$t^{1/3}$-scaling of current fluctuations: coupling results and problems

Joint with
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Márton Balázs

MTA-BME Stochastics Research Group

Happy birthday Jóska!

Budapest, March 2008

The models
- ASEP
- Zero range
- Bricklayers

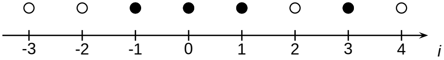
Current variance

Hydrodynamics
- Characteristics

Tool: the second class particle
- Single
- Many second class particles

Results
- Current fluctuations
- Microscopic convexity
- Coupling results
- Nonconvex, nonconcave

Asymmetric simple exclusion



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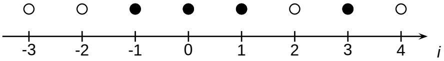

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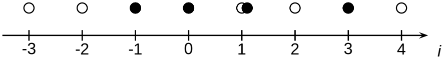

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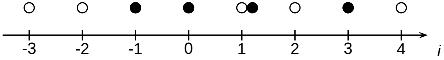

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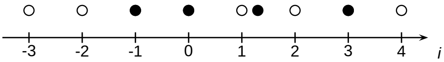

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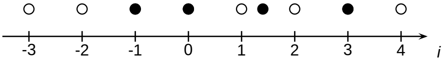

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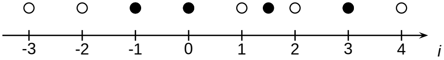

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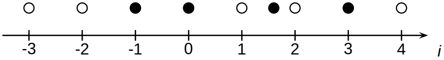

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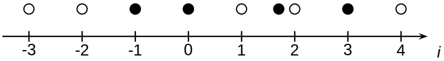

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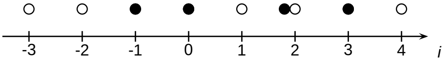

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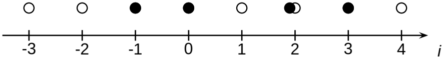

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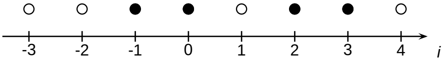

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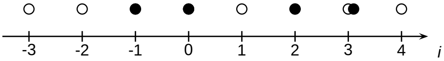

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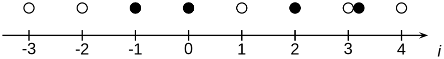

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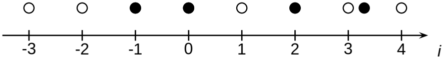

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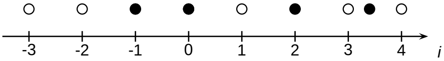

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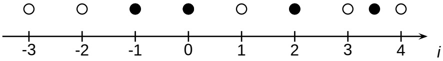

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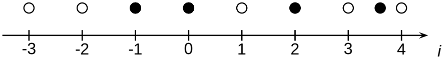

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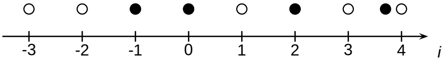

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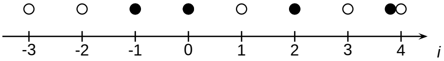

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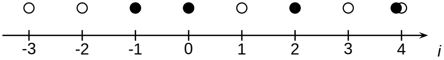

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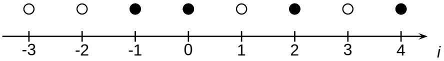

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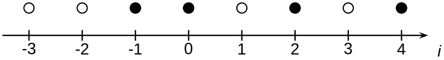

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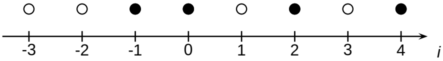

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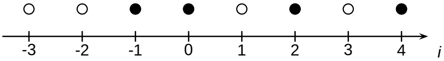

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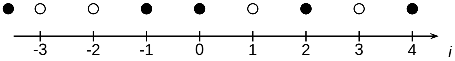

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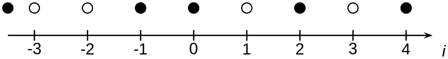

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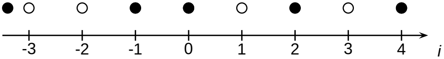

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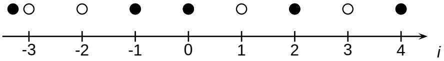

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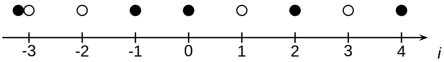

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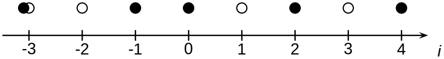

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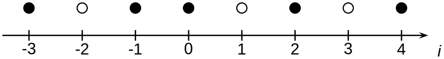

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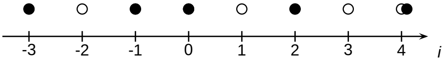

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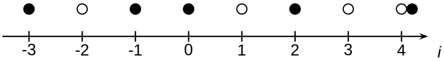

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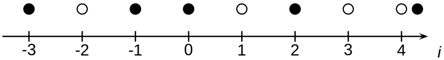

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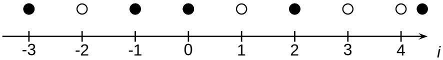

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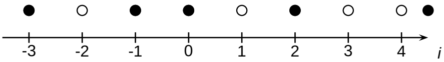

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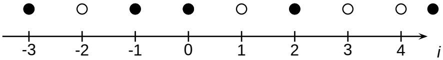

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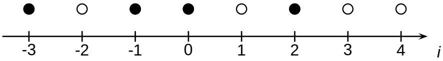

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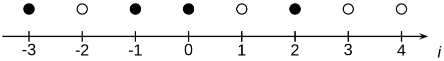

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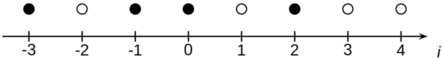

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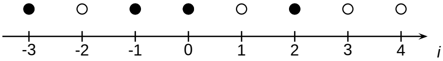

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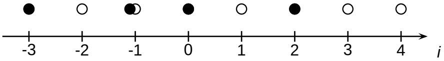

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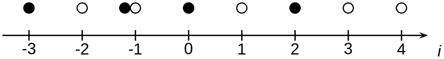

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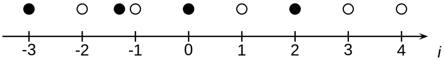


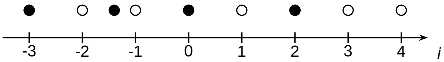
Bernoulli(ϱ) distribution; $\omega_i = 0$ or 1.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



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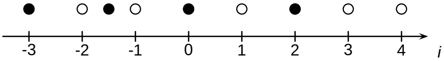

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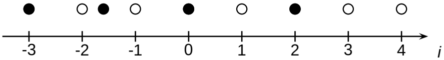

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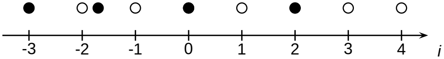

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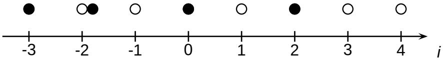

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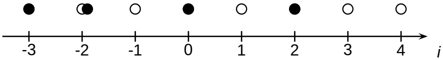

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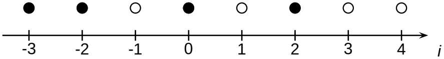

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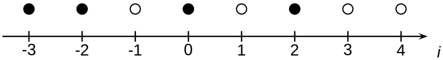

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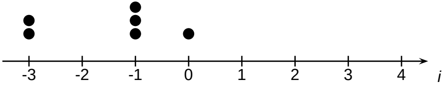
The Bernoulli(ϱ) distribution is time-stationary for any $(0 \leq \varrho \leq 1)$. Any translation-invariant stationary distribution is a mixture of Bernoullis.

The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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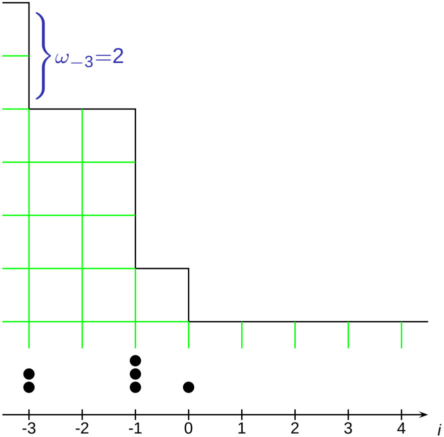

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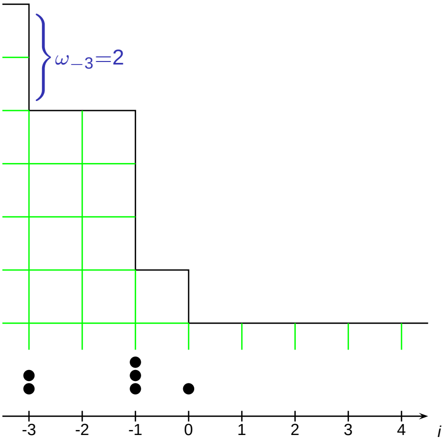

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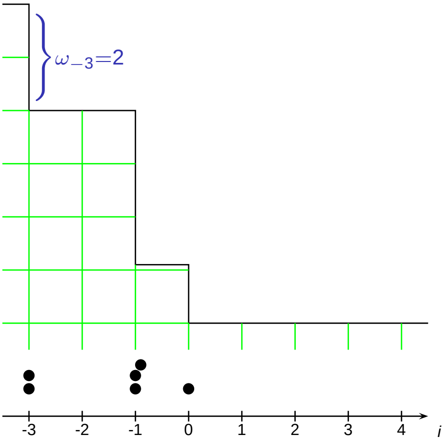

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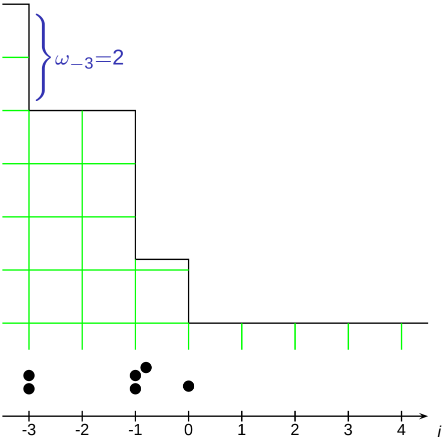

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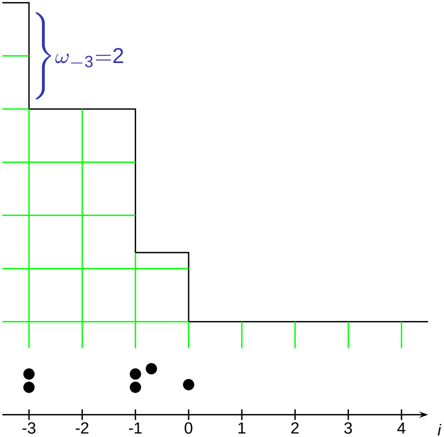

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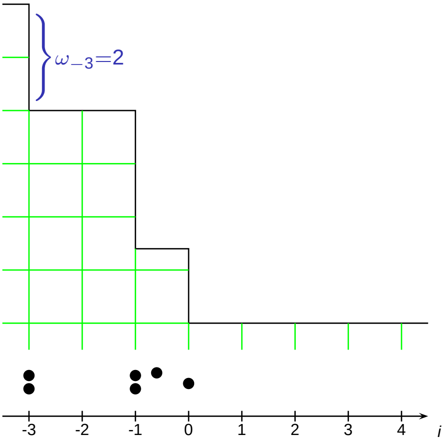

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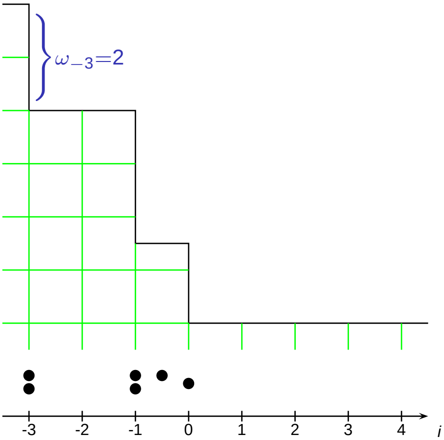

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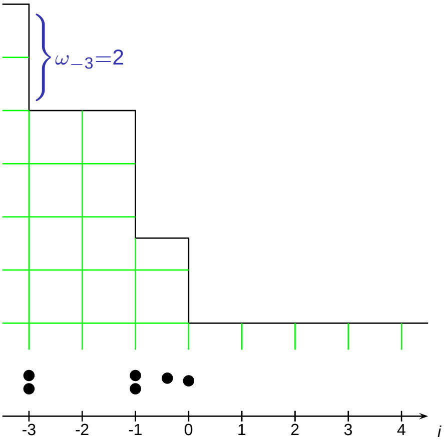

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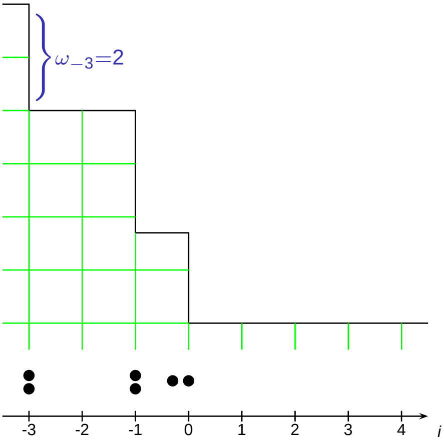

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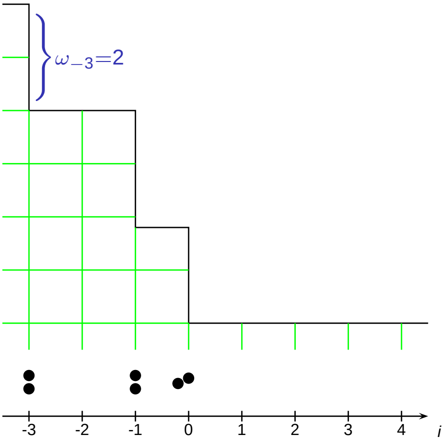

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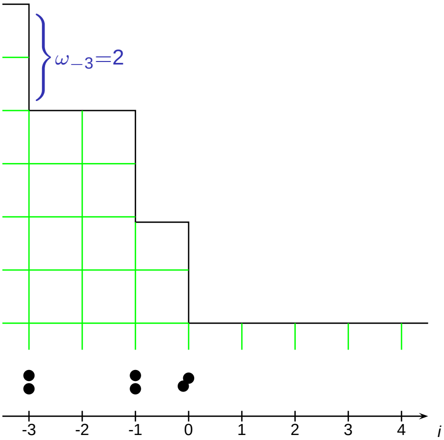

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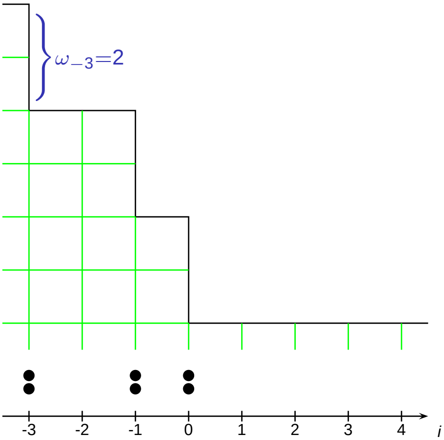

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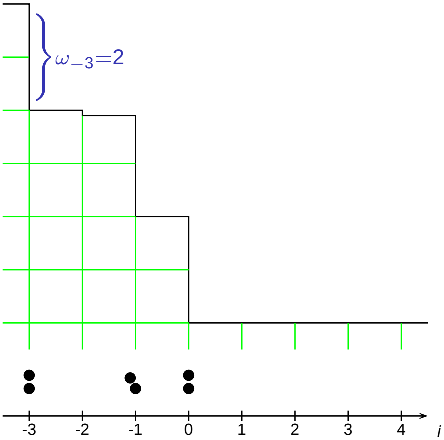

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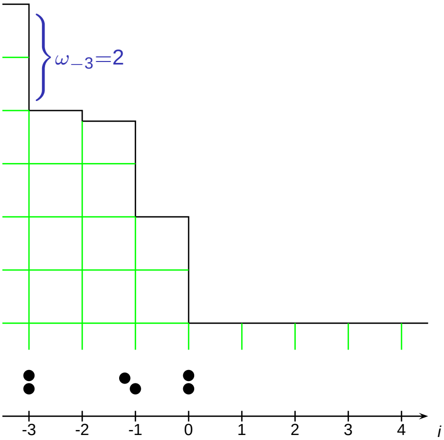

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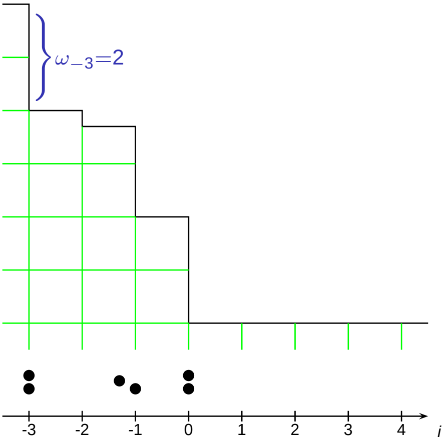

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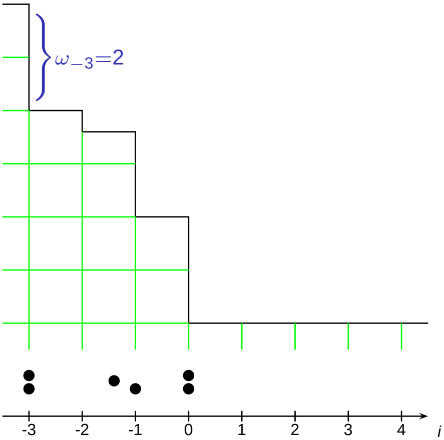

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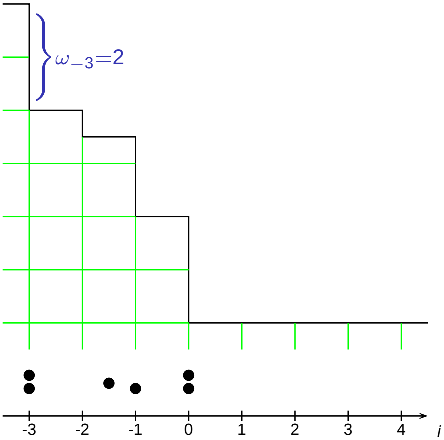

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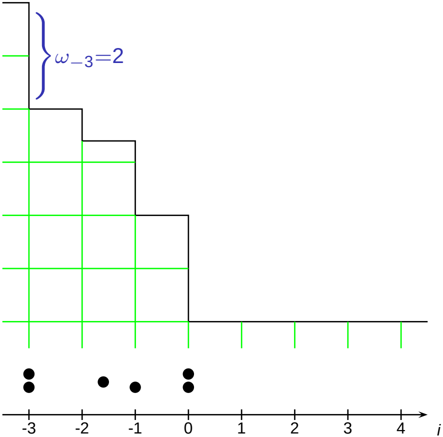

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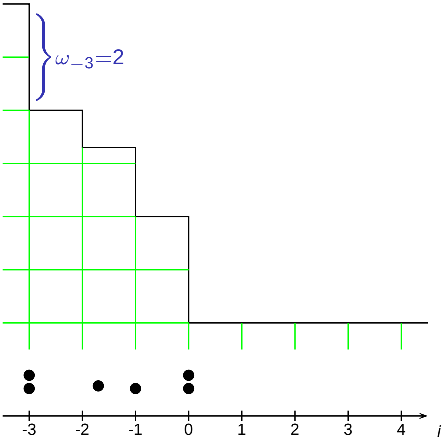

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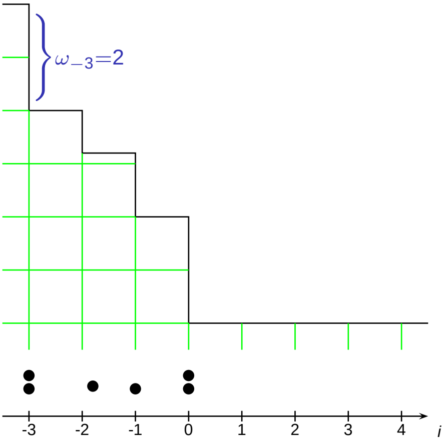

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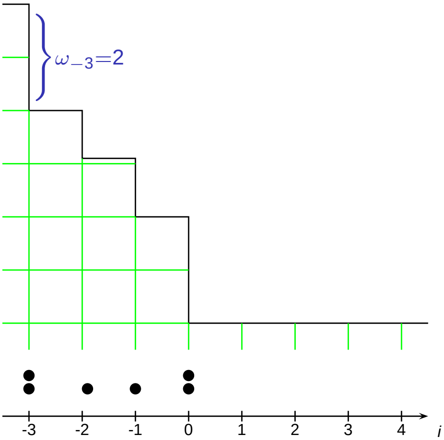

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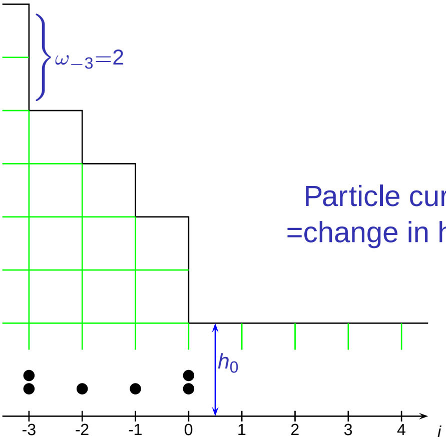

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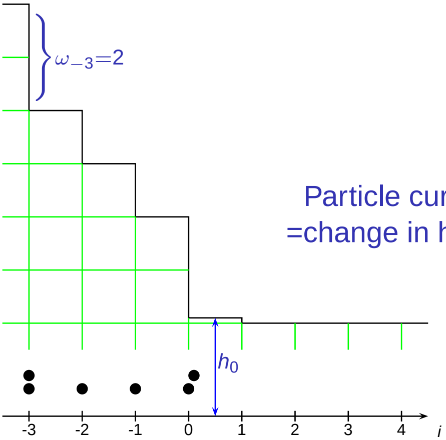

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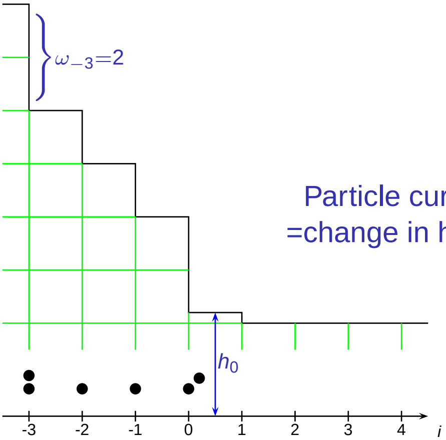

