Second class particles can perform simple random walks (in some cases)

Joint with
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Márton Balázs

Budapest University of Technology and Economics

BME Stochastics Seminar
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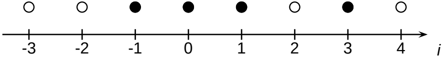
Asymmetric simple exclusion



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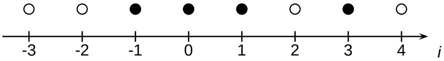

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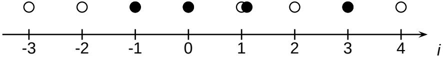

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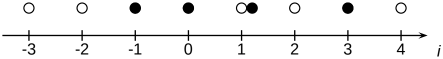

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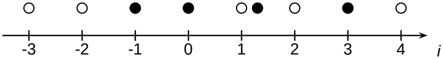

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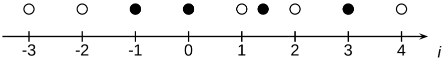

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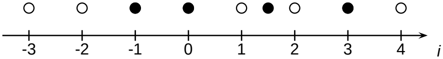

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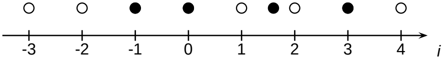

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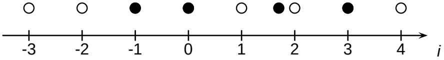

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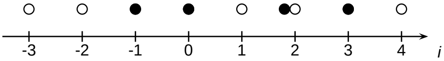

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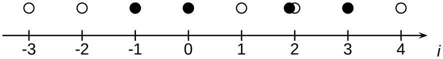

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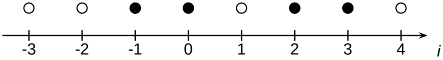

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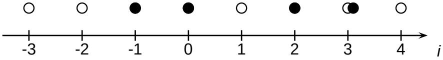

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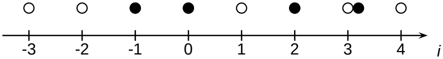

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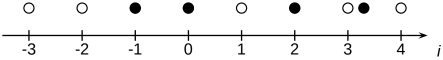

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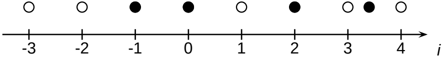

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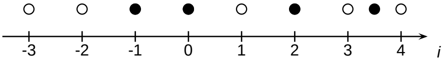

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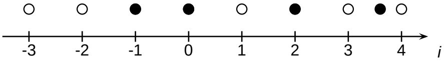

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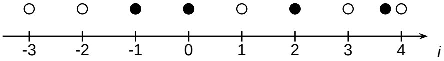

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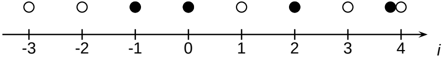

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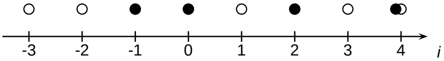

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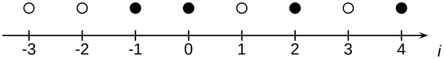

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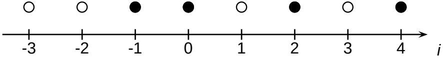

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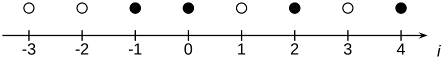

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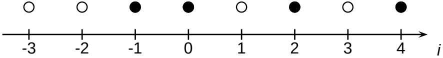

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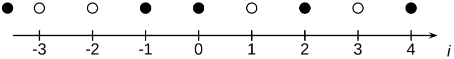

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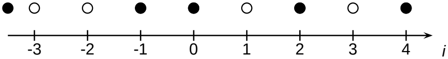

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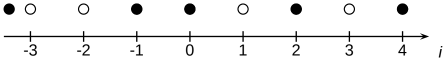

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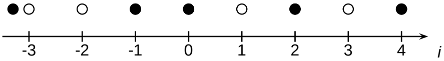

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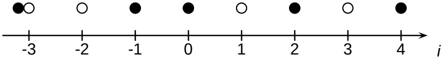

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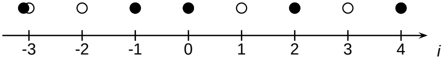

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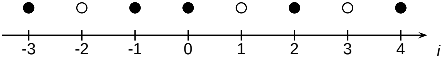

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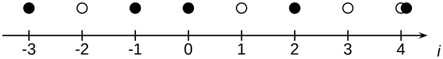

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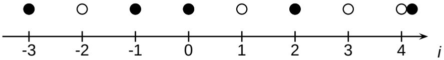

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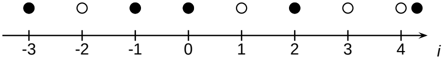

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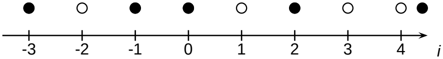

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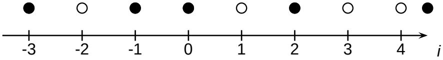

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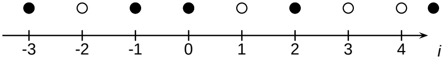

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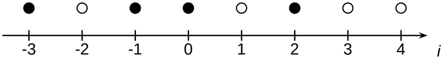

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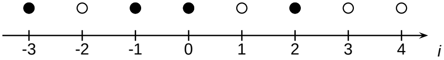

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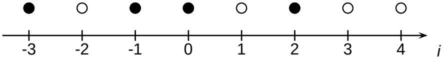

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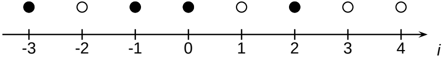

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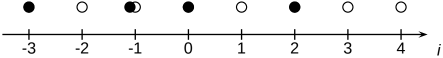

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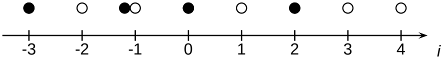

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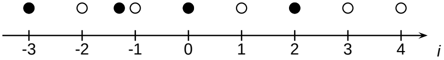

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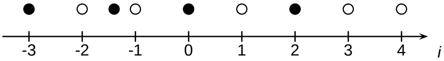

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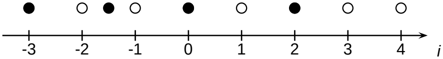

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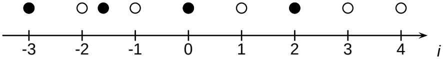

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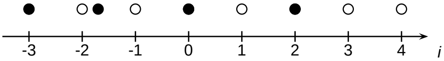

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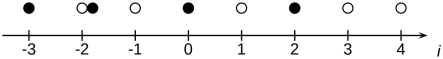

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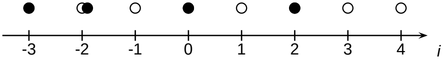

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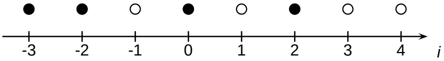
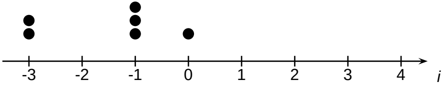


Particles try to jump

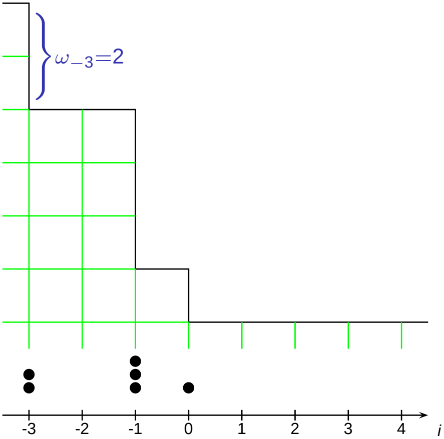
to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

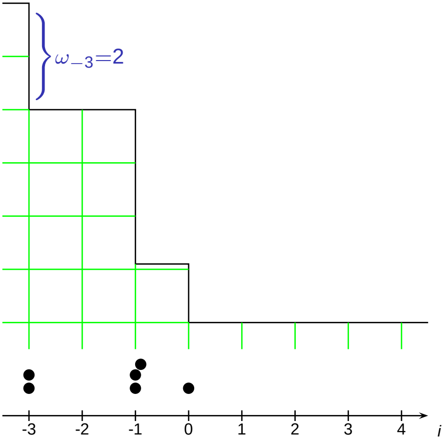
The totally asymmetric zero range process



The totally asymmetric zero range process



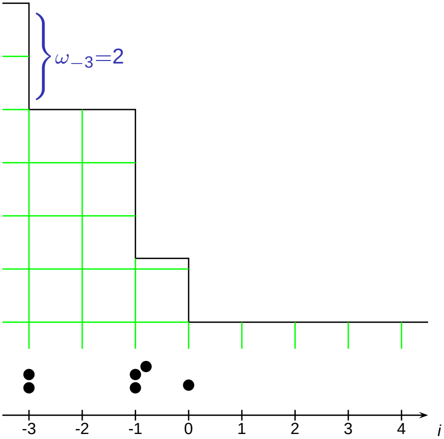
The totally asymmetric zero range process



Particles jump to the right from site i with rate $r(\omega_i)$

(r non-decreasing).

The totally asymmetric zero range process



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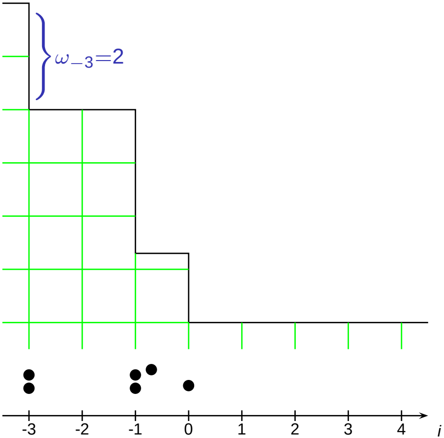
The totally asymmetric zero range process



Particles jump to the right from site i with rate $r(\omega_i)$

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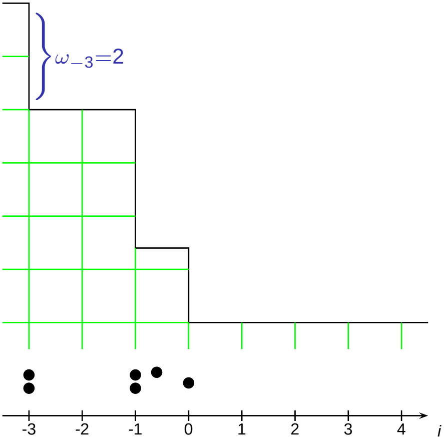
The totally asymmetric zero range process



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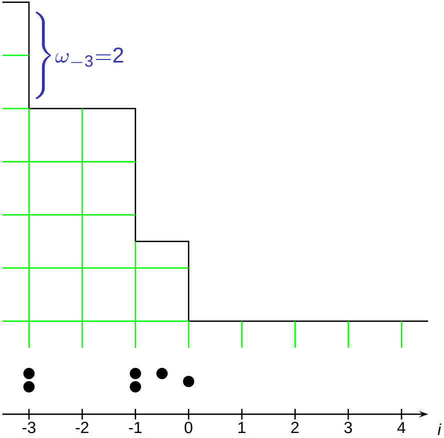
The totally asymmetric zero range process



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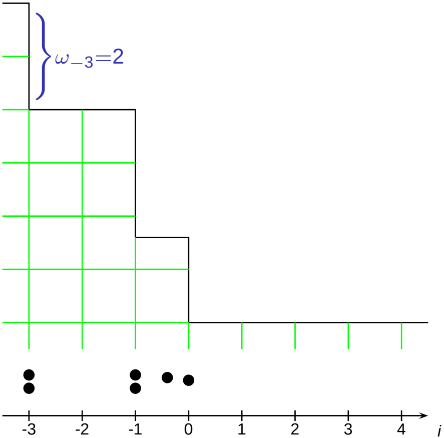
The totally asymmetric zero range process



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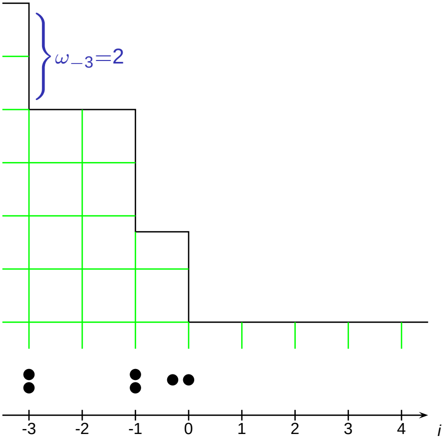
The totally asymmetric zero range process



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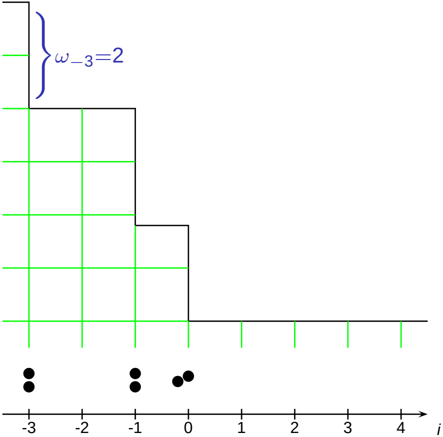
The totally asymmetric zero range process



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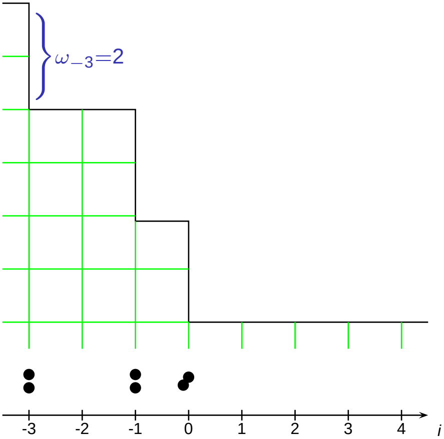
The totally asymmetric zero range process



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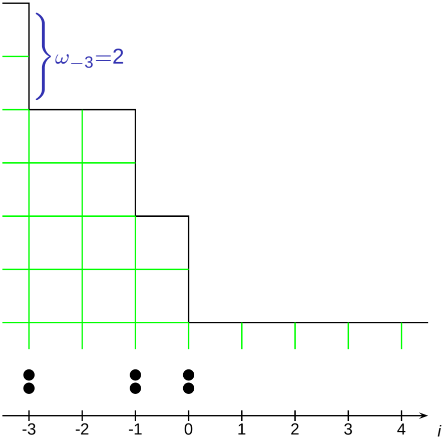
The totally asymmetric zero range process



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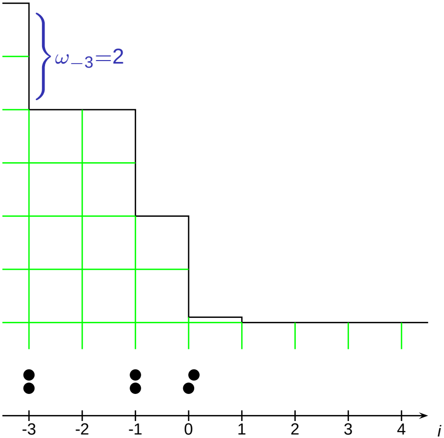
(r non-decreasing).

