## TASEP and the corner growth model

## TASEP: Interacting particles



$$
\text { Bernoulli( } \varrho) \text { distribution }
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## TASEP: Interacting particles



## Bernoulli( $\varrho$ ) distribution

(particle, hole) pairs become (hole, particle) pairs with rate 1.

## TASEP: Interacting particles



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Particles try to jump to the right, but block each other.

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The Bernoulli( $\varrho$ ) distribution is time-stationary for any $(0 \leq \varrho \leq 1)$. Any translation-invariant stationary distribution is a mixture of Bernoullis.

## TASEP: Surface growth



Bernoulli( $\varrho$ ) distribution

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Bernoulli( $\varrho$ ) distribution

TASEP: Last passage percolation


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Occupation of $(i, j)=$ jump of $P_{j}$ over $H_{i}$. Occupation of $(2,1)=$ jump of $P_{1}$ over $H_{2}$.


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The time when this happens $=: G_{i j}$.



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Therefore:

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\left.\begin{array}{rl}
Q & \sim \text { Exponential }(1-\varrho) \\
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\end{array}\right\} \text { independently }
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The last passage model


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© starts ticking when its west neighbor becomes occupied

## The last passage model



Q Starts ticking when its west neighbor becomes occupied
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M. Prähofer and H. Spohn 2002

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## The competition interface



Ferrari, Martin, Pimentel (2005)
Which squares are infected via $(1,0)$ and via $(0,1)$ ?

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If it passes left of $(m, n)$, then $G_{m n}$ is not sensitive to decreasing the weights on the $j$-axis. If it passes below $(m, n)$, then $G_{m n}$ is not sensitive to decreasing the $\otimes$ weights on the $i$-axis.

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## Thank you.