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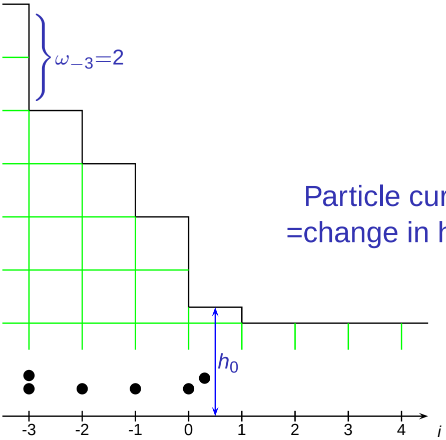

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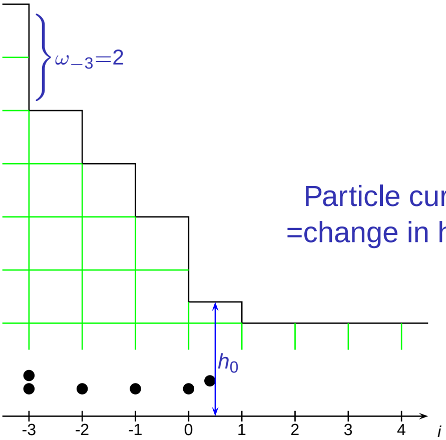

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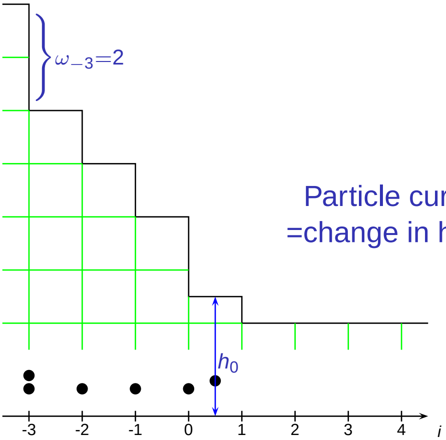

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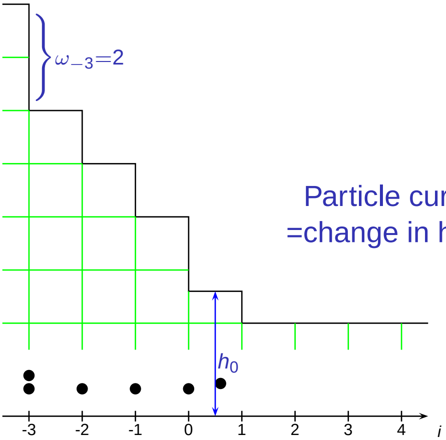

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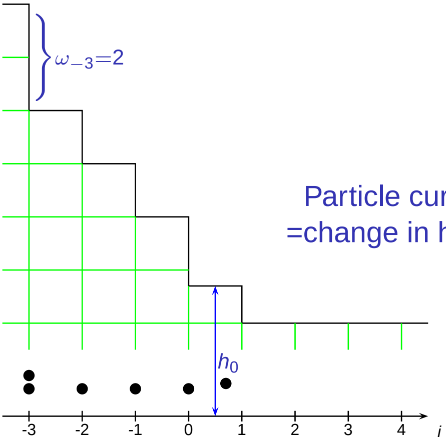

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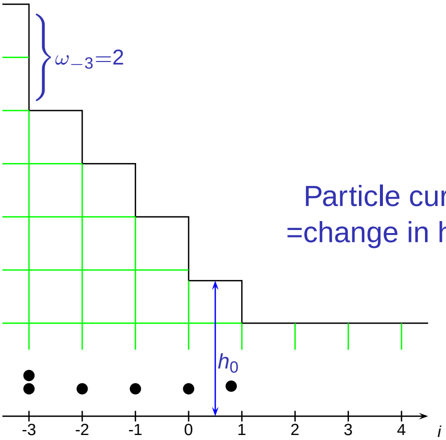

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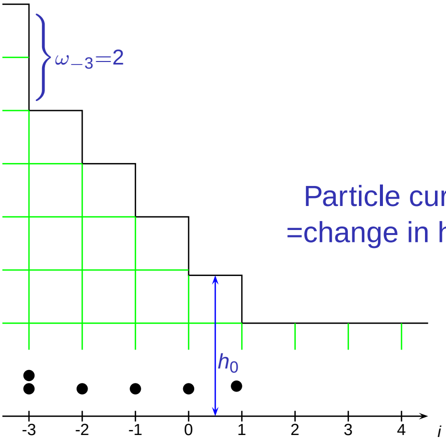

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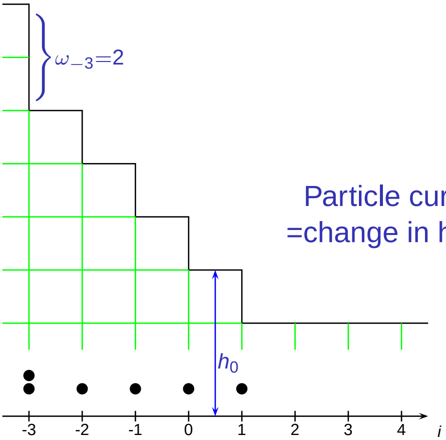

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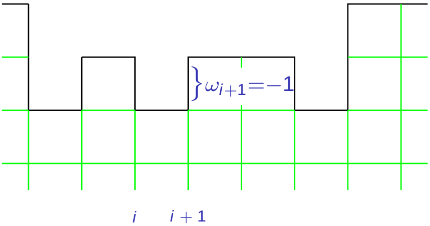
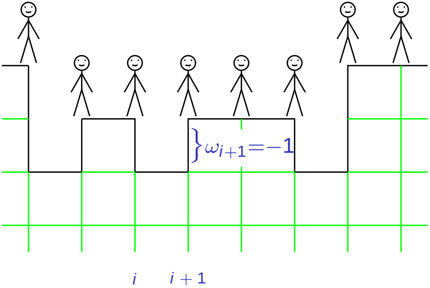
The asymmetric bricklayers process



$$\begin{matrix}\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix}\\ \| \\ \begin{pmatrix}1\\ -1\end{pmatrix}\end{matrix}$$

Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

The asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} \\ \| \\ \begin{pmatrix}1\\ -1\end{pmatrix}$$

Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

a brick is added with rate $p \cdot [r(\omega_i) + r(-\omega_{i+1})]$
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$(r(\omega) \cdot r(1-\omega) = 1; \quad r$ non-decreasing; $\quad q = 1 - p < p)$.

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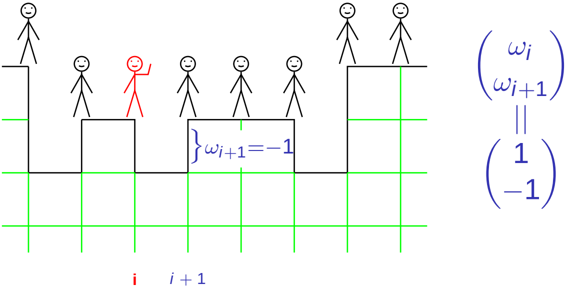

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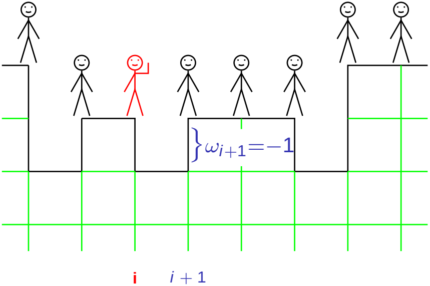

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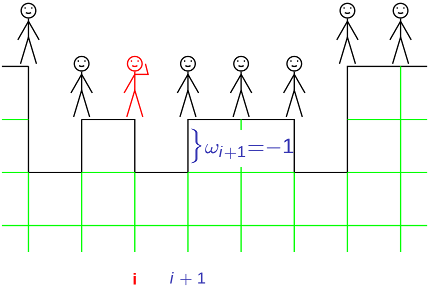

$$\begin{array}{c}\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix}\\ \Vert\\ \begin{pmatrix}1\\ -1\end{pmatrix}\end{array}$$

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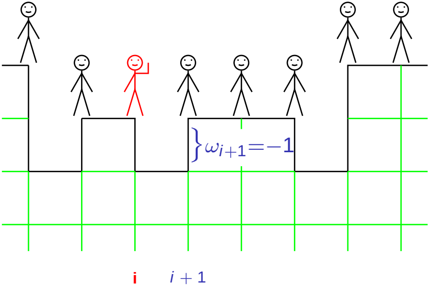

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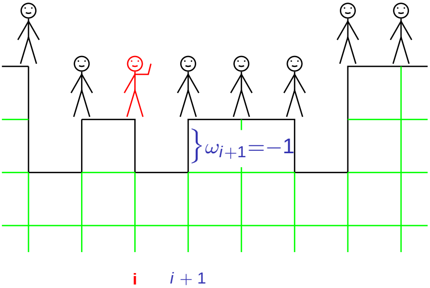

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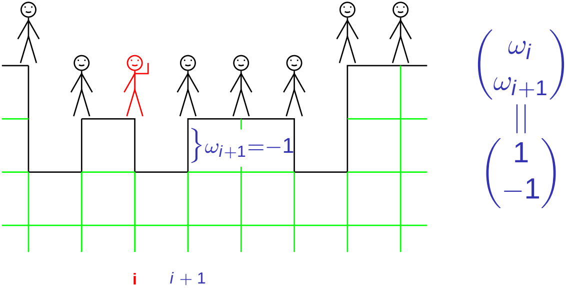

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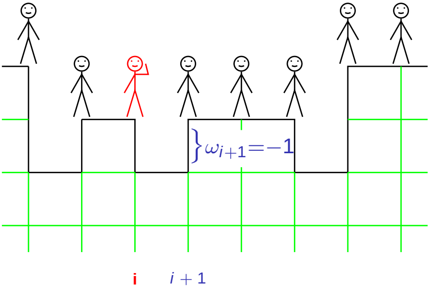

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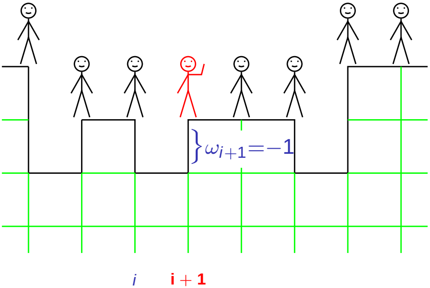

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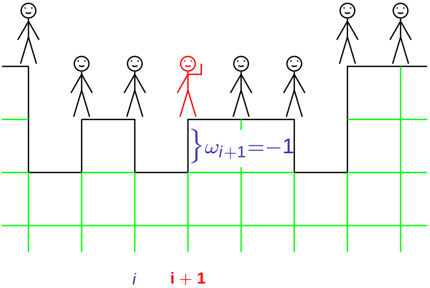

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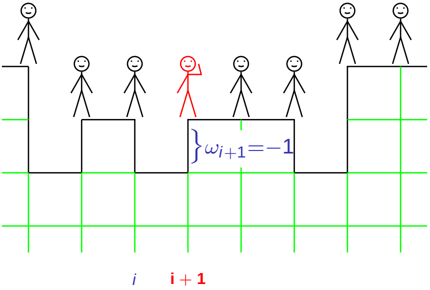

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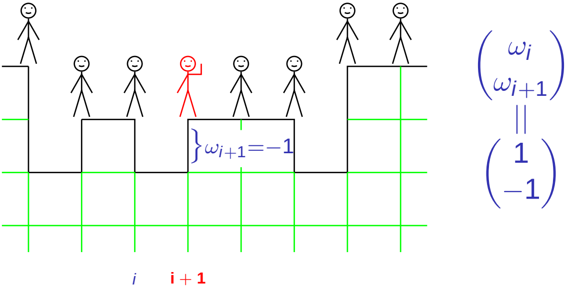

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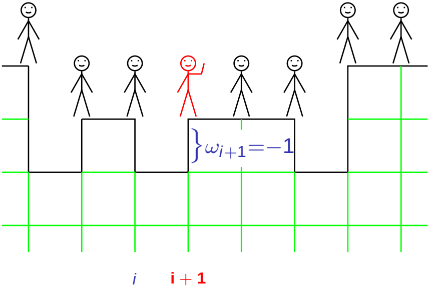

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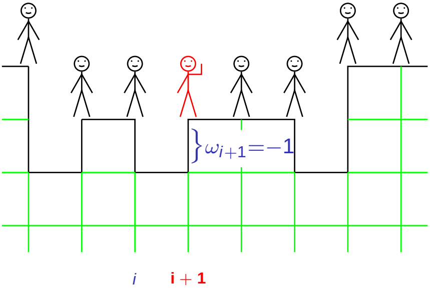

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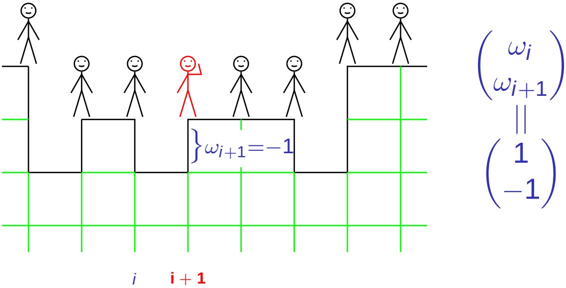

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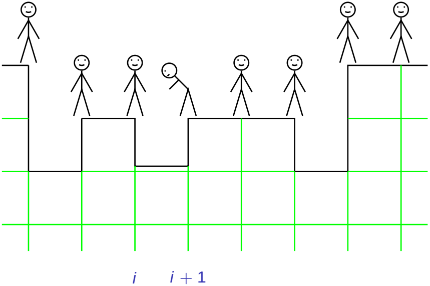

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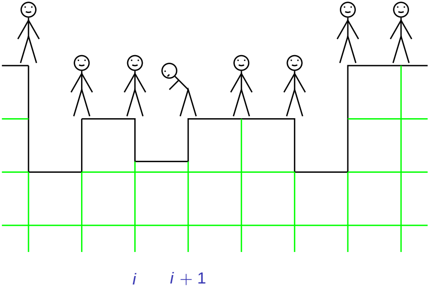

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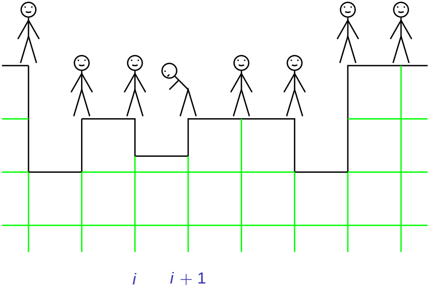

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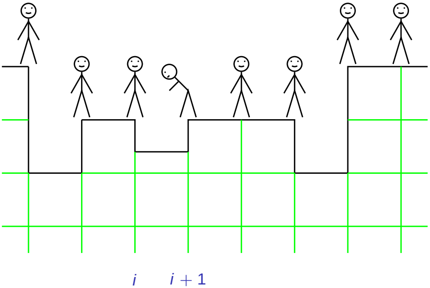

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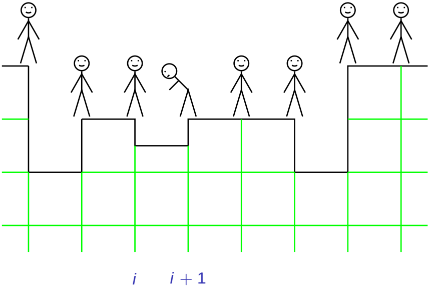

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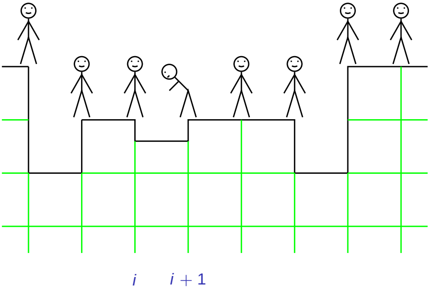

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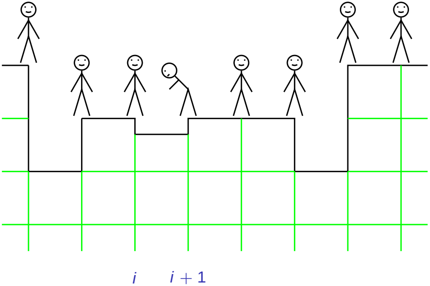

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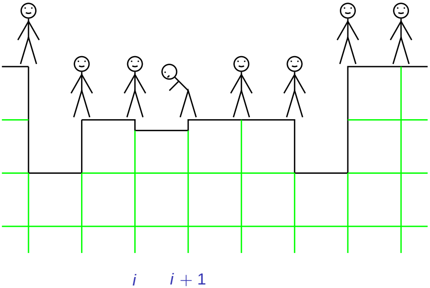

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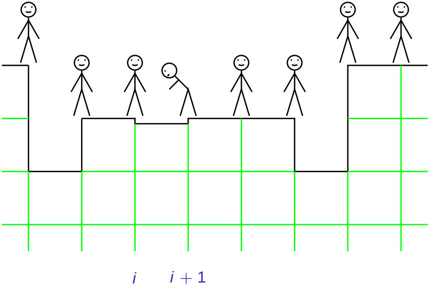

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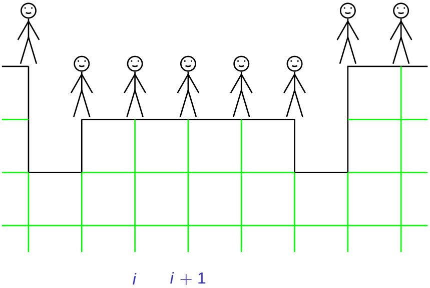

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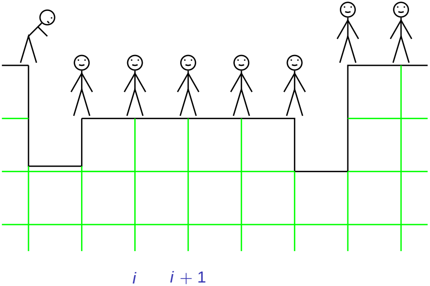

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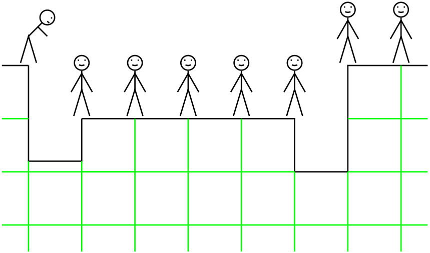

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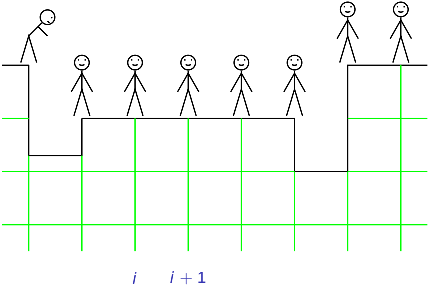

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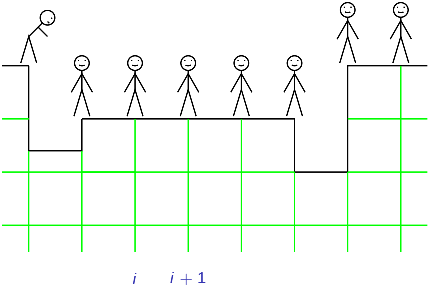

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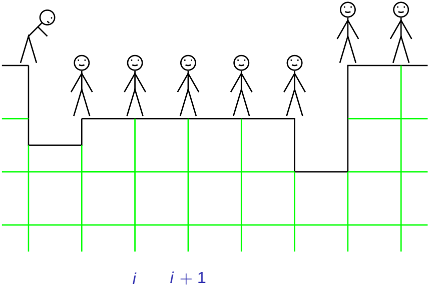

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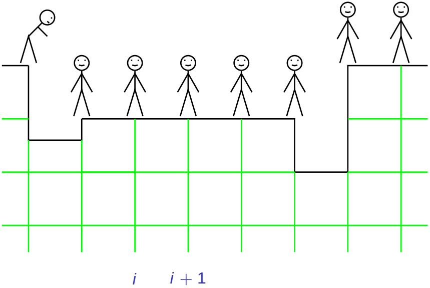

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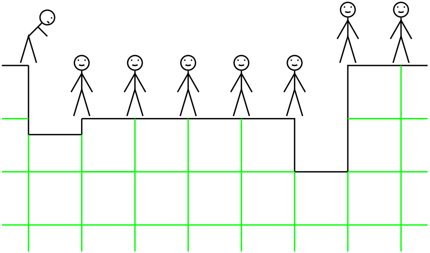

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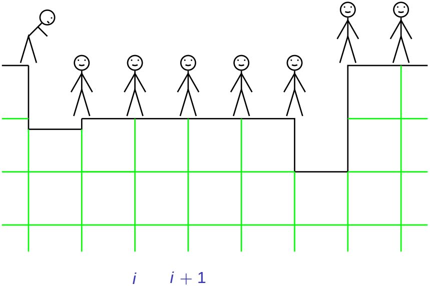

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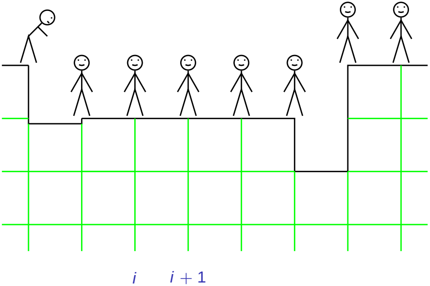

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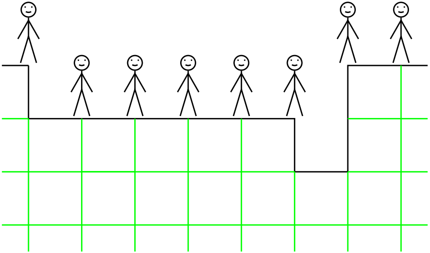

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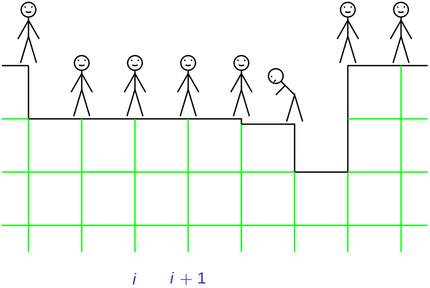

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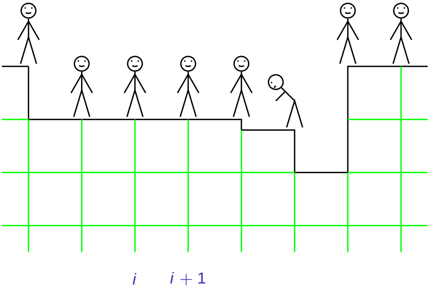

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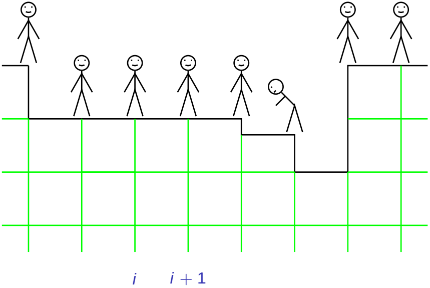