The totally asymmetric zero range process



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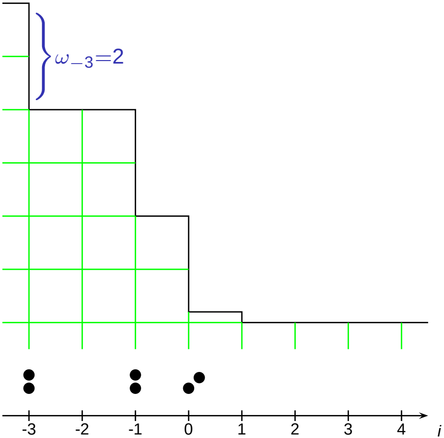
The totally asymmetric zero range process



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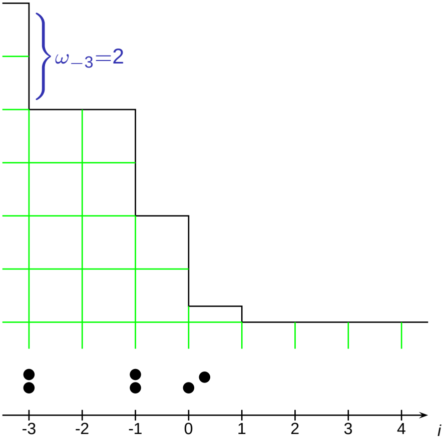
The totally asymmetric zero range process



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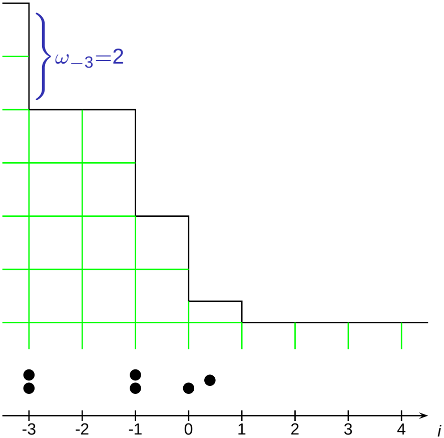

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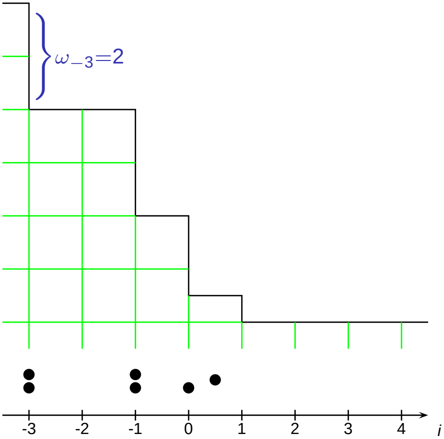

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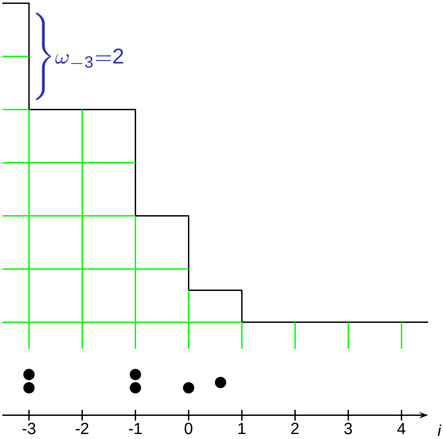

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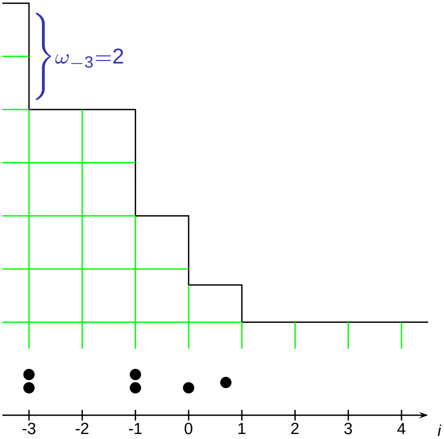
The totally asymmetric zero range process



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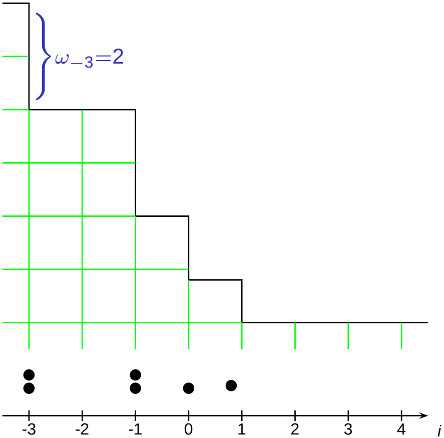

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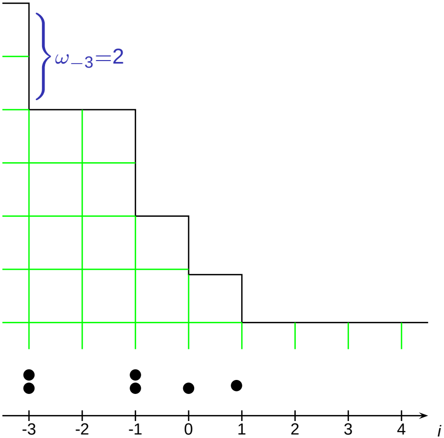
The totally asymmetric zero range process



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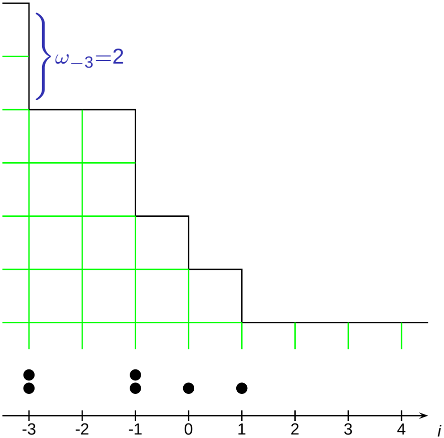
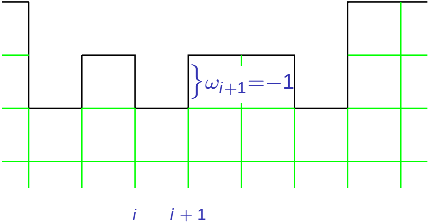


Particles jump to the right from site i with rate $r(\omega_i)$

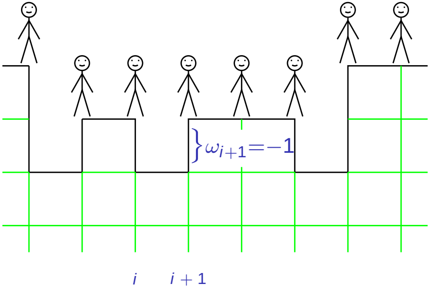
(r non-decreasing).

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \| \begin{pmatrix}1\\ -1\end{pmatrix}$$

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$

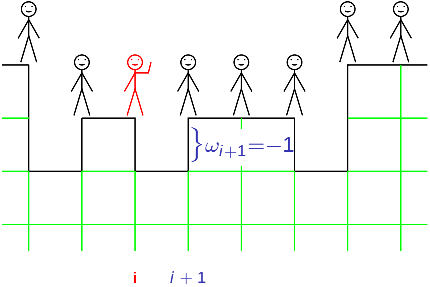


$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ \| \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $r(\omega_i)$.

(r non-decreasing).

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$

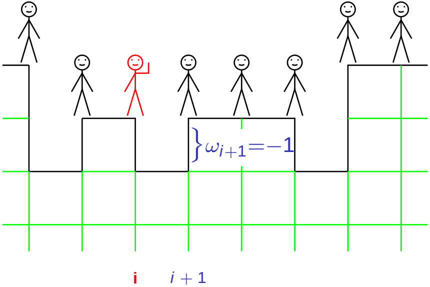


$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \,\|\, \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $\mathbf{r}(\omega_\mathbf{i})$.

(r non-decreasing).

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



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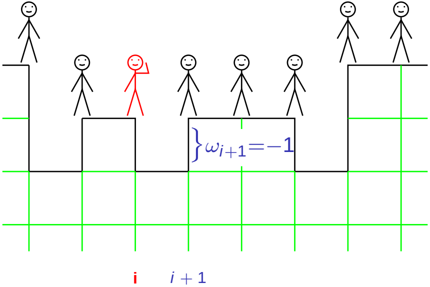

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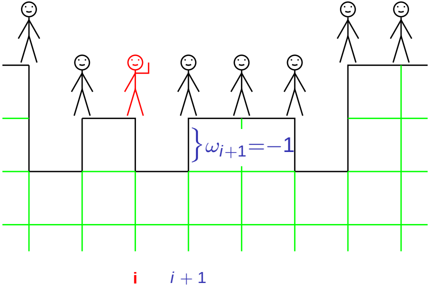

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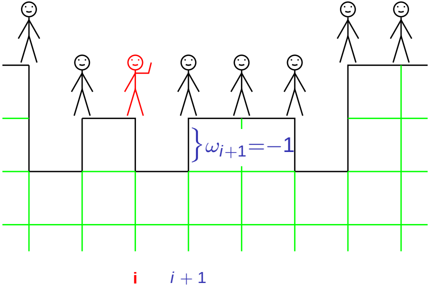

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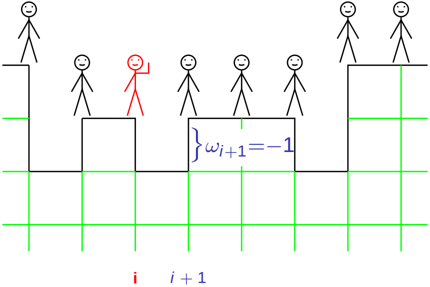

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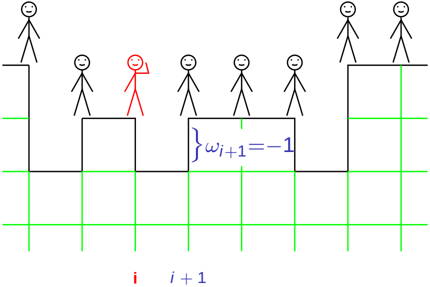

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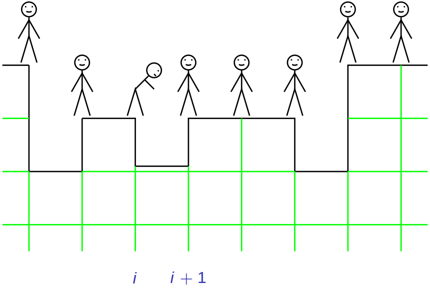
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

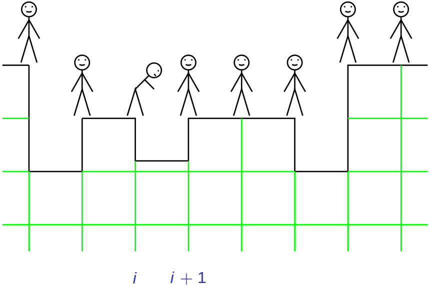
$$\| \qquad\qquad \|$$

$$\begin{pmatrix}1\\ -1\end{pmatrix} \to \begin{pmatrix}0\\ 0\end{pmatrix}$$

a brick is added with rate $r(\omega_i)$.

(r non-decreasing).

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1} + 1\end{pmatrix}$$

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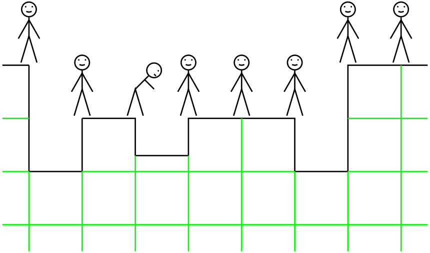


$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1} + 1\end{pmatrix}$$

$$\| \qquad\qquad \|$$

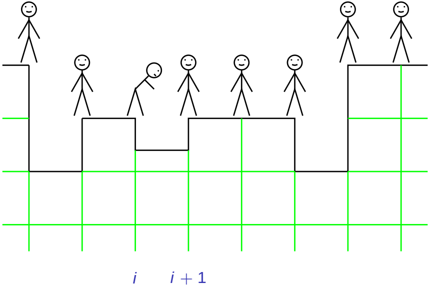
$$\begin{pmatrix}1\\ -1\end{pmatrix} \to \begin{pmatrix}0\\ 0\end{pmatrix}$$

i $\quad$ $i+1$

a brick is added with rate $r(\omega_i)$.

