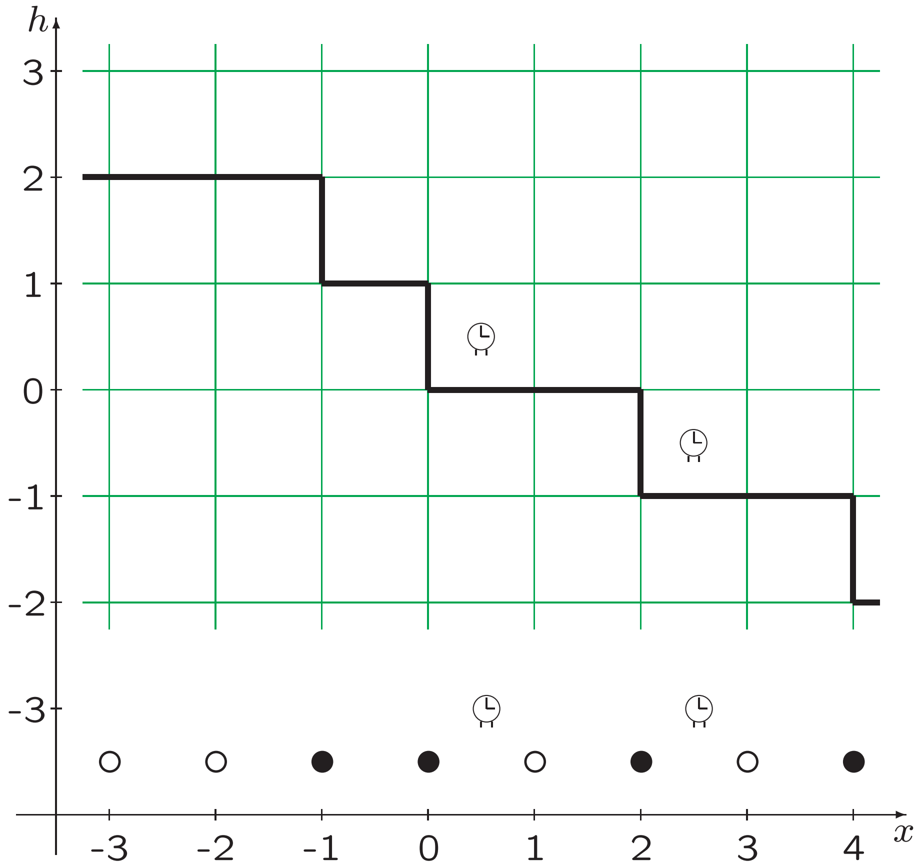

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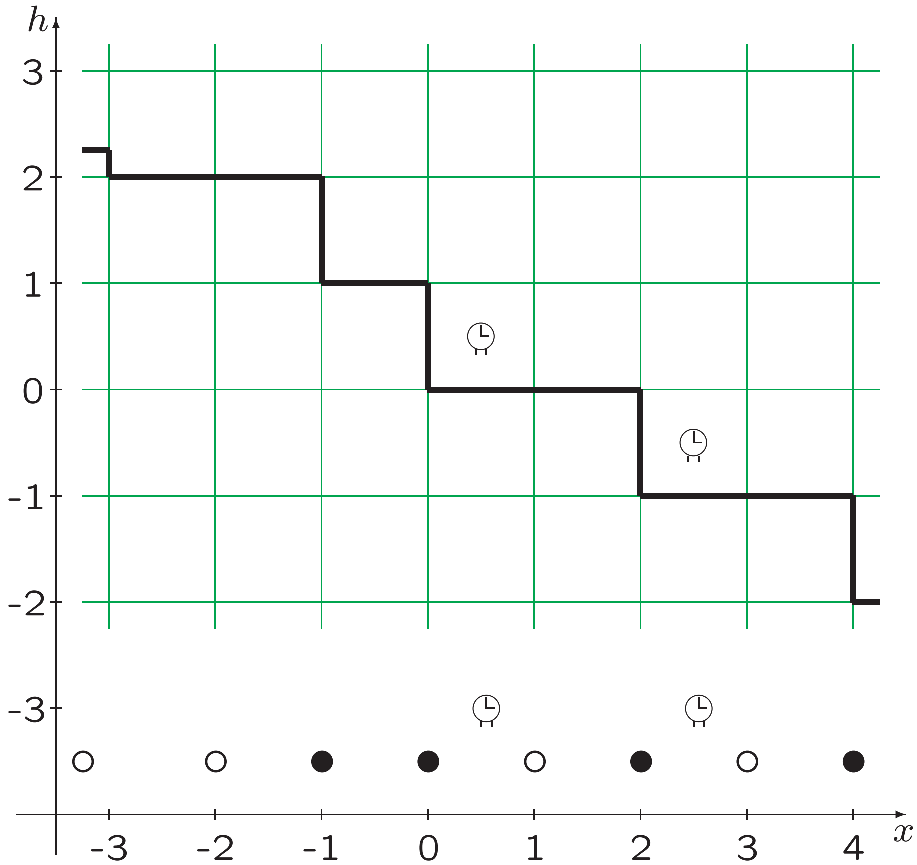

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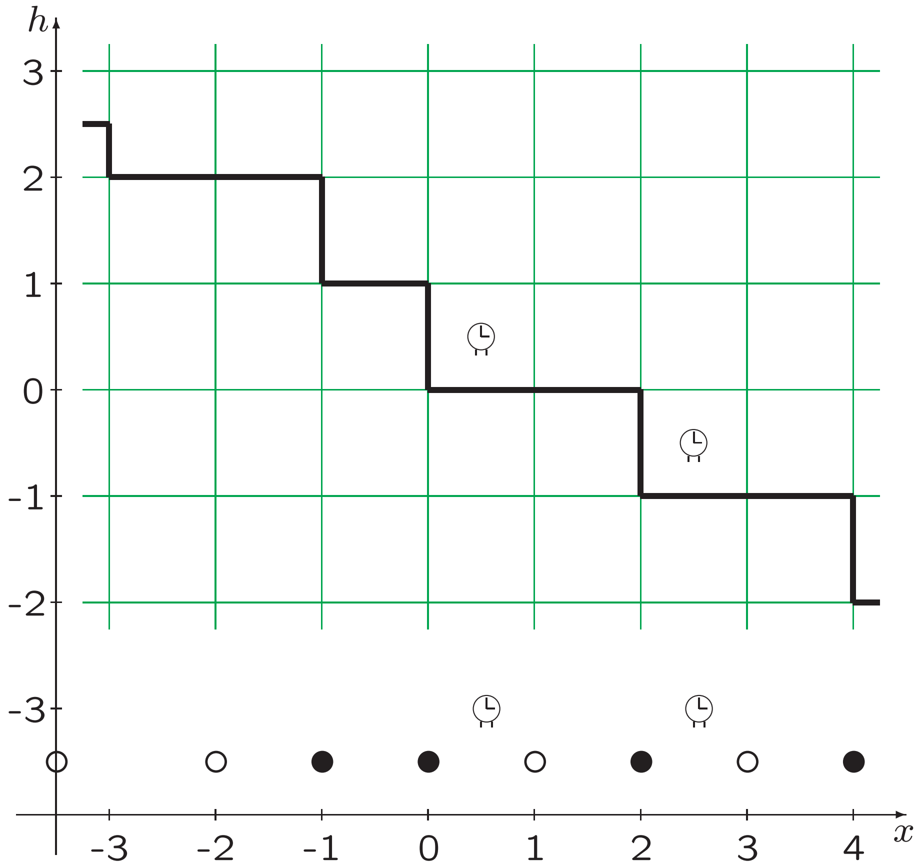

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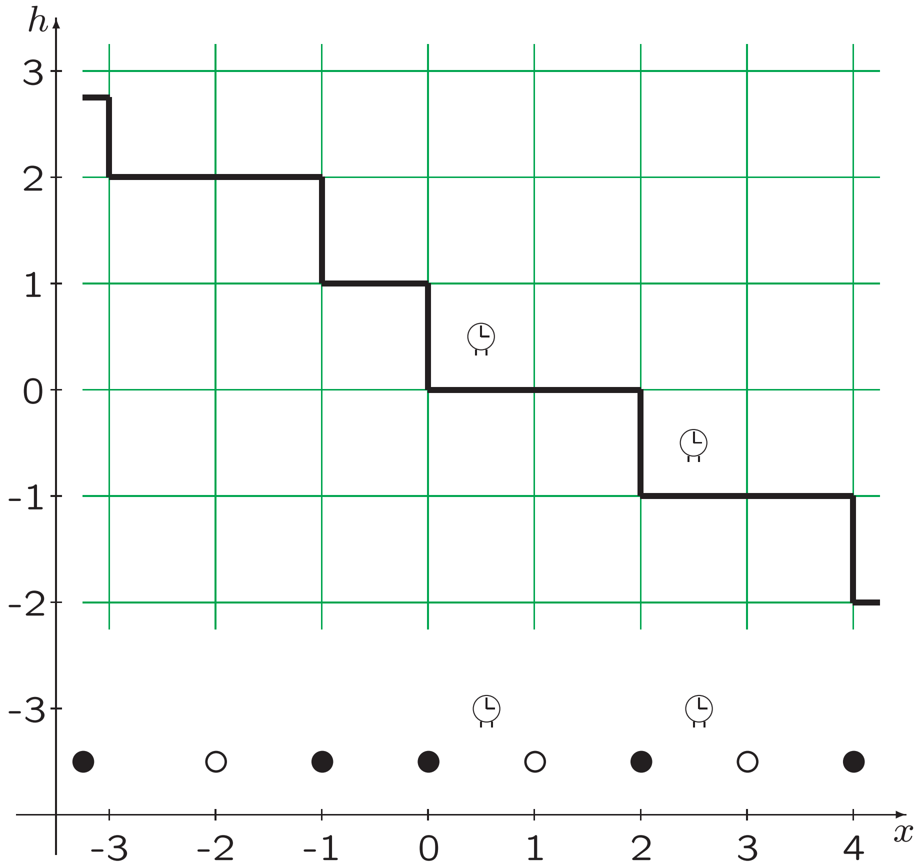

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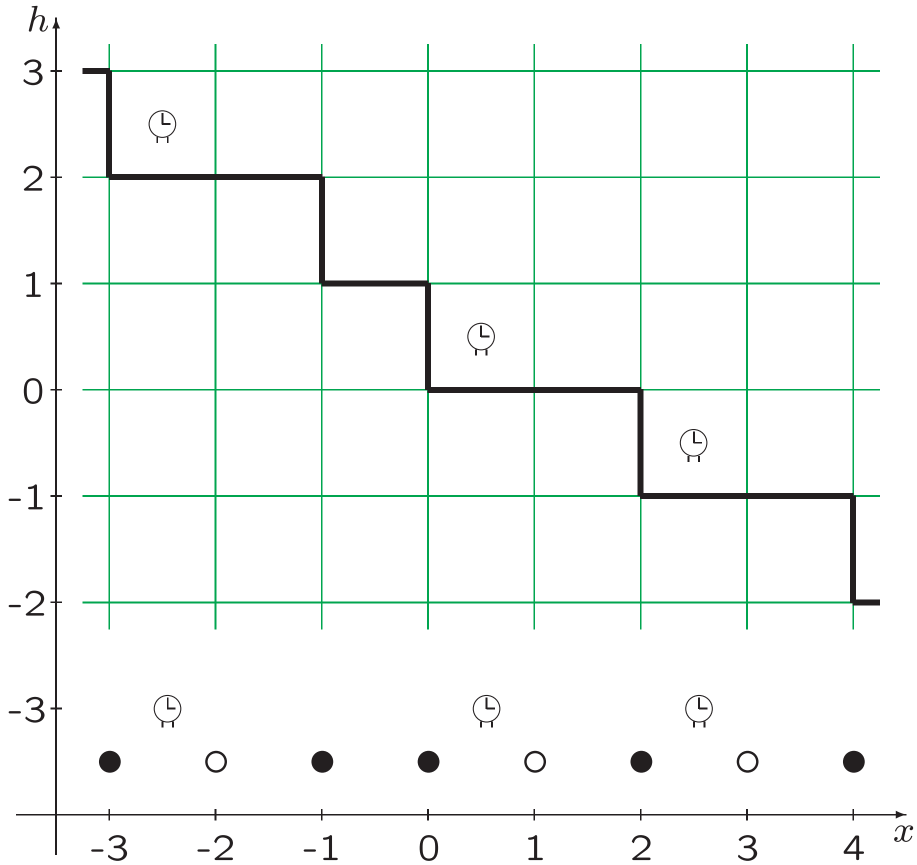

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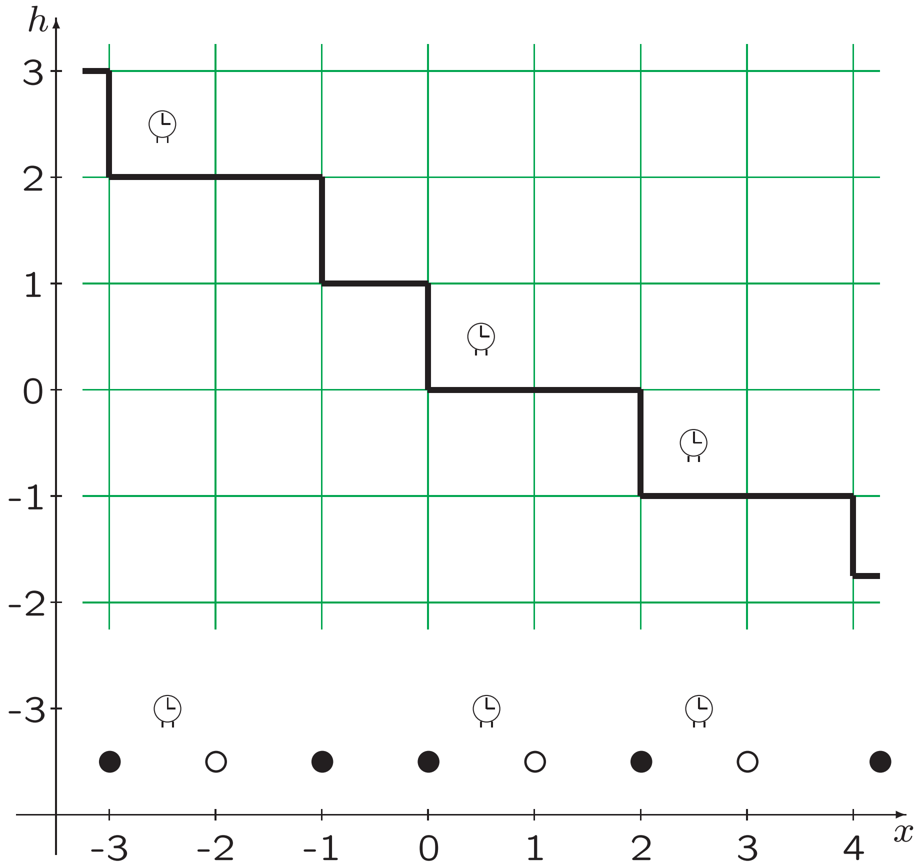

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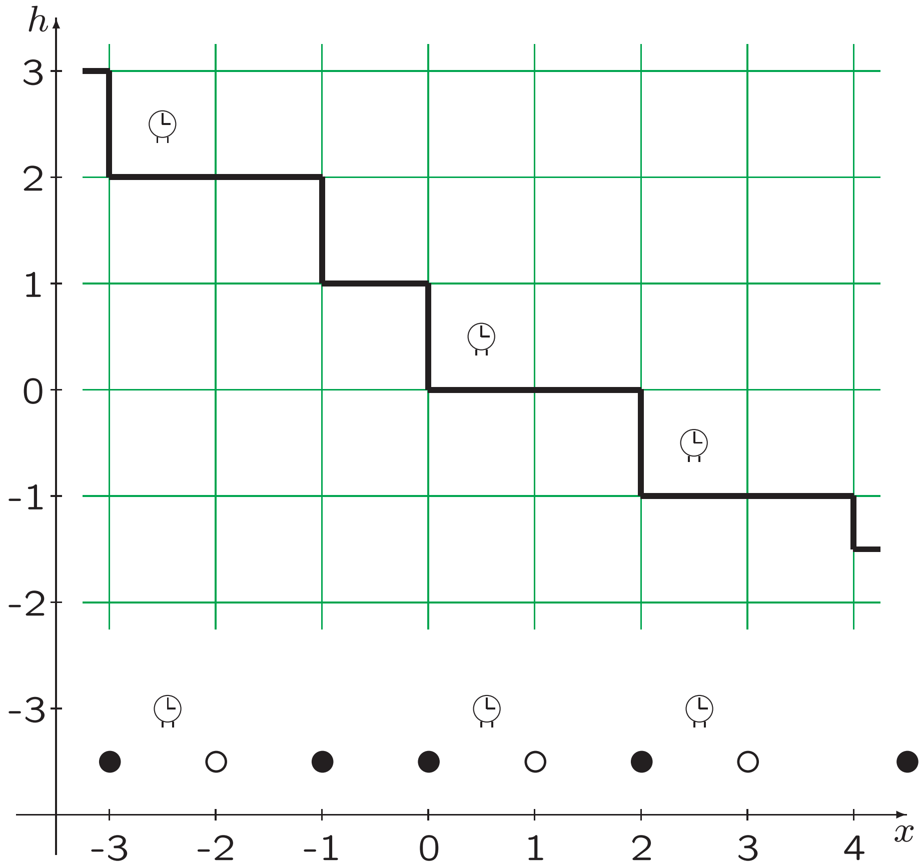

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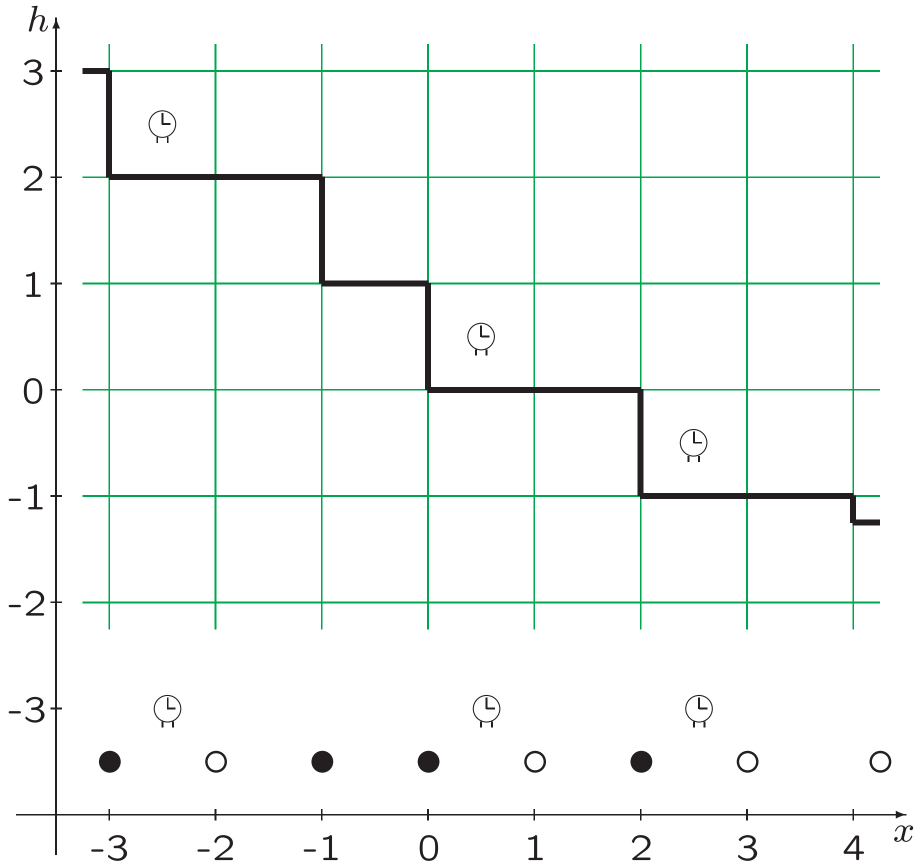

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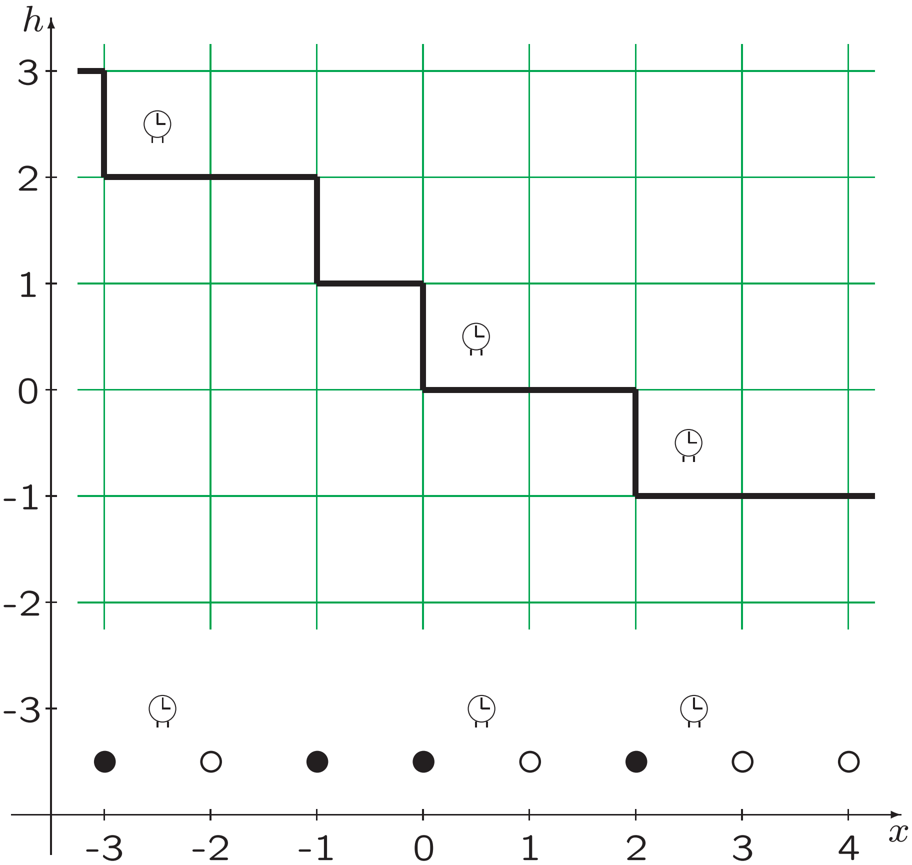

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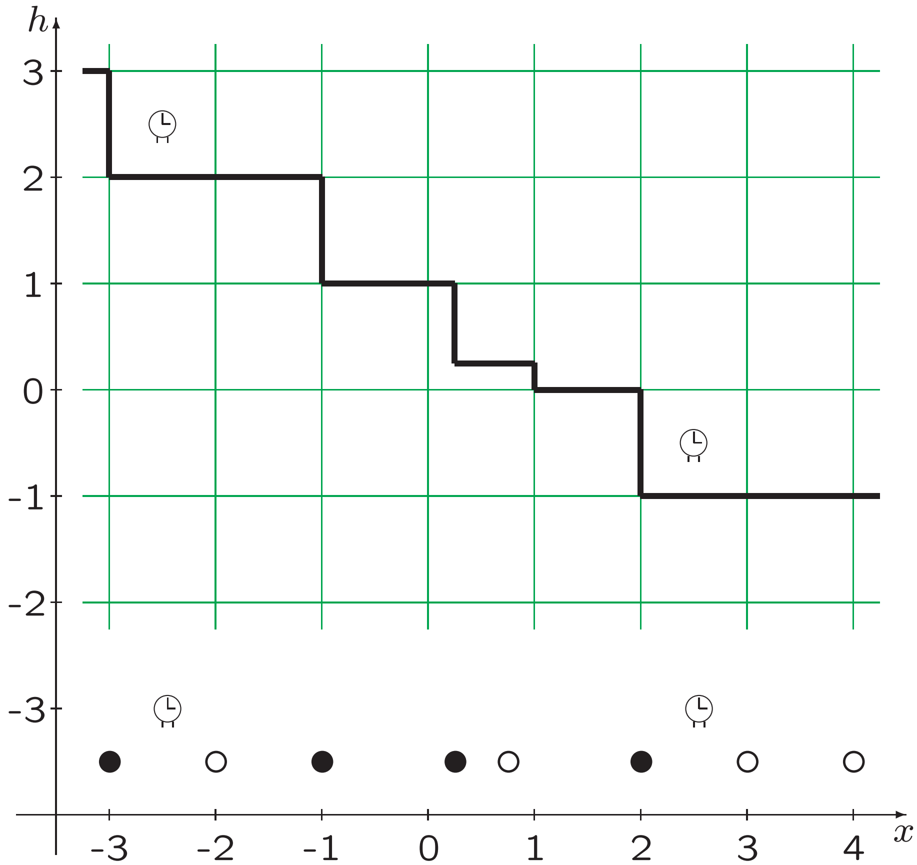

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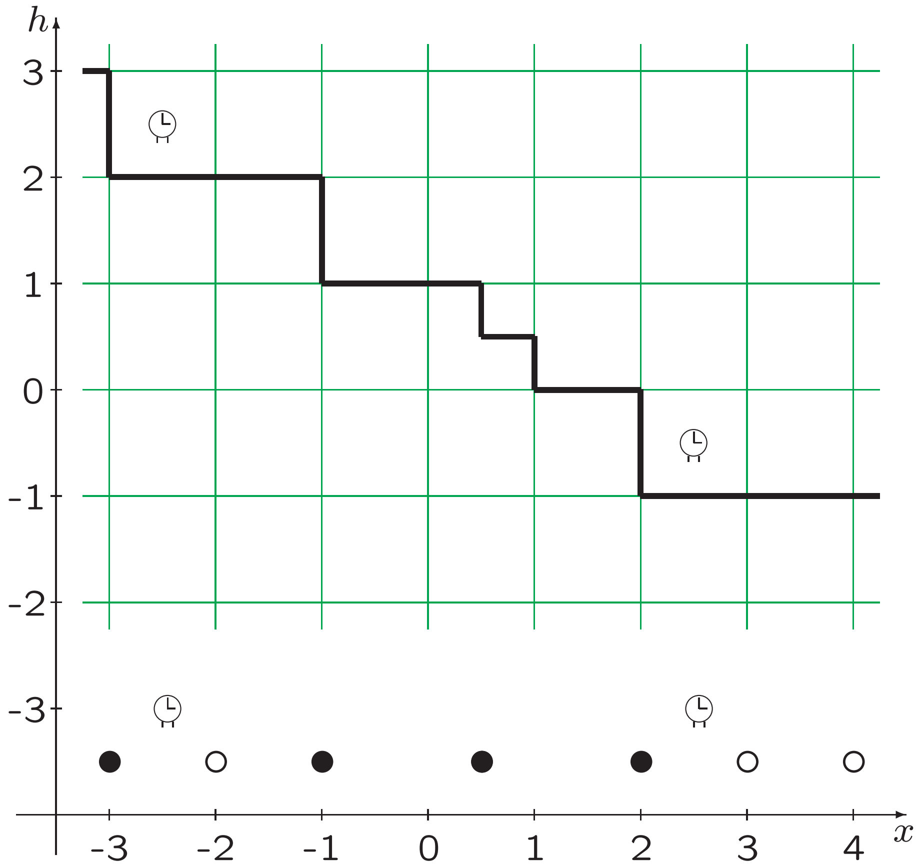