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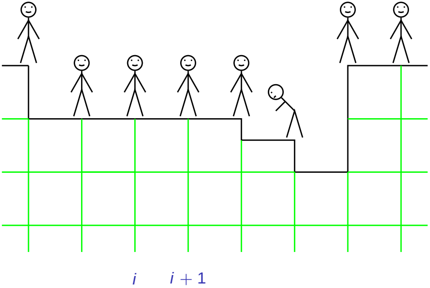

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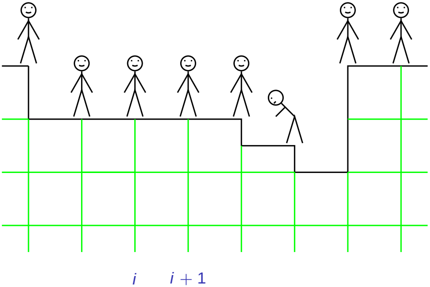

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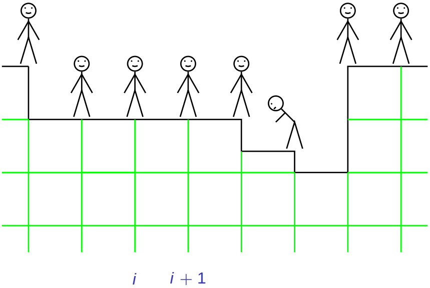

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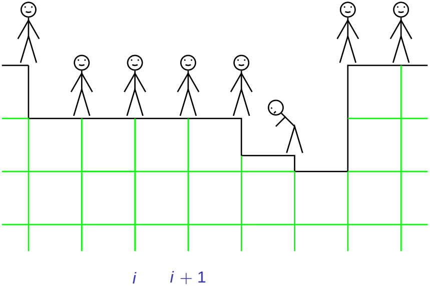

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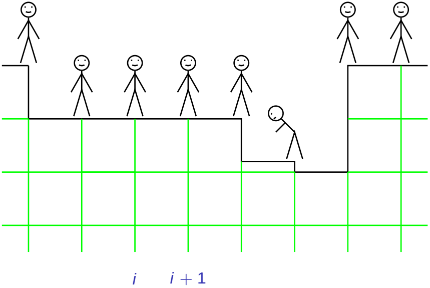

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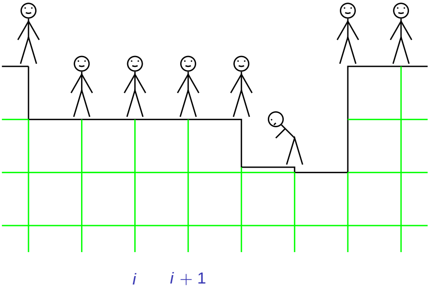

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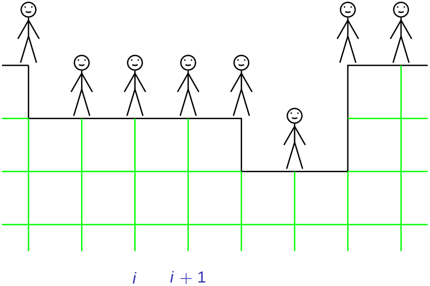

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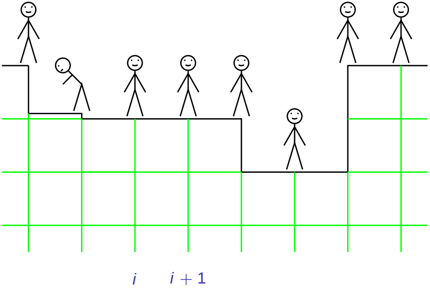

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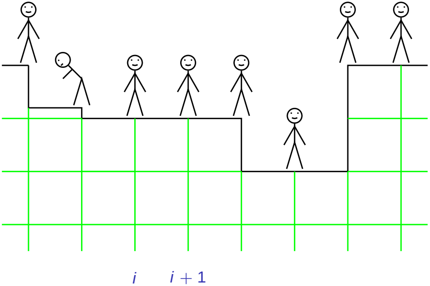

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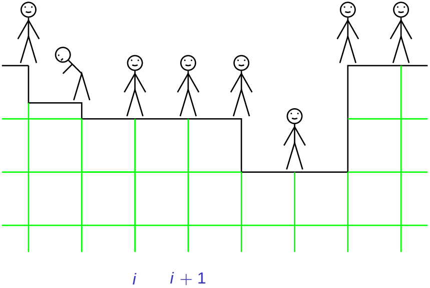

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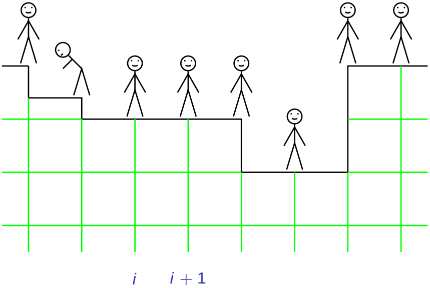

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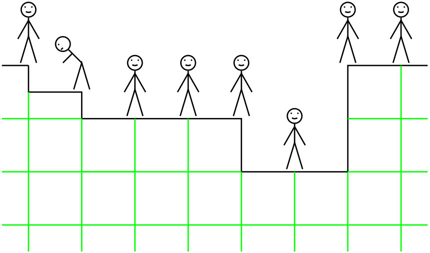

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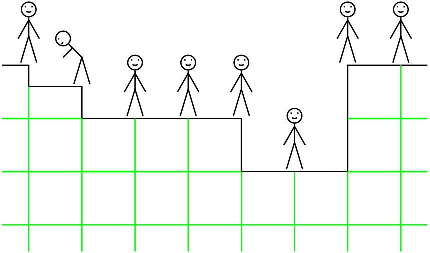

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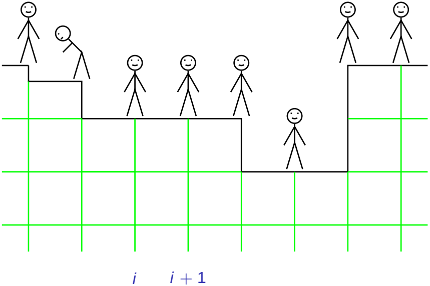

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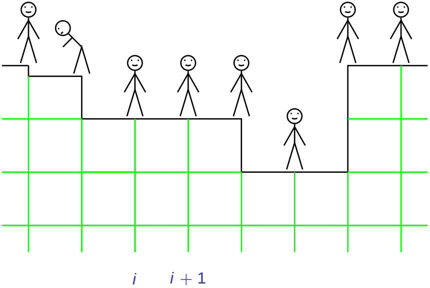

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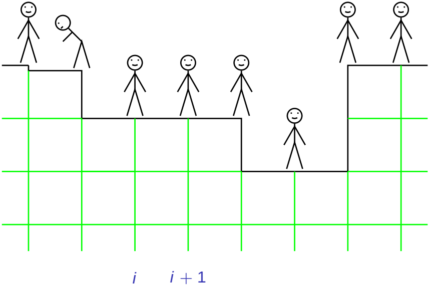

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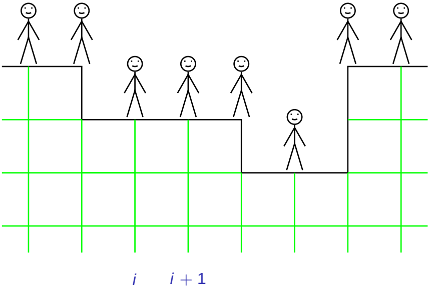

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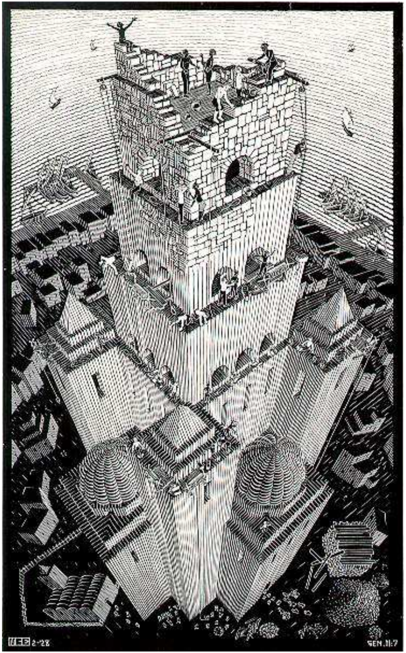
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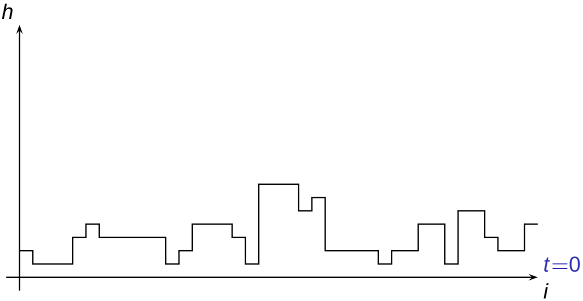

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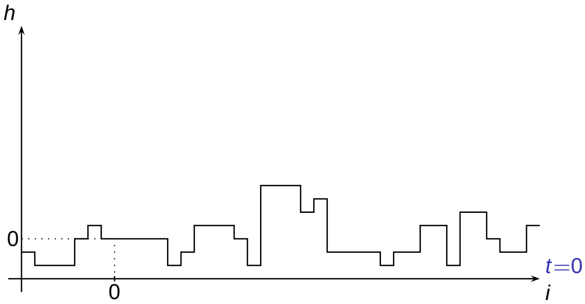

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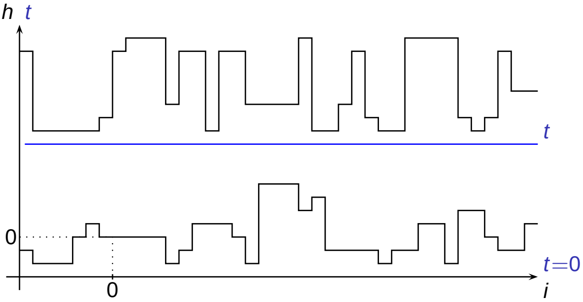

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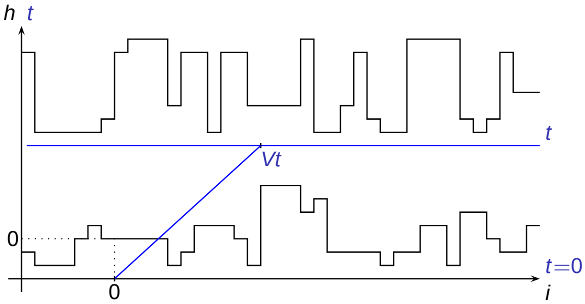

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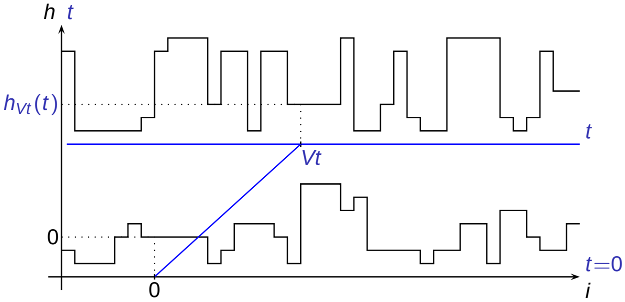

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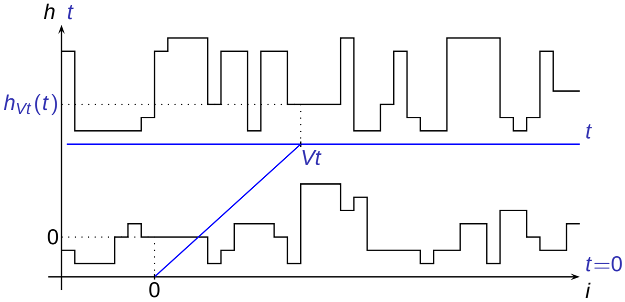

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Question: What is the time-order of **Var**($h_{Vt}(t)$)?

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The *density* $\varrho := \mathbf{E}(\omega)$ and the *hydrodynamic flux* $H := \mathbf{E}$[growth rate] both depend on a parameter of the stationary distribution.

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- The *characteristics* is a path $X(T)$ where $\varrho(T, X(T))$ is constant.

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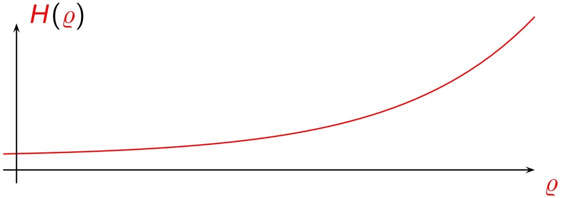
$$R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}.$$

This would be the speed of a shock of densities ϱ and λ.

Characteristics (very briefly)

Convex flux (some cases of AZRP, ABLP):



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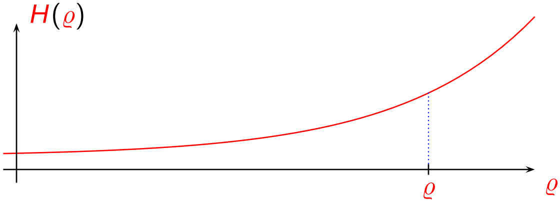

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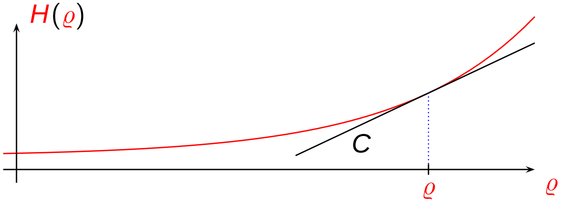
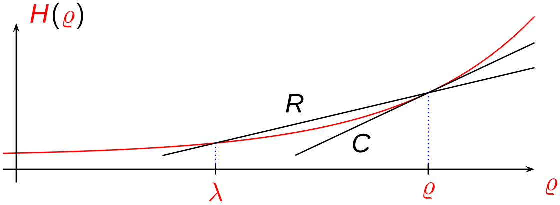


$$C = H'(\varrho)$$

Characteristics (very briefly)

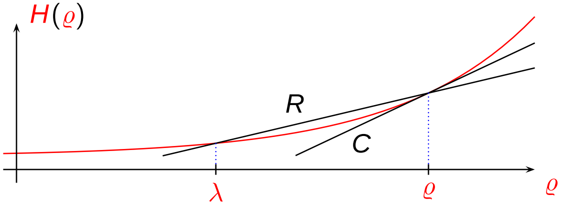
Convex flux (some cases of AZRP, ABLP):



$$C = H'(\varrho) \qquad R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Characteristics (very briefly)

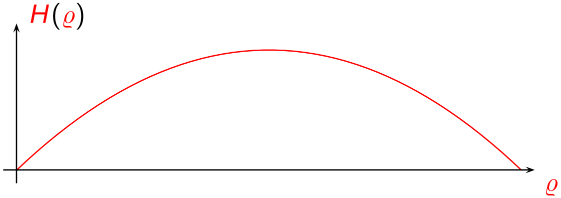
Convex flux (some cases of AZRP, ABLP):



$$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

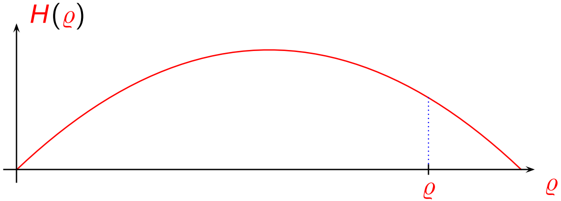
Characteristics (very briefly)

Concave flux (ASEP):



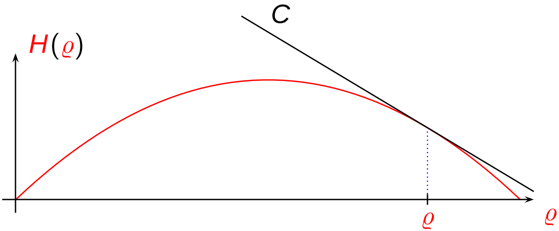
Characteristics (very briefly)

Concave flux (ASEP):



Characteristics (very briefly)

Concave flux (ASEP):



$$C = H'(\varrho)$$

Characteristics (very briefly)

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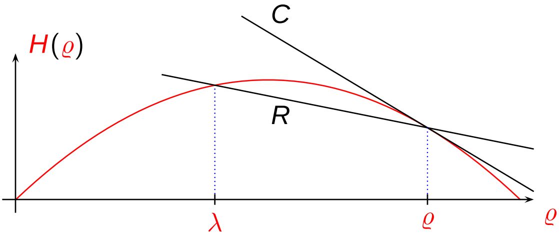

$$C = H'(\varrho) \qquad R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Characteristics (very briefly)

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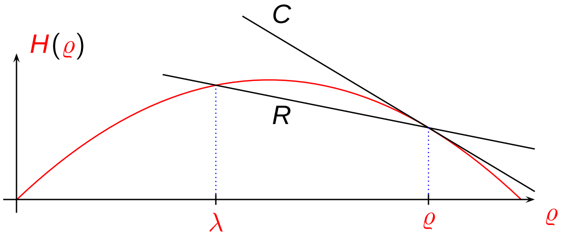
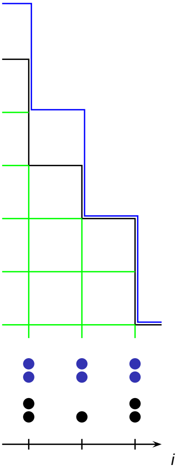


$$C = H'(\varrho) < R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Tool: the second class particle

States ω and ω only differ at one site.



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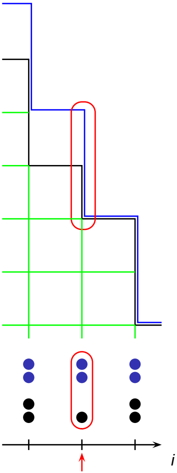

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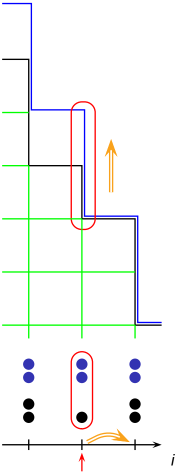
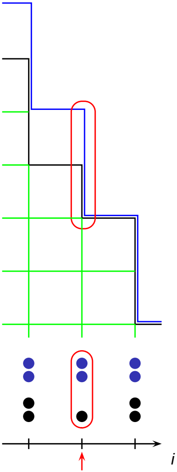


Growth on the right:

rate≤rate

Tool: the second class particle

States ω and ω only differ at one site.



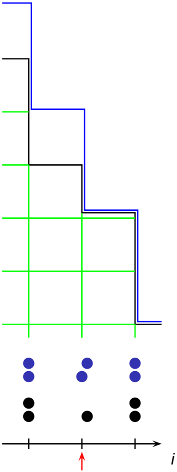
Growth on the right:

rate$\leq$rate

with rate:

Tool: the second class particle

States ω and ω only differ at one site.



Growth on the right:

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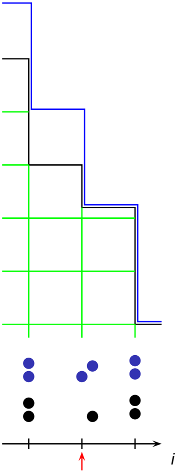

Growth on the right:

rate≤rate

with rate:

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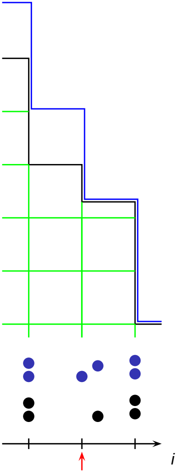

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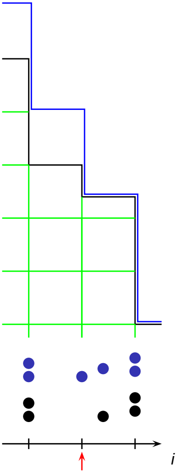

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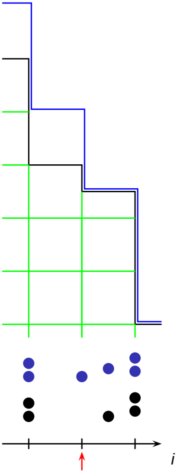

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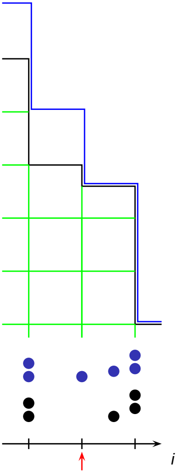

Growth on the right:

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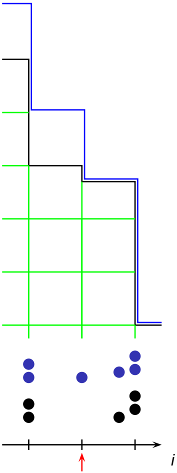

Growth on the right:

rate$\leq$rate

with rate:

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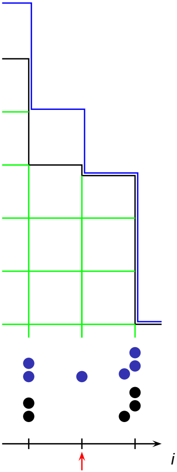

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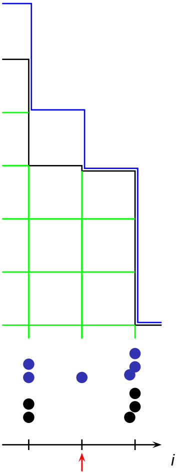

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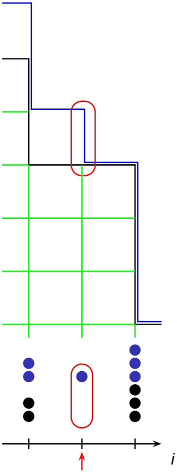


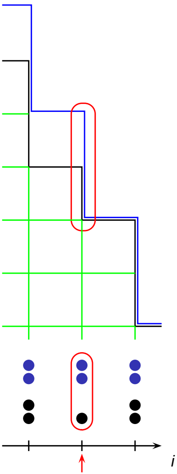
Growth on the right:

rate≤rate

with rate:

Tool: the second class particle

States ω and ω only differ at one site.



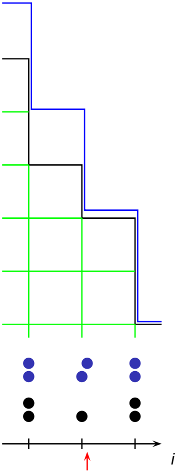
Growth on the right:

rate≤rate

with rate-rate:

Tool: the second class particle

States ω and ω only differ at one site.