(r non-decreasing).

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$

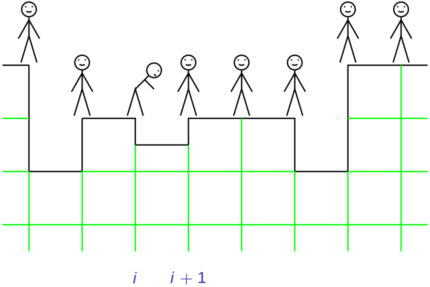


$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \rightarrow \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$
$$\begin{pmatrix}1\\ -1\end{pmatrix} \rightarrow \begin{pmatrix}0\\ 0\end{pmatrix}$$

a brick is added with rate $r(\omega_i)$.

(r non-decreasing).

A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

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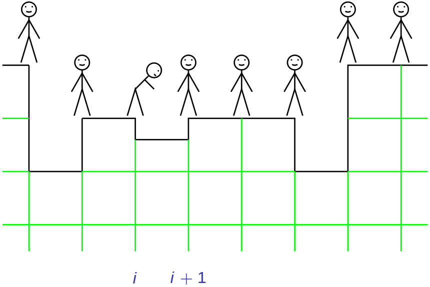
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



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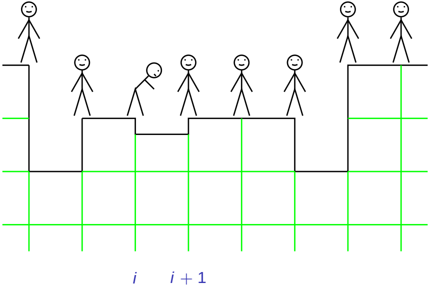

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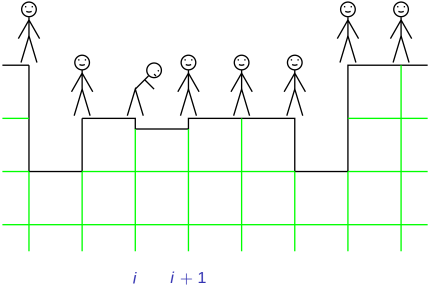
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



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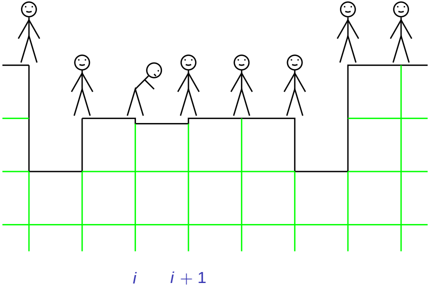
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}$$
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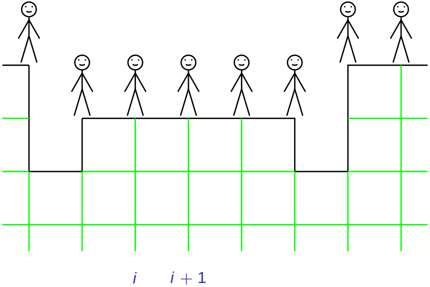

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1} + 1\end{pmatrix}$$

$$\Vert \qquad\qquad\qquad \Vert$$

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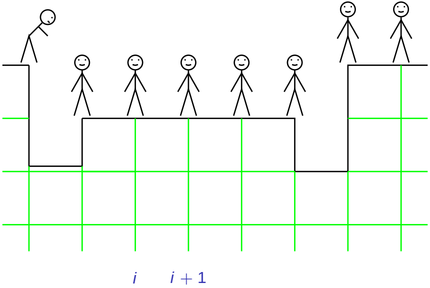
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



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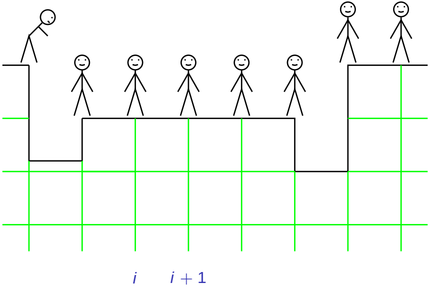
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \rightarrow \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\begin{matrix} \| & & \| \\ \begin{pmatrix}1\\ -1\end{pmatrix} & \rightarrow & \begin{pmatrix}0\\ 0\end{pmatrix}\end{matrix}$$

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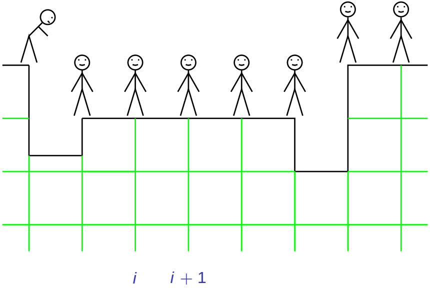

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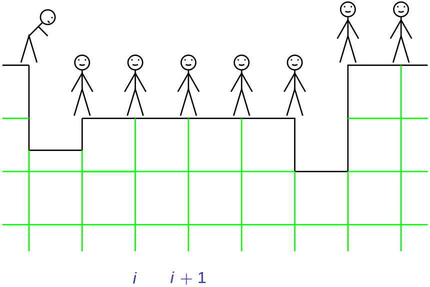
A generalized totally asymmetric zero range process: $\omega_i \in \mathbb{Z}$



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\begin{matrix} \| & & \| \\ \begin{pmatrix}1\\ -1\end{pmatrix} & \to & \begin{pmatrix}0\\ 0\end{pmatrix}\end{matrix}$$

a brick is added with rate $r(\omega_i)$.

(r non-decreasing).

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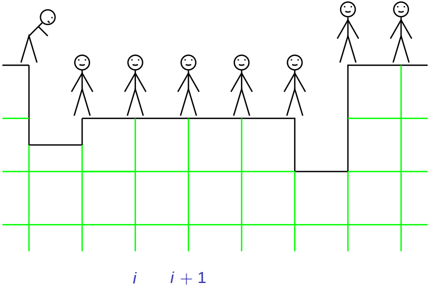

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$
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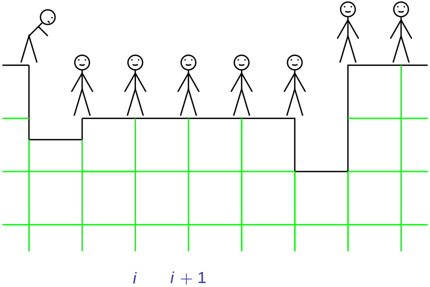

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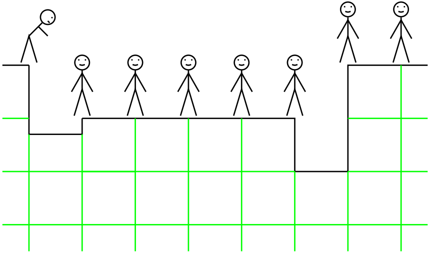

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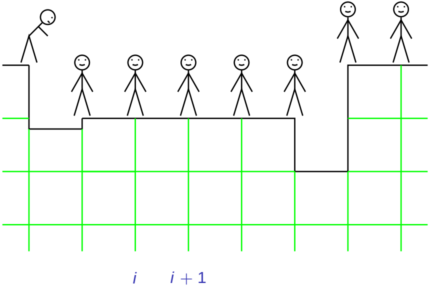

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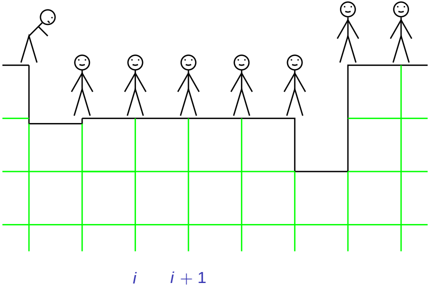

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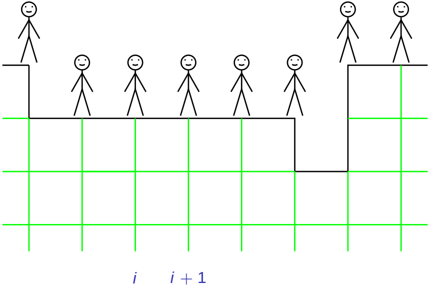

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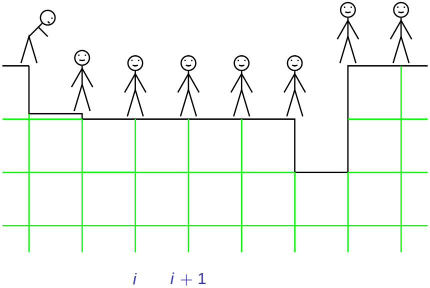

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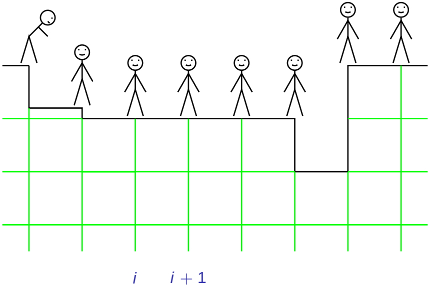

$$\begin{array}{ccc}\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} & \longrightarrow & \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}\\ \| & & \| \\ \begin{pmatrix}1\\ -1\end{pmatrix} & \longrightarrow & \begin{pmatrix}0\\ 0\end{pmatrix}\end{array}$$

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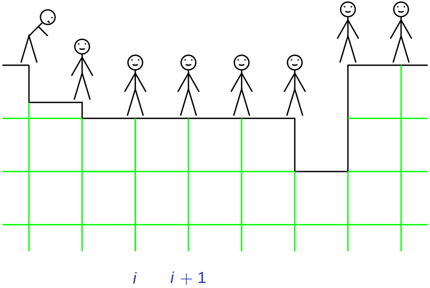

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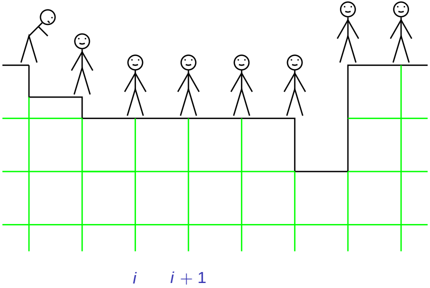

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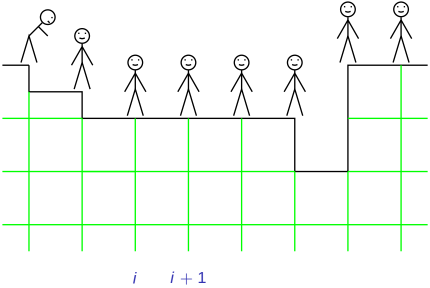

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1} + 1\end{pmatrix}$$

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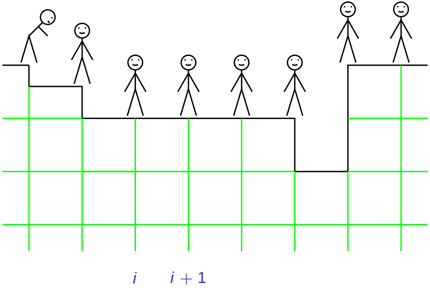

$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \to \begin{pmatrix} \omega_i - 1 \\ \omega_{i+1} + 1 \end{pmatrix}$$

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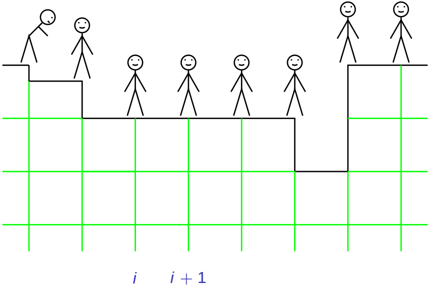

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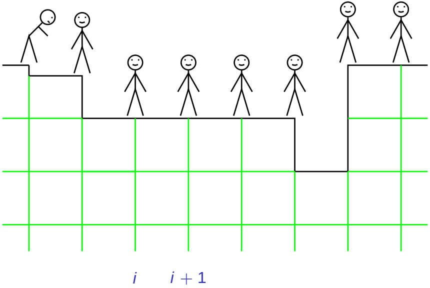

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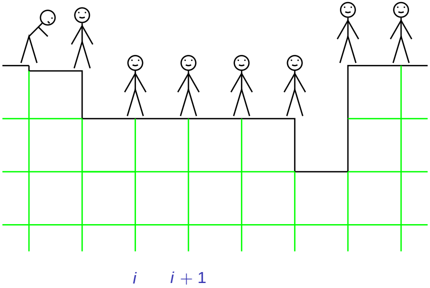

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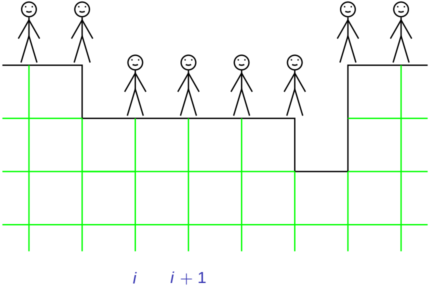

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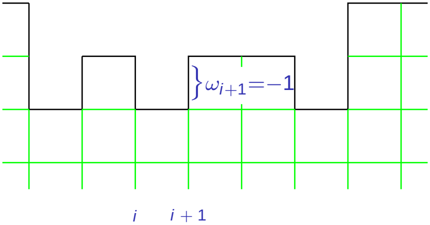
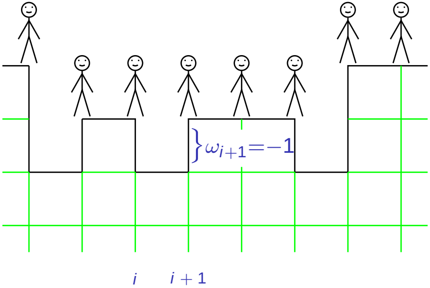
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix}$$
$$\|$$
$$\begin{pmatrix}1\\ -1\end{pmatrix}$$

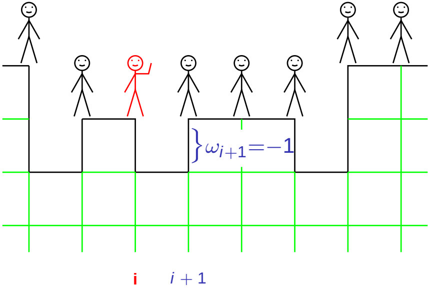
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \,\|\, \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $r(\omega_i)+r(-\omega_{i+1})$.