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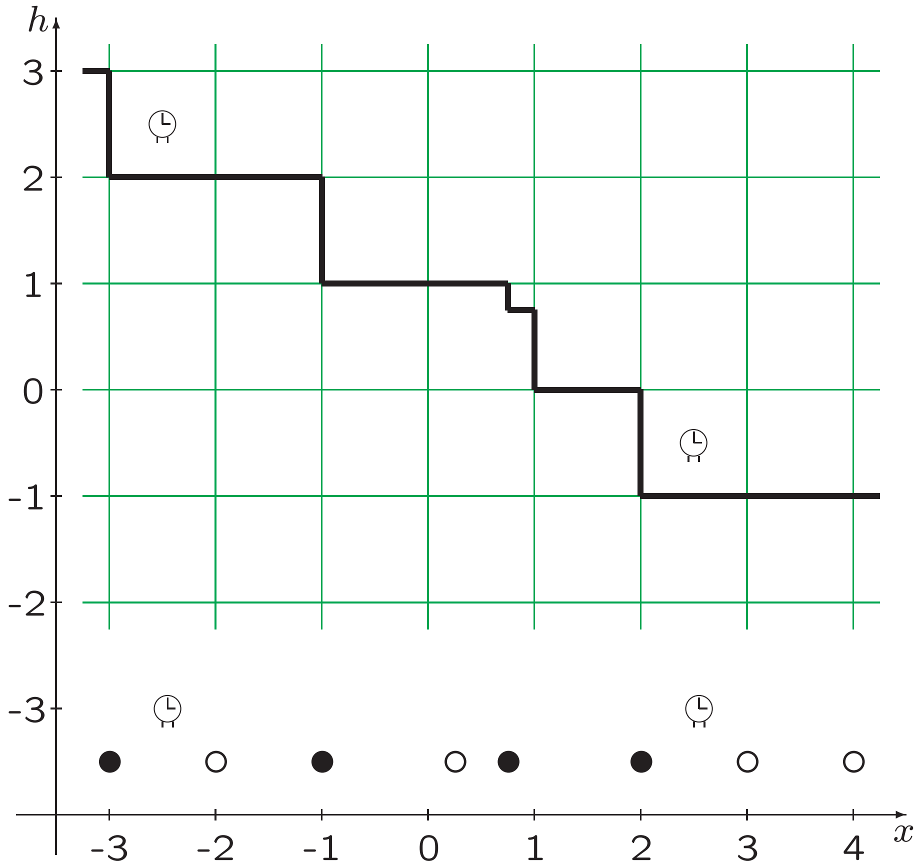

Bernoulli(ϱ) distribution

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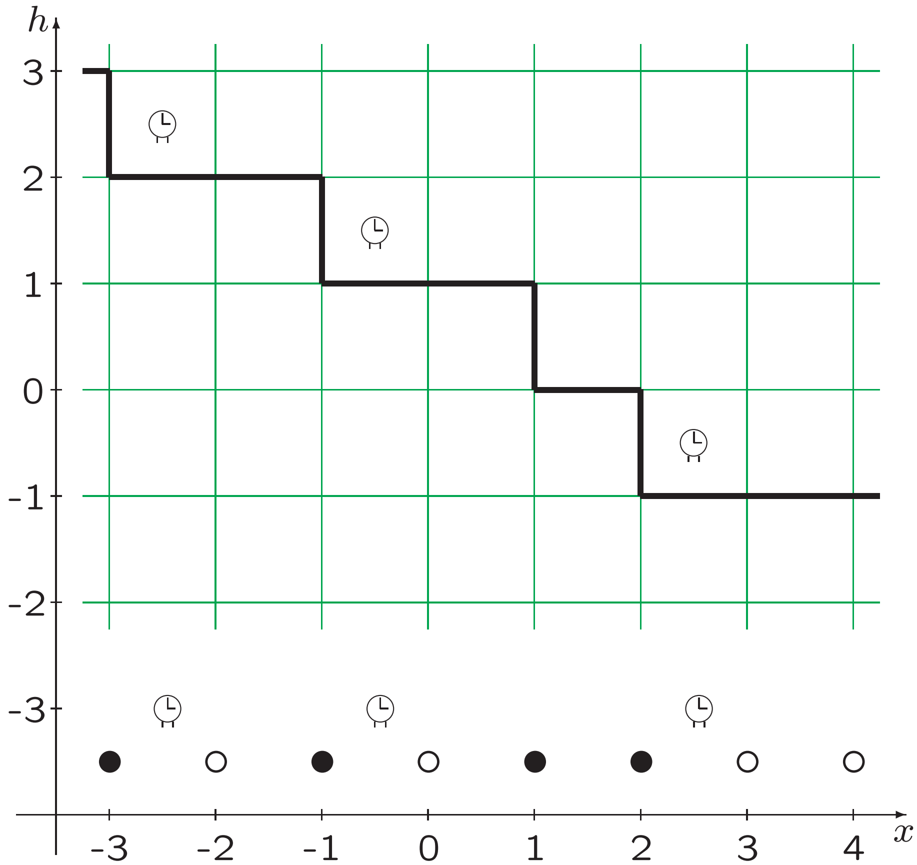
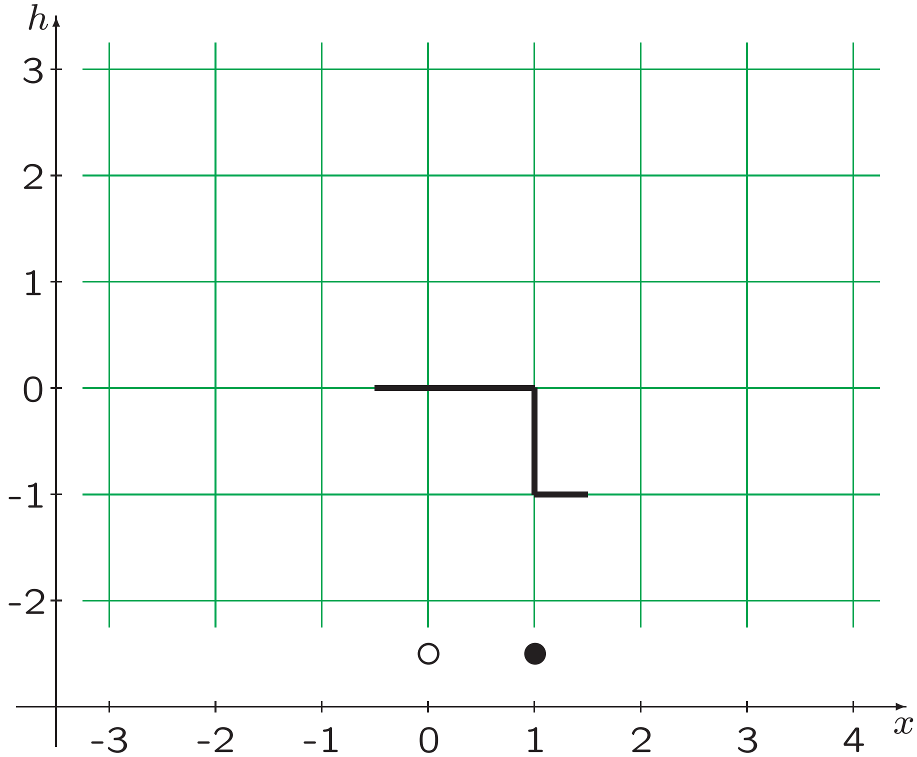


Bernoulli(ϱ) distribution

TASEP: Last passage percolation



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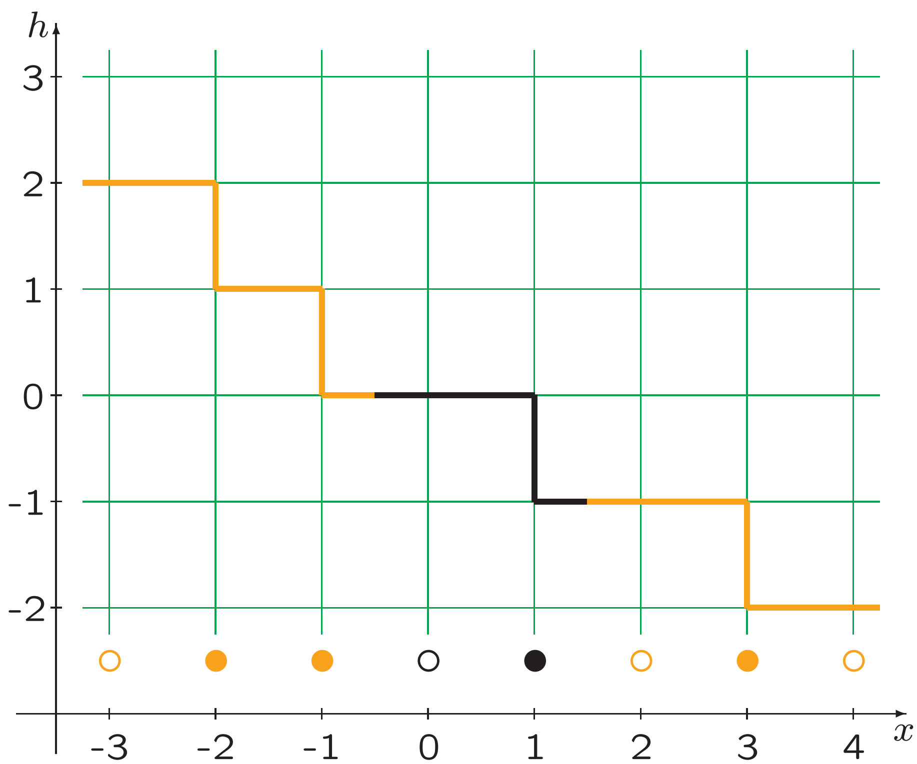

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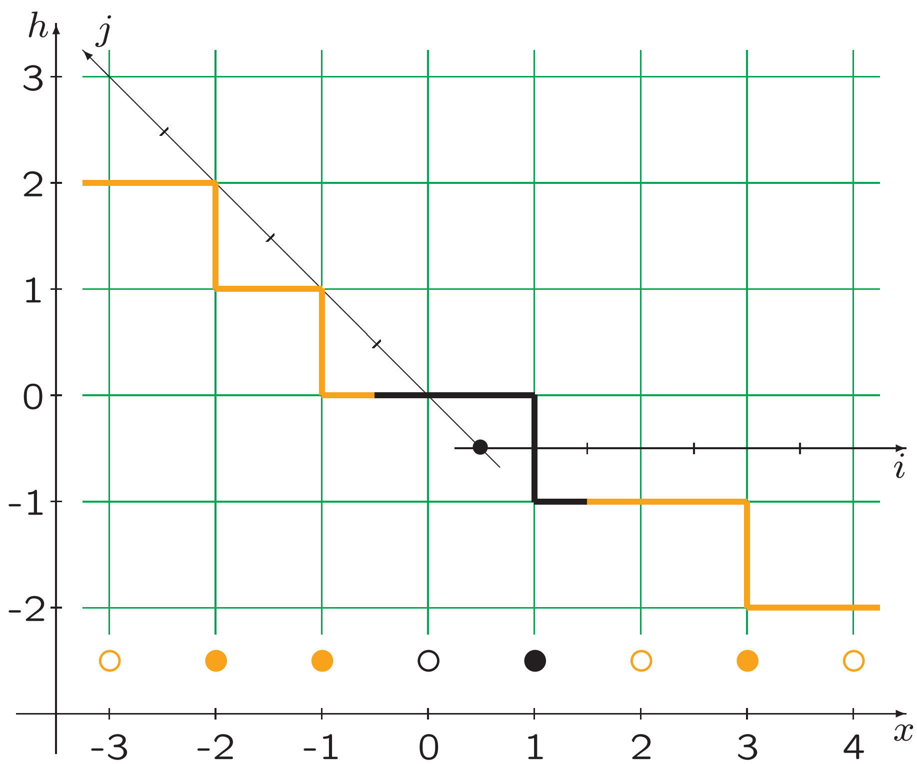

Bernoulli(ϱ) distribution

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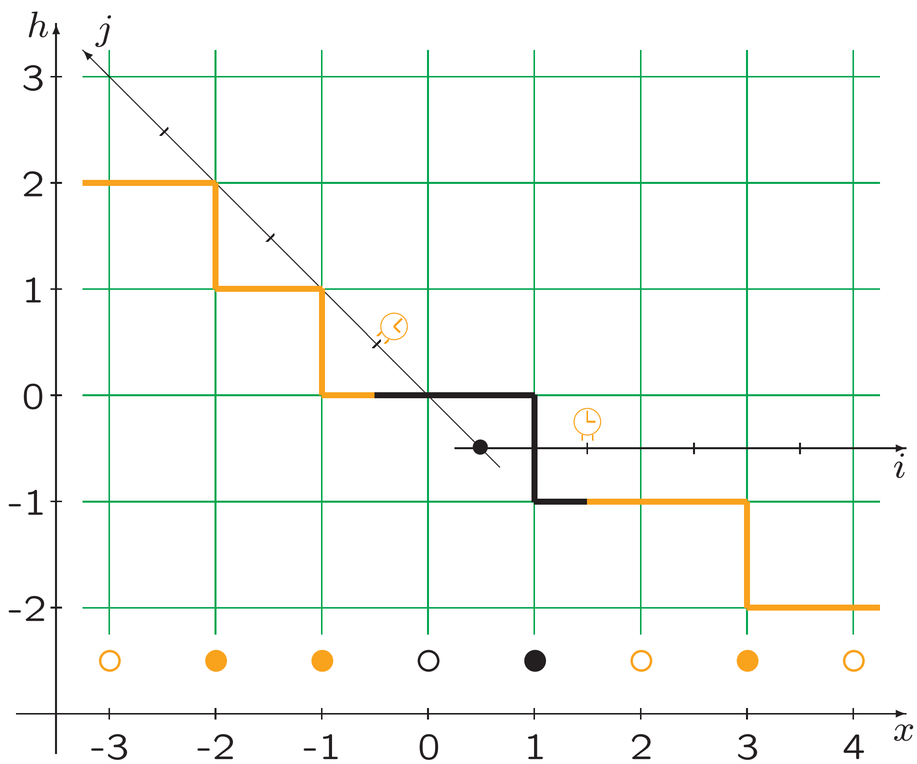

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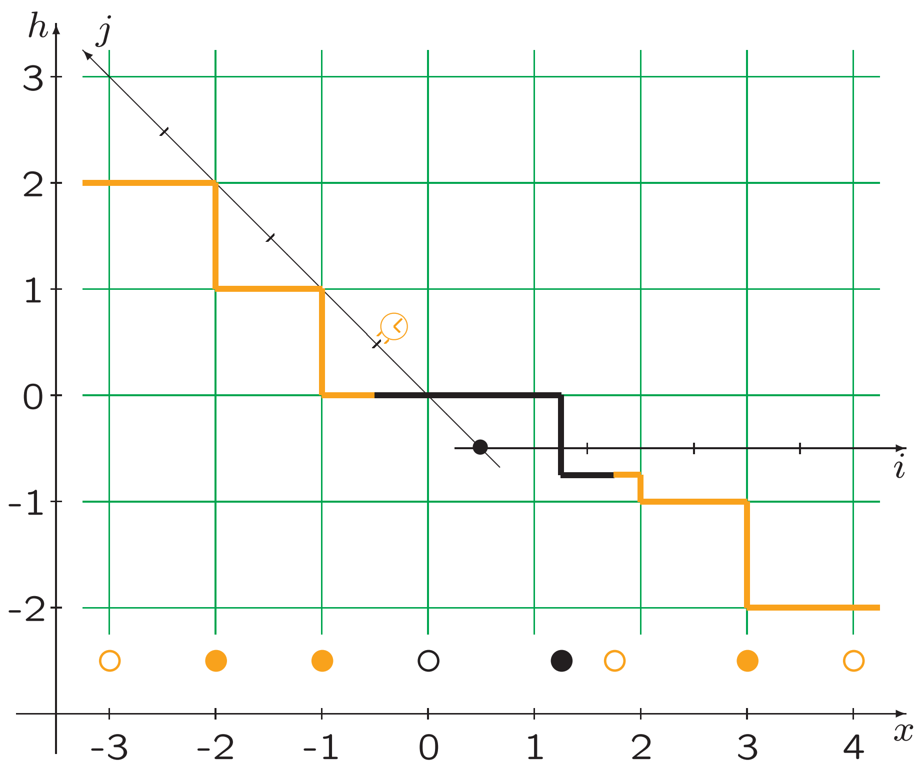

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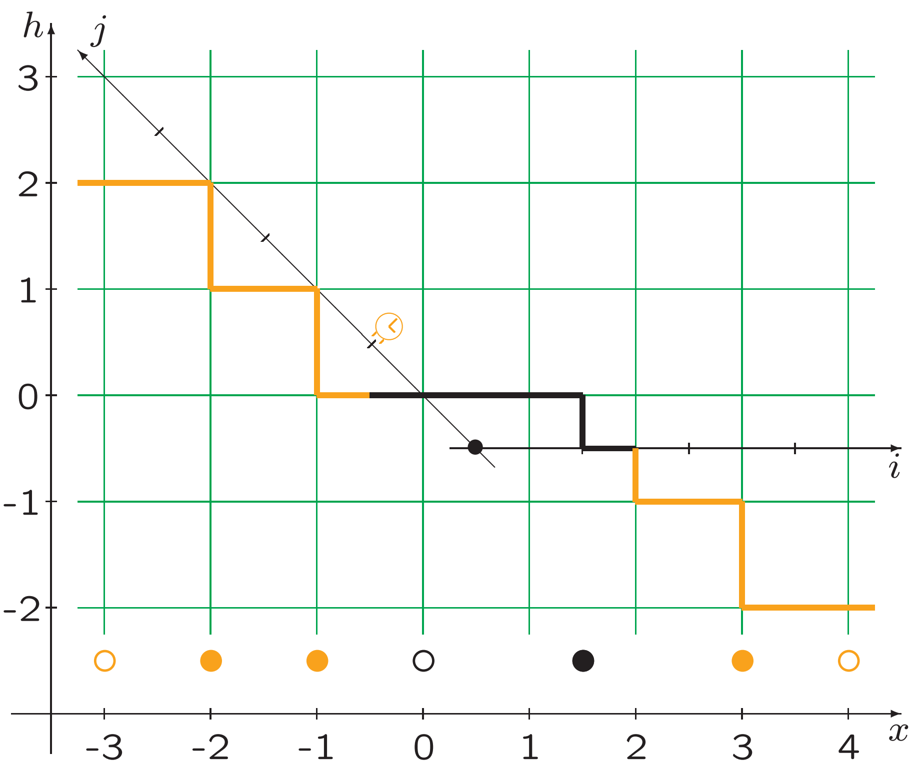

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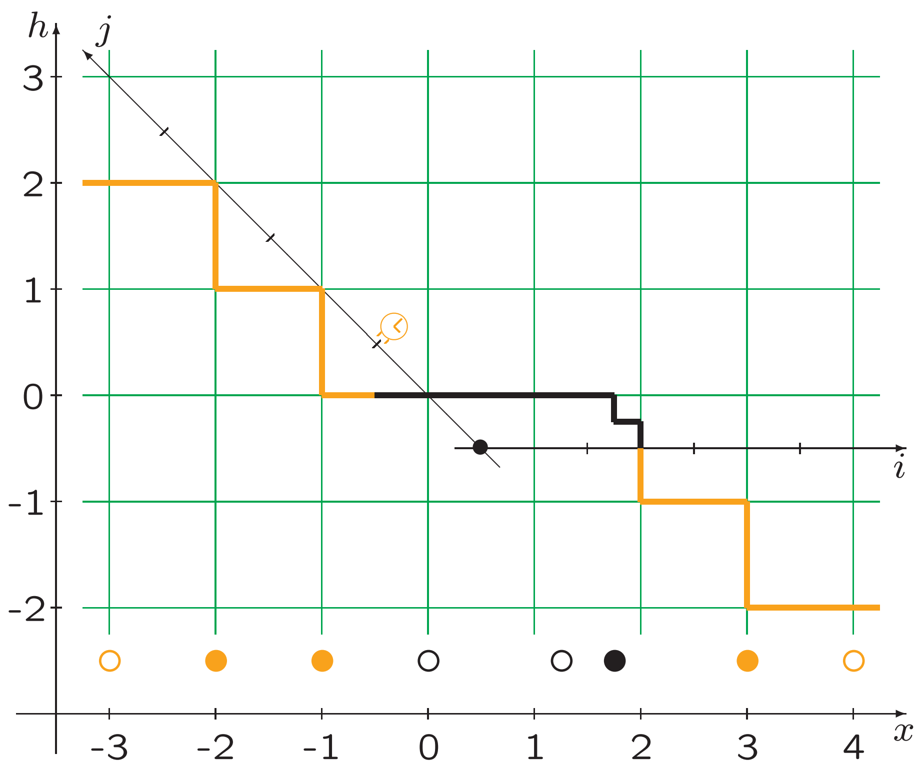

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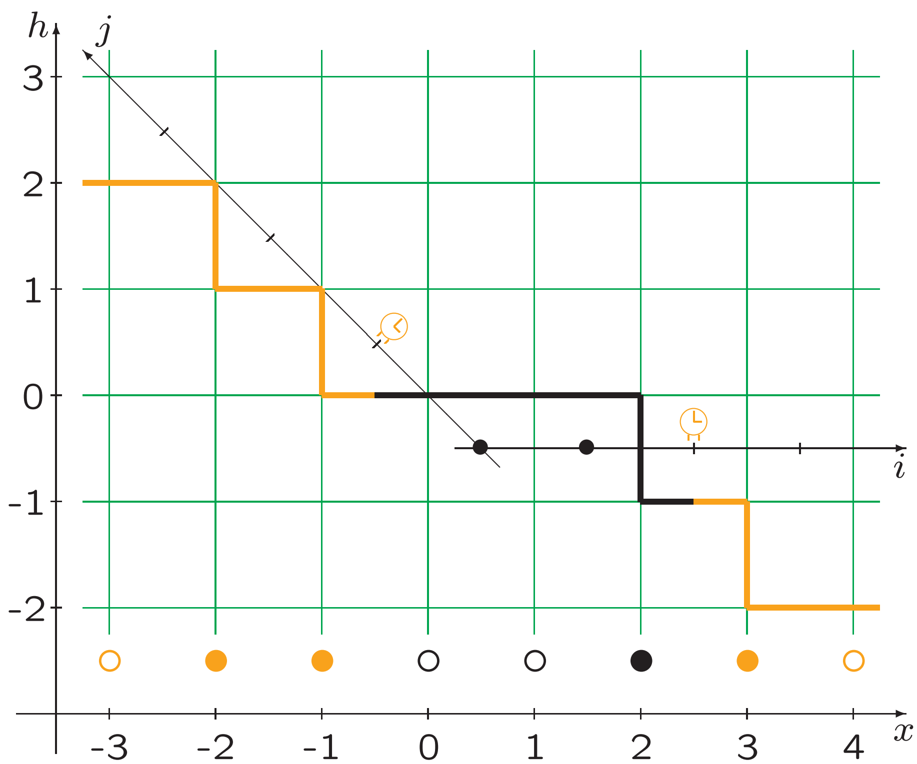

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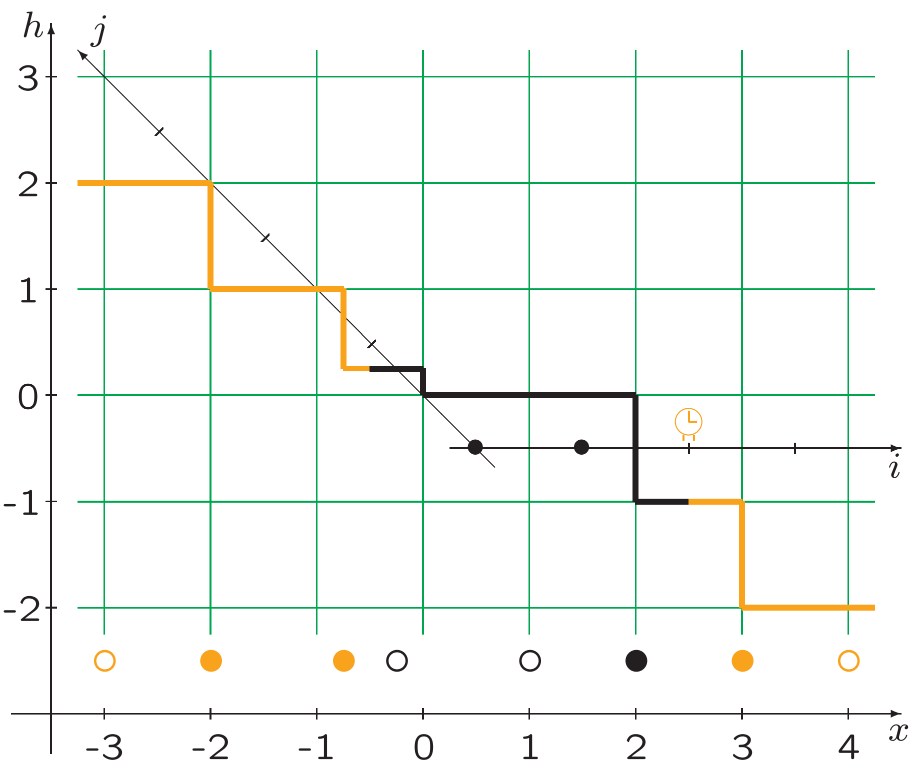

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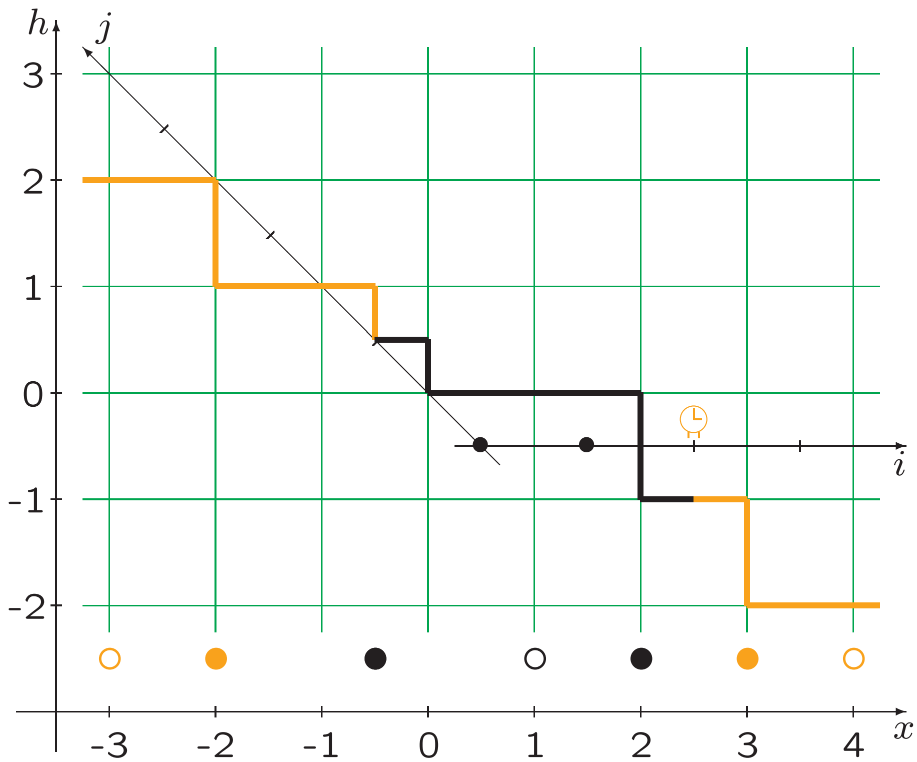

