

# Second class particles can perform simple random walks (in some cases)

Joint with

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Márton Balázs

University of Bristol

Warwick Probability Seminar  
October 23, 2013.

## The models

- Asymmetric simple exclusion process

- Zero range process

- Generalized ZRP

- Bricklayers process

- Stationary distributions

## Hydrodynamics

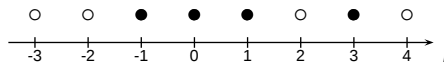
### The second class particle

### Earlier results

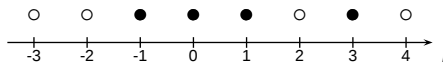
### The question

### Branching coalescing random walk

# Asymmetric simple exclusion



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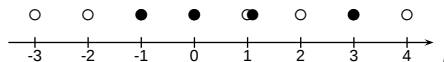
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# Asymmetric simple exclusion



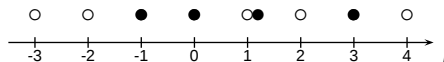
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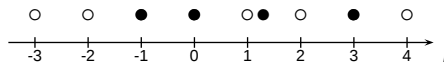
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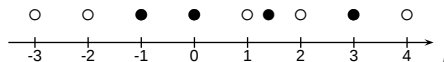
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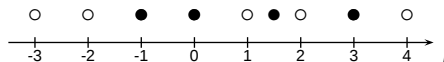
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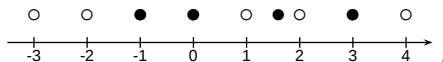
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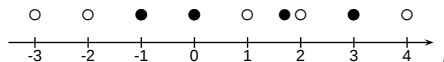
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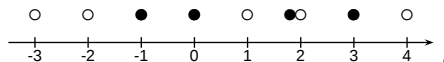
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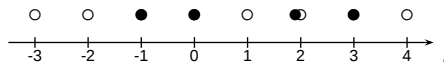
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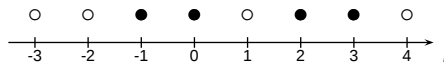
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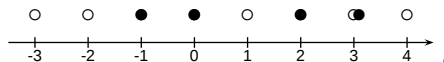
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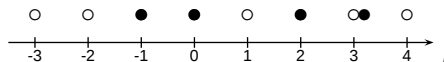
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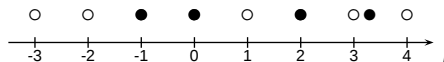
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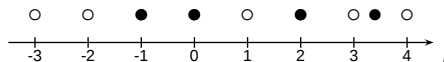
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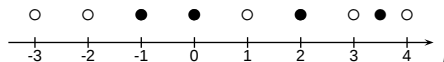
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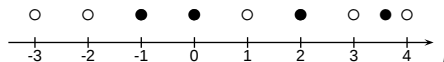
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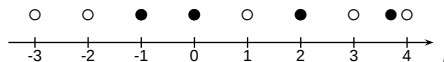
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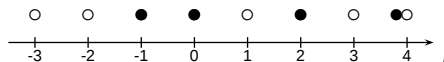
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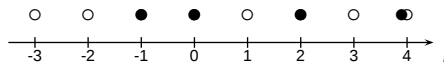
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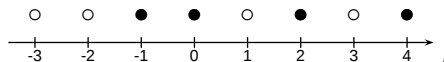
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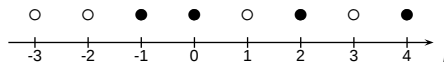
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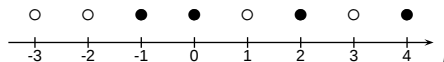
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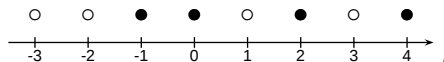
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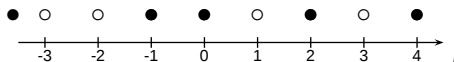
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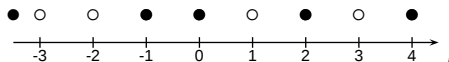
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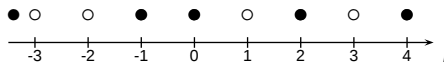
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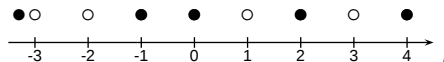
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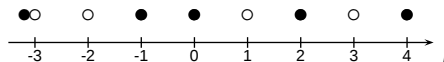
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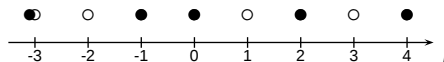
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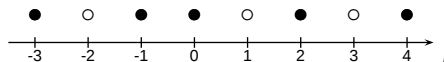
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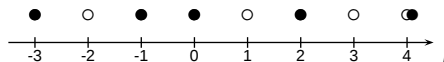
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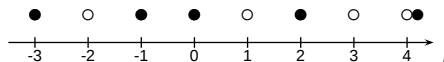
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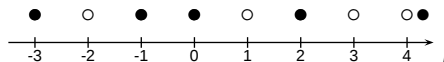
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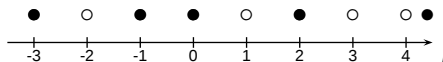
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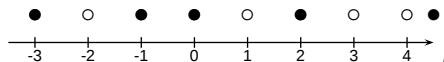
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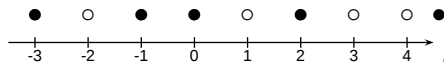
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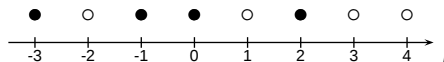
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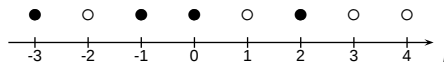
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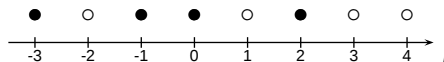
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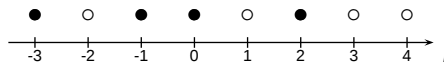
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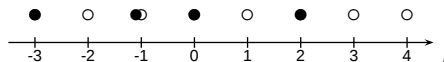
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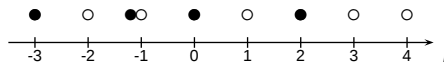
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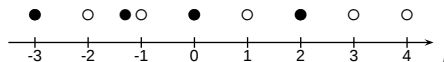
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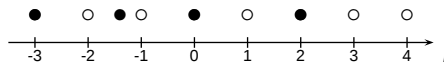
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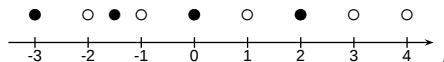
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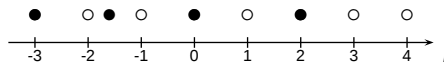
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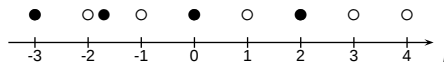
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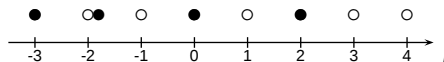
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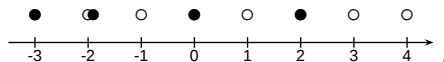
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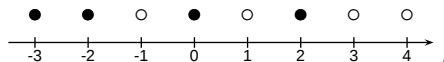
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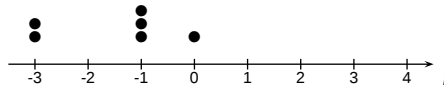
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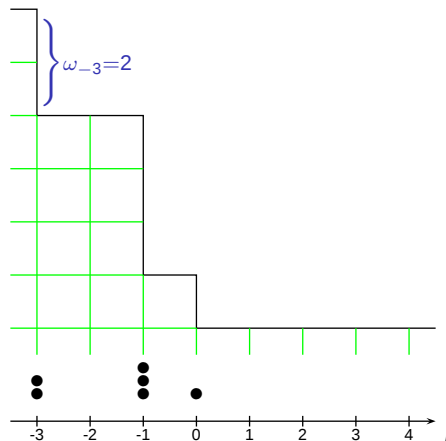
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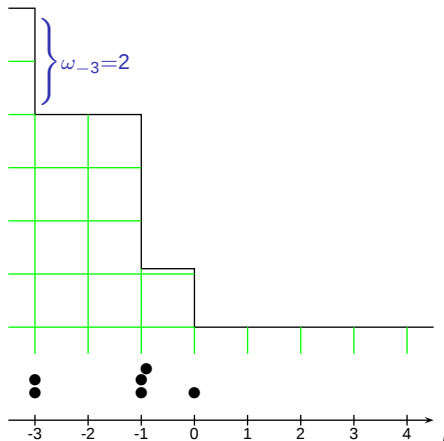
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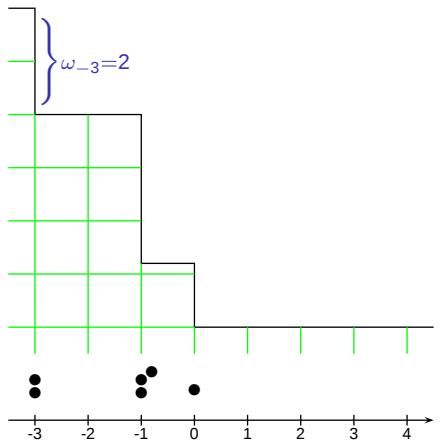
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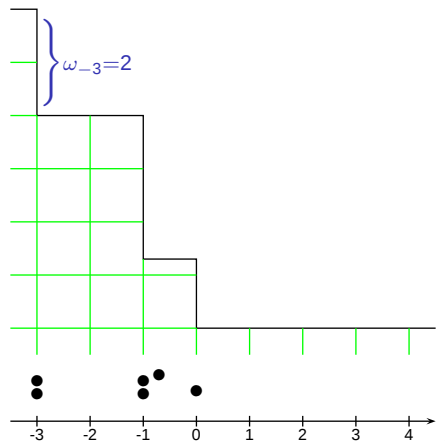
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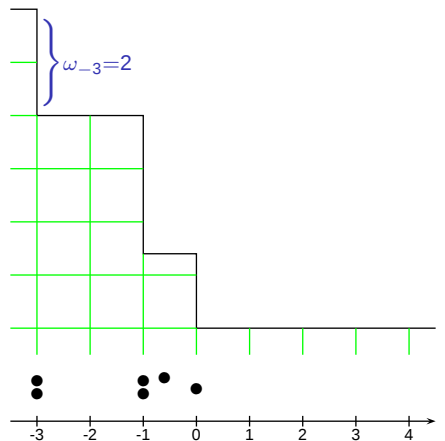
# The totally asymmetric zero range process



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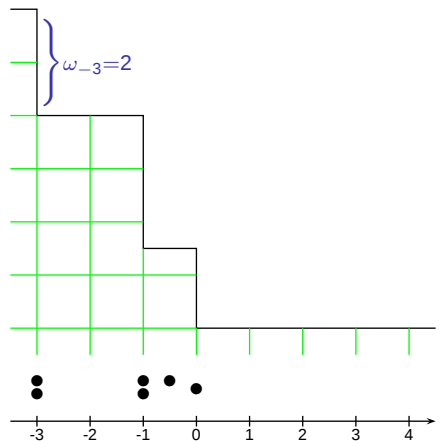
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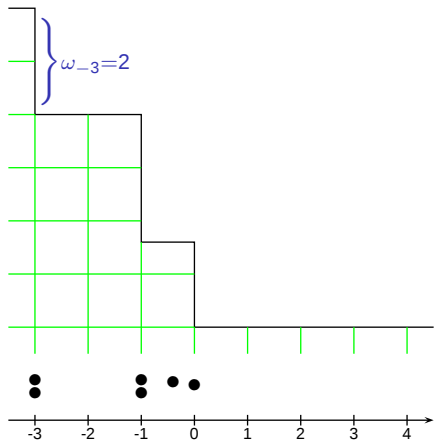
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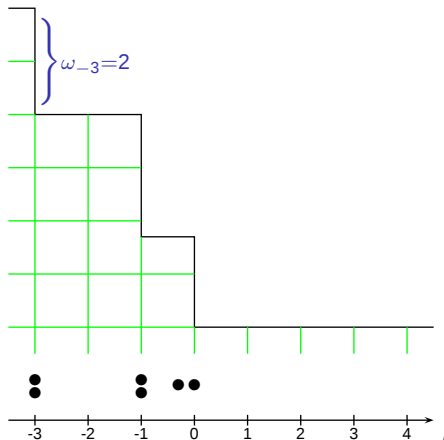
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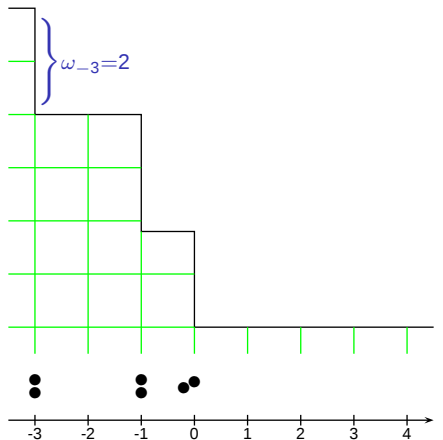
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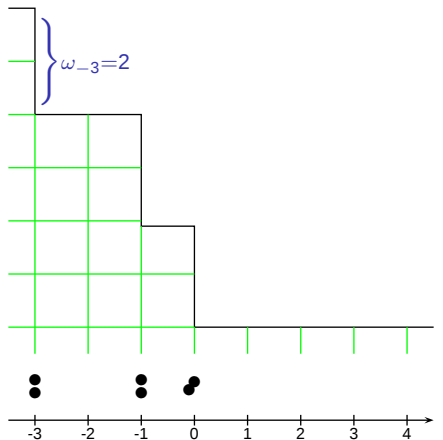
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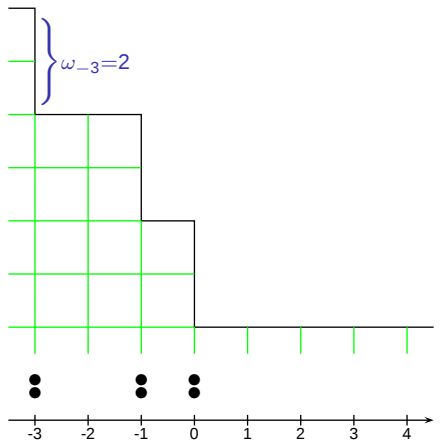
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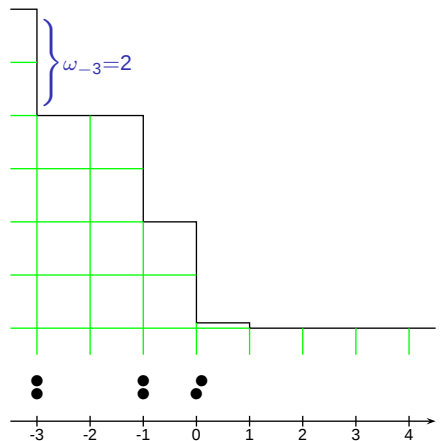
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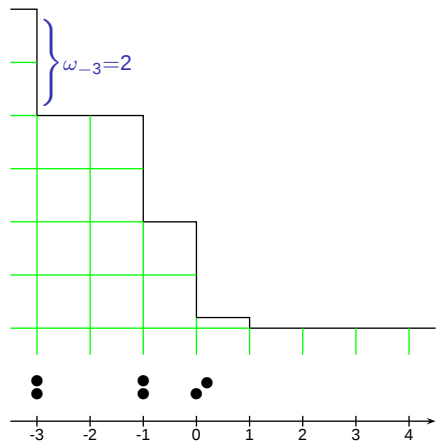
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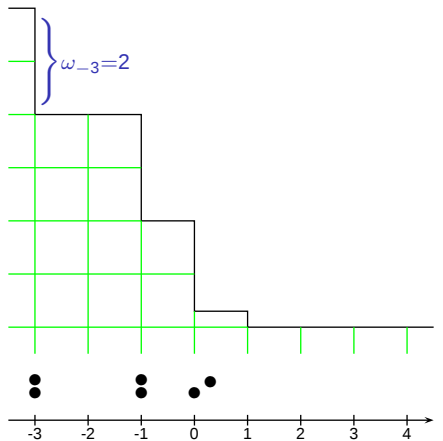
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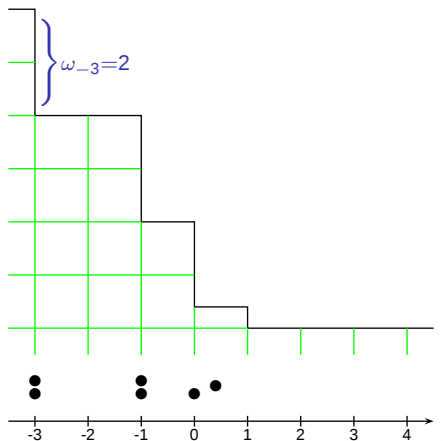
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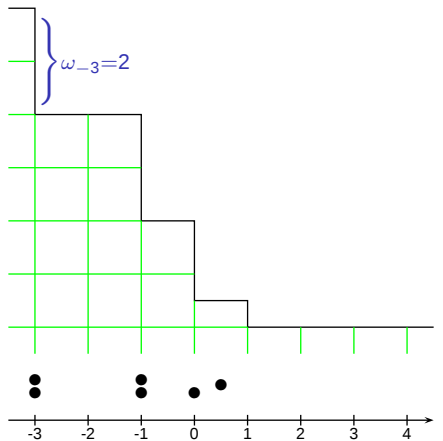
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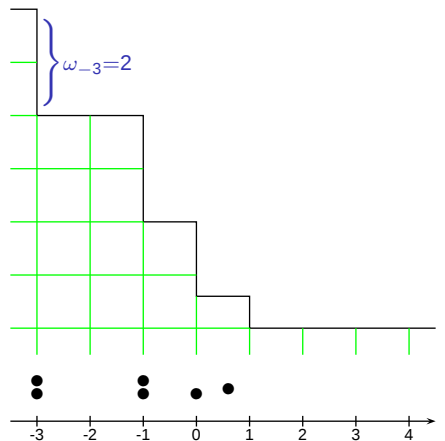
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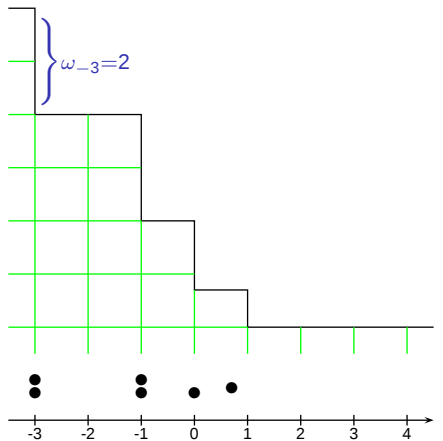
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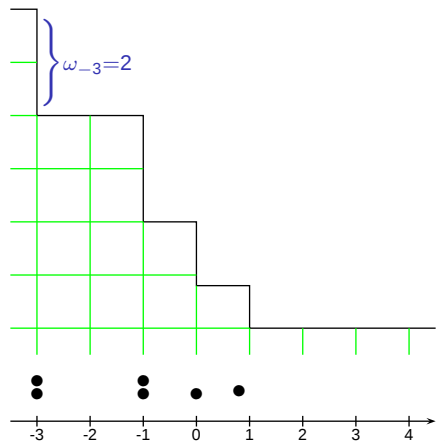
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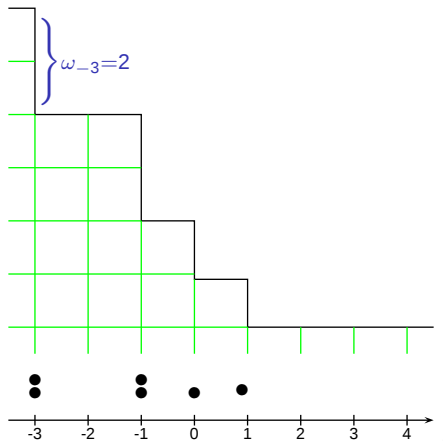
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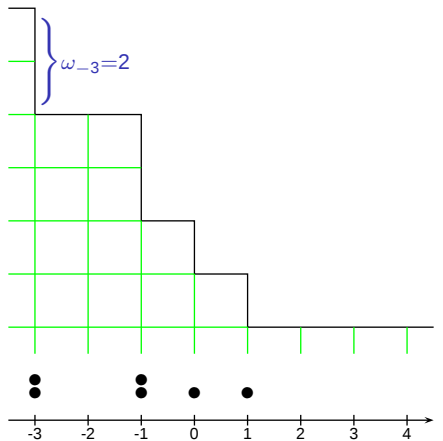
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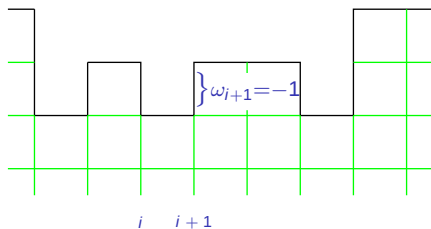