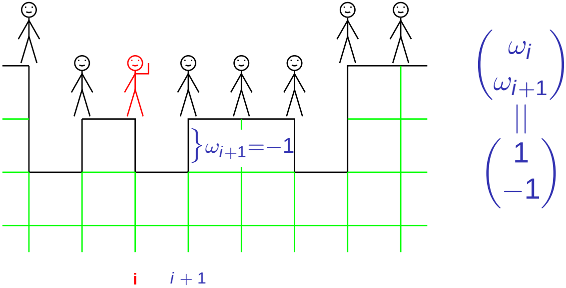
The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \,\|\, \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $\mathbf{r}(\omega_\mathbf{i}) + r(-\omega_{i+1})$.

The totally asymmetric bricklayers process



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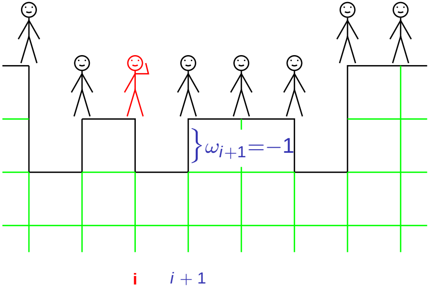

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \Vert \begin{pmatrix}1\\ -1\end{pmatrix}$$

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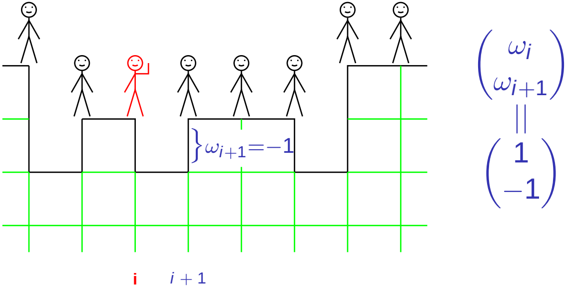

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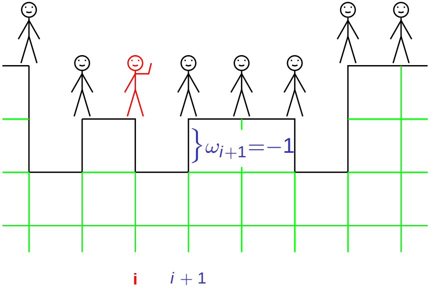
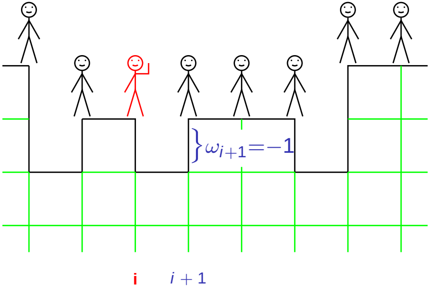


$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $\mathbf{r}(\omega_{\mathbf{i}}) + r(-\omega_{i+1})$.

The totally asymmetric bricklayers process



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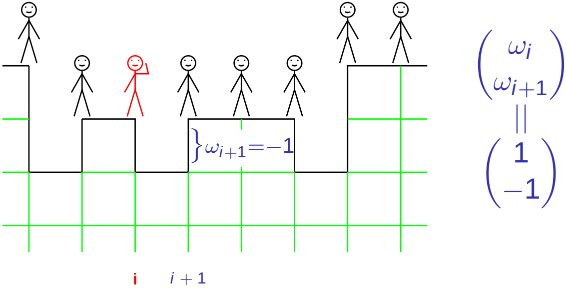

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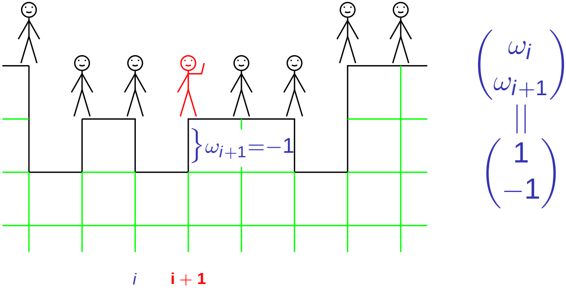

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

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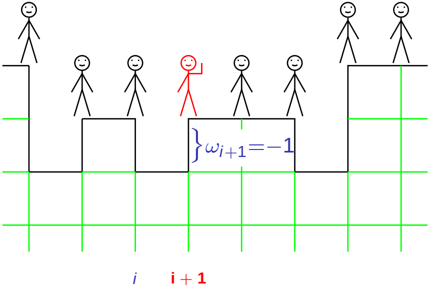

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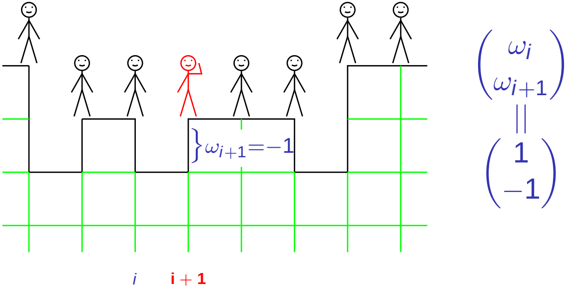

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ || \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

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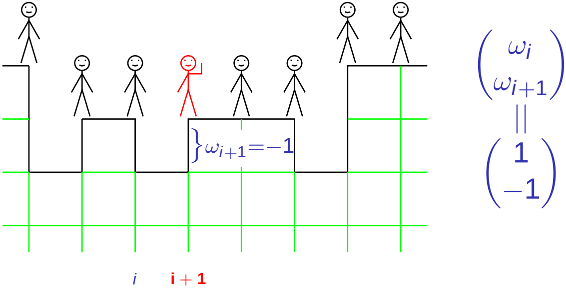
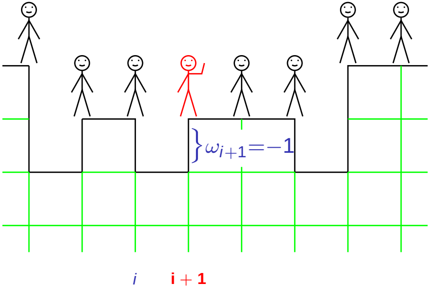


a brick is added with rate $r(\omega_i)+\mathbf{r}(-\omega_{\mathbf{i+1}})$.

The totally asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \Big\| \begin{pmatrix}1\\ -1\end{pmatrix}$$

a brick is added with rate $r(\omega_i) + \mathbf{r(-\omega_{i+1})}$.

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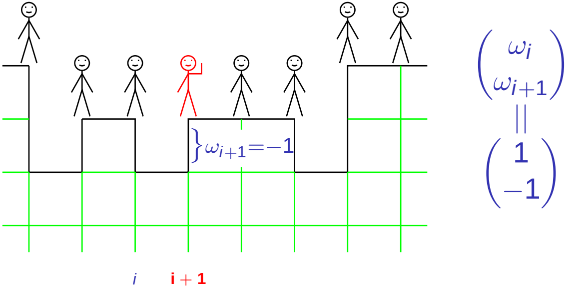

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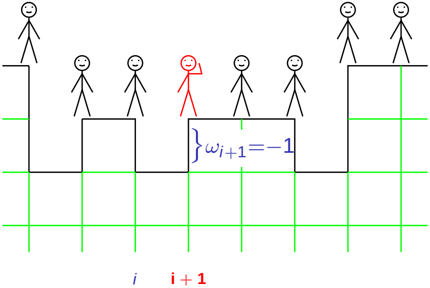

$$\begin{array}{c}\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix}\\ \| \\ \begin{pmatrix}1\\ -1\end{pmatrix}\end{array}$$

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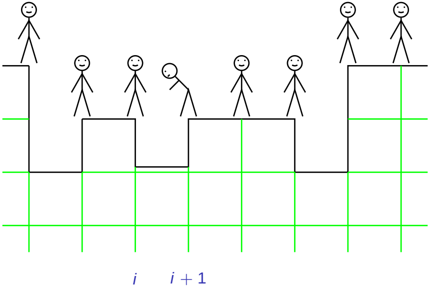

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\|\qquad\qquad\qquad\|$$

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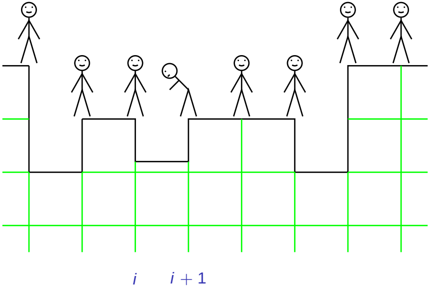

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$$\| \qquad\qquad \|$$

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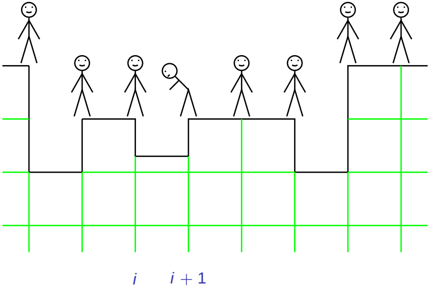
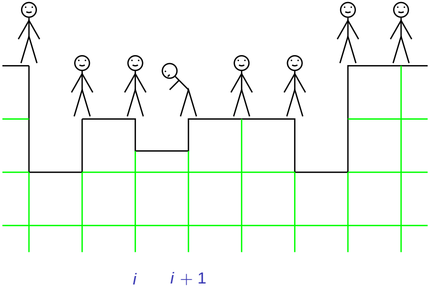


$$\begin{array}{ccc} \begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} & \rightarrow & \begin{pmatrix} \omega_i - 1 \\ \omega_{i+1} + 1 \end{pmatrix} \\ \| & & \| \\ \begin{pmatrix} 1 \\ -1 \end{pmatrix} & \rightarrow & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{array}$$

a brick is added with rate $r(\omega_i) + r(-\omega_{i+1})$.

The totally asymmetric bricklayers process



$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \to \begin{pmatrix} \omega_i - 1 \\ \omega_{i+1} + 1 \end{pmatrix}$$

$$\begin{matrix} \| & & \| \end{matrix}$$

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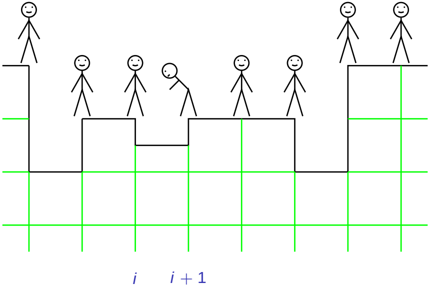

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

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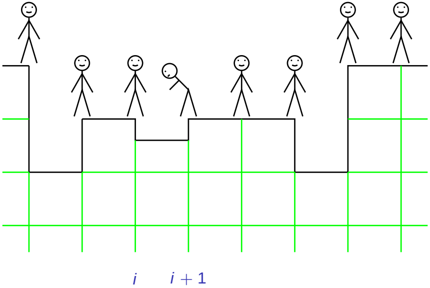

$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \to \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix}$$

$$\Vert \qquad\qquad \Vert$$

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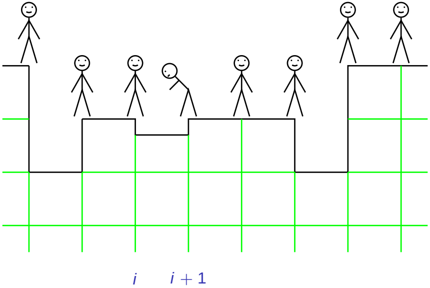
The totally asymmetric bricklayers process



$$\begin{array}{ccc} \begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} & \to & \begin{pmatrix}\omega_i - 1\\ \omega_{i+1}+1\end{pmatrix} \\ \| & & \| \\ \begin{pmatrix}1\\ -1\end{pmatrix} & \to & \begin{pmatrix}0\\ 0\end{pmatrix} \end{array}$$

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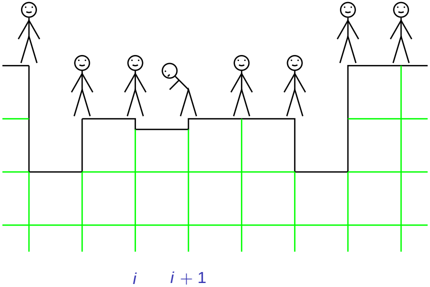

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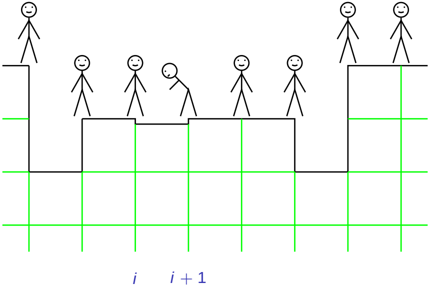

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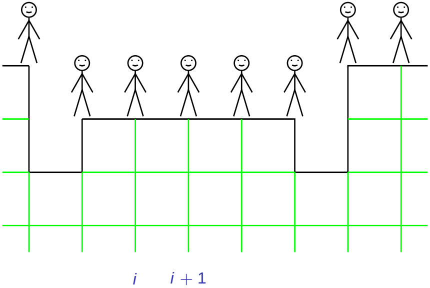