Growth on the right:

rate≤rate

with rate-rate:

Tool: the second class particle

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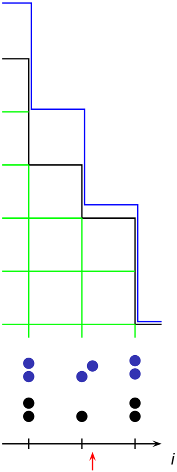

Growth on the right:

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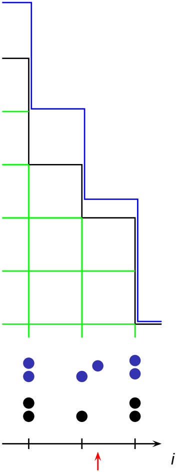

Growth on the right:

rate≤rate

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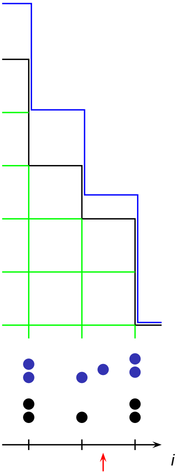

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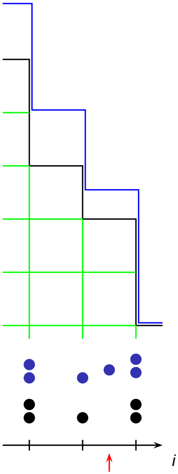

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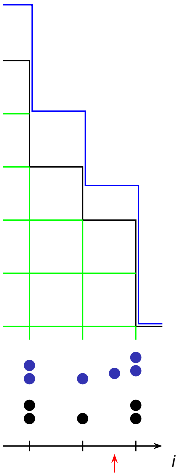

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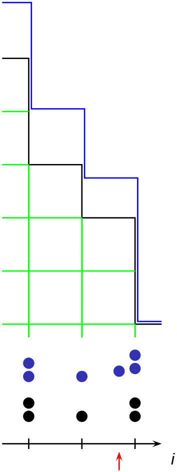

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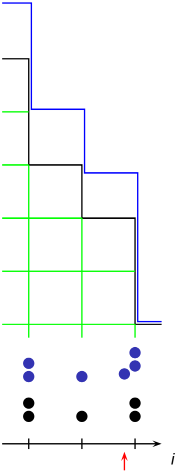

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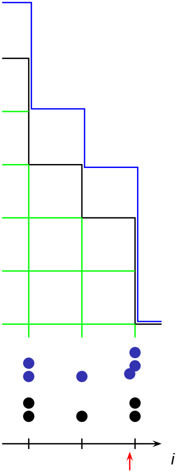

Growth on the right:

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with rate-rate:

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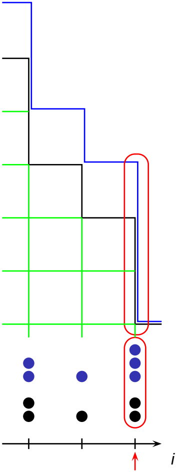


Growth on the right:

rate≤rate

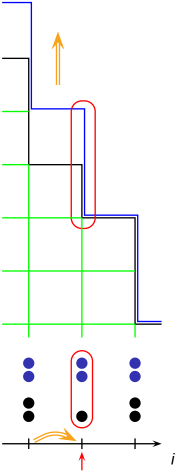
with rate-rate:

Tool: the second class particle

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate



Tool: the second class particle

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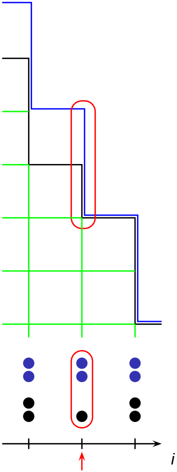
Growth on the left:

rate$\geq$rate

with rate:



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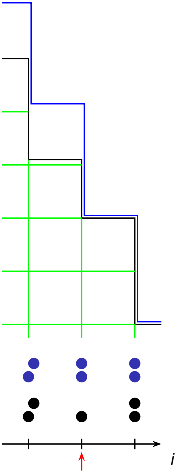

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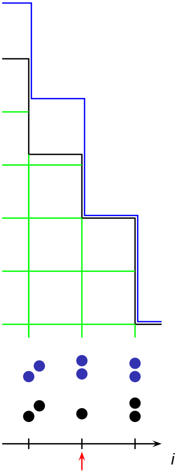

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Tool: the second class particle

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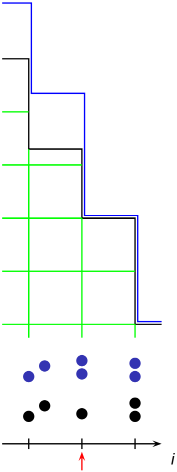

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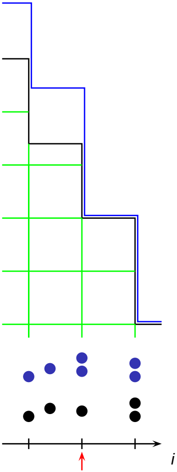

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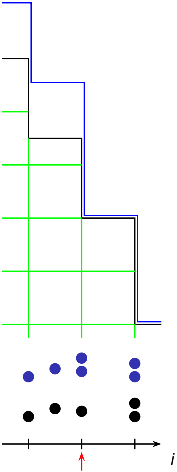

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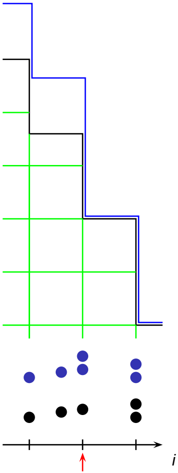

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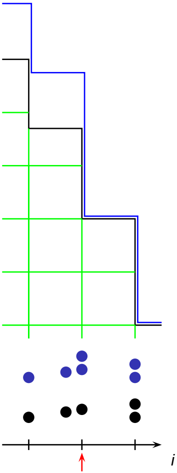

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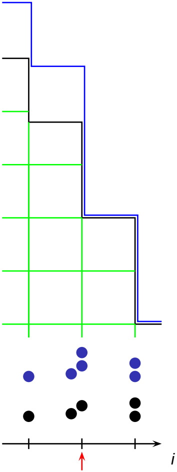

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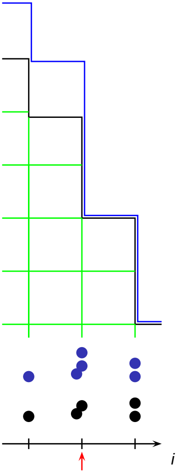

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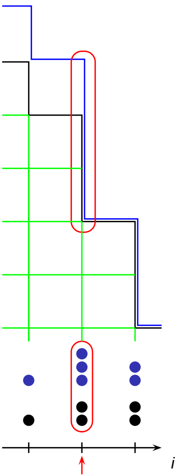

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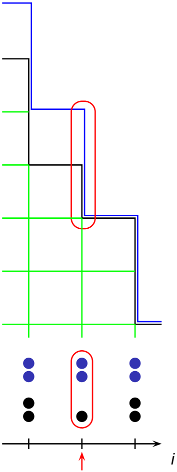

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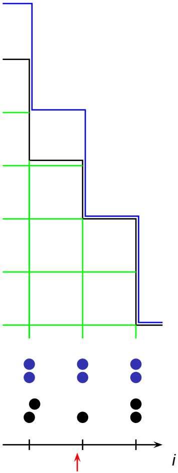

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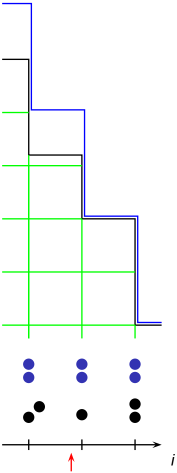

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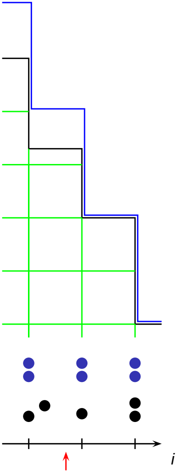

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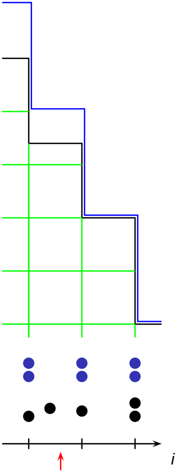

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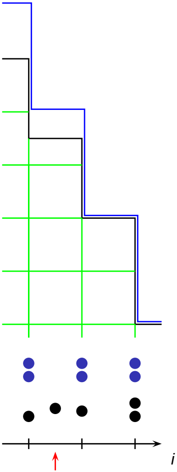

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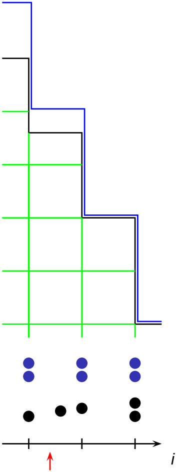

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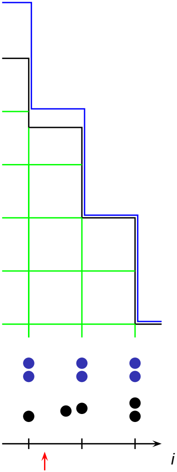

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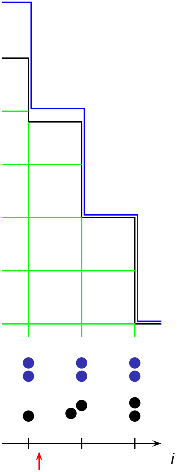
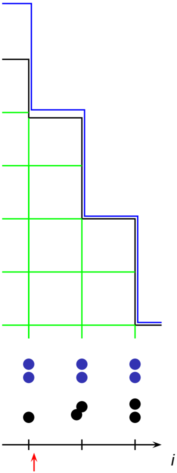


Tool: the second class particle

States ω and ω only differ at one site.



Growth on the left:

rate$\geq$rate

with rate-rate:

Tool: the second class particle

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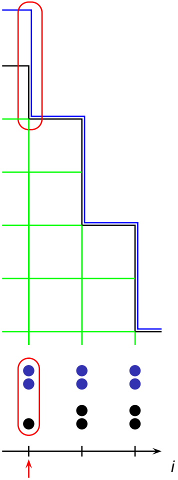

Growth on the left:

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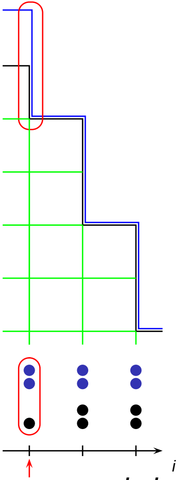

A single discrepancy↑, the *second class particle*, is conserved.

Its position at time t is $Q(t)$.

Tool: the second class particle

Theorem (B. - Seppäläinen; also ideas from Bálint, Herbert, Michael Prähofer)

Started from (almost) equilibrium,

$$\mathbf{E}(Q(t)) = C \cdot t$$

in a whole bunch of processes.

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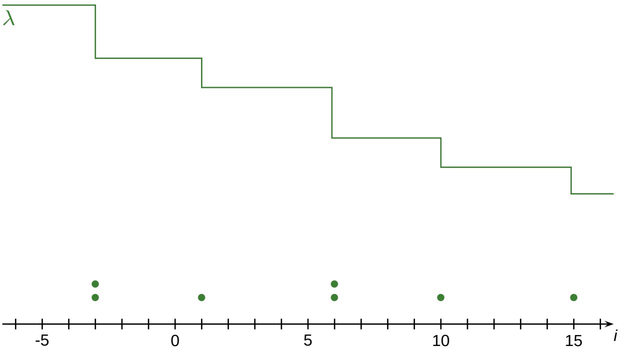
The second class particle follows the characteristics, people have known this for a long time.

Many second class particles



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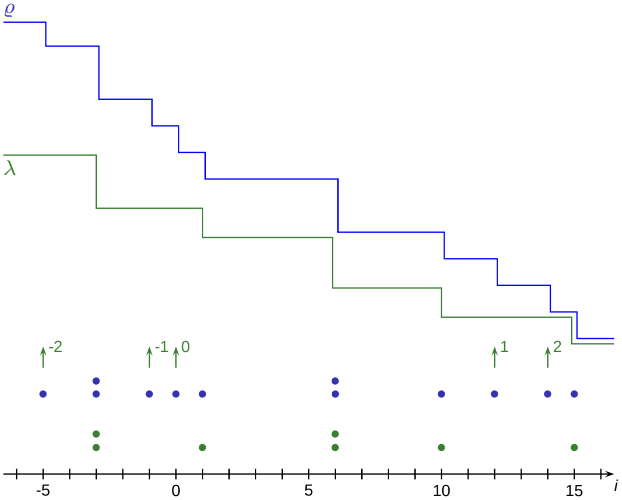

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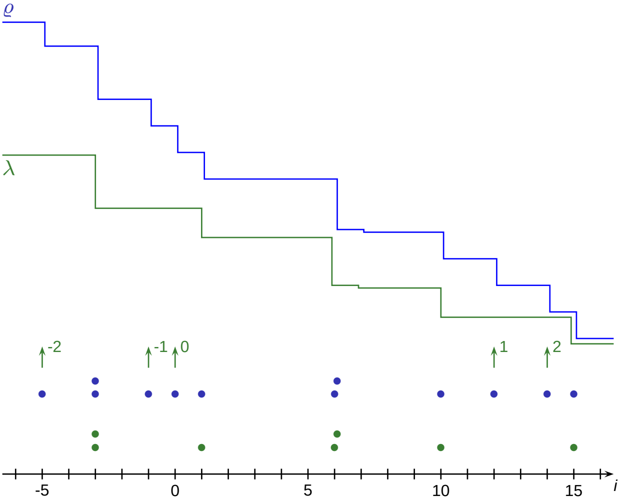

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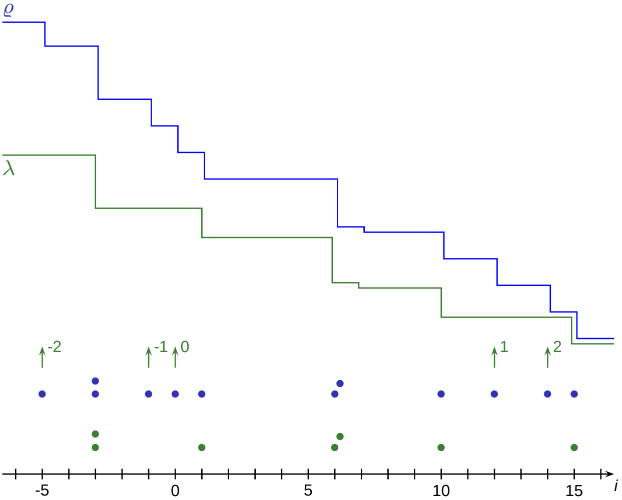

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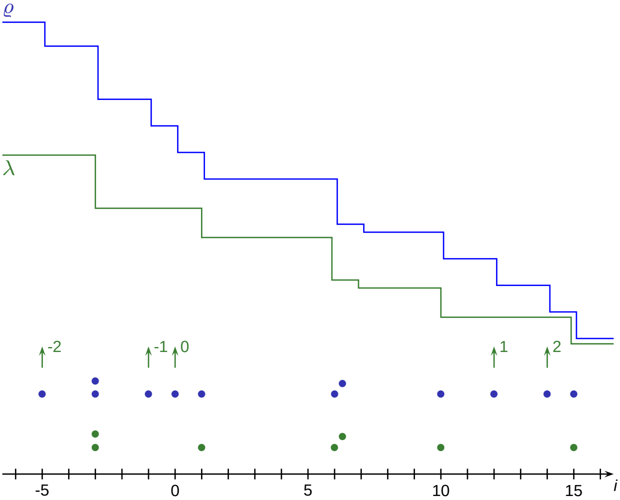

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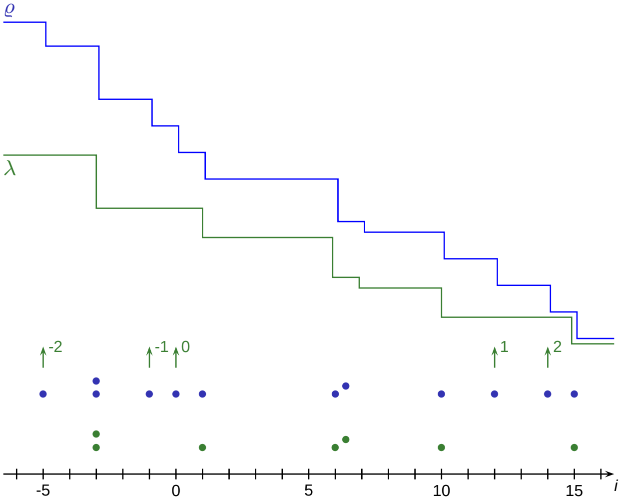

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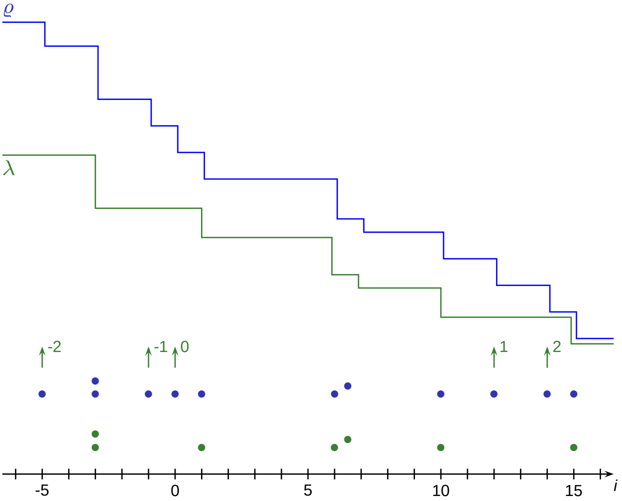

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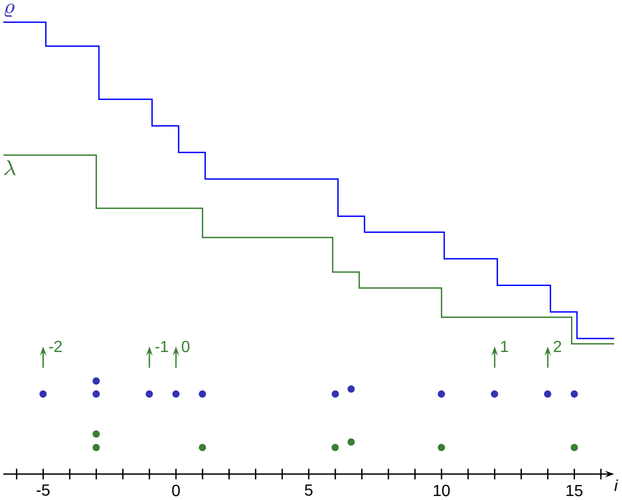

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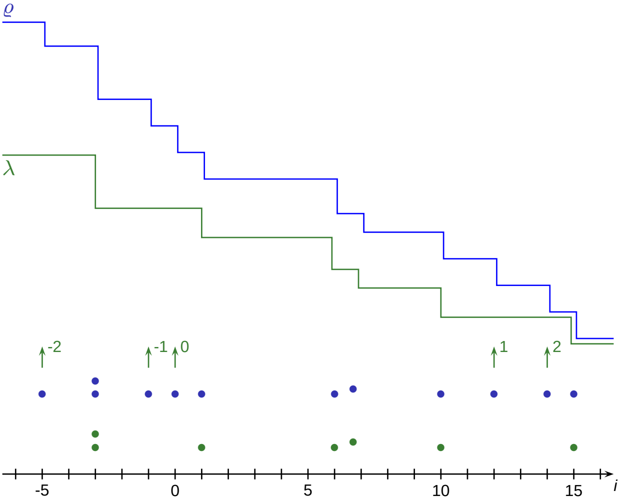

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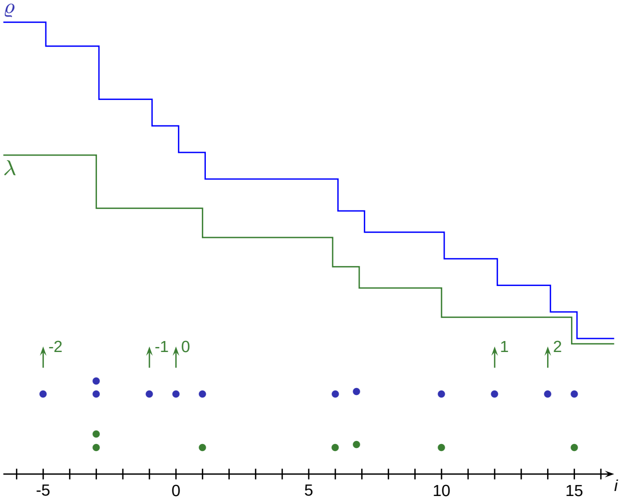

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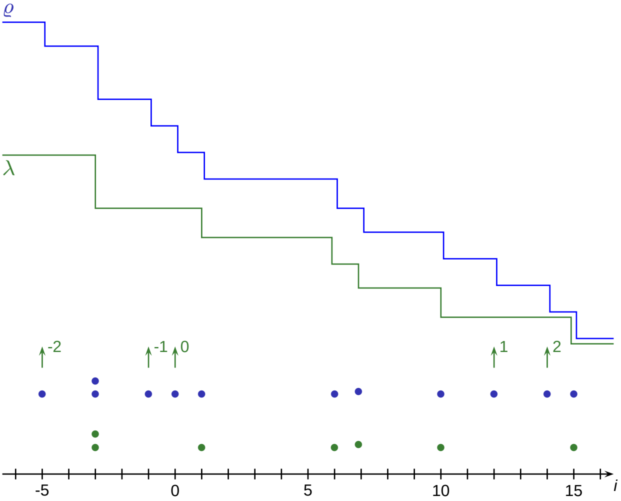

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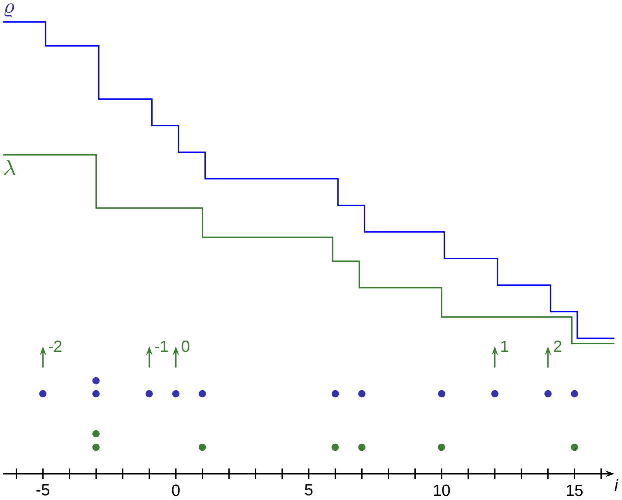