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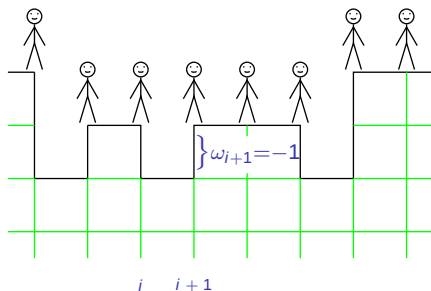
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$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

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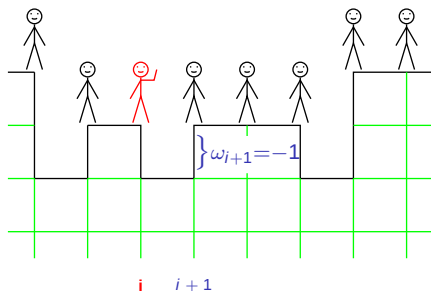
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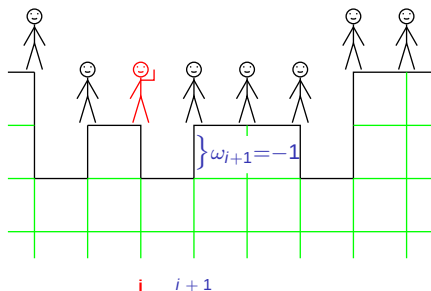
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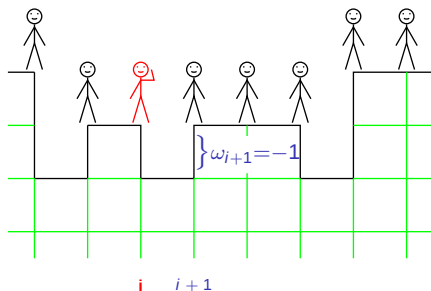
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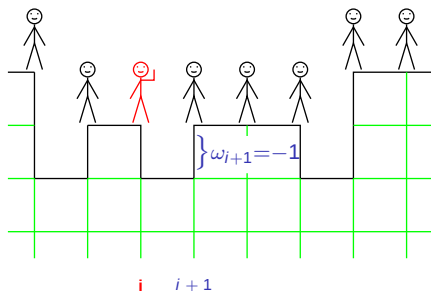
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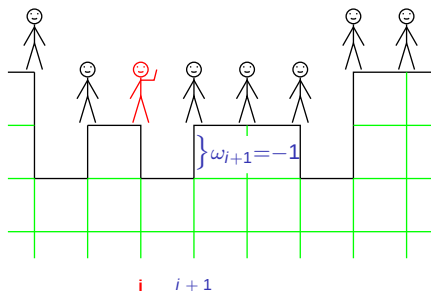
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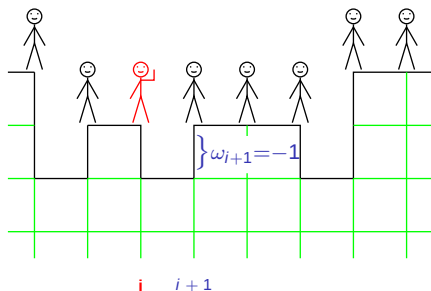
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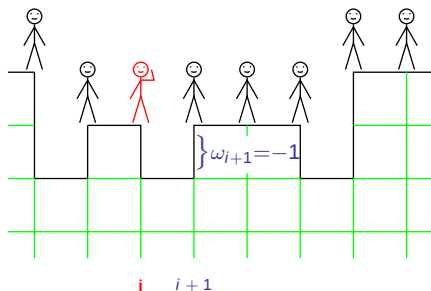
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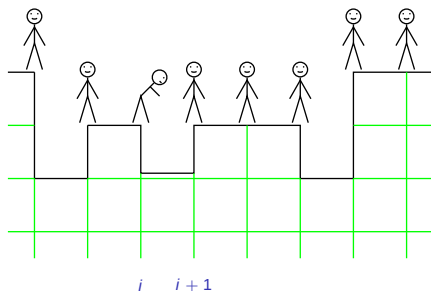
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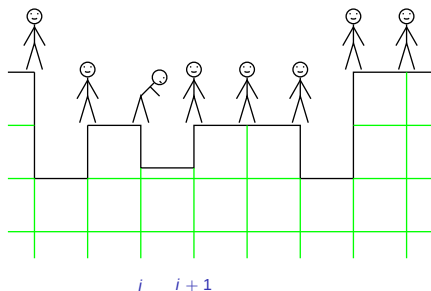
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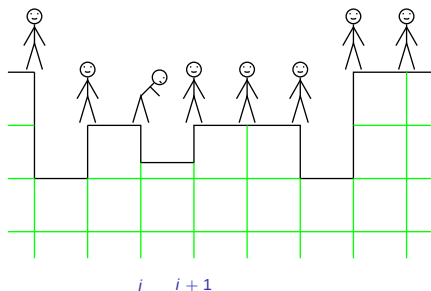
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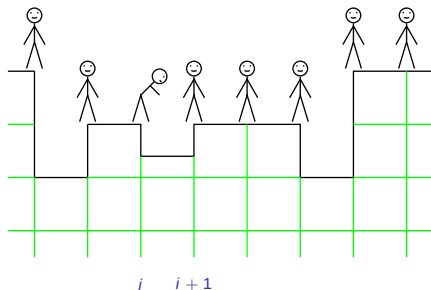
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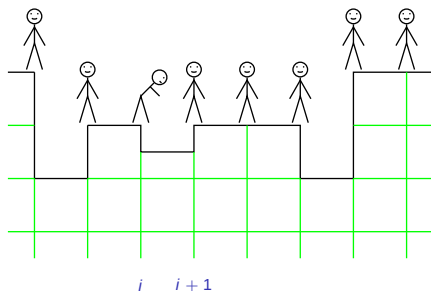
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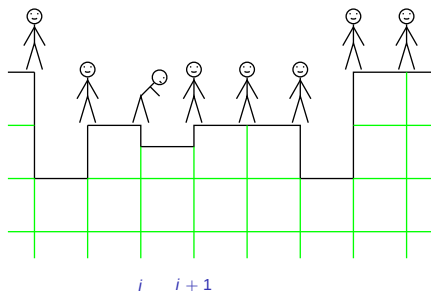
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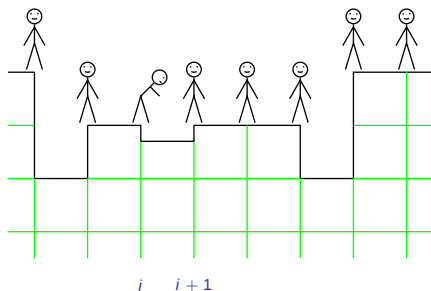
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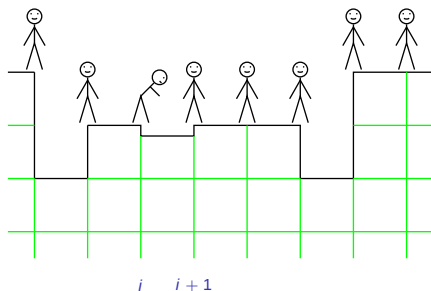
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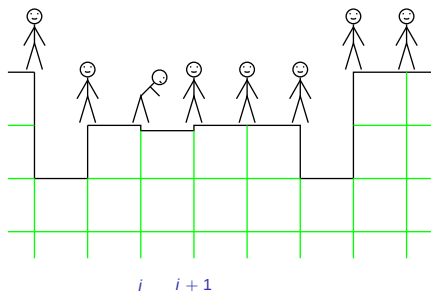
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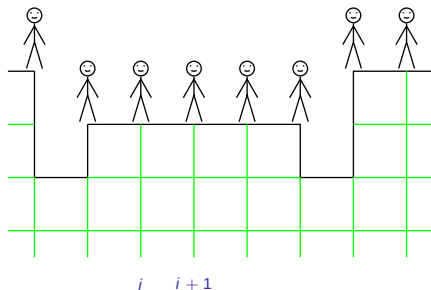
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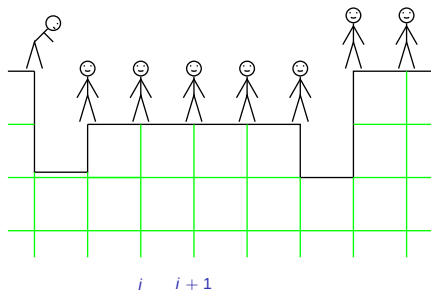
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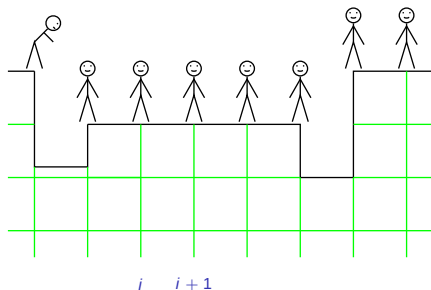
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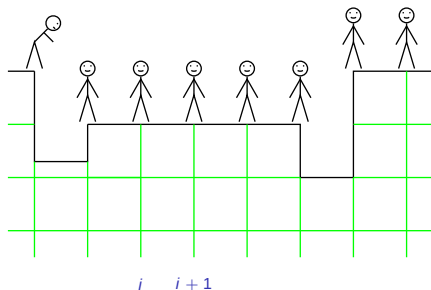
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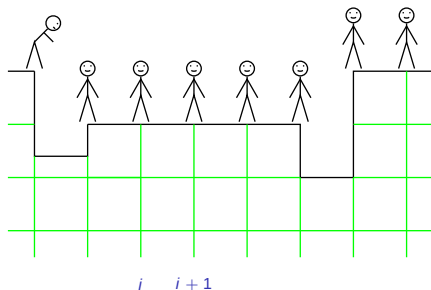
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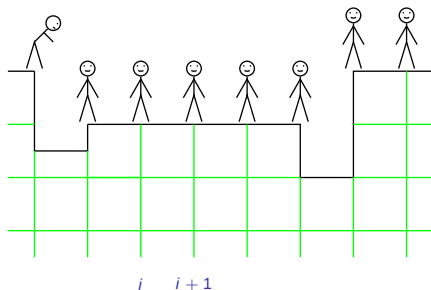
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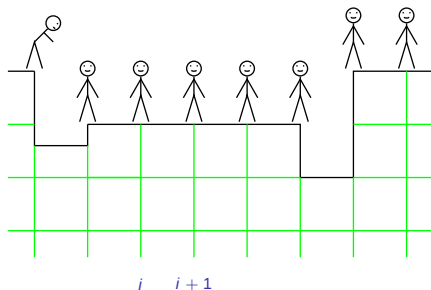
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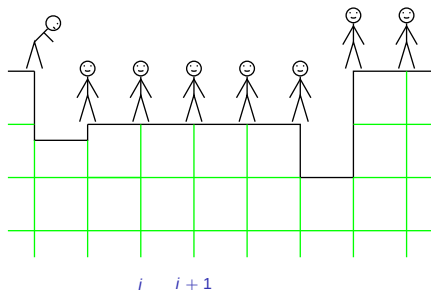
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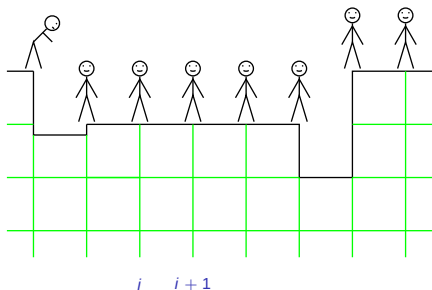
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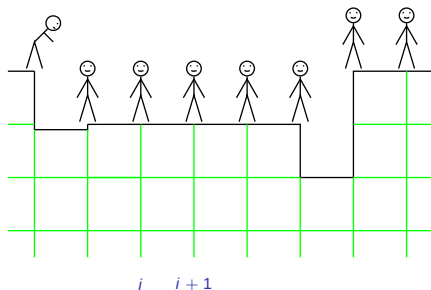
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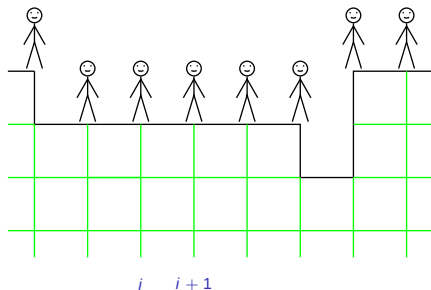
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$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \rightarrow \begin{pmatrix} \omega_i - 1 \\ \omega_{i+1} + 1 \end{pmatrix}$$

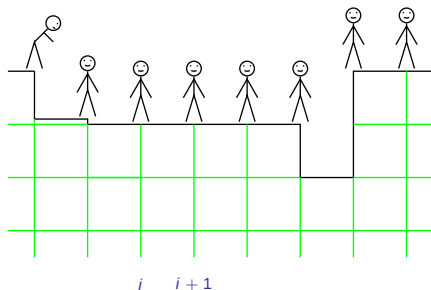
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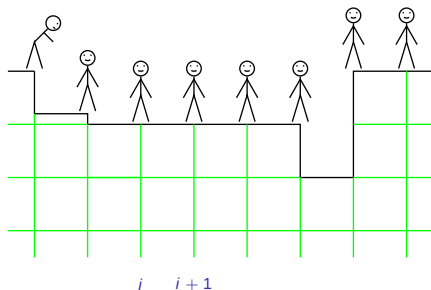
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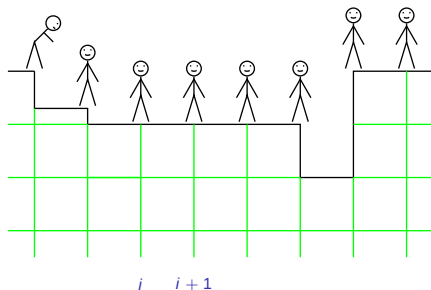
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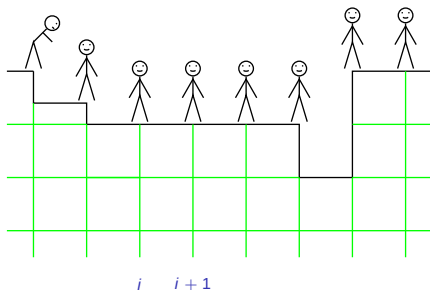
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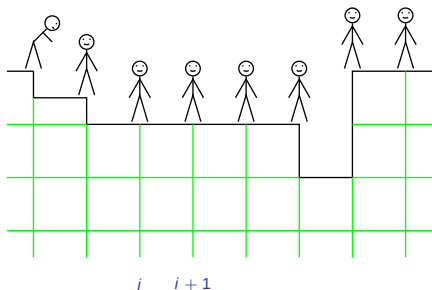
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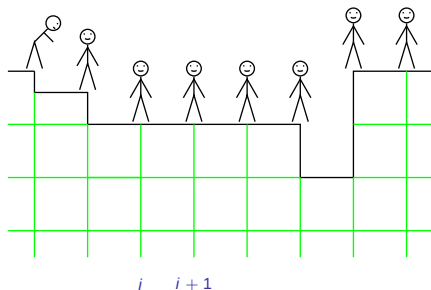
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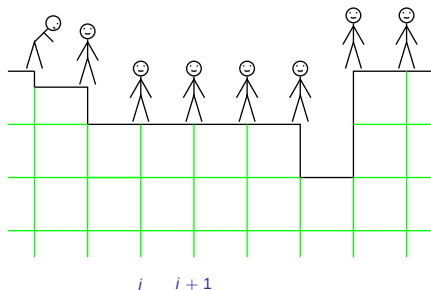
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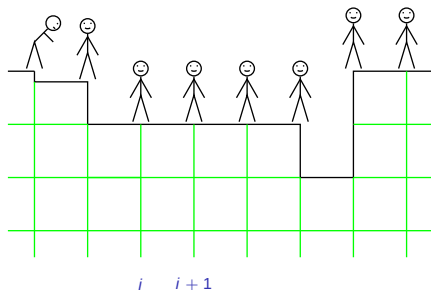
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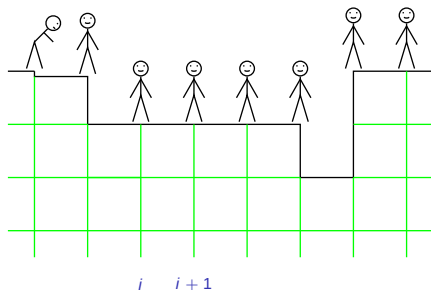
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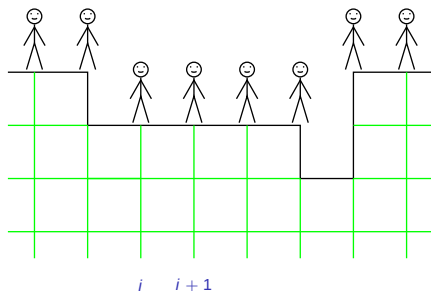
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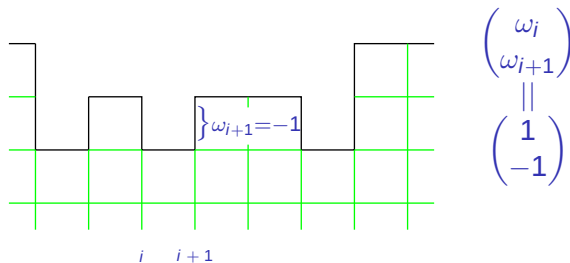
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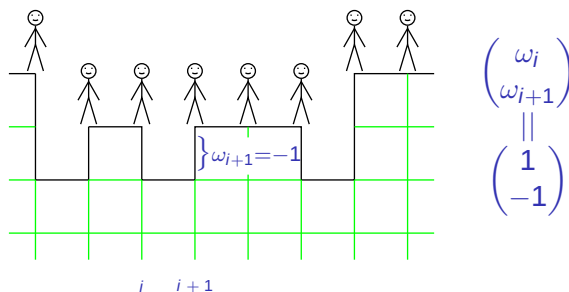
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# The totally asymmetric bricklayers process