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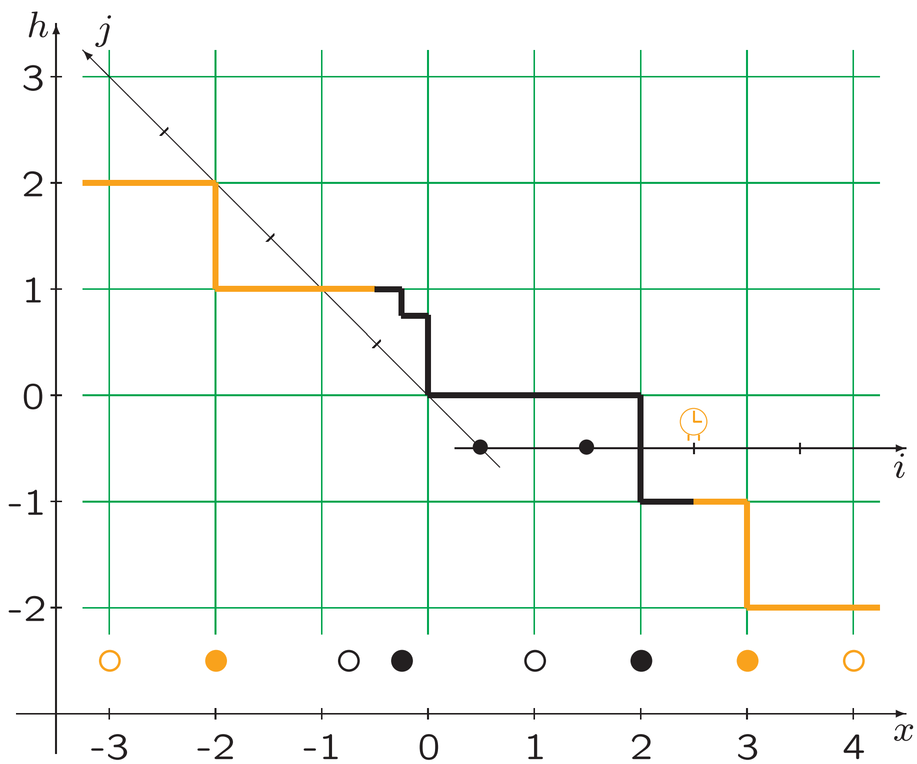

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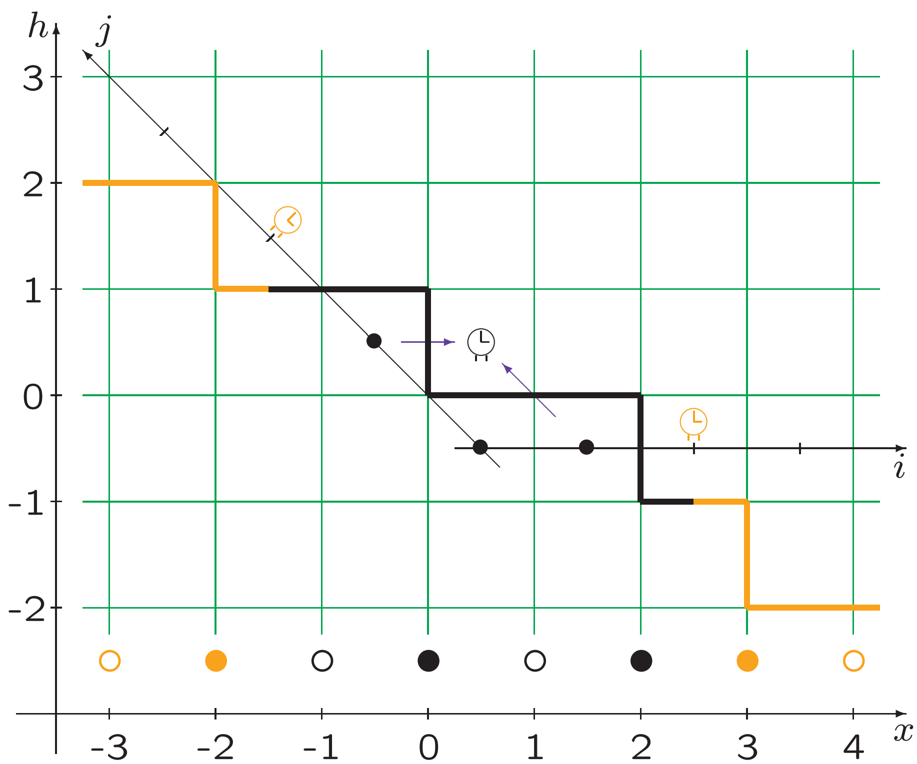

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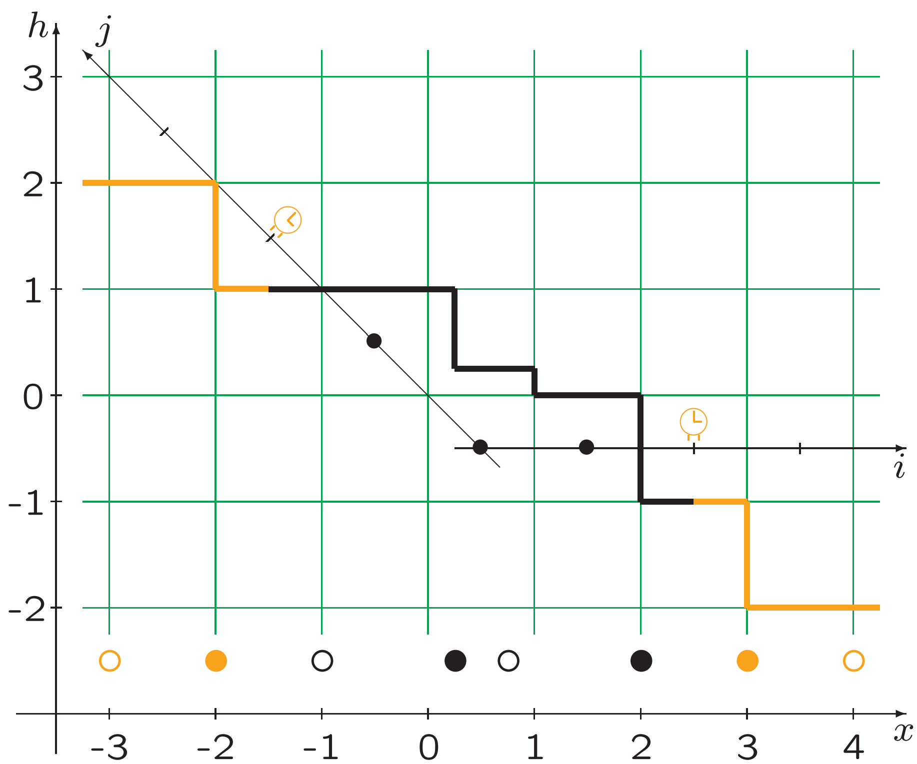

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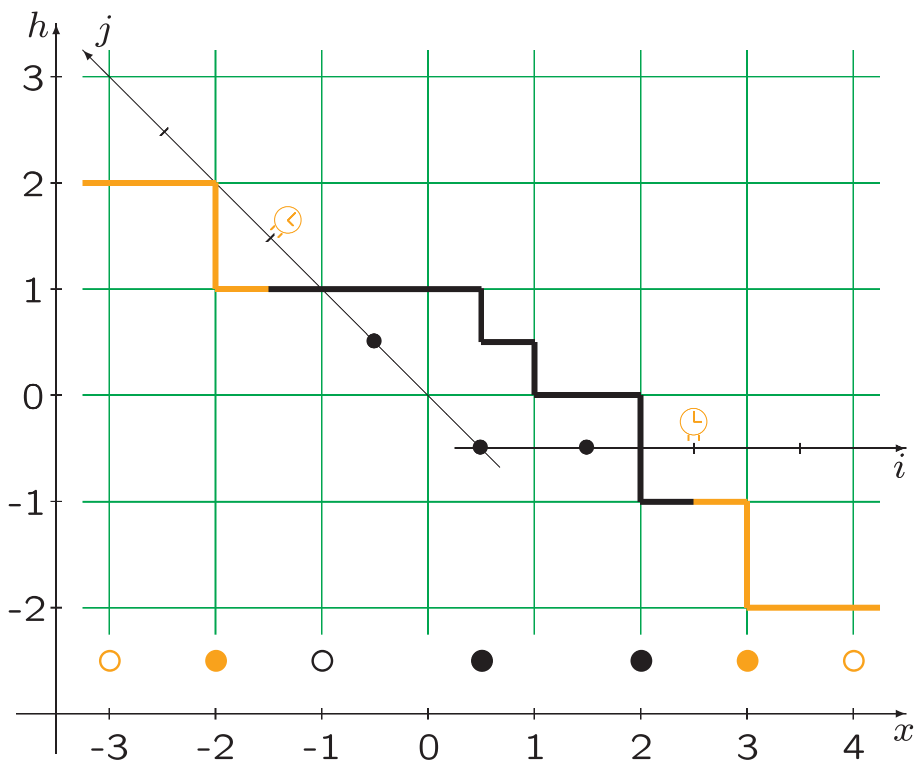

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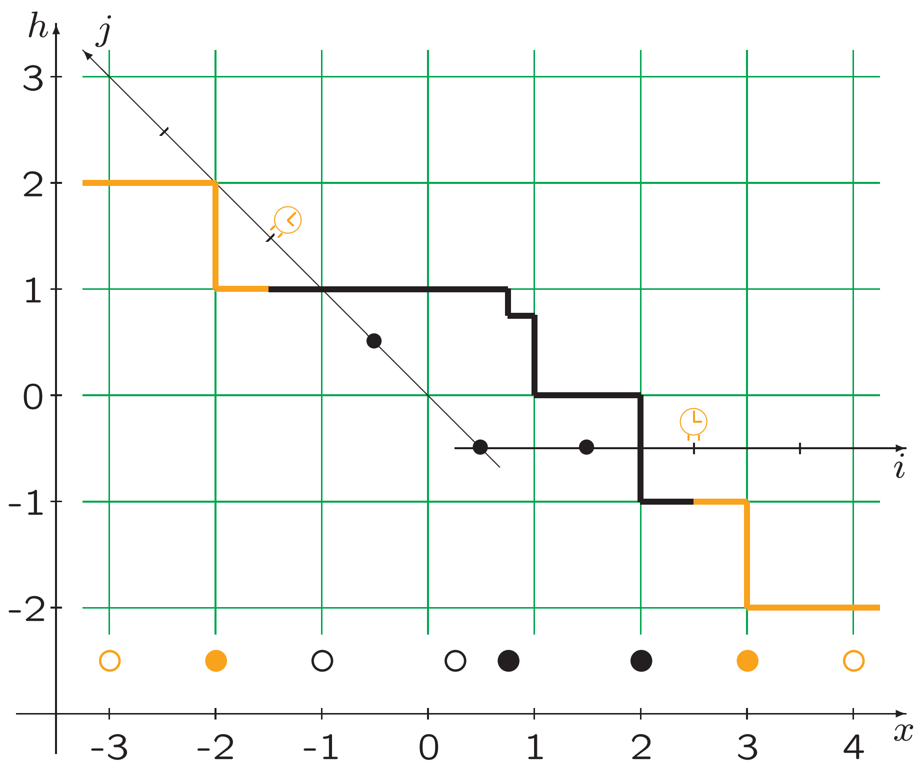

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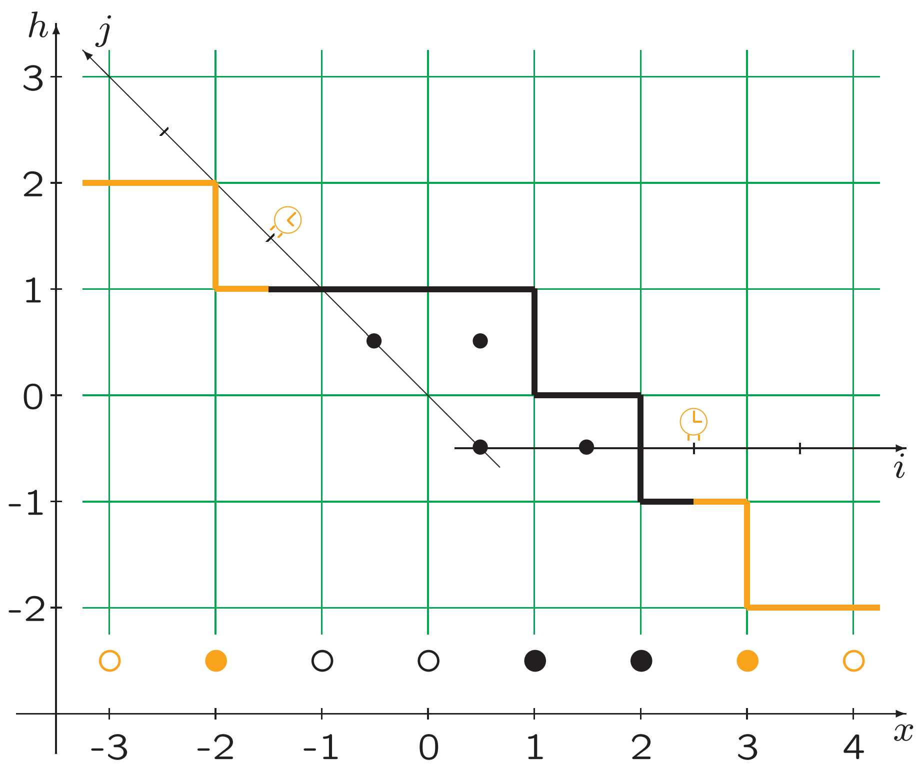

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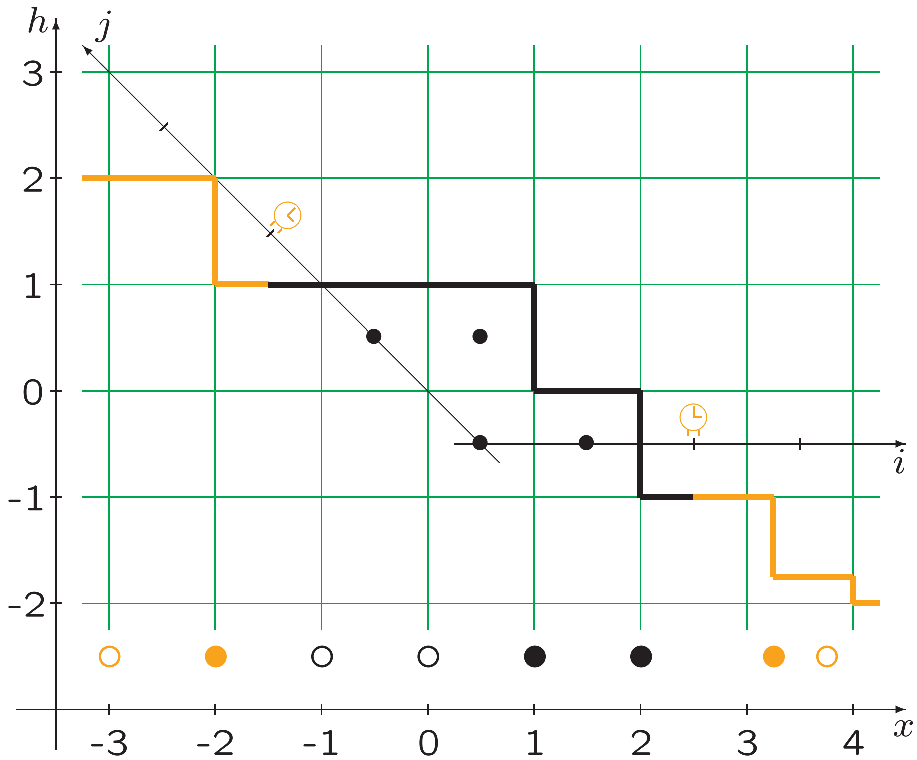

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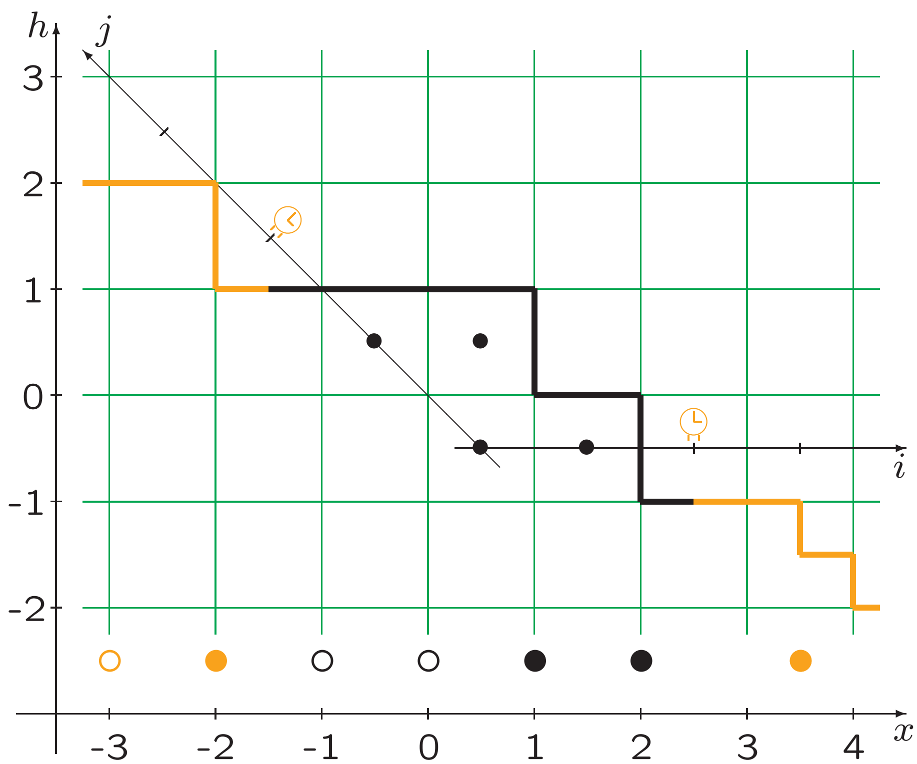

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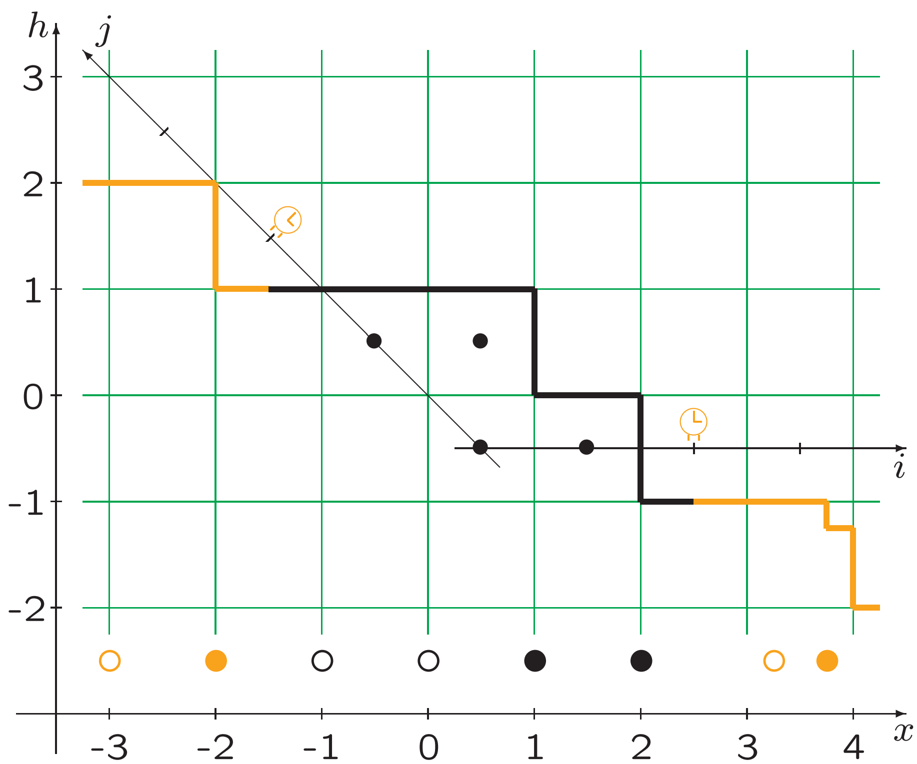

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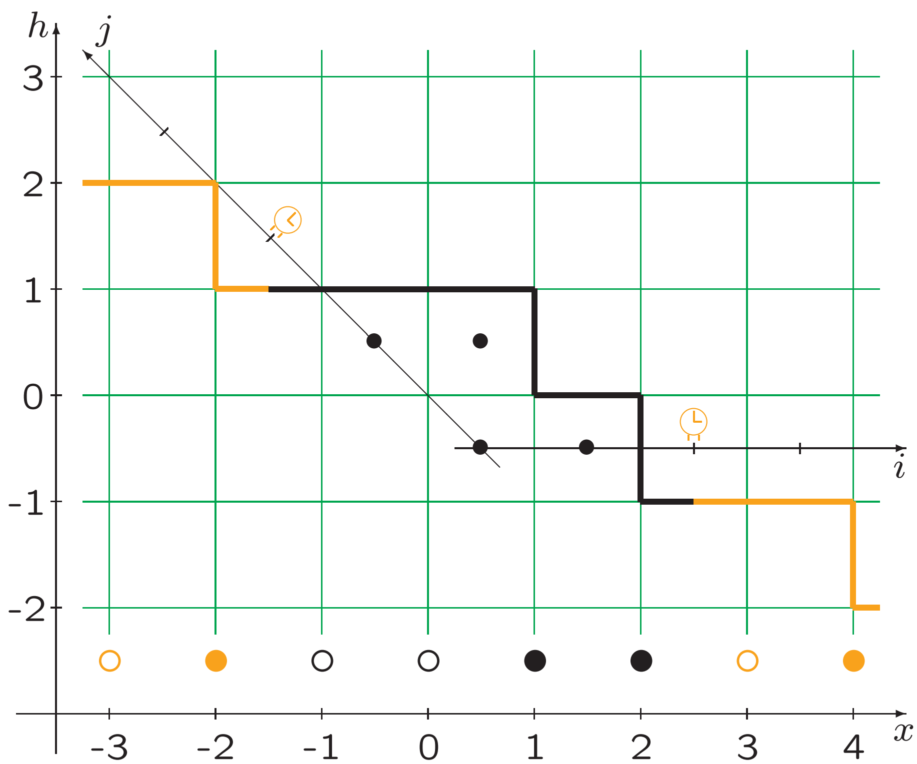

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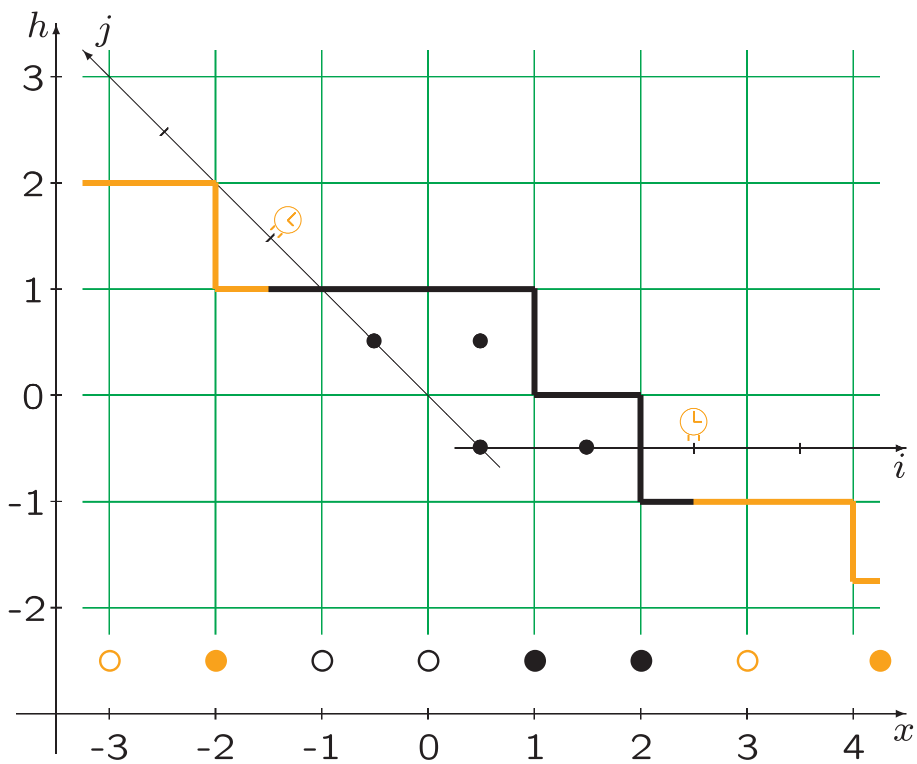

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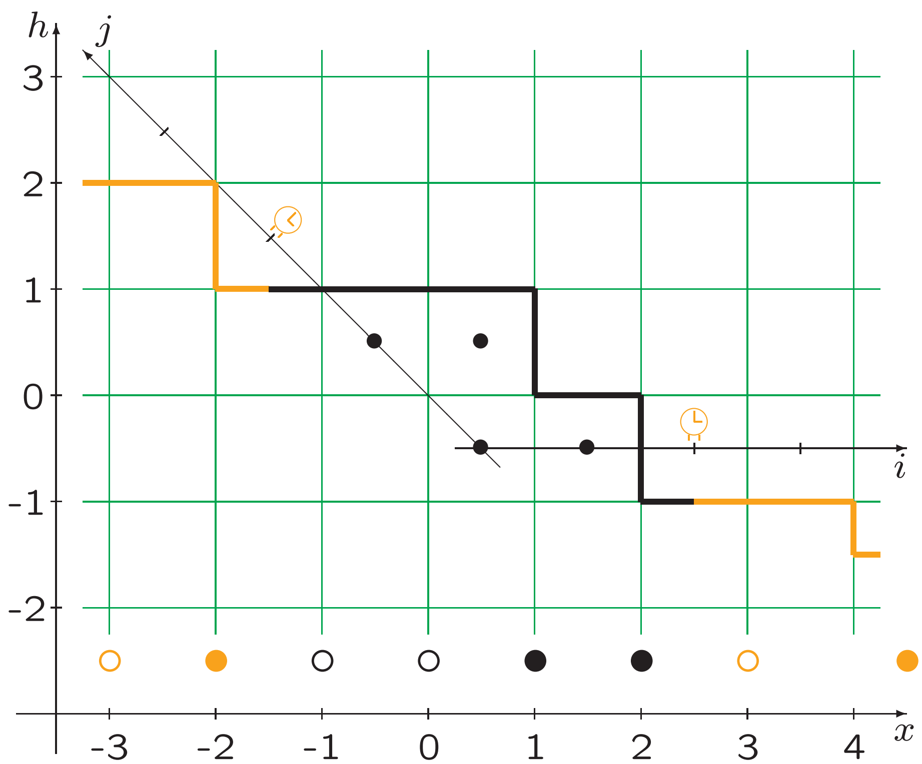