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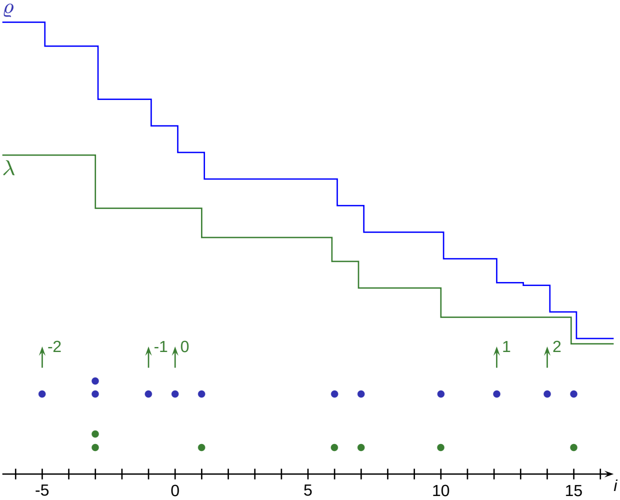

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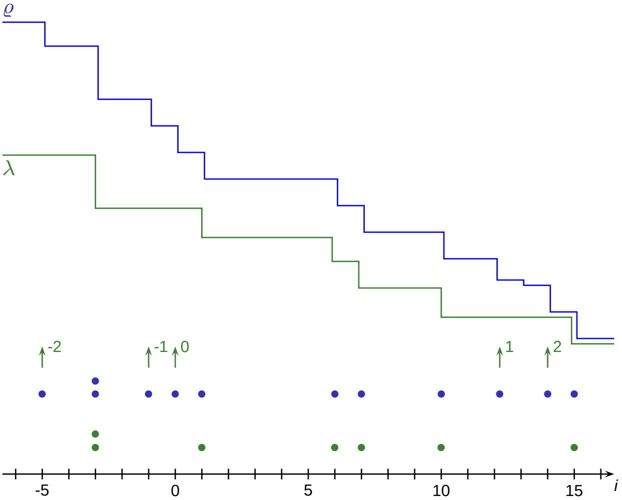

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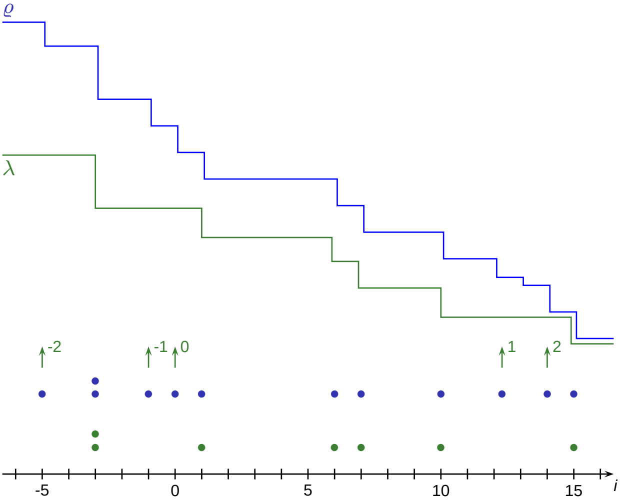

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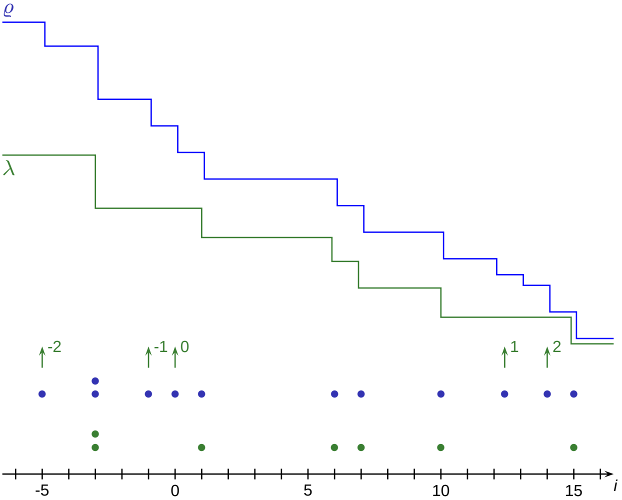

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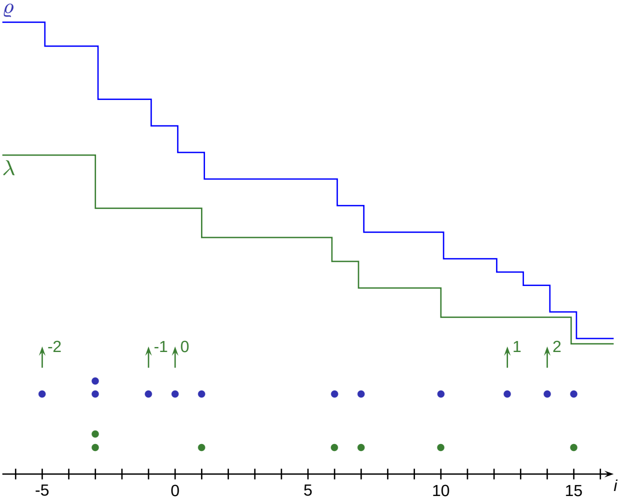

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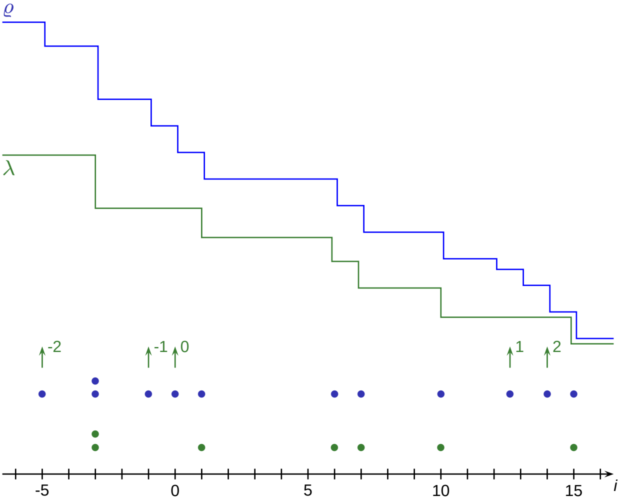

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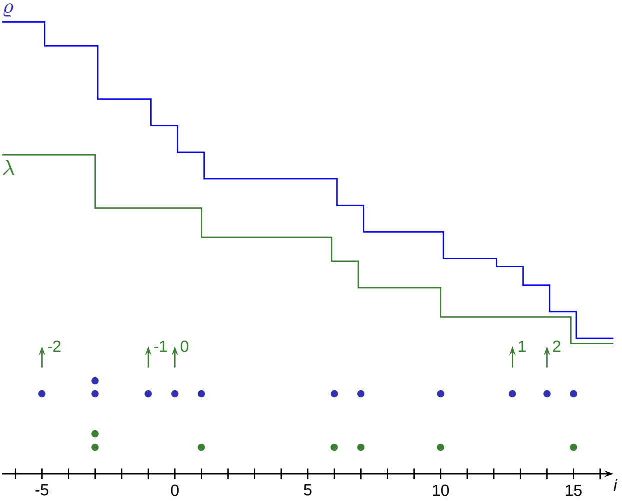

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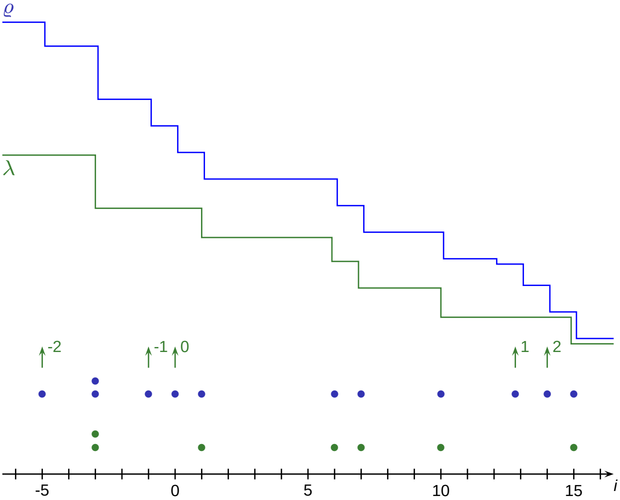

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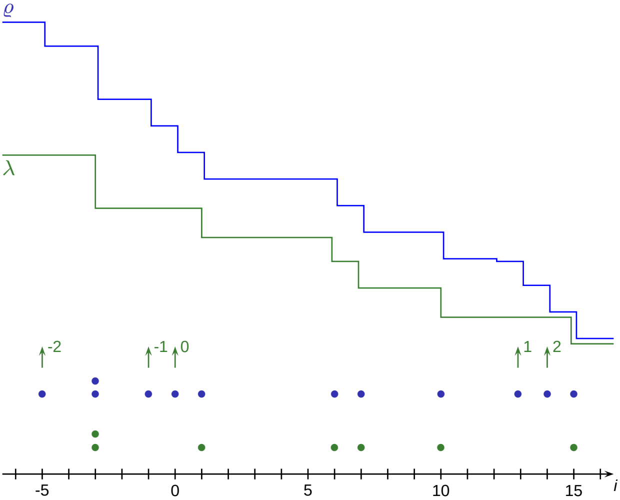

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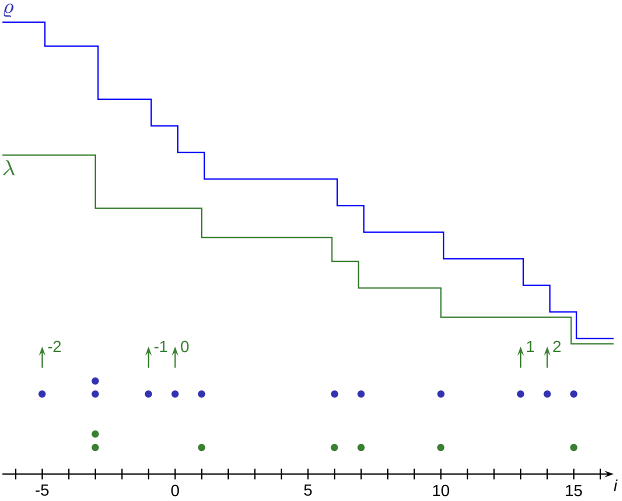

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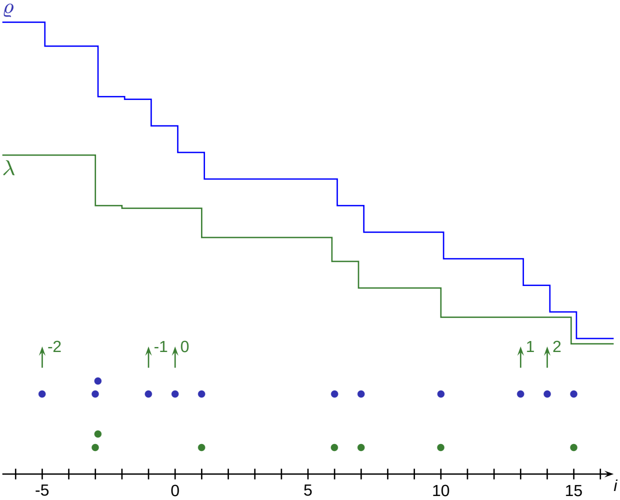

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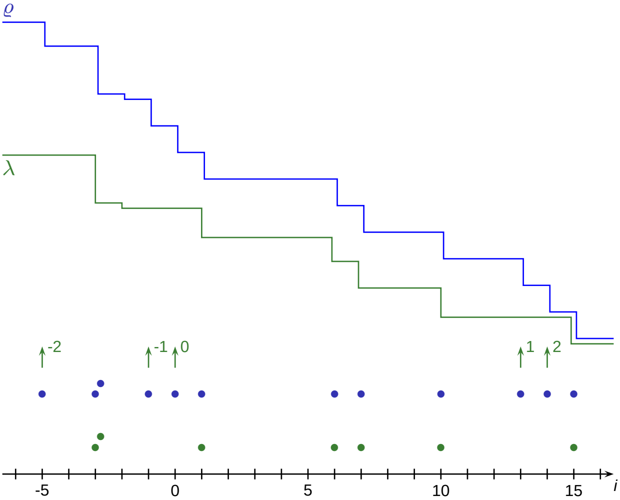

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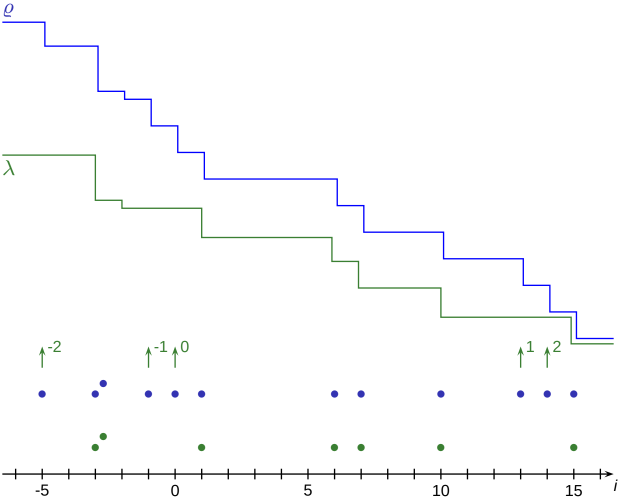

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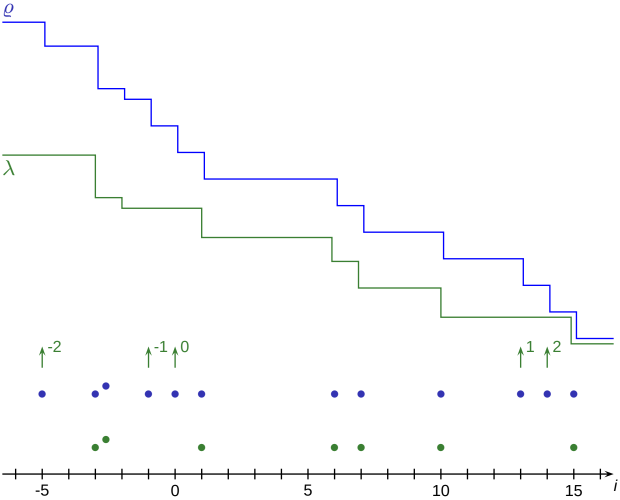

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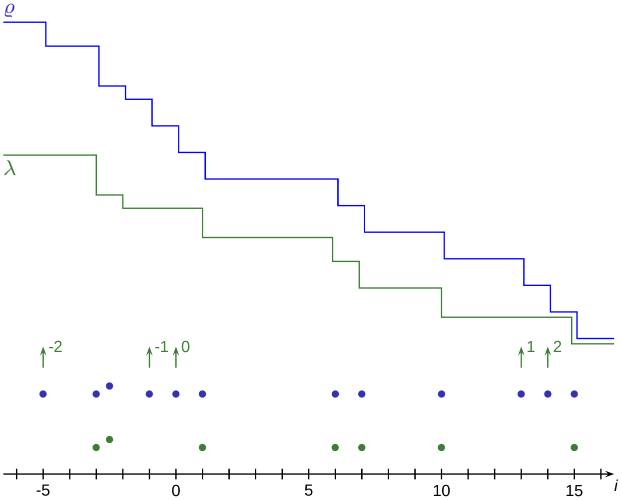

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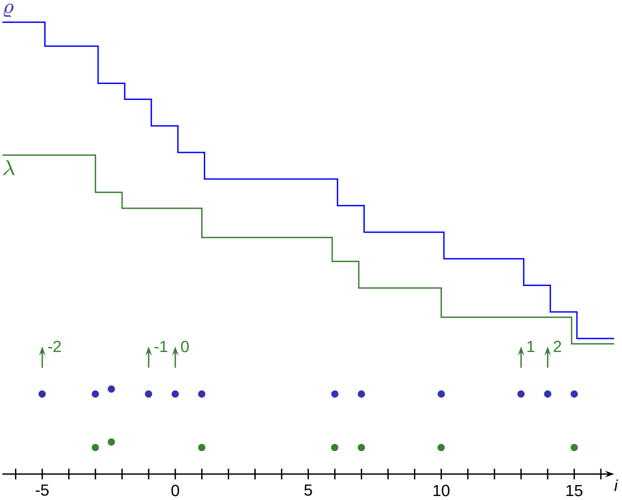

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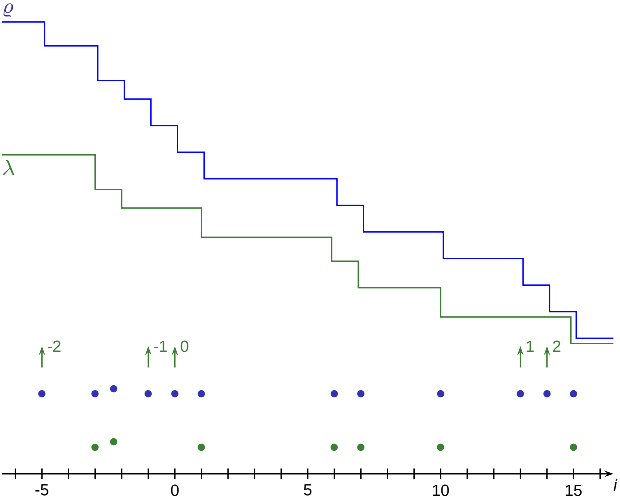

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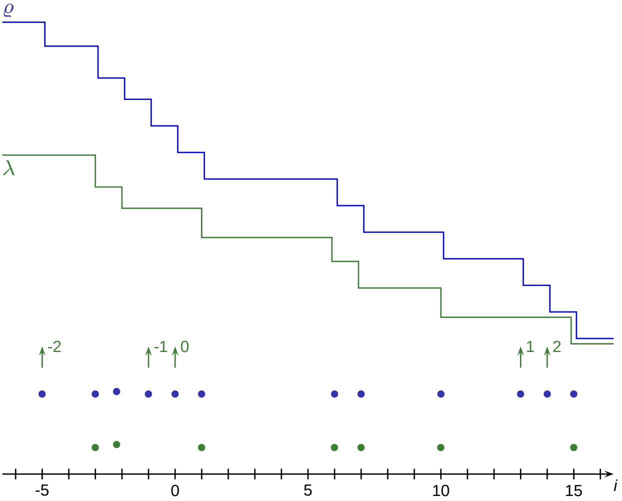

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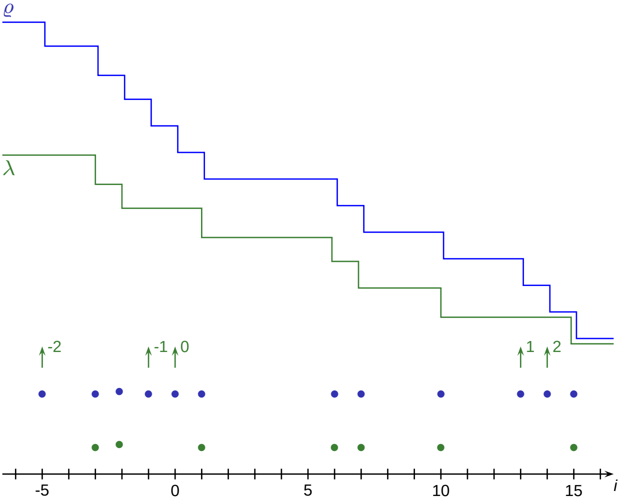

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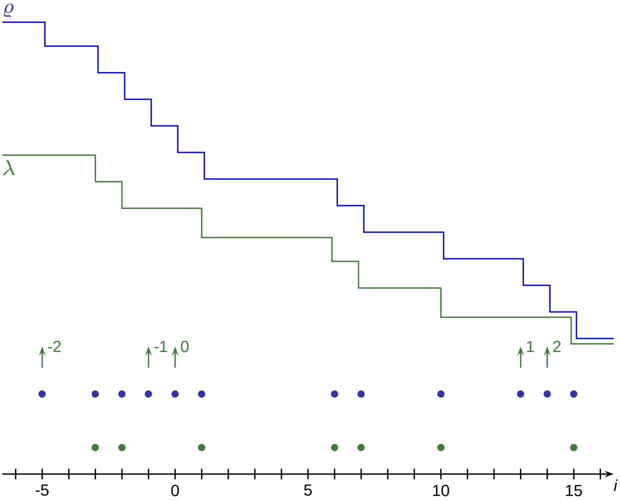

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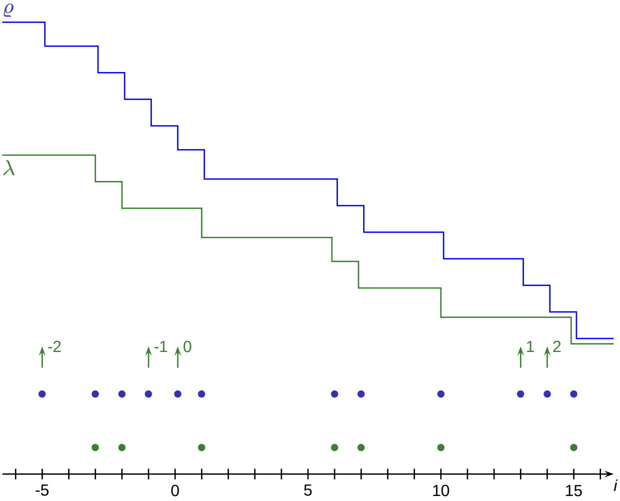

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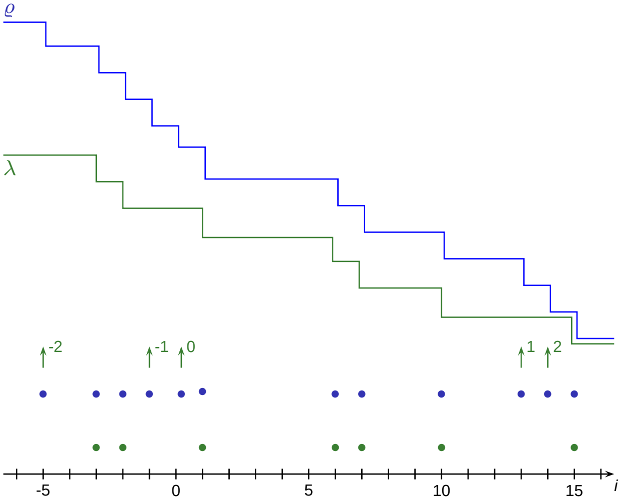

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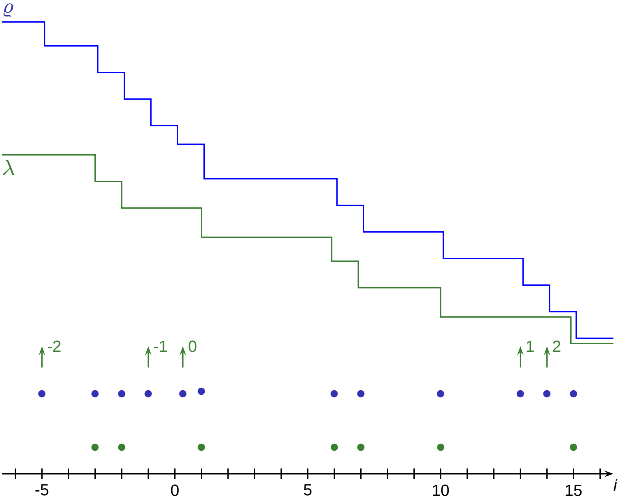