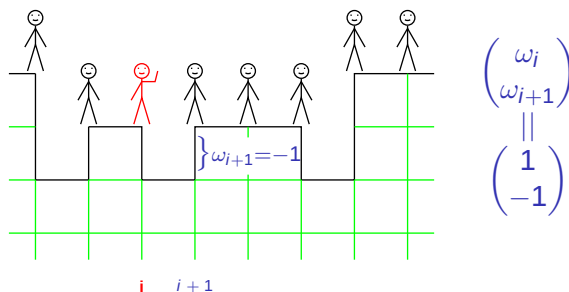


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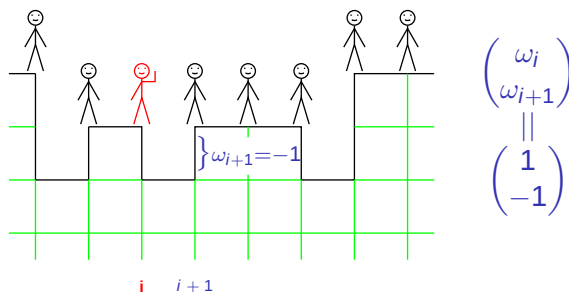
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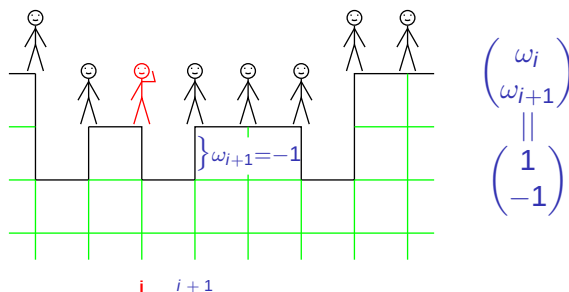
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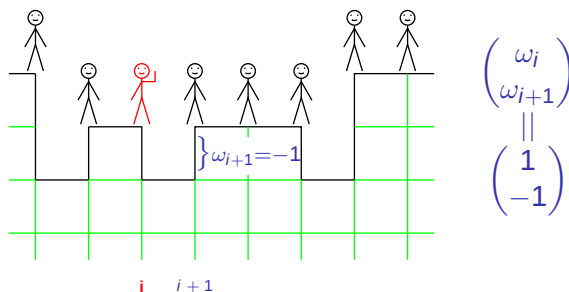
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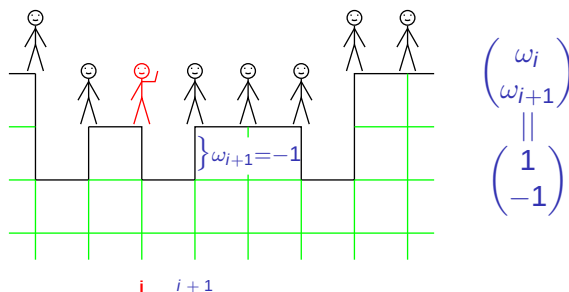
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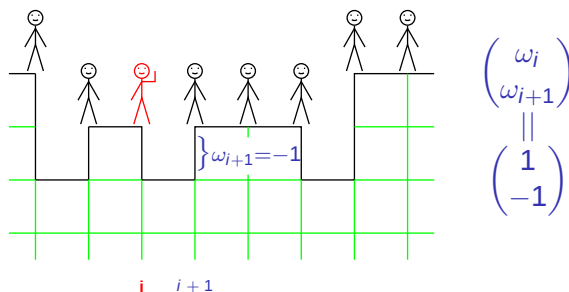
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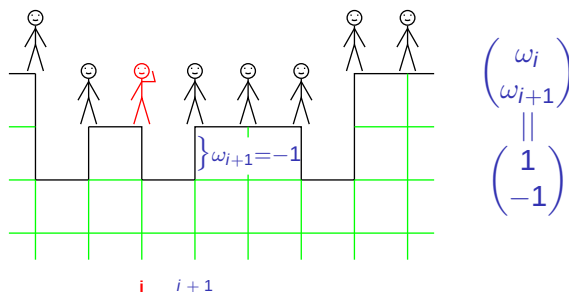
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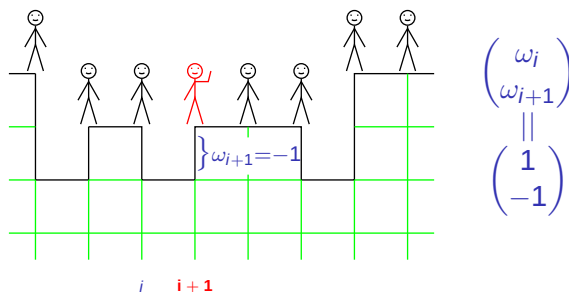
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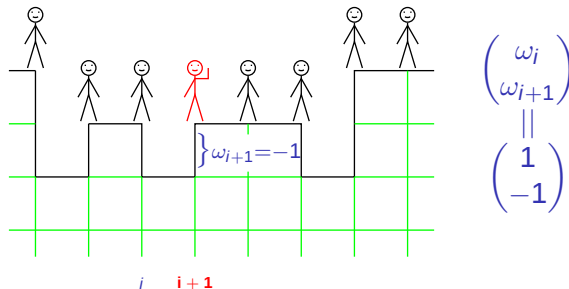
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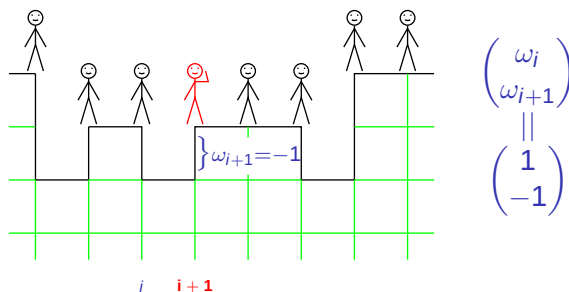
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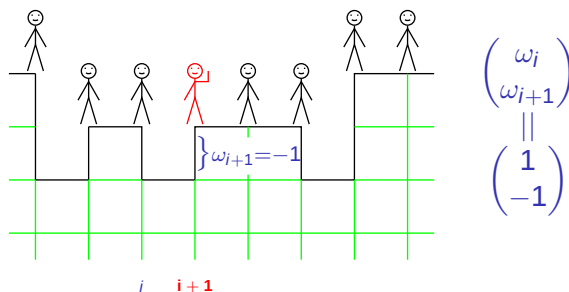
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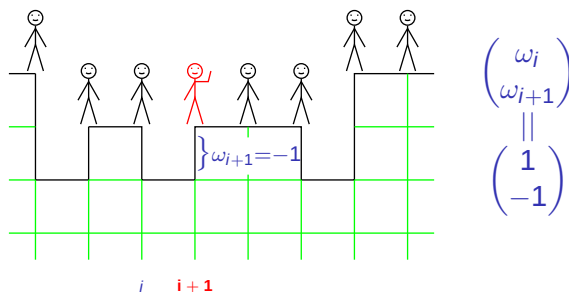
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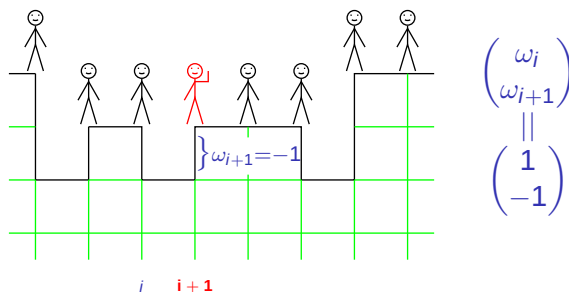
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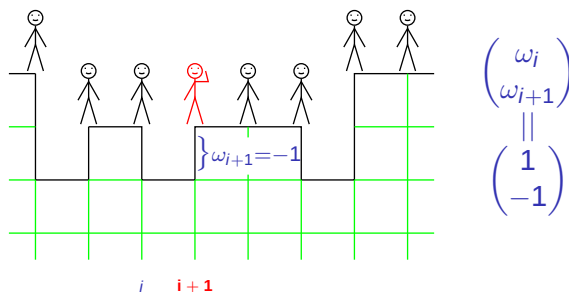
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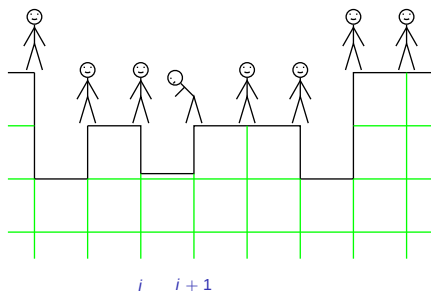
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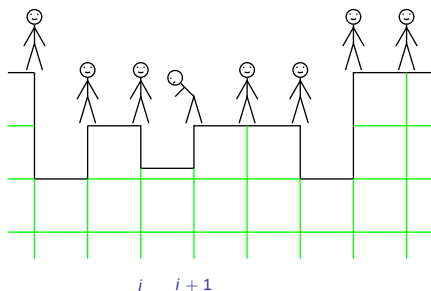


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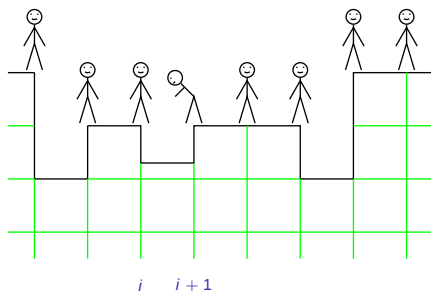


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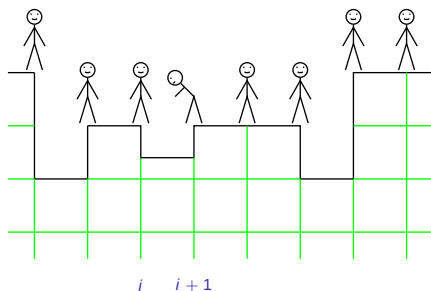


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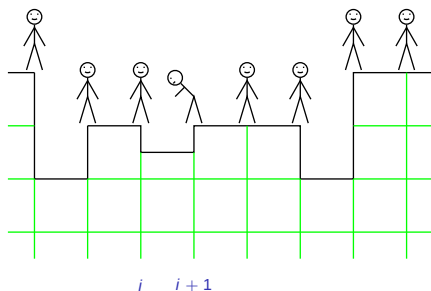


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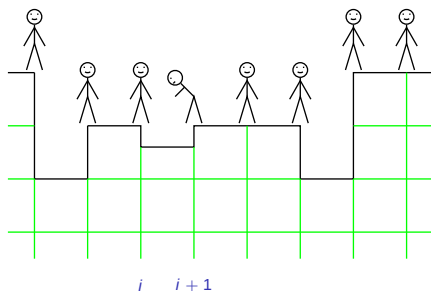


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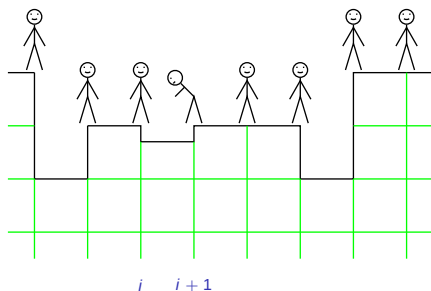


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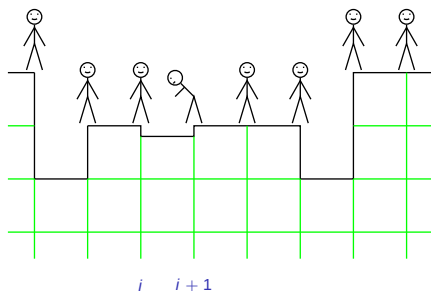


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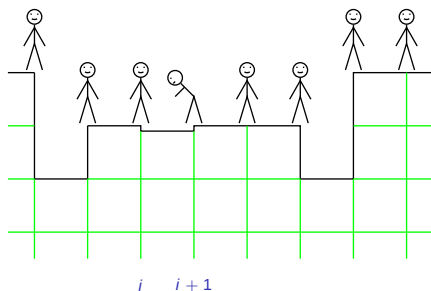


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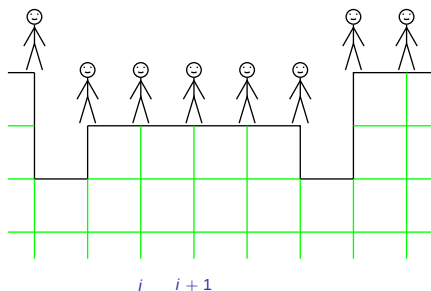


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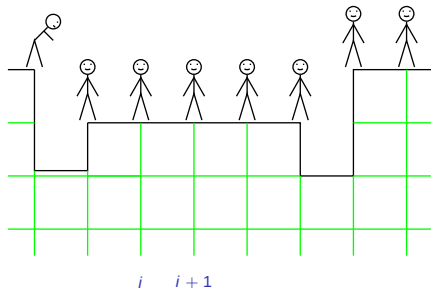


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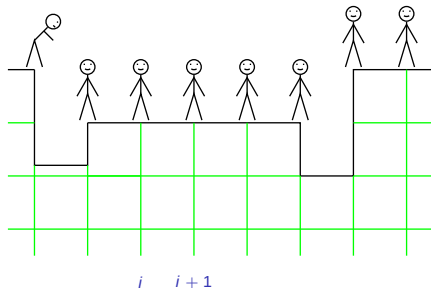


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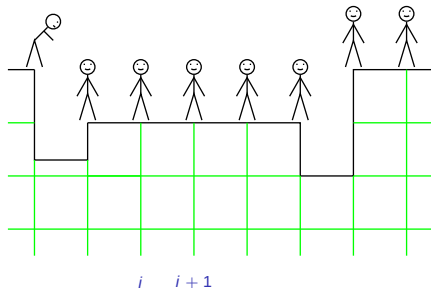


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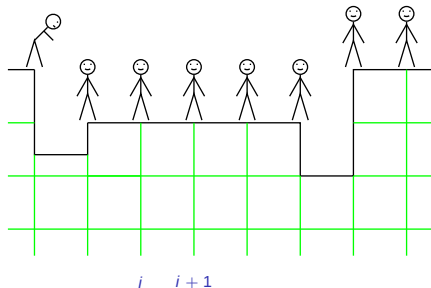


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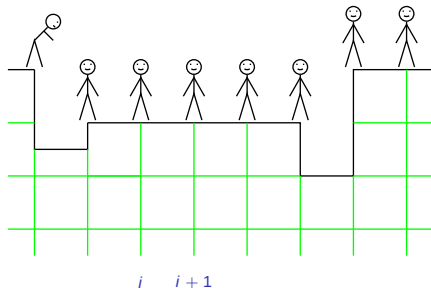


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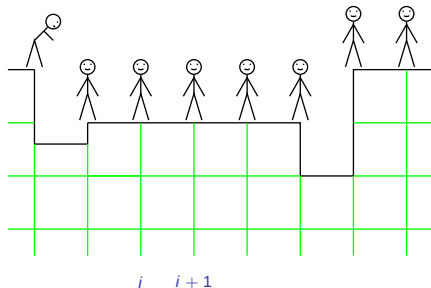


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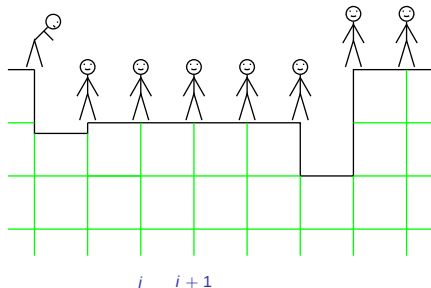
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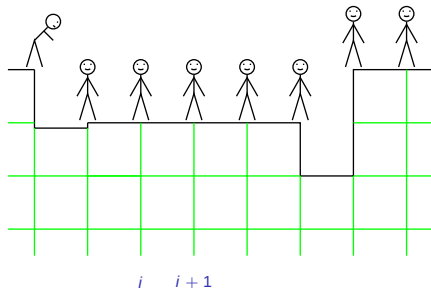


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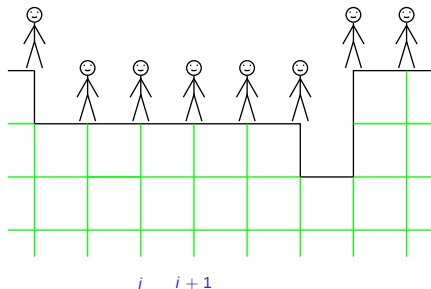
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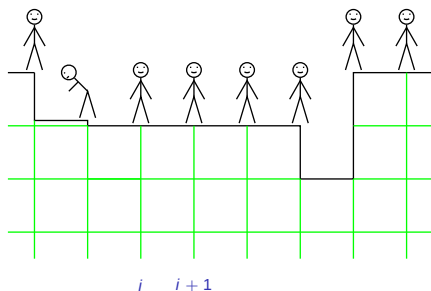


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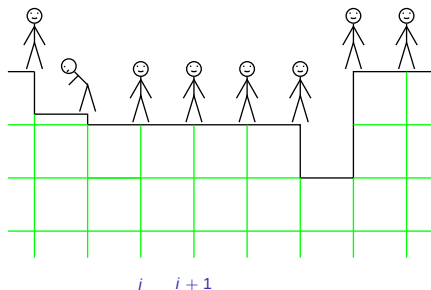


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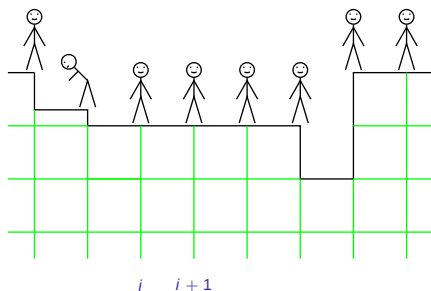


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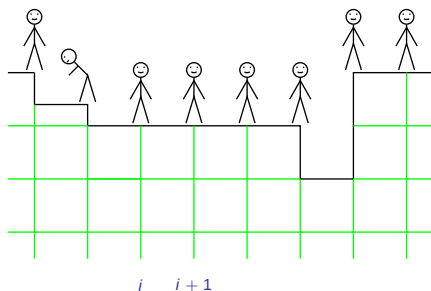


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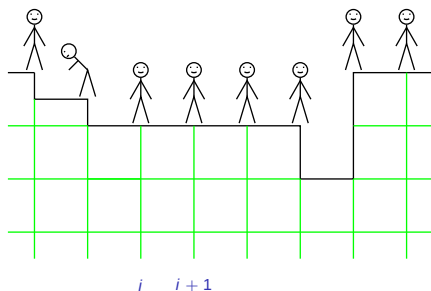


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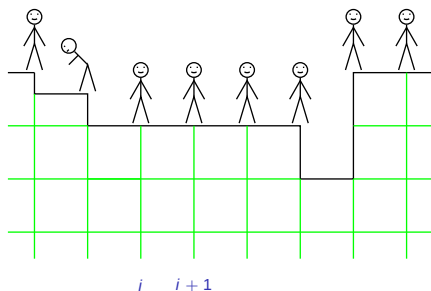


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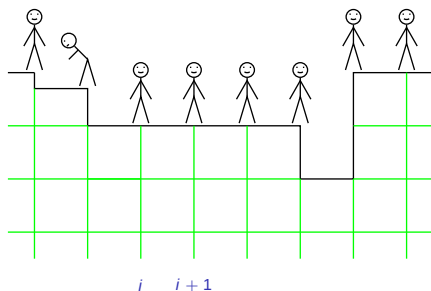


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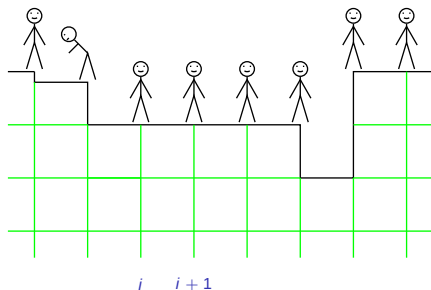


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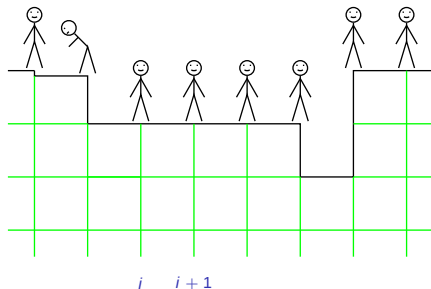


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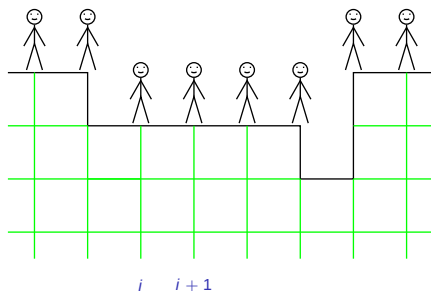


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A mirror-symmetrized version of the extended zero range. Left and right jumps of the dynamics cooperate, if  $(r(\omega) \cdot r(1 - \omega) = 1; \quad r \text{ non-decreasing})$ .