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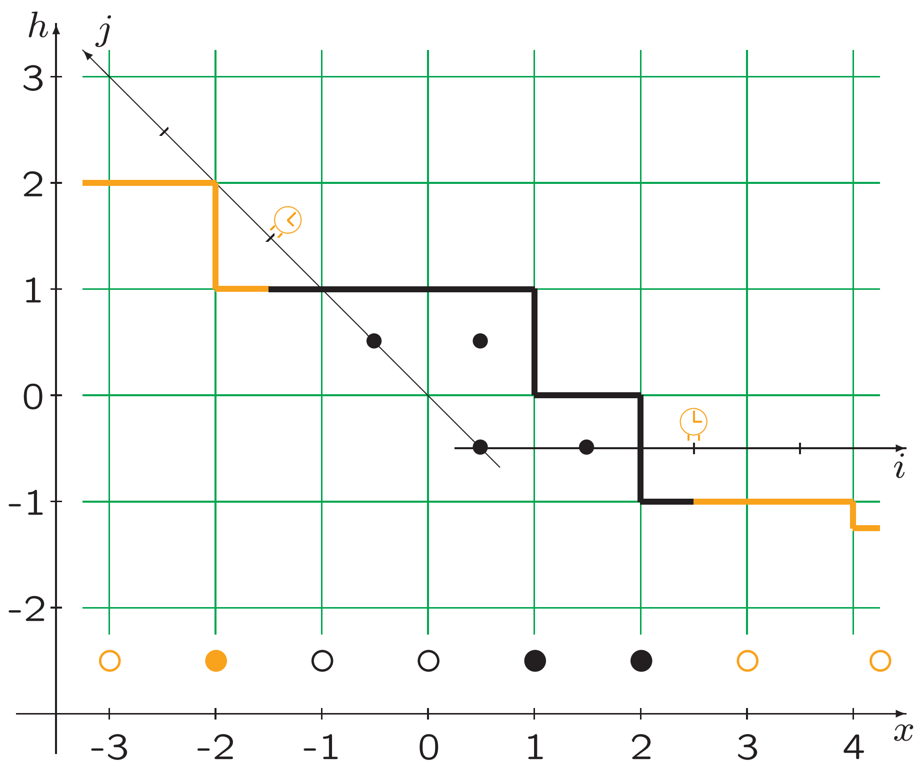

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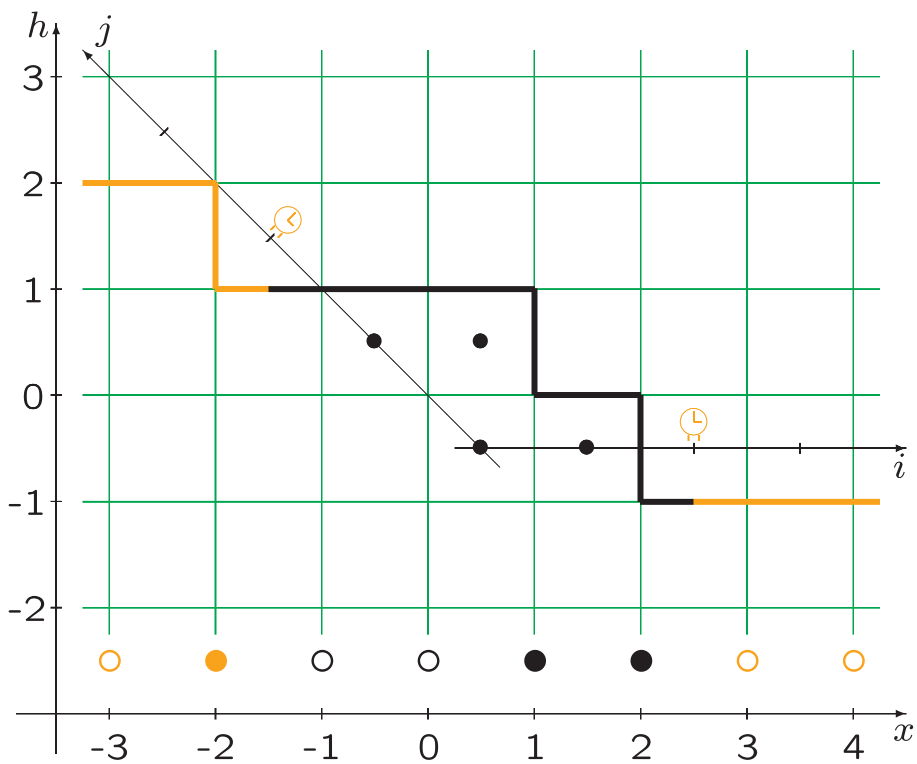

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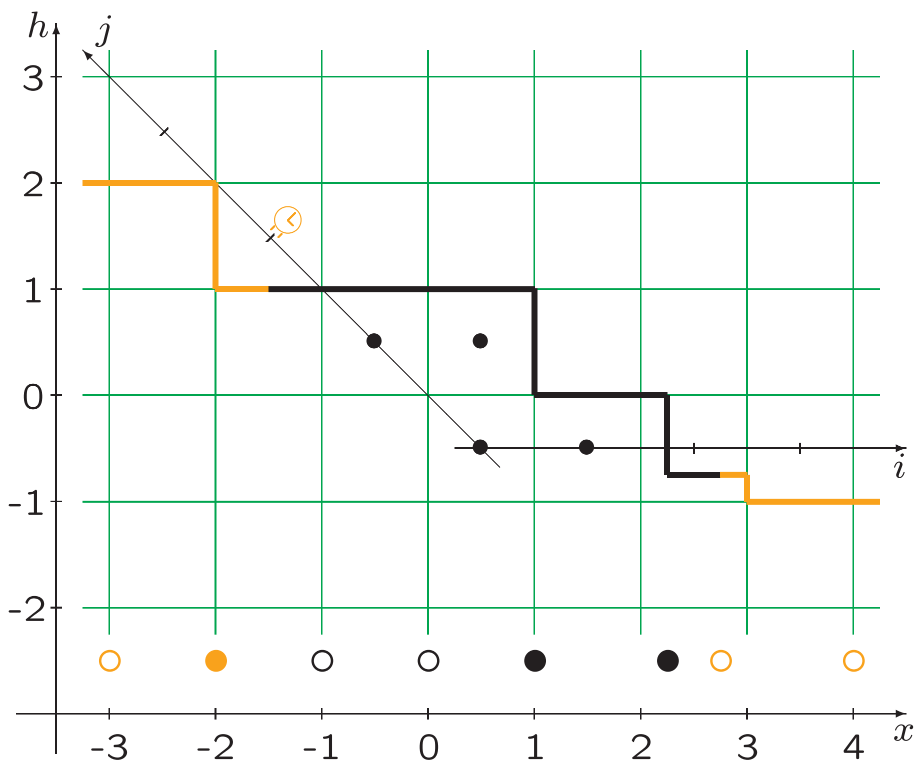

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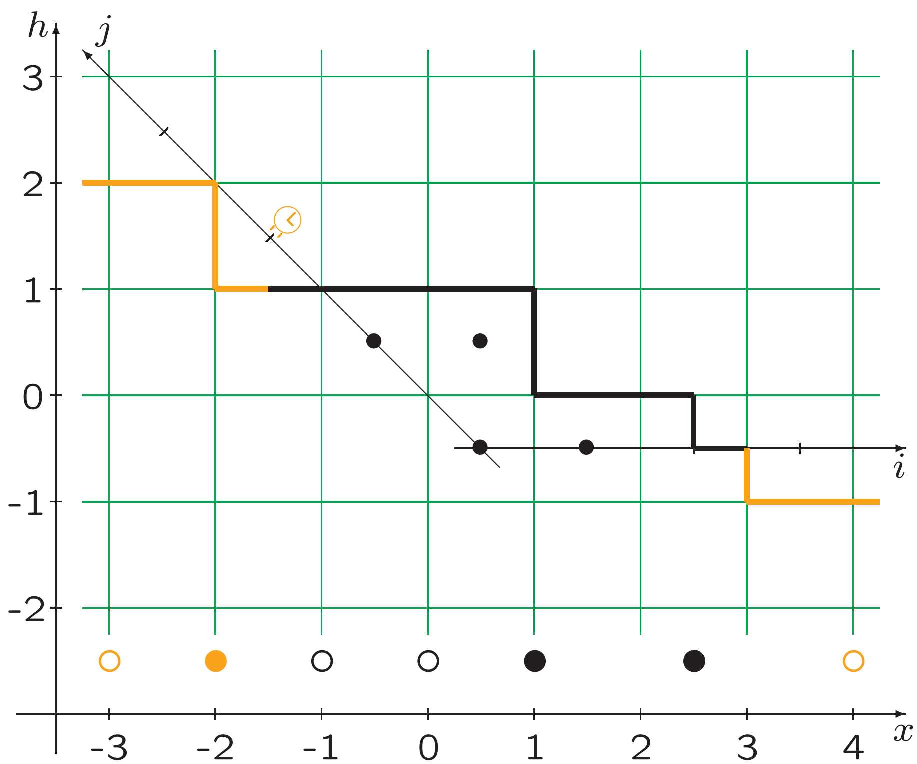

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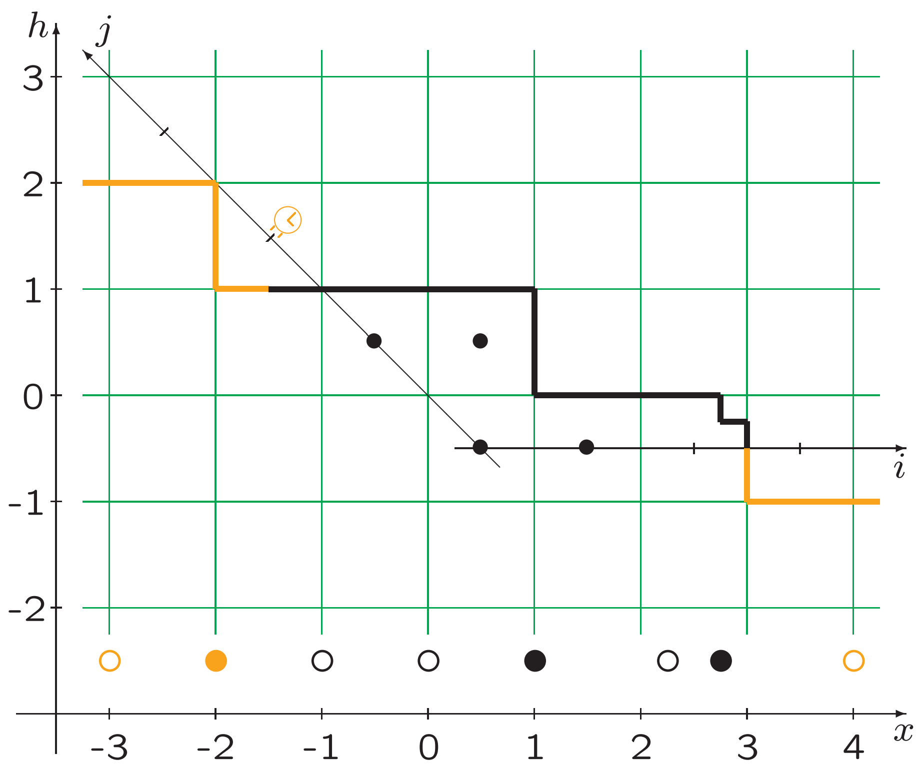
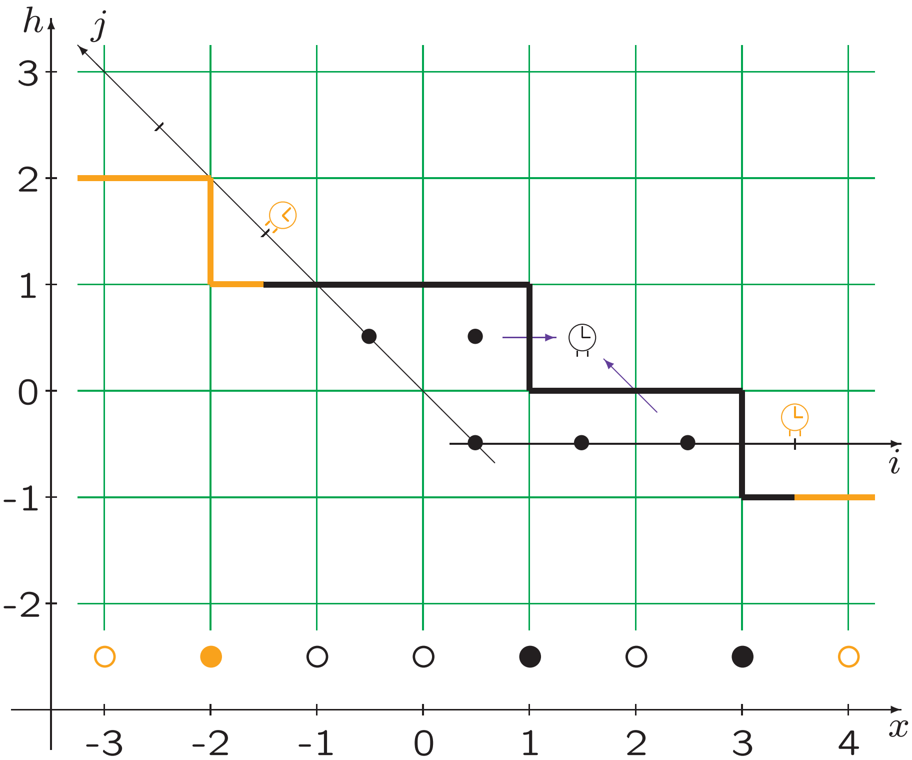


TASEP: Last passage percolation



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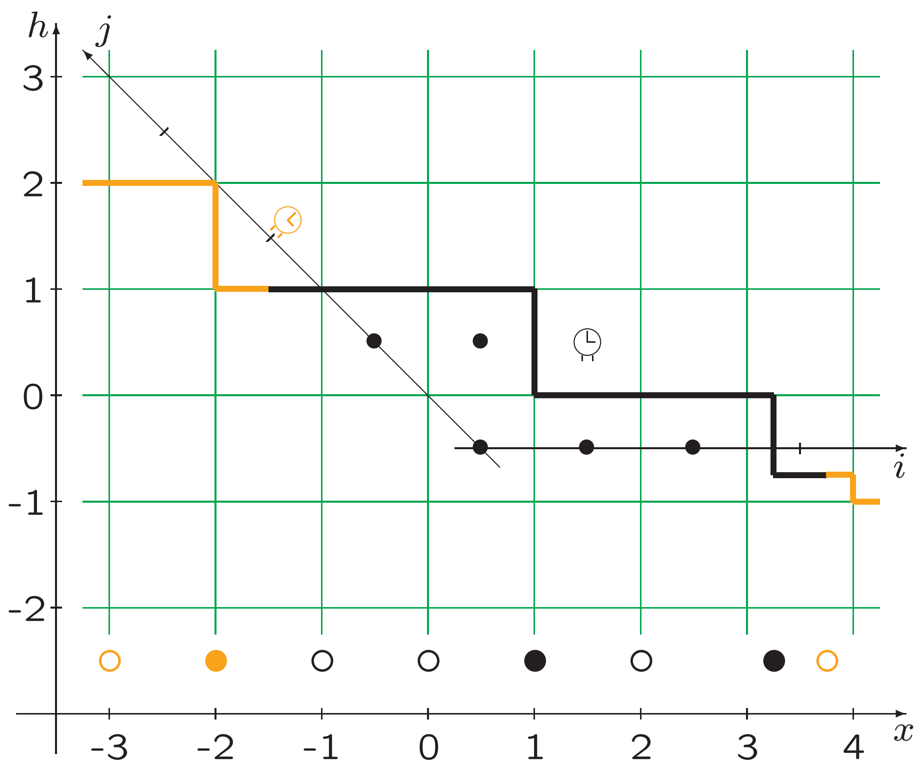

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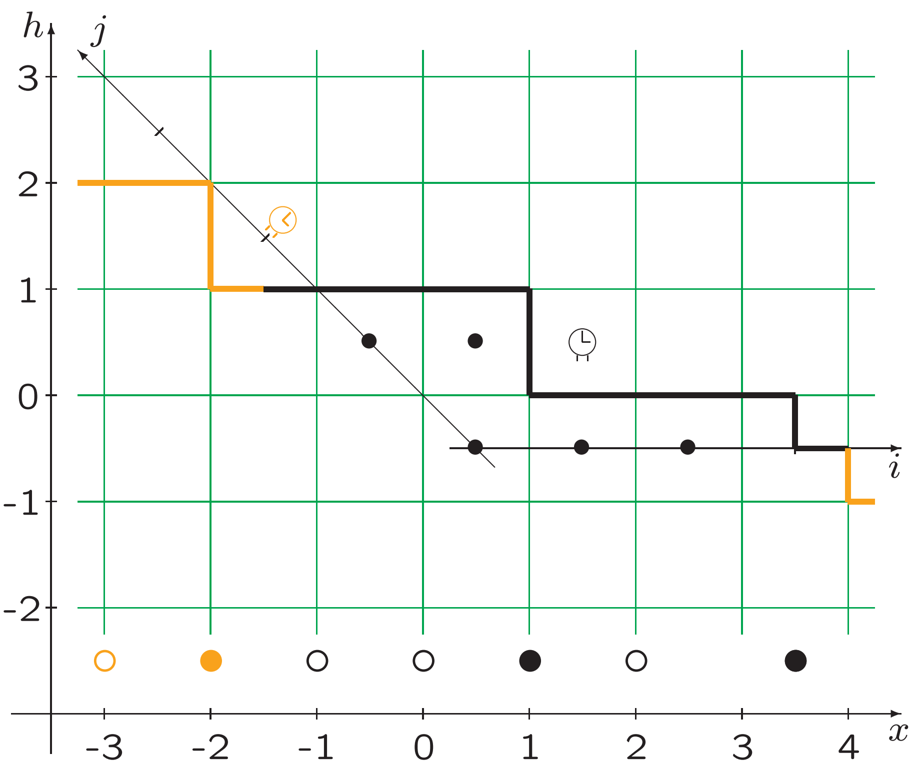

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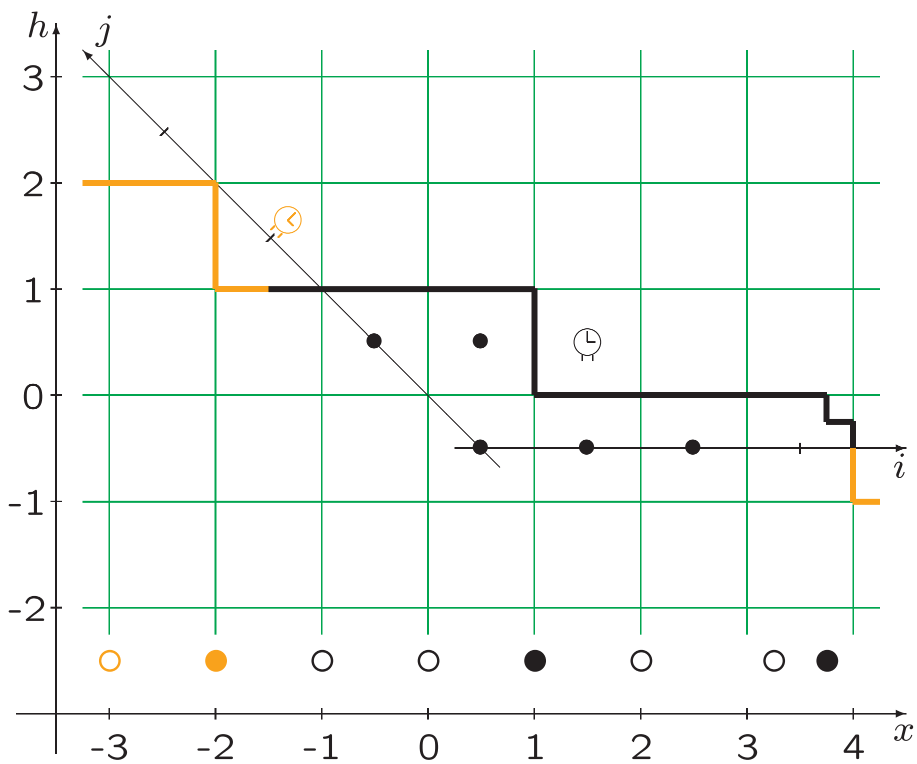

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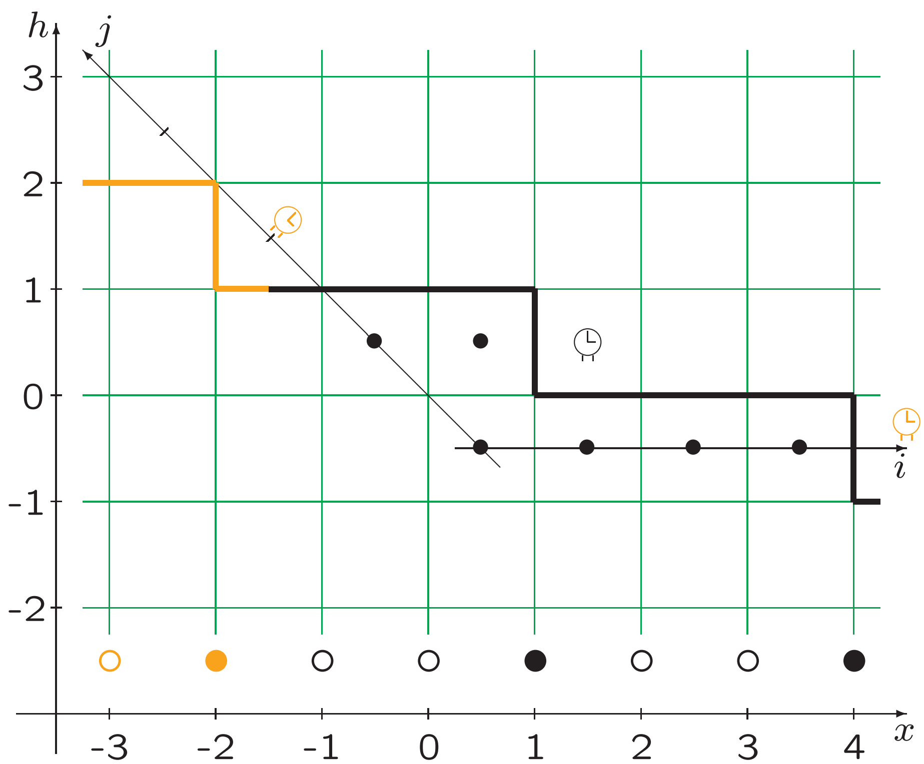

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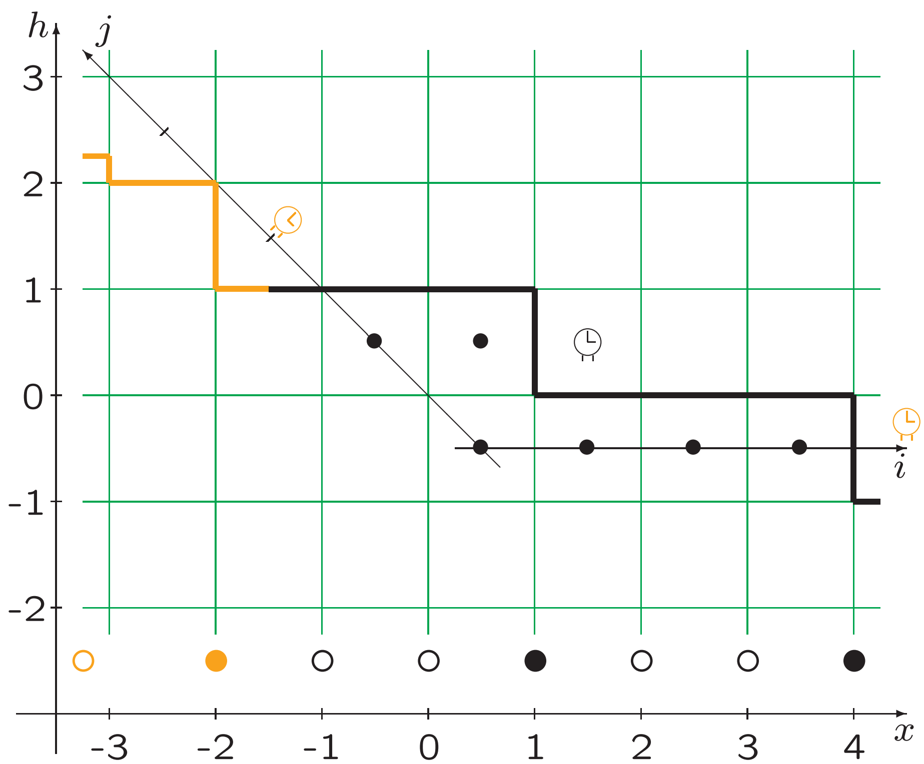

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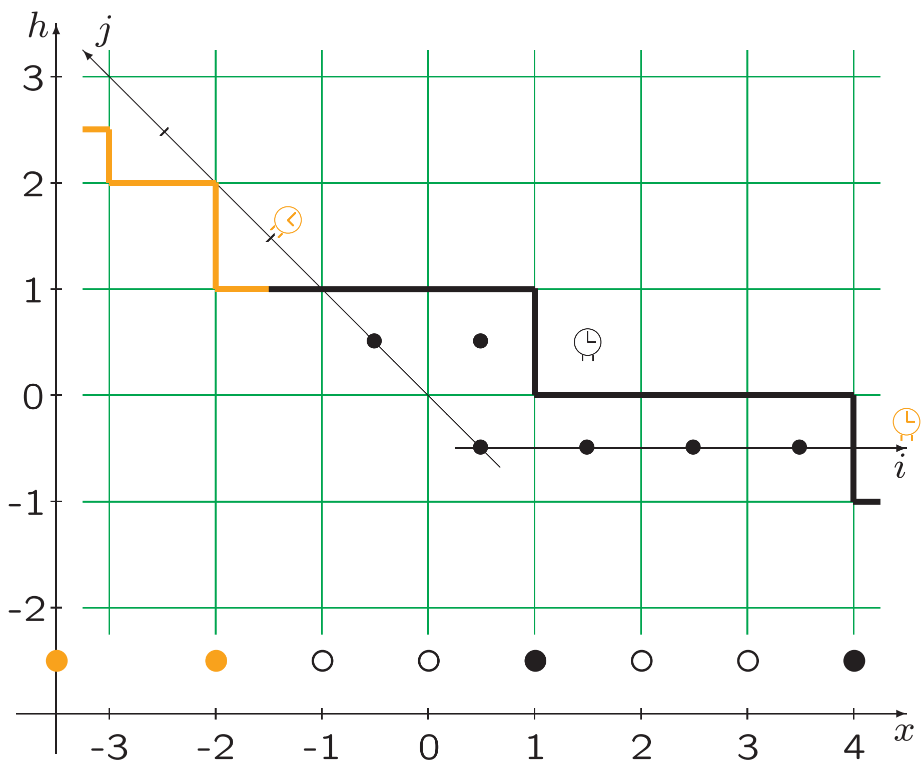