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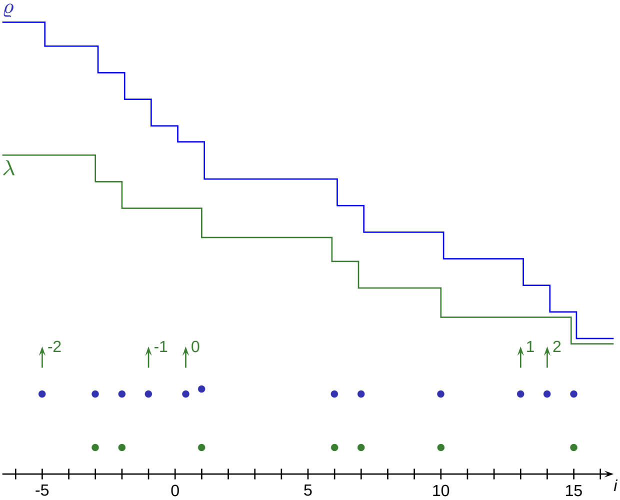

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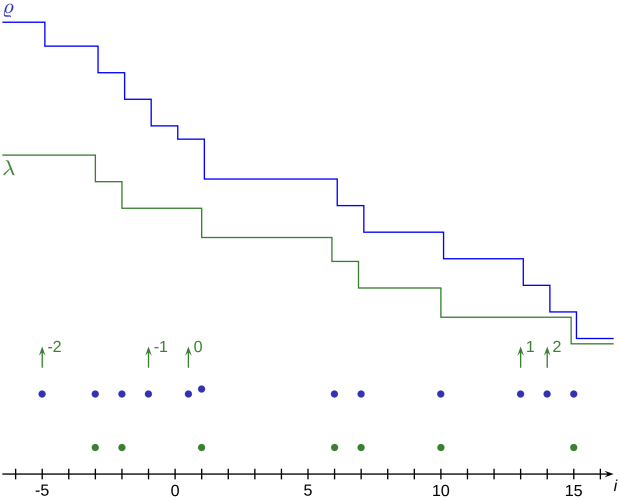

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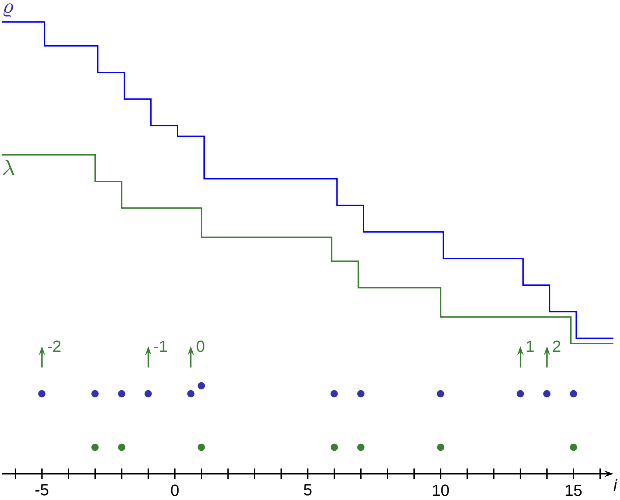

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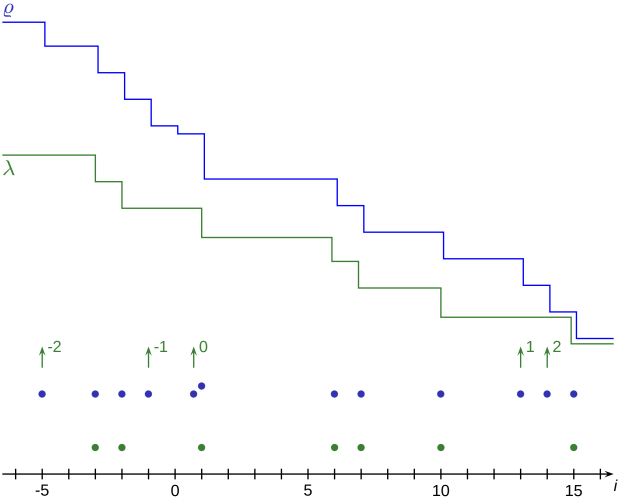

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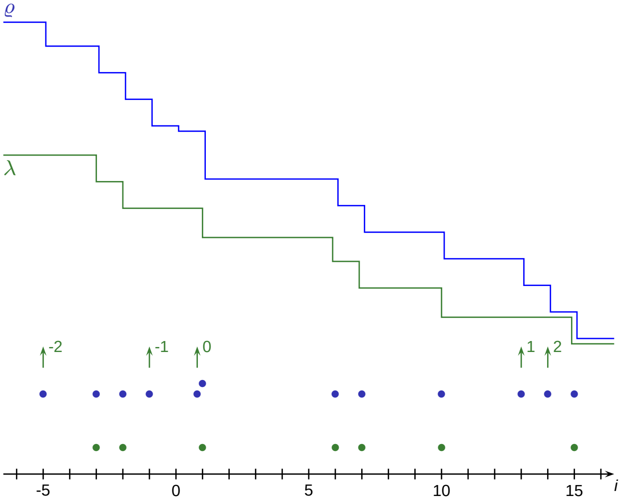

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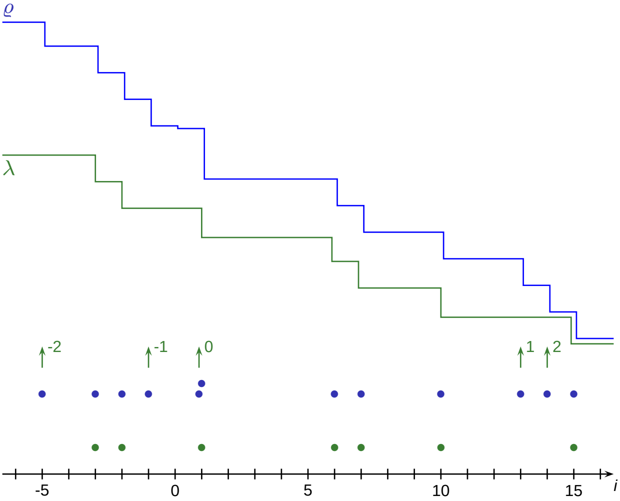

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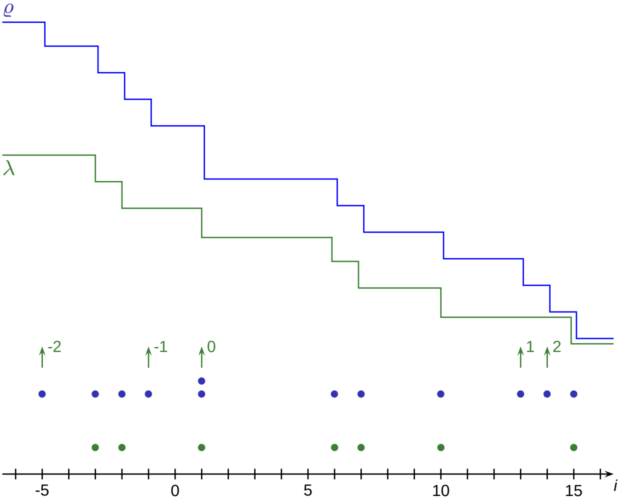

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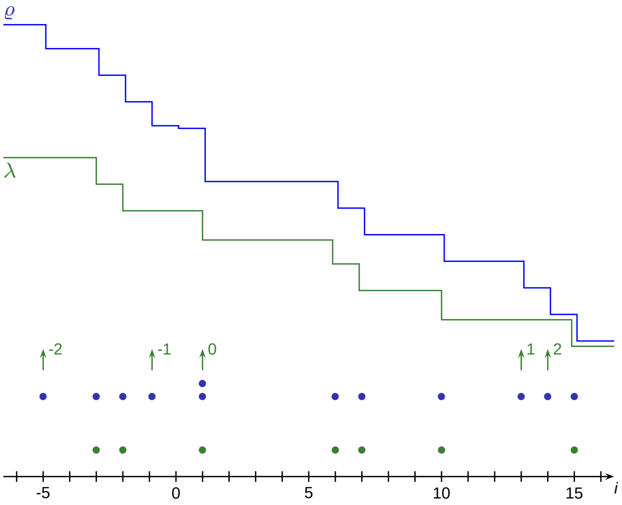

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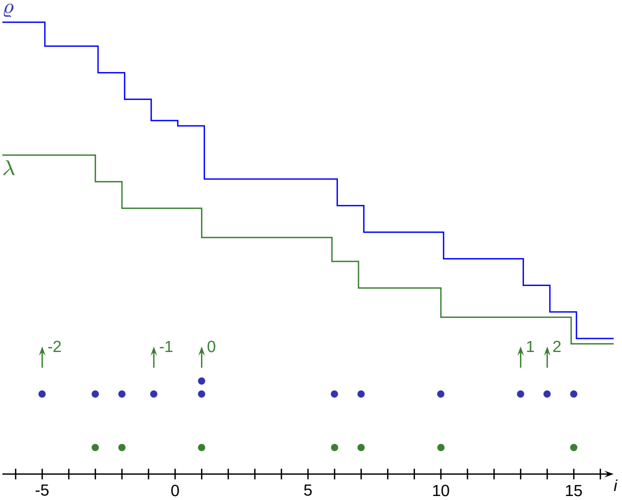

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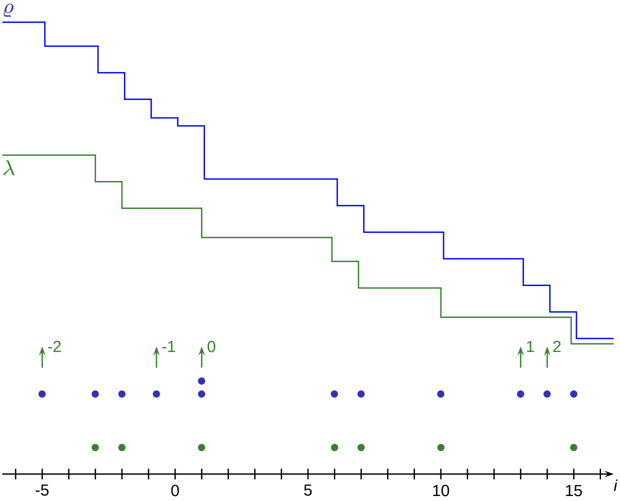

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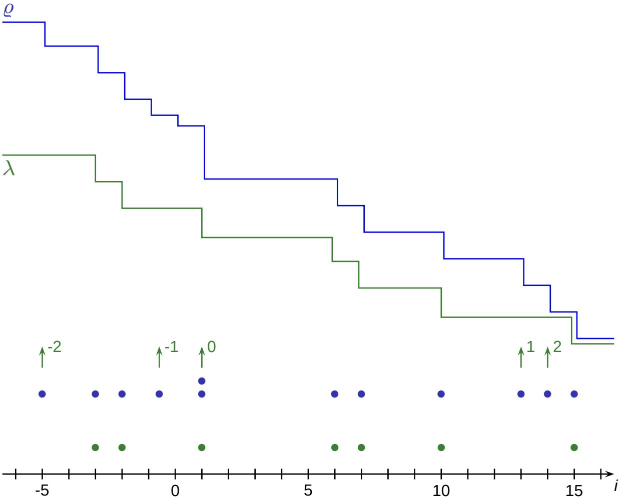

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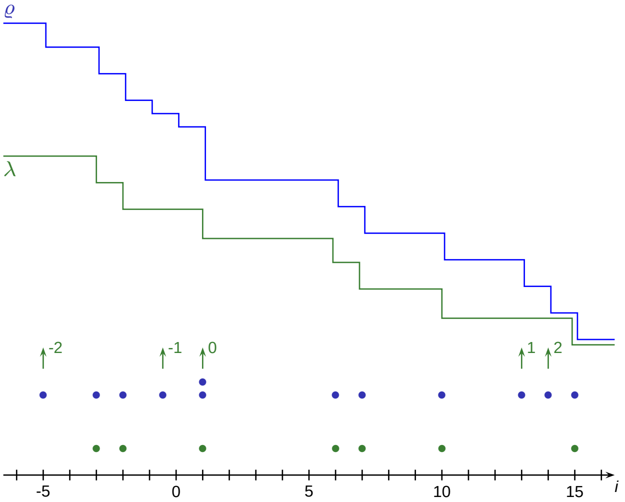

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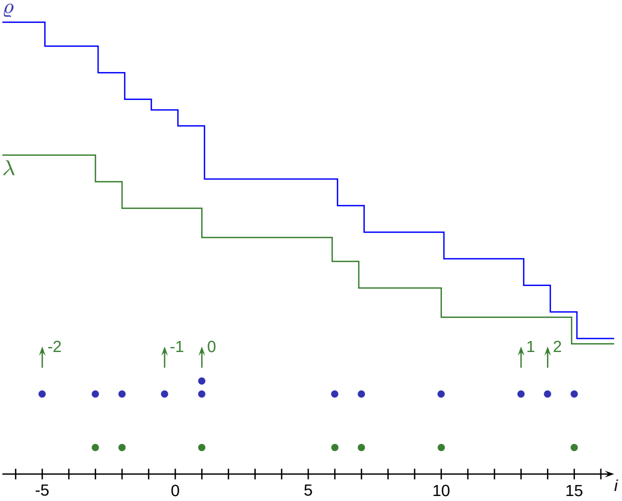

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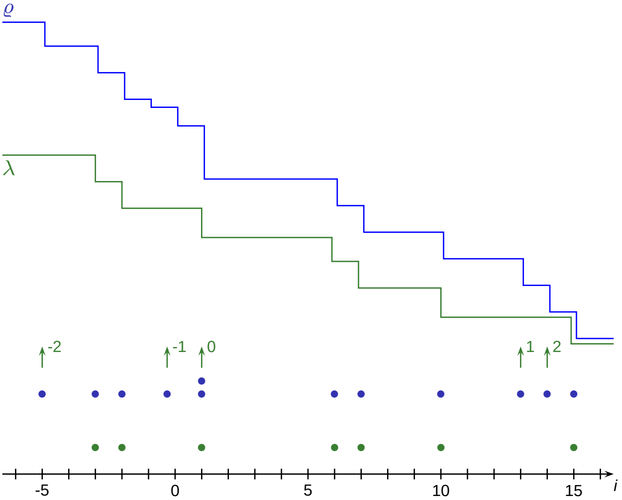

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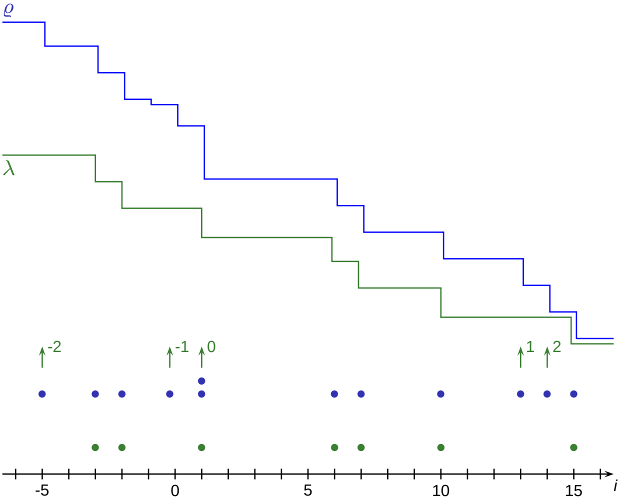

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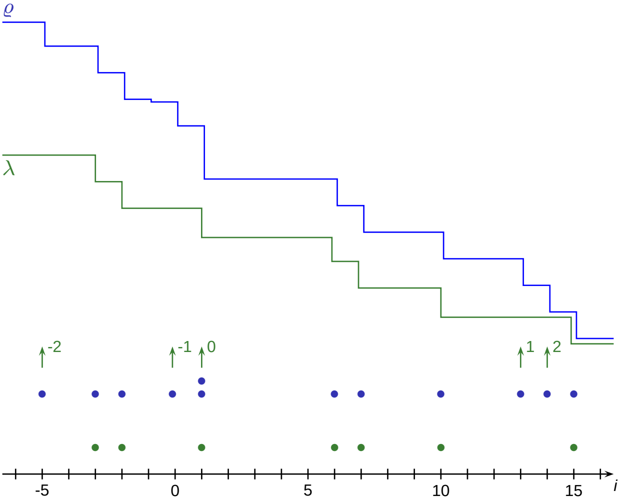

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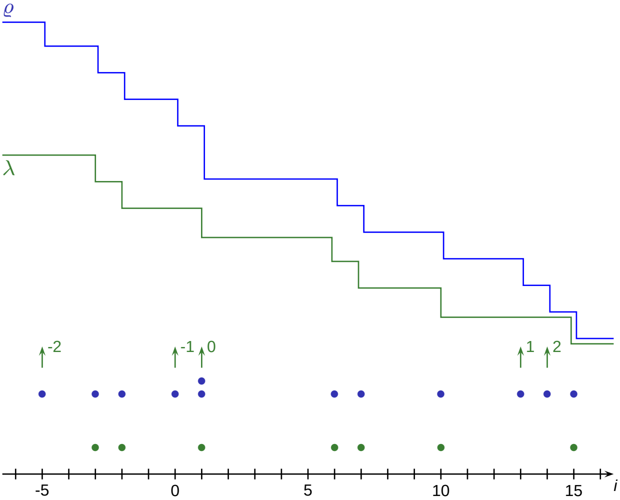

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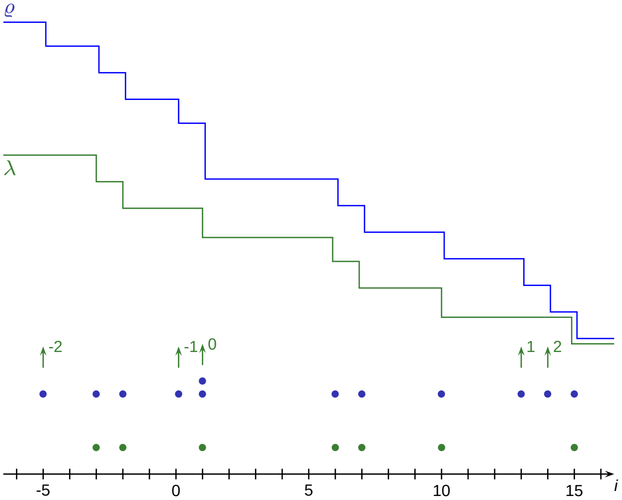

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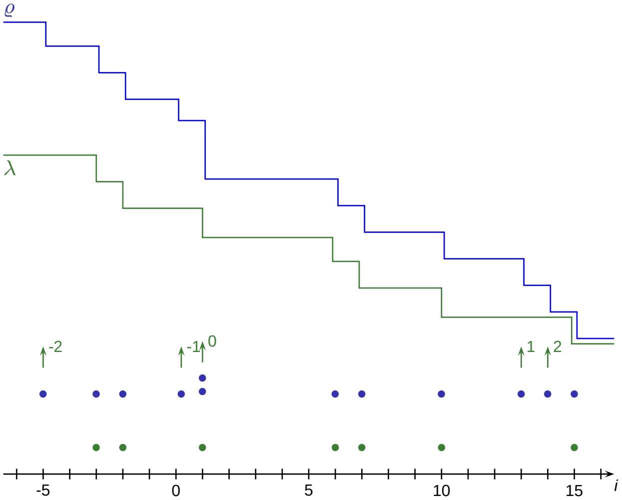

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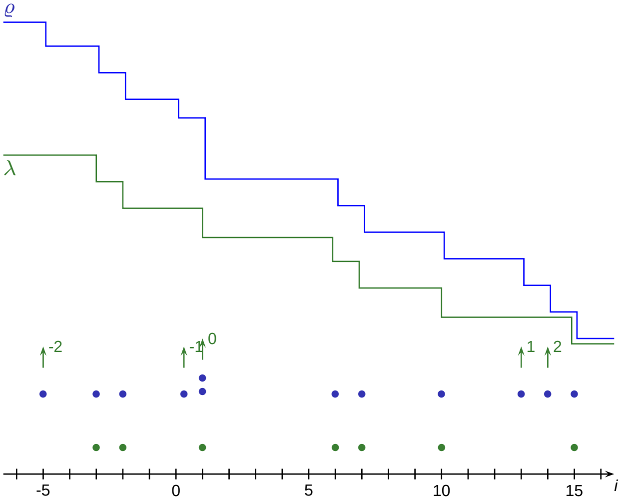

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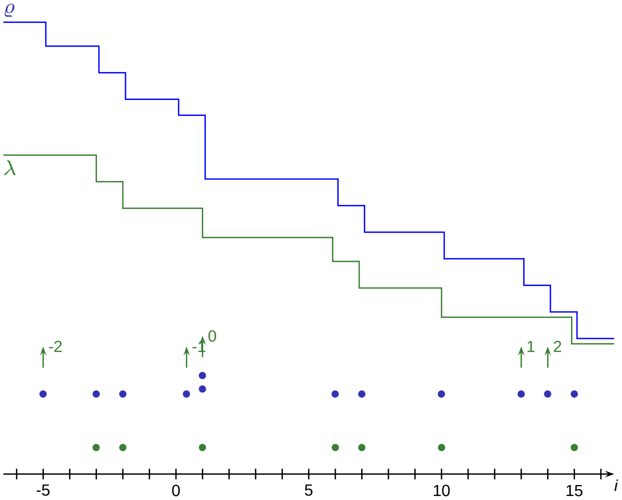

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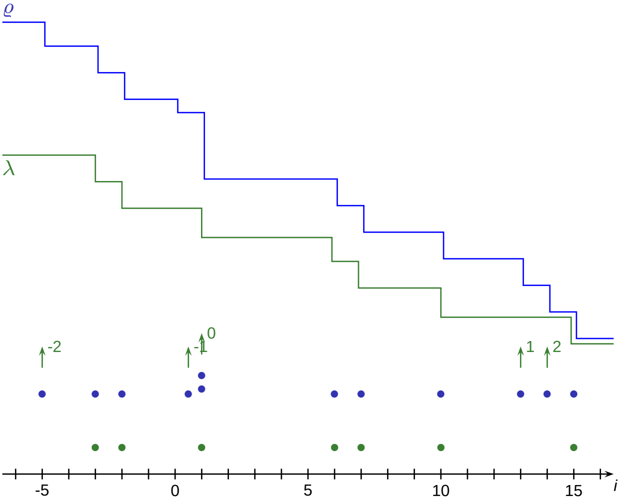