# Stationary product distributions

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Here  $r(0)! := 1$ , and  $r(z+1)! = r(z)! \cdot r(z+1)$  for all  $z \in \mathbb{Z}$ .

## Hydrodynamics (very briefly)

The *density*  $\varrho = \varrho(\theta) := \mathbf{E}^\theta(\omega)$  and the *hydrodynamic flux*  $H = H^\theta := \mathbf{E}^\theta[\text{growth rate}]$  both depend on a parameter  $\varrho$  or  $\theta$  of the stationary distribution.

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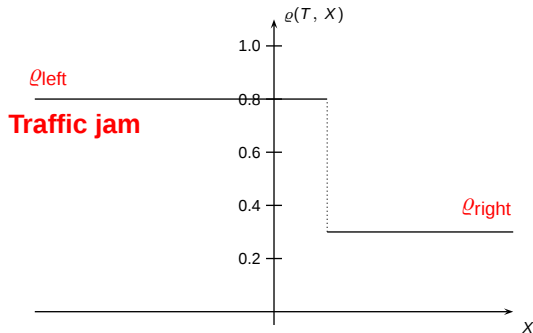
↪ Either convex or concave, discontinuous shock solutions exist.

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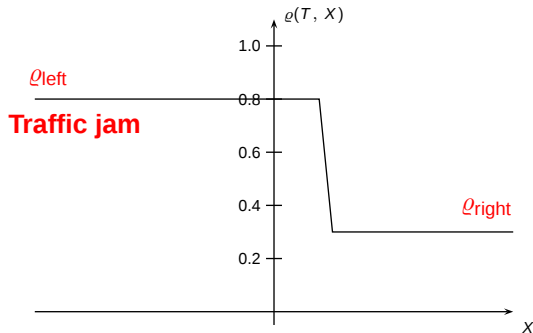
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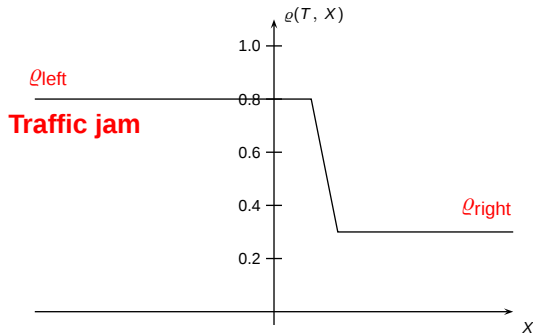
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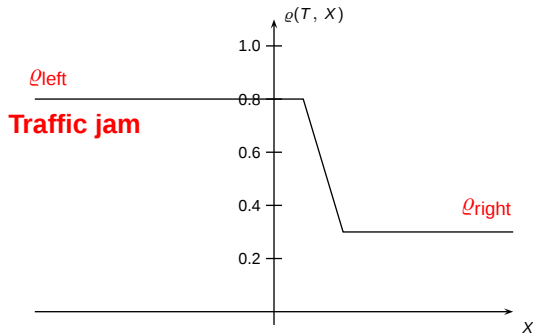
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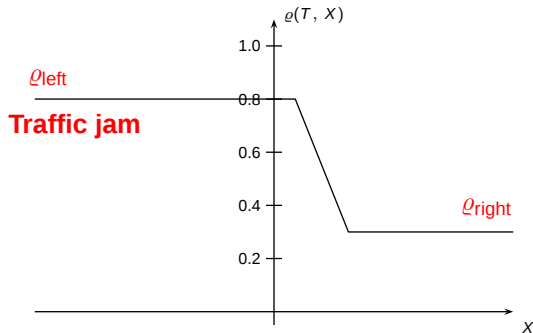
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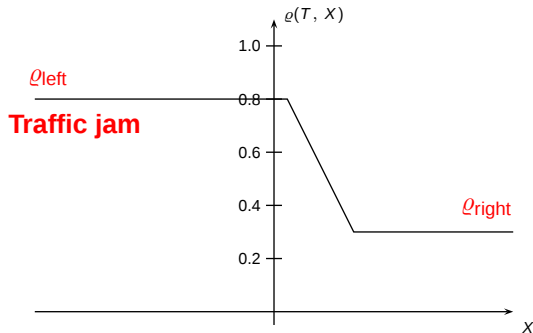
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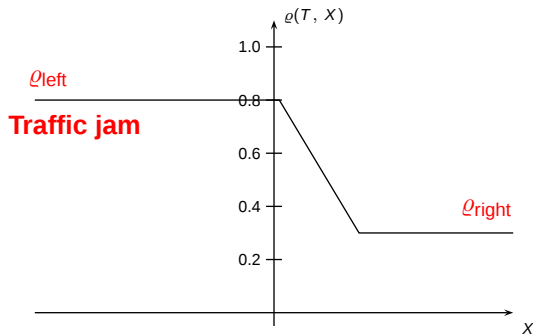
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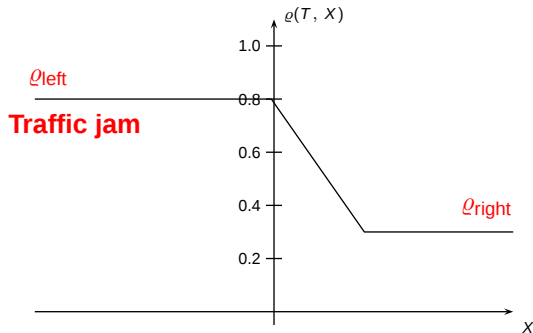
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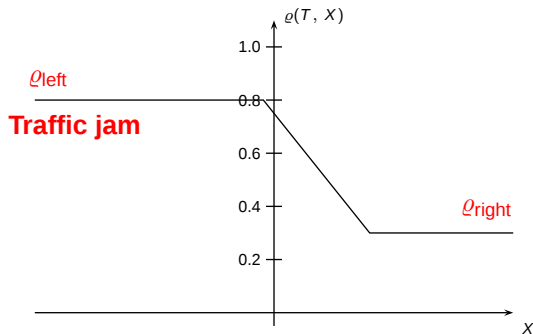
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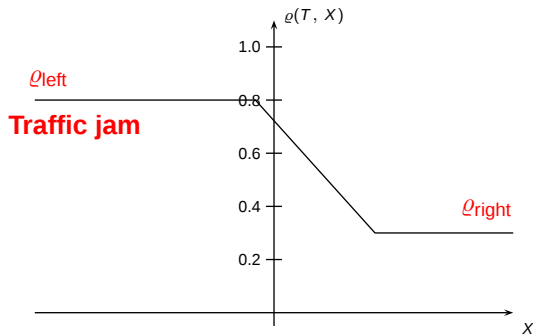
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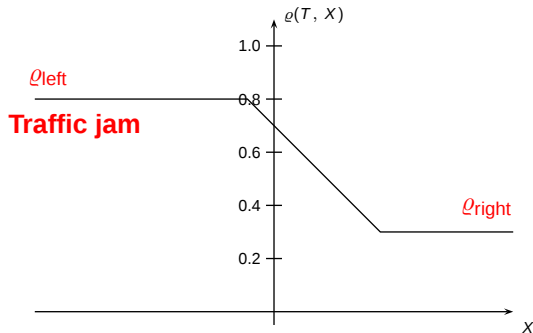
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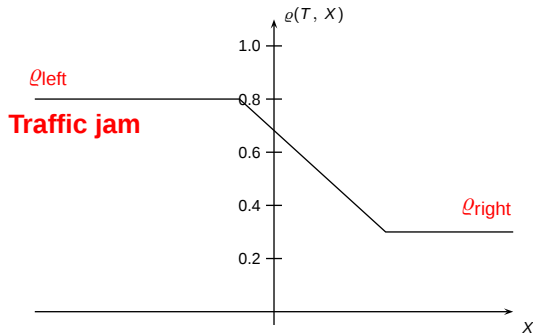
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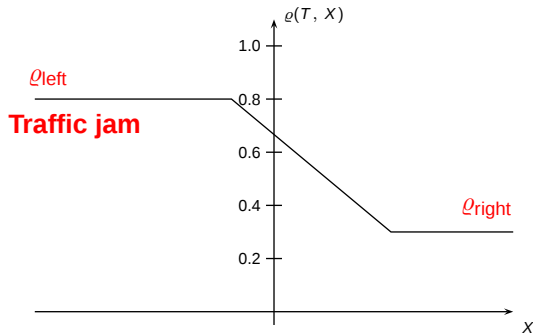
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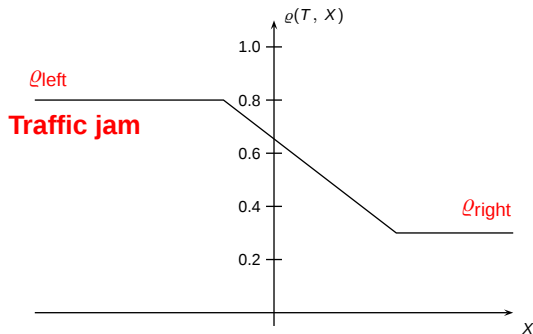
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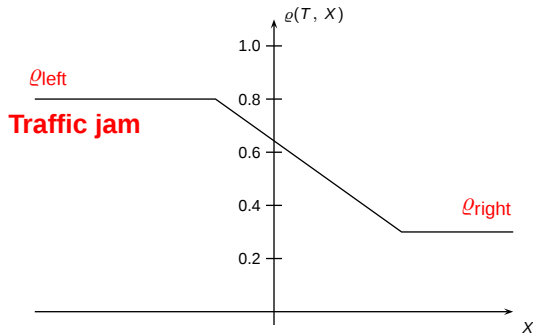
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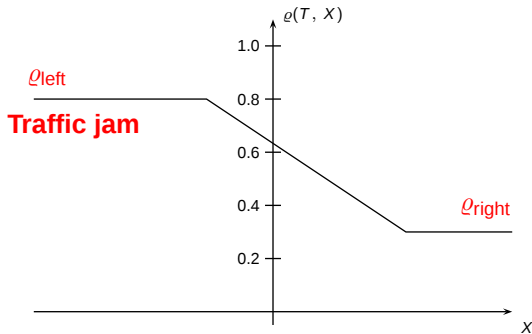
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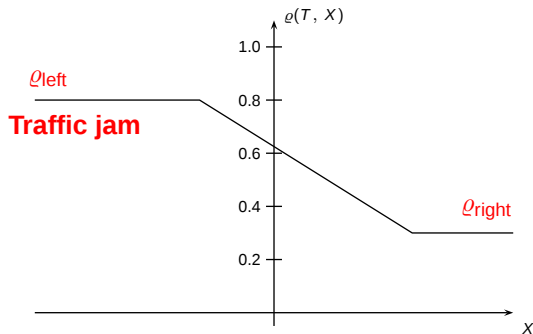
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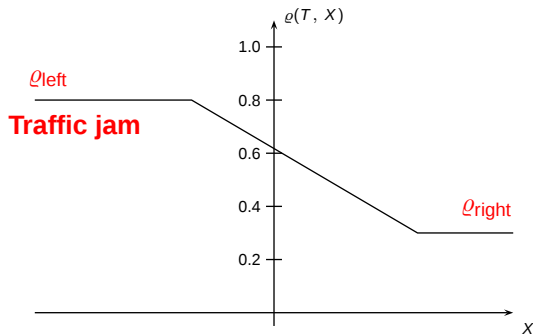
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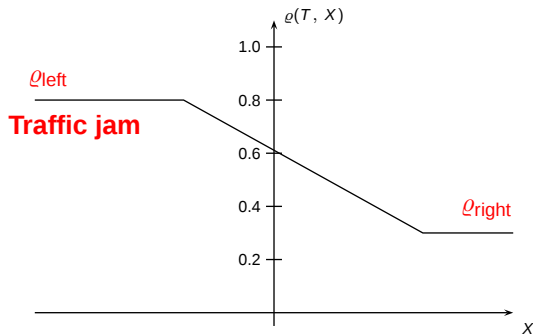
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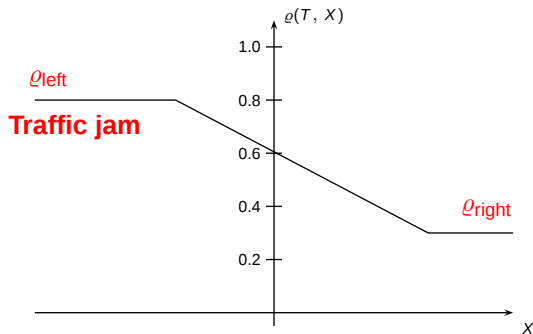
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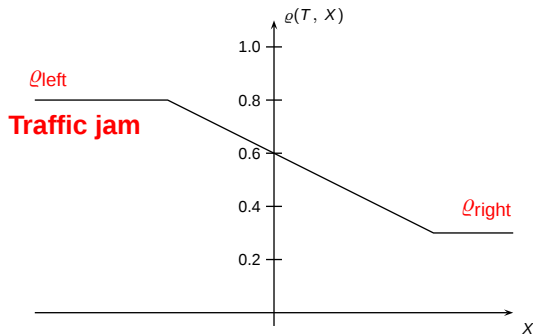
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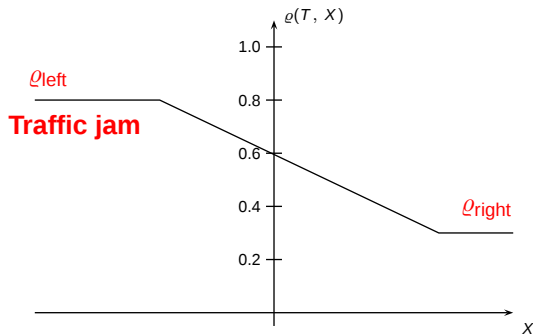
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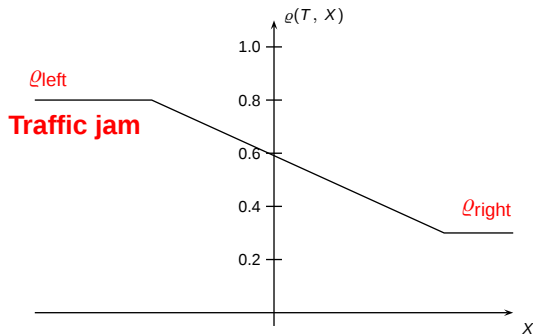
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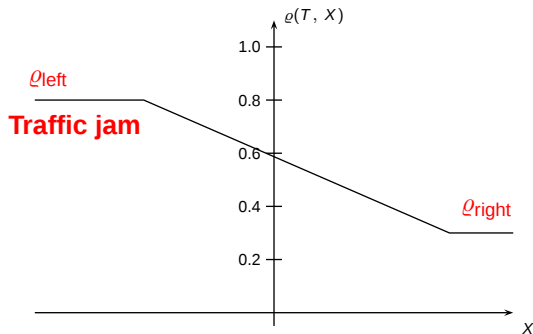
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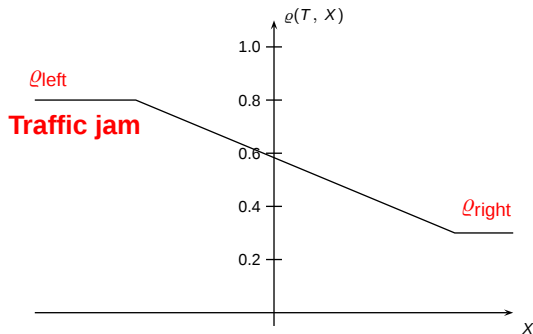
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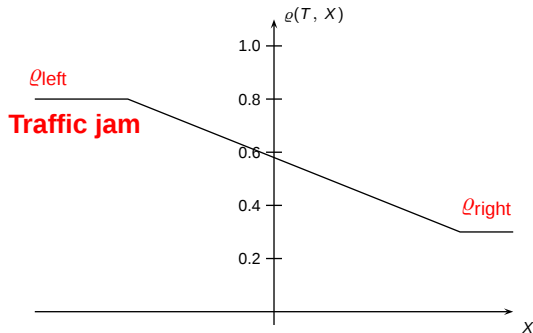
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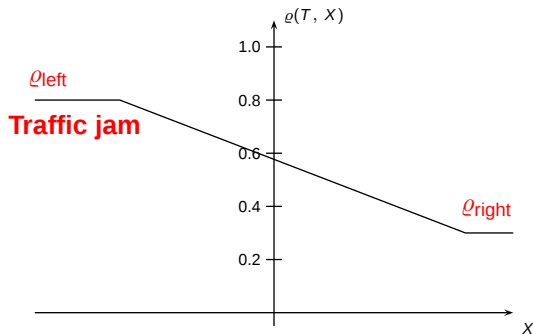
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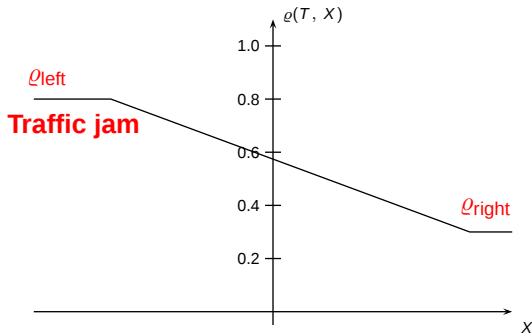
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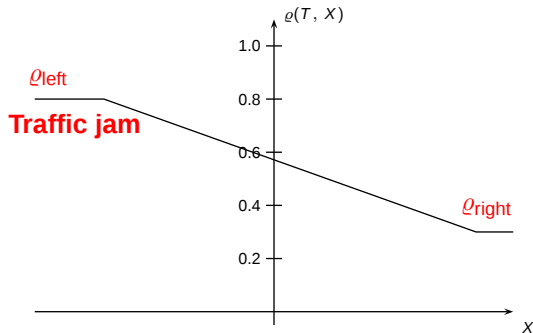
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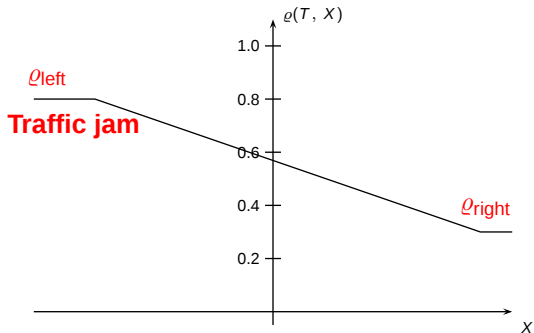
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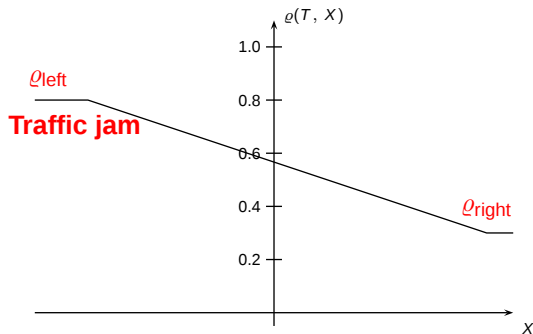
# Rarefaction wave