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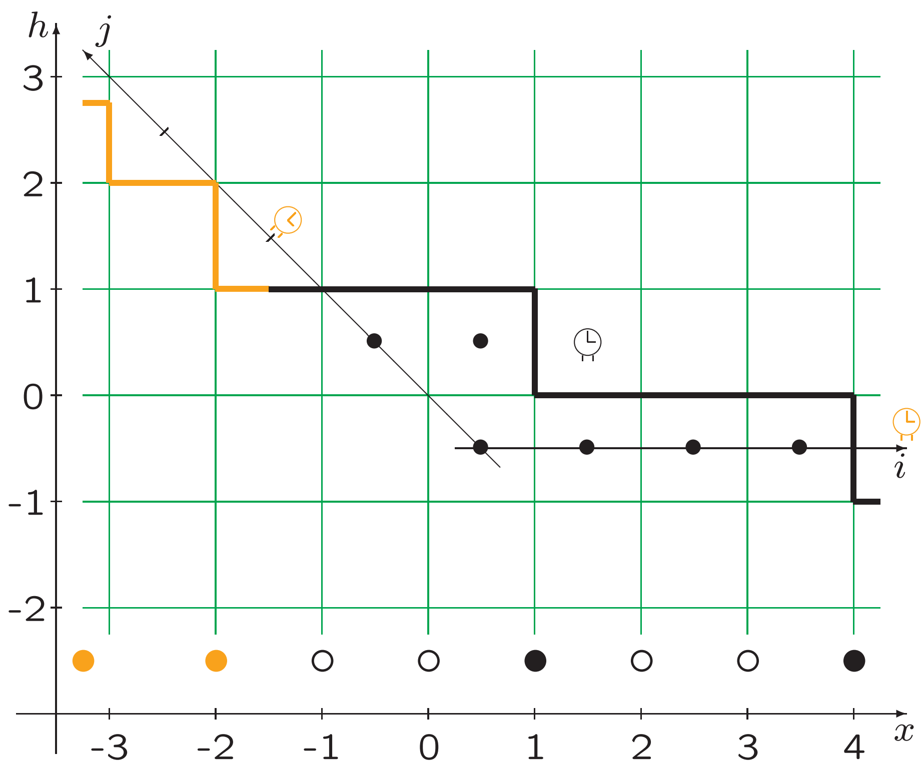

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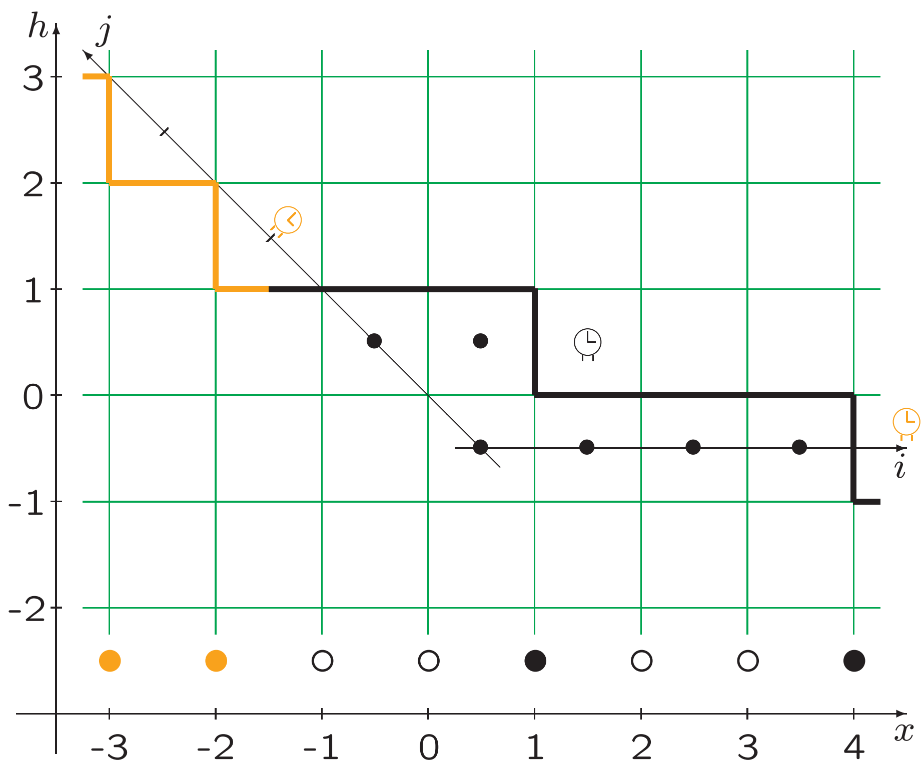

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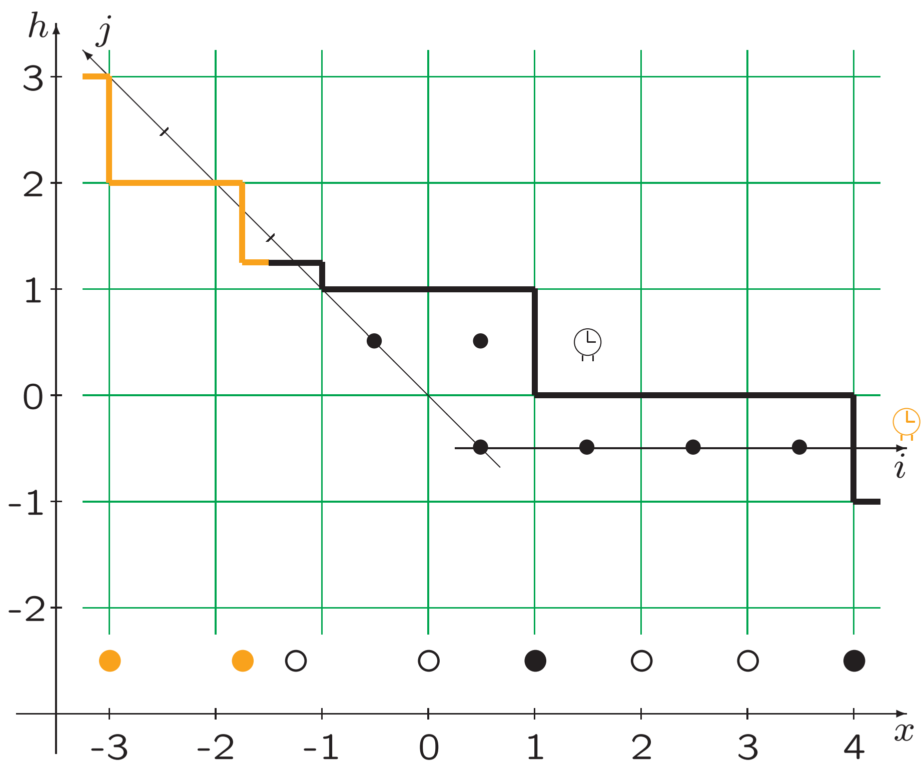

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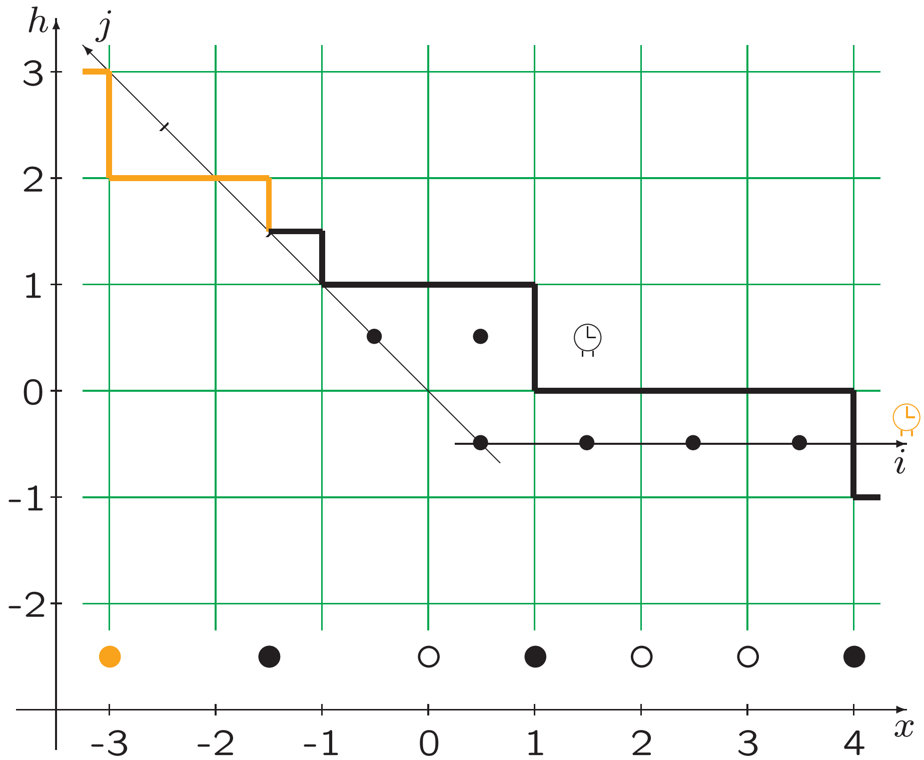
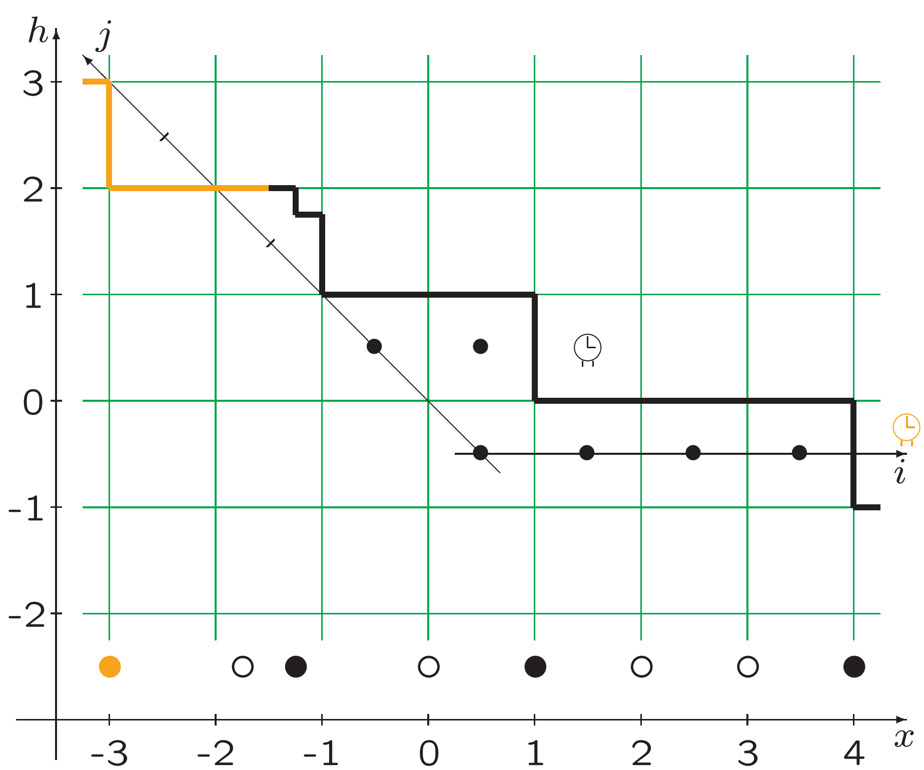


TASEP: Last passage percolation



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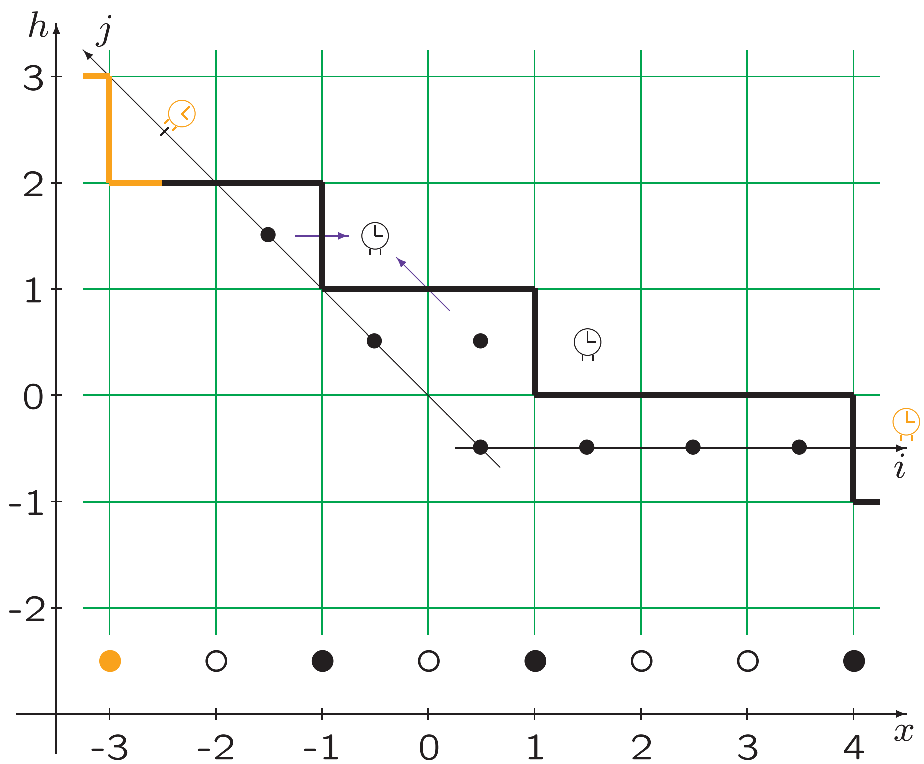

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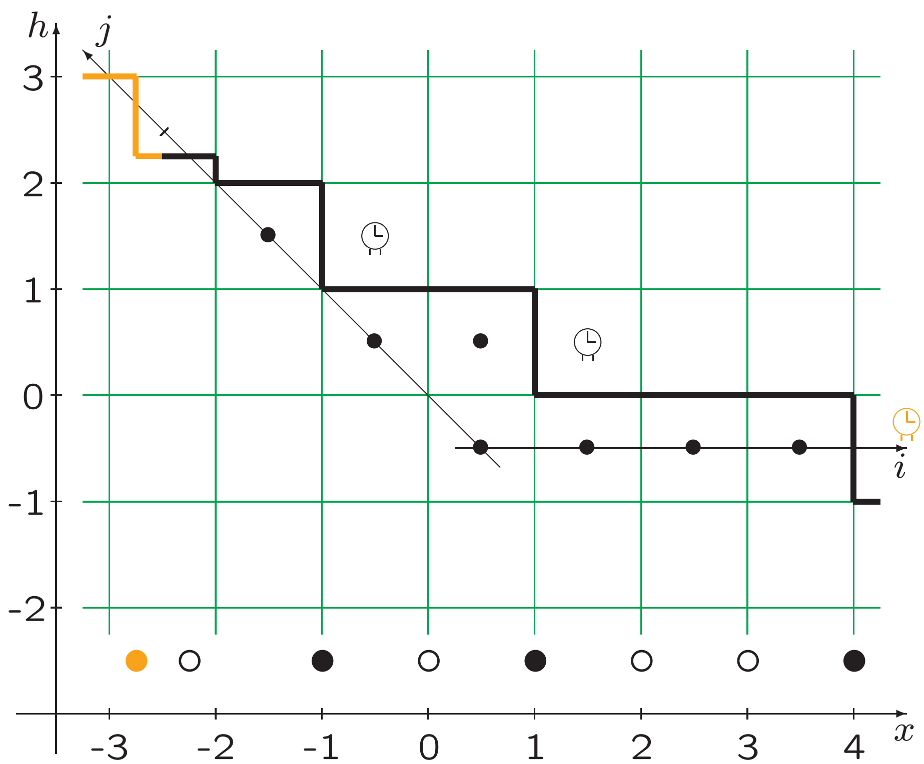

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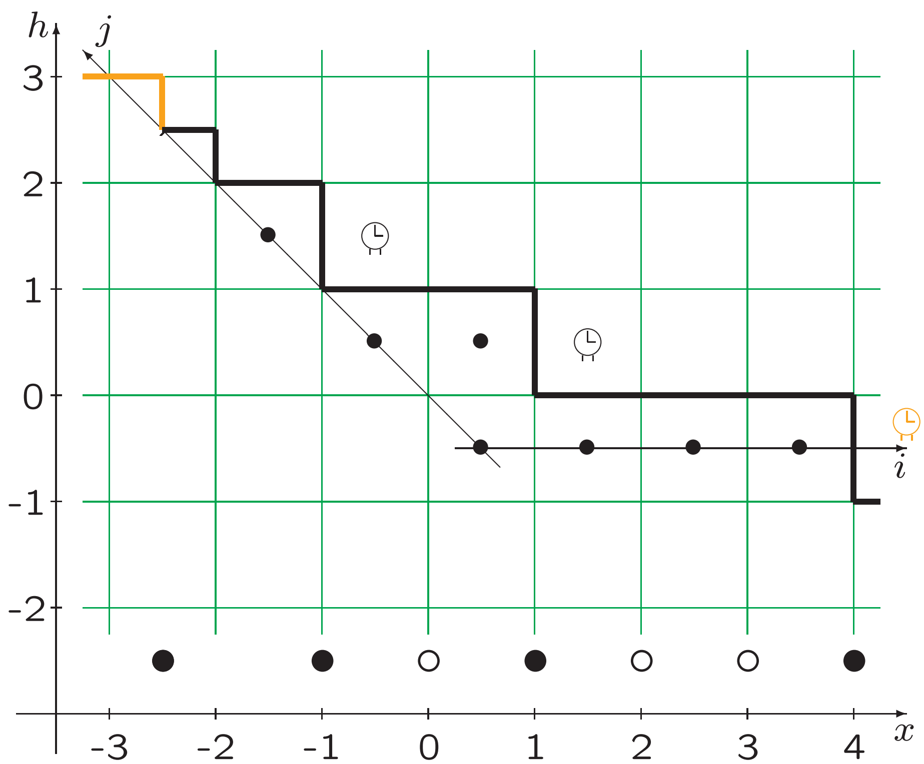
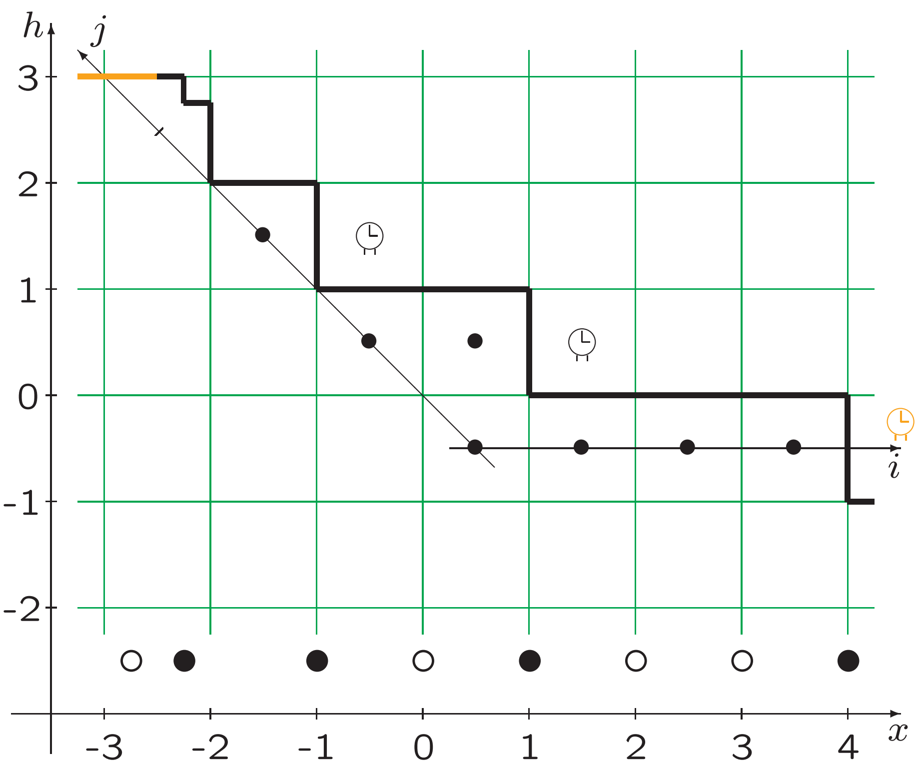


TASEP: Last passage percolation



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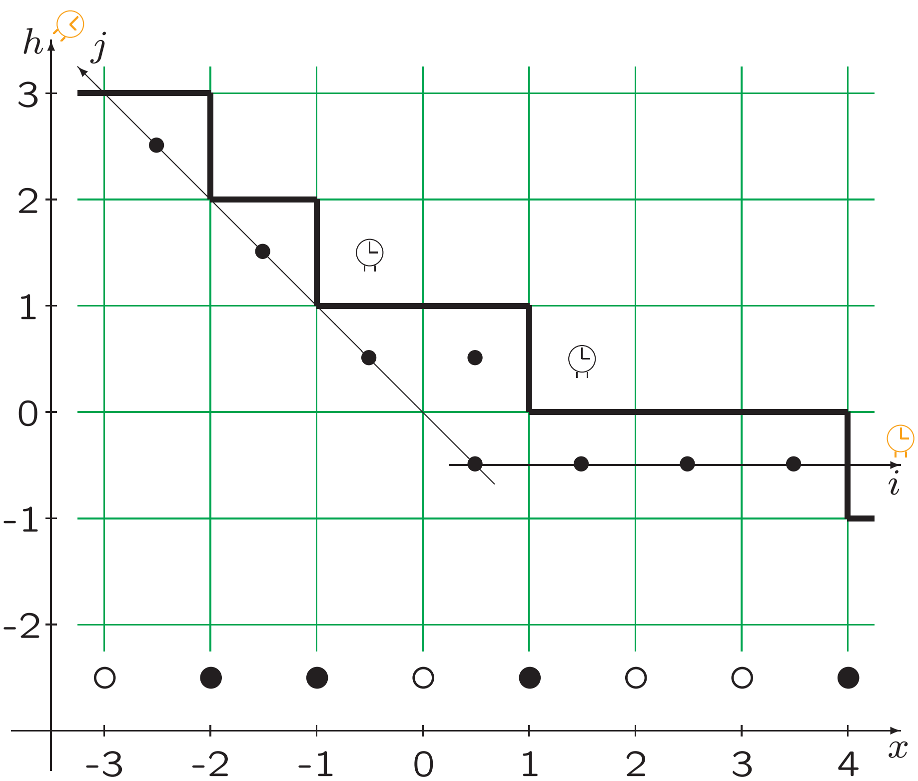
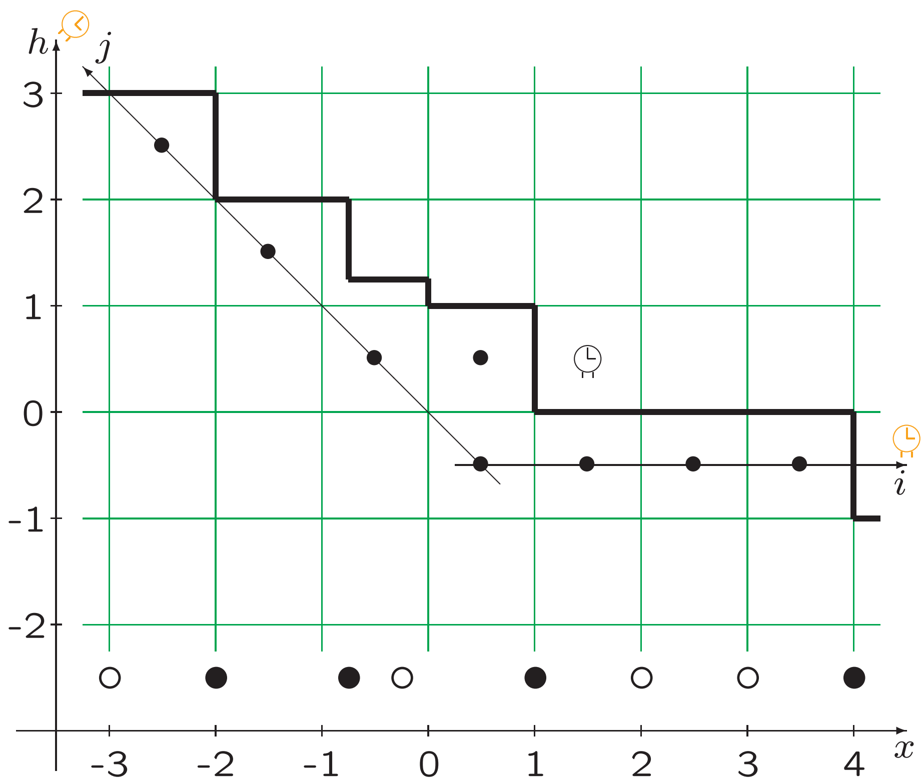
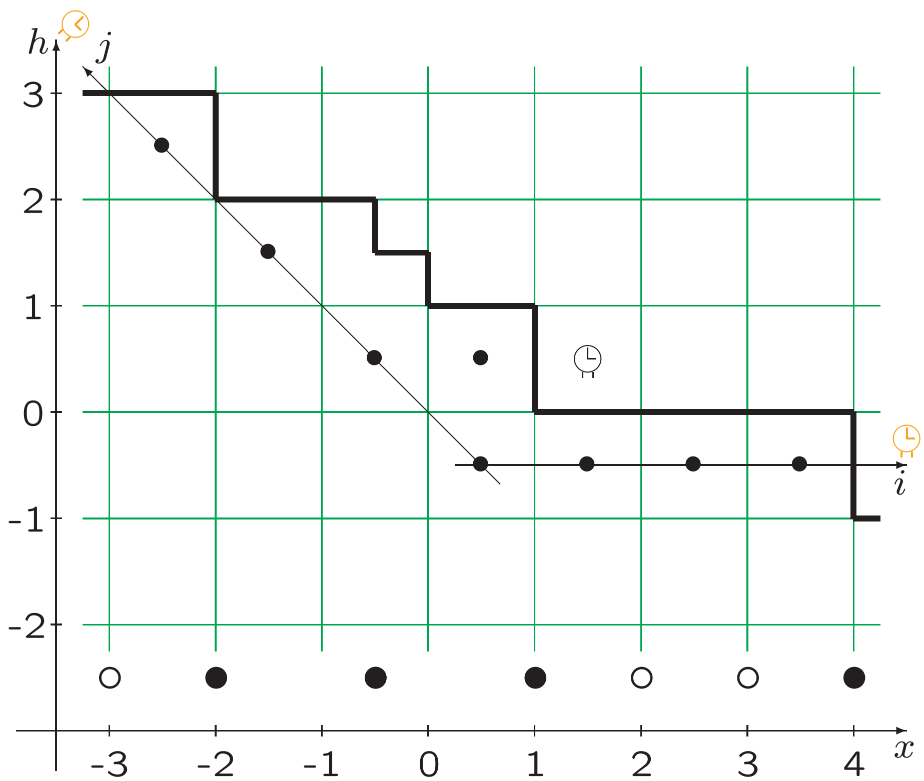