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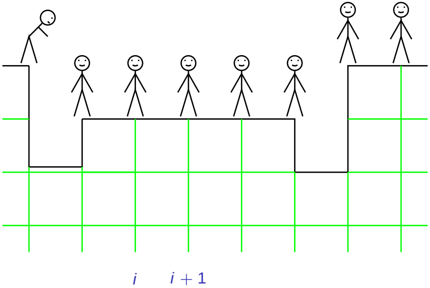

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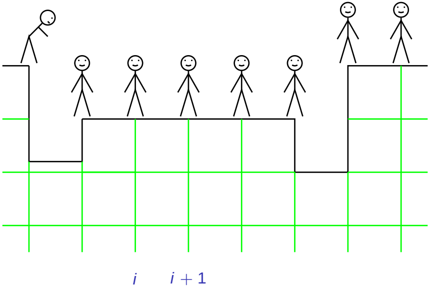

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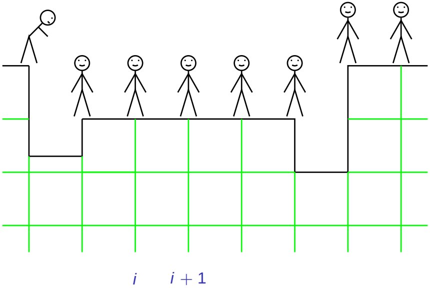

$$\begin{tikzcd} \begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \arrow[r] \arrow[d, equal] & \begin{pmatrix}\omega_i-1\\ \omega_{i+1}+1\end{pmatrix} \arrow[d, equal] \\ \begin{pmatrix}1\\ -1\end{pmatrix} \arrow[r] & \begin{pmatrix}0\\ 0\end{pmatrix} \end{tikzcd}$$

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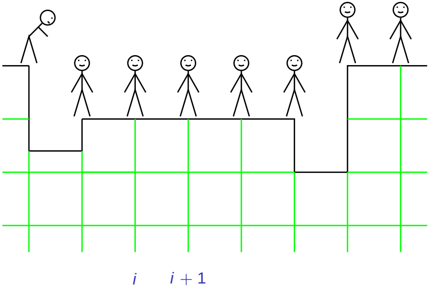

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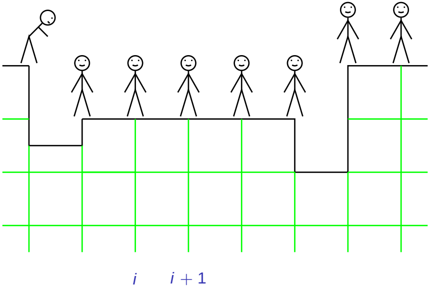

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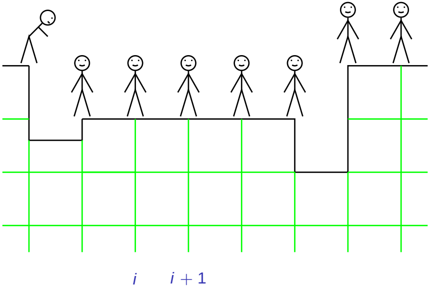

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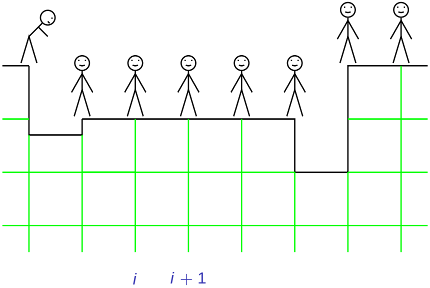

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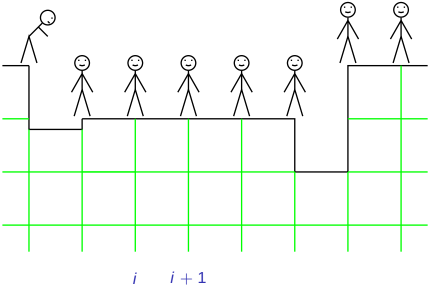

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(with $\|$ between each upper and lower vector)

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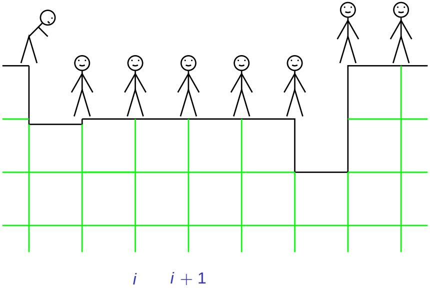

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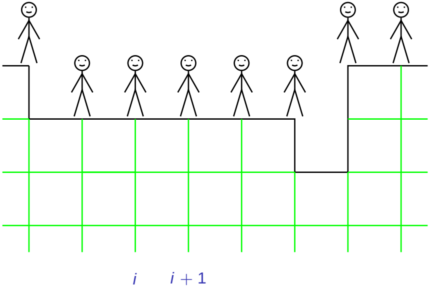

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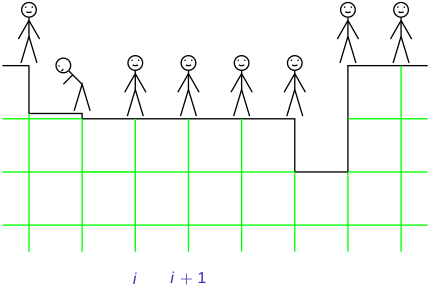

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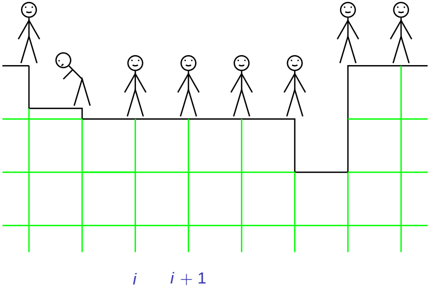

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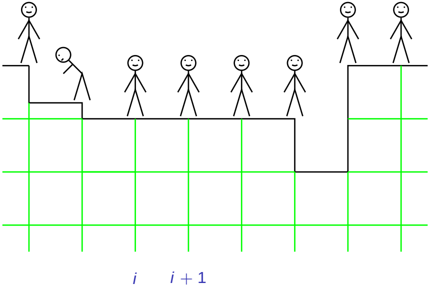

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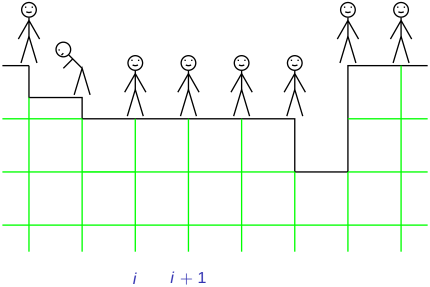

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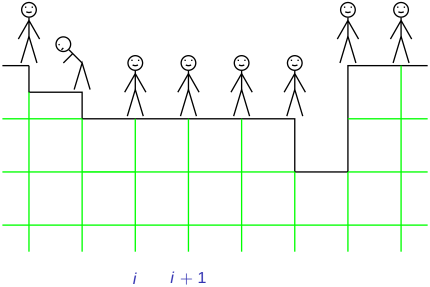

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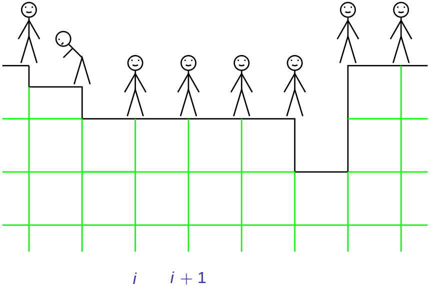

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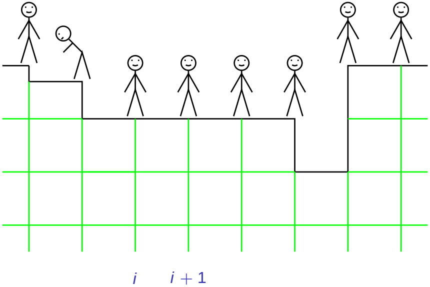

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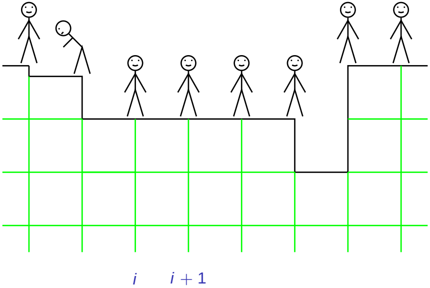

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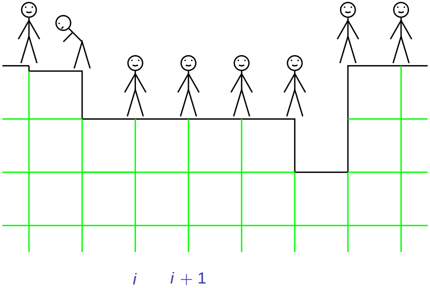

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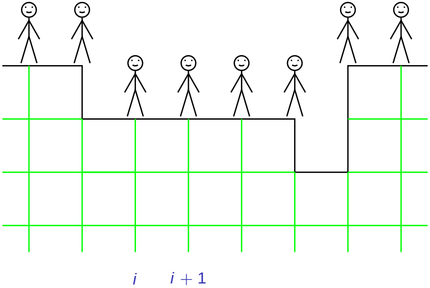

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A mirror-symmetrized version of the extended zero range. Left and right jumps of the dynamics cooperate, if
$(r(\omega)\cdot r(1-\omega)=1; \quad r$ non-decreasing$)$.

Stationary product distributions

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Here $r(0)! := 1$, and $r(z+1)! = r(z)! \cdot r(z+1)$ for all $z \in \mathbb{Z}$.

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The *density* $\varrho := \mathbf{E}(\omega)$ and the *hydrodynamic flux* $H := \mathbf{E}[\text{growth rate}]$ both depend on a parameter ϱ or θ of the stationary distribution.

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- $H(\varrho)$ is the *hydrodynamic flux function*.
- If the process is *locally* in equilibrium, but changes over some *large scale* (variables $X = \varepsilon i$ and $T = \varepsilon t$), then

$$\partial_T \varrho(T, X) + \partial_X H(\varrho(T, X)) = 0 \qquad \text{(conservation law).}$$

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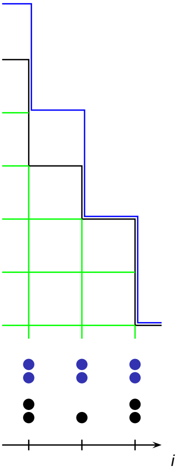
$\rightsquigarrow$ Either convex or concave, discontinuous shock solutions exist. Let's look for the corresponding microscopic structure.

The second class particle

States ω and ω only differ at one site.



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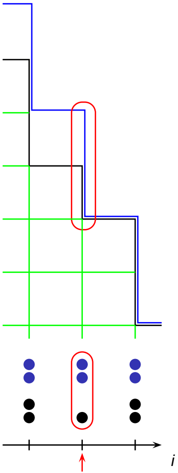

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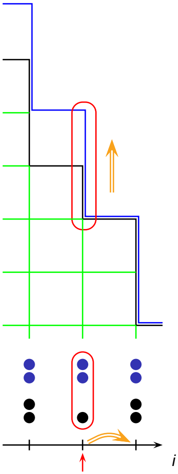

Growth on the right:

rate≤rate

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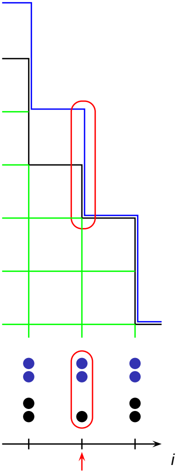

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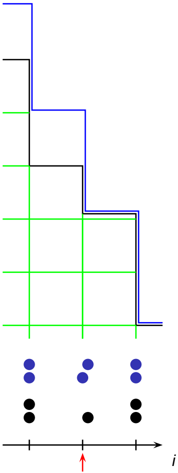

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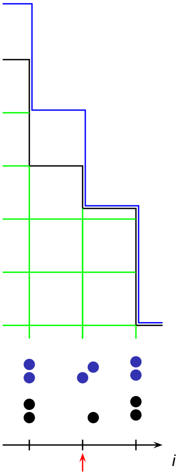

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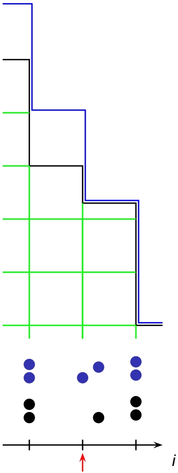

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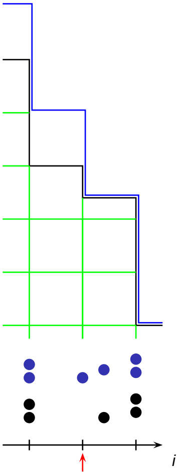

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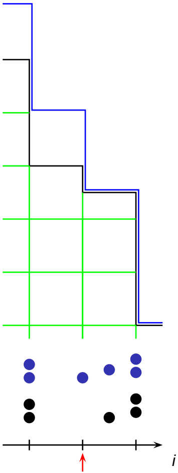

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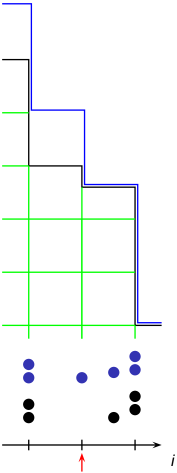

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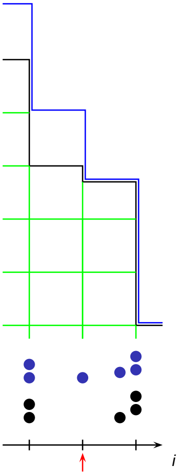

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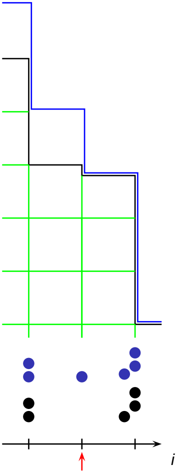
rate$\leq$rate

with rate:

The second class particle

States ω and ω only differ at one site.