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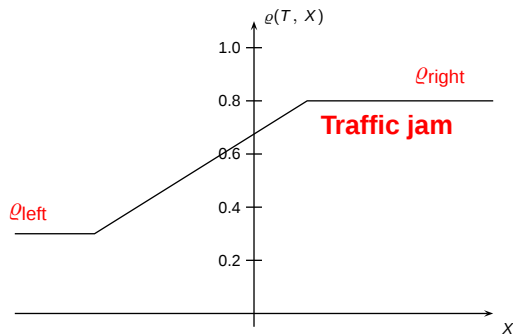
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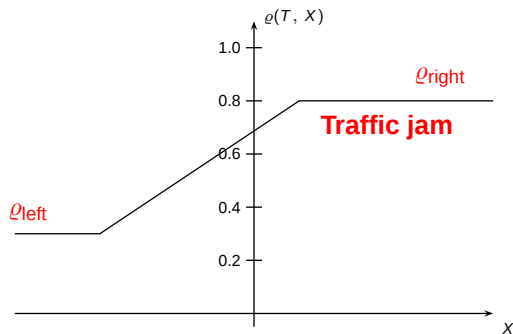
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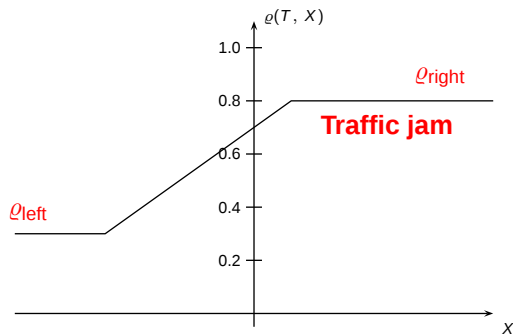
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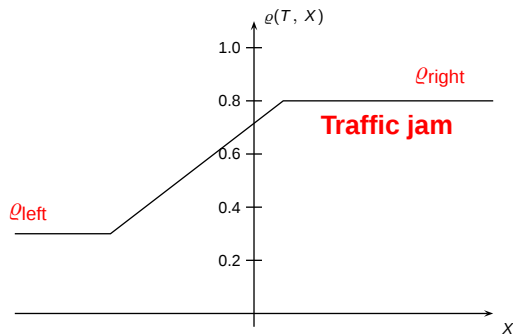
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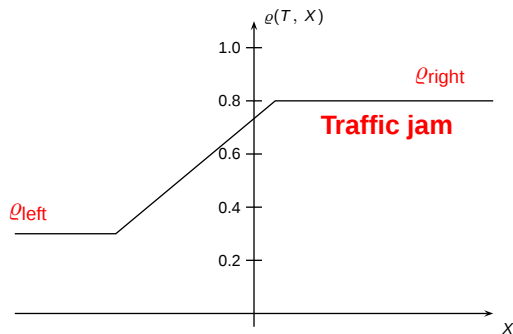
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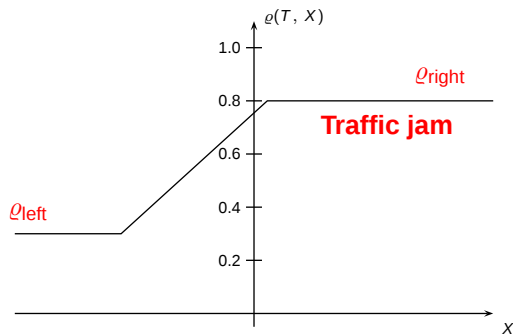
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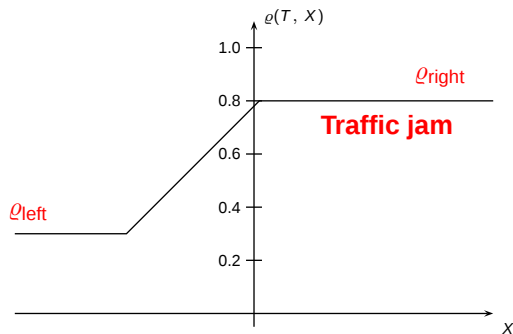
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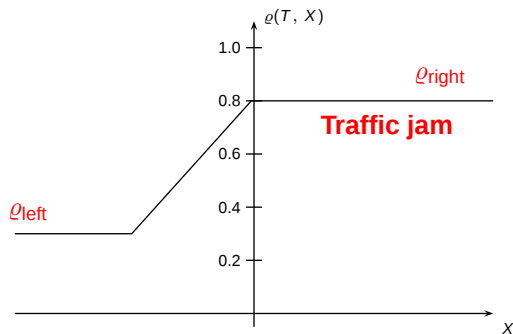
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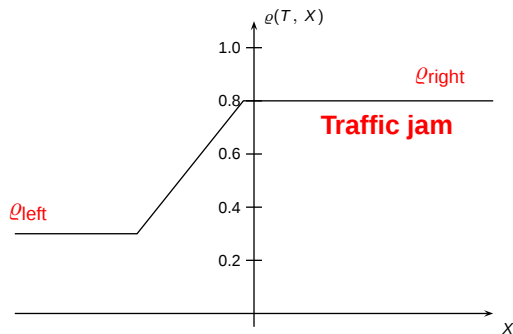
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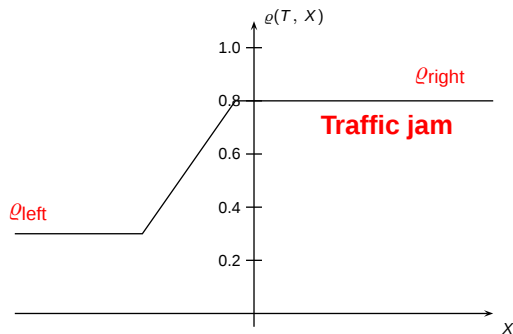
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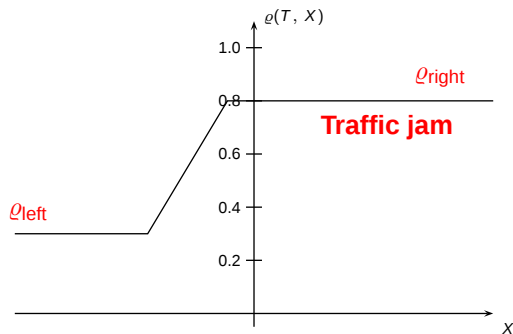
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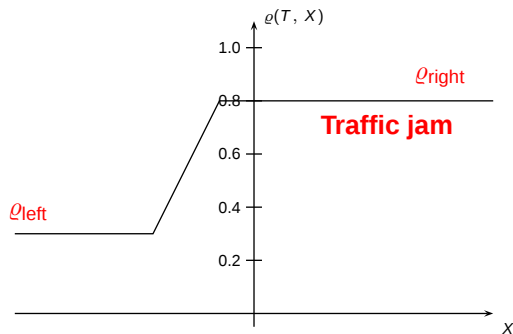
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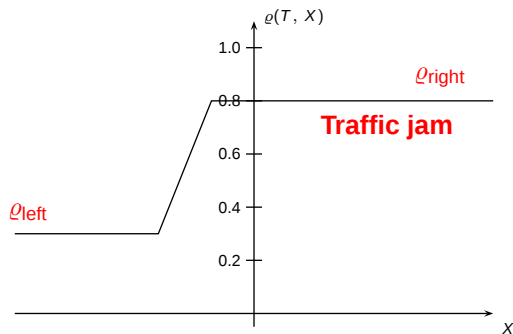
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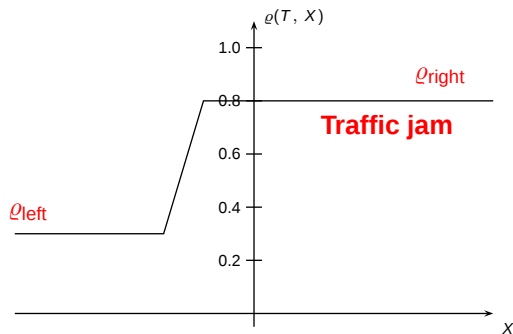
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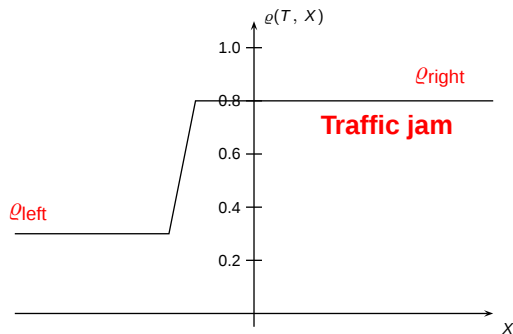
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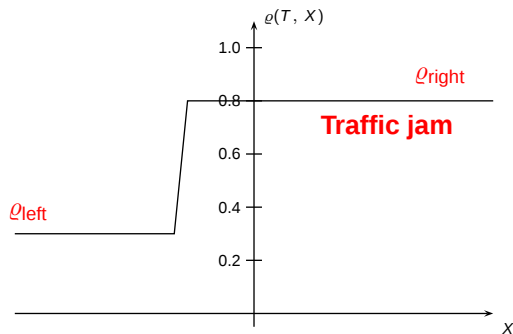
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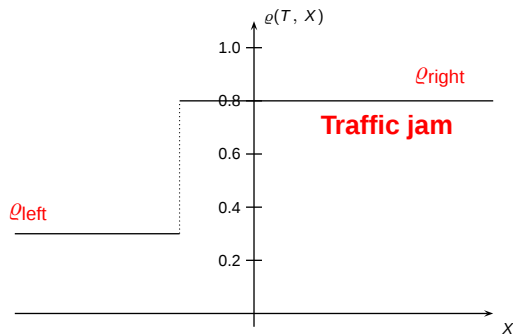
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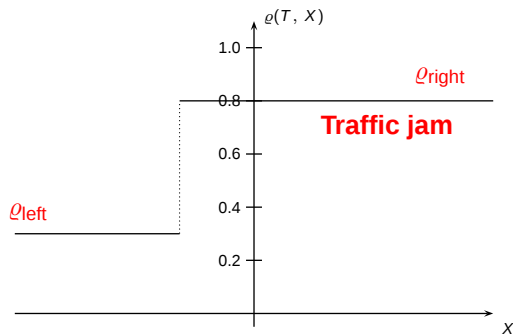
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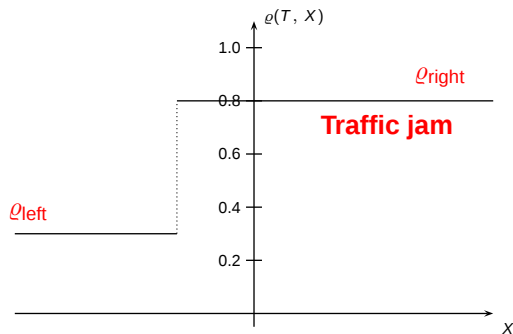
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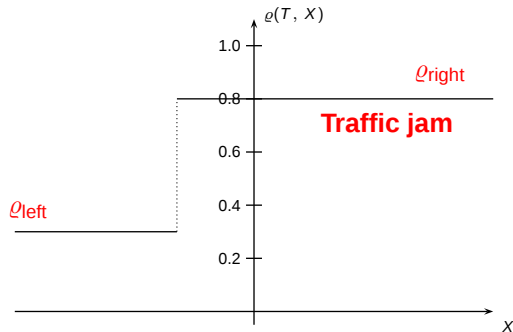
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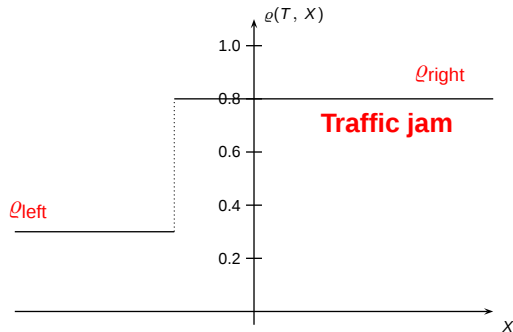
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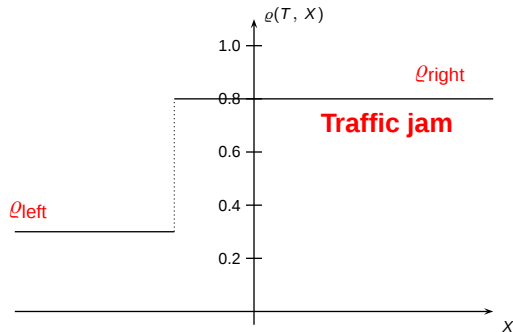
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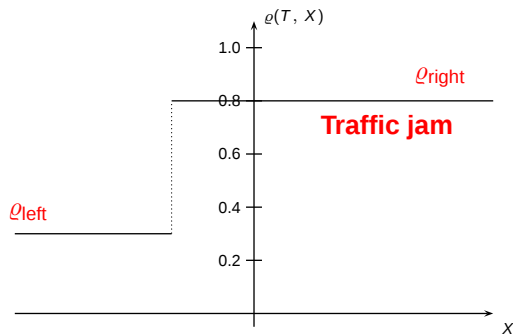
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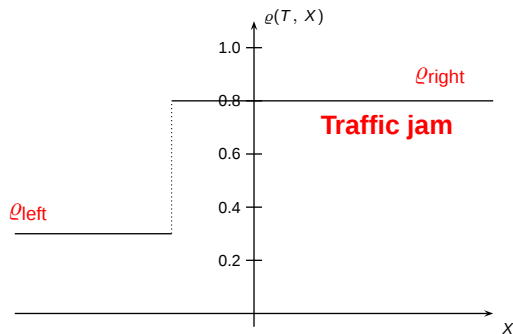
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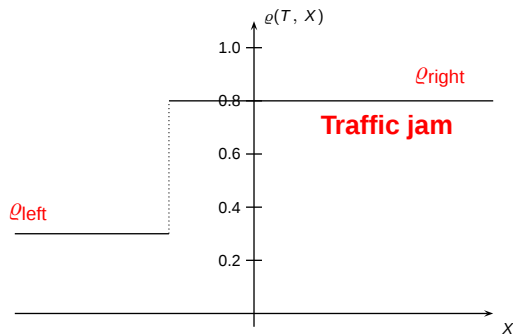
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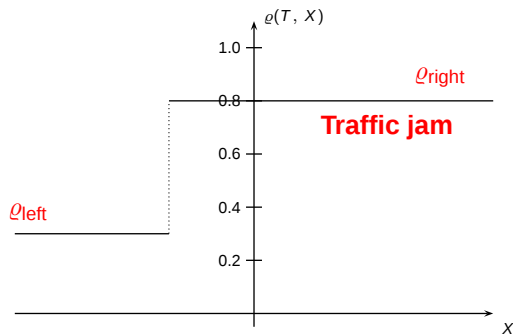
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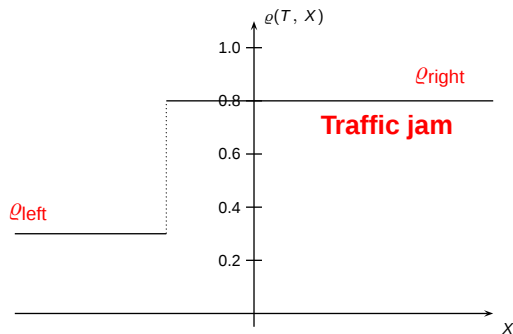
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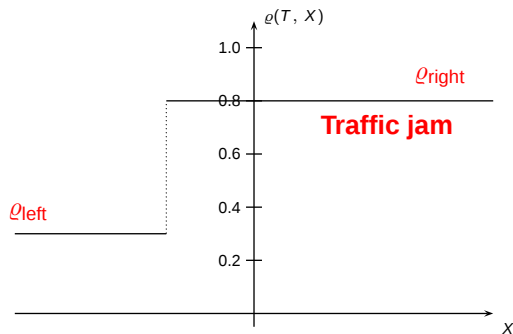
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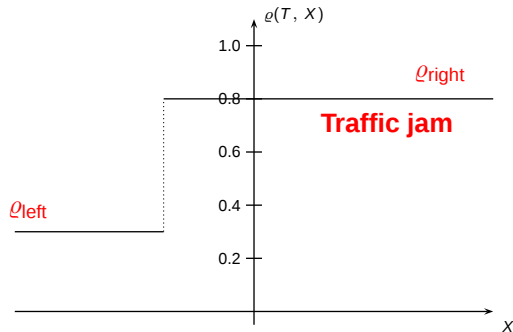
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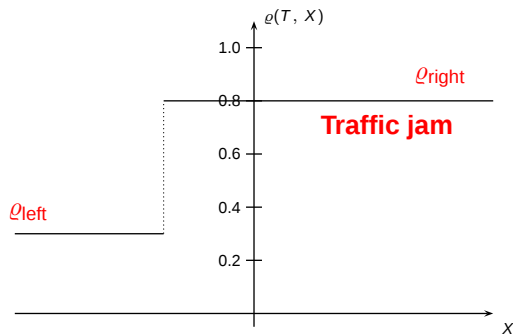
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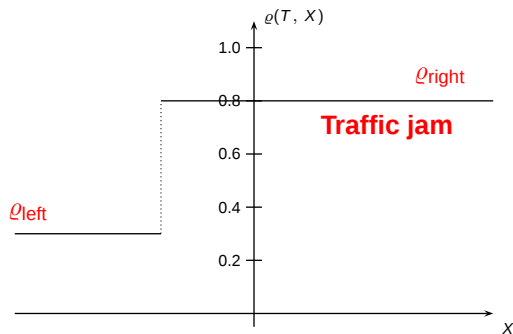
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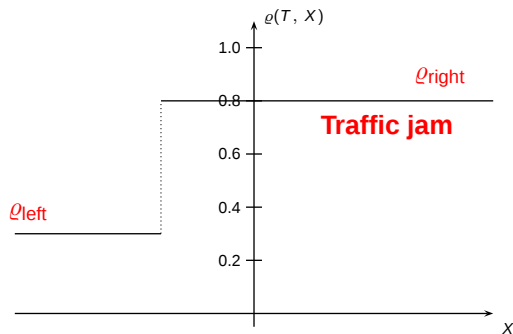
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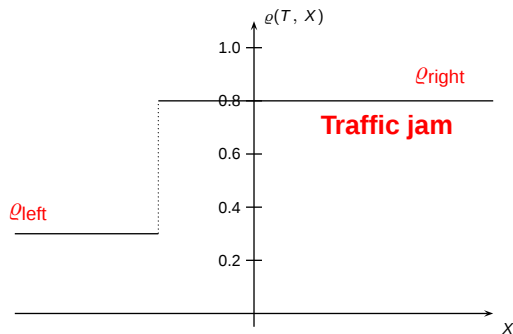
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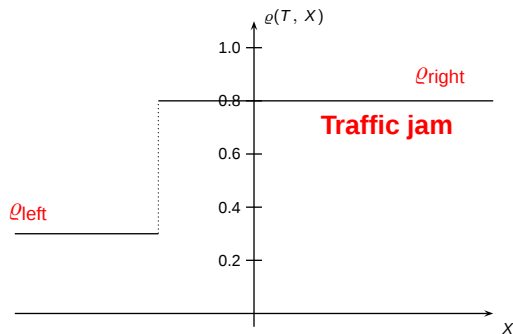
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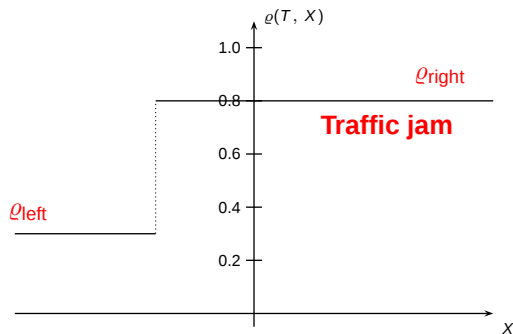
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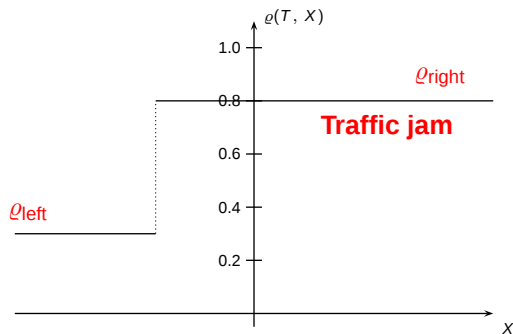
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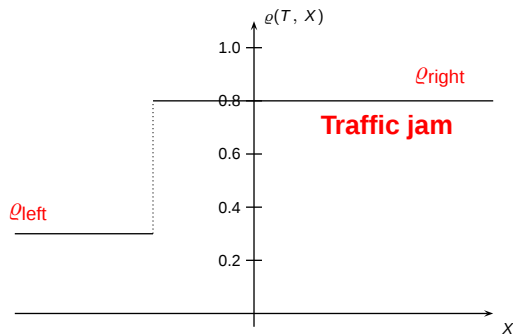
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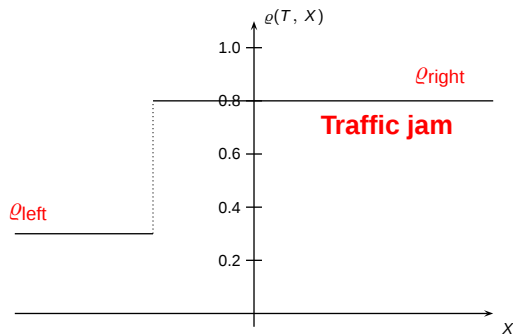
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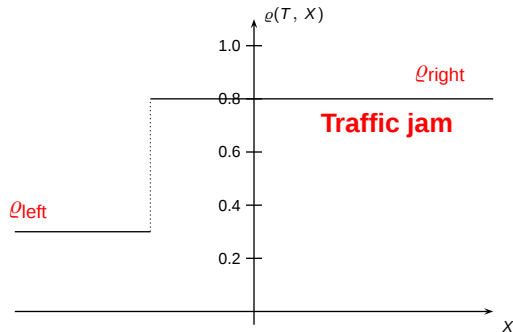
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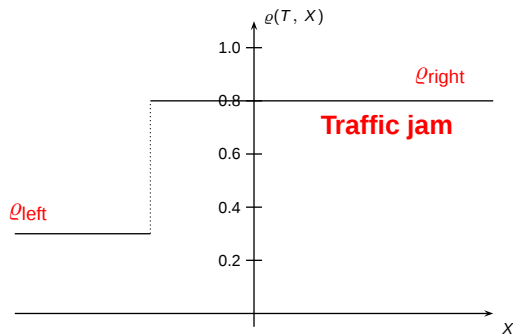
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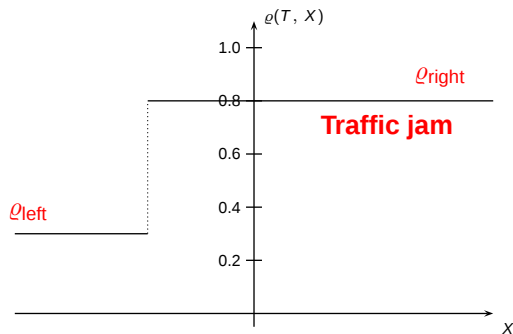
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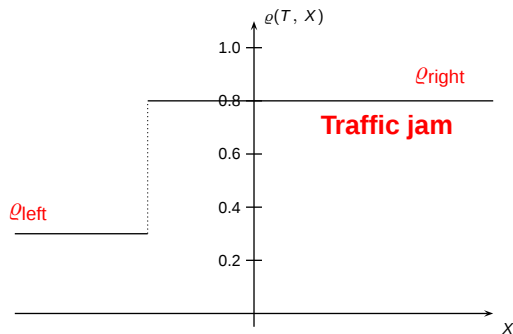
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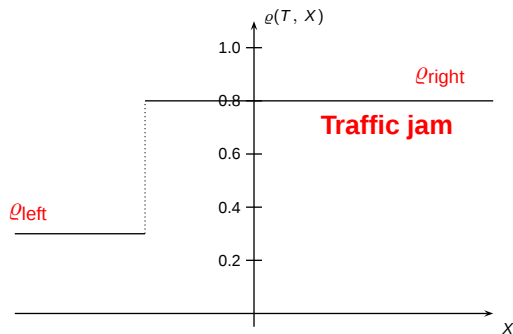
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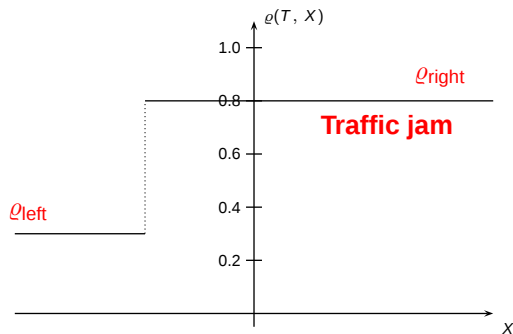
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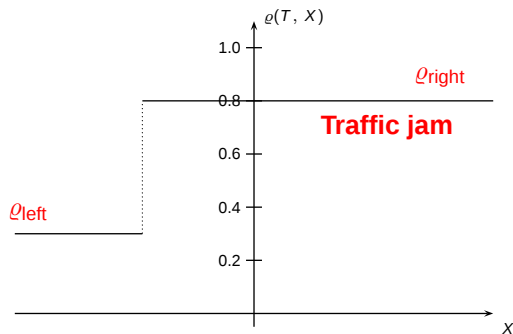
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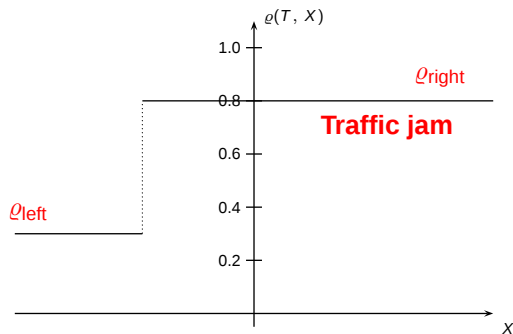
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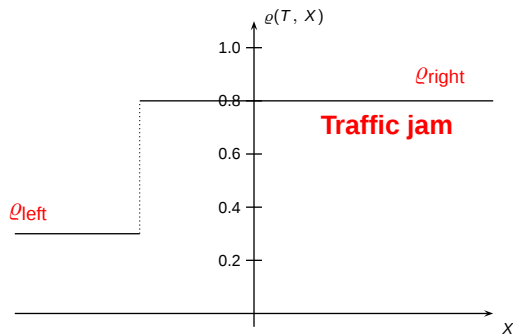
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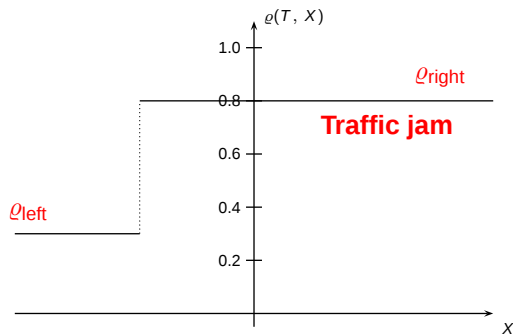
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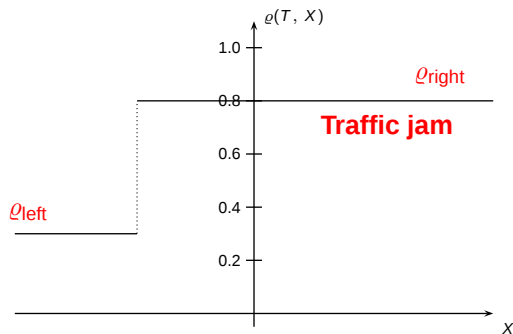
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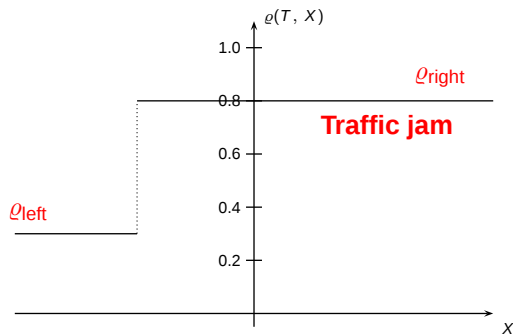
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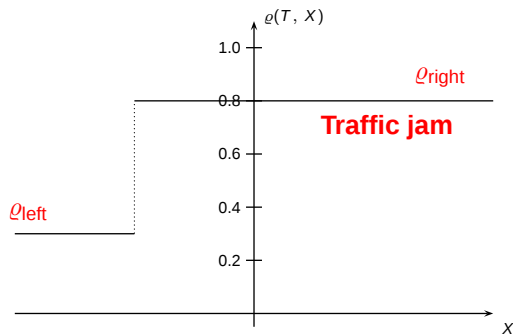
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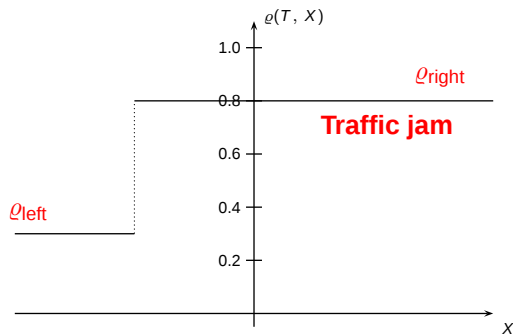