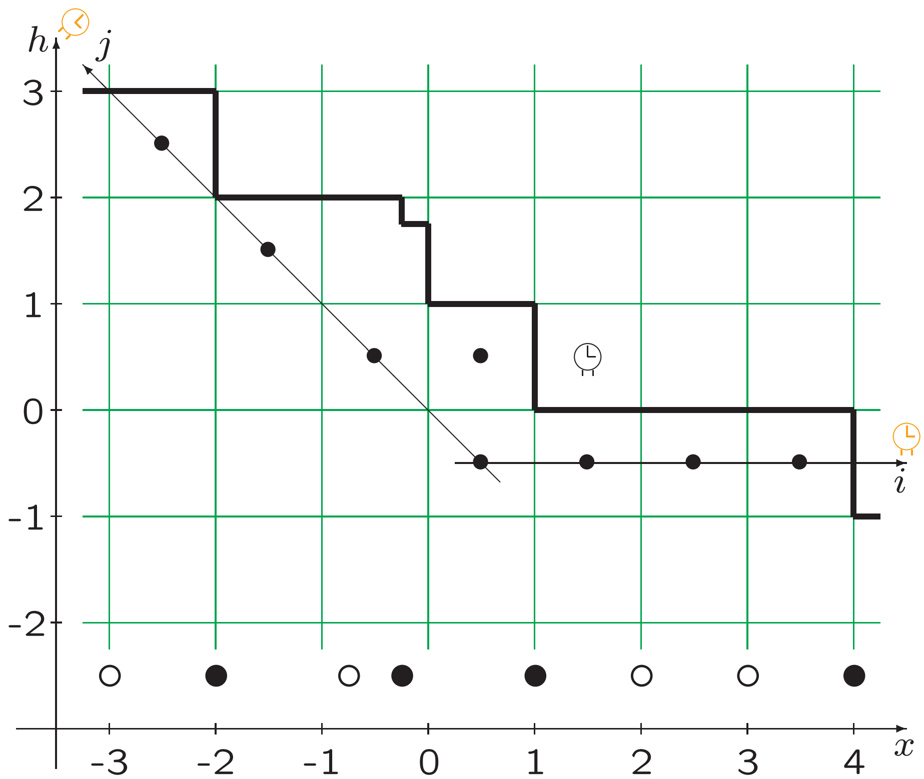
TASEP: Last passage percolation



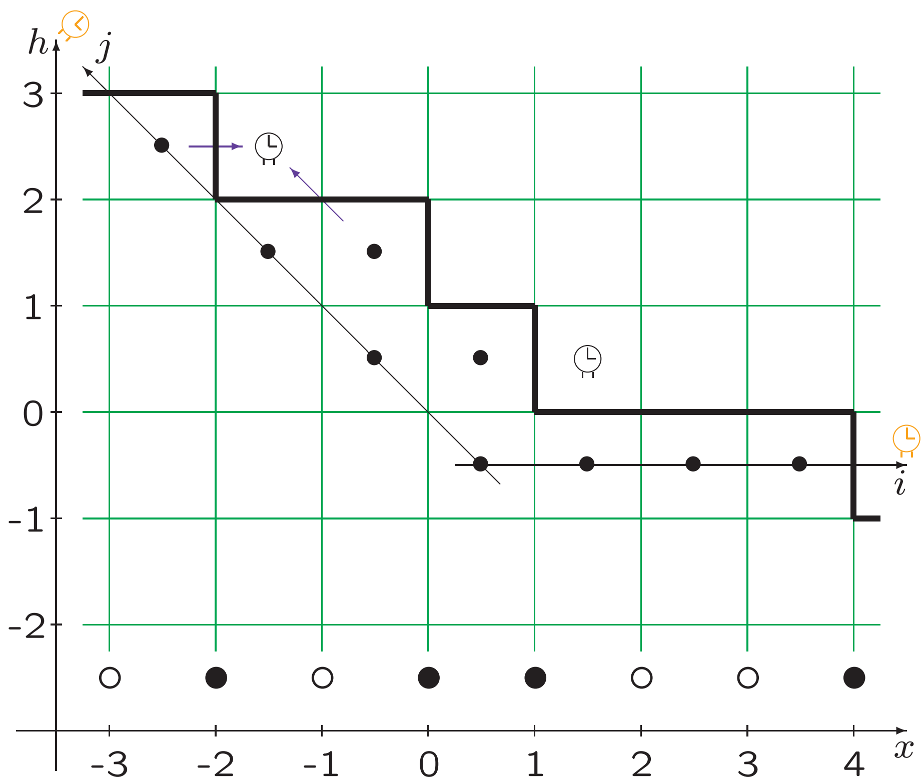
TASEP: Last passage percolation

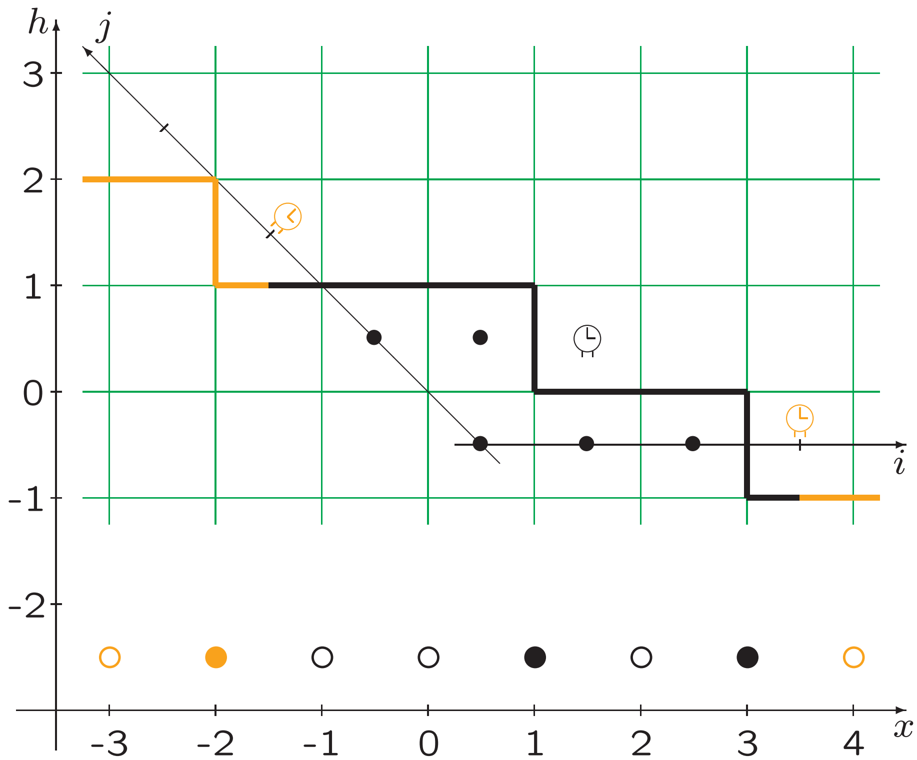


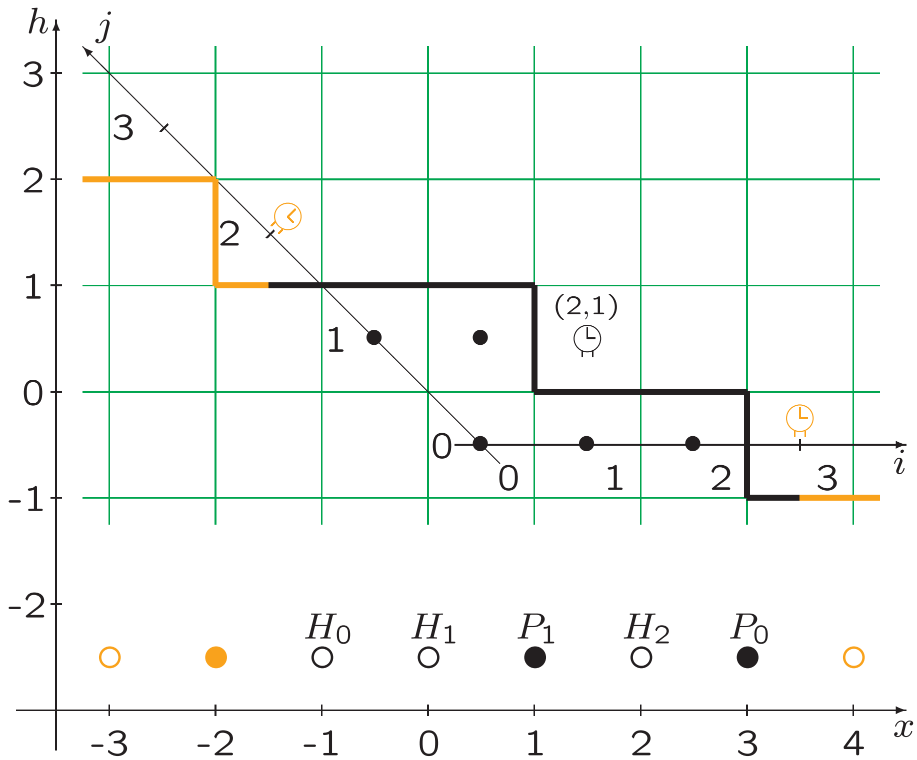
TASEP: Last passage percolation

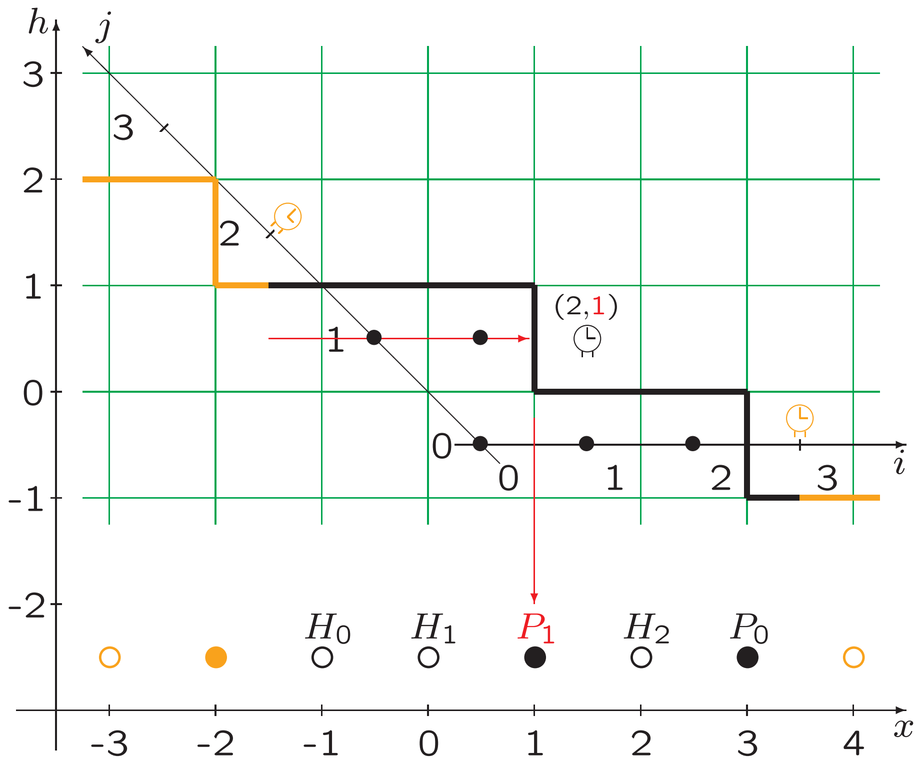


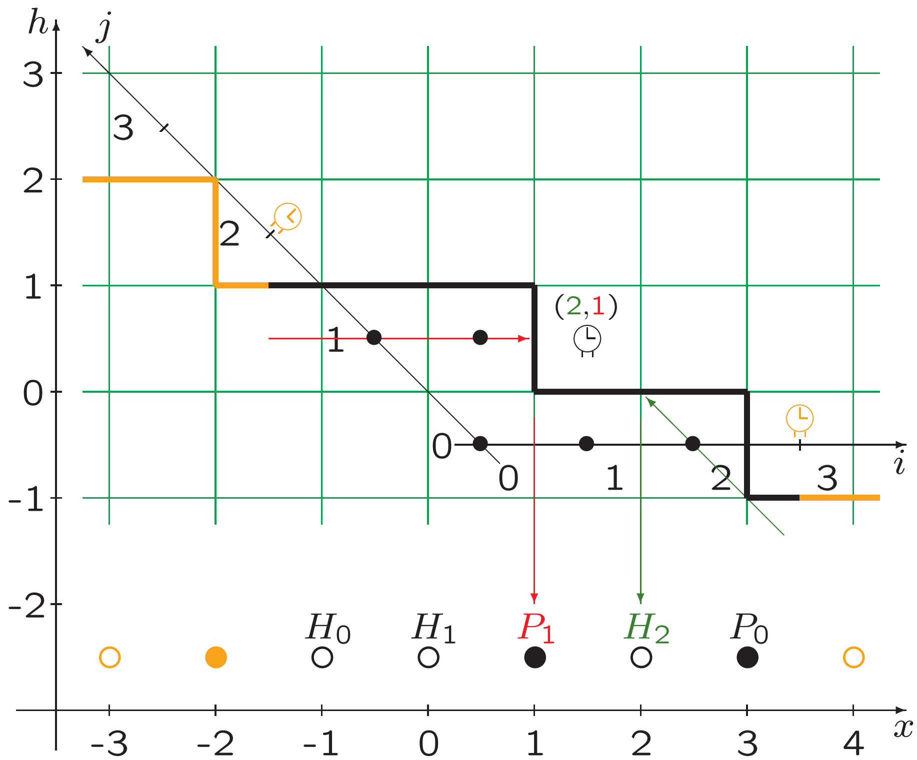
TASEP: Last passage percolation




h
j
3
2
1
0
-1
-2
i
-3
-2
-1
0
1
2
3
4
x


h
j
i
x
3
2
1
0
-1
-2
(2,1)
H_0
H_1
P_1
H_2
P_0
-3
-2
-1
0
1
2
3
4


h
j
i
x
(2,1)
H0
H1
P1
H2
P0


h
j
3
2
1
0
-1
-2
(2,1)
0
1
2
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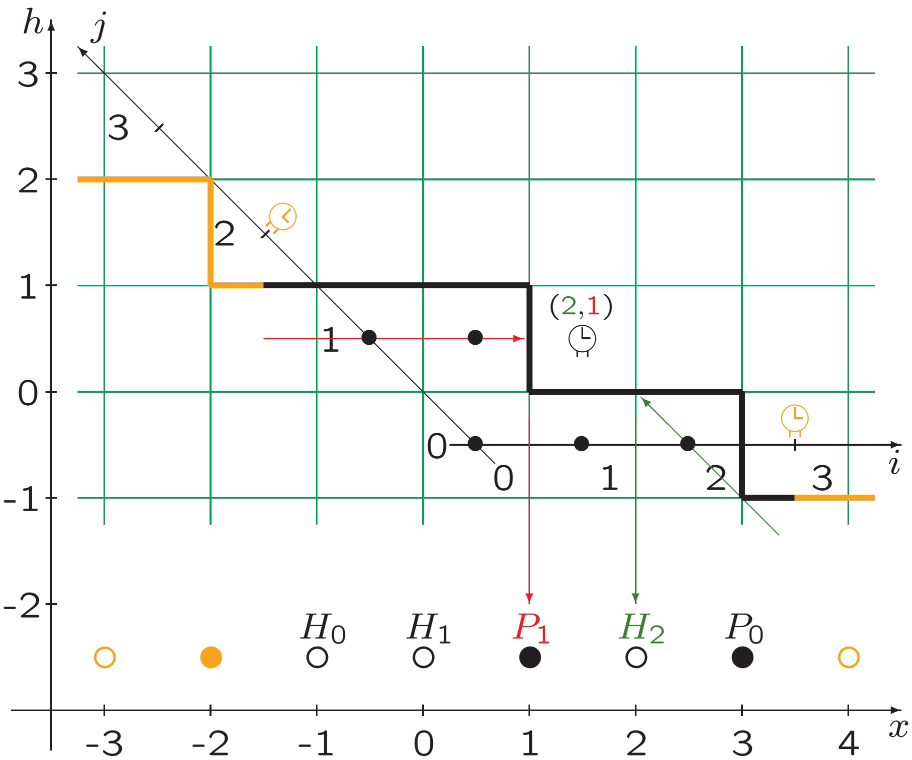
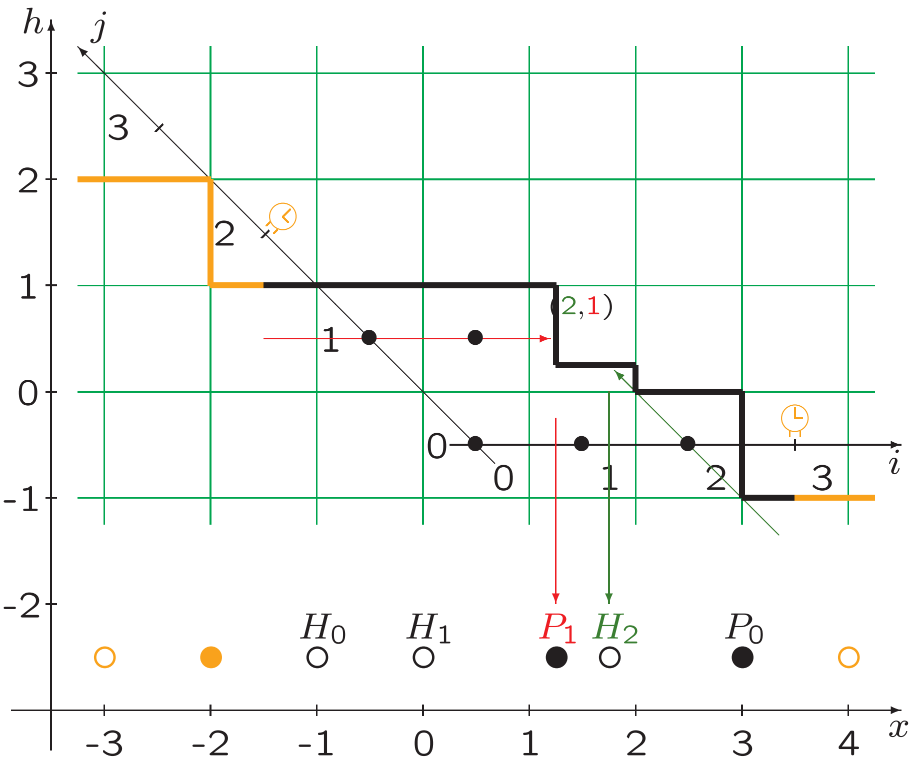
i
H_0
H_1
P_1
H_2
P_0
-3
-2
-1
0
1
2
3
4
x



Occupation of (i, j) = jump of P_j over H_i.

Occupation of $(2, 1)$ = jump of P_1 over H_2.



Occupation of (i, j) = jump of P_j over H_i.

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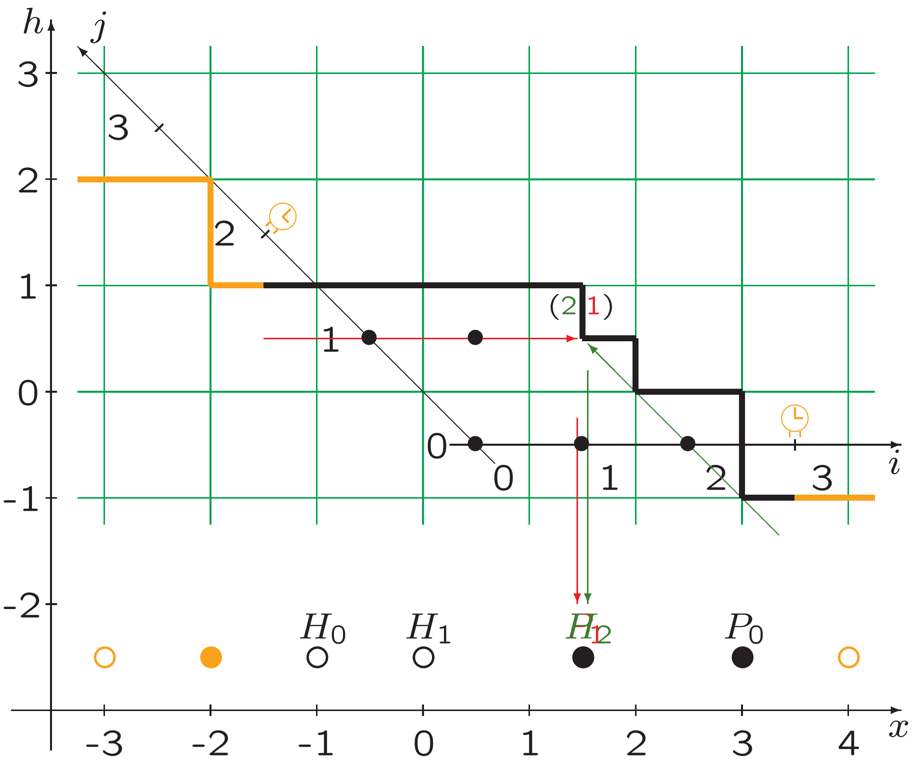

Occupation of (i, j) = jump of P_j over H_i.

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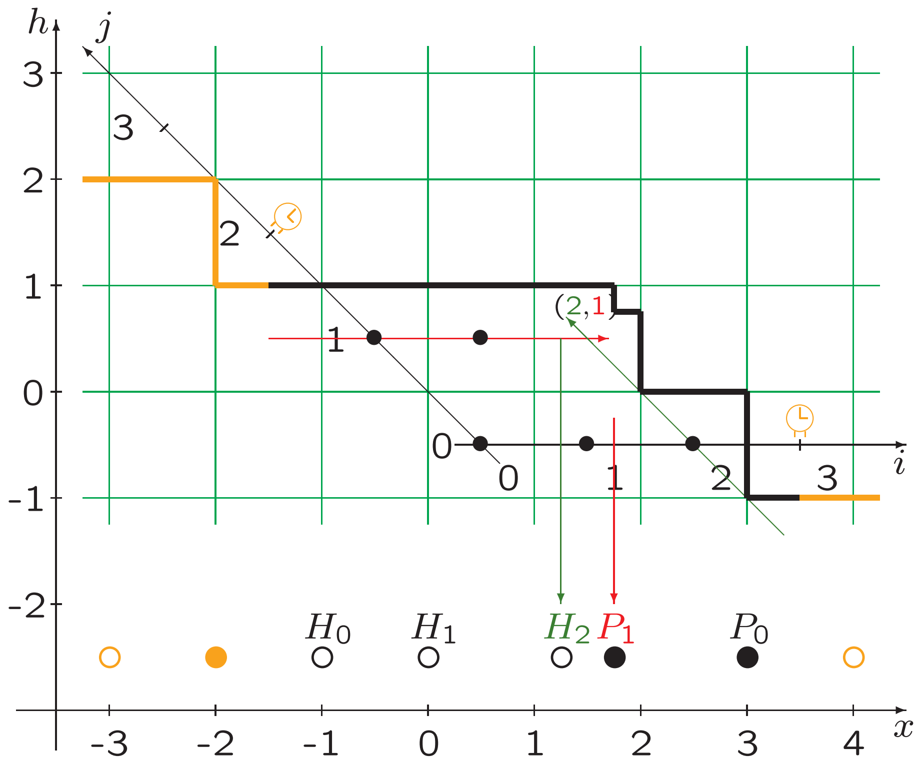

Occupation of (i, j) = jump of P_j over H_i.

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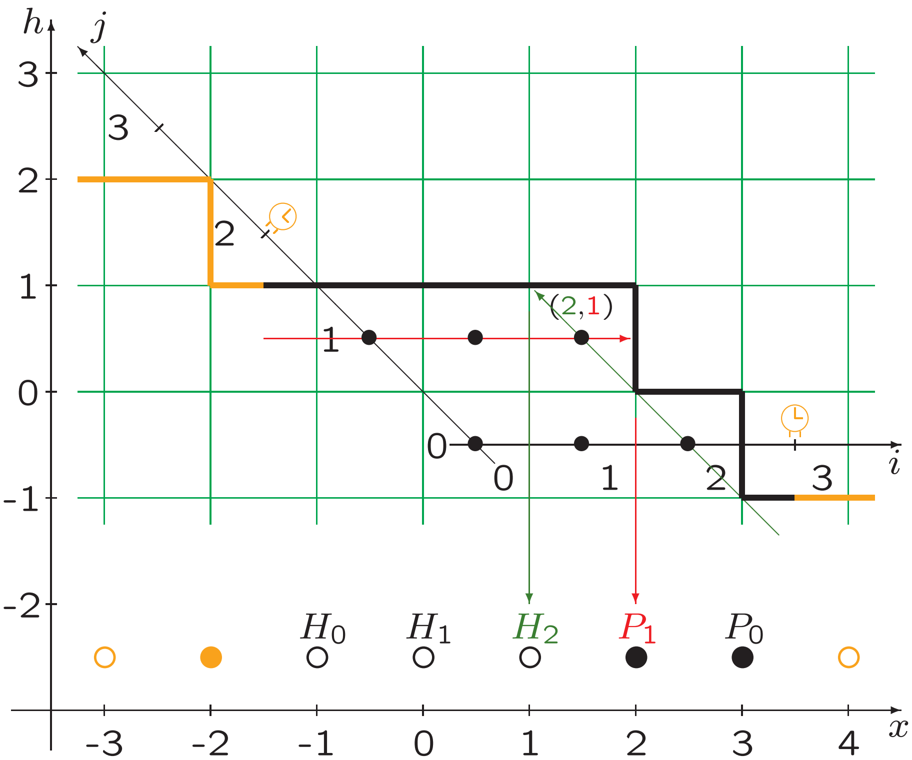

Occupation of (i, j) = jump of P_j over H_i.