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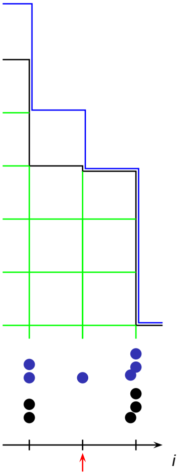

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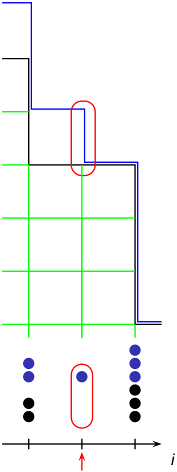

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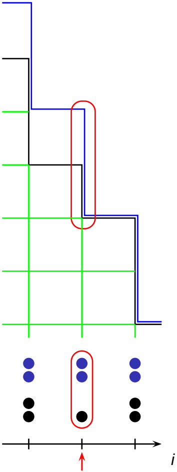

Growth on the right:

rate≤rate

with rate-rate:

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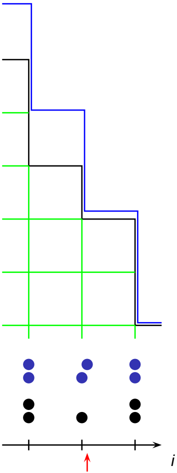

Growth on the right:

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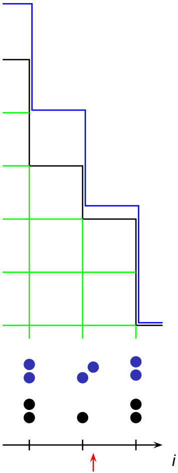

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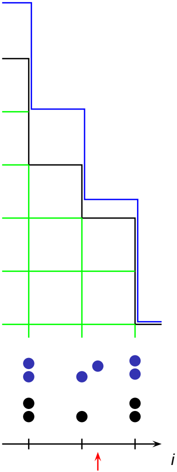

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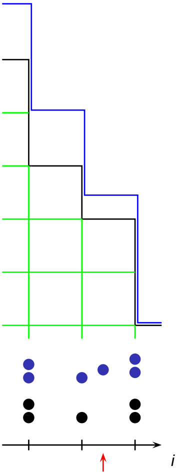

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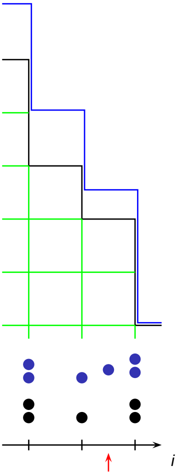

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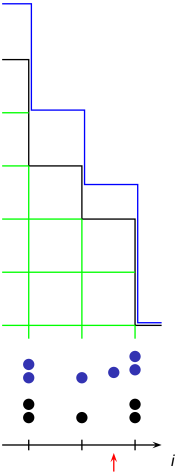

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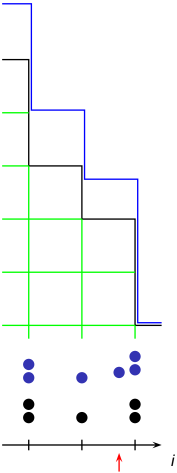

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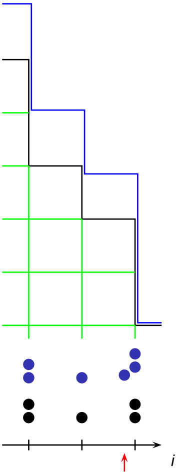

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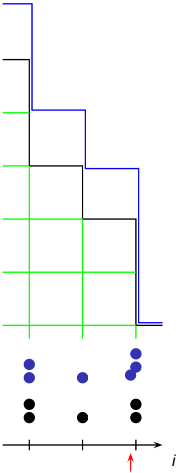

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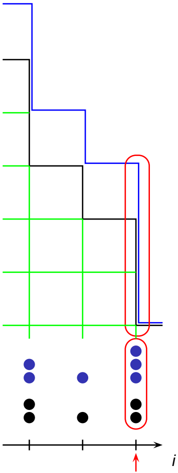

Growth on the right:

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with rate-rate:

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Growth on the left:

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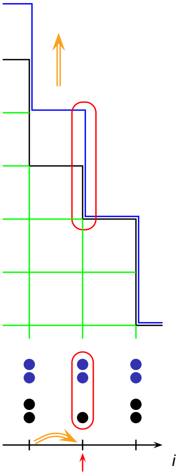

The second class particle

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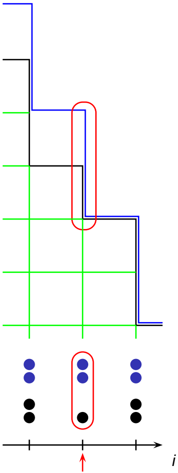
with rate:



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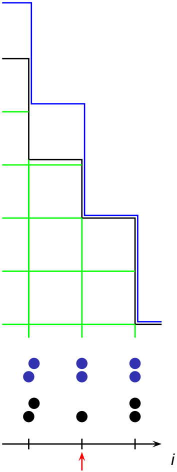

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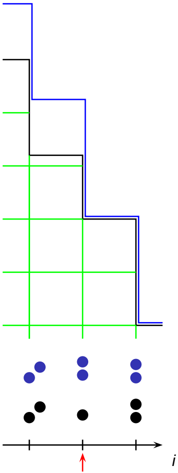

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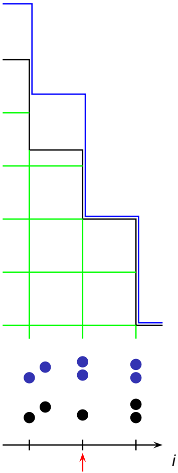

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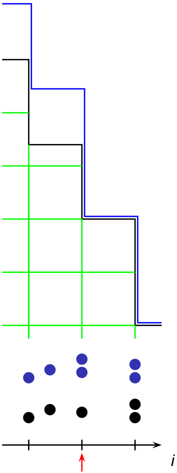

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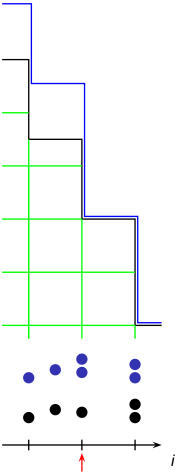

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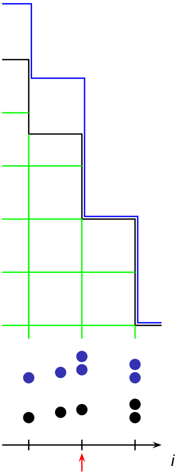

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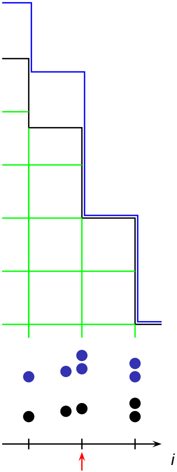

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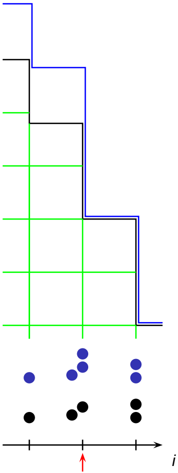

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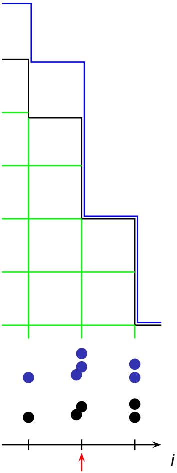

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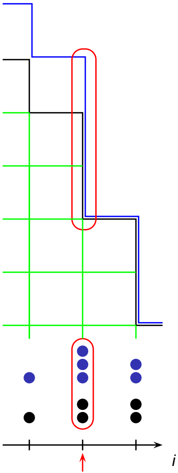

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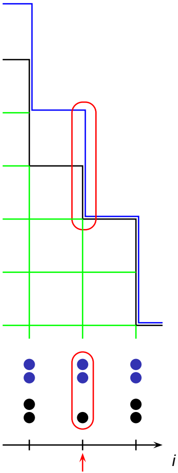

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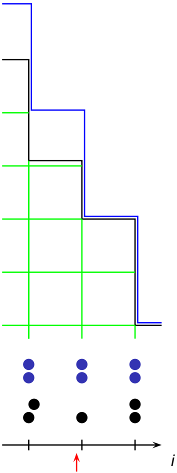

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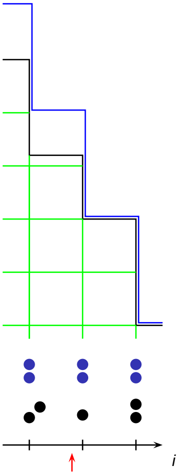

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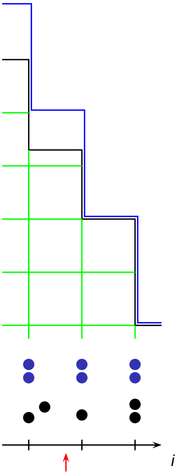

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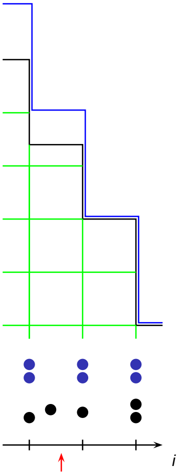

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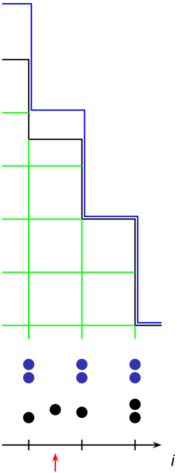

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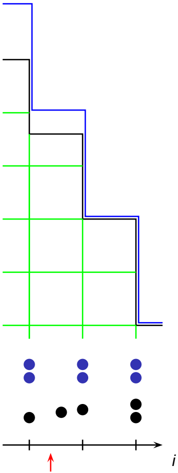

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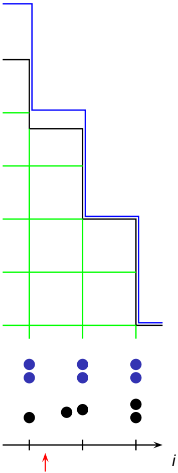

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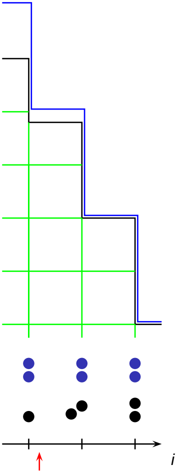

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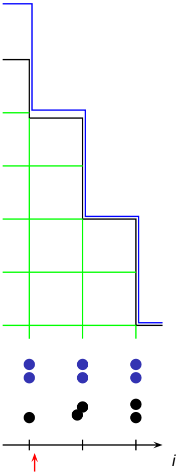

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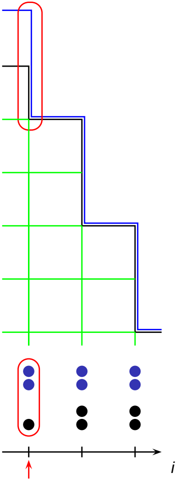

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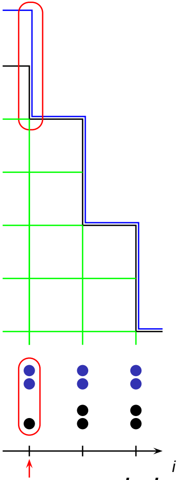

Growth on the left:
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with rate-rate:

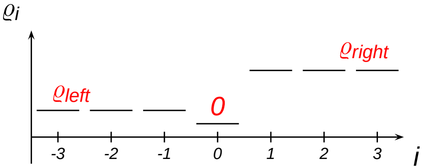
A single discrepancy ↑, the *second class particle*, is conserved.

Earlier results: as seen by the second class particle

From now on: ASEP, TAGEZRP, TAEBLP only; “E”=exponential.

Theorem (Derrida, Lebowitz, Speer ’97)

For the ASEP, the Bernoulli product distribution with densities



is stationary for the process, as seen from the second class particle, if

$$\frac{\varrho_{right}\cdot(1-\varrho_{left})}{\varrho_{left}\cdot(1-\varrho_{right})}=\frac{p}{q}.$$

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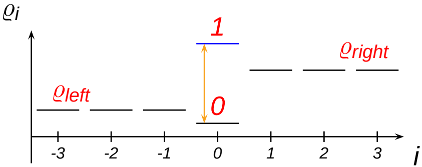

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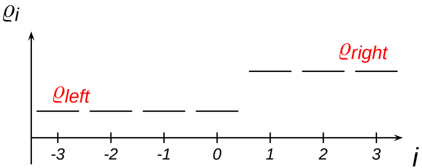
Earlier results: random walking shocks

Theorem (Belitsky and Schütz '02)

For the ASEP with the very same parameters, the Bernoulli product distribution μ_0 *with densities*



evolves according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = p \cdot \frac{\varrho_{left}}{\varrho_{right}} \cdot [\mu_{-1} - \mu_0] + q \cdot \frac{\varrho_{right}}{\varrho_{left}} \cdot [\mu_1 - \mu_0].$$

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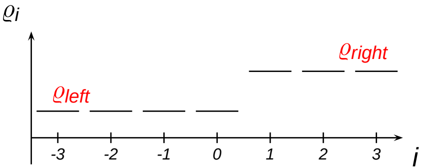

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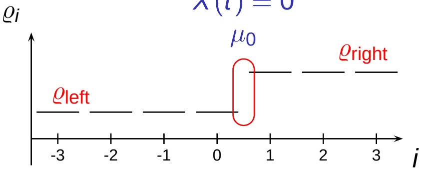
Multiple shocks and their interactions are also handled.