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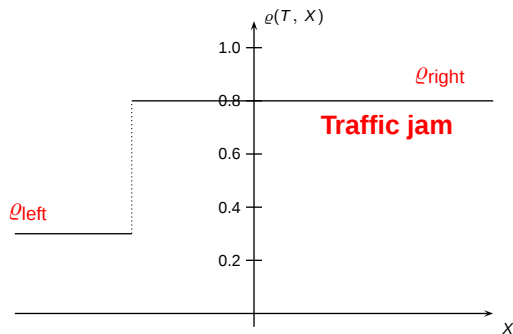
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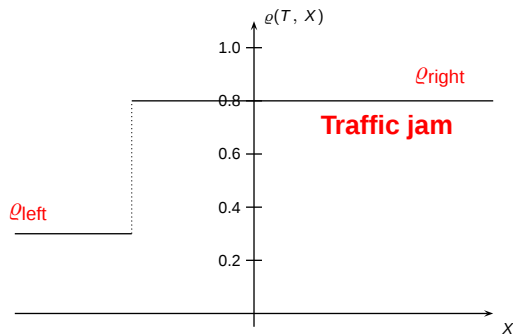
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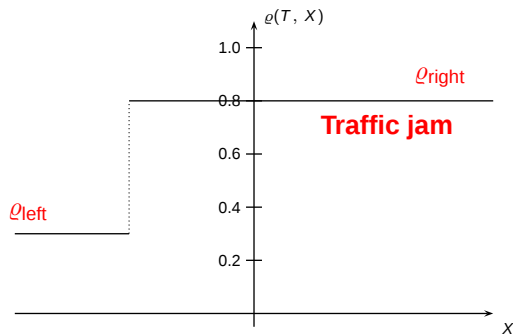
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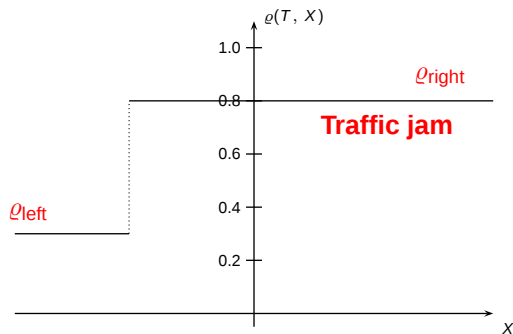
# Shock wave



$$\partial_T \rho + \partial_X H(\rho) = 0$$

$$\rho = \mathbf{E}(\omega), \quad H(\rho) = \mathbf{E}^{\theta(\rho)}[\text{growth rate}]$$

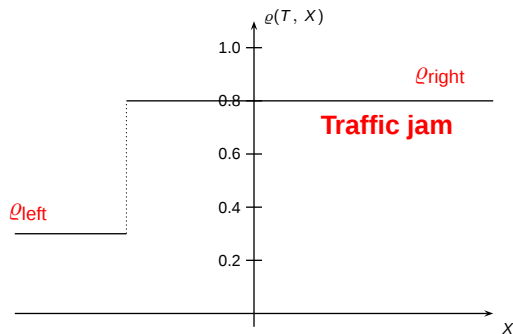
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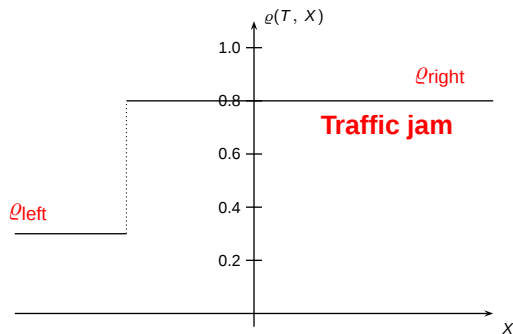
# Shock wave



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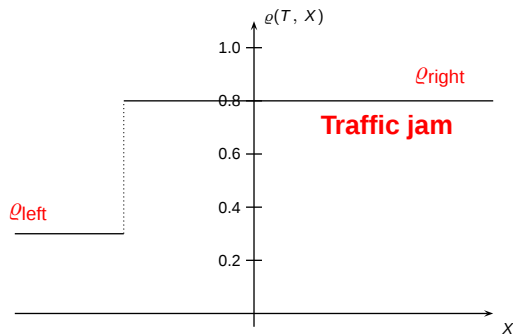
# Shock wave



$$\partial_T \rho + \partial_X H(\rho) = 0$$

$$\rho = \mathbf{E}(\omega), \quad H(\rho) = \mathbf{E}^{\theta(\rho)}[\text{growth rate}]$$

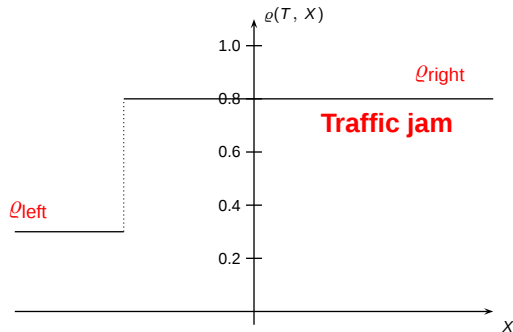
# Shock wave



$$\partial_T \rho + \partial_X H(\rho) = 0$$

$$\rho = \mathbf{E}(\omega), \quad H(\rho) = \mathbf{E}^{\theta(\rho)}[\text{growth rate}]$$

# Shock wave

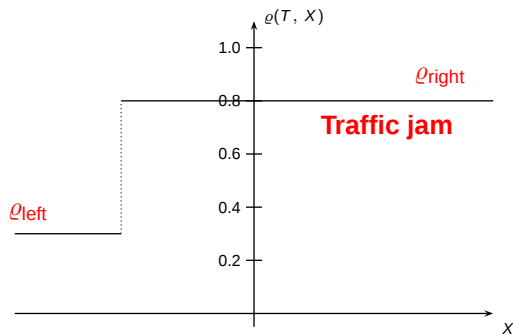


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$$\partial_T \rho + \partial_X H(\rho) = 0$$

$$\rho = \mathbf{E}(\omega), \quad H(\rho) = \mathbf{E}^{\theta(\rho)}[\text{growth rate}]$$

# Shock wave

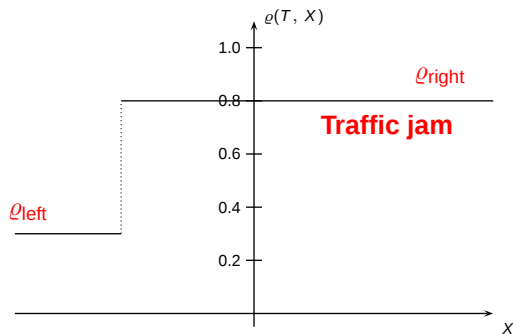


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$$\rho = \mathbf{E}(\omega), \quad H(\rho) = \mathbf{E}^{\theta(\rho)}[\text{growth rate}]$$

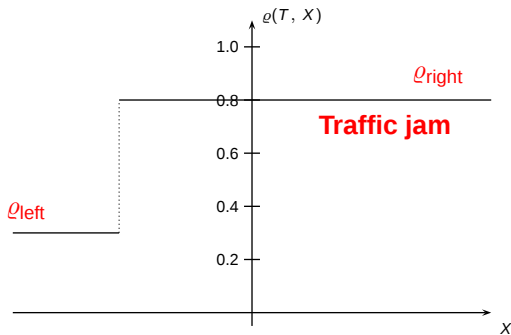
# Shock wave



$$\partial_T \rho + \partial_X H(\rho) = 0$$

$$\rho = \mathbf{E}(\omega), \quad H(\rho) = \mathbf{E}^{\theta(\rho)}[\text{growth rate}]$$

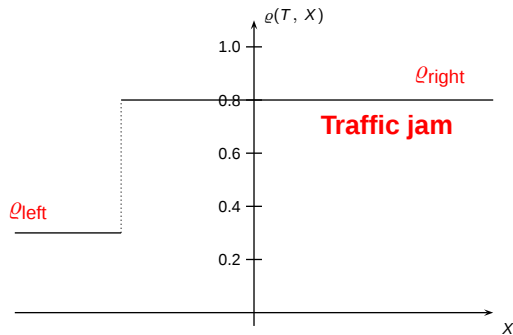
# Shock wave



Discontinuous shock appears. Its velocity is given by the *Rankine-Hugoniot speed* for densities  $\rho_{\text{left}}$  and  $\rho_{\text{right}}$

$$R = \frac{H(\rho_{\text{left}}) - H(\rho_{\text{right}})}{\rho_{\text{left}} - \rho_{\text{right}}}.$$

# Shock wave



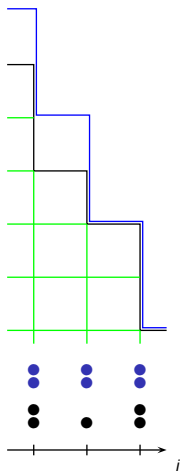
Discontinuous shock appears. Its velocity is given by the *Rankine-Hugoniot speed for densities  $\rho_{\text{left}}$  and  $\rho_{\text{right}}$*

$$R = \frac{H(\rho_{\text{left}}) - H(\rho_{\text{right}})}{\rho_{\text{left}} - \rho_{\text{right}}}.$$

Let's look for the corresponding microscopic structure.

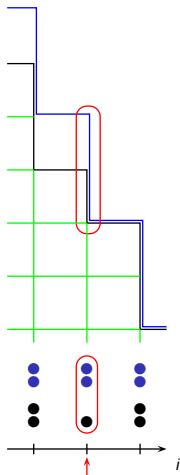
# The second class particle

States  $\omega$  and  $\tilde{\omega}$  only differ at one site.



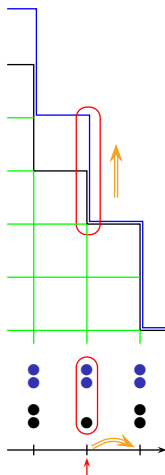
# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



# The second class particle

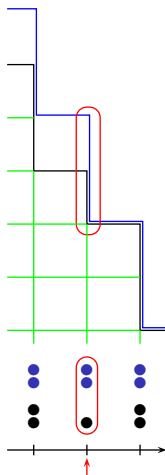
States  $\omega$  and  $\omega'$  only differ at one site.



Growth on the right:  
 $\text{rate} \leq \text{rate}$

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



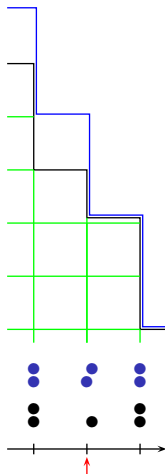
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



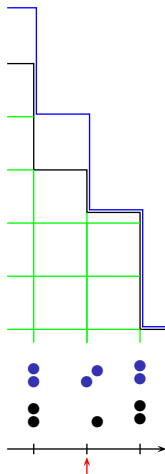
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



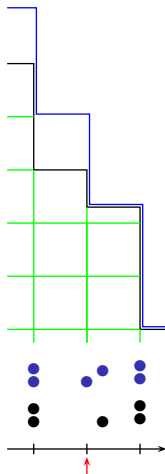
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



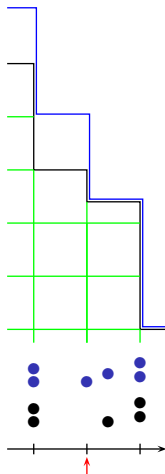
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



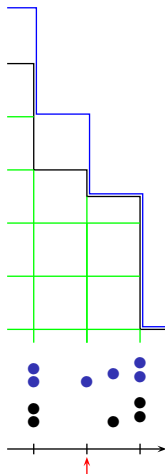
Growth on the right:

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with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



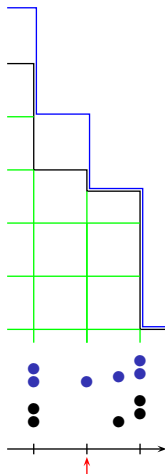
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



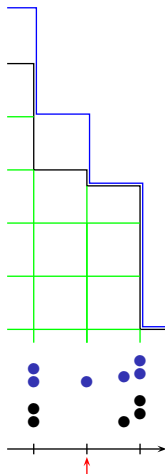
Growth on the right:

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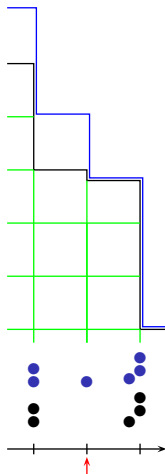
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



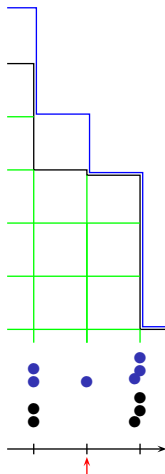
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



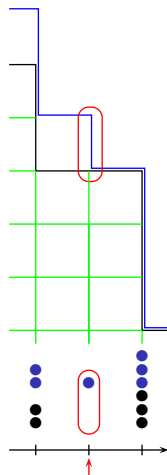
Growth on the right:

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with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



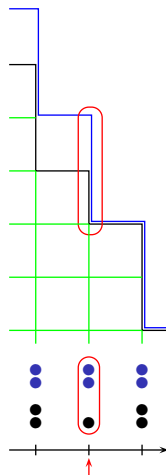
Growth on the right:

$\text{rate} \leq \text{rate}$

with rate:

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



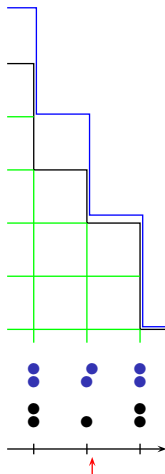
Growth on the right:

$\text{rate} \leq \text{rate}$

with  $\text{rate-rate}$ :

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



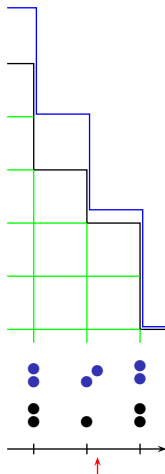
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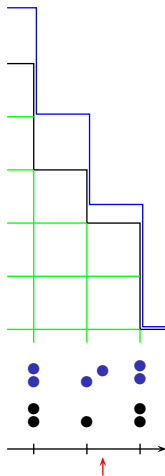
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$\text{rate} \leq \text{rate}$

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States  $\omega$  and  $\omega'$  only differ at one site.



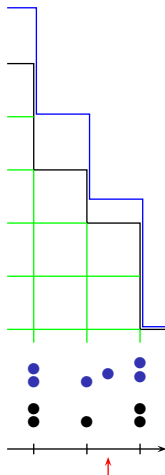
Growth on the right:

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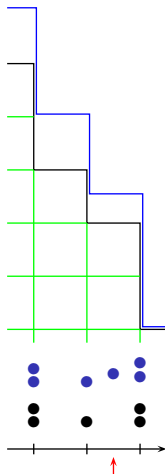
Growth on the right:

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# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.



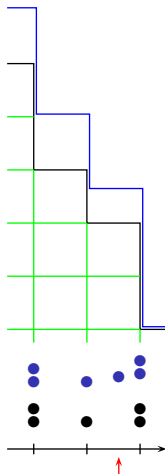
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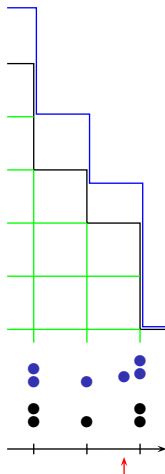
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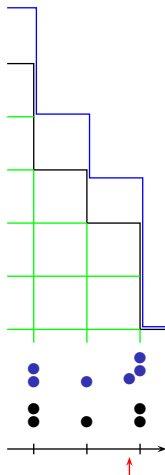
Growth on the right:

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with  $\text{rate-rate}$ :

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States  $\omega$  and  $\omega'$  only differ at one site.



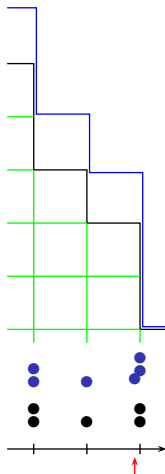
Growth on the right:

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with  $\text{rate-rate}$ :

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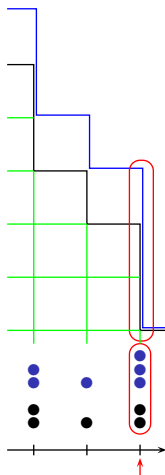
Growth on the right:

$\text{rate} \leq \text{rate}$

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States  $\omega$  and  $\omega'$  only differ at one site.



Growth on the right:

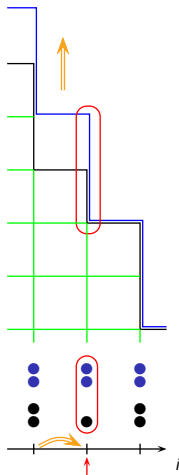
$\text{rate} \leq \text{rate}$

with  $\text{rate-rate}$ :

# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

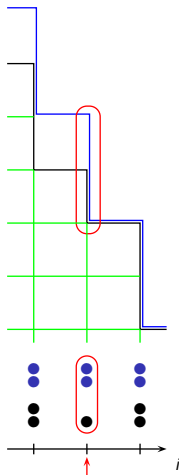
Growth on the left:  
rate  $\geq$  rate



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate}$ :



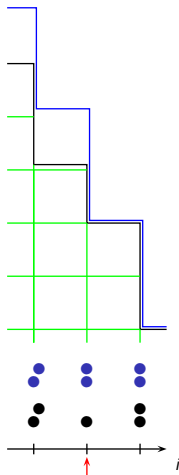
# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

rate  $\geq$  rate

with rate:



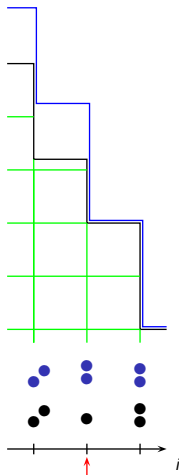
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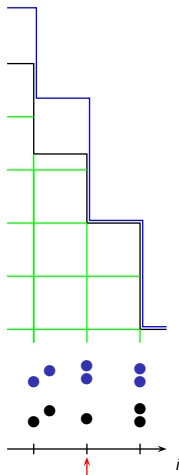
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with rate:



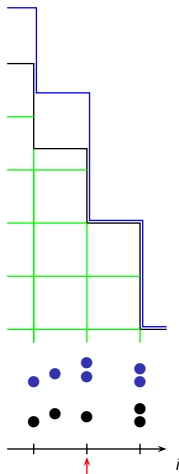
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States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

rate  $\geq$  rate

with rate:



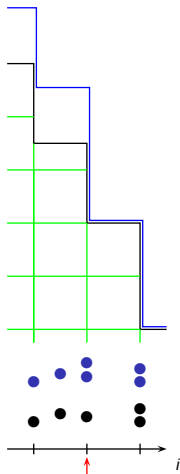
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States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

rate  $\geq$  rate

with rate:



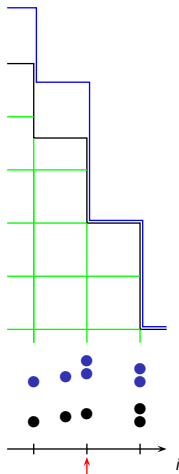
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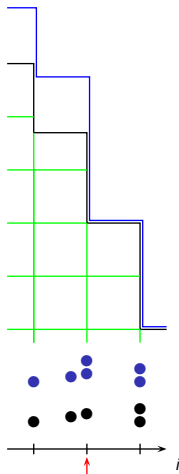
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States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

rate  $\geq$  rate

with rate:



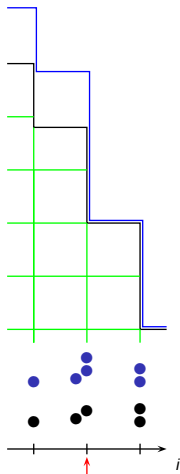
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States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

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with rate:



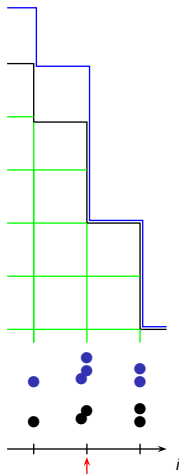
# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

rate  $\geq$  rate

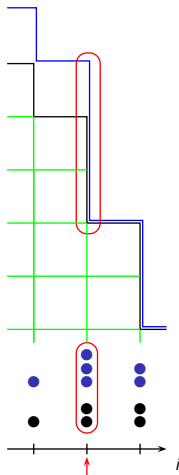
with rate:



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

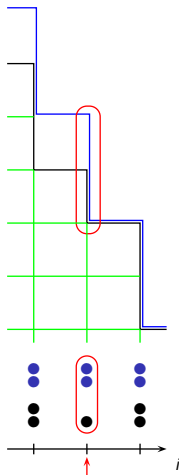
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate}$ :



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

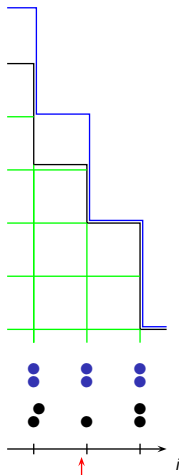
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

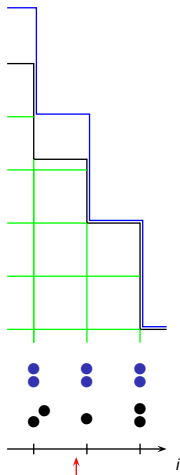
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 $\text{rate} \geq \text{rate}$   
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# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

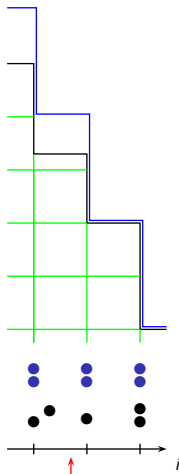
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



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States  $\omega$  and  $\omega'$  only differ at one site.

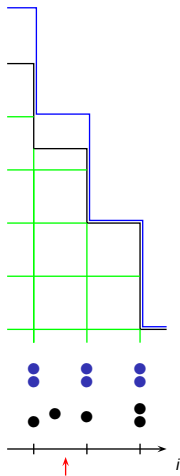
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

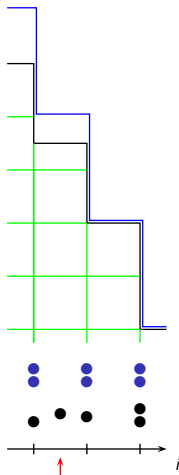
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

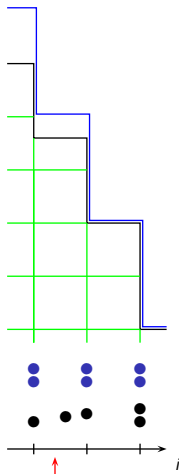
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

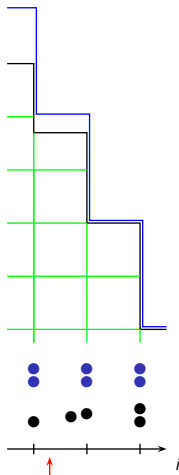
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



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States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:  
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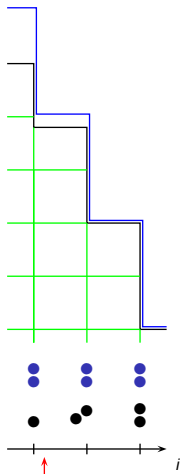
# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

Growth on the left:

rate  $\geq$  rate

with rate-rate:



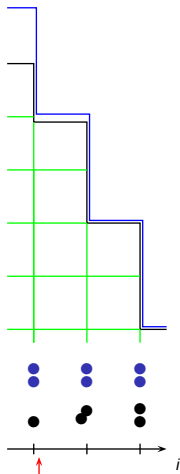
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Growth on the left:

rate  $\geq$  rate

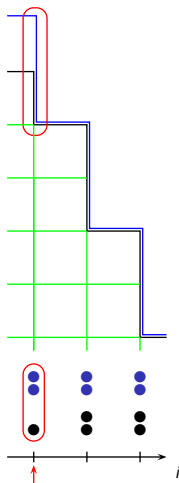
with rate-rate:



# The second class particle

States  $\omega$  and  $\omega'$  only differ at one site.

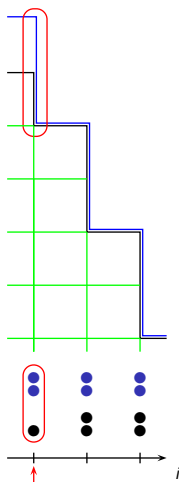
Growth on the left:  
 $\text{rate} \geq \text{rate}$   
 with  $\text{rate} - \text{rate}$ :



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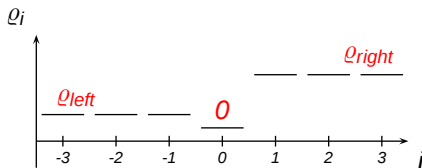
A single discrepancy  $\uparrow$ , the *second class particle*, is conserved.

## Earlier results: as seen by the second class particle

From now on: ASEP, TAG $\textcolor{brown}{E}$ ZRP, TA $\textcolor{brown}{E}$ BLP only; “E”=exponential.

### Theorem (Derrida, Lebowitz, Speer '97)

*For the ASEP, the Bernoulli product distribution with densities*



*is stationary for the process, as seen from the second class particle, if*

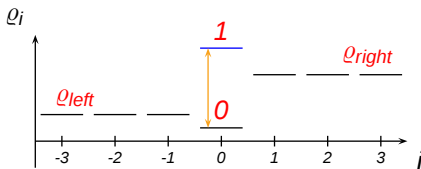
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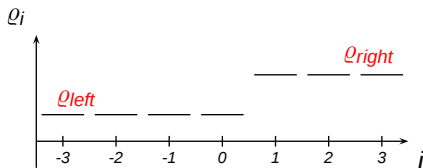
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### Theorem (Belitsky and Schütz '02)

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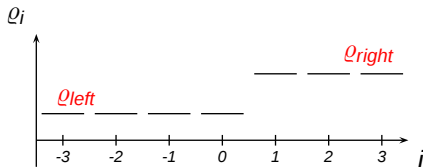
*evolves according to*

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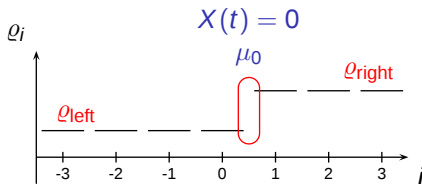
$$\frac{d}{dt} \mu_0 = p \cdot \frac{\rho_{\text{left}}}{\rho_{\text{right}}} \cdot [\mu_{-1} - \mu_0] + q \cdot \frac{\rho_{\text{right}}}{\rho_{\text{left}}} \cdot [\mu_1 - \mu_0].$$

*Multiple shocks and their interactions are also handled.*

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Interpretation: random walking shock  $\mu(t) = \mu_{X(t)}$ :

with rate  $p \cdot \frac{q_{\text{left}}}{q_{\text{right}}}$ :

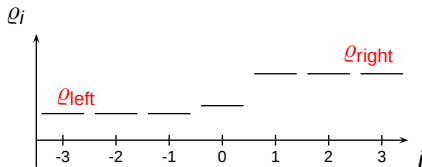


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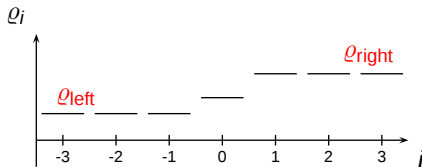


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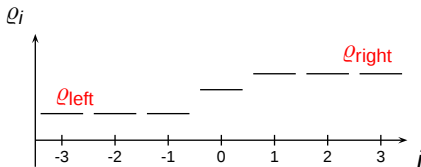


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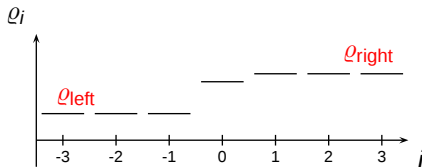


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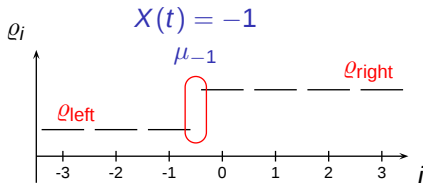


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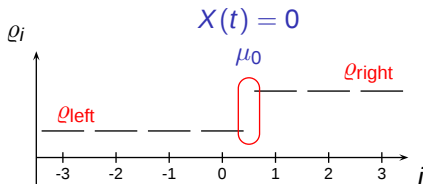


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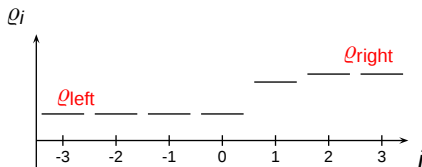


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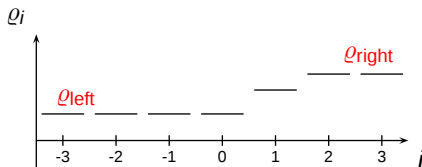


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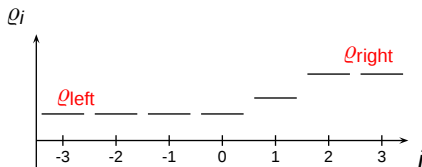


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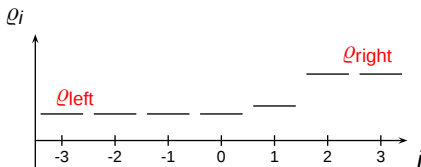


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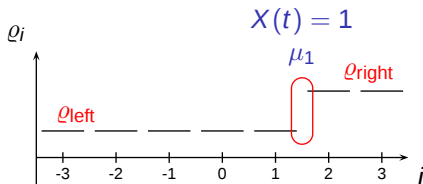


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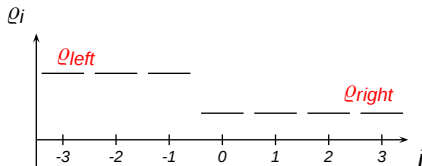


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For the TAEBLP, the product distribution of marginals  $\mu^{q_i}$  with densities



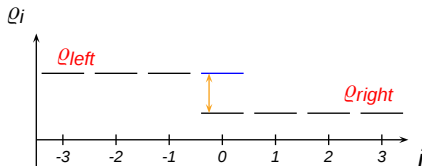
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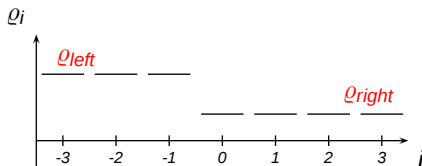
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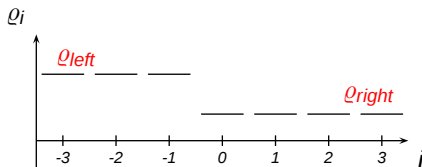
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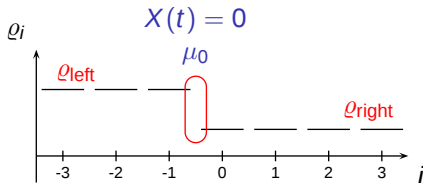
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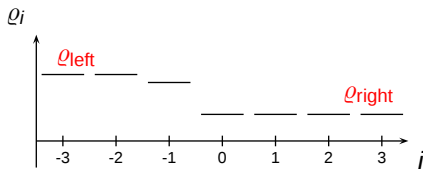


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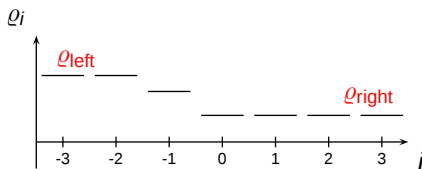


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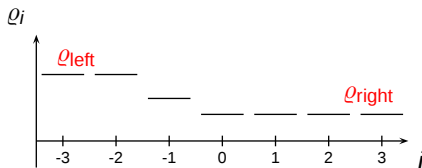


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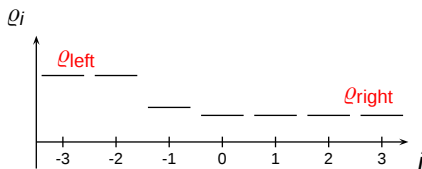


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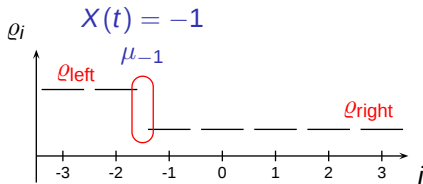


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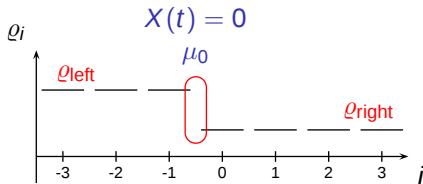


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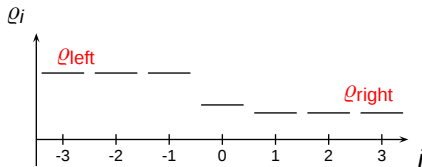


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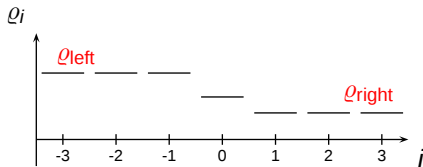


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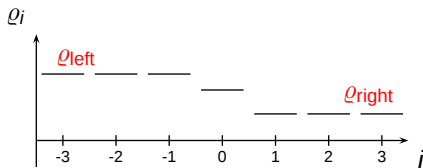


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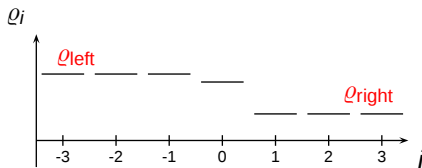


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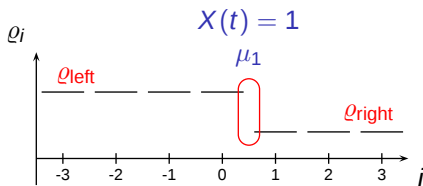


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Is it the second class particle that performs the simple random walk in the middle of a shock?

In what sense? Annealed w.r.t. the initial shock distribution...  
But what does this mean?

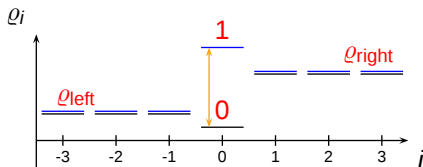
## Here is the question:

For the ASEP, let  $\nu_0$  be the Bernoulli product distribution

$$\nu_0 = \left( \bigotimes_{i < 0} \mu^{\varrho_{\text{left}}} \right) \otimes (\delta) \otimes \left( \bigotimes_{i > 0} \mu^{\varrho_{\text{right}}} \right),$$

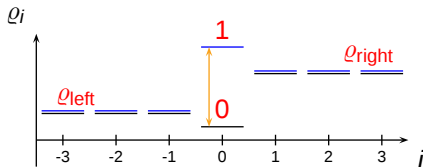
where

$$\mu^{\varrho}(\omega = \omega) = \begin{cases} \varrho, & \text{if } \omega = 1, \\ 1 - \varrho, & \text{if } \omega = 0; \end{cases} \quad \delta(0, 1) = 1.$$



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Does it satisfy

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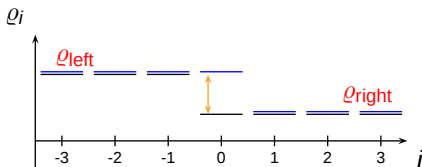
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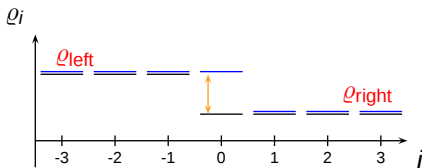
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The presence of a second class particle in the measure significantly simplifies the computations.  $\rightsquigarrow$  This is how we discovered the TAGEZRP.

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It also gives a rough tail bound for the second class particle in a *flat initial distribution*; essential in the  $t^{2/3}$  proofs for the **exponential** models.

## Interactions:

We also see that shocks+second class particles

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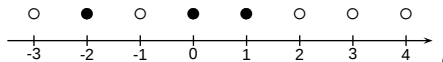
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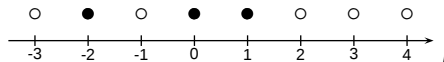
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*Macroscopically it's one shock after all.*

## A similar result: branching coalescing random walk

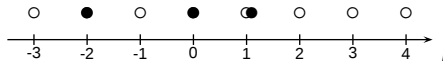


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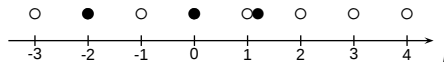
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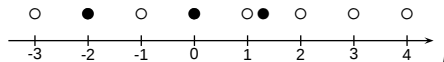
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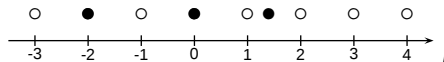
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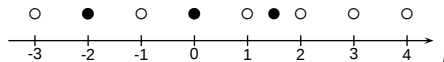
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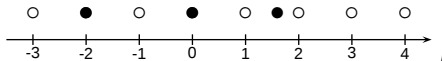
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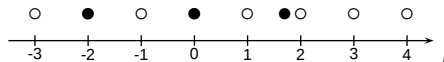
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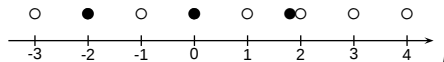
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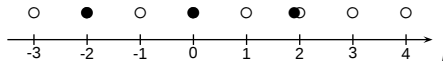
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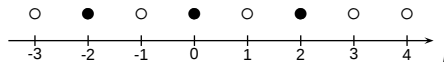
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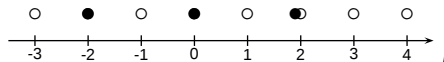
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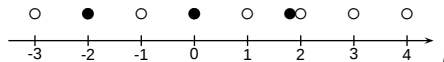
With rate  $q$ : jump to the left

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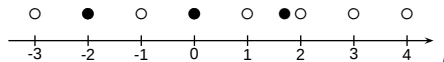
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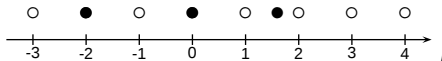
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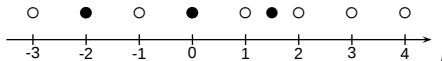
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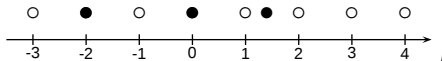
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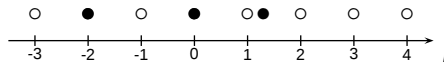
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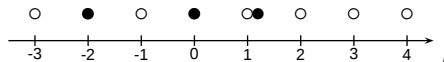
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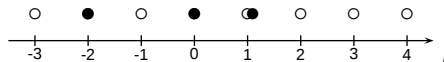
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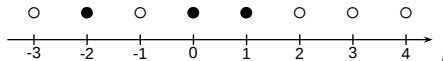
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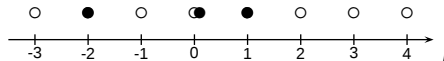
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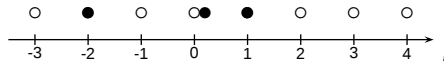
With rate  $c_r$ : coalescence to the right

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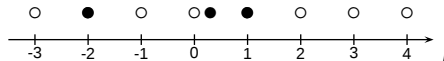
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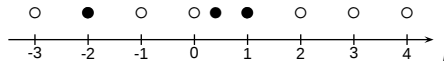
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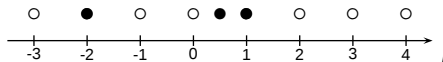
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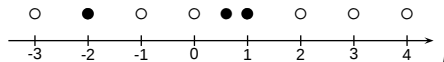
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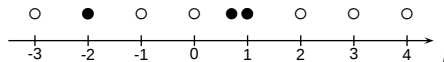
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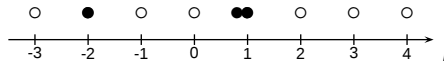
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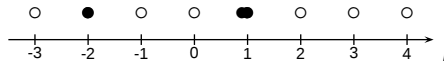
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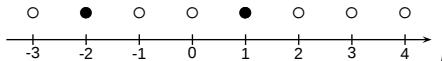
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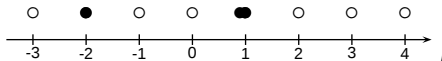
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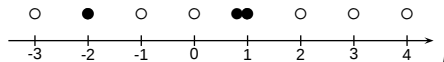
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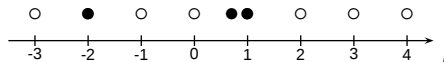
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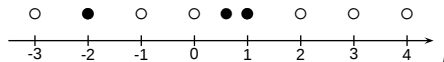
With rate  $b_l$ : branching to the left

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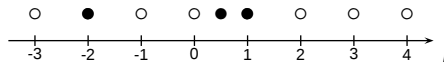
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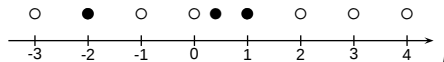
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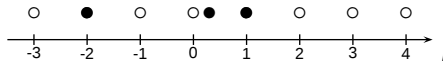
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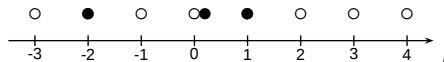
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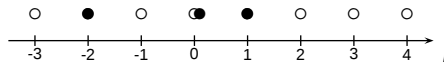
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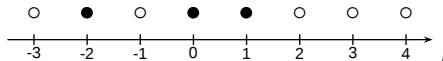
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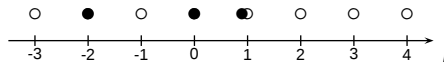
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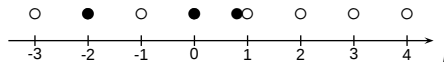
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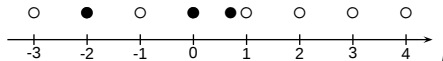
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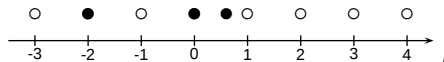
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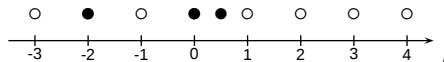
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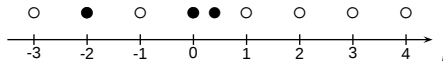
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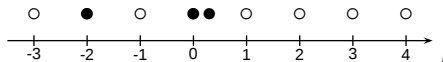
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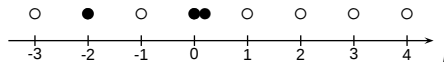
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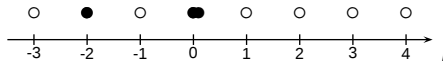
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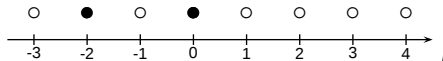
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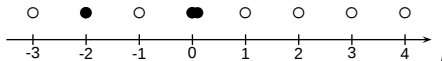
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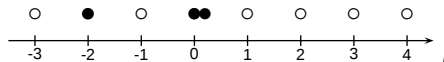
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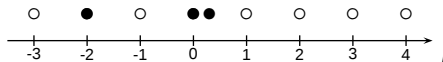
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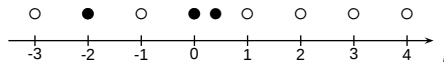
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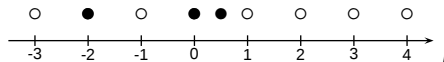
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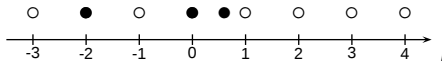
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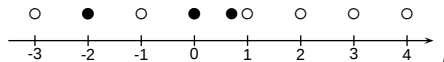
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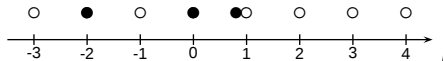
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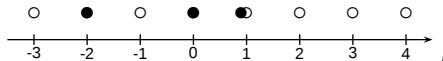
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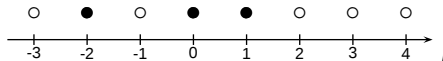
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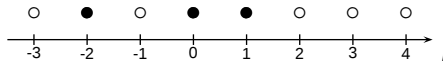
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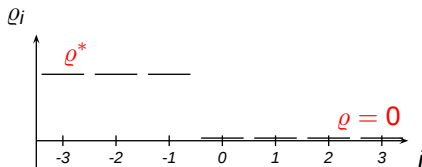
The  $\text{Bernoulli}(\varrho^*)$  distribution is stationary for

$$\varrho^* = \frac{b_l + b_r}{b_l + b_r + c_l + c_r}.$$

# Earlier results: as seen by the rightmost particle

## Theorem

*For the BCRW, the Bernoulli product distribution with densities*

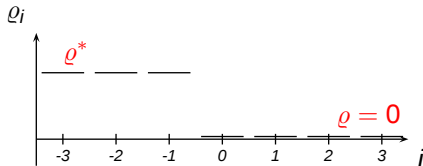


*is stationary for the process, as seen from the rightmost particle.*

## Earlier results: random walking shocks

### Theorem (Krebs, Jafarpour and Schütz '03)

*For the BCRW with the very same parameters, the Bernoulli product distribution  $\mu_0$  with densities*



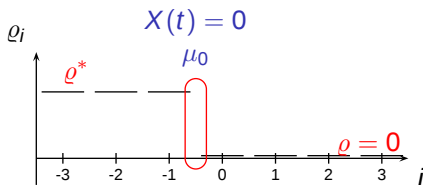
*evolves according to*

$$\begin{aligned} \frac{d}{dt} \mu_0 &= \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\mu_{-1} - \mu_0] \\ &+ p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\mu_1 - \mu_0]. \end{aligned}$$

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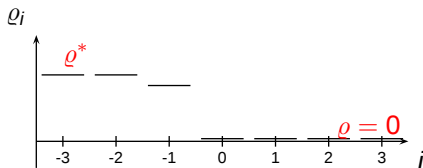


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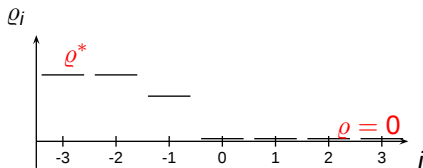


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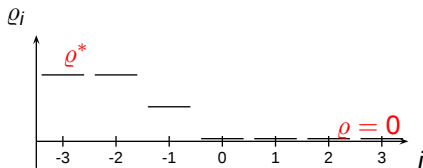


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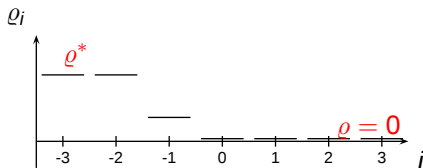


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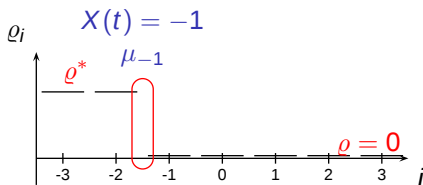


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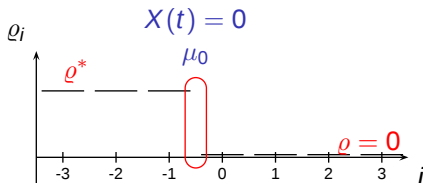


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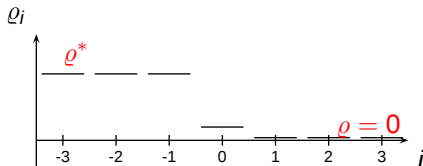


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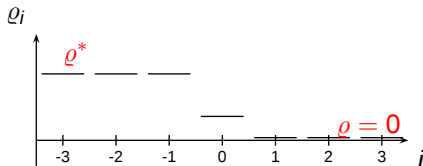


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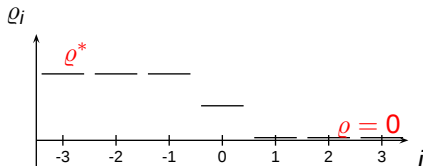


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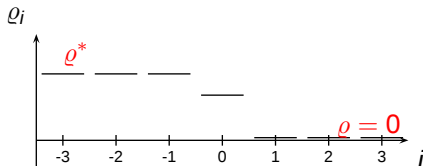


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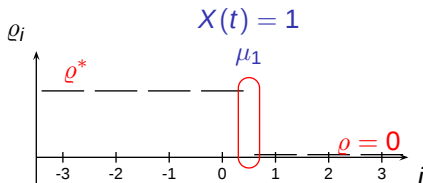


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# The question:

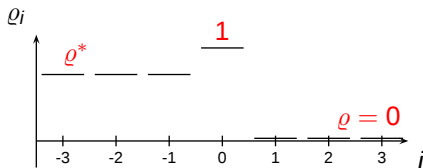
Is it the rightmost particle that performs the random walk?

## Here is the question:

For the BCRW, let  $\nu_0$  be the Bernoulli product distribution

$$\nu_0 = \left( \bigotimes_{i < 0} \mu^{\varrho^*} \right) \otimes (\delta) \otimes \left( \bigotimes_{i > 0} \mu^0 \right),$$

where  $\delta(0) = 1$ .



Does it satisfy

$$\begin{aligned} \frac{d}{dt} \nu_0 &= \frac{c_l \cdot (b_l + b_r) + q \cdot (c_l + c_r)}{b_l + b_r + c_l + c_r} \cdot [\nu_{-1} - \nu_0] \\ &\quad + p \cdot \frac{b_l + b_r + c_l + c_r}{c_l + c_r} \cdot [\nu_1 - \nu_0]? \end{aligned}$$

## The answer

- ▶ ... is, of course, yes again. [Gy. Farkas, P. Kovács, A. Rákos, B. '10]

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Thank you.