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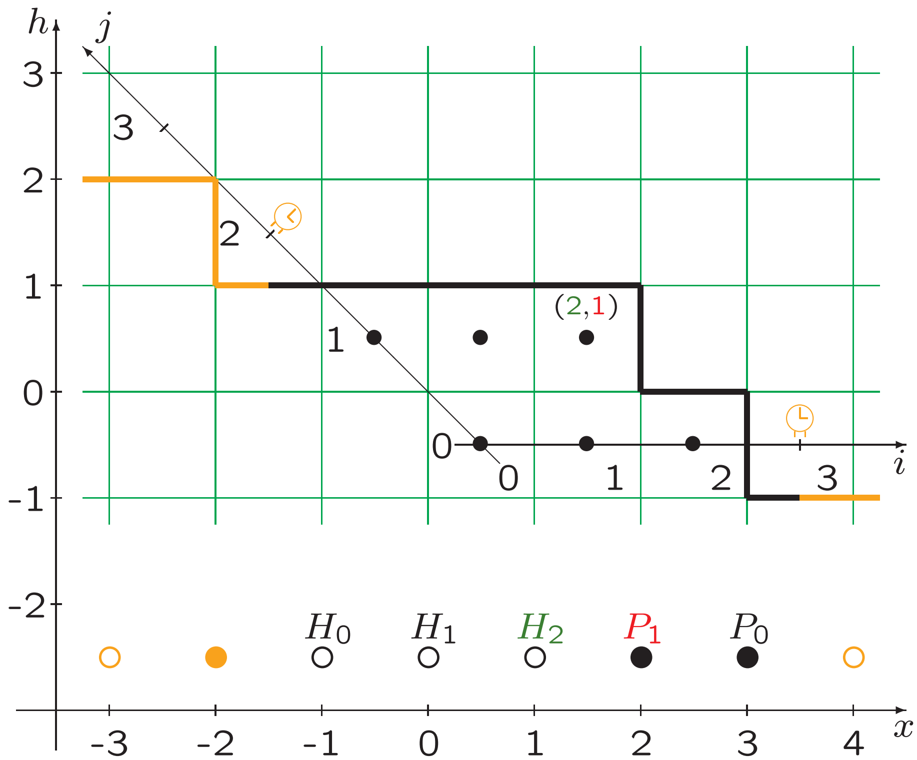
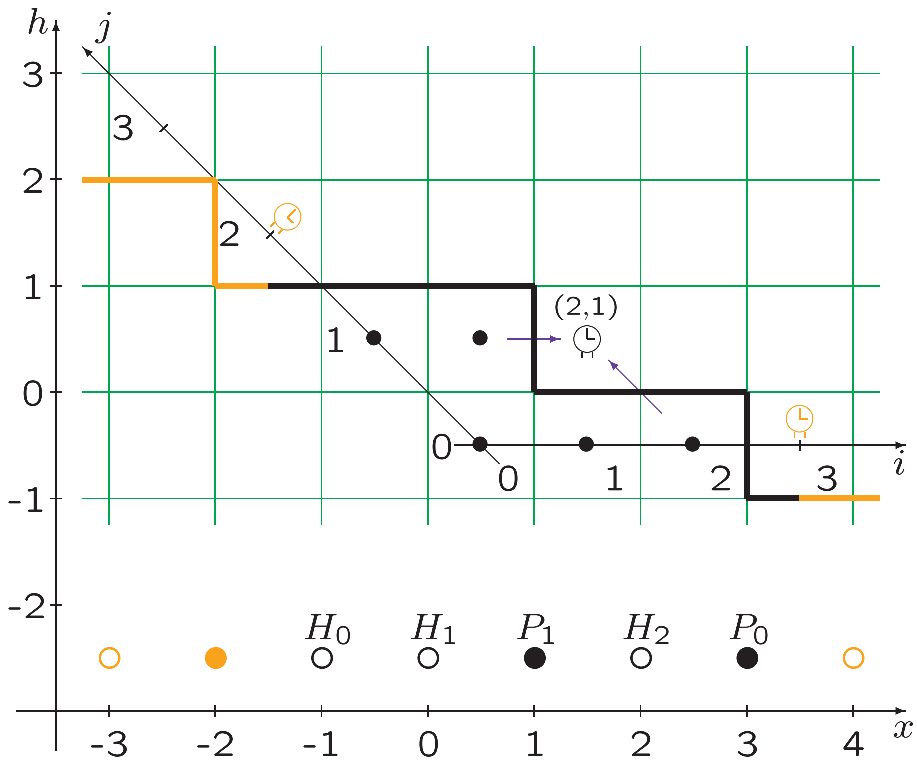


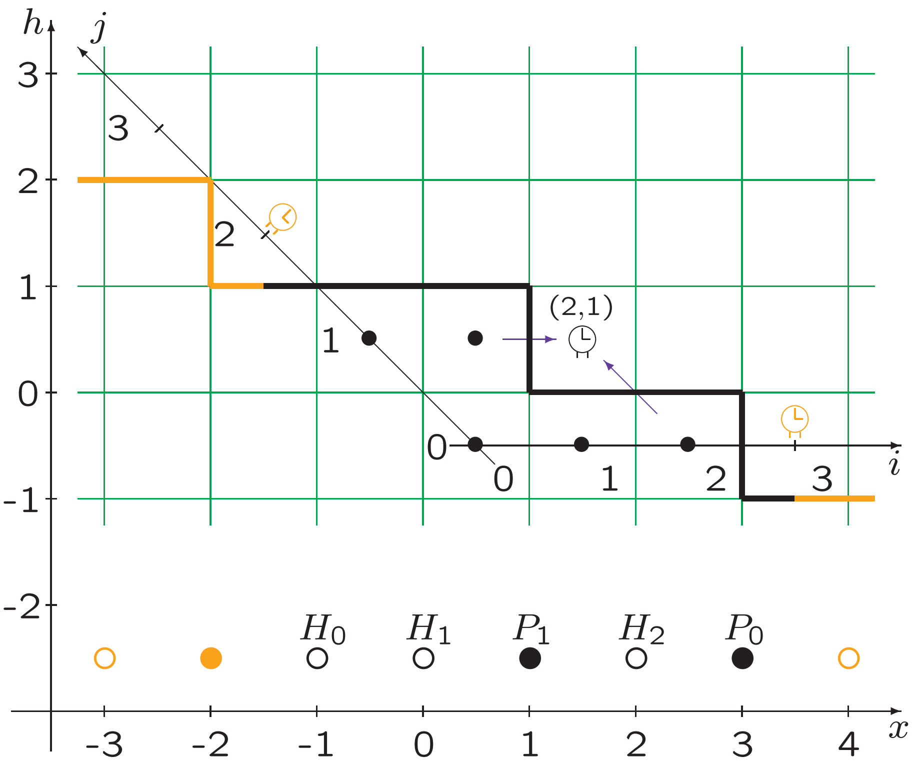
Occupation of (i, j) = jump of P_j over H_i.

Occupation of $(2, 1)$ = jump of P_1 over H_2.

The time when this happens $=: G_{ij}$.



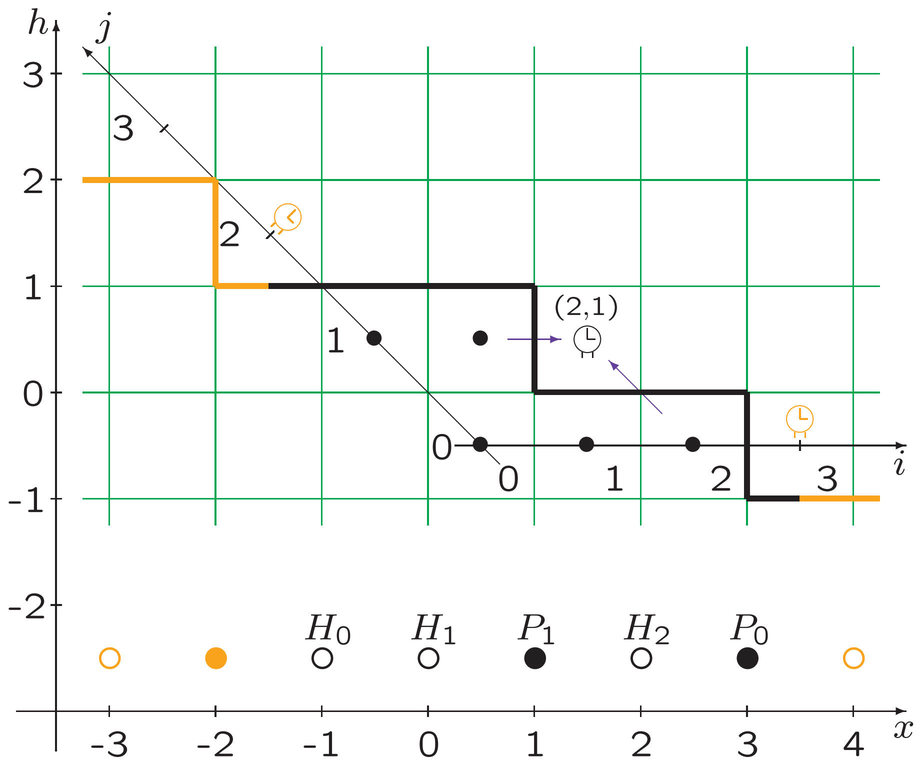
h
j
i
x
(2,1)
H_0
H_1
P_1
H_2
P_0



Burke's Theorem:

P_0 jumps according to a Poisson($1-\varrho$) process,

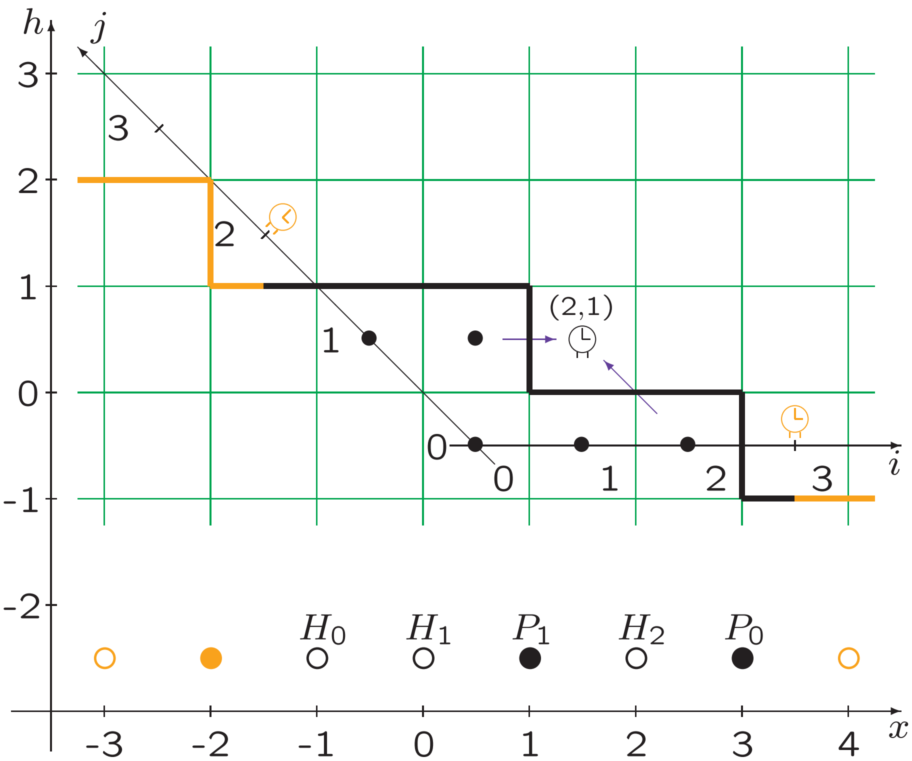
governed by the right orange part



<u>Burke's Theorem:</u>

P_0 jumps according to a Poisson($1-\varrho$) process,
governed by the right orange part

H_0 jumps according to a Poisson(ϱ) process,
governed by the left orange part



Burke's Theorem:

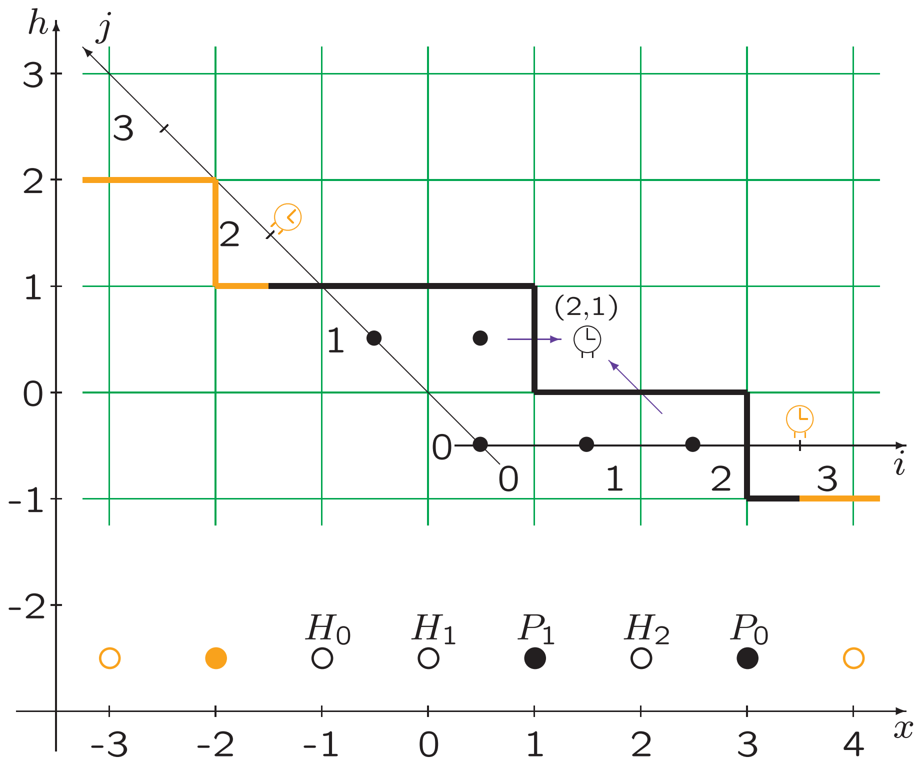
P_0 jumps according to a Poisson($1-\varrho$) process,

governed by the right orange part

H_0 jumps according to a Poisson(ϱ) process,

governed by the left orange part

independently of the ⏲'s.

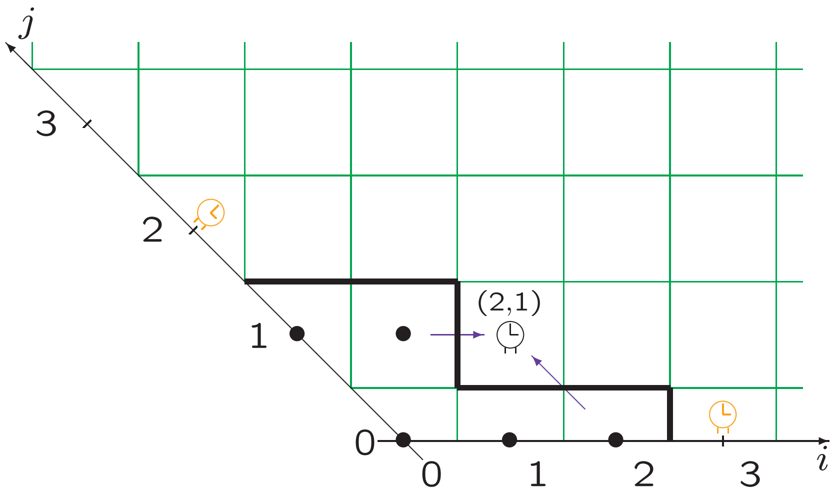


Burke's Theorem:

P_0 jumps according to a Poisson$(1-\varrho)$ process,
governed by the right orange part
H_0 jumps according to a Poisson(ϱ) process,
governed by the left orange part
independently of the ⏲'s.

Therefore:

$$\left.\begin{array}{l} ⏲ \sim \text{Exponential}(1-\varrho) \\ ⏲ \sim \text{Exponential}(\varrho) \\ ⏲ \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$



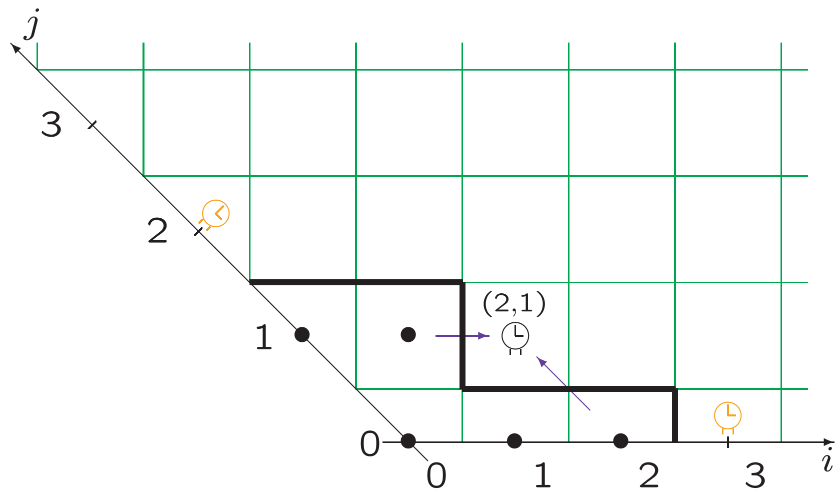
 $\sim$ Exponential$(1-\varrho)$

 $\sim$ Exponential(ϱ)

 $\sim$ Exponential(1)

(all three) independently

The last passage model



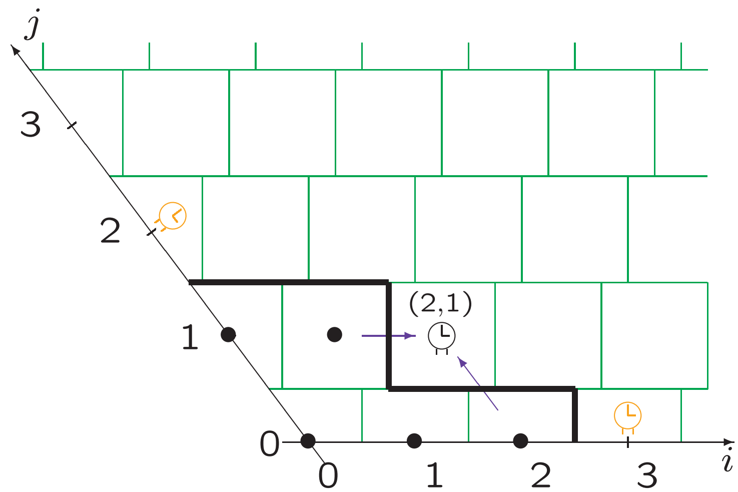
$\sim$ Exponential$(1 - \varrho)$

$\sim$ Exponential(ϱ)

$\sim$ Exponential(1)

} independently

The last passage model



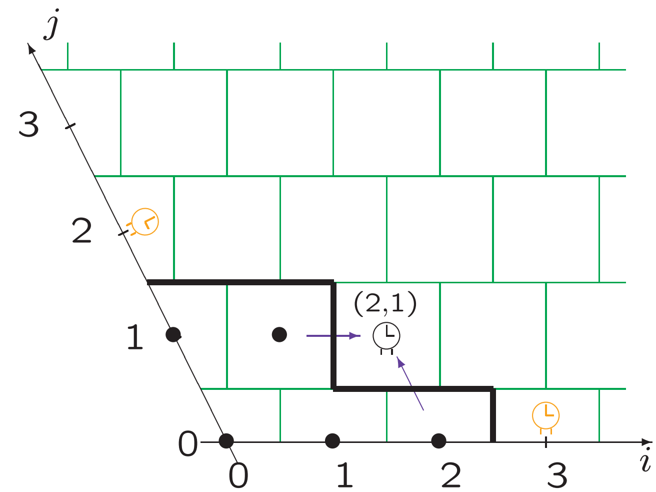
$\sim$ Exponential$(1-\varrho)$

$\sim$ Exponential(ϱ)

$\sim$ Exponential(1)

independently

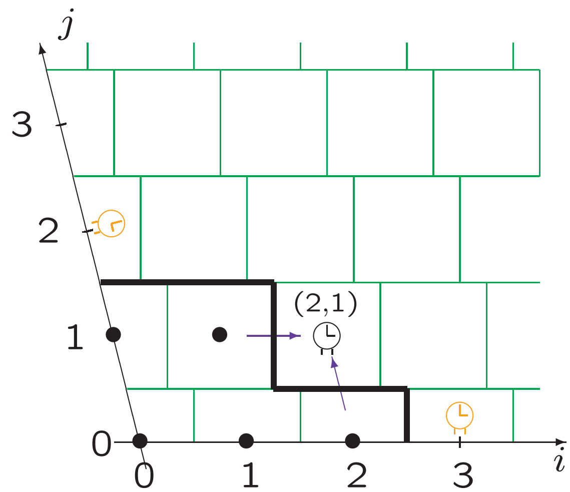
The last passage model



$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)

independently

The last passage model



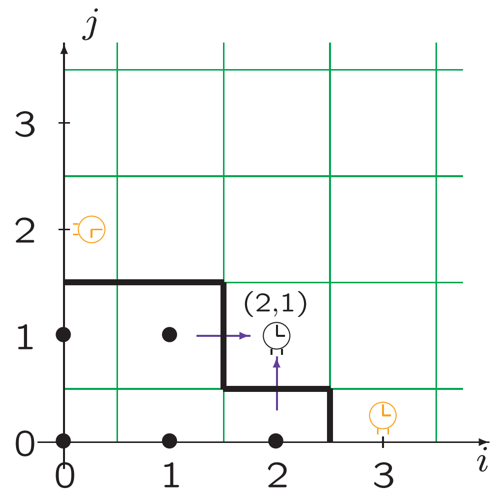
$\sim$ Exponential$(1-\varrho)$

$\sim$ Exponential(ϱ)

$\sim$ Exponential(1)

} independently

The last passage model



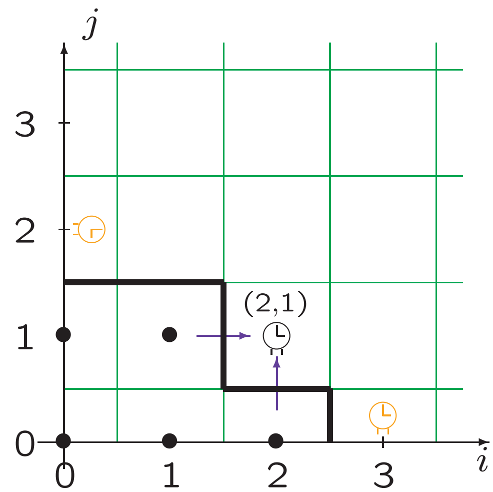
 $\sim$ Exponential($1 - \varrho$)

 $\sim$ Exponential(ϱ)

 $\sim$ Exponential(1)

} independently

The last passage model



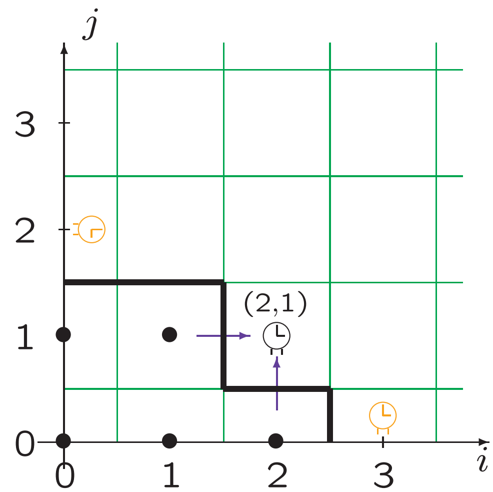
$\sim$ Exponential$(1-\varrho)$

$\sim$ Exponential(ϱ) } independently

$\sim$ Exponential(1)

starts ticking when its west neighbor becomes occupied

The last passage model



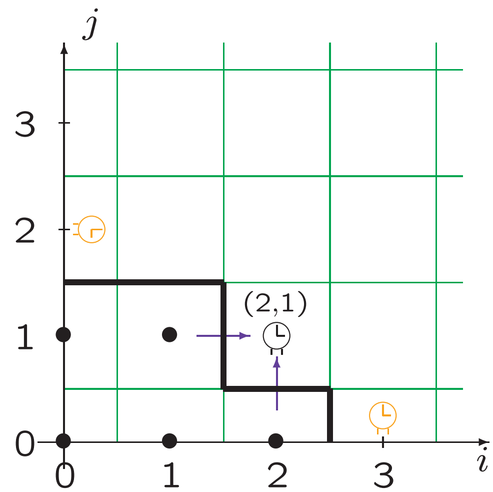
$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)

} independently

starts ticking when its west neighbor becomes occupied

starts ticking when its south neighbor becomes occupied

The last passage model



$\sim$ Exponential($1-\varrho$)
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)

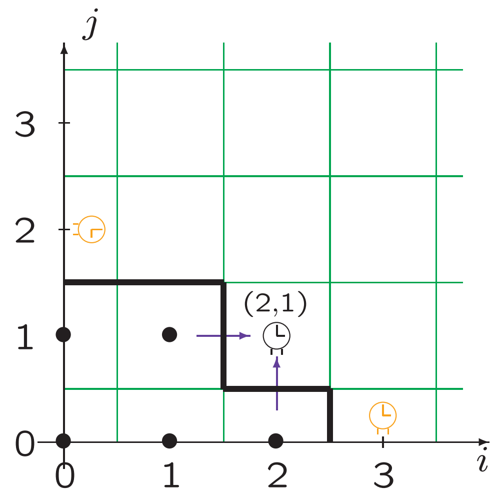
} independently

starts ticking when its west neighbor becomes occupied

starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

The last passage model



M. Prähofer and H. Spohn 2002

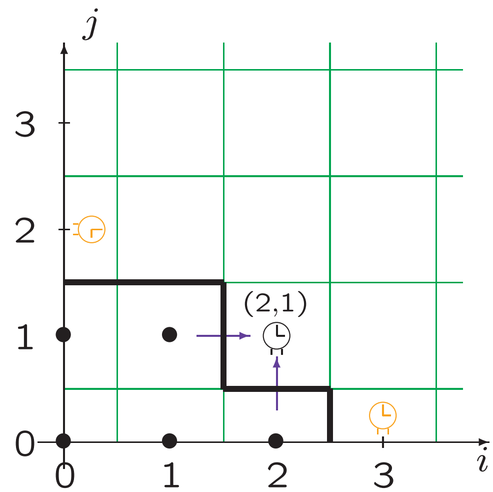
(orange clock) $\sim$ Exponential$(1-\varrho)$
(orange side clock) $\sim$ Exponential(ϱ)
(black clock) $\sim$ Exponential(1)
} independently

(orange clock) starts ticking when its west neighbor becomes occupied

(orange side clock) starts ticking when its south neighbor becomes occupied

(black clock) starts ticking when both its west and south neighbors become occupied

The last passage model



M. Prähofer and H. Spohn 2002

$$\left.\begin{array}{l} \sim \text{Exponential}(1-\varrho) \\ \sim \text{Exponential}(\varrho) \\ \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$

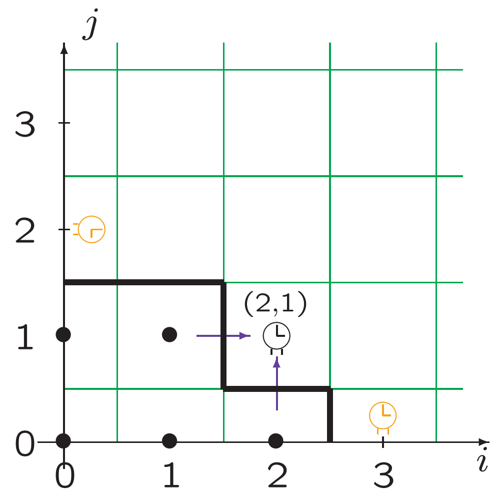
starts ticking when its west neighbor becomes occupied

starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i, j)

The last passage model



M. Prähofer and H. Spohn 2002

$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)
} independently

starts ticking when its west neighbor becomes occupied

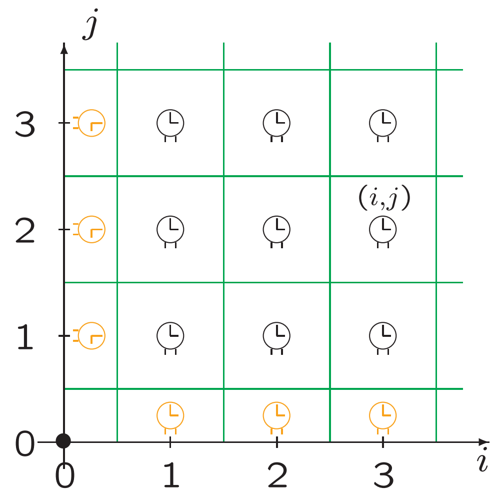
starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i,j)

G_{ij} = the maximum weight collected by a north-east path from $(0,0)$ to (i,j).