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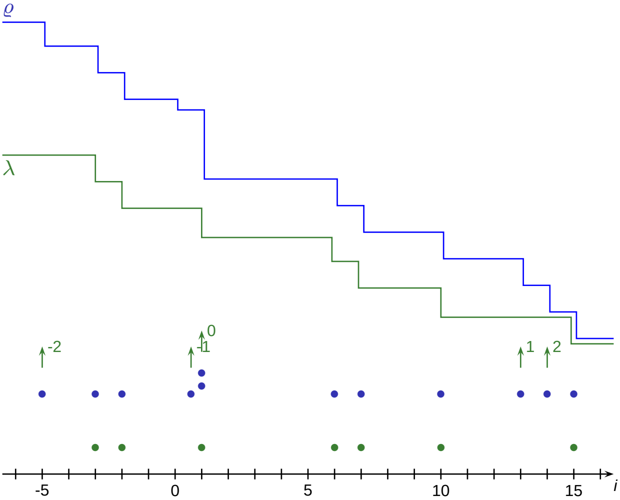

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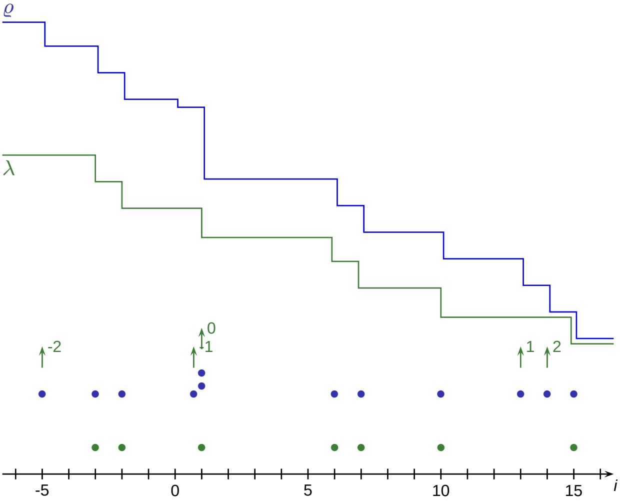

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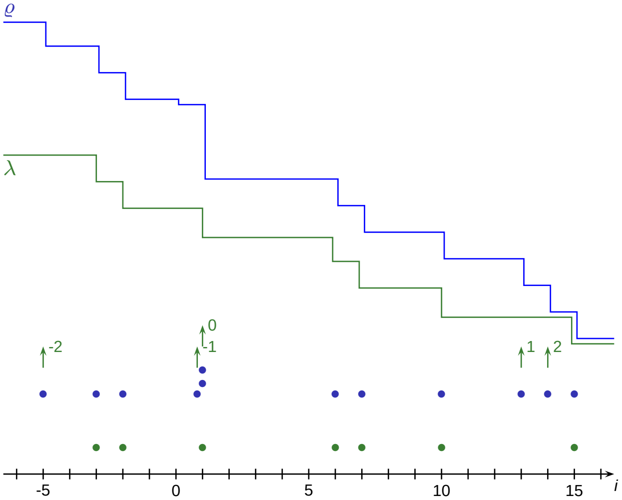

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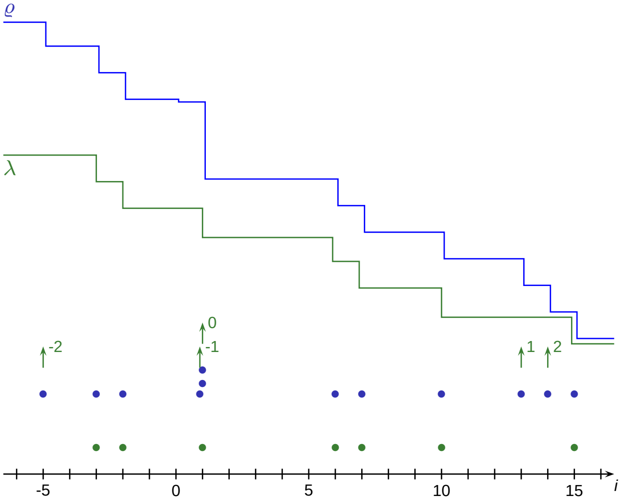

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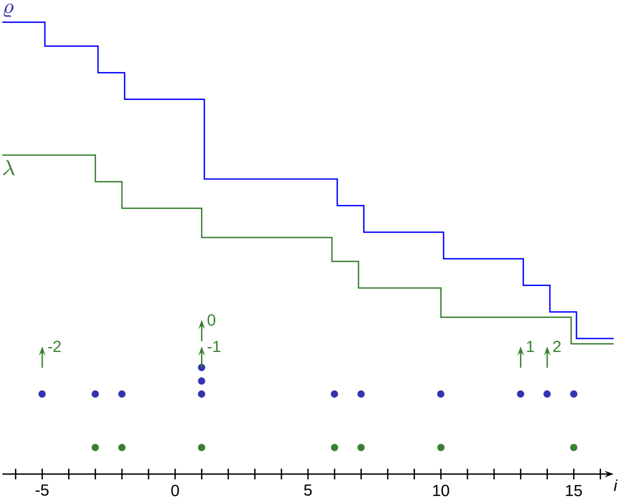

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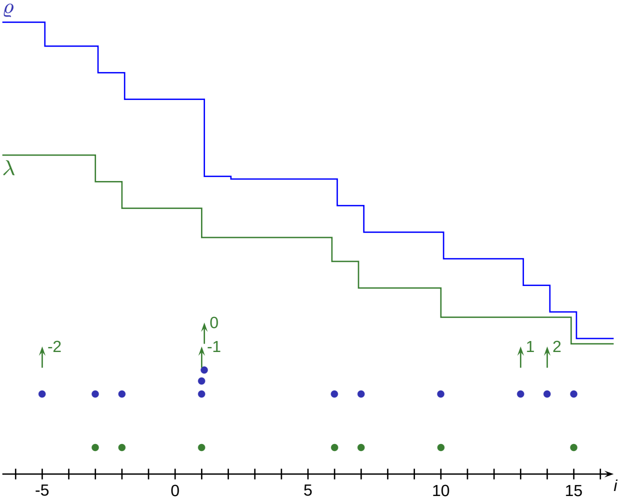

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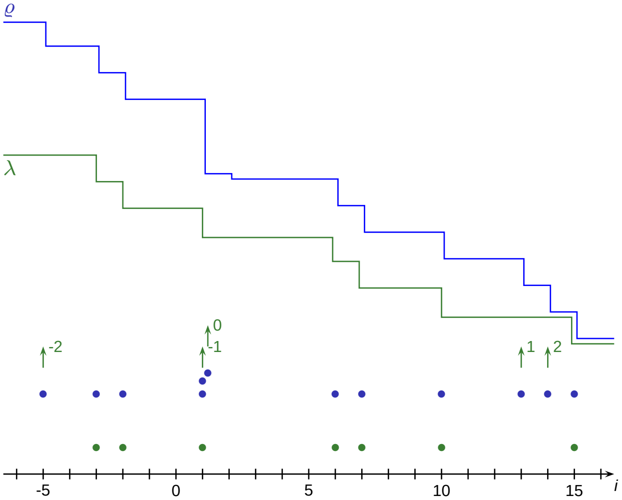

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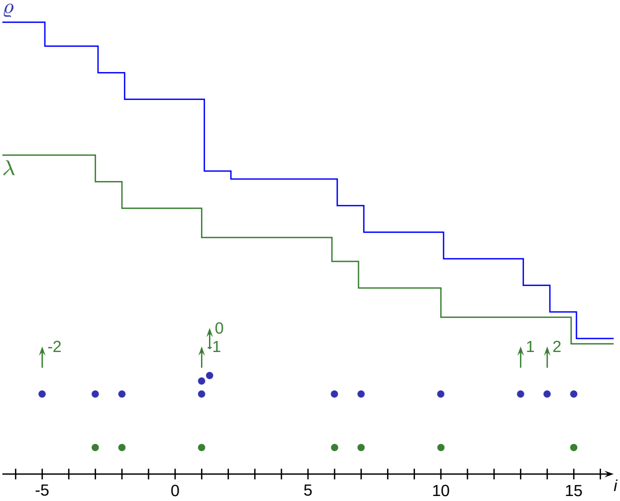

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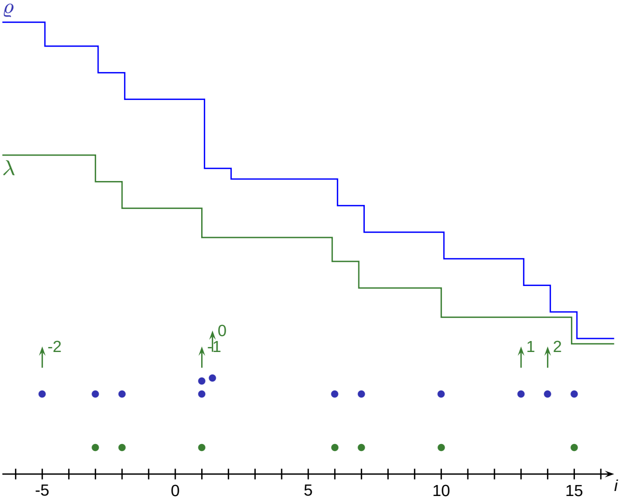

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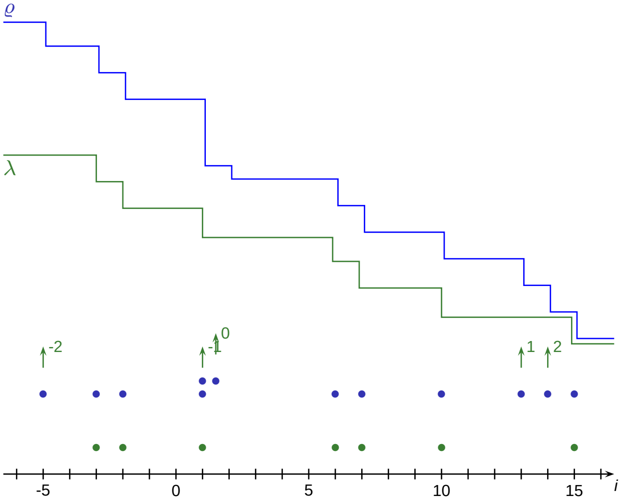

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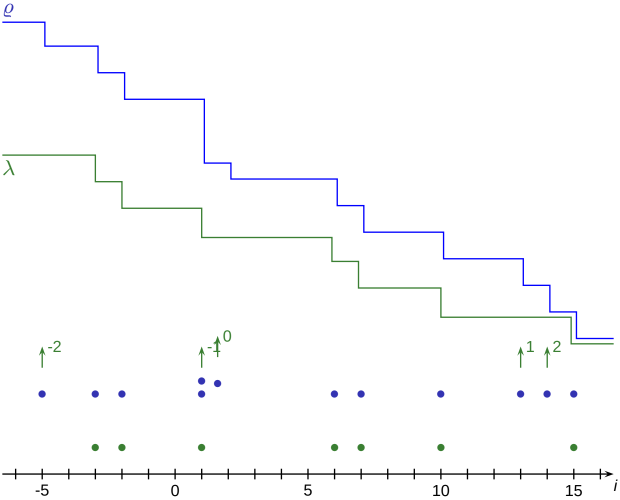

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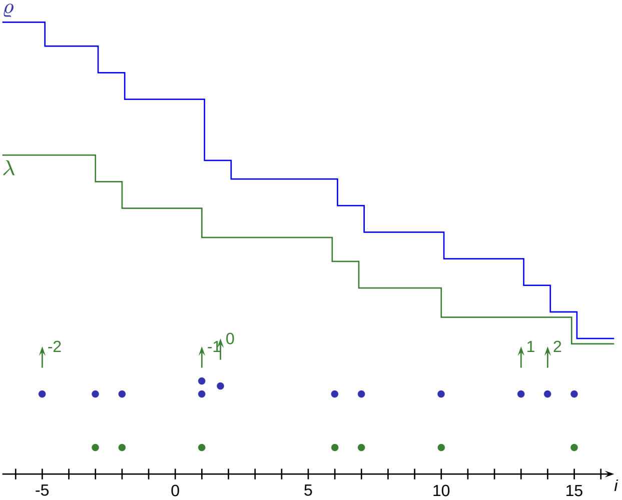

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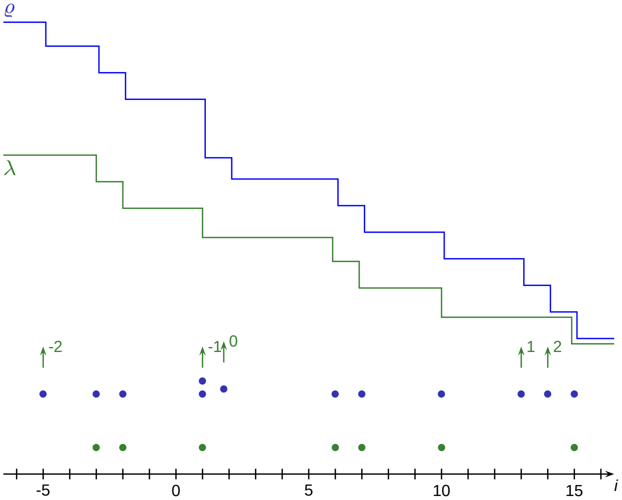

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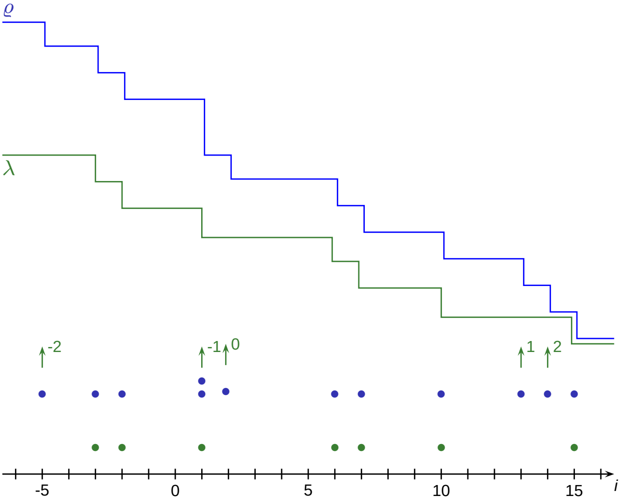

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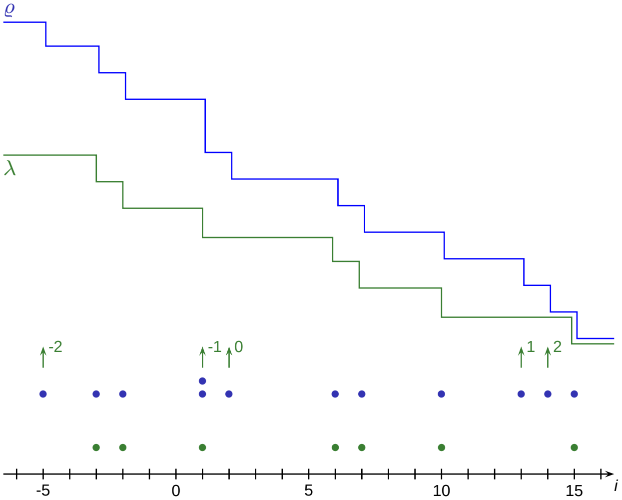

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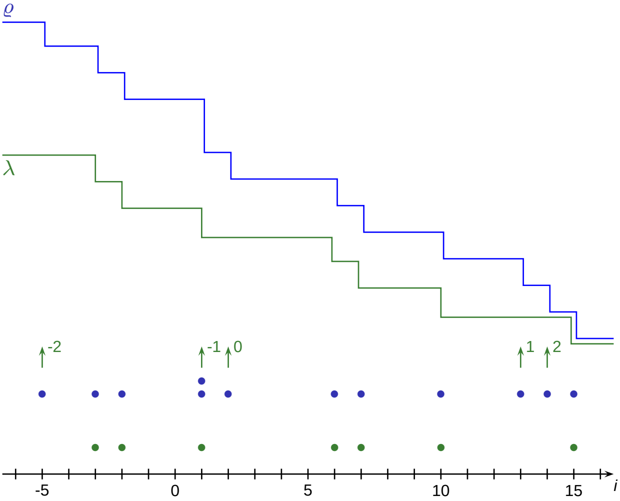
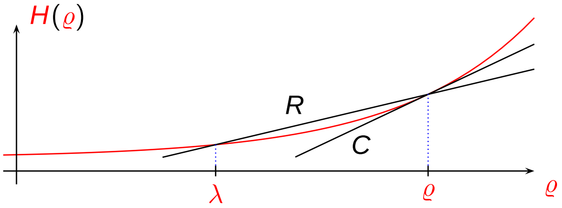


Picture:

The position $X(t)$ of $\uparrow^0$ follows the Rankine-Hugoniot speed R.

Characteristics (very briefly)

Convex flux (some cases of AZRP, ABLP):



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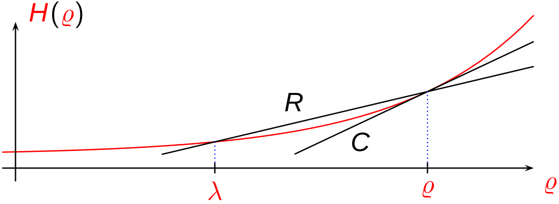

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Do we have $Q(t) \overset{?}{\geq} X(t)$

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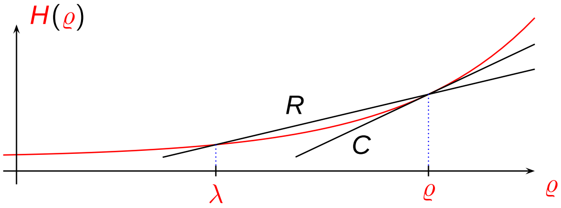

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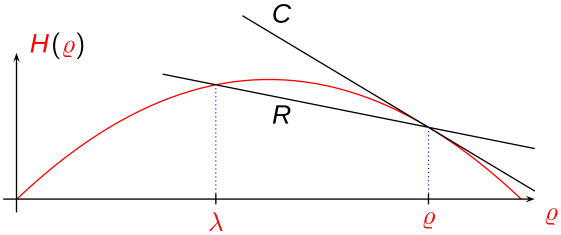
Concave flux (ASEP):



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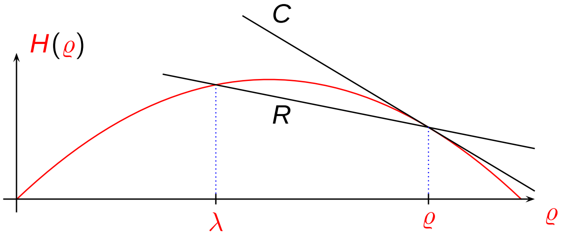

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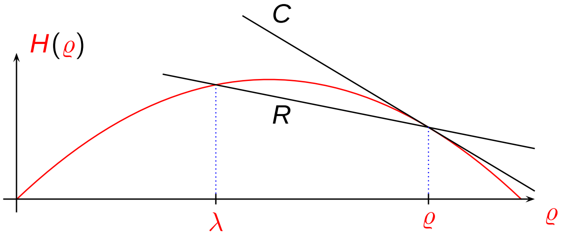

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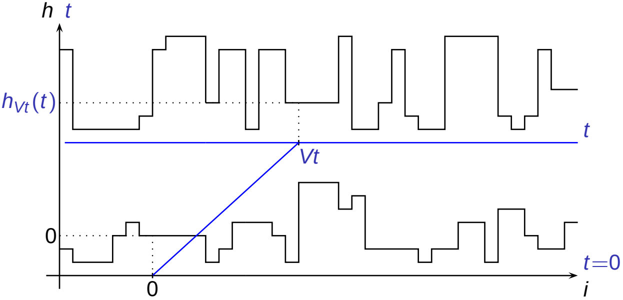

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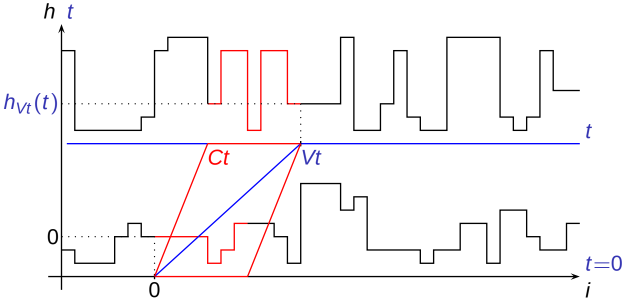

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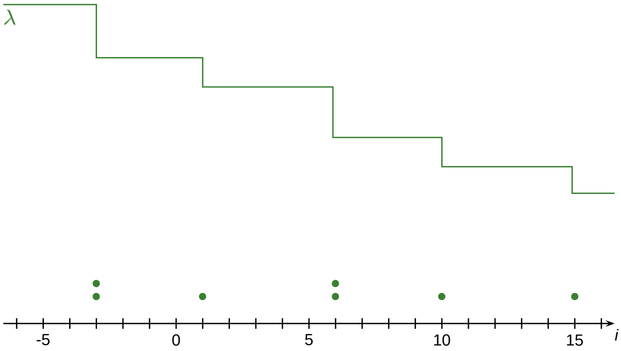
There are limit distribution results for TASEP by Johansson 2000, Prähofer and Spohn 2001, Ferrari and Spohn 2006. Their methods are completely different, relying on combinatorial tricks and asymptotic analysis of certain determinants.

Many second class particles



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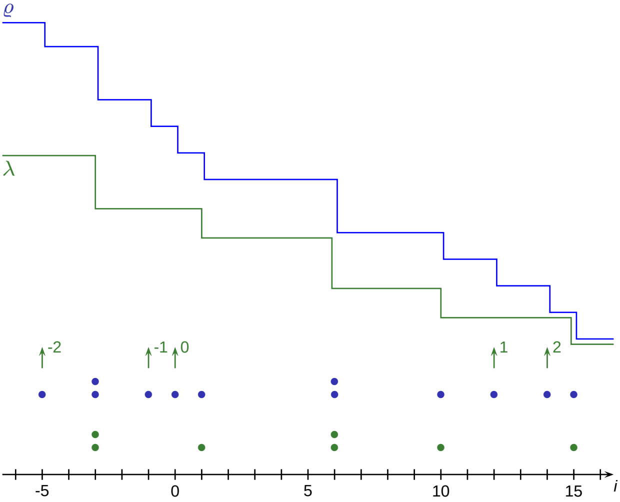
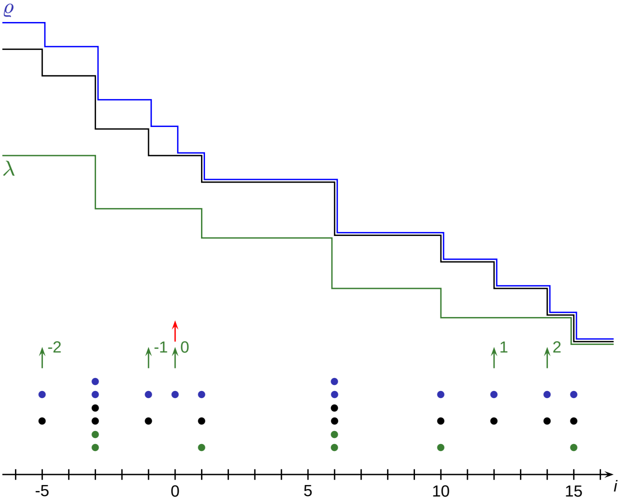


Many second class particles plus one



Couple three processes, and $X(t)$ to $Q(t)$.