Earlier results: random walking shocks

Interpretation: random walking shock $\mu(t) = \mu_{X(t)}$:

with rate $p \cdot \frac{\varrho_{\text{left}}}{\varrho_{\text{right}}}$:

$X(t) = 0$



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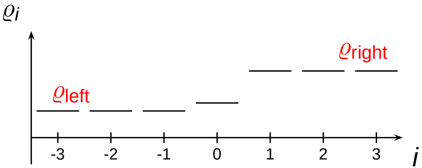

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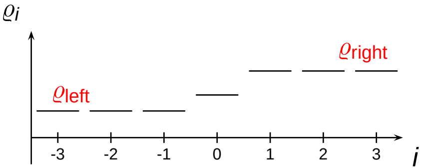

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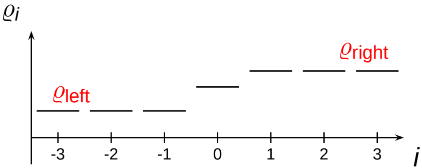

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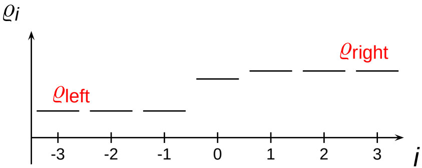

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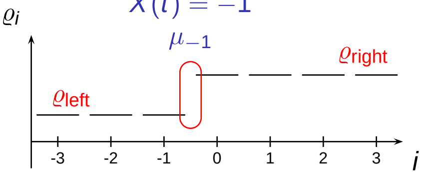

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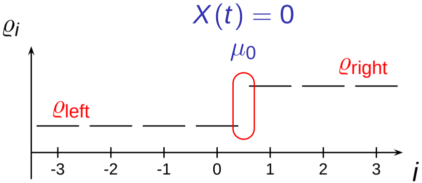

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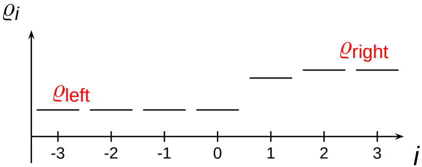

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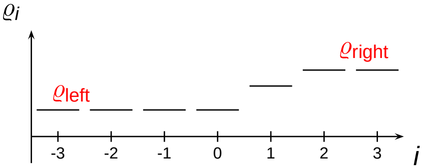

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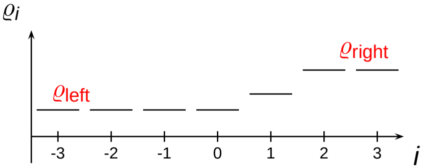

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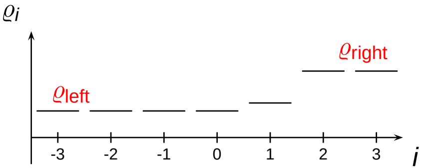

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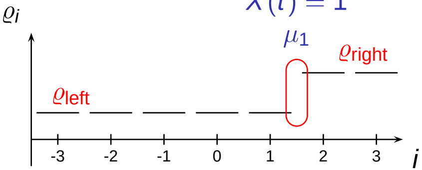

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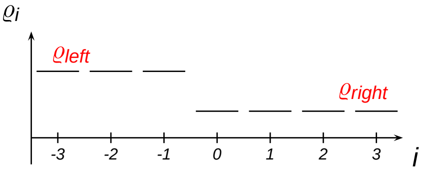
Theorem (B. '01)

For the TAEBLP, the product distribution of marginals μ^{ϱ_i} *with densities*



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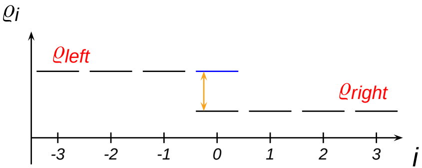

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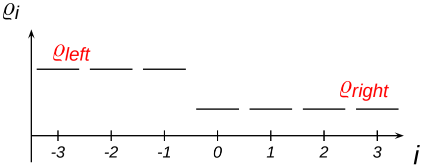

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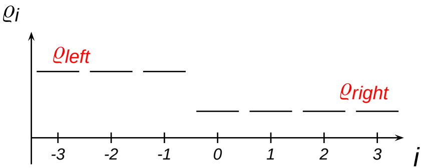

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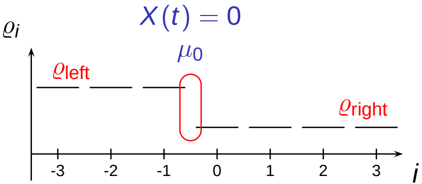

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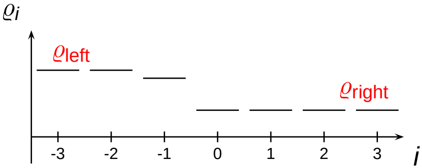

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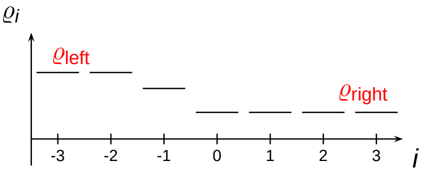

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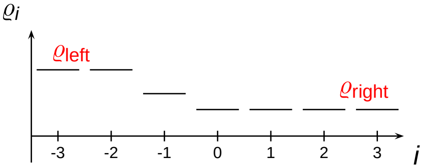

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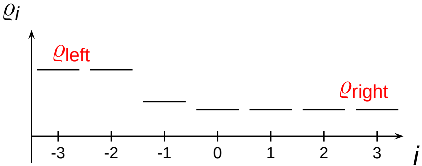

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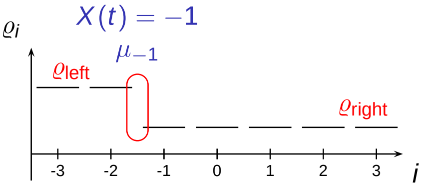

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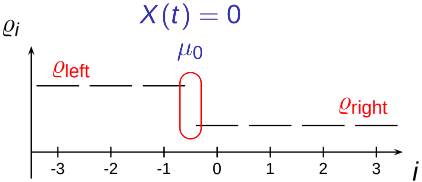

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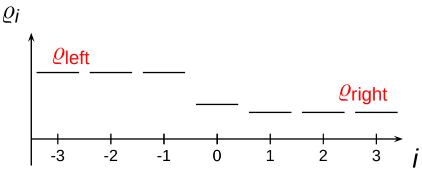

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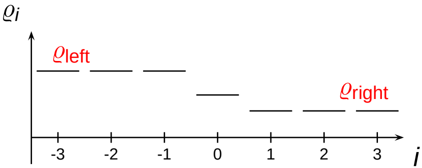

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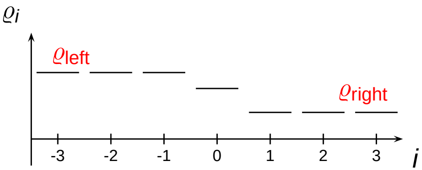

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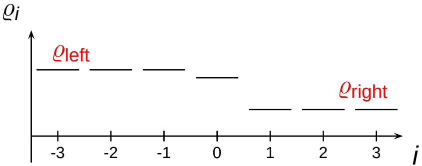

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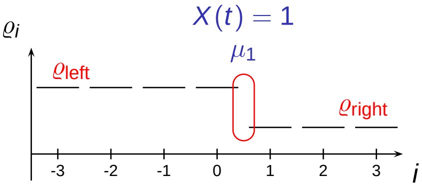

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Is it the second class particle that performs the simple random walk in the middle of a shock?

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Is it the second class particle that performs the simple random walk in the middle of a shock?

In what sense? Annealed w.r.t. the initial shock distribution...
But what does this mean?

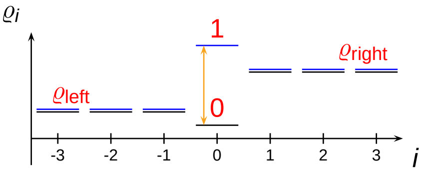
Here is the question:

For the ASEP, let ν_0 be the Bernoulli product distribution

$$\nu_0 = \Big(\bigotimes_{i<0} \mu^{\varrho_{\text{left}}}\Big) \otimes \big(\delta\big) \otimes \Big(\bigotimes_{i>0} \mu^{\varrho_{\text{right}}}\Big),$$

where

$$\mu^{\varrho}(\omega = \omega) = \begin{cases} \varrho, & \text{if } \omega = 1, \\ 1 - \varrho, & \text{if } \omega = 0; \end{cases} \qquad \delta(0,\, 1) = 1.$$



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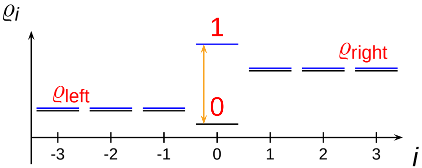

Does it satisfy

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when

$$\frac{\varrho_{\text{right}} \cdot (1 - \varrho_{\text{left}})}{\varrho_{\text{left}} \cdot (1 - \varrho_{\text{right}})} = \frac{p}{q} \quad ?$$

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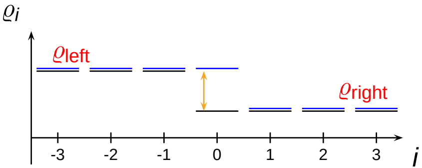

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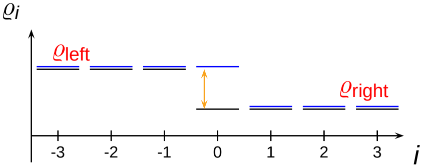

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Here is the answer:

Theorem (Gy. Farkas, P. Kovács, A. Rákos, B. ’09)

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This explains both types of the previous results.

The presence of a second class particle in the measure significantly simplifies the computations. $\rightsquigarrow$ This is how we discovered the TAGEZRP.

Nice, since

This might open up the path for applying methods physicists like (e.g. Bethe Ansatz).

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It also gives a rough tail bound for the second class particle in a *flat initial distribution*; essential in the $t^{2/3}$ proofs for the exponential models.

Interactions:

We also see that shocks+second class particles

- locally interact by exclusion in ASEP, and don’t locally interact in TAGEZRP, TAEBLP, *but*

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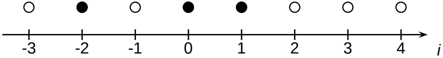
Macroscopically it's one shock after all.

A similar result: branching coalescing random walk



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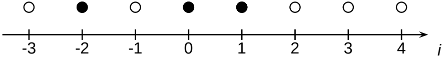

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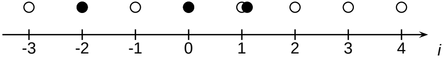

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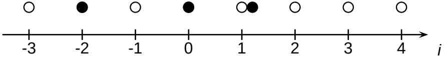

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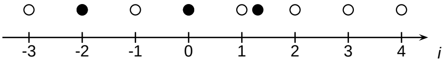

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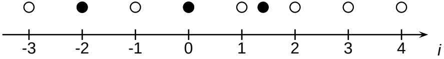

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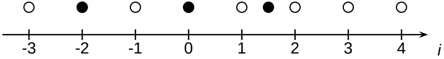

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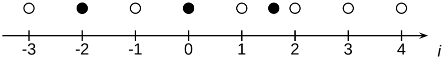

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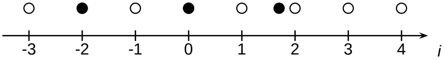

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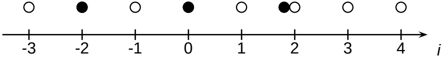

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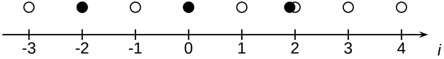
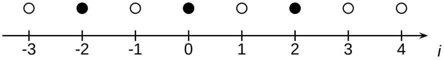


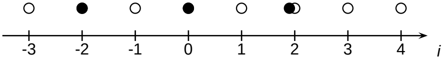
With rate p: jump to the right

A similar result: branching coalescing random walk



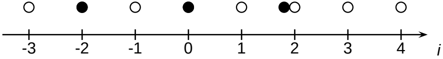
With rate q: jump to the left

A similar result: branching coalescing random walk



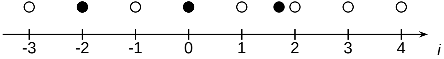
With rate q: jump to the left

A similar result: branching coalescing random walk



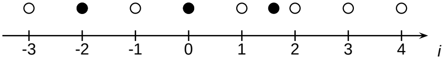
With rate q: jump to the left

A similar result: branching coalescing random walk



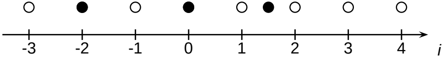
With rate q: jump to the left

A similar result: branching coalescing random walk



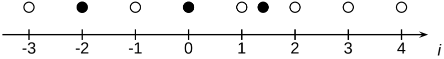
With rate q: jump to the left

A similar result: branching coalescing random walk



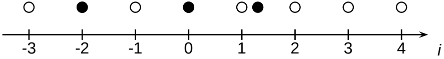
With rate q: jump to the left

A similar result: branching coalescing random walk



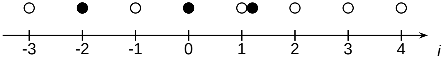
With rate q: jump to the left

A similar result: branching coalescing random walk



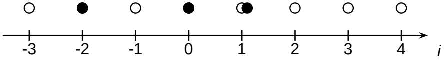
With rate q: jump to the left

A similar result: branching coalescing random walk



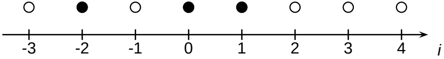
With rate q: jump to the left

A similar result: branching coalescing random walk



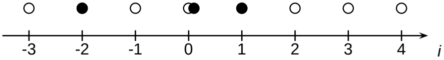
With rate q: jump to the left

A similar result: branching coalescing random walk



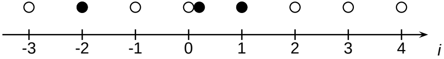
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