The last passage model



M. Prähofer and H. Spohn 2002

 $\sim$ Exponential$(1-\varrho)$
 $\sim$ Exponential(ϱ)
 $\sim$ Exponential(1)

} independently

 starts ticking when its west neighbor becomes occupied

 starts ticking when its south neighbor becomes occupied

 starts ticking when both its west and south neighbors become occupied

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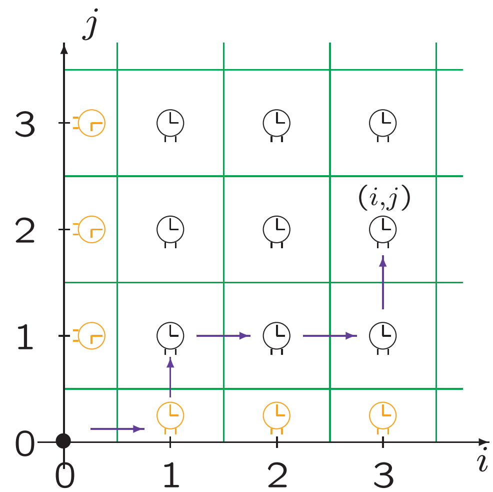
The last passage model



M. Prähofer and H. Spohn 2002

$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ) } independently
$\sim$ Exponential(1)

starts ticking when its west neighbor becomes occupied

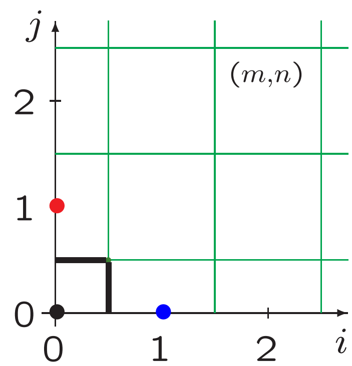
starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i, j)

G_{ij} = the maximum weight collected by a north-east path from $(0, 0)$ to (i, j).

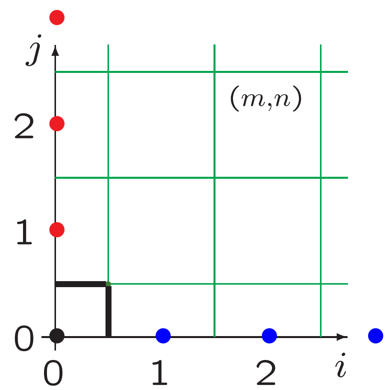
The competition interface



Ferrari, Martin, Pimentel (2005)

Which squares are infected via $(1,0)$ and via $(0,1)$?

The competition interface



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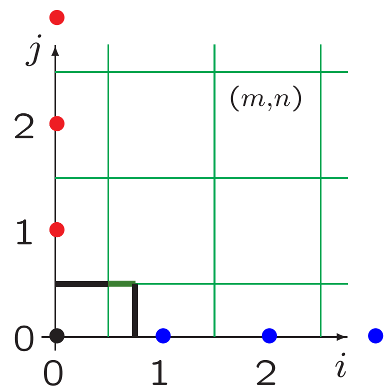

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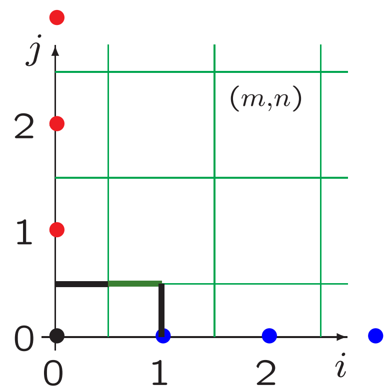

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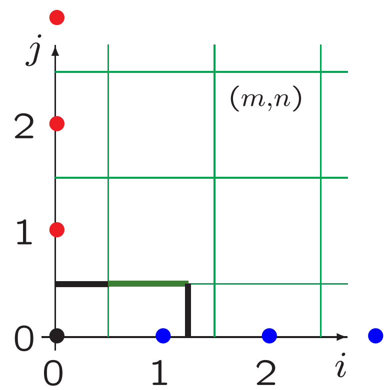

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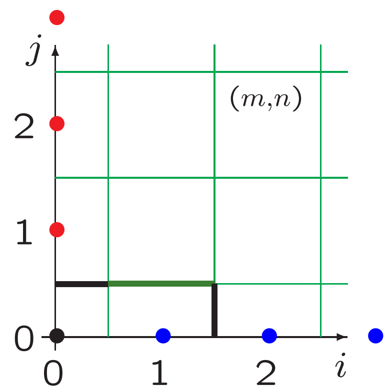

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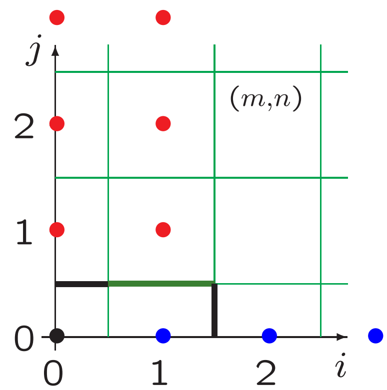

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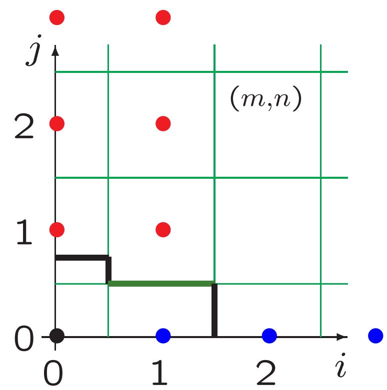

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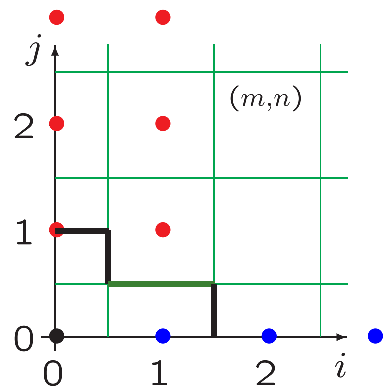

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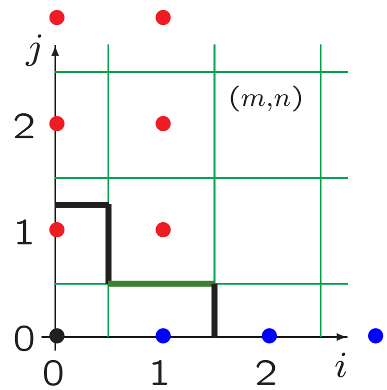

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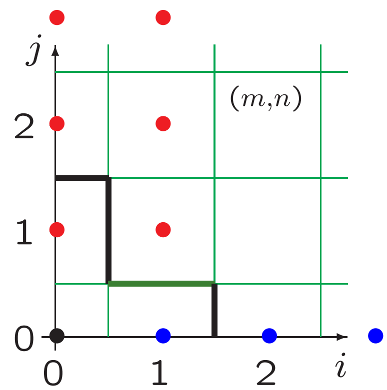
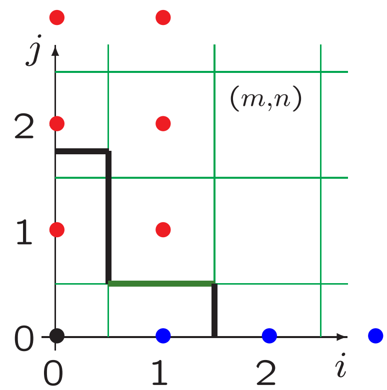


Ferrari, Martin, Pimentel (2005)

Which squares are infected via $(1, 0)$ and via $(0, 1)$?

The competition interface



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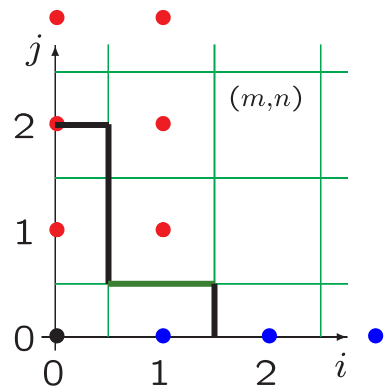

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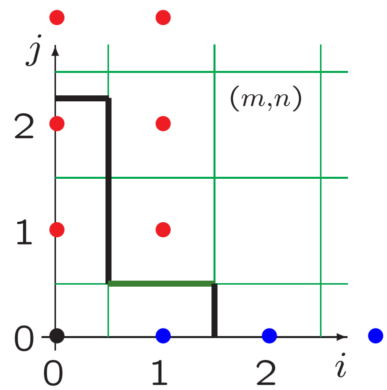

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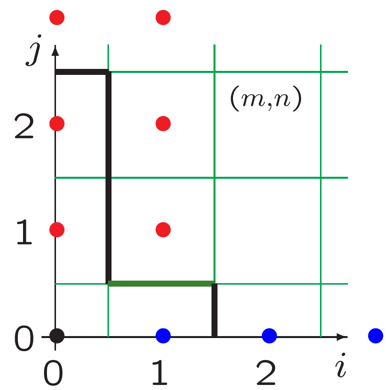

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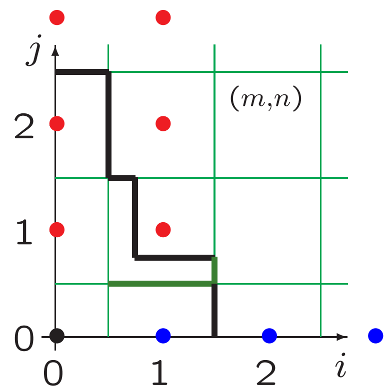

Ferrari, Martin, Pimentel (2005)

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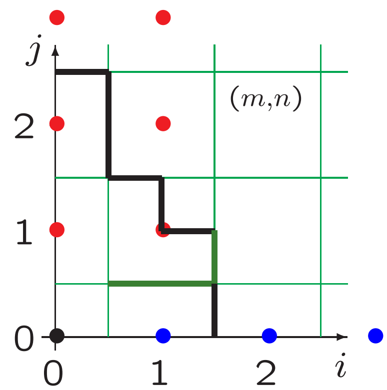
The competition interface



Ferrari, Martin, Pimentel (2005)

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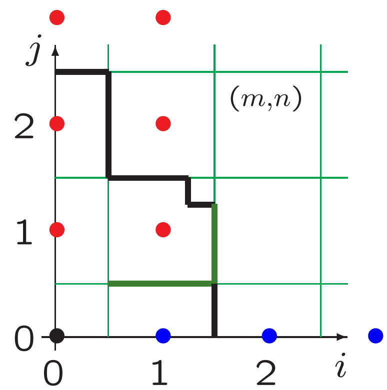
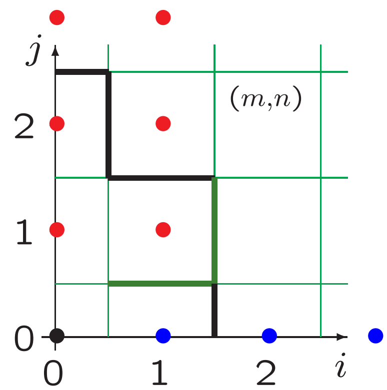
The competition interface



Ferrari, Martin, Pimentel (2005)

Which squares are infected via $(1, 0)$ and via $(0, 1)$?

The competition interface



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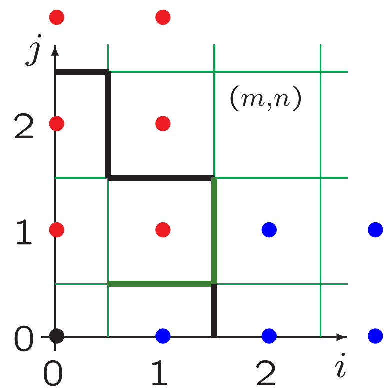

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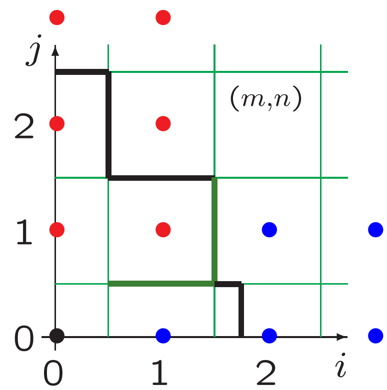

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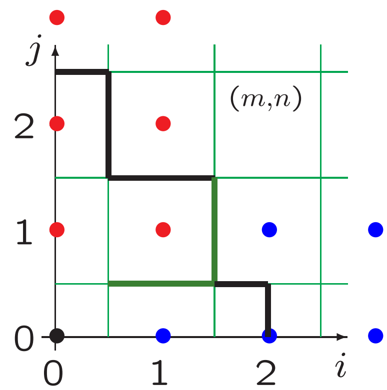

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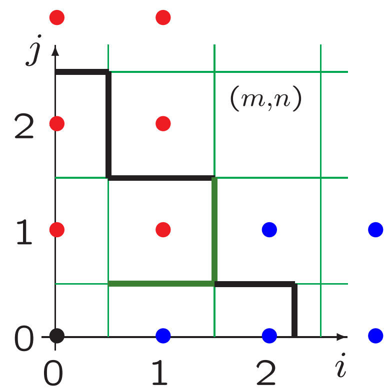

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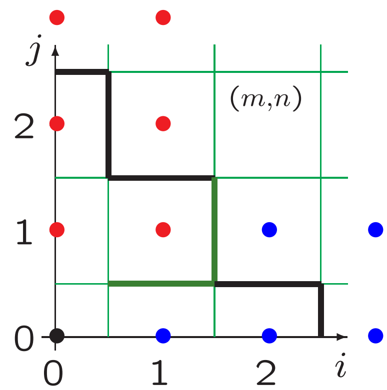

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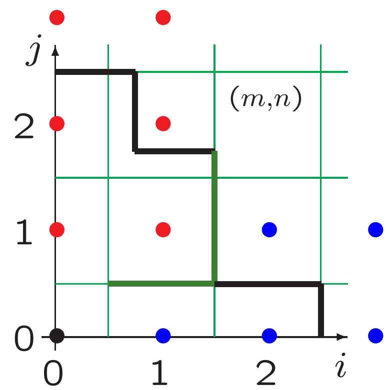

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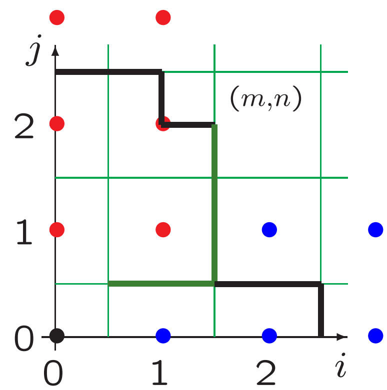

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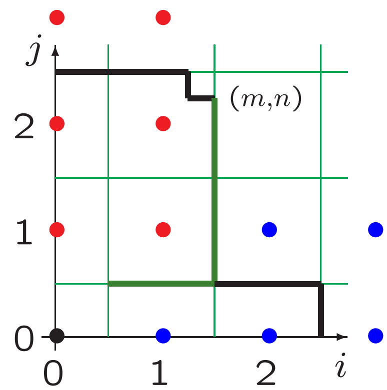

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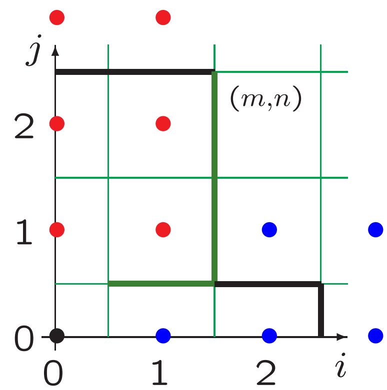

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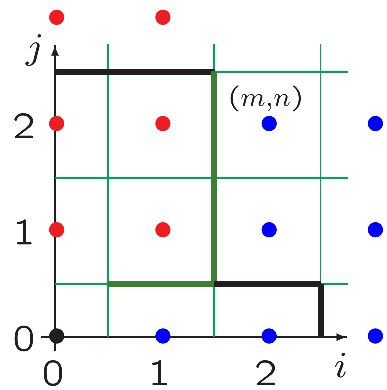

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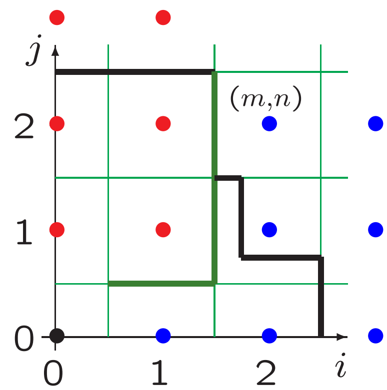

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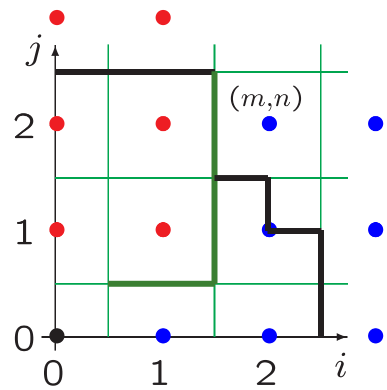

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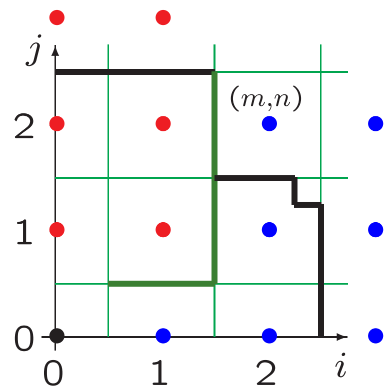

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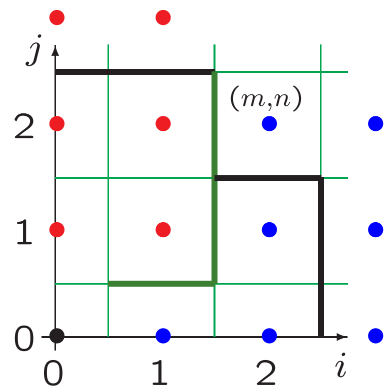

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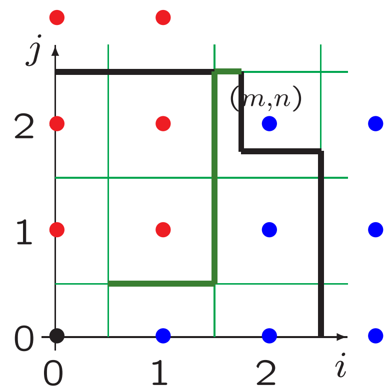

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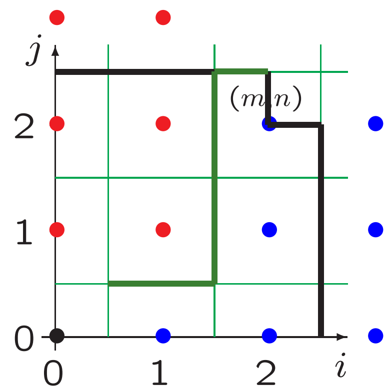

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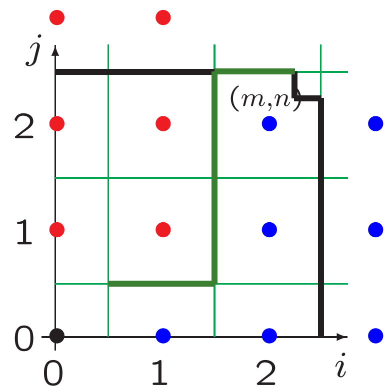

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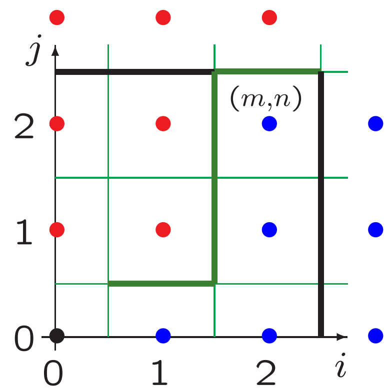

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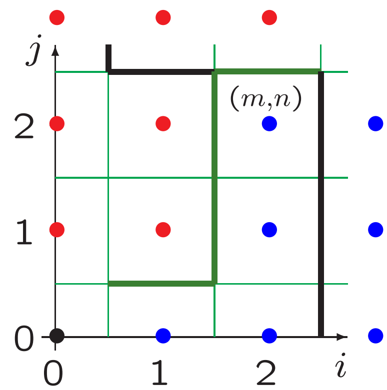

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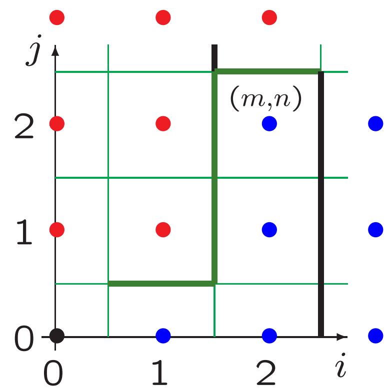

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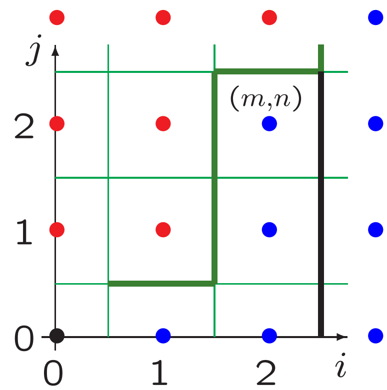

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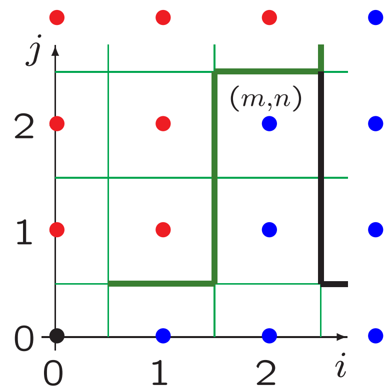

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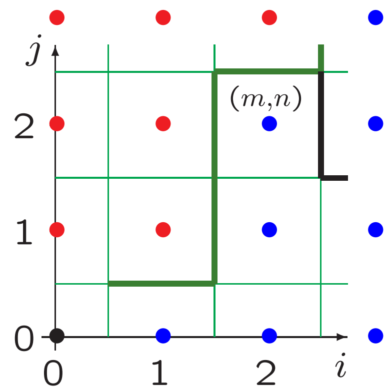

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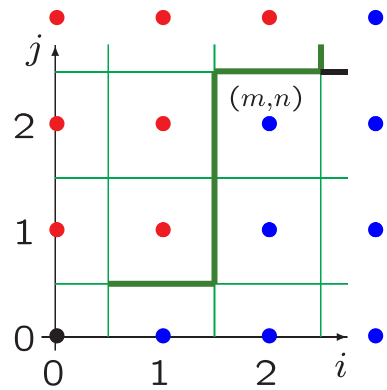

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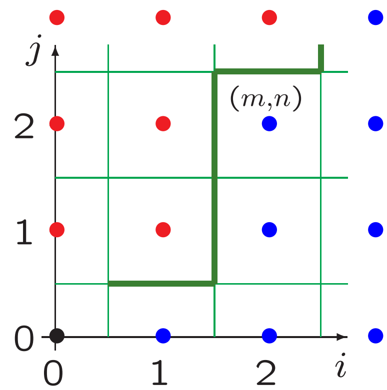

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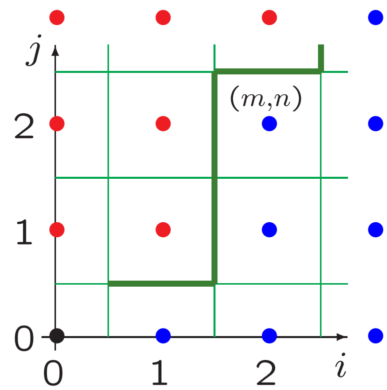

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The competition interface follows the same rules as the *second class particle* of simple exclusion.

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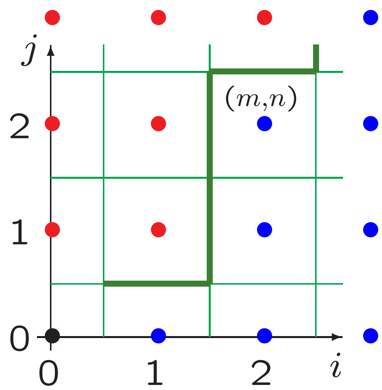

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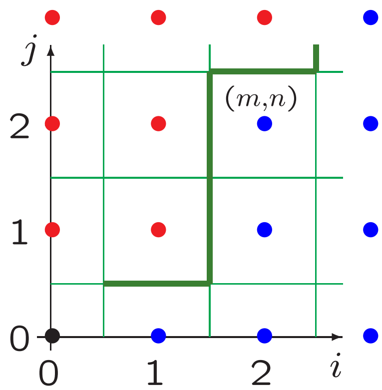
Which squares are infected via $(1,0)$ and via $(0,1)$?

The competition interface follows the same rules as the *second class particle* of simple exclusion.

If it passes left of (m,n), then G_{mn} is not sensitive to decreasing the  weights on the j-axis.

If it passes below (m,n), then G_{mn} is not sensitive to decreasing the  weights on the i-axis.

The competition interface



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Thank you.