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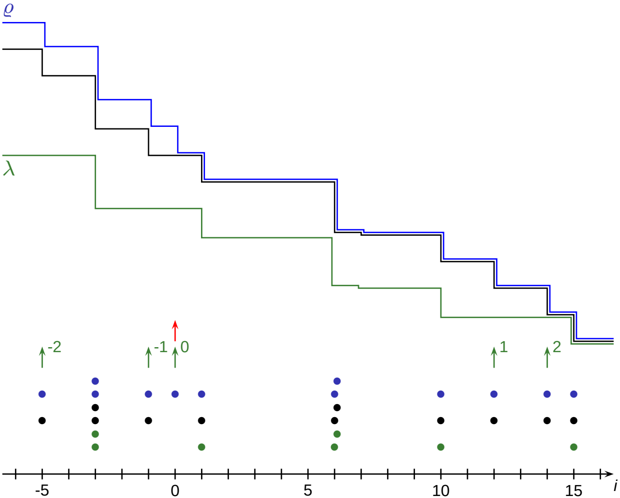

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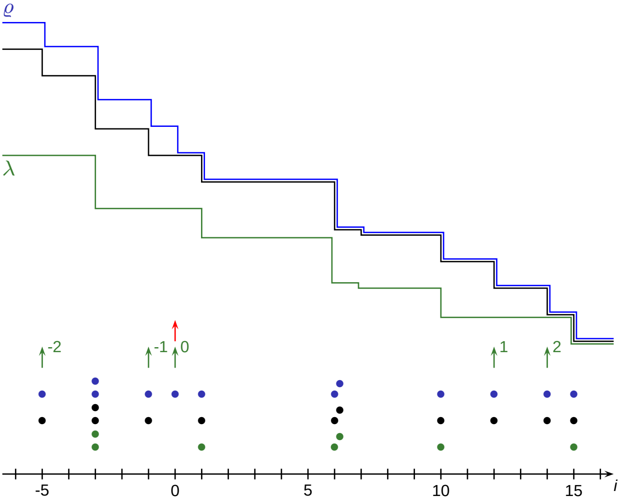

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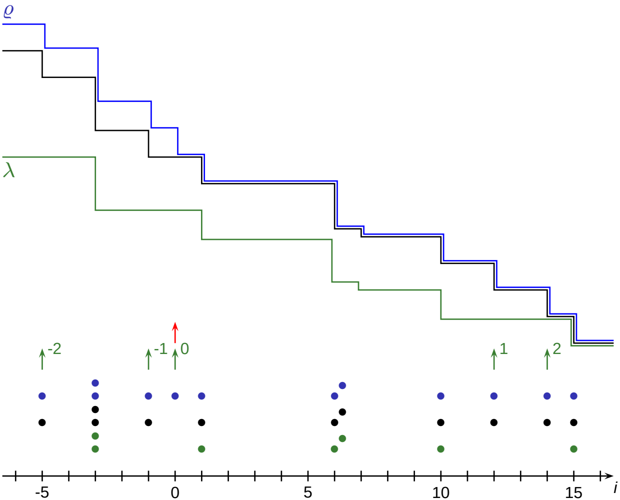

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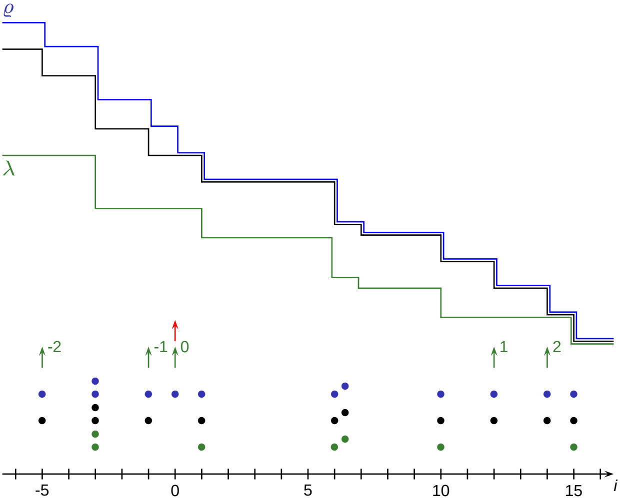

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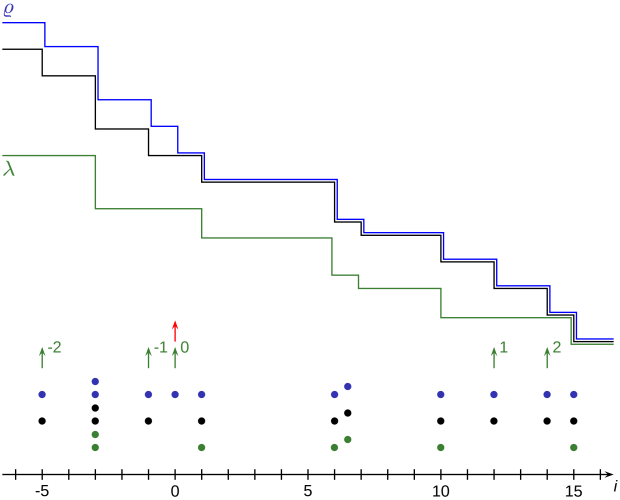

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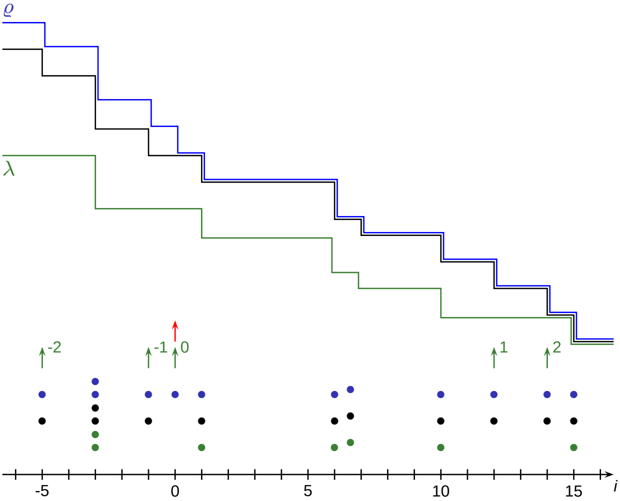

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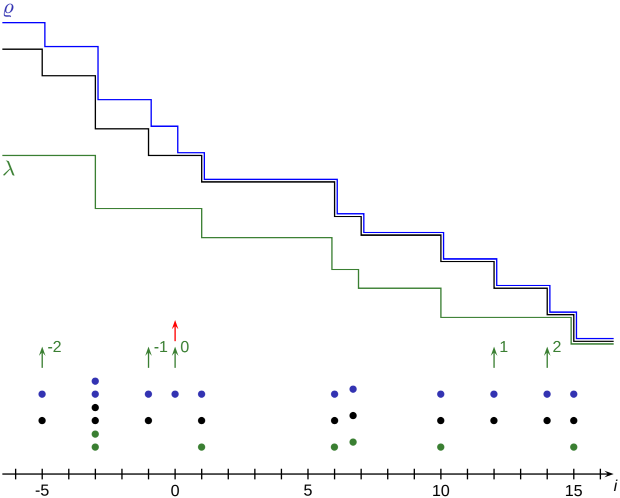

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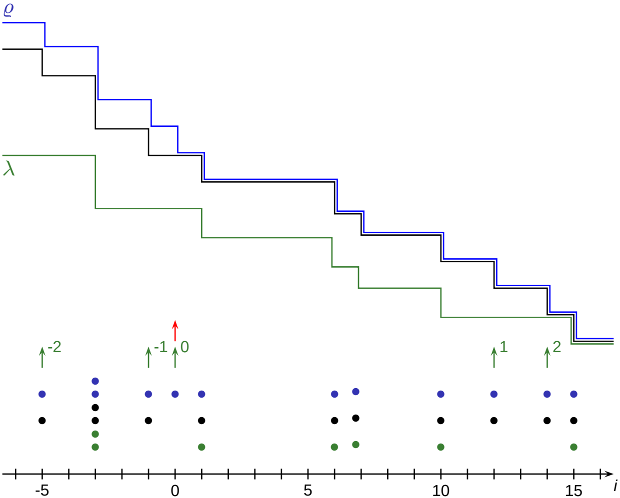

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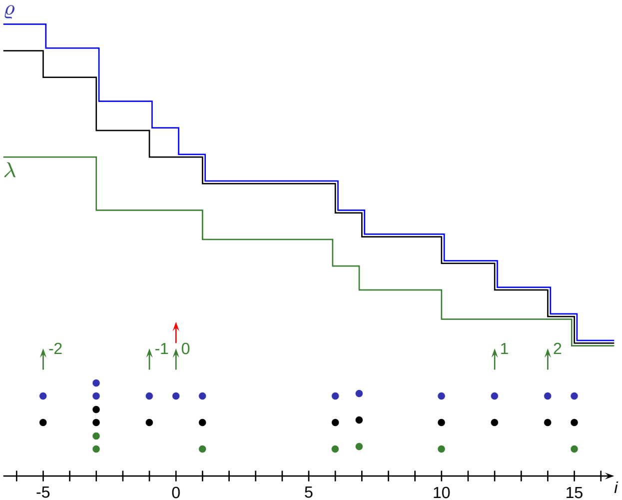

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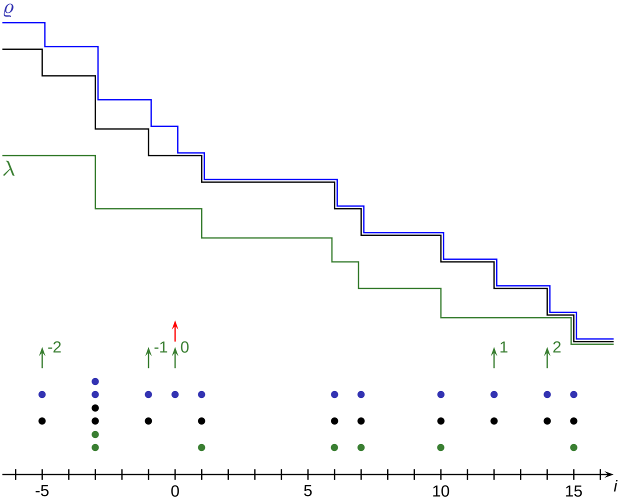

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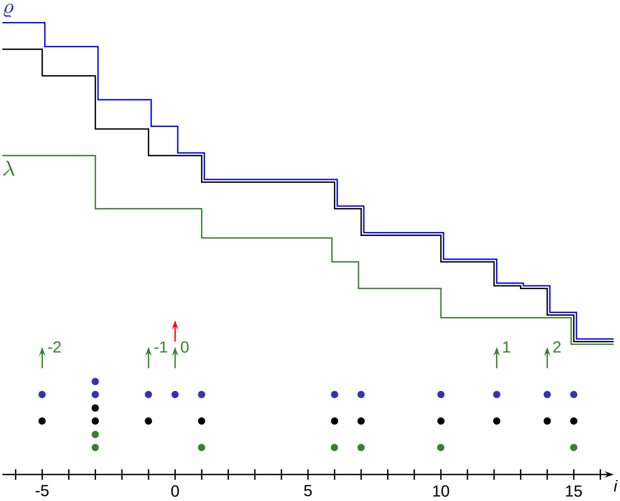

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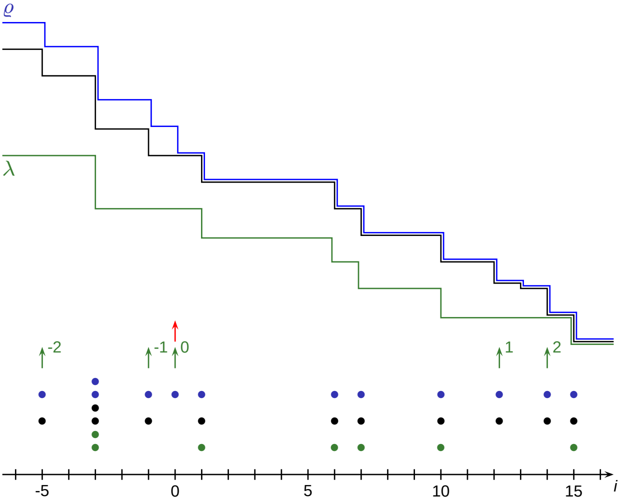

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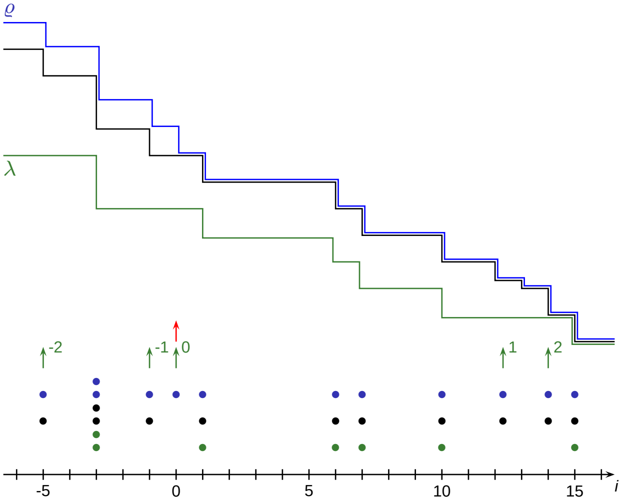

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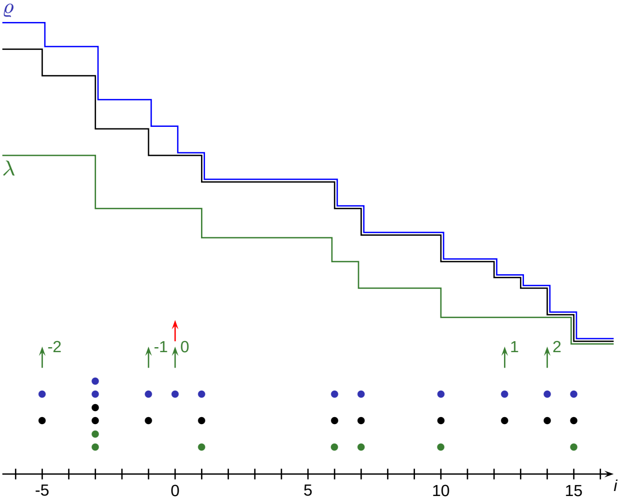

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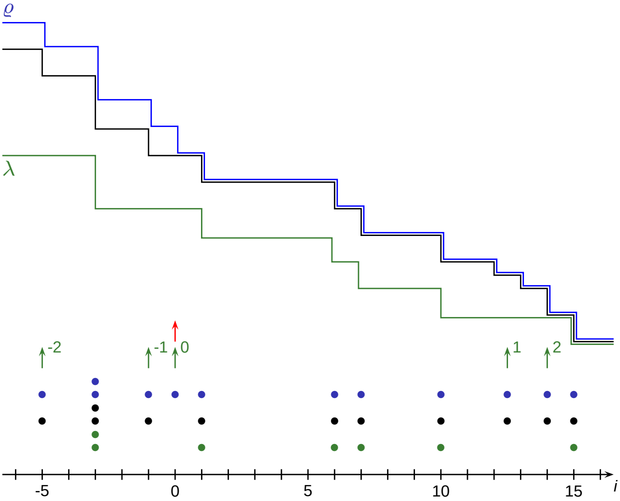

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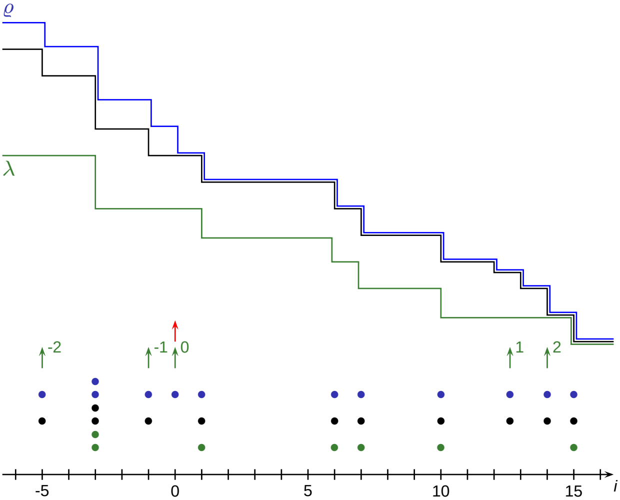

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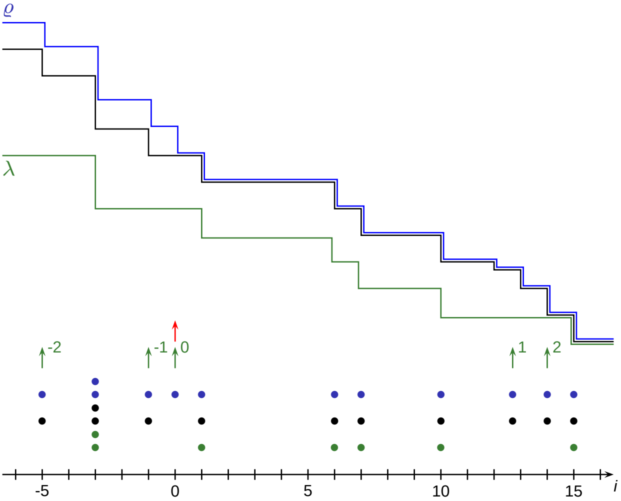

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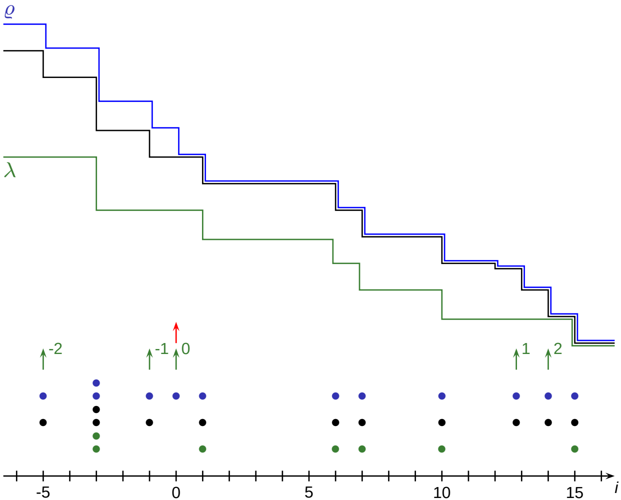

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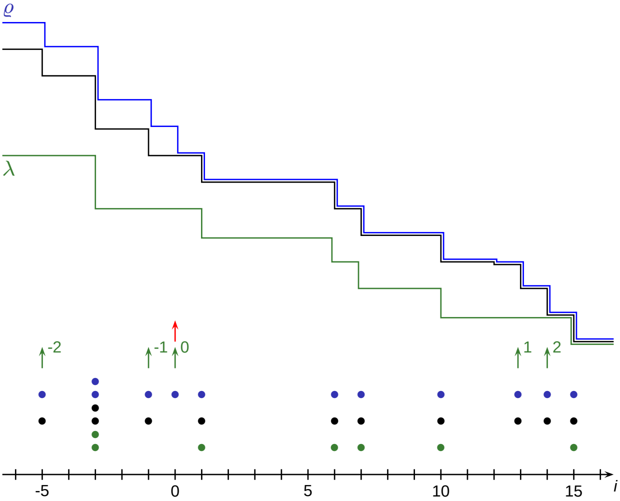

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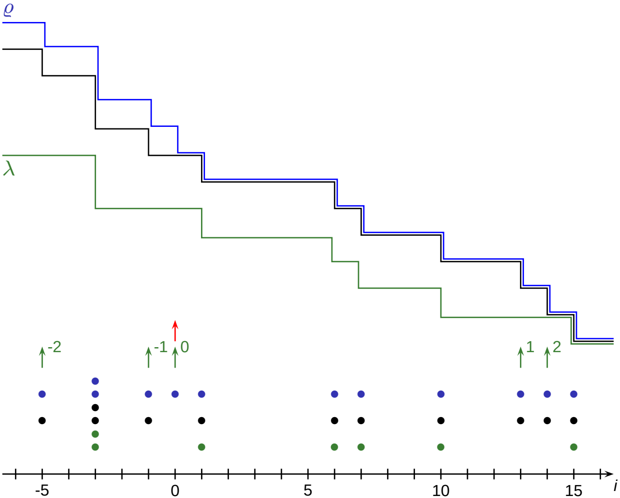

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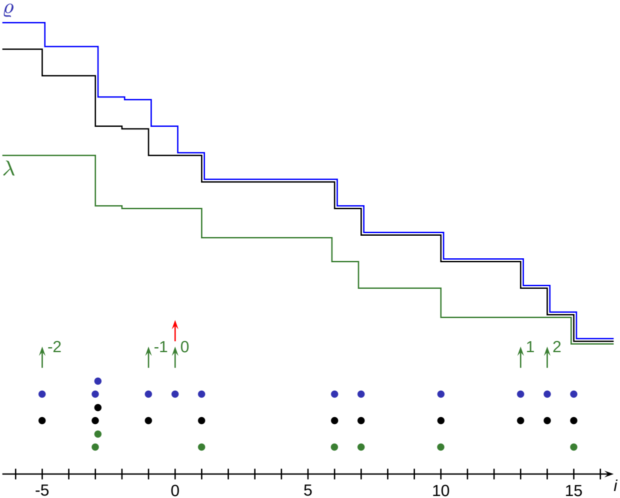

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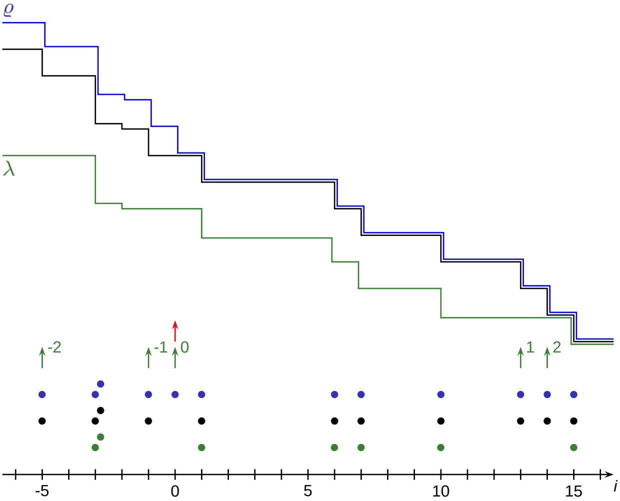

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