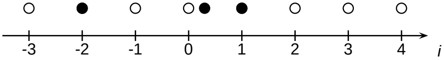
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



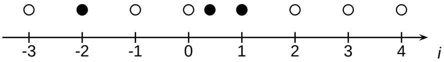
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



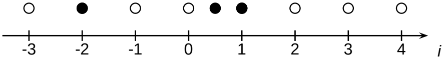
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



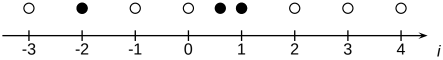
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



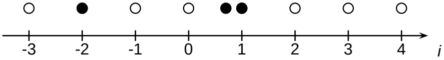
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



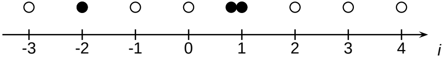
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



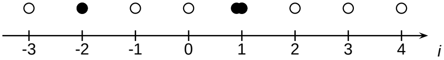
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



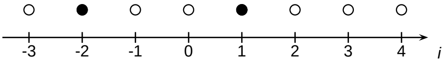
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



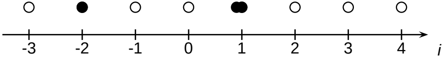
With rate c_r: coalescence to the right

A similar result: branching coalescing random walk



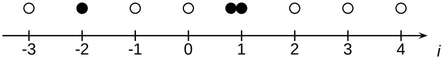
With rate b_l: branching to the left

A similar result: branching coalescing random walk



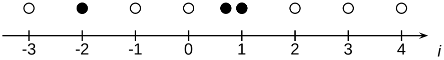
With rate b_l: branching to the left

A similar result: branching coalescing random walk



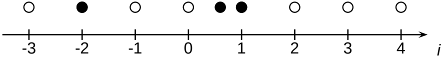
With rate b_l: branching to the left

A similar result: branching coalescing random walk



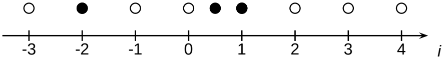
With rate b_l: branching to the left

A similar result: branching coalescing random walk



With rate b_l: branching to the left

A similar result: branching coalescing random walk



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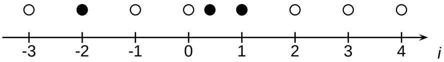
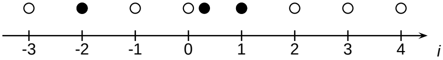
A similar result: branching coalescing random walk



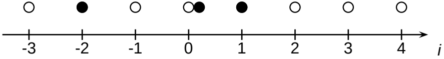
With rate b_l: branching to the left

A similar result: branching coalescing random walk



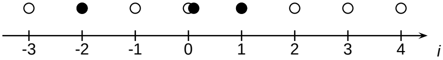
With rate b_l: branching to the left

A similar result: branching coalescing random walk



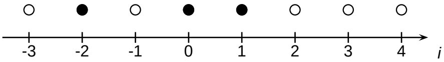
With rate b_l: branching to the left

A similar result: branching coalescing random walk



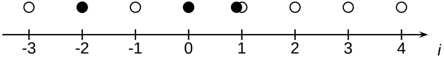
With rate b_l: branching to the left

A similar result: branching coalescing random walk



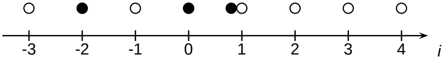
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



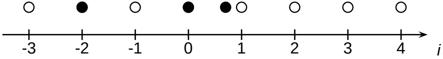
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



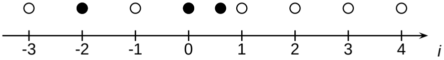
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



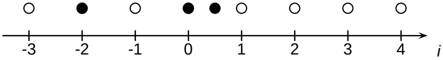
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



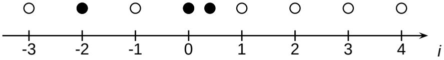
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



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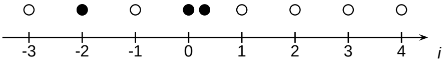

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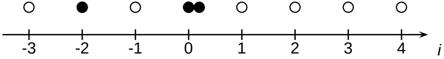
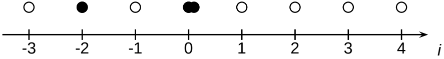


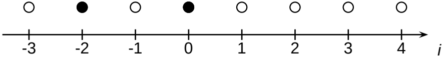
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



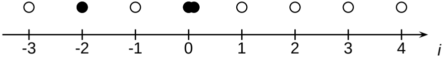
With rate c_l: coalescence to the left

A similar result: branching coalescing random walk



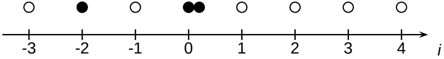
With rate b_r: branching to the right

A similar result: branching coalescing random walk



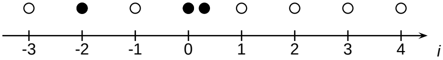
With rate b_r: branching to the right

A similar result: branching coalescing random walk



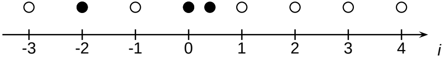
With rate b_r: branching to the right

A similar result: branching coalescing random walk



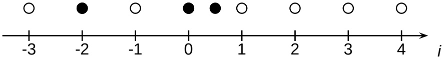
With rate b_r: branching to the right

A similar result: branching coalescing random walk



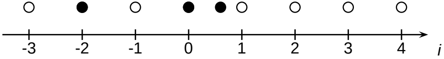
With rate b_r: branching to the right

A similar result: branching coalescing random walk



With rate b_r: branching to the right

A similar result: branching coalescing random walk



With rate b_r: branching to the right

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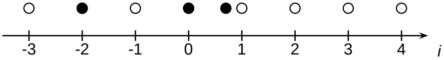
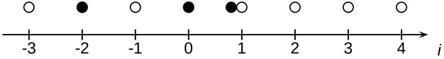


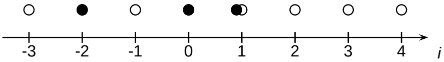
With rate b_r: branching to the right

A similar result: branching coalescing random walk



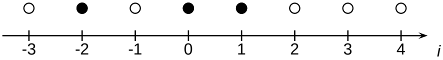
With rate b_r: branching to the right

A similar result: branching coalescing random walk



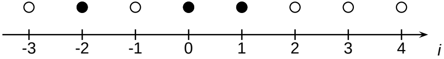
With rate b_r: branching to the right

A similar result: branching coalescing random walk



With rate b_r: branching to the right

A similar result: branching coalescing random walk



With rate b_r: branching to the right

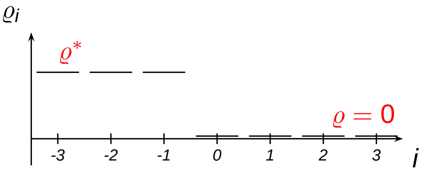
The Bernoulli(ϱ^*) distribution is stationary for

$$\varrho^* = \frac{b_l + b_r}{b_l + b_r + c_l + c_r}.$$

Earlier results: as seen by the rightmost particle

Theorem

For the BCRW, the Bernoulli product distribution with densities

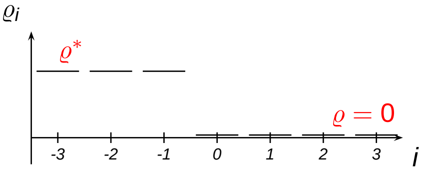


is stationary for the process, as seen from the rightmost particle.

Earlier results: random walking shocks

Theorem (Krebs, Jafarpour and Schütz '03)

For the BCRW with the very same parameters, the Bernoulli product distribution μ_0 *with densities*



evolves according to

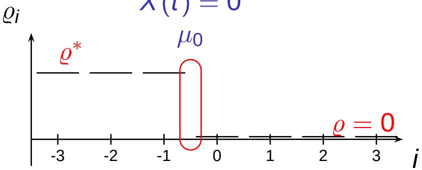
$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\mu_{-1} - \mu_0]$$
$$+ p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\mu_1 - \mu_0].$$

Earlier results: random walking shocks

Interpretation: random walking shock $\mu(t) = \mu_{X(t)}$:

with rate $\frac{c_l\cdot(b_l+b_r)+q\cdot(c_l+c_r)}{b_l+b_r+c_l+c_r}$:

$X(t) = 0$



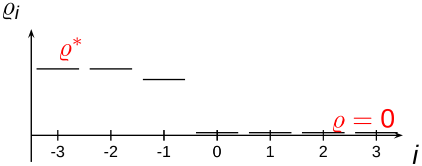
$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = \frac{c_l\cdot(b_l+b_r)+q\cdot(c_l+c_r)}{b_l+b_r+c_l+c_r}\cdot[\mu_{-1}-\mu_0]$$

$$+p\cdot\frac{b_l+b_r+c_l+c_r}{c_l+c_r}\cdot[\mu_1-\mu_0].$$

Earlier results: random walking shocks

Interpretation: random walking shock $\mu(t) = \mu_{X(t)}$:

with rate $\frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r}$:



$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_0 = \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\mu_{-1} - \mu_0] + p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\mu_1 - \mu_0].$$

Earlier results: random walking shocks

Interpretation: random walking shock $\mu(t) = \mu_{X(t)}$:

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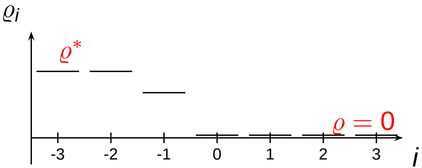

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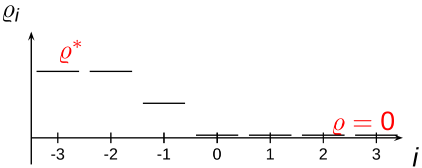

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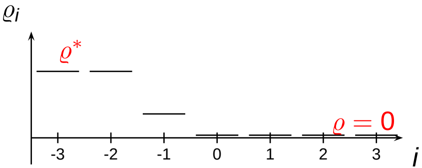

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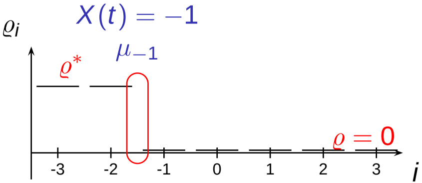

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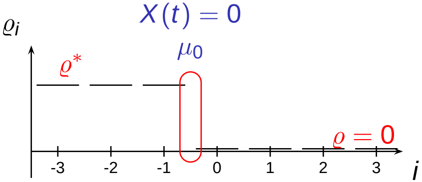

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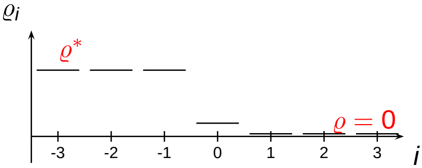

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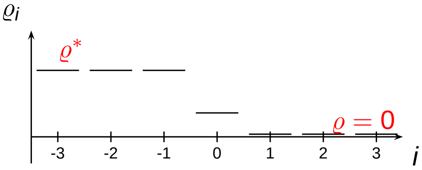

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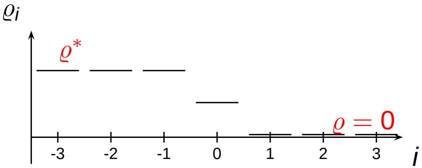

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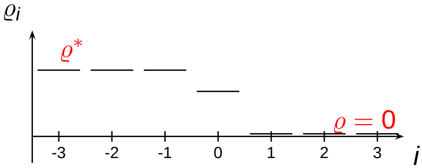

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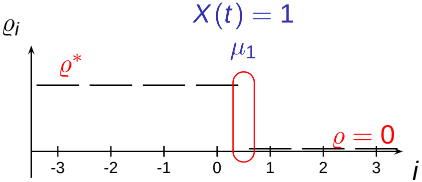

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The question:

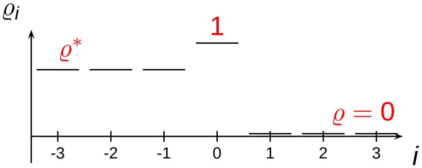
Is it the rightmost particle that performs the random walk?

Here is the question:

For the BCRW, let ν_0 be the Bernoulli product distribution

$$\nu_0 = \Big(\bigotimes_{i<0} \mu^{\varrho^*}\Big) \otimes \Big(\delta\Big) \otimes \Big(\bigotimes_{i>0} \mu^{0}\Big),$$

where $\delta(0) = 1$.



Does it satisfy

$$\frac{\mathrm{d}}{\mathrm{d}t}\nu_0 = \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\nu_{-1} - \nu_0]$$
$$+ p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\nu_1 - \nu_0]?$$

The answer

- ... is, of course, yes again. [Gy. Farkas, P. Kovács, A. Rákos, B. '09]

The answer

- ... is, of course, yes again. [Gy. Farkas, P. Kovács, A. Rákos, B. '09]
- Fronts of the other direction: $0 - 1 - \varrho^*$ can also be handled.

Thank you.