

Local convergence in t-PNG

joint with Ruby Bestwick, Artem Borisov, Elnur Emrah, and
Jessica Jay

Márton Balázs

University of Bristol

BUTE Stochastics Seminar
8th April 2026.

Polynuclear Growth

t-PNG

Stationary *t*-PNG

Local convergence

Ingredients

Proof

Polynuclear Growth (PNG) [Hammersley '72]

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Polynuclear Growth (PNG) [Hammersley '72]

- ▶ Poisson space-time points initiate nucleations
 - ▶ Nucleations spread at unit speed
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Polynuclear Growth (PNG) [Hammersley '72]

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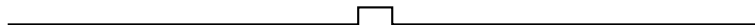
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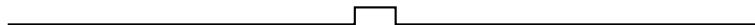
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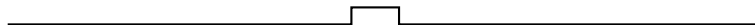
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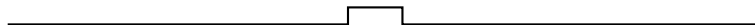
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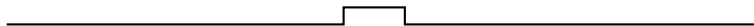
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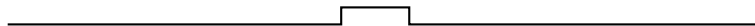
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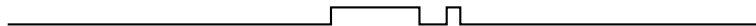
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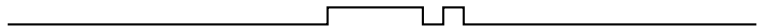
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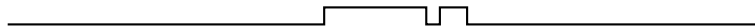
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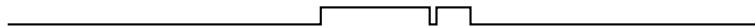
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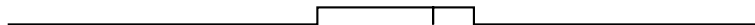
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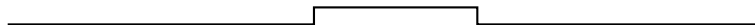
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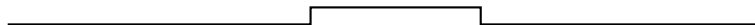
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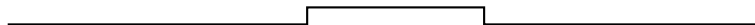
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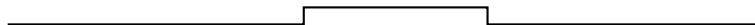
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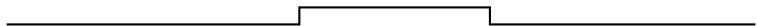
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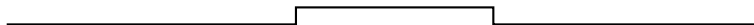
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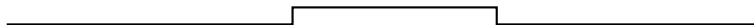
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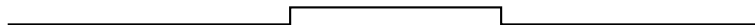
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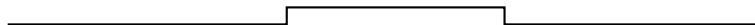
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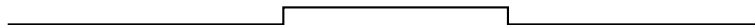
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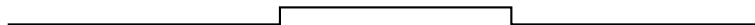
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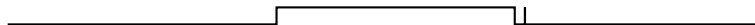
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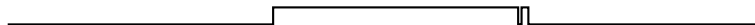
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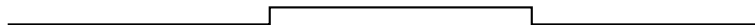
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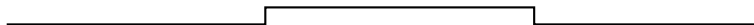
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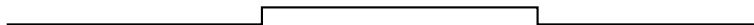
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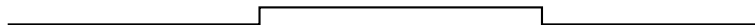
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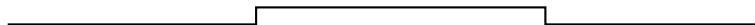
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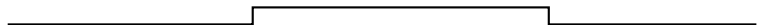
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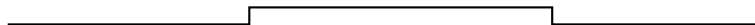
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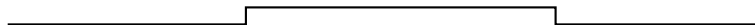
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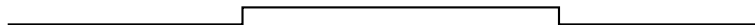
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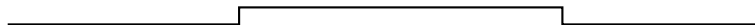
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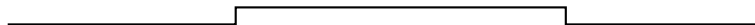
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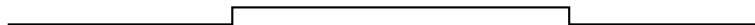
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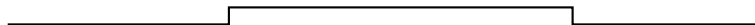
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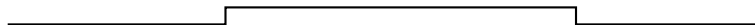
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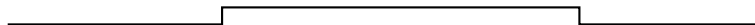
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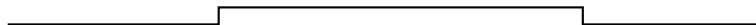
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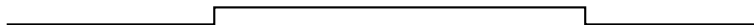
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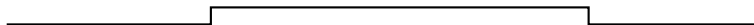
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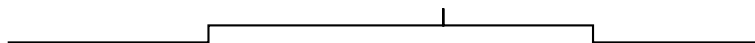
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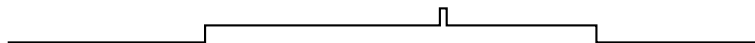
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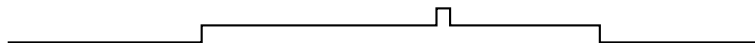
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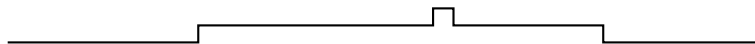
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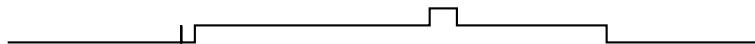
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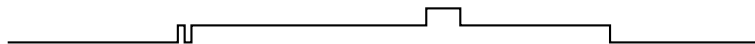
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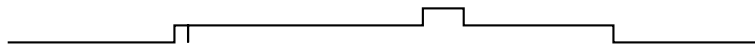
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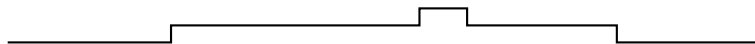
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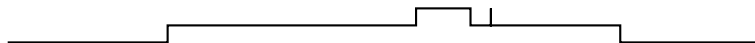
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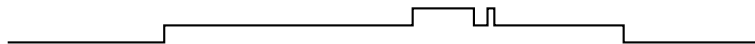
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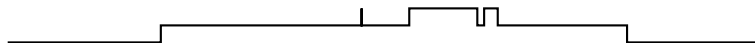
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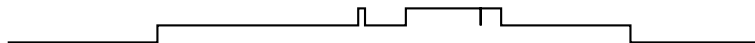
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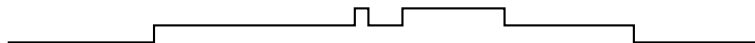
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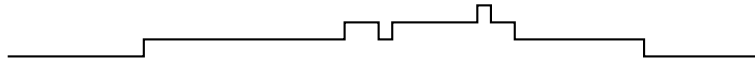
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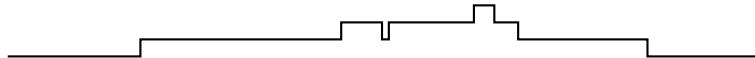
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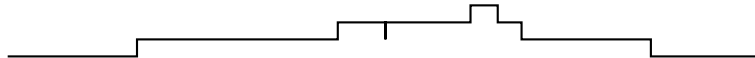
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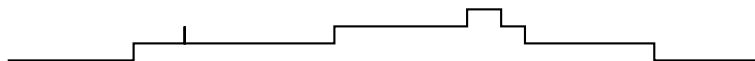
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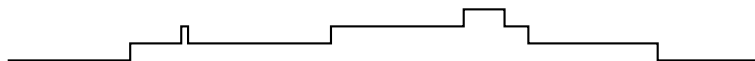
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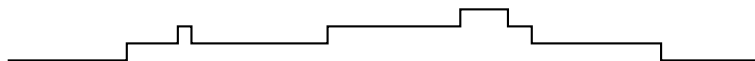
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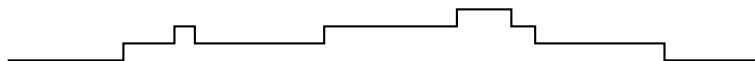
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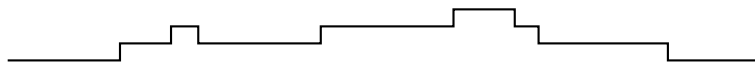
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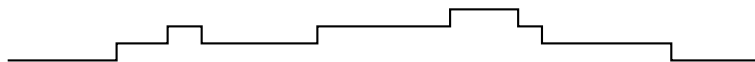
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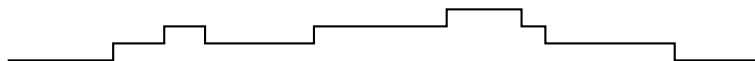
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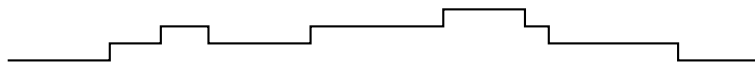
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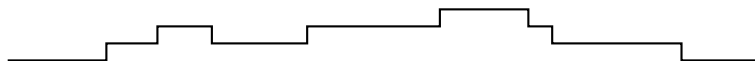
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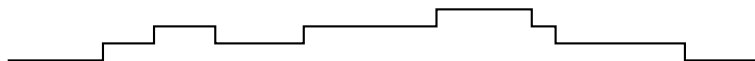
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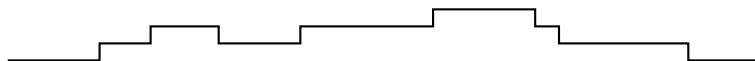
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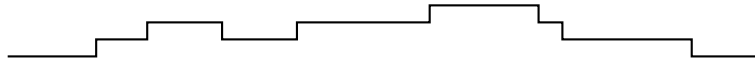
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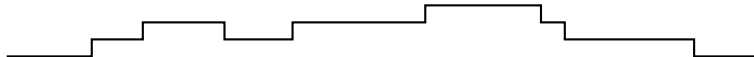
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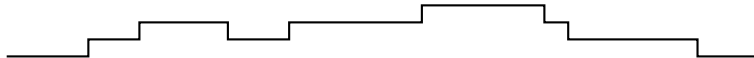
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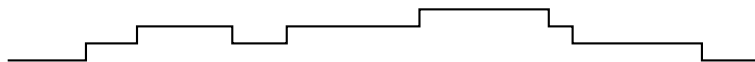
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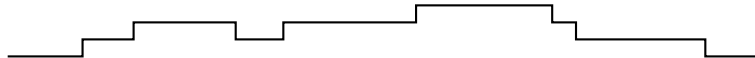
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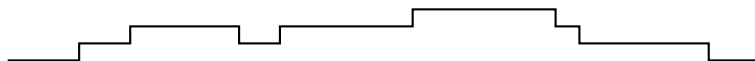
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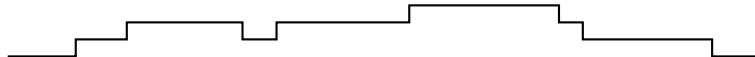
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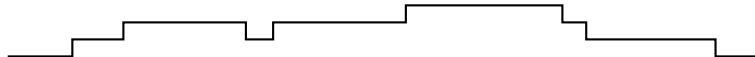
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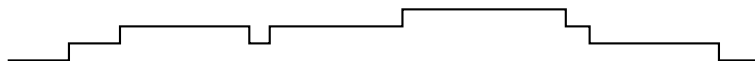
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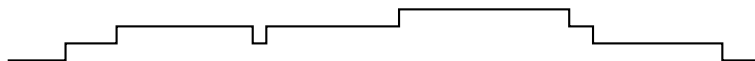
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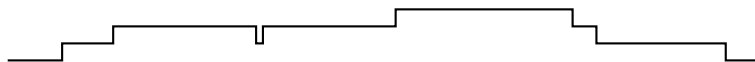
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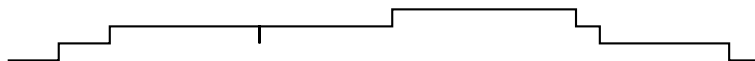
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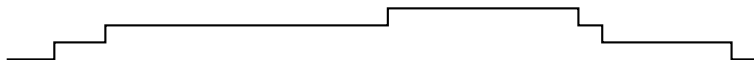
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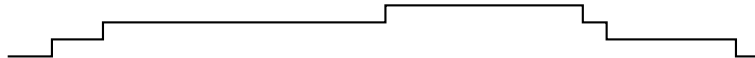
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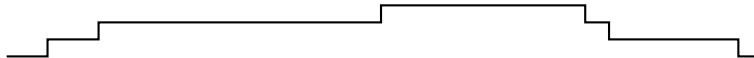
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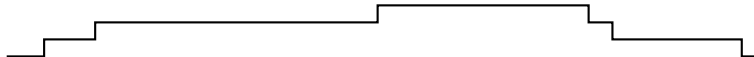
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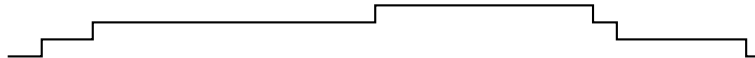
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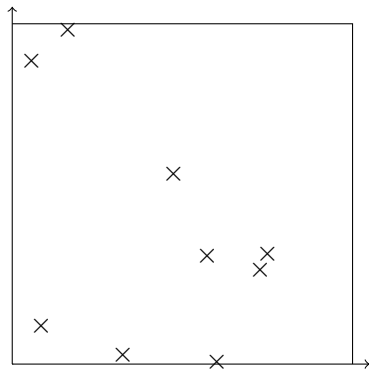
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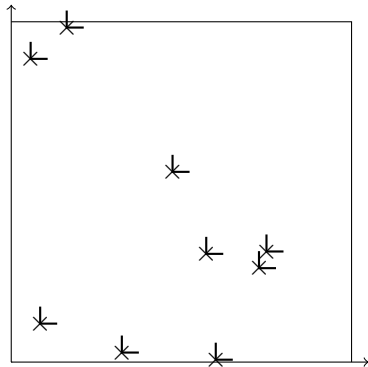


- ▶ Member of the KPZ Universality class ($\frac{1}{3}$ -power scalings, GO/UE limit distributions) [Krug-Spohn '92]

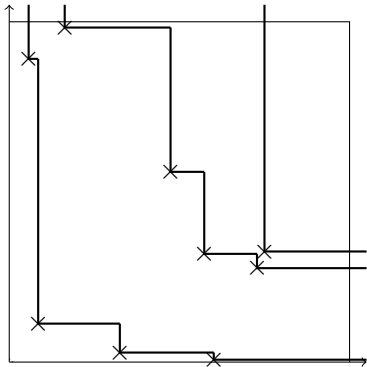
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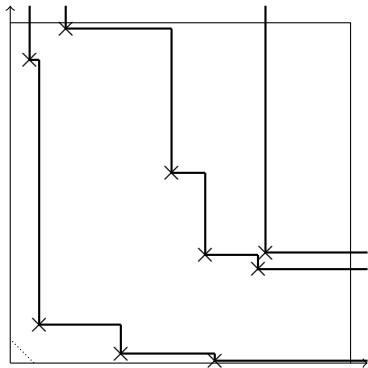
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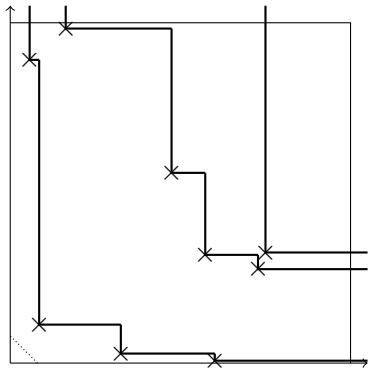
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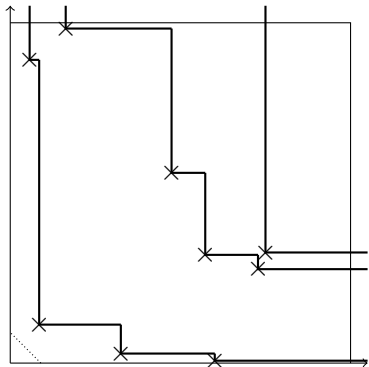
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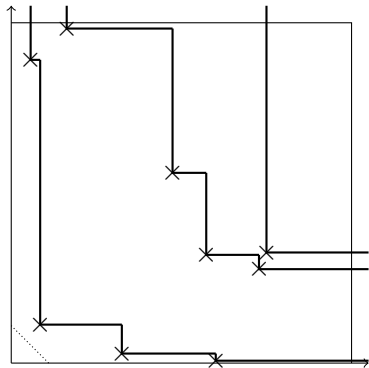
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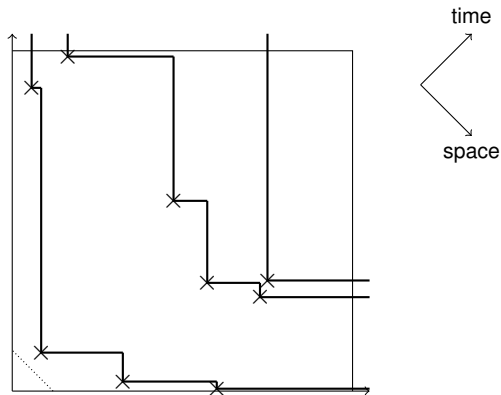
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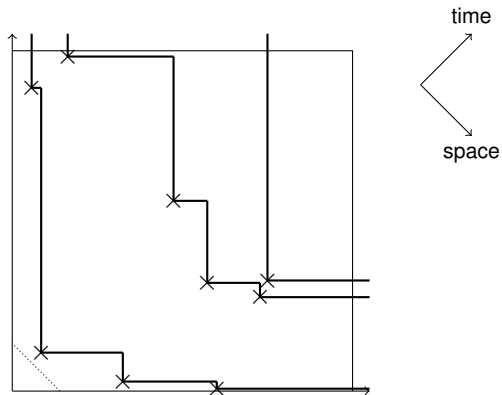
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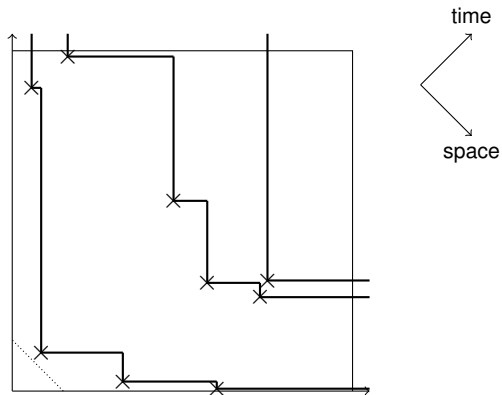
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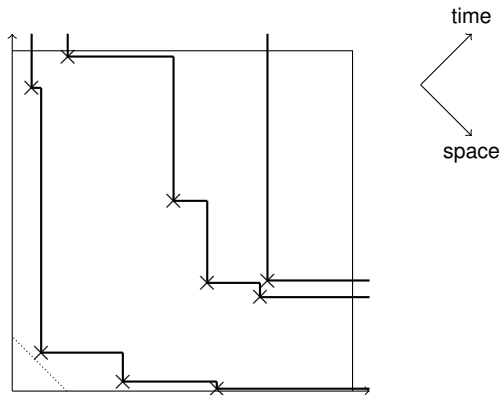
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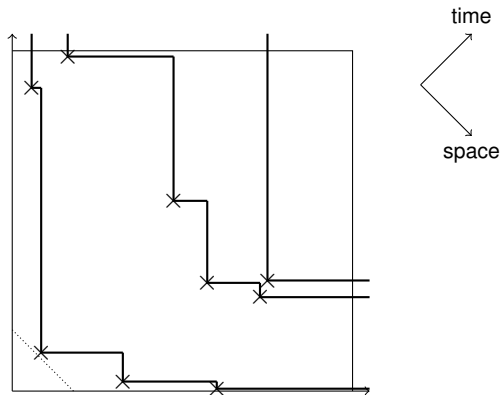
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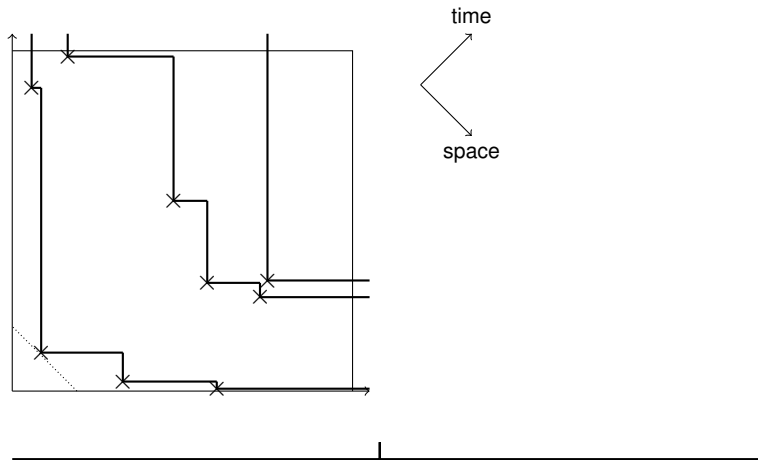
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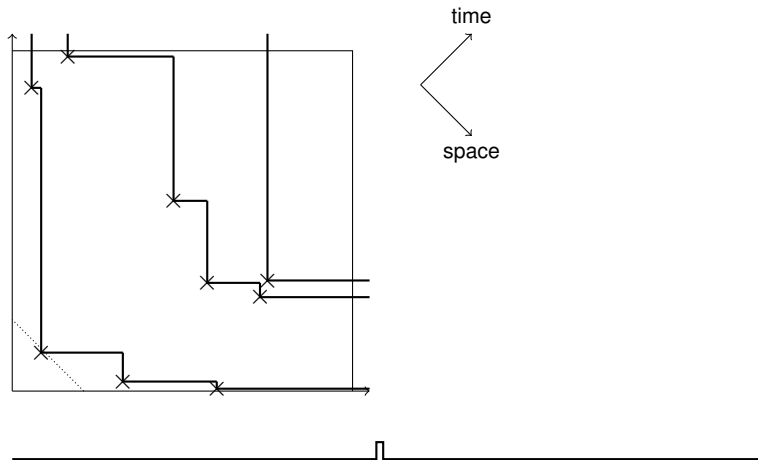
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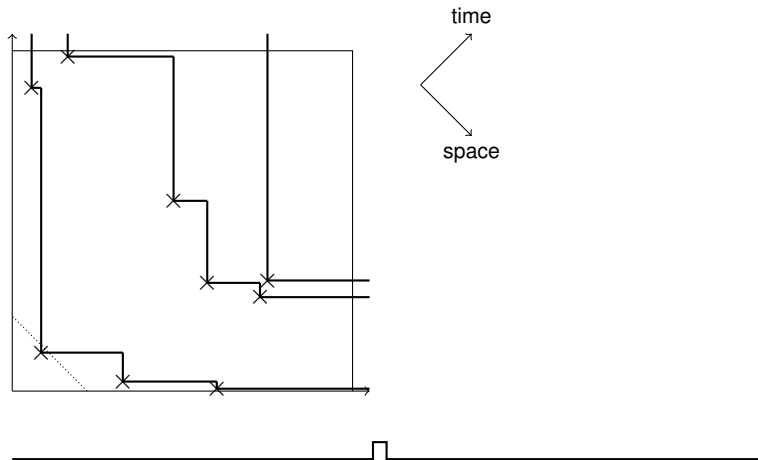
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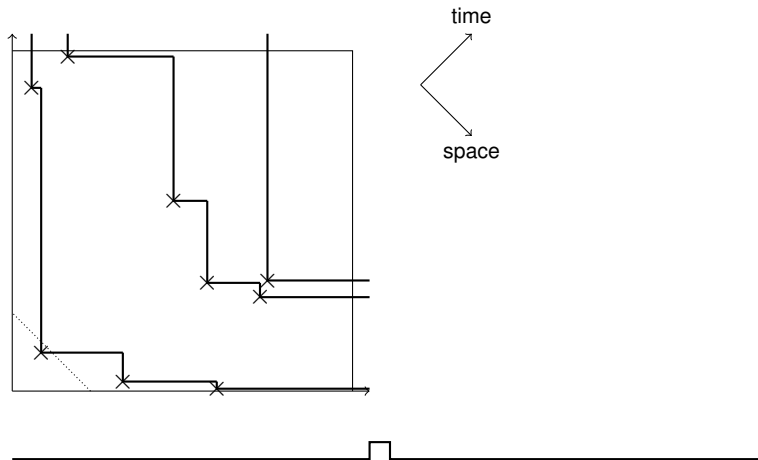
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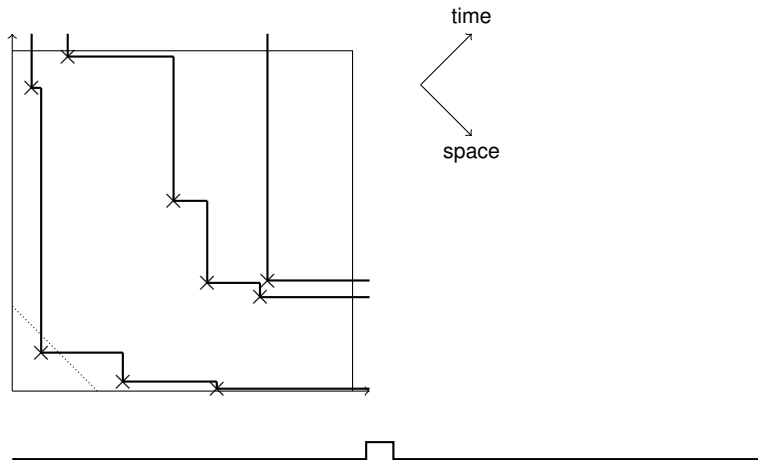
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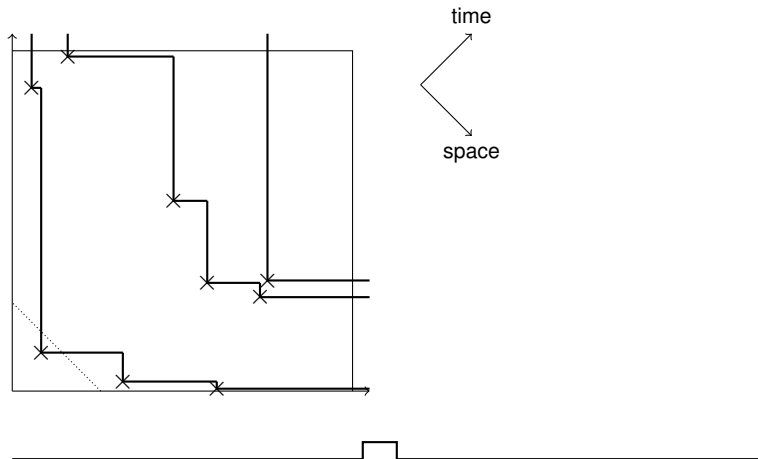
Polynuclear Growth (PNG) [Hammersley '72]



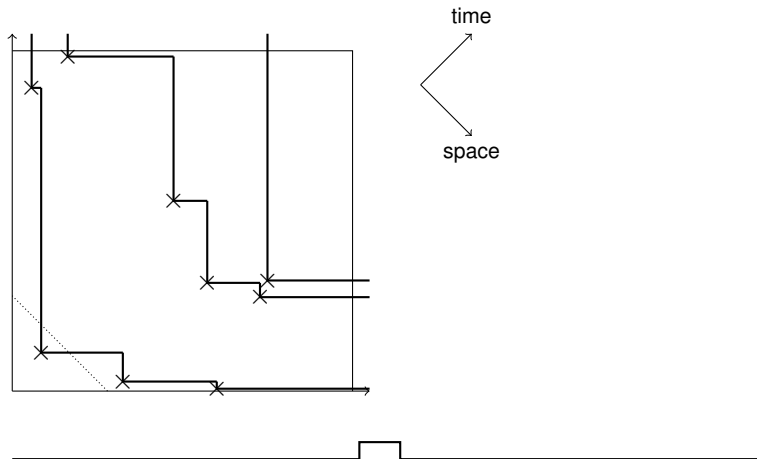
Polynuclear Growth (PNG) [Hammersley '72]



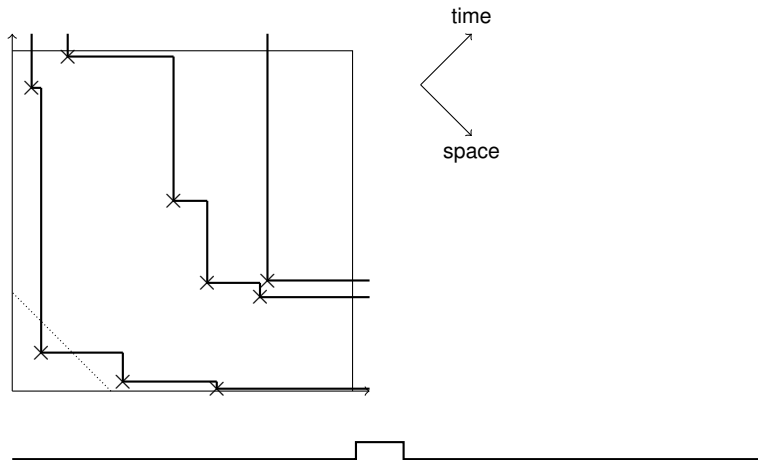
Polynuclear Growth (PNG) [Hammersley '72]



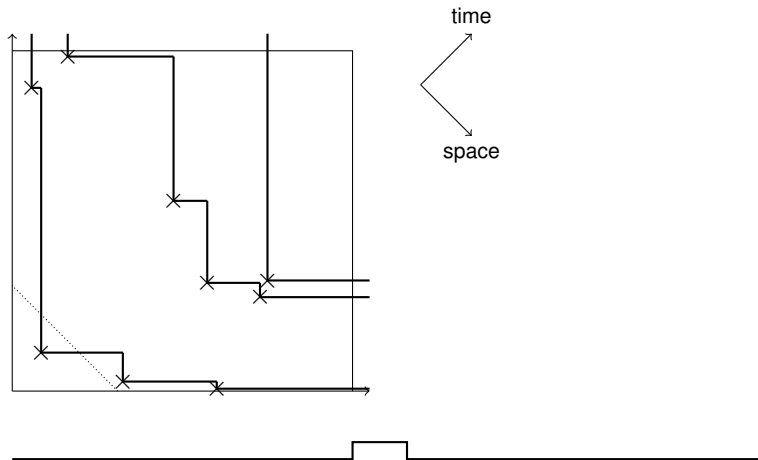
Polynuclear Growth (PNG) [Hammersley '72]



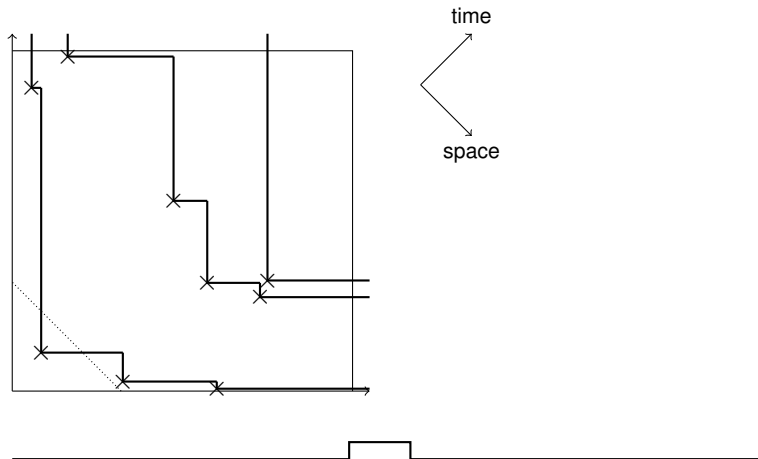
Polynuclear Growth (PNG) [Hammersley '72]



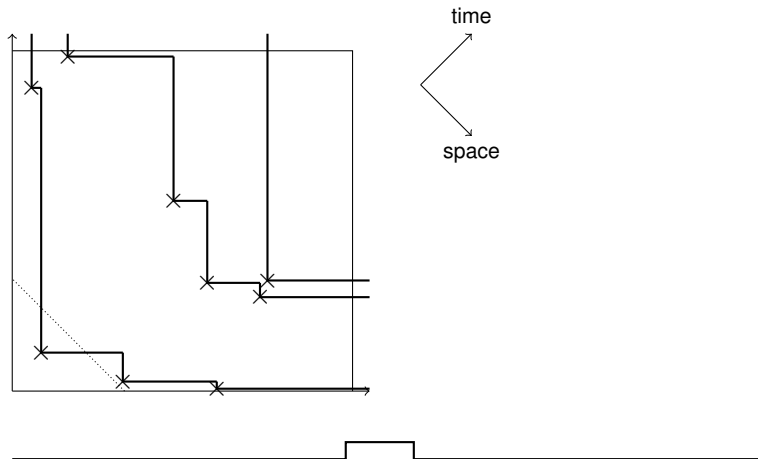
Polynuclear Growth (PNG) [Hammersley '72]



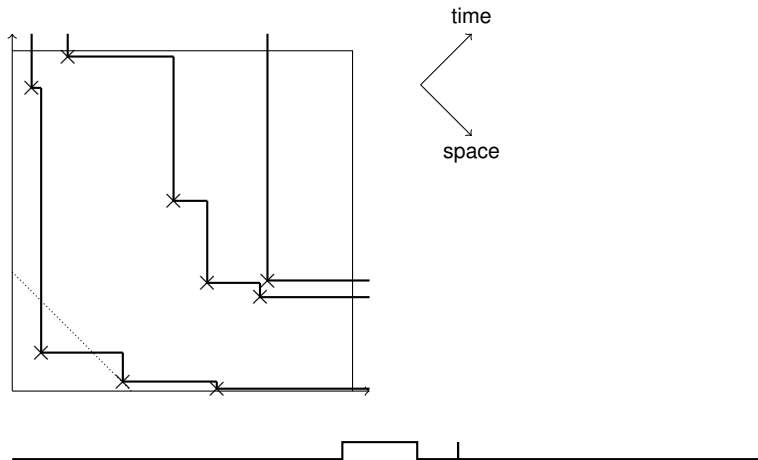
Polynuclear Growth (PNG) [Hammersley '72]



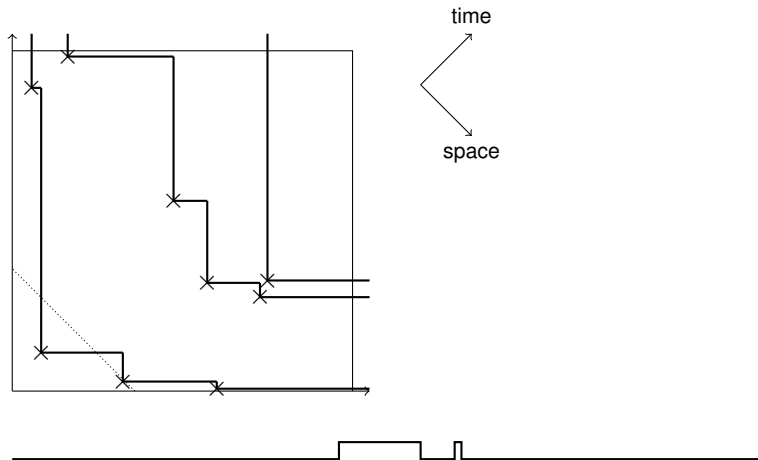
Polynuclear Growth (PNG) [Hammersley '72]



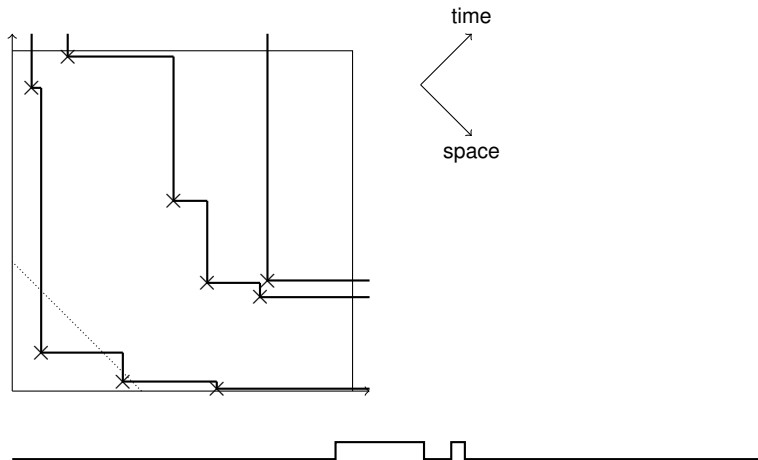
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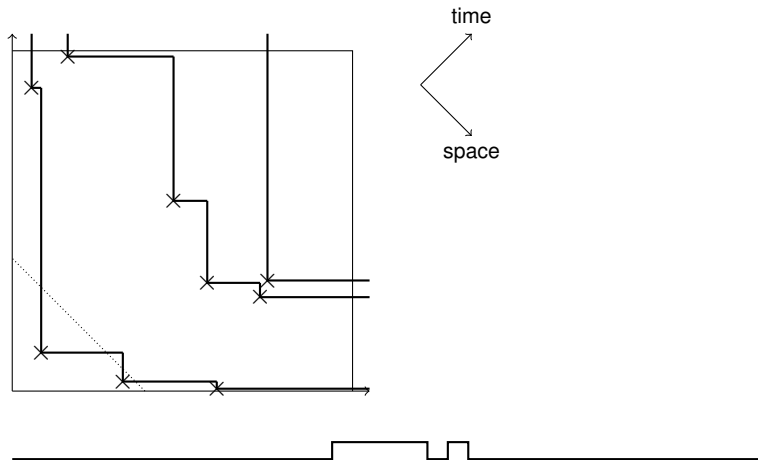
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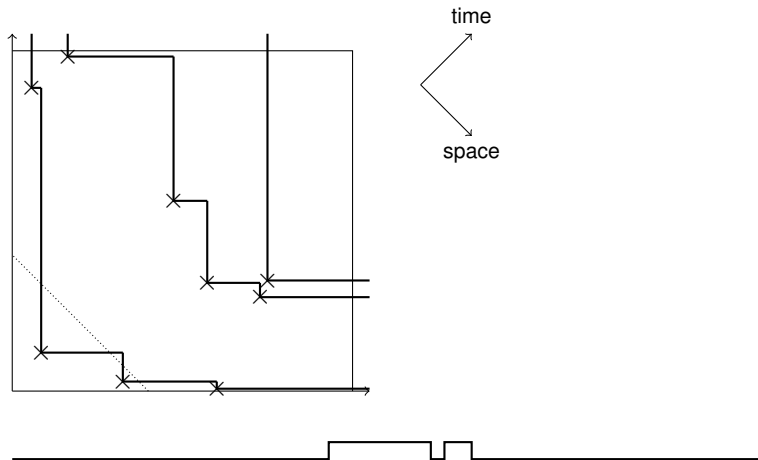
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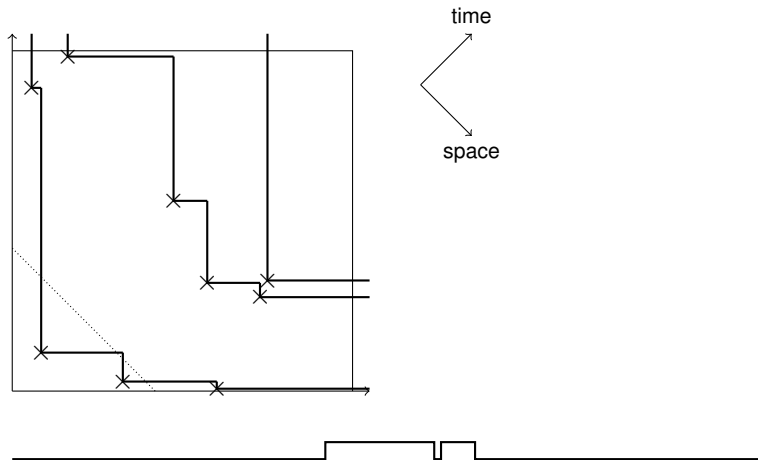
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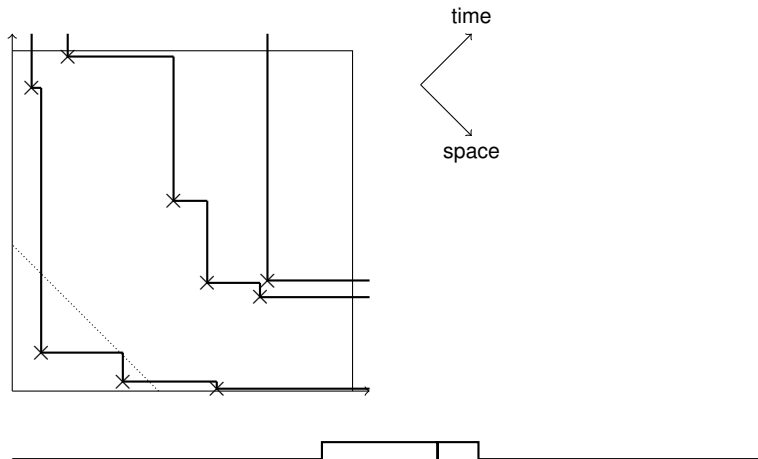
Polynuclear Growth (PNG) [Hammersley '72]



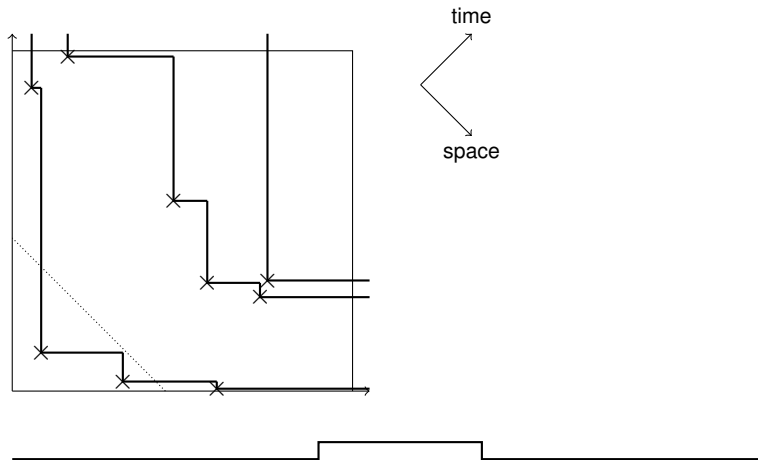
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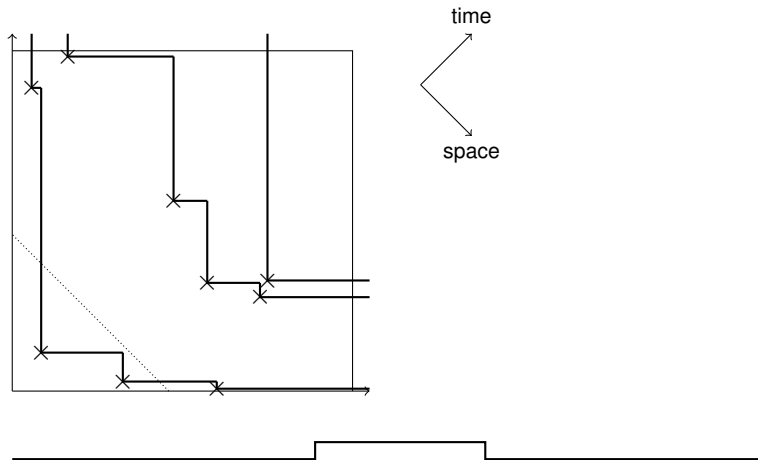
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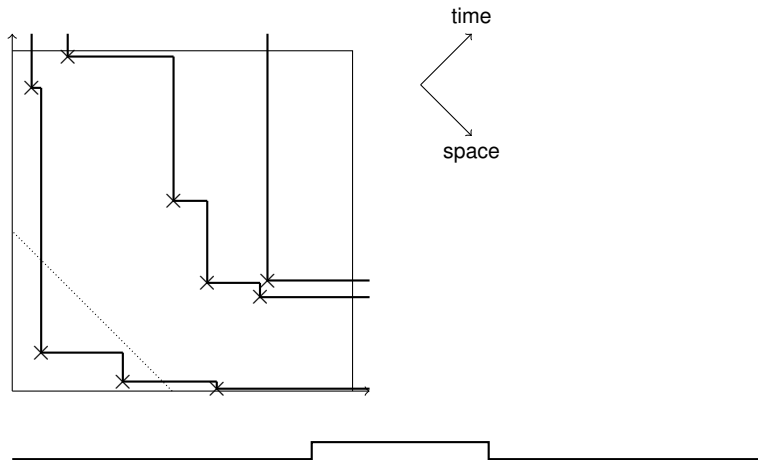
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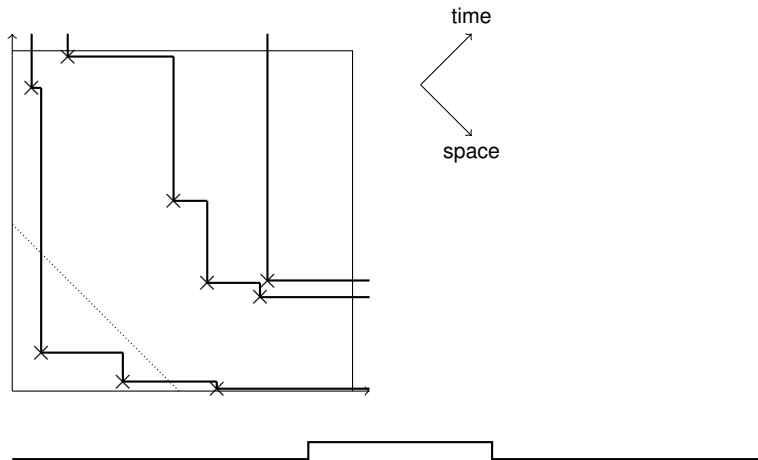
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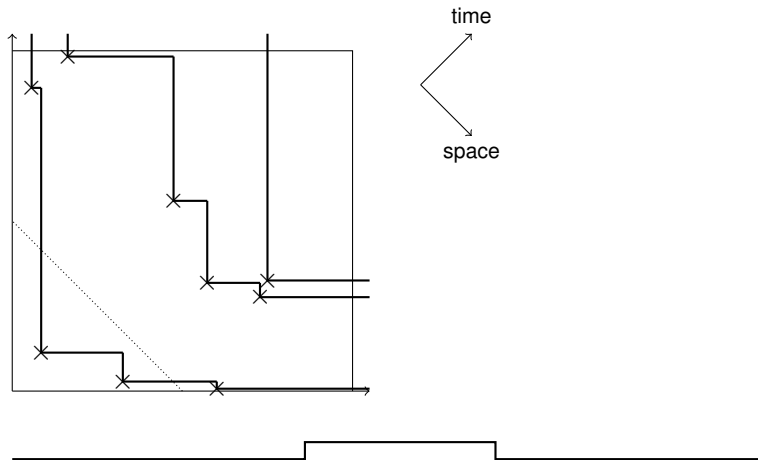
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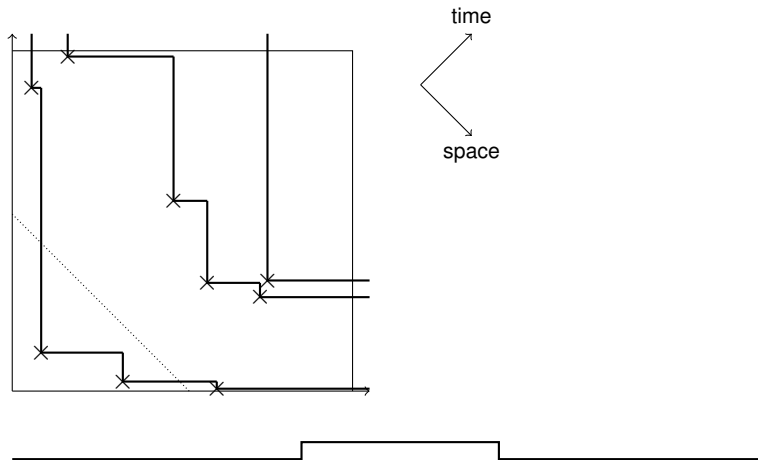
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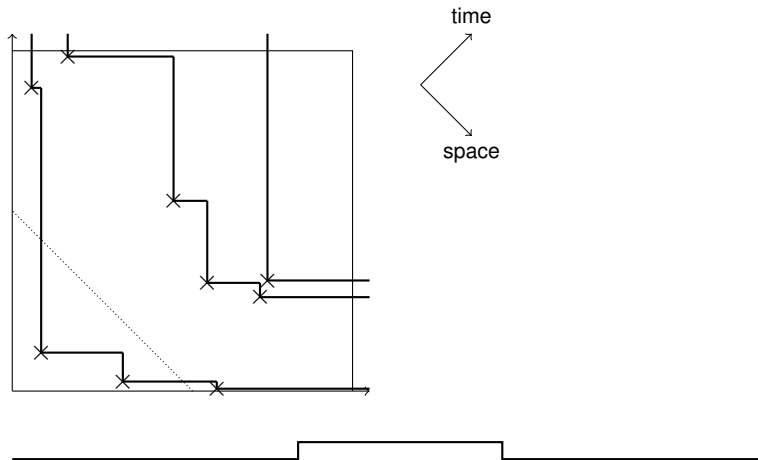
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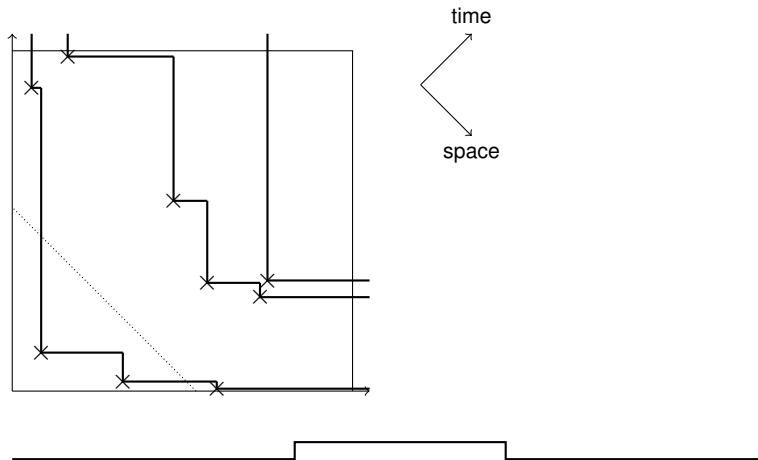
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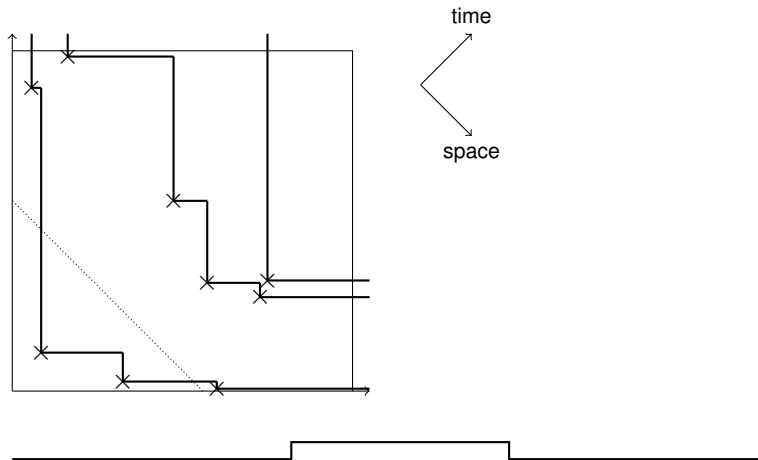
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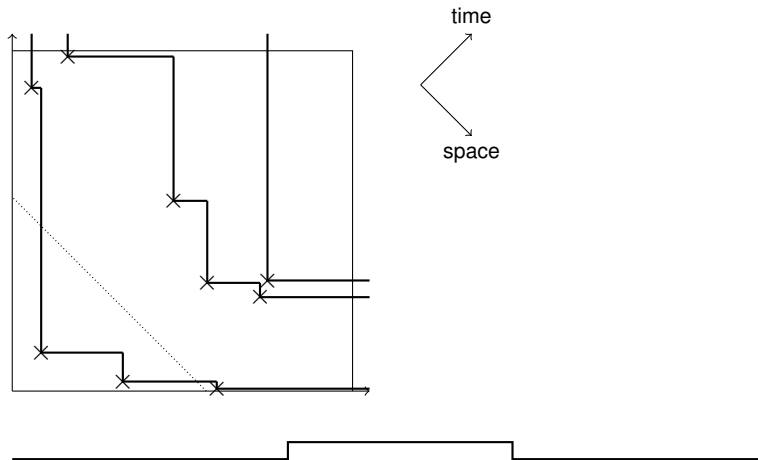
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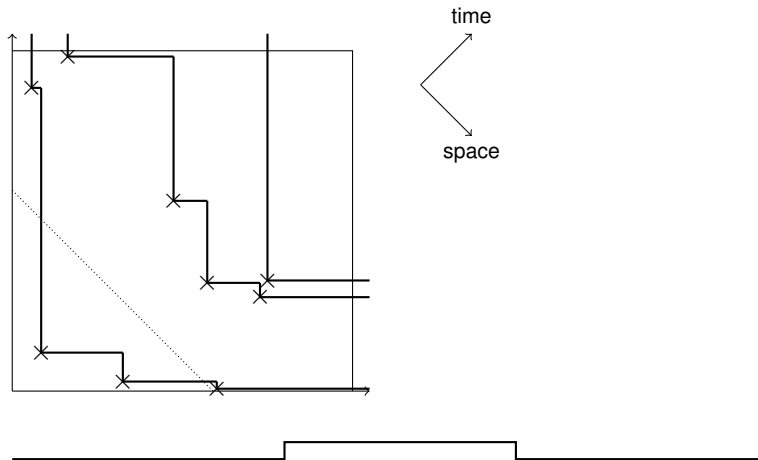
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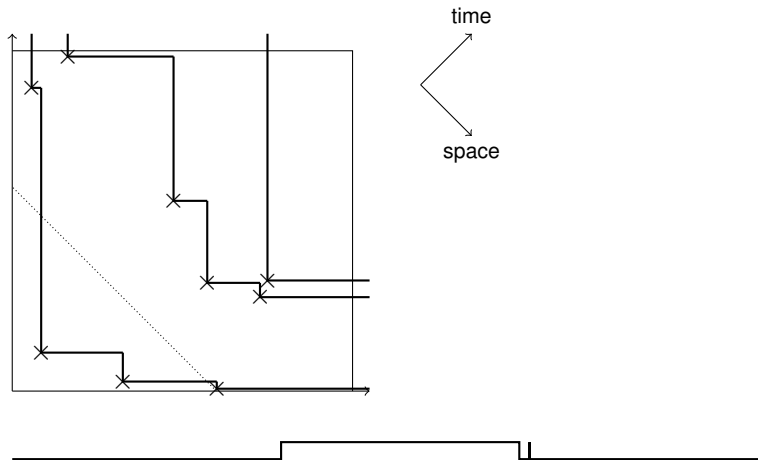
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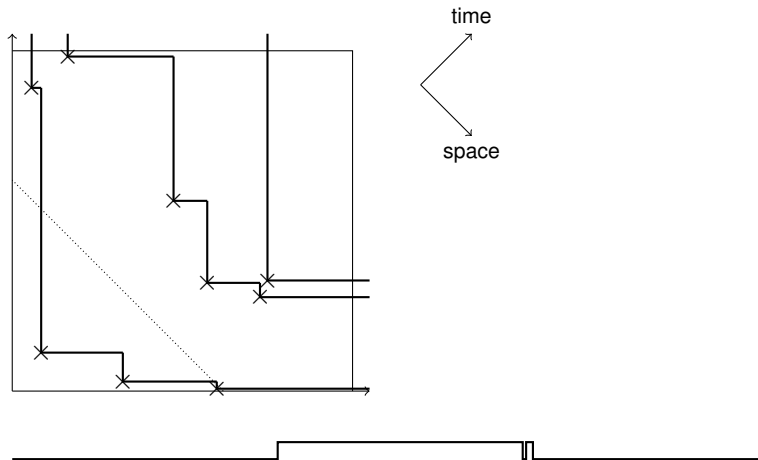
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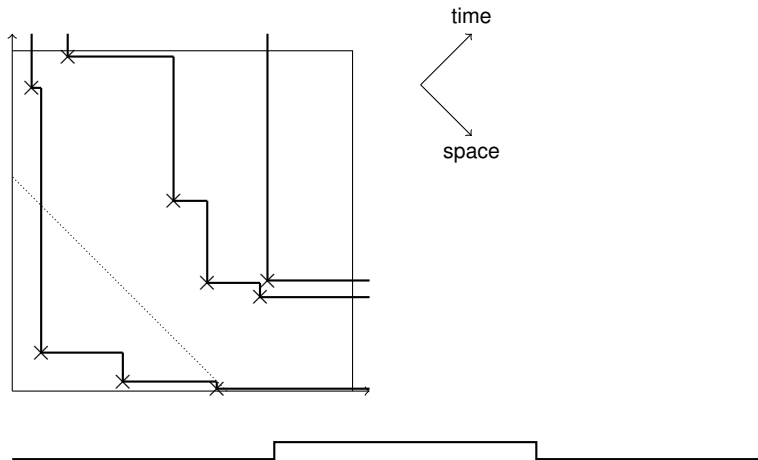
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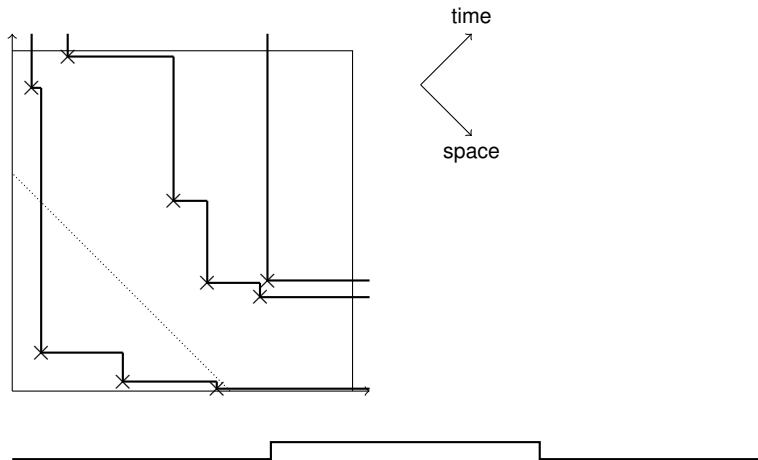
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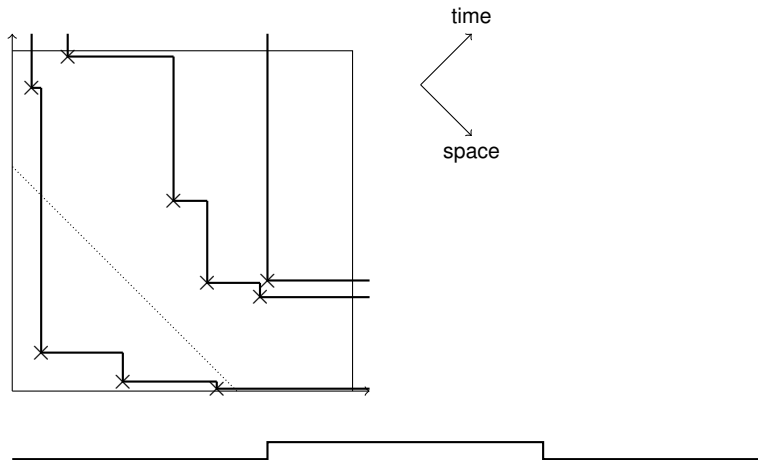
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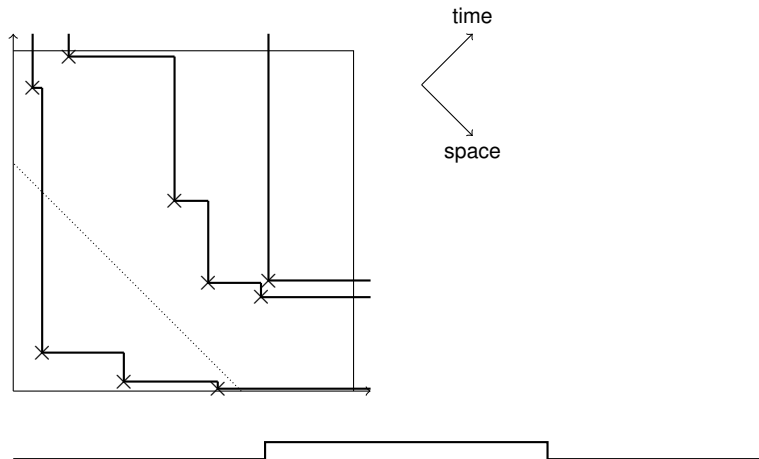
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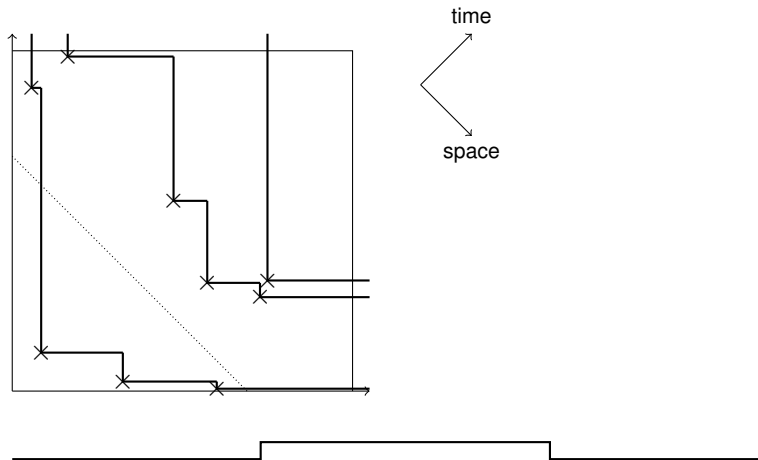
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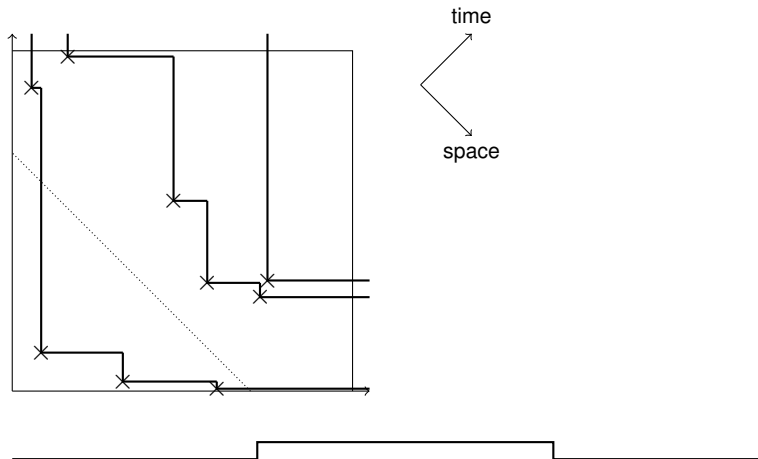
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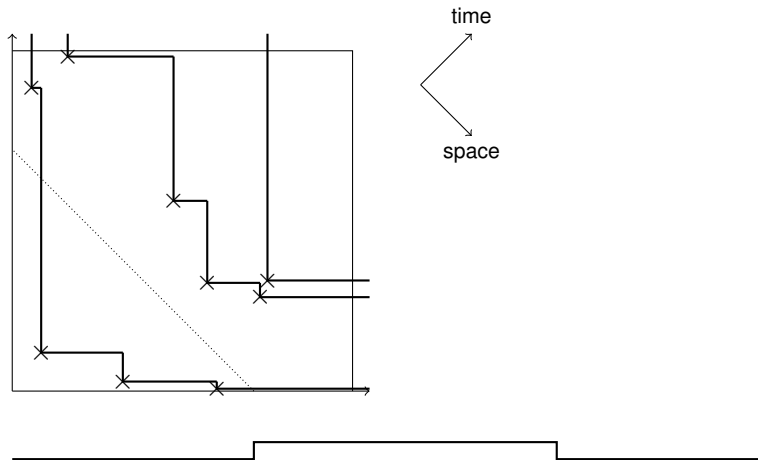
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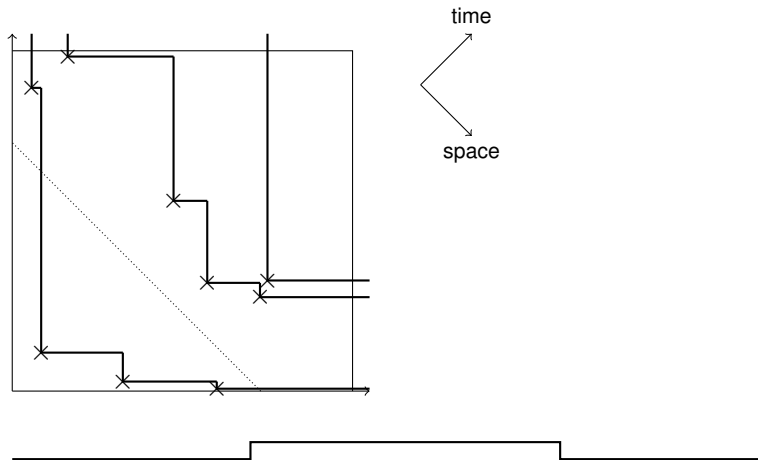
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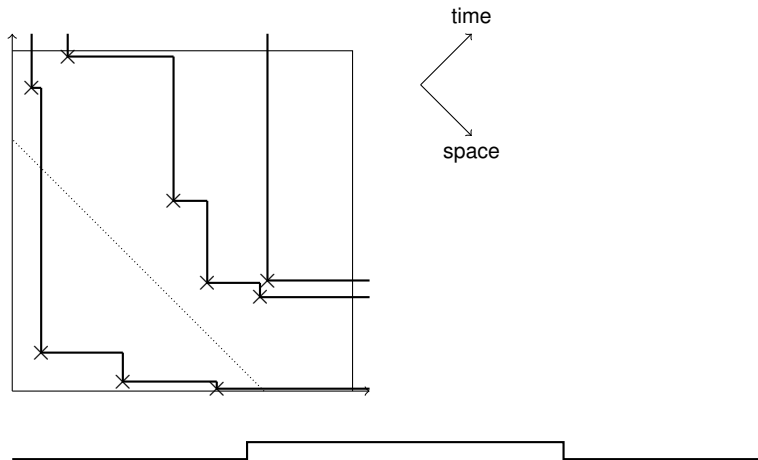
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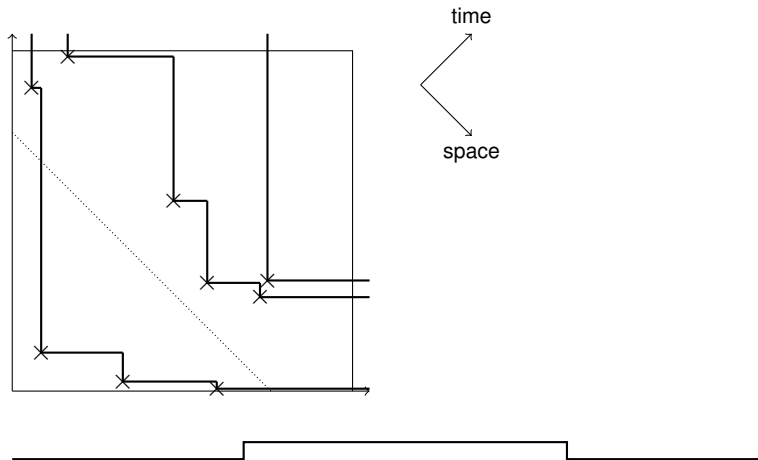
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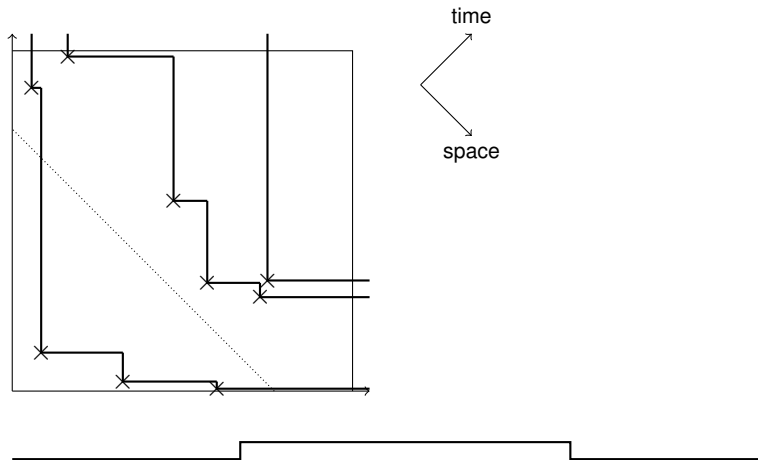
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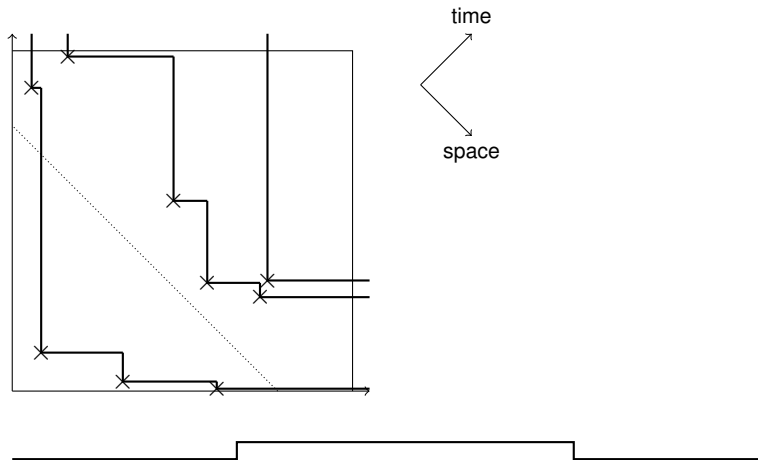
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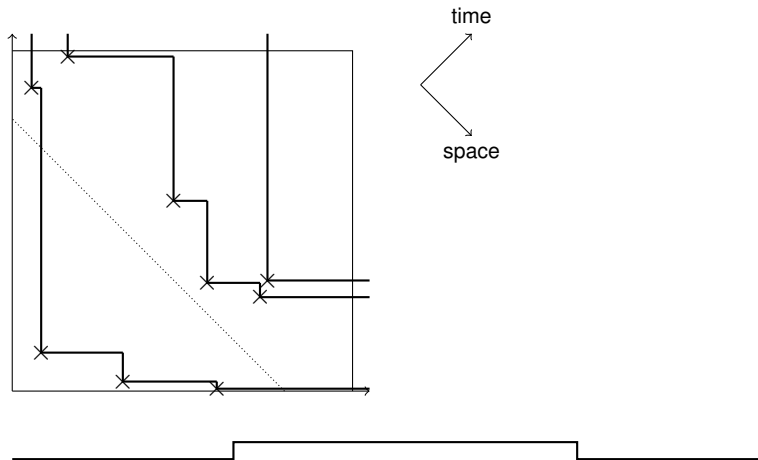
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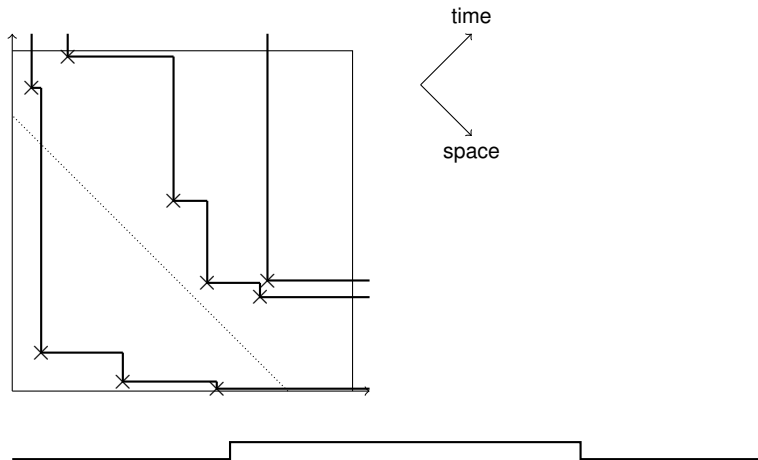
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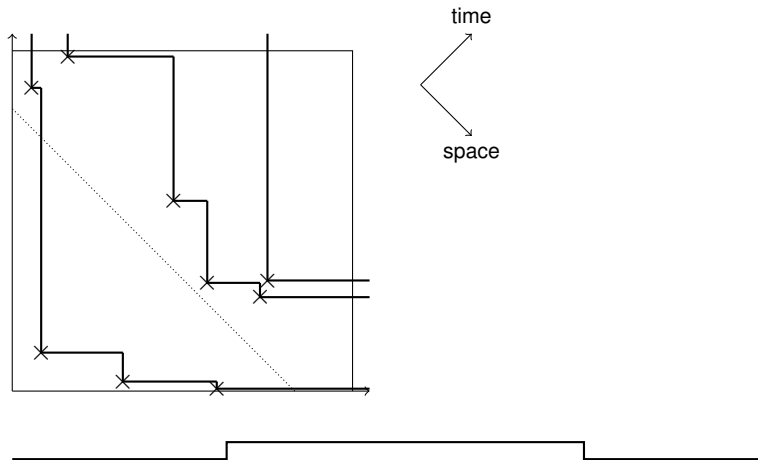
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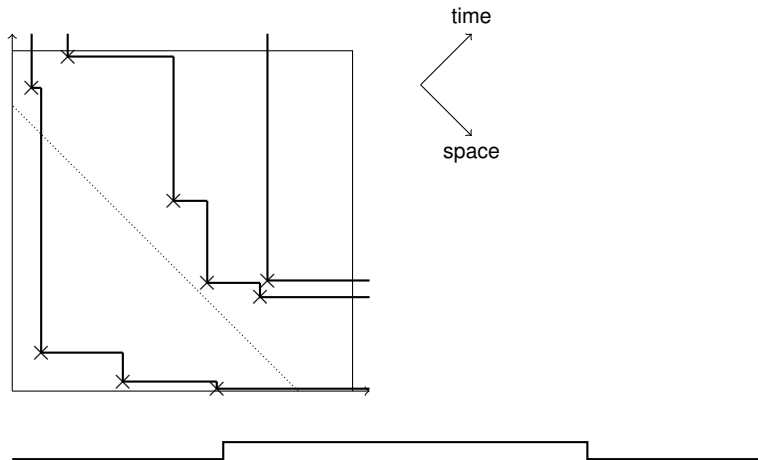
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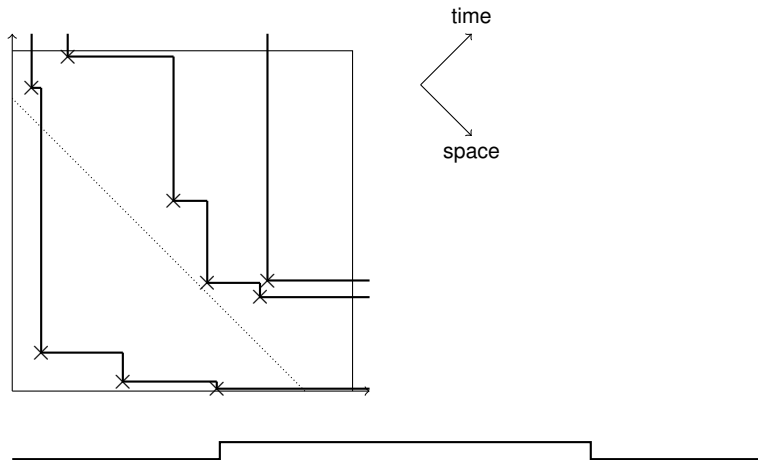
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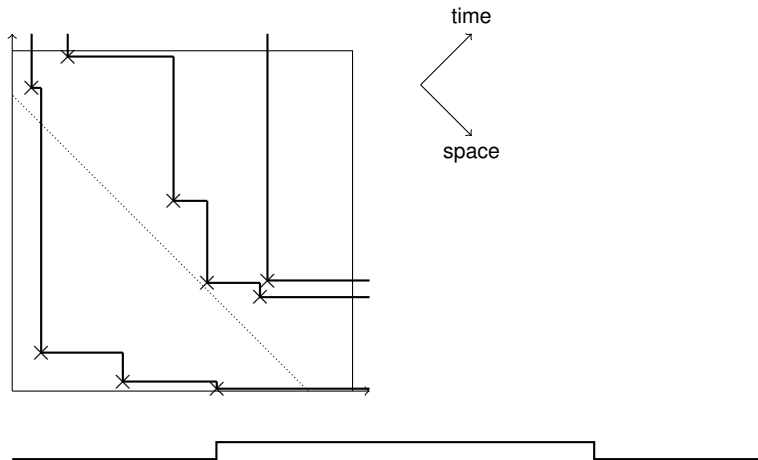
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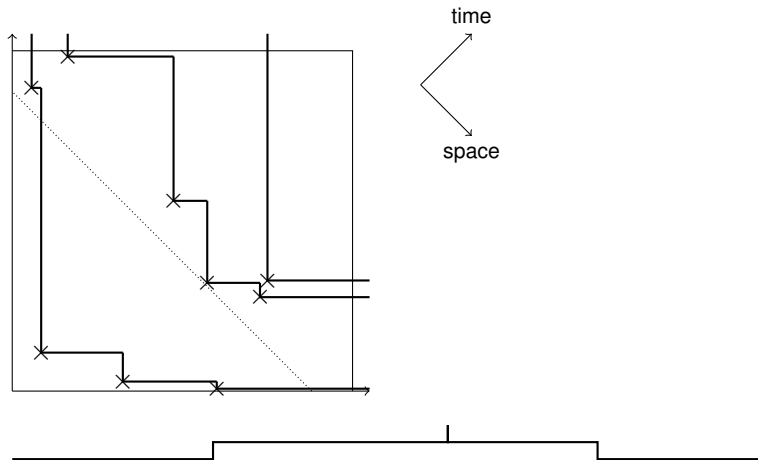
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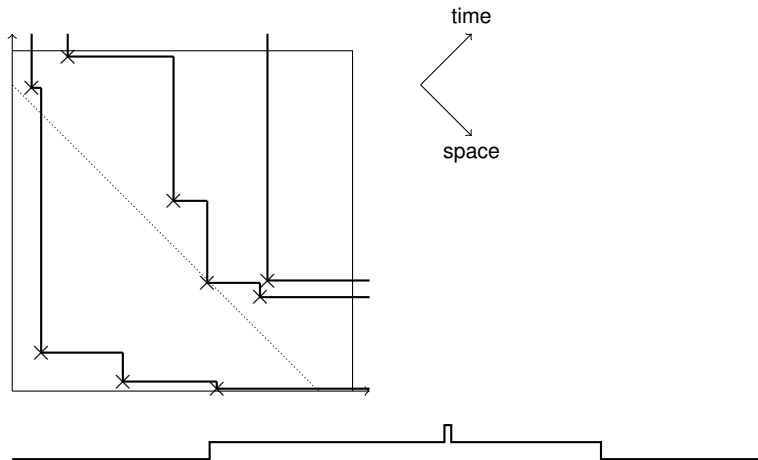
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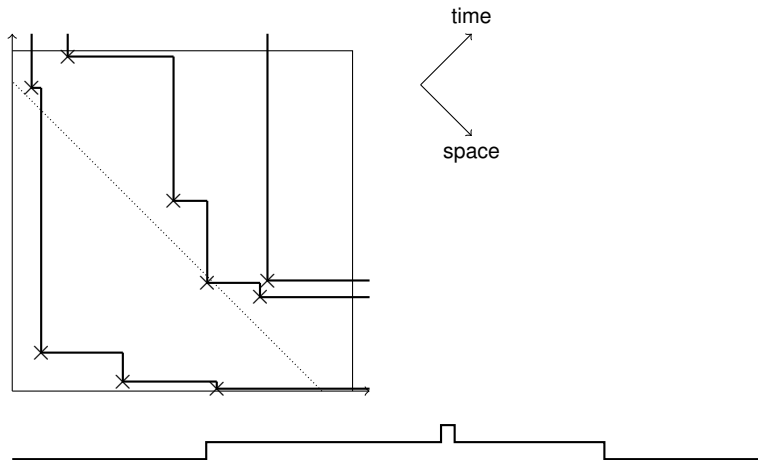
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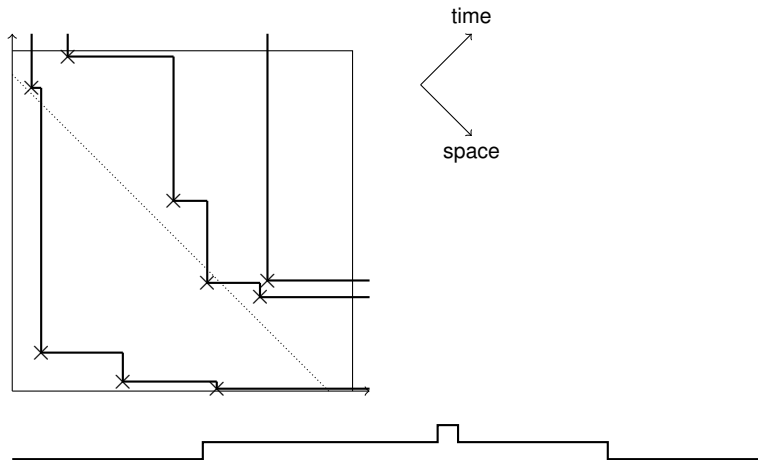
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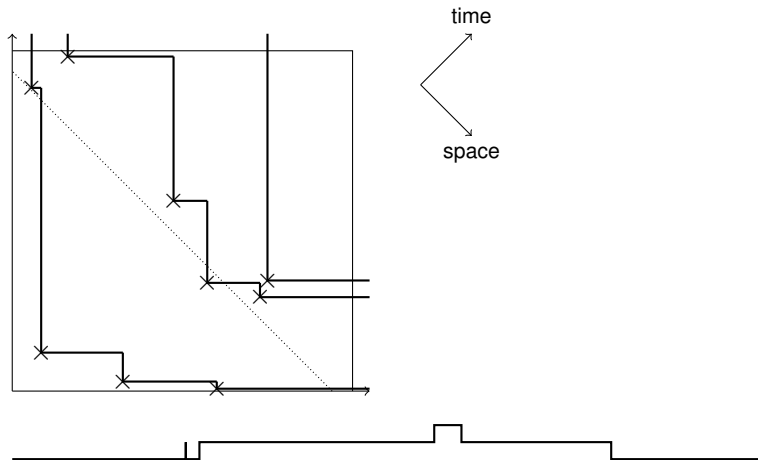
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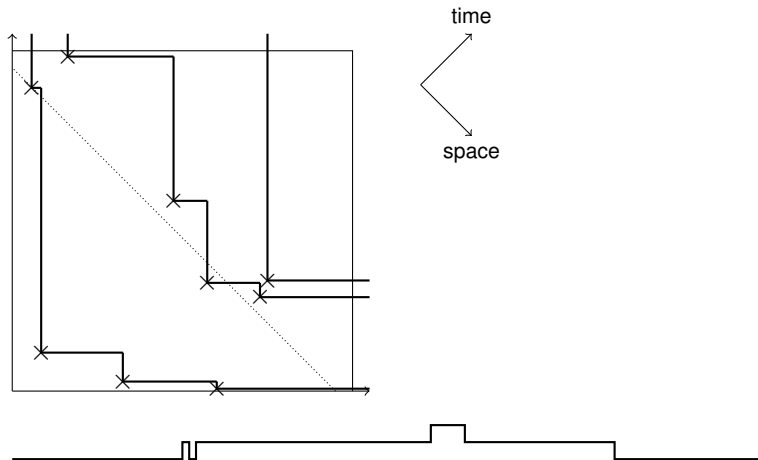
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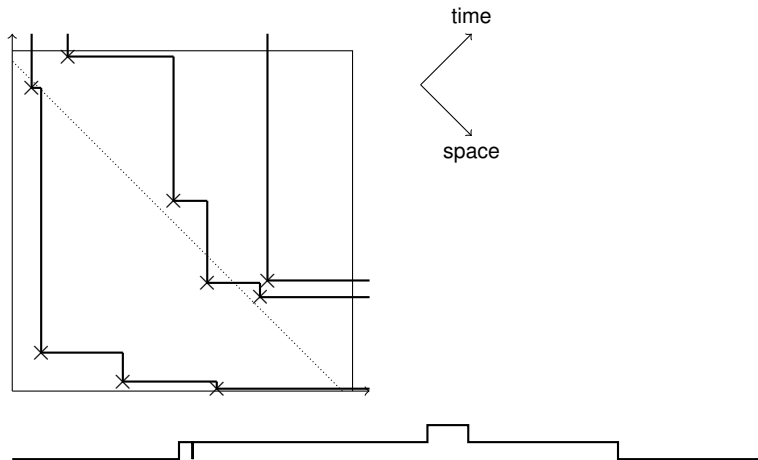
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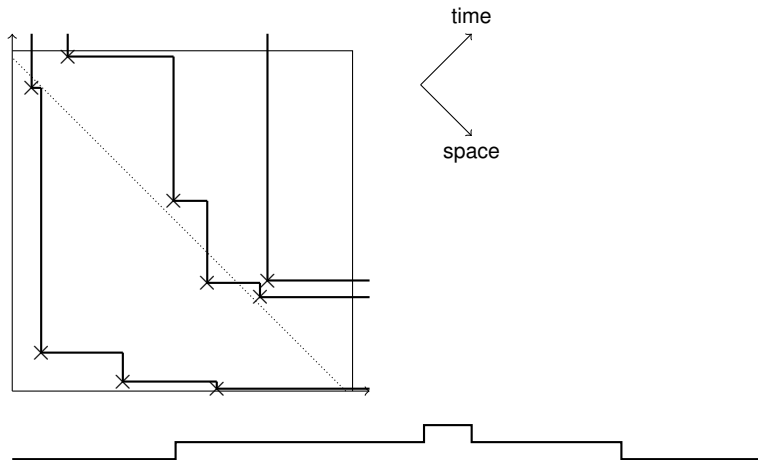
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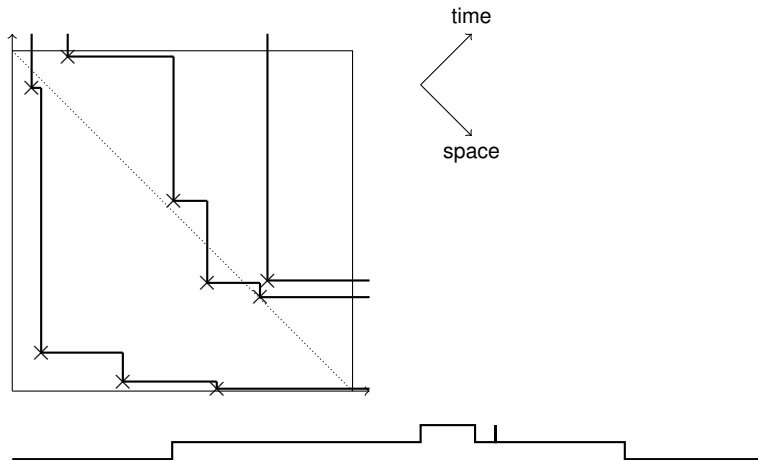
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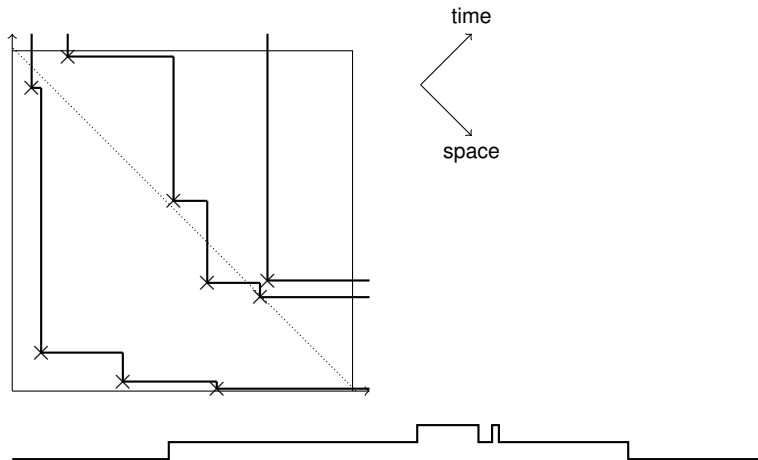
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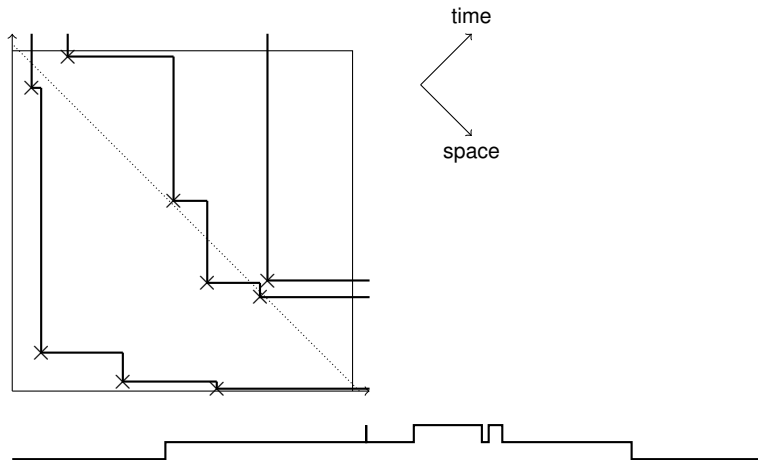
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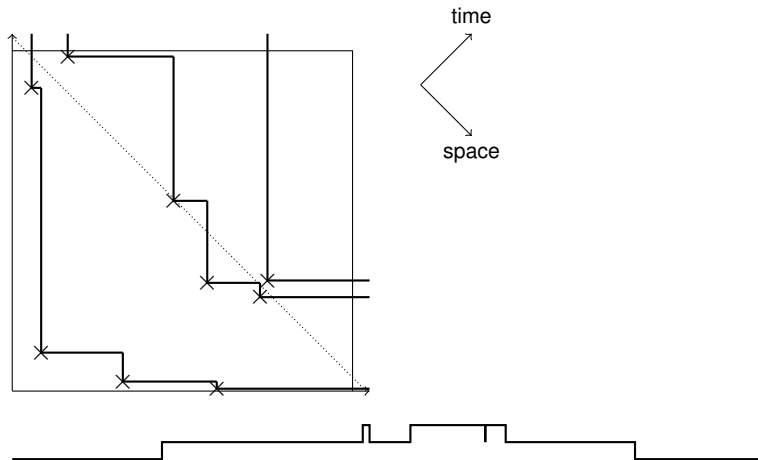
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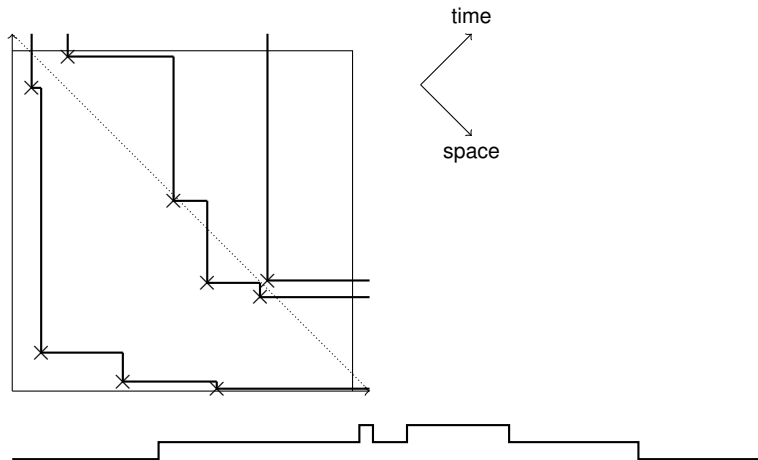
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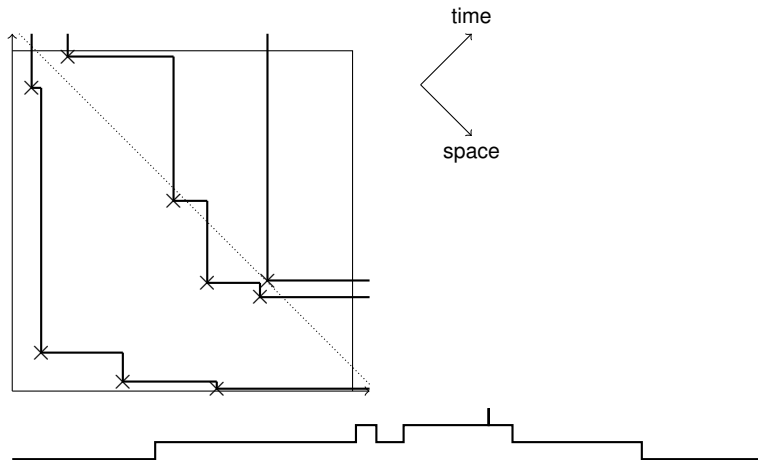
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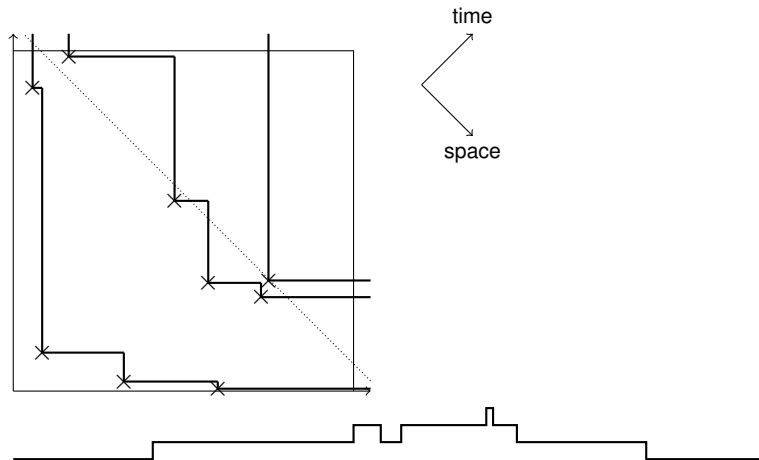
Polynuclear Growth (PNG) [Hammersley '72]



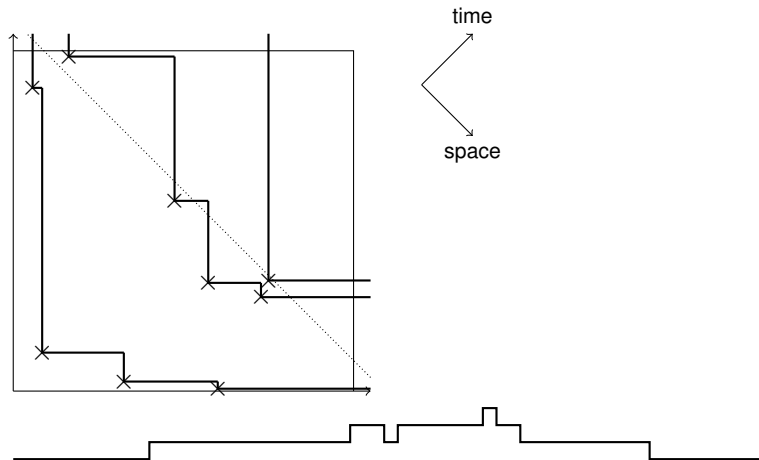
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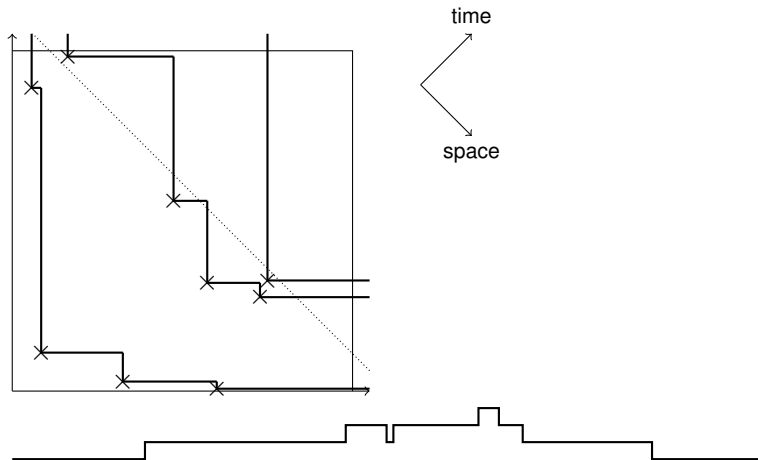
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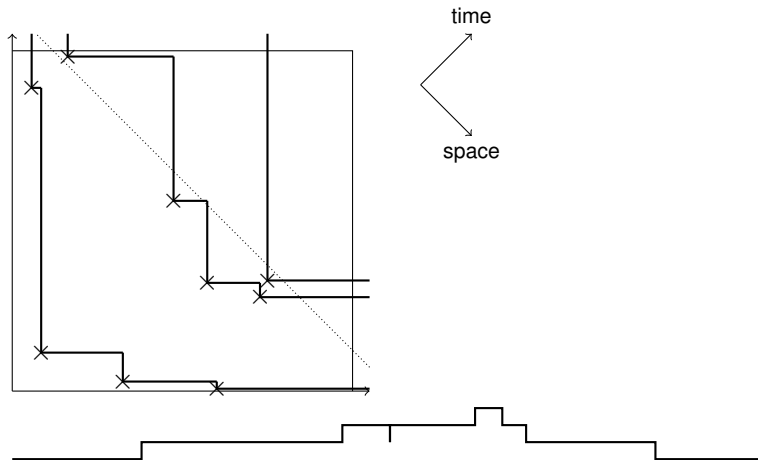
Polynuclear Growth (PNG) [Hammersley '72]



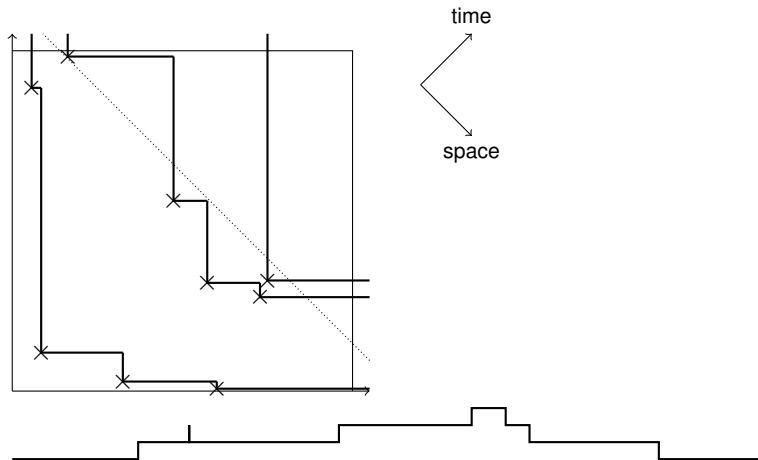
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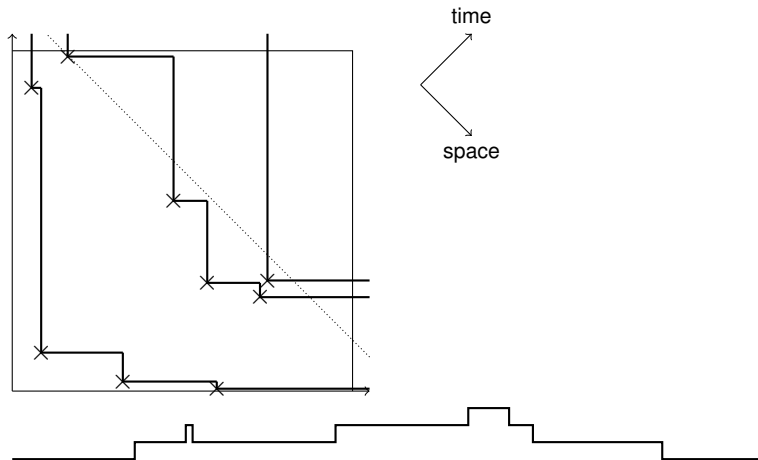
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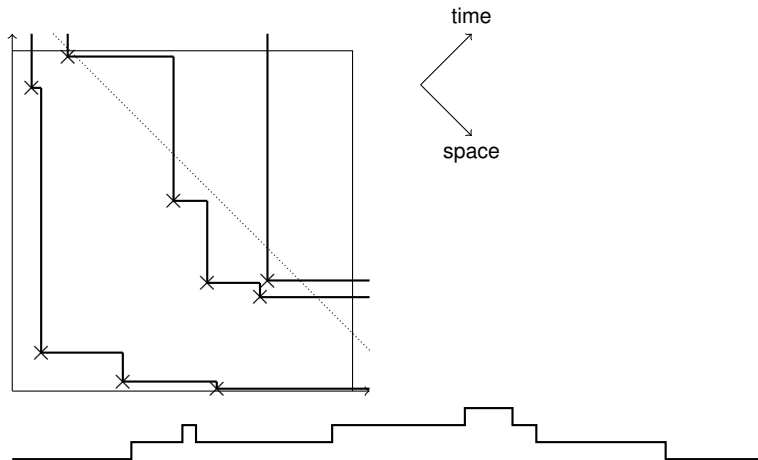
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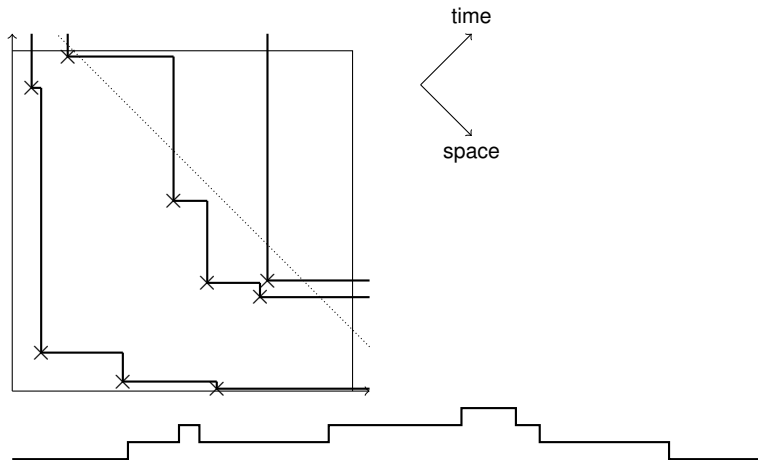
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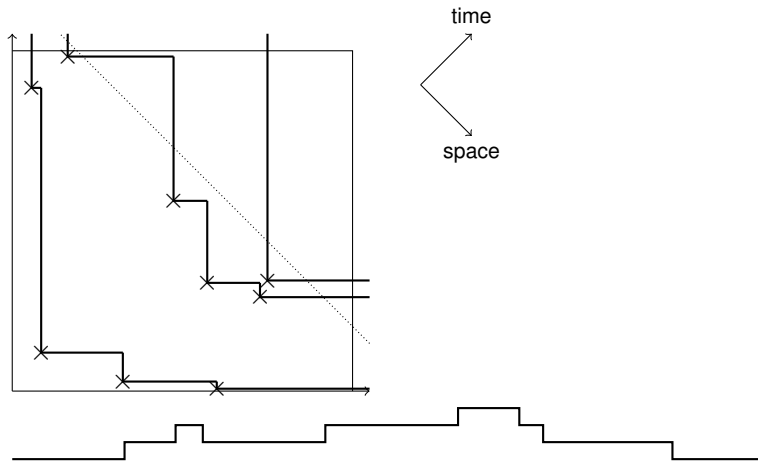
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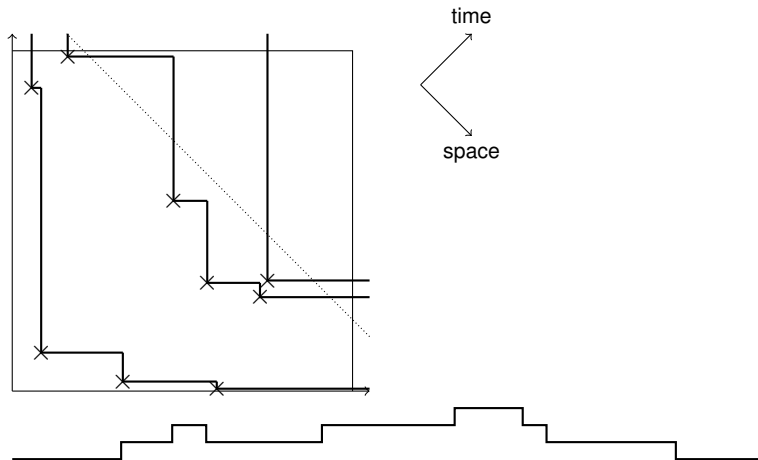
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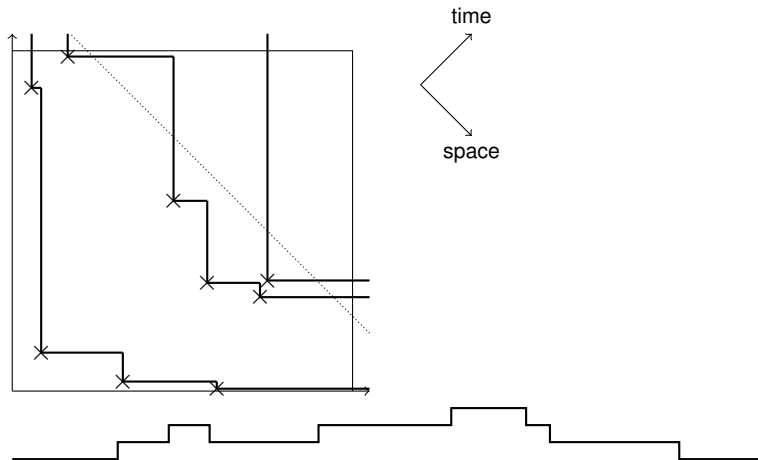
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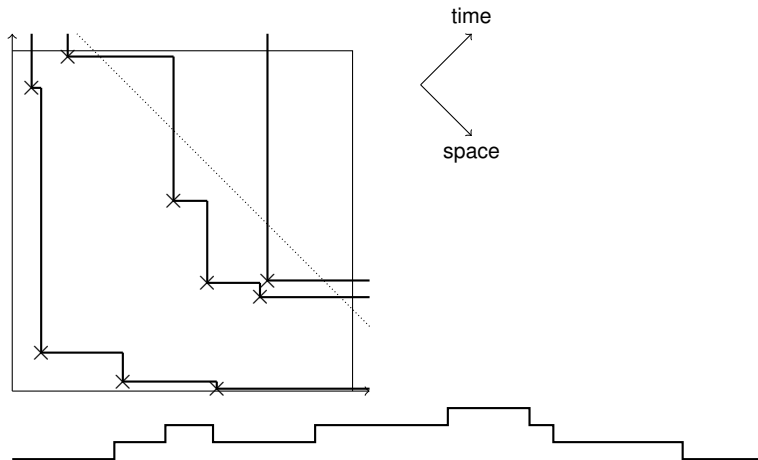
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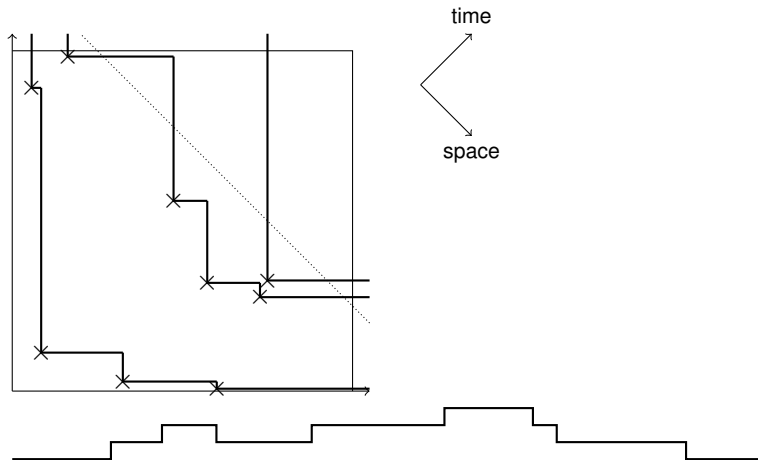
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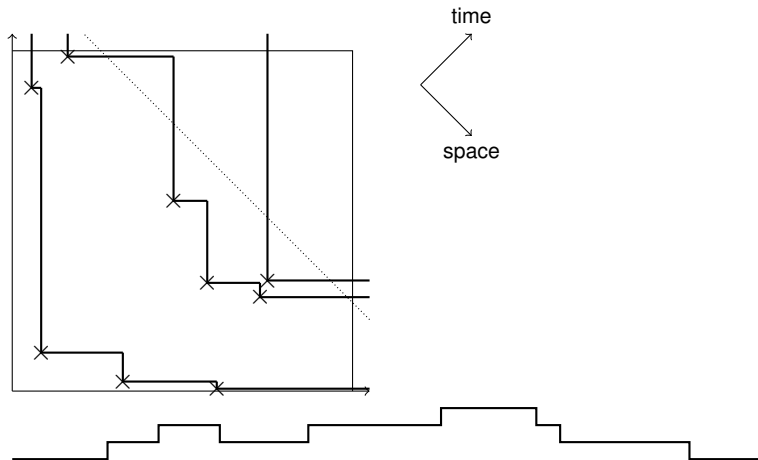
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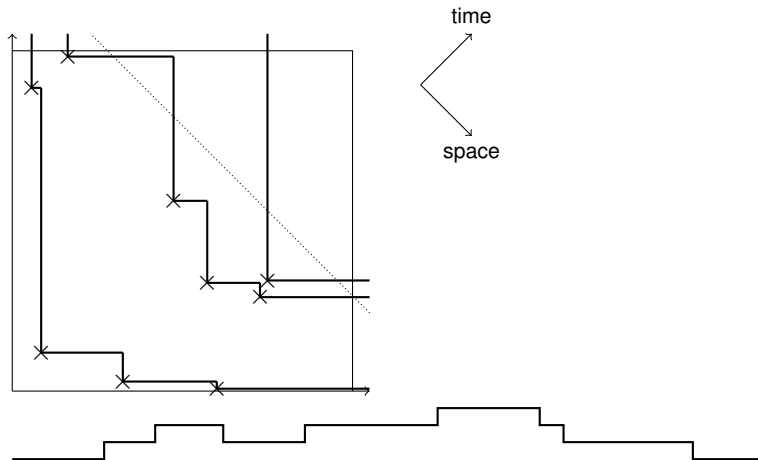
Polynuclear Growth (PNG) [Hammersley '72]



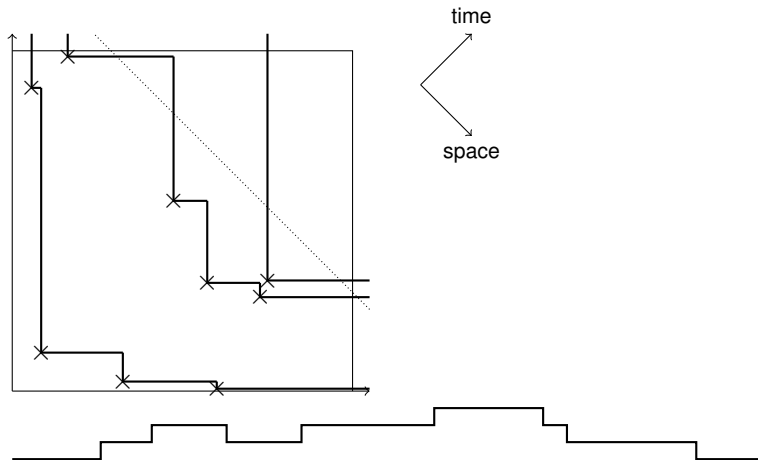
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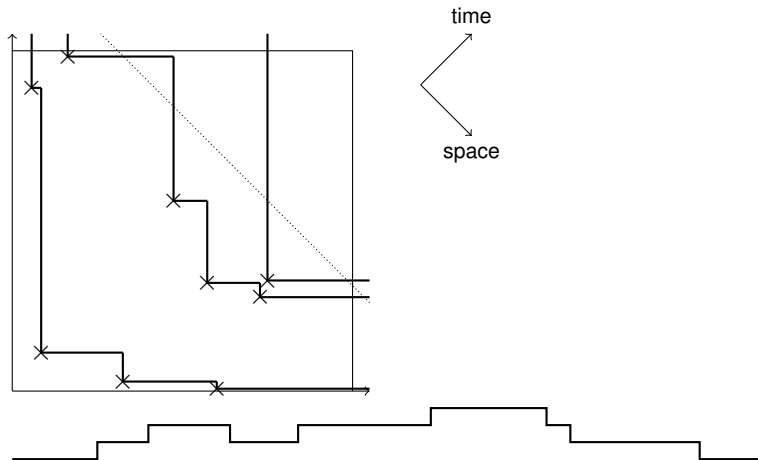
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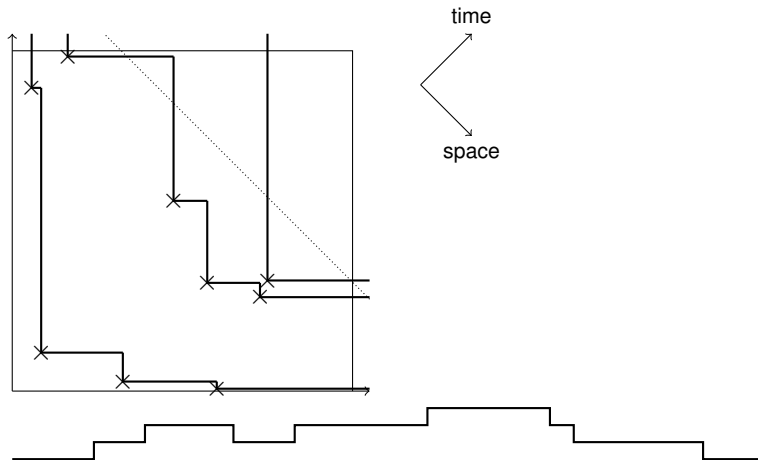
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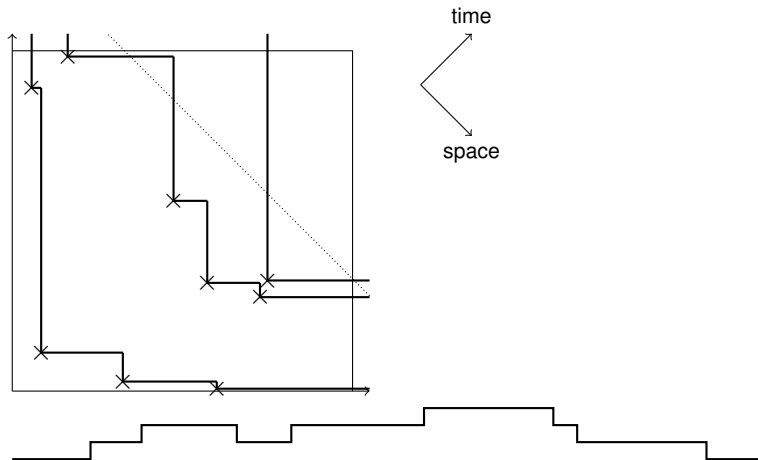
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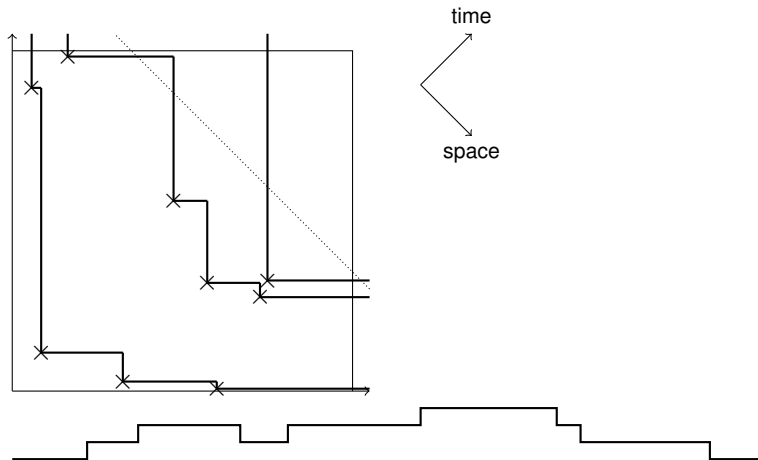
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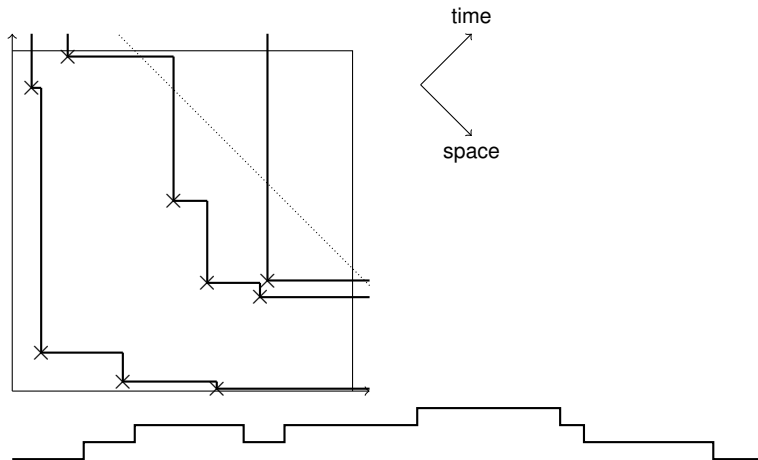
Polynuclear Growth (PNG) [Hammersley '72]



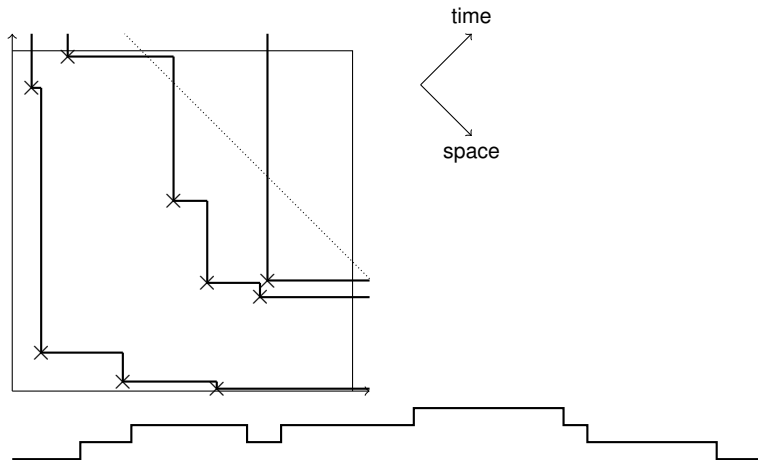
Polynuclear Growth (PNG) [Hammersley '72]



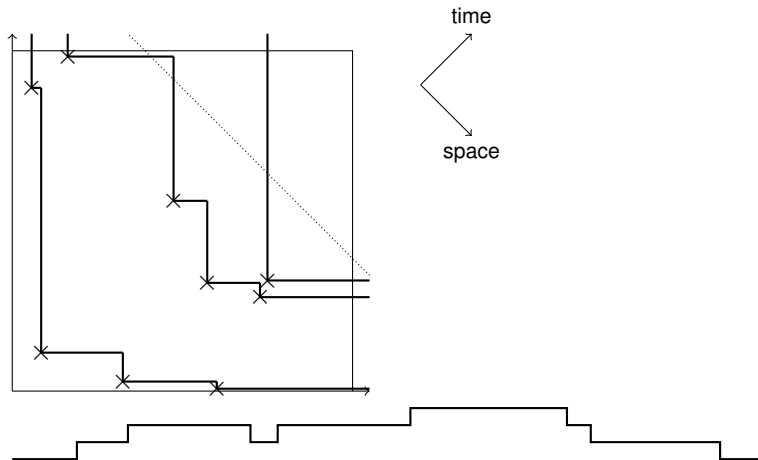
Polynuclear Growth (PNG) [Hammersley '72]



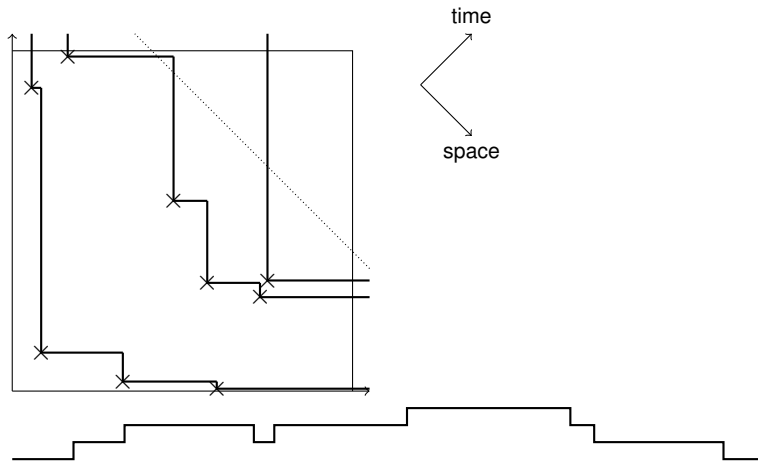
Polynuclear Growth (PNG) [Hammersley '72]



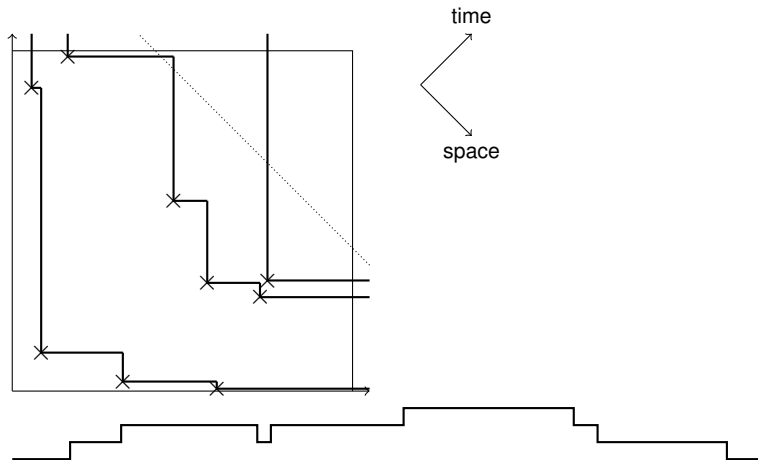
Polynuclear Growth (PNG) [Hammersley '72]



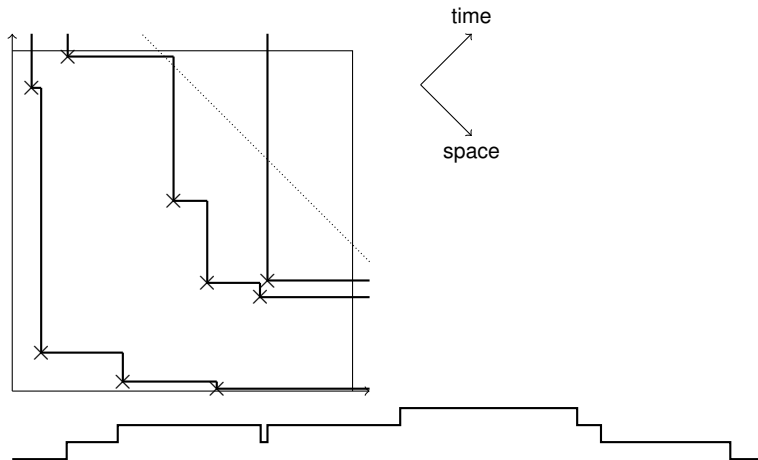
Polynuclear Growth (PNG) [Hammersley '72]



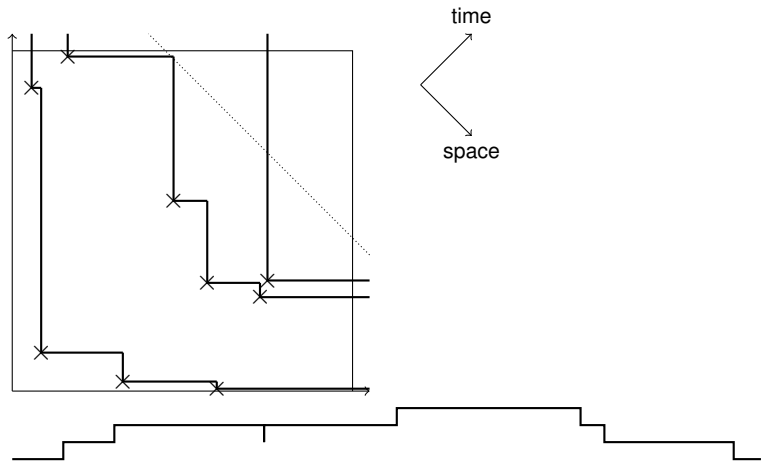
Polynuclear Growth (PNG) [Hammersley '72]



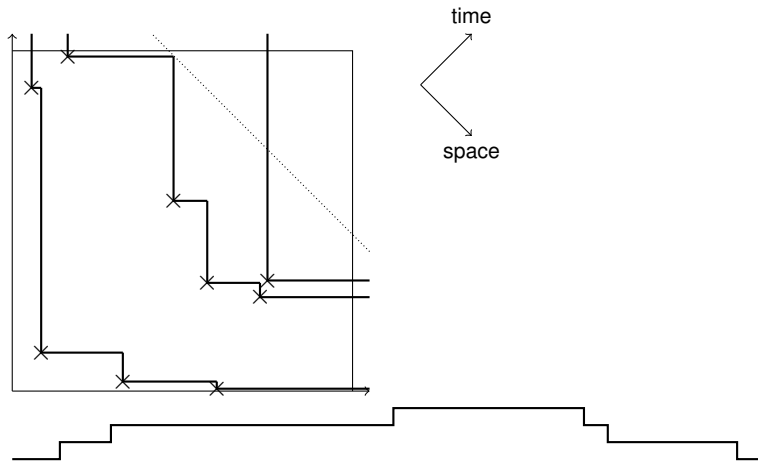
Polynuclear Growth (PNG) [Hammersley '72]



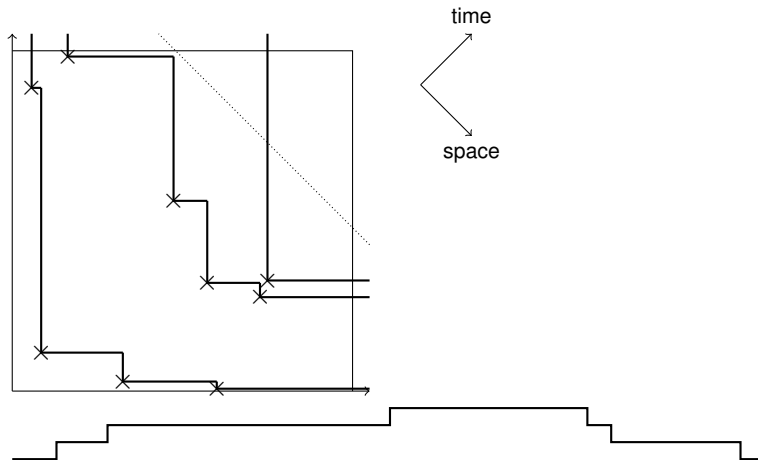
Polynuclear Growth (PNG) [Hammersley '72]



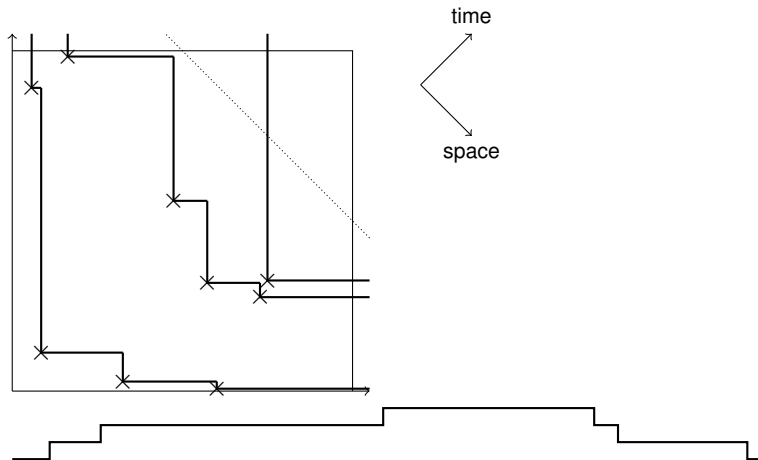
Polynuclear Growth (PNG) [Hammersley '72]



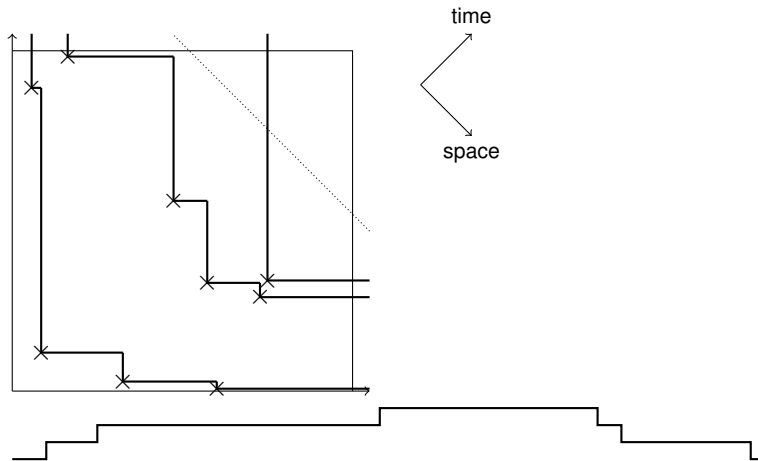
Polynuclear Growth (PNG) [Hammersley '72]



Polynuclear Growth (PNG) [Hammersley '72]

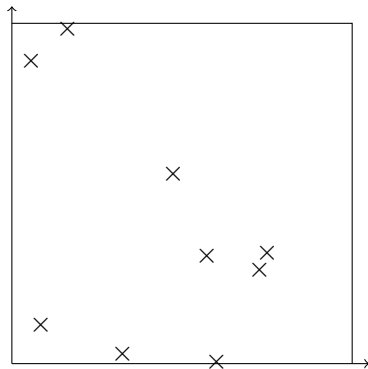


Polynuclear Growth (PNG) [Hammersley '72]



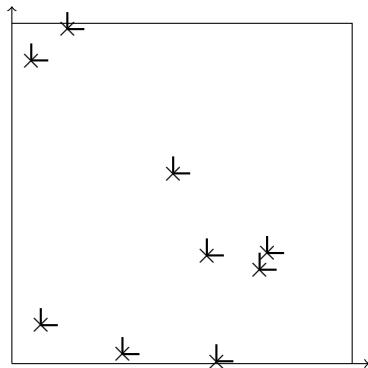
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



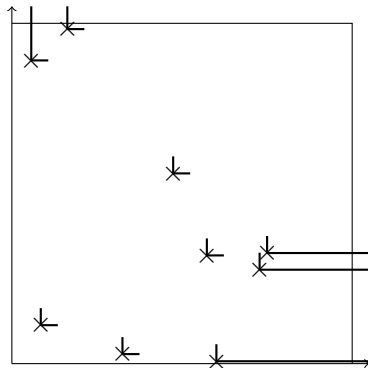
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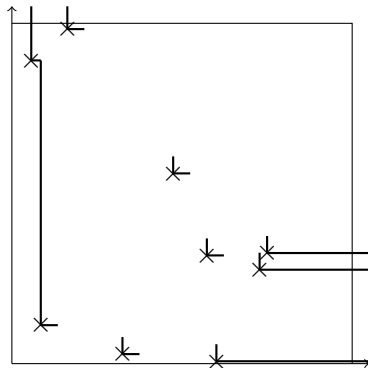
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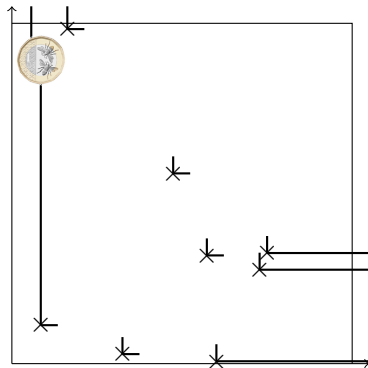
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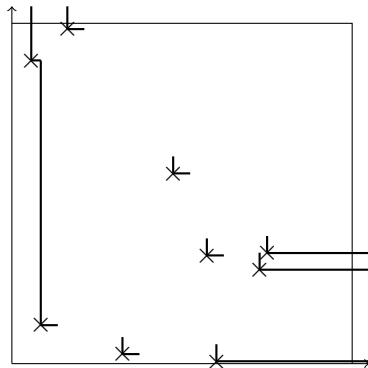
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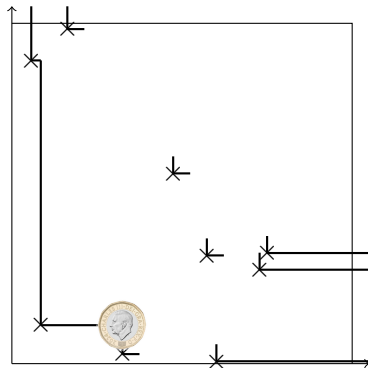
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



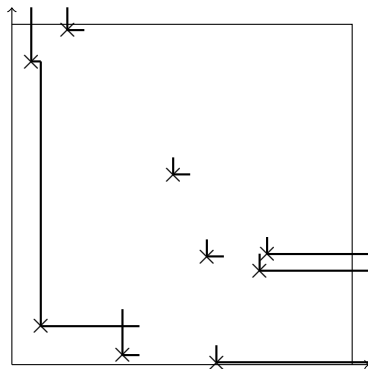
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



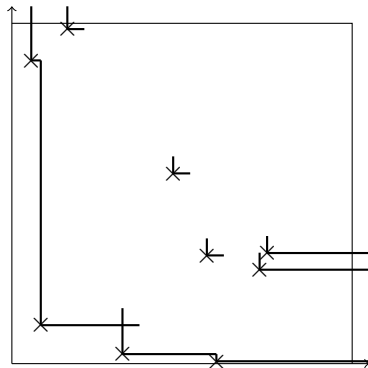
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



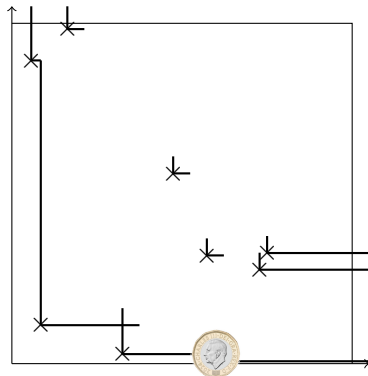
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



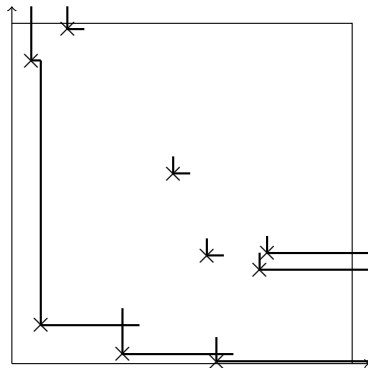
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



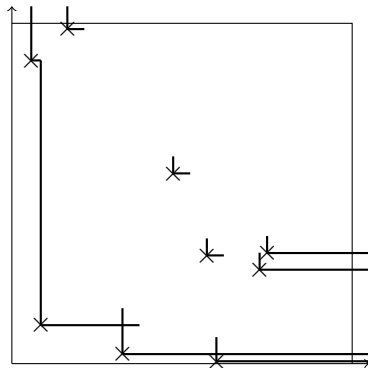
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



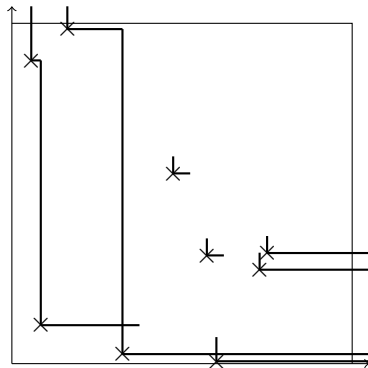
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



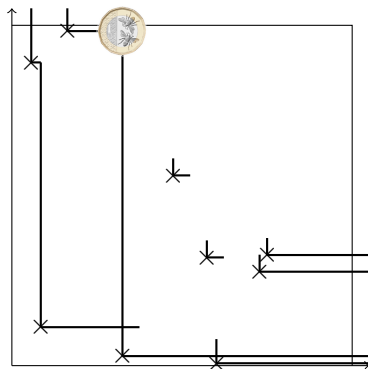
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



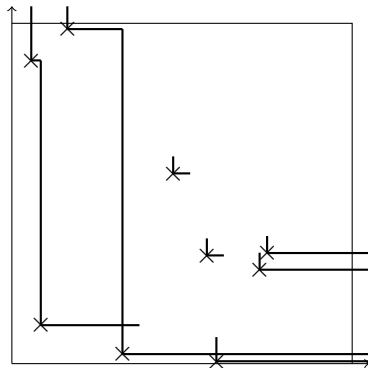
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



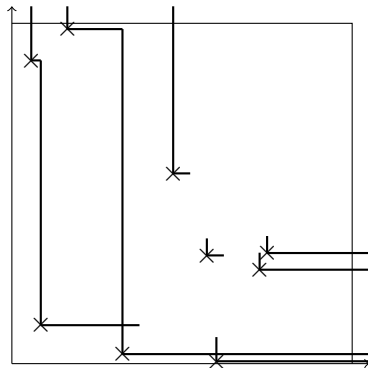
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



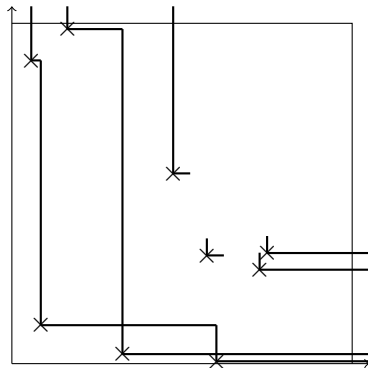
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



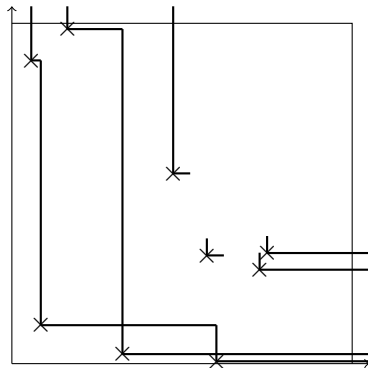
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



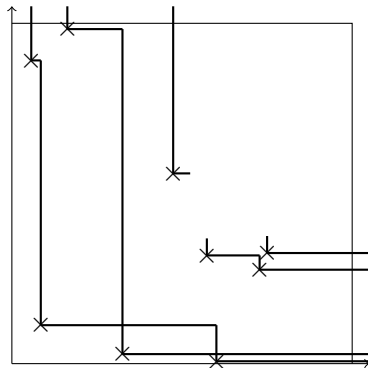
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



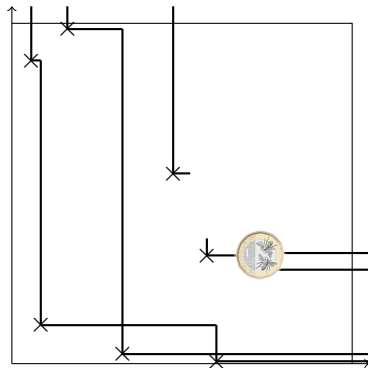
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



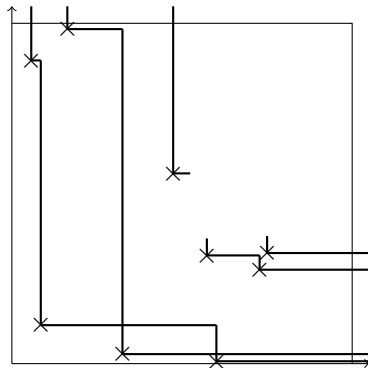
t -PNG [Aggarwal, Borodin, Wheeler '23]

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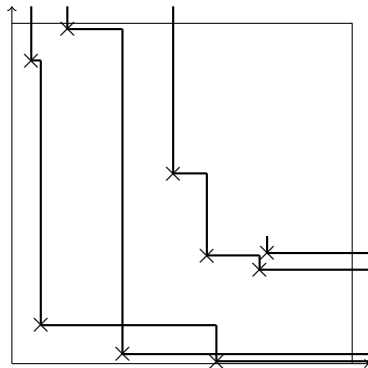
t -PNG [Aggarwal, Borodin, Wheeler '23]

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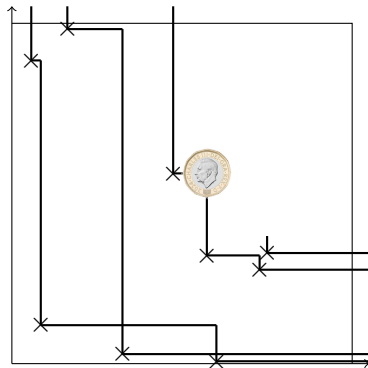
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



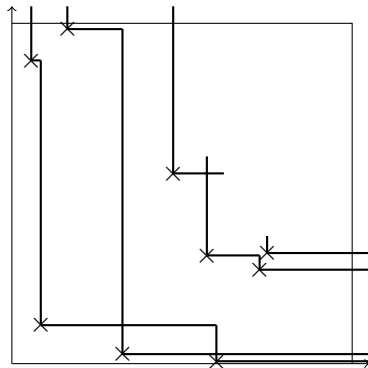
t -PNG [Aggarwal, Borodin, Wheeler '23]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



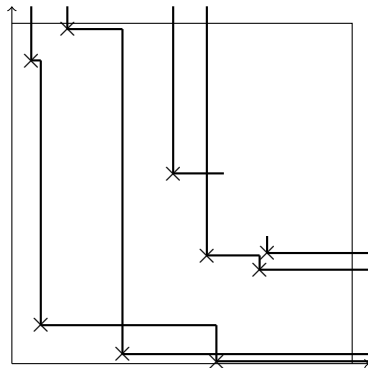
t -PNG [Aggarwal, Borodin, Wheeler '23]

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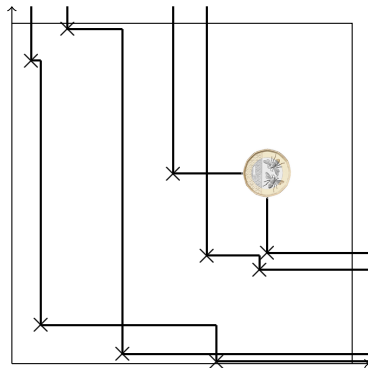
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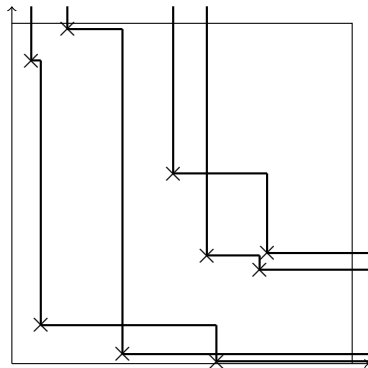
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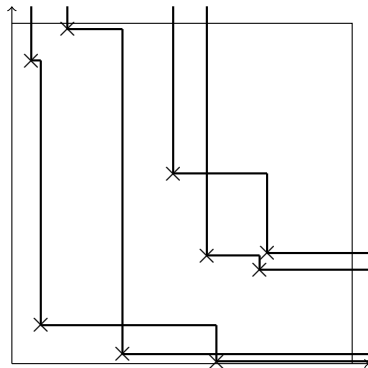
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t -PNG [Aggarwal, Borodin, Wheeler '23]

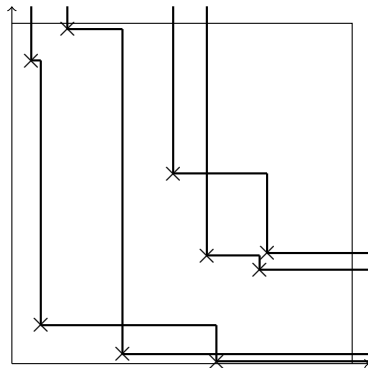
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- Fluctuations: in KPZ-class [Aggarwal, Borodin, Wheeler '23]

t -PNG [Aggarwal, Borodin, Wheeler '23]

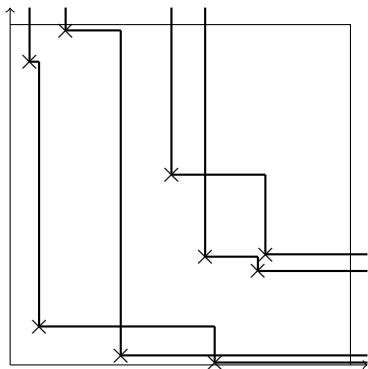
Flip coin: cross with prob. t , corner with prob. $1 - t$.



- ▶ Fluctuations: in KPZ-class [Aggarwal, Borodin, Wheeler '23]
- ▶ Colliding nucleations don't always annihilate (coin)

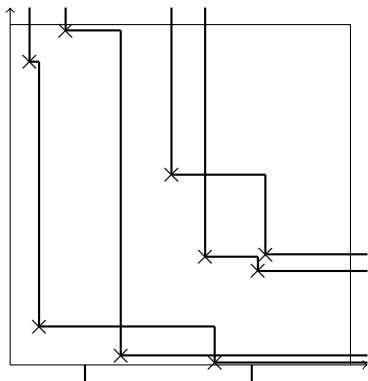
Stationary t -PNG [Drillick, Lin '24]

Flip coin: cross with prob. t , corner with prob. $1 - t$.



Stationary t -PNG [Drillick, Lin '24]

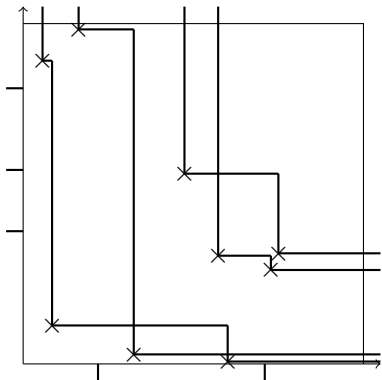
Flip coin: cross with prob. t , corner with prob. $1 - t$.



- Add **sources** on the bottom, and

Stationary t -PNG [Drillick, Lin '24]

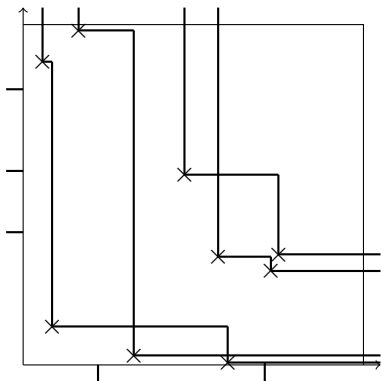
Flip coin: cross with prob. t , corner with prob. $1 - t$.



- ▶ Add **sources** on the bottom, and
- ▶ **sinks** on the left.

Stationary t -PNG [Drillick, Lin '24]

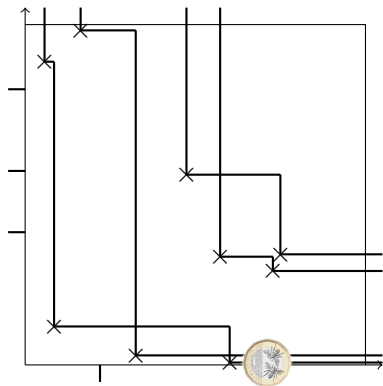
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Stationary t -PNG [Drillick, Lin '24]

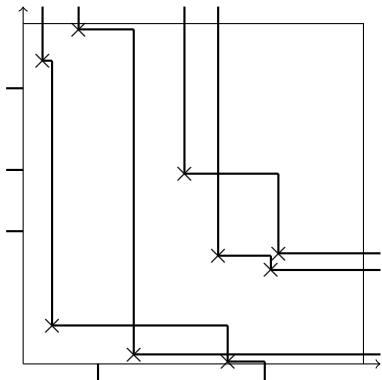
Flip coin: cross with prob. t , corner with prob. $1 - t$.



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Stationary t -PNG [Drillick, Lin '24]

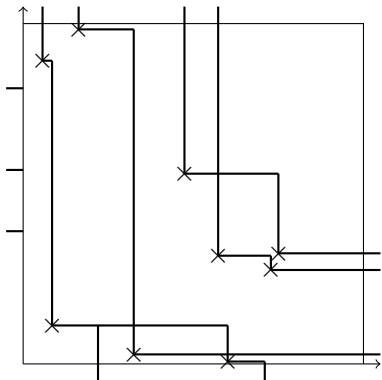
Flip coin: cross with prob. t , corner with prob. $1 - t$.



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Stationary t -PNG [Drillick, Lin '24]

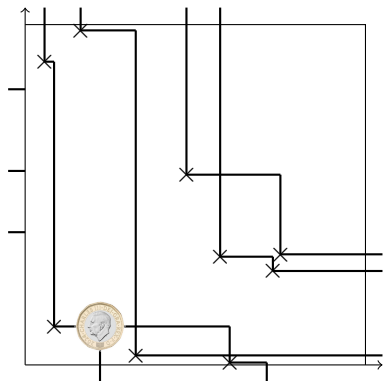
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Stationary t -PNG [Drillick, Lin '24]

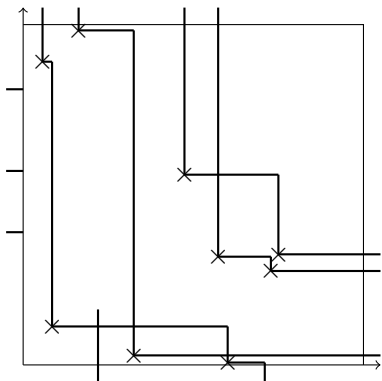
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Stationary t -PNG [Drillick, Lin '24]

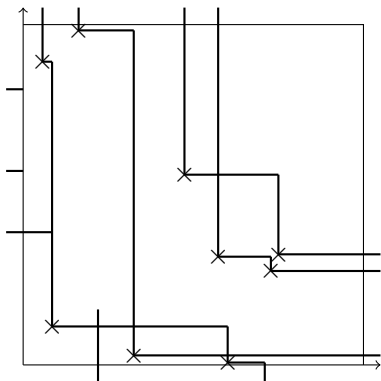
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Stationary t -PNG [Drillick, Lin '24]

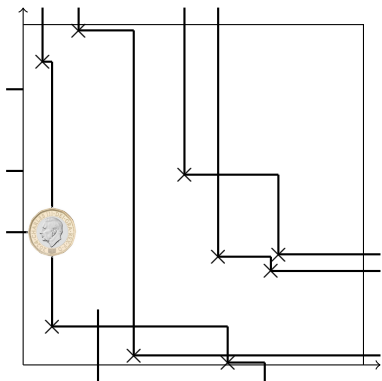
Flip coin: cross with prob. t , corner with prob. $1 - t$.



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Stationary t -PNG [Drillick, Lin '24]

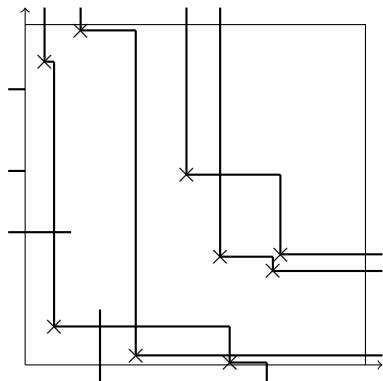
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Stationary t -PNG [Drillick, Lin '24]

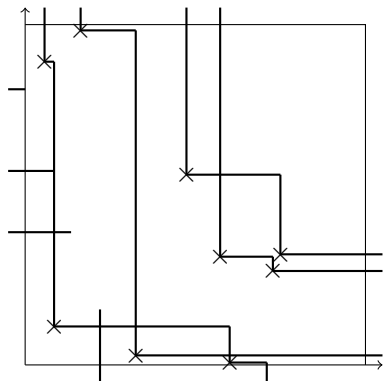
Flip coin: cross with prob. t , corner with prob. $1 - t$.



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Stationary t -PNG [Drillick, Lin '24]

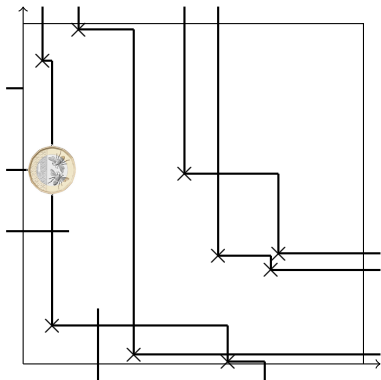
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Stationary t -PNG [Drillick, Lin '24]

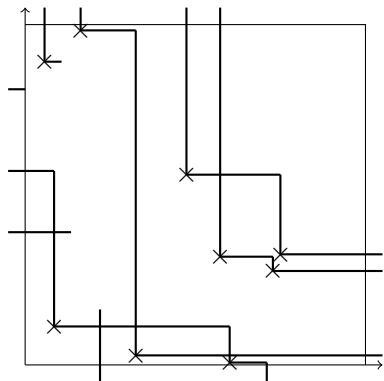
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Stationary t -PNG [Drillick, Lin '24]

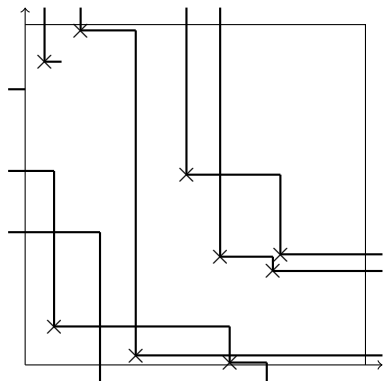
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Stationary t -PNG [Drillick, Lin '24]

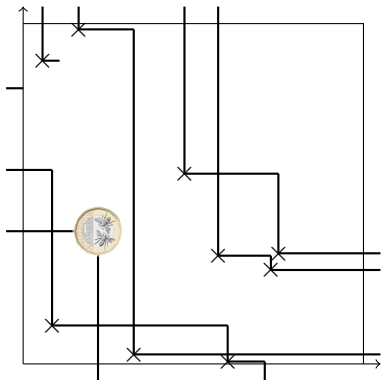
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Stationary t -PNG [Drillick, Lin '24]

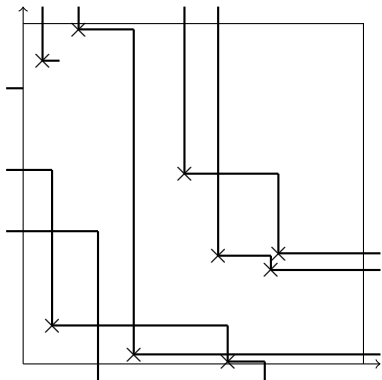
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Stationary t -PNG [Drillick, Lin '24]

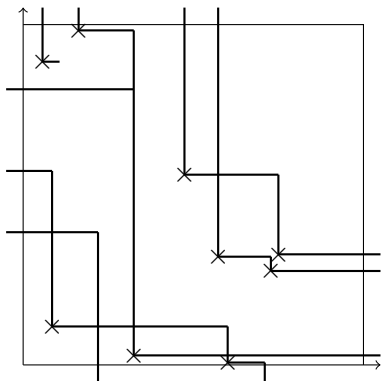
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Stationary t -PNG [Drillick, Lin '24]

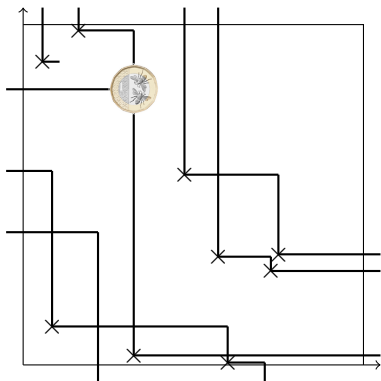
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Stationary t -PNG [Drillick, Lin '24]

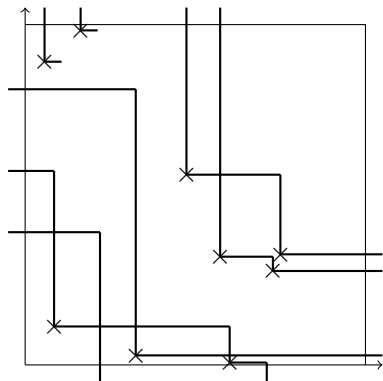
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Stationary t -PNG [Drillick, Lin '24]

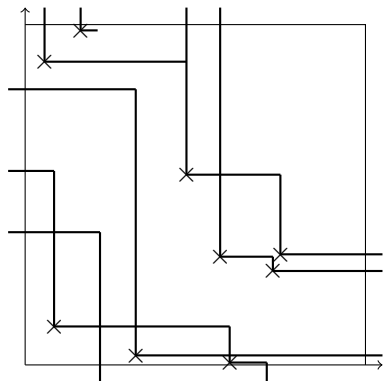
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- ▶ **sinks** on the left.

Stationary t -PNG [Drillick, Lin '24]

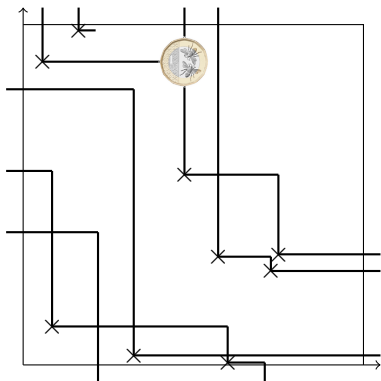
Flip coin: cross with prob. t , corner with prob. $1 - t$.



- ▶ Add **sources** on the bottom, and
- ▶ **sinks** on the left.

Stationary t -PNG [Drillick, Lin '24]

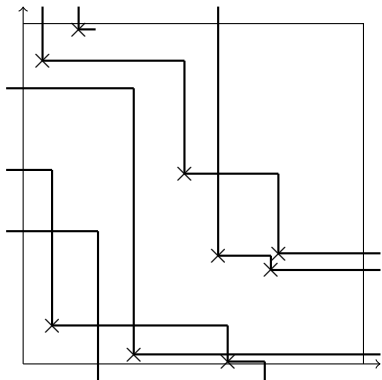
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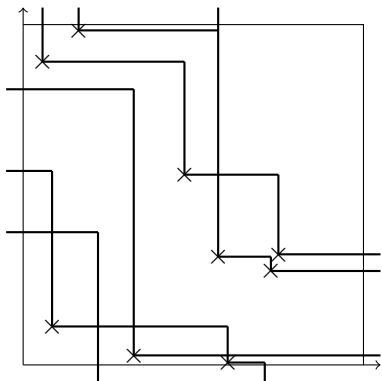
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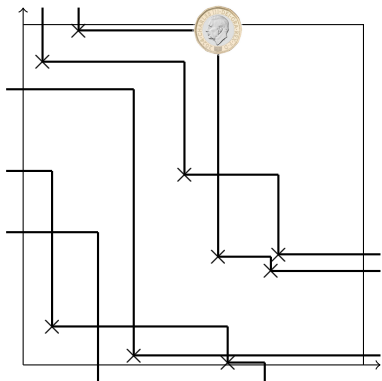
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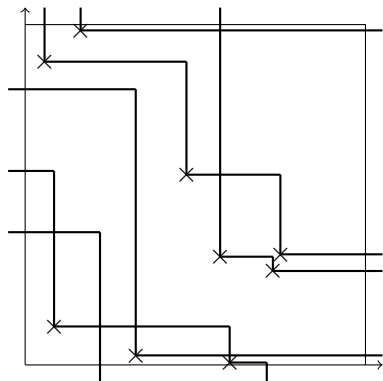
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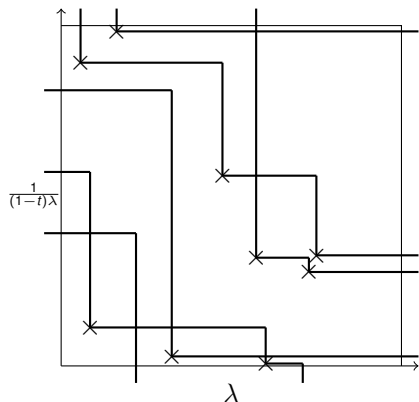
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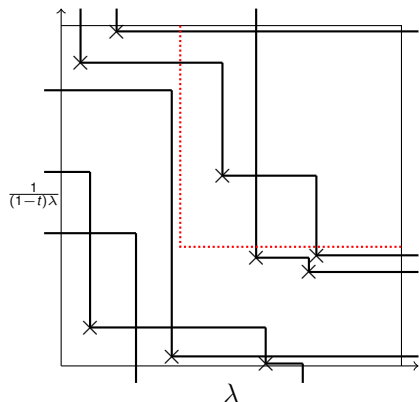
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Theorem

- ▶ If *sources* $\sim \text{Poi}(\lambda)$, and
- ▶ *sinks* $\sim \text{Poi}(\frac{1}{(1-t)\lambda})$ independent,

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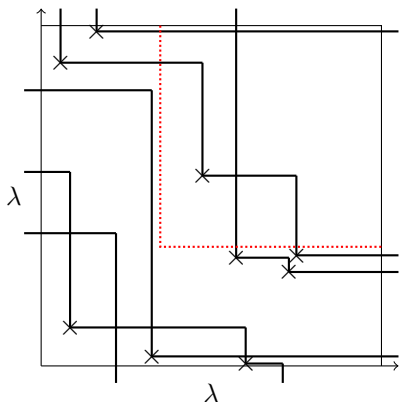
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the no of lines entering

- ▶ on the bottom $\sim \text{Poi}(\lambda)$
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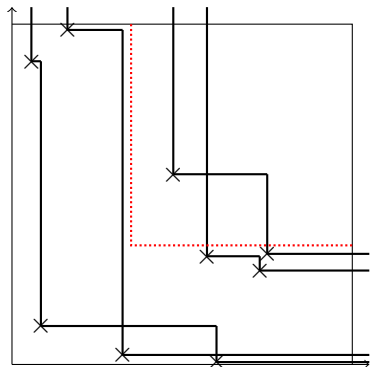
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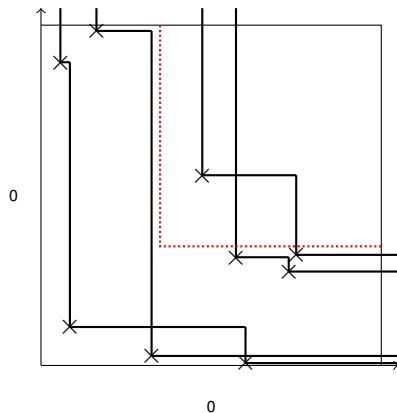
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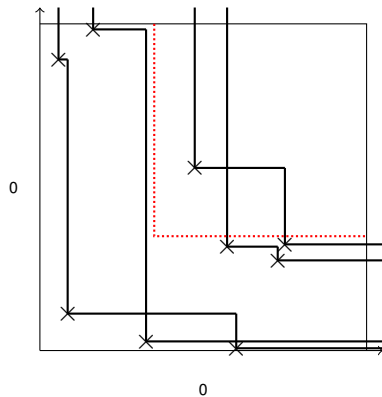
Local convergence



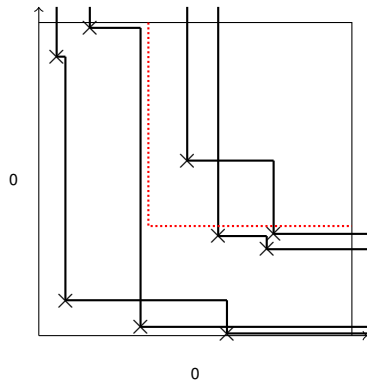
Local convergence



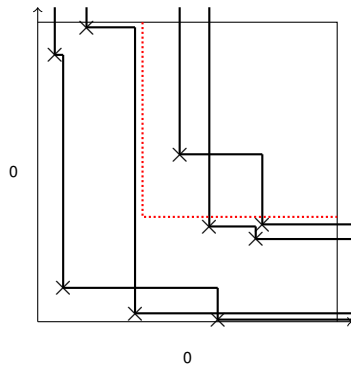
Local convergence



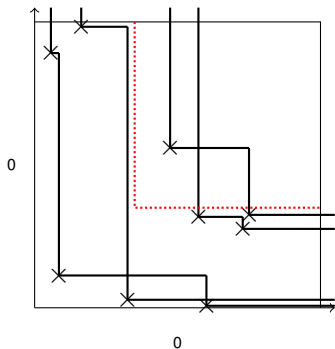
Local convergence



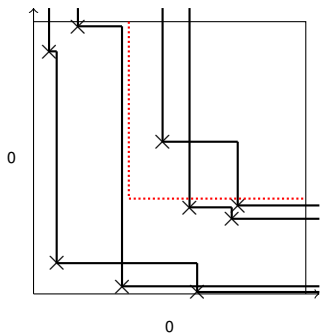
Local convergence



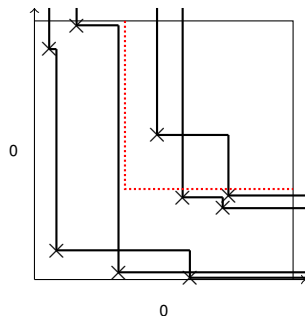
Local convergence



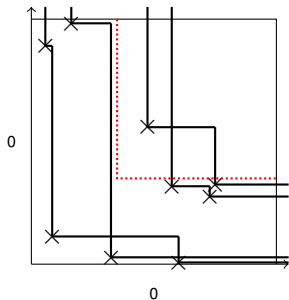
Local convergence



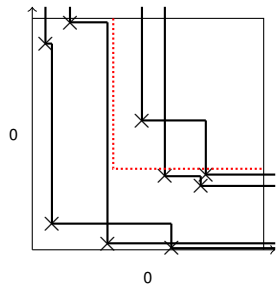
Local convergence



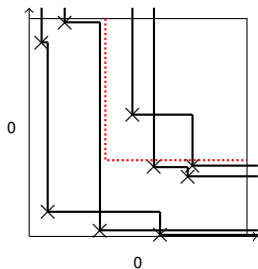
Local convergence



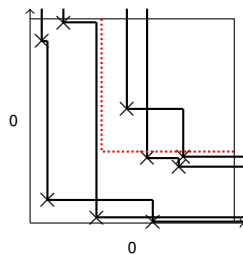
Local convergence



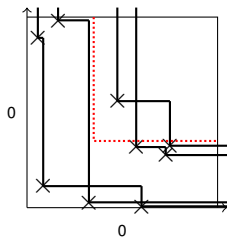
Local convergence



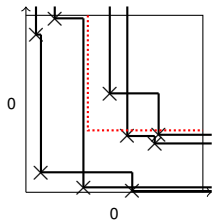
Local convergence



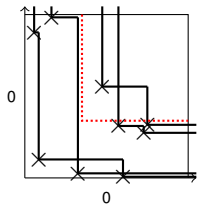
Local convergence



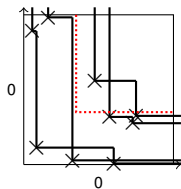
Local convergence



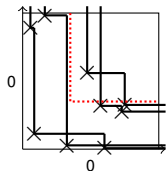
Local convergence



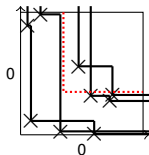
Local convergence



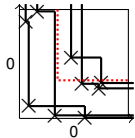
Local convergence



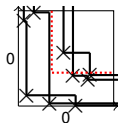
Local convergence



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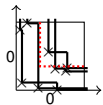
Local convergence



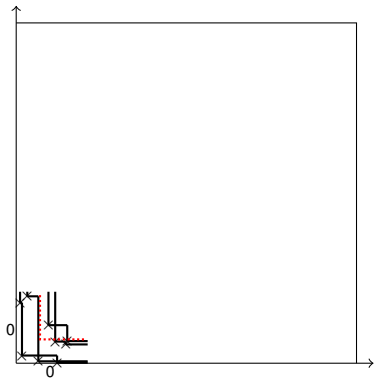
Local convergence



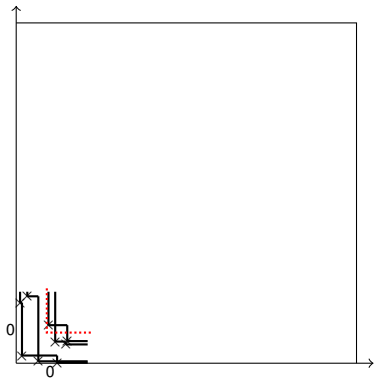
Local convergence



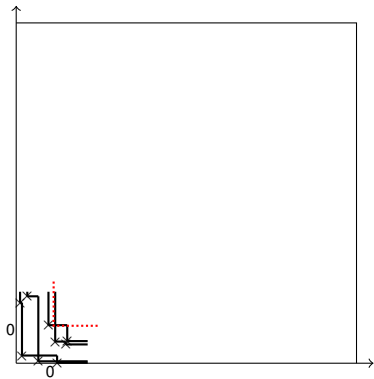
Local convergence



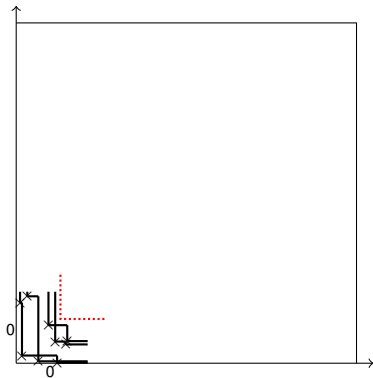
Local convergence



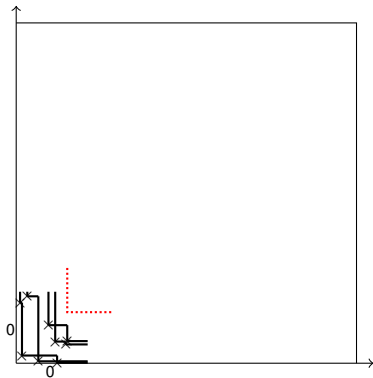
Local convergence



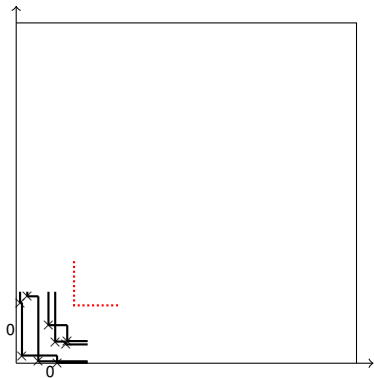
Local convergence



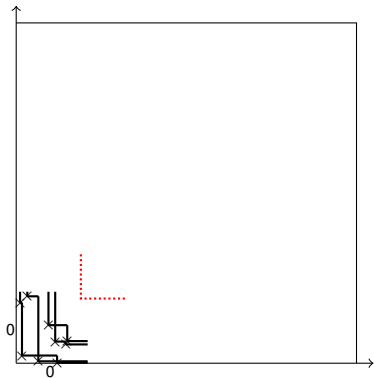
Local convergence



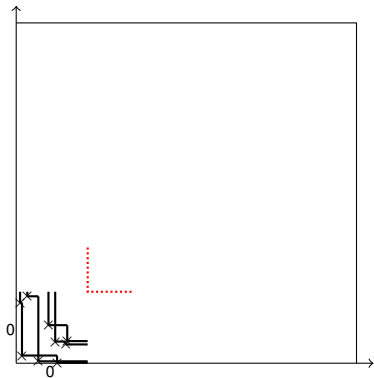
Local convergence



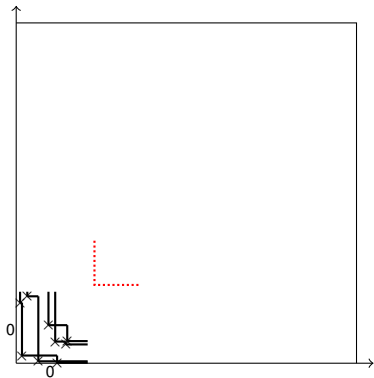
Local convergence



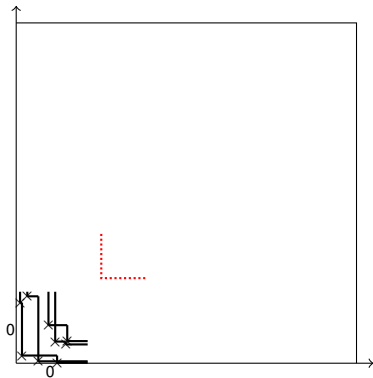
Local convergence



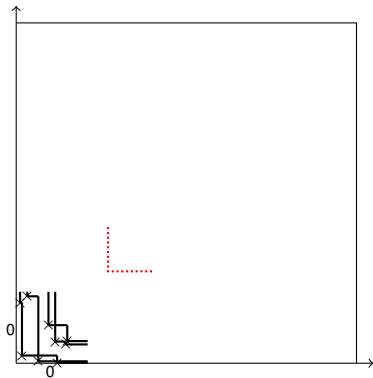
Local convergence



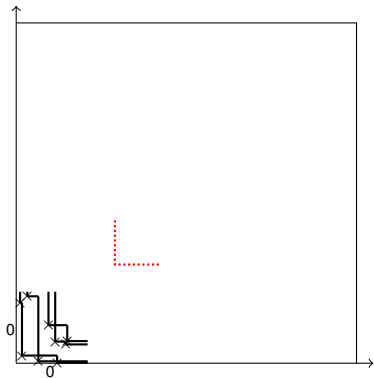
Local convergence



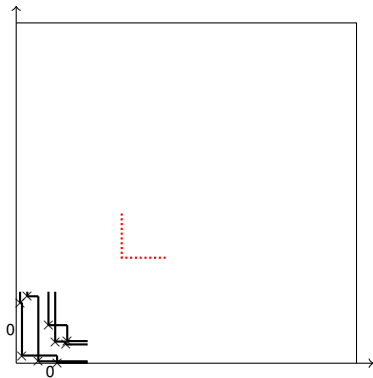
Local convergence



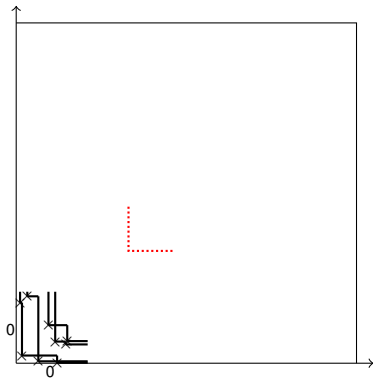
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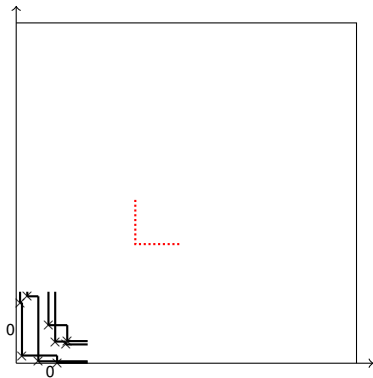
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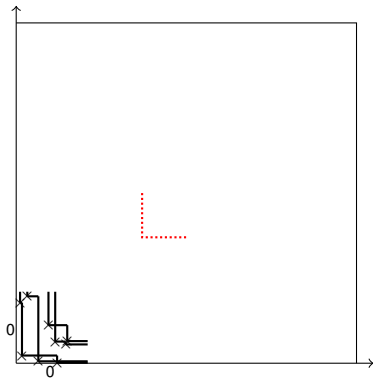
Local convergence



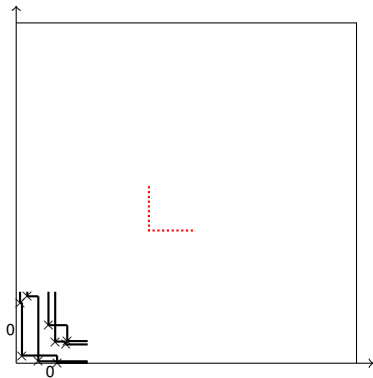
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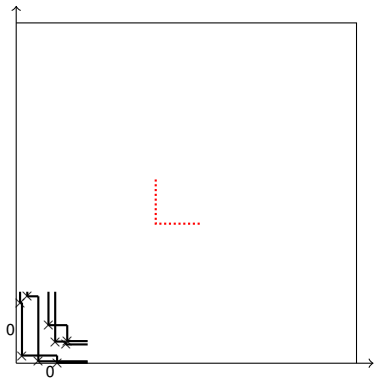
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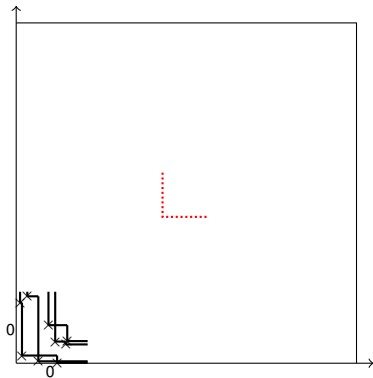
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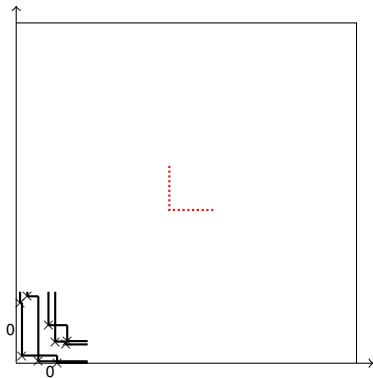
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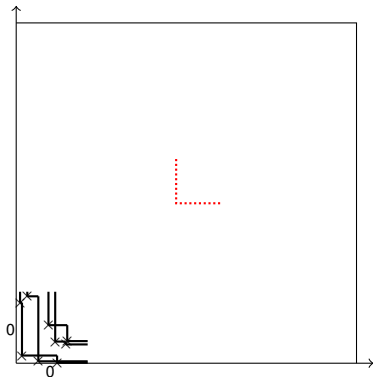
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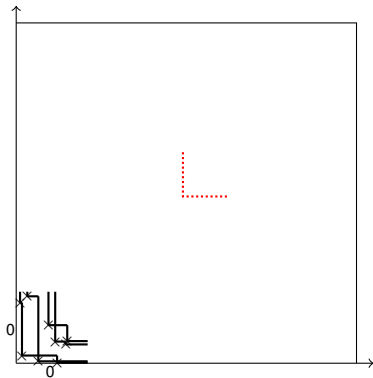
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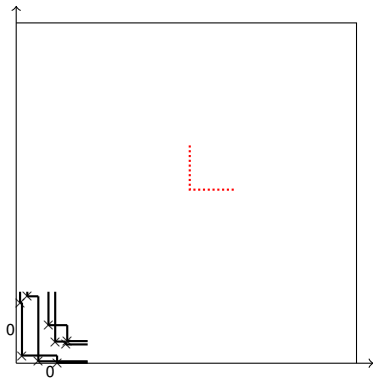
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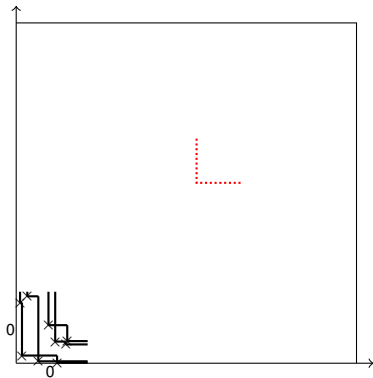
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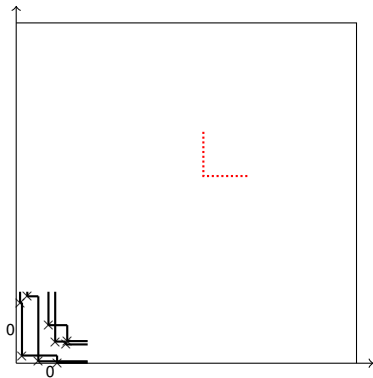
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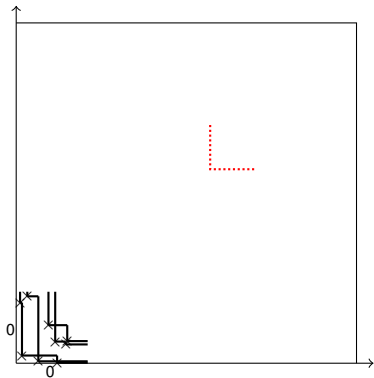
Local convergence



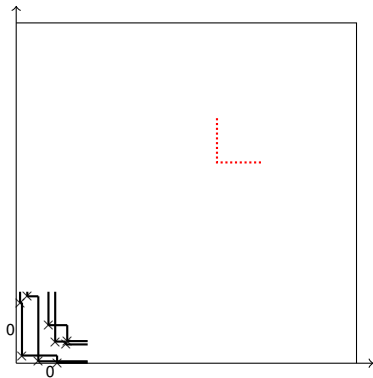
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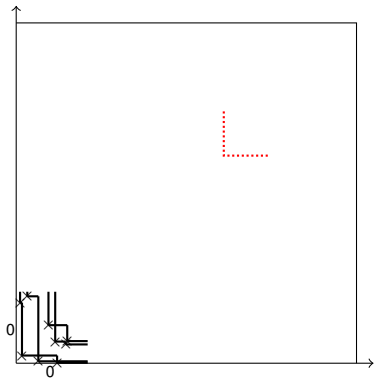
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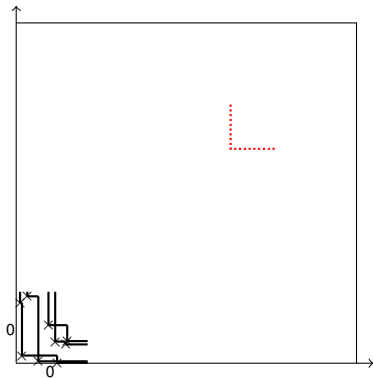
Local convergence



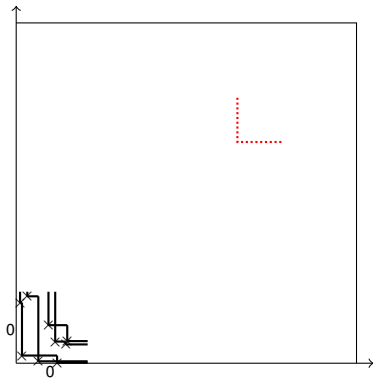
Local convergence



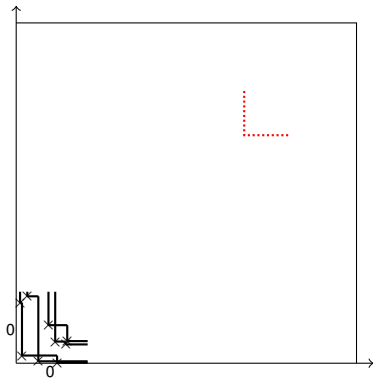
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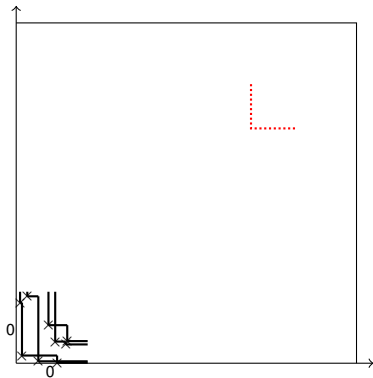
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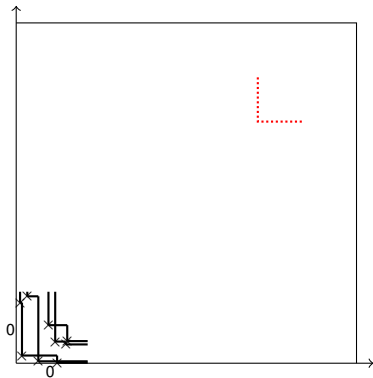
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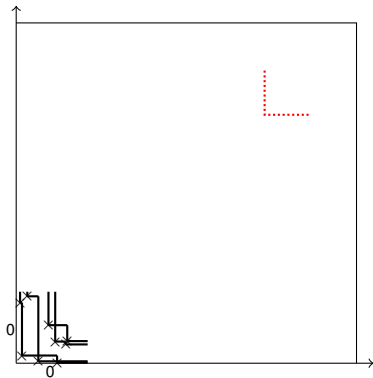
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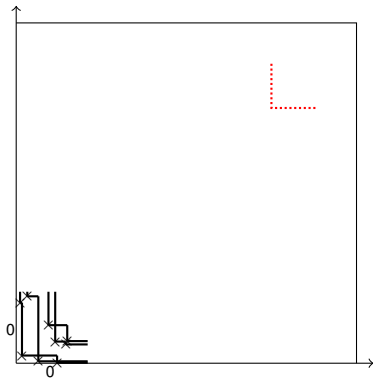
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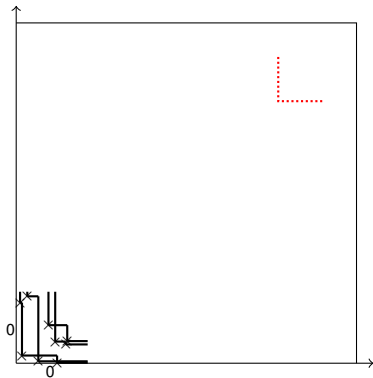
Local convergence



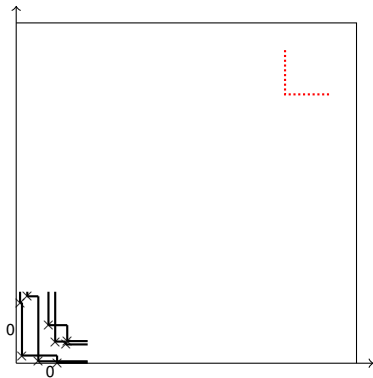
Local convergence



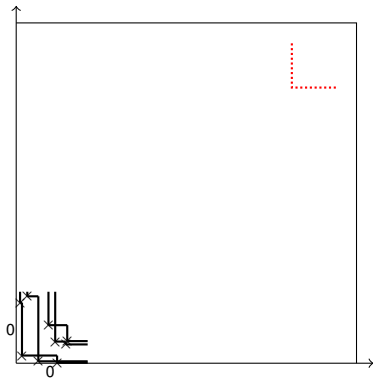
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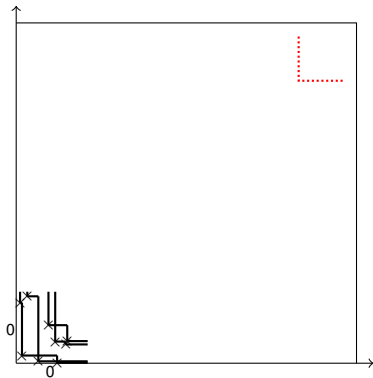
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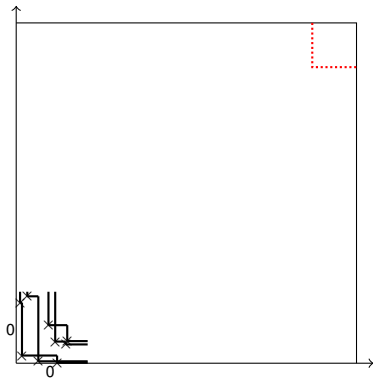
Local convergence



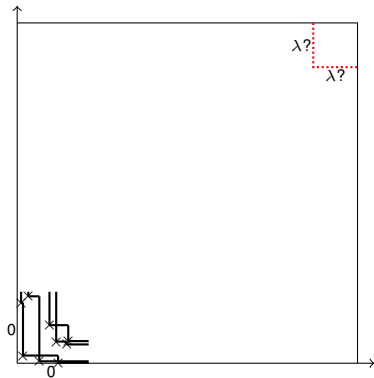
Local convergence



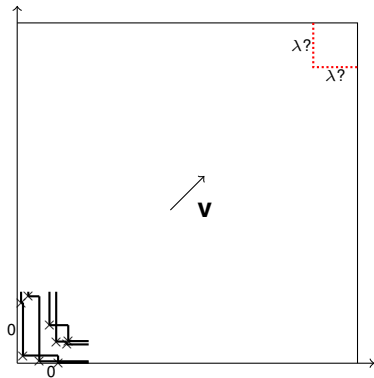
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*Yes, the boundaries of the small box converge in distribution to the two independent stationary Poisson processes with an explicit $\lambda = \lambda(\mathbf{v})$ (*direction*).*

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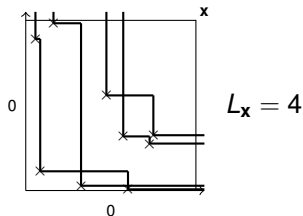
*Yes, the boundaries of the small box converge in distribution to the two independent stationary Poisson processes with an explicit $\lambda = \lambda(\mathbf{v})$ (*direction*).*

- ▶ Confirms the conjecture from [Drillick, Lin '24].
- ▶ Hence the full t -PNG inside the small box converges to stationary t -PNG.
- ▶ The actual limit intensities are $\frac{1}{\sqrt{1-t}} \sqrt{\frac{v_2}{v_1}}$ for **sources** and $\frac{1}{\sqrt{1-t}} \sqrt{\frac{v_1}{v_2}}$ for **sinks**.

Shape theorem [Drillick, Lin '24]

Define $L_x :=$ no. of lines entering the bottom or right edge.

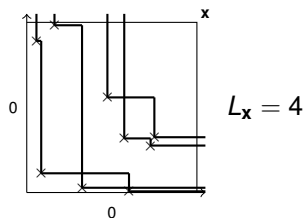
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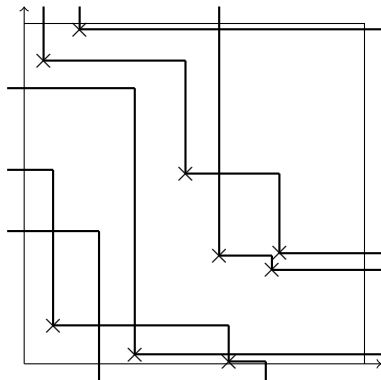


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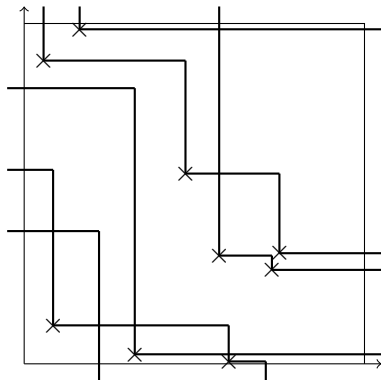
With zero boundaries,

$$\lim_{\mathbf{x} \rightarrow \infty} \frac{L_{\mathbf{x}}}{|\mathbf{x}|} = 2 \sqrt{\frac{x_1 x_2}{|\mathbf{x}|^2 \cdot (1-t)}}$$

Second class particles

 α

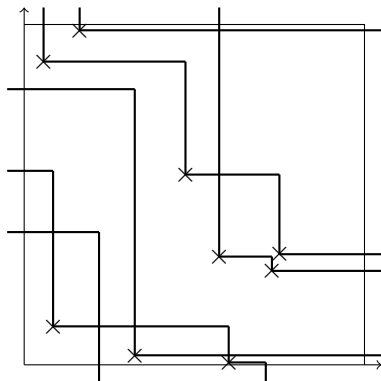
Second class particles



Definition: $\alpha \leq \omega$, if

- ▶ more verticals in ω ,
- ▶ more horizontals in α .

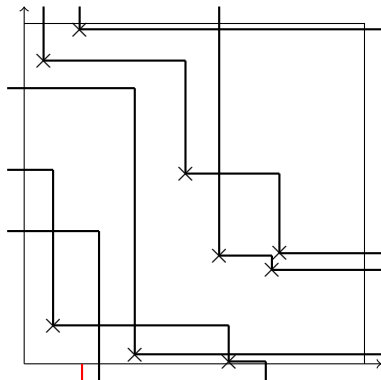
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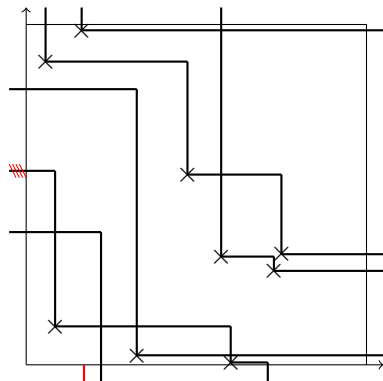
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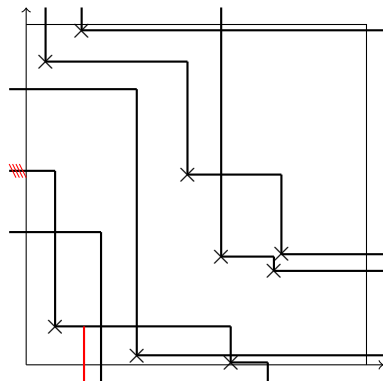
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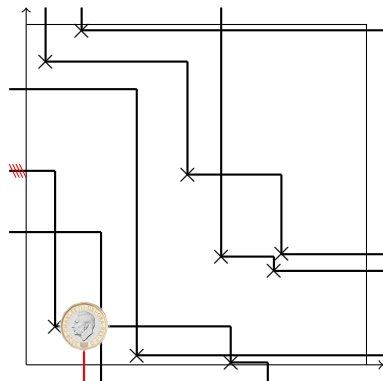
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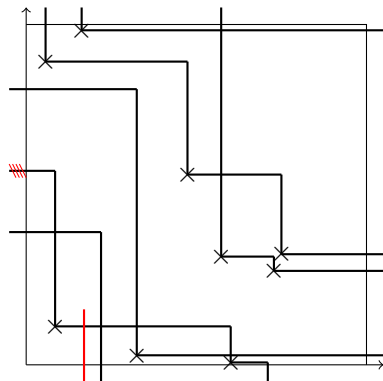
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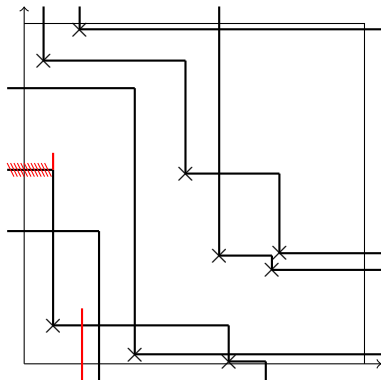
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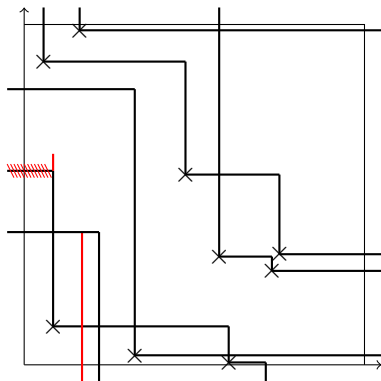
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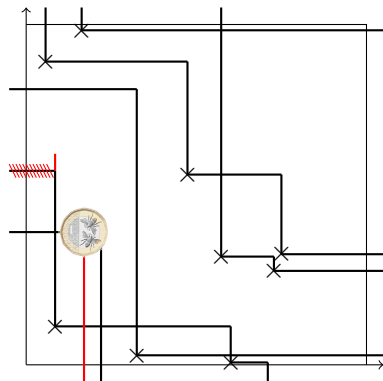
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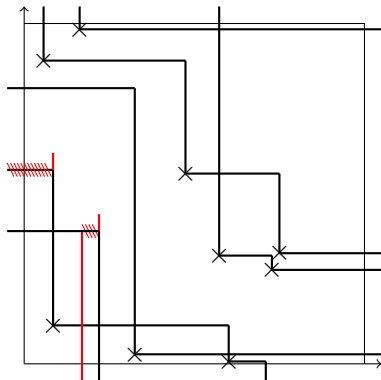
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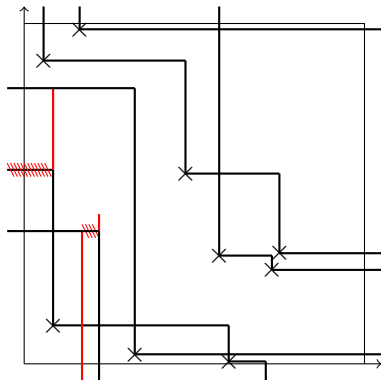
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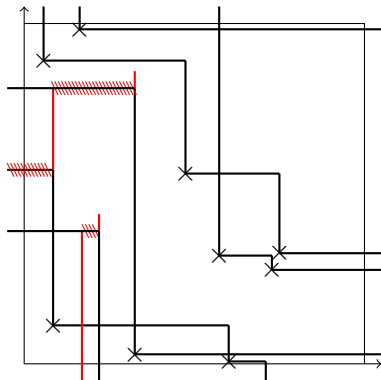
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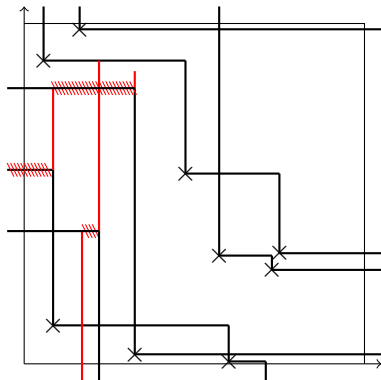
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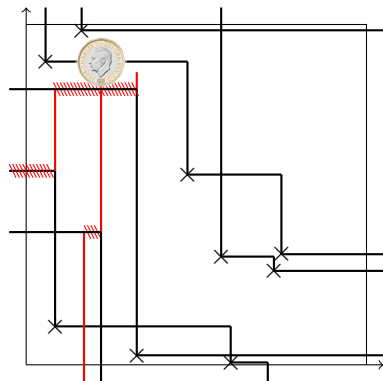
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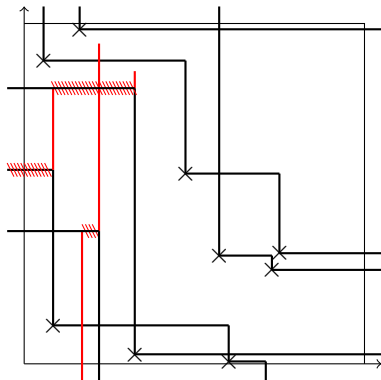
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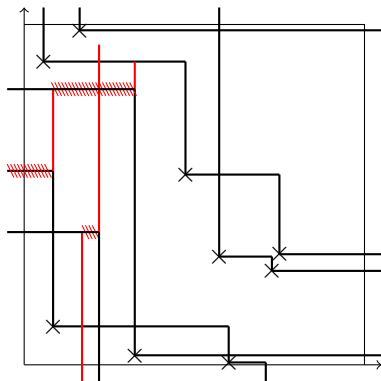
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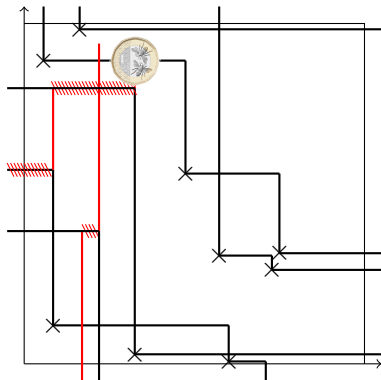
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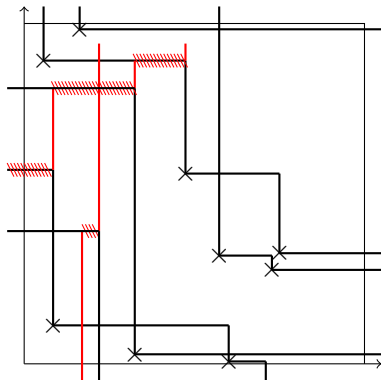
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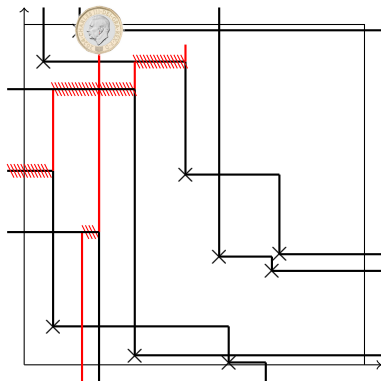
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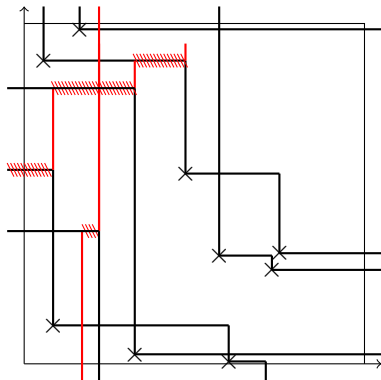
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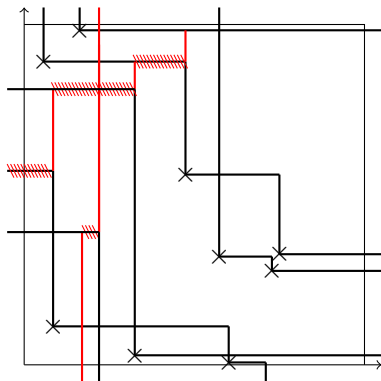
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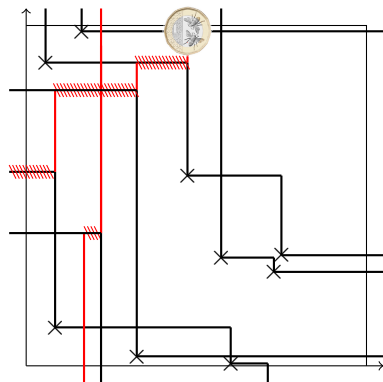
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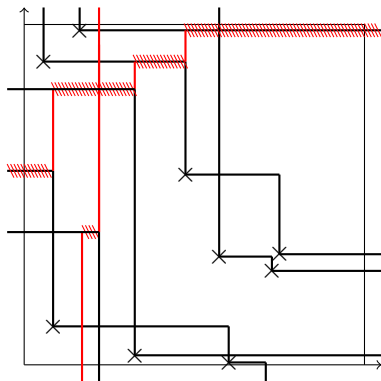
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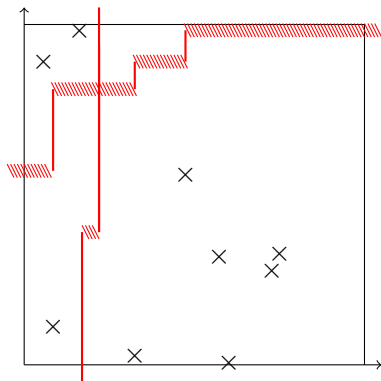
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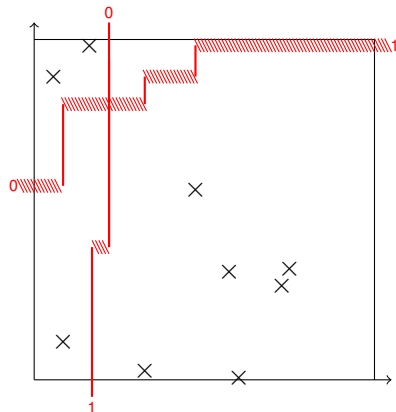


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$\frac{\alpha}{\omega}$ second class particles:
upright paths

Second class particles

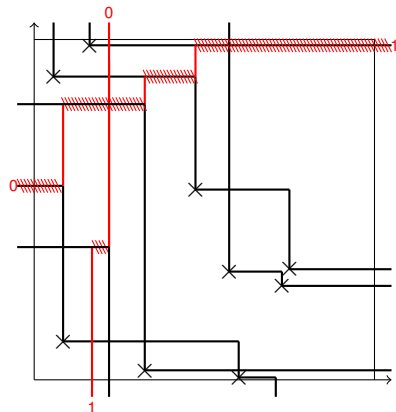


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$\frac{\alpha}{\omega}$ second class particles:
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Second class particles

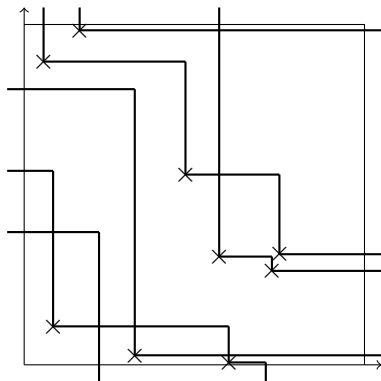


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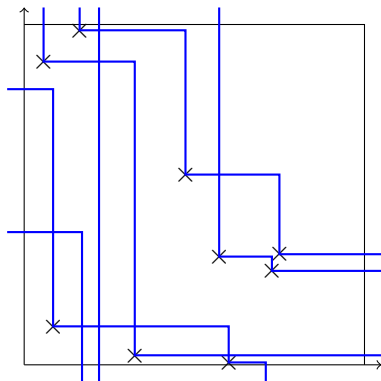


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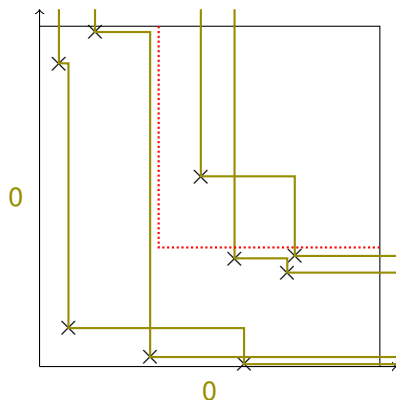


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The main actors

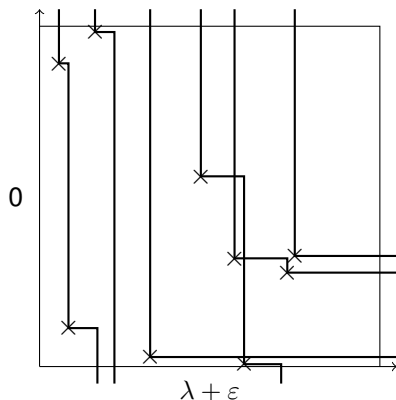


WTS: no. of lines crossing

→ $\text{stati}(\lambda(\mathbf{v}))$ Poisson

$\theta^{0,0}$

The main actors



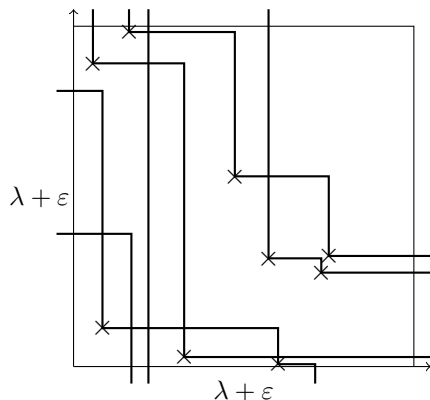
WTS: no. of lines crossing

$\rightarrow \text{stati}(\lambda(\mathbf{v}))$ Poisson

$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0}$$

$$\omega^{\lambda+\varepsilon,0}$$

The main actors

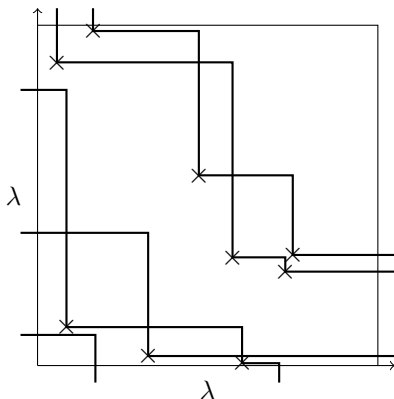


WTS: no. of lines crossing
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$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0}$$

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The main actors

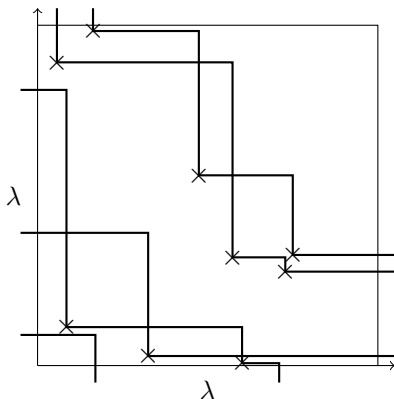


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The main actors



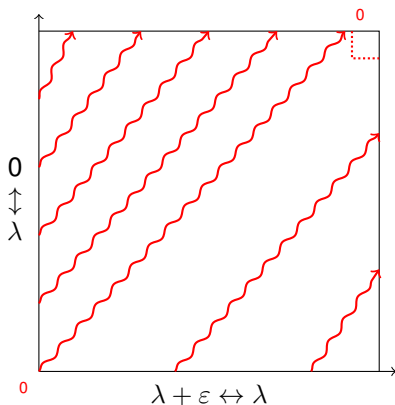
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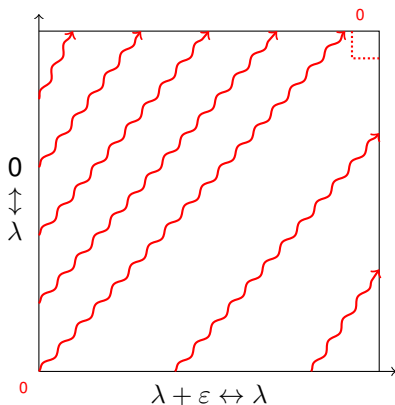
1. $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$ 2nd class particles and the small box



Second class particle 0 goes above the small box.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \underline{\alpha^{\lambda,\lambda}}$$

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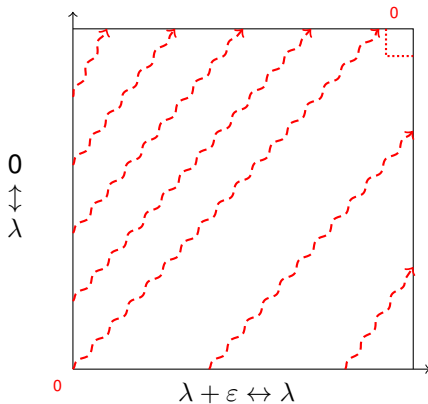


Second class particle 0 goes above the small box.

That's because of the shape theorem for each $\omega^{\lambda+\varepsilon,0}$ and $\alpha^{\lambda,\lambda}$, and the ε leeway.

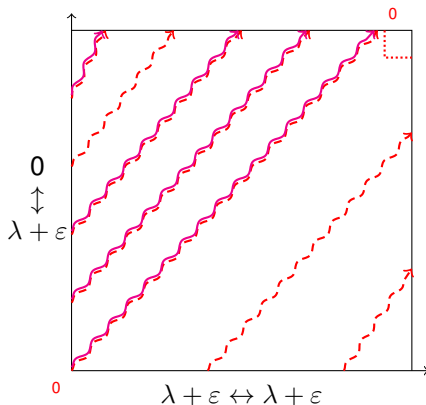
$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \underline{\alpha^{\lambda,\lambda}}$$

2. $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ on $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$



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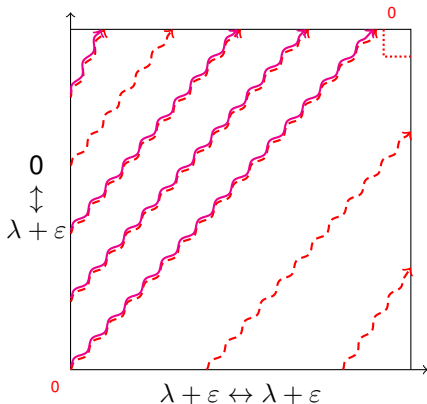


$$\left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \right\} \subseteq \left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}} \right\}$$

and only start from the left boundary.

$$\theta^{0,0} \leq \omega^{\lambda+\varepsilon,0} \geq \eta^{\lambda+\varepsilon,\lambda+\varepsilon} \geq \alpha^{\lambda,\lambda}$$

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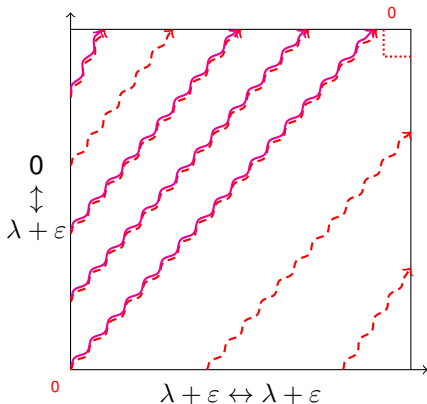
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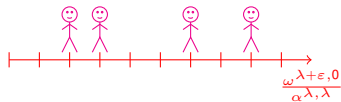
\rightsquigarrow there is a rightmost $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ second class particle, jumping on $\left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}} \right\}$.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \underline{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

3. $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ on $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$: we have drift!



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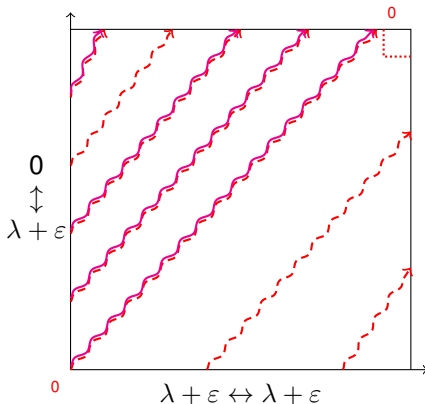


↖: prob. 1

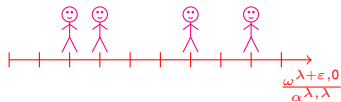
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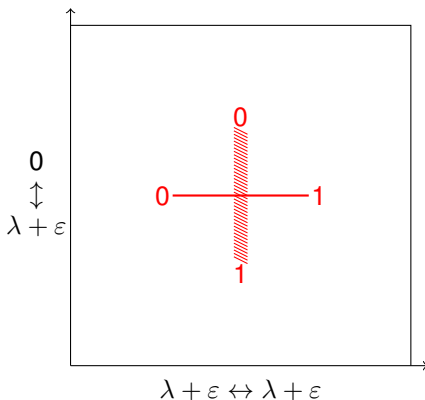
↪: prob. 1

↪: prob. t

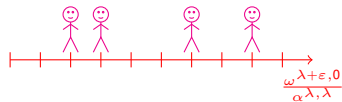
↪ rightmost $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ second class still left of the small box whp.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \underline{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

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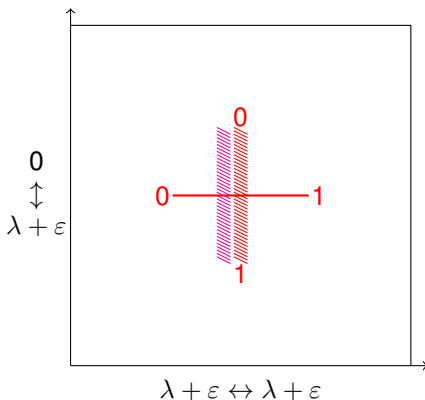
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↷: prob. t

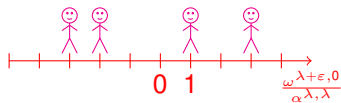
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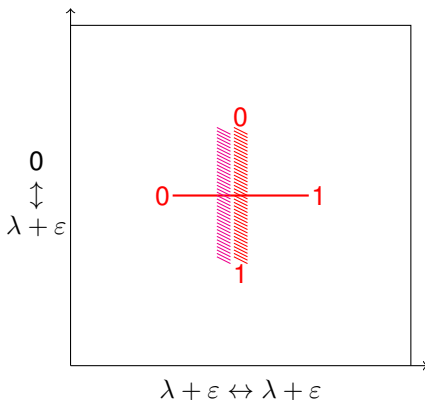
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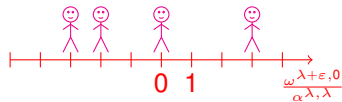
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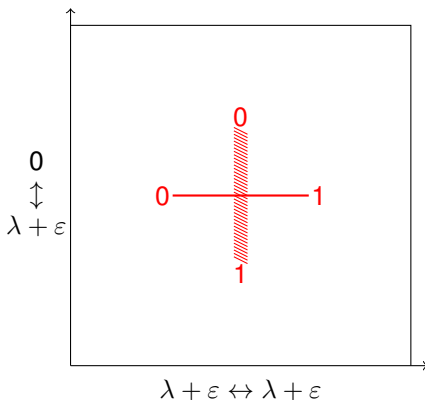
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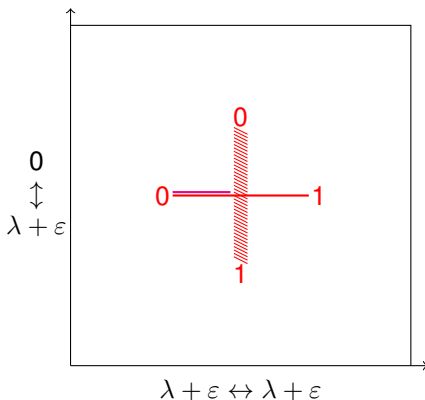
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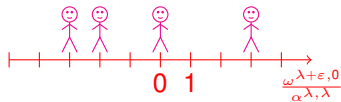
↷ rightmost $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ second class still left of the small box whp.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \underline{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

3. $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ on $\frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}}$: we have drift!



$$\left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \right\} \subseteq \left\{ \frac{\omega^{\lambda+\varepsilon,0}}{\alpha^{\lambda,\lambda}} \right\}$$



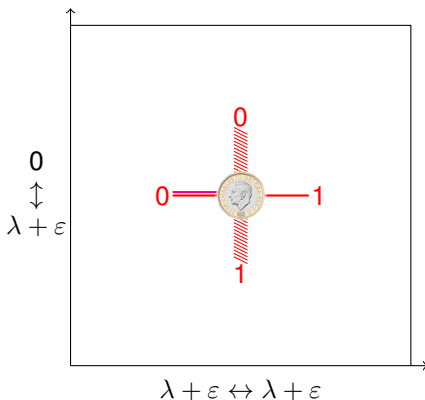
↶: prob. 1

↷: prob. t

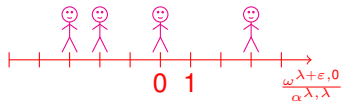
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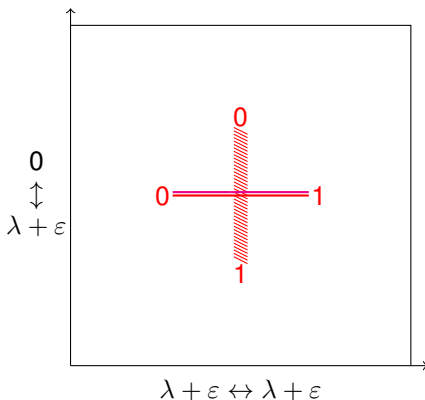
↶: prob. 1

↷: prob. t

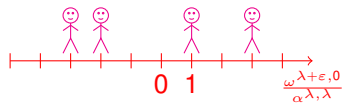
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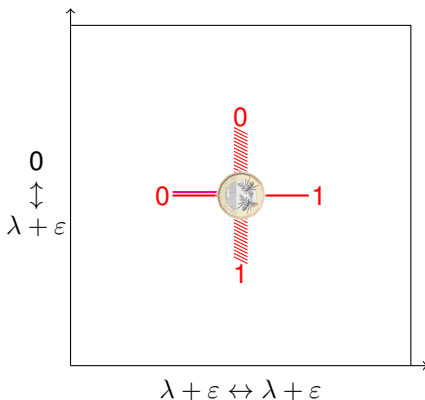
↶: prob. 1

↷: prob. t

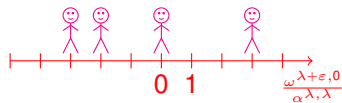
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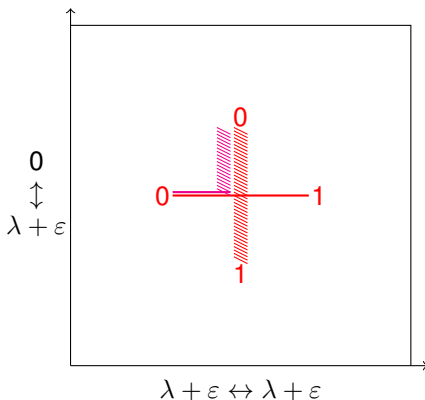
↶: prob. 1

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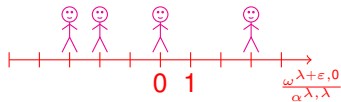
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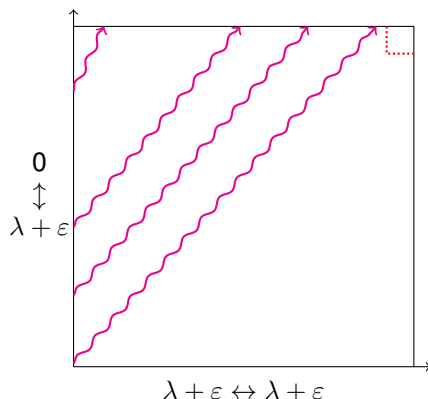
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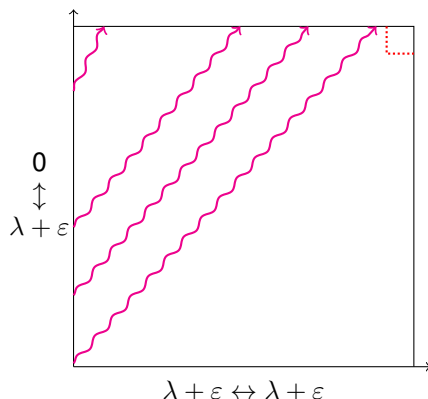
4. Conclusion



Rightmost $\frac{\omega^{\lambda+\epsilon,0}}{\eta^{\lambda+\epsilon,\lambda+\epsilon}}$ second class still left of the small box whp.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\epsilon,0}} \geq \underline{\eta^{\lambda+\epsilon,\lambda+\epsilon}} \geq \alpha^{\lambda,\lambda}$$

4. Conclusion

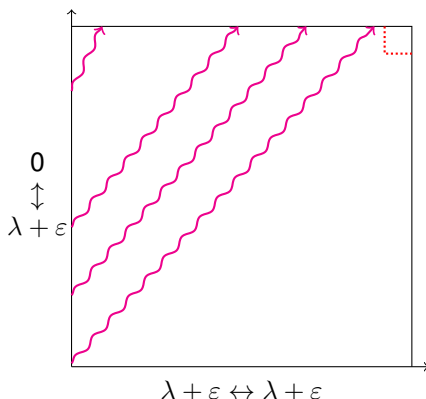


Rightmost $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ second class still left of the small box whp.

$\rightsquigarrow \omega^{\lambda+\varepsilon,0} = \eta^{\lambda+\varepsilon,\lambda+\varepsilon}$ in the small box whp.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \underline{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

4. Conclusion



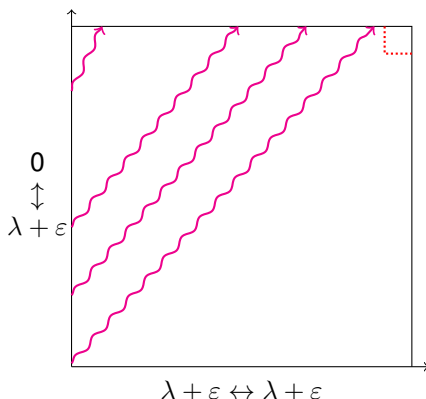
Rightmost $\frac{\omega^{\lambda+\epsilon,0}}{\eta^{\lambda+\epsilon,\lambda+\epsilon}}$ second class still left of the small box whp.

$\rightsquigarrow \omega^{\lambda+\epsilon,0} = \eta^{\lambda+\epsilon,\lambda+\epsilon}$ in the small box whp.

\rightsquigarrow but $\eta^{\lambda+\epsilon,\lambda+\epsilon}$ is stationary so has $\text{Poi}(\lambda + \epsilon)$ number of lines entering the box whp.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\epsilon,0}} \geq \underline{\eta^{\lambda+\epsilon,\lambda+\epsilon}} \geq \alpha^{\lambda,\lambda}$$

4. Conclusion



Rightmost $\frac{\omega^{\lambda+\varepsilon,0}}{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}}$ second class still left of the small box whp.

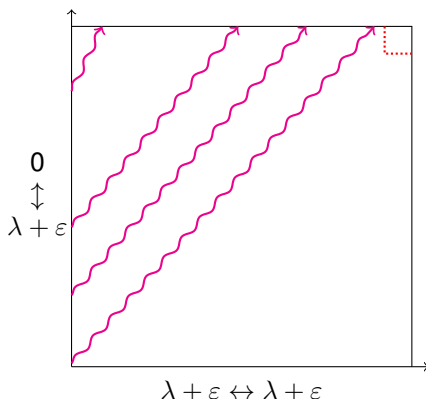
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\rightsquigarrow whp. $\theta^{0,0} \leq \text{Poi}(\lambda + \varepsilon)$
 $\forall \varepsilon > 0$.

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\varepsilon,0}} \geq \underline{\eta^{\lambda+\varepsilon,\lambda+\varepsilon}} \geq \alpha^{\lambda,\lambda}$$

4. Conclusion



Thank you.

Rightmost $\frac{\omega^{\lambda+\epsilon,0}}{\eta^{\lambda+\epsilon,\lambda+\epsilon}}$ second class still left of the small box whp.

$\rightsquigarrow \omega^{\lambda+\epsilon,0} = \eta^{\lambda+\epsilon,\lambda+\epsilon}$ in the small box whp.

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 $\forall \epsilon > 0.$

$$\theta^{0,0} \leq \underline{\omega^{\lambda+\epsilon,0}} \geq \underline{\eta^{\lambda+\epsilon,\lambda+\epsilon}} \geq \alpha^{\lambda,\lambda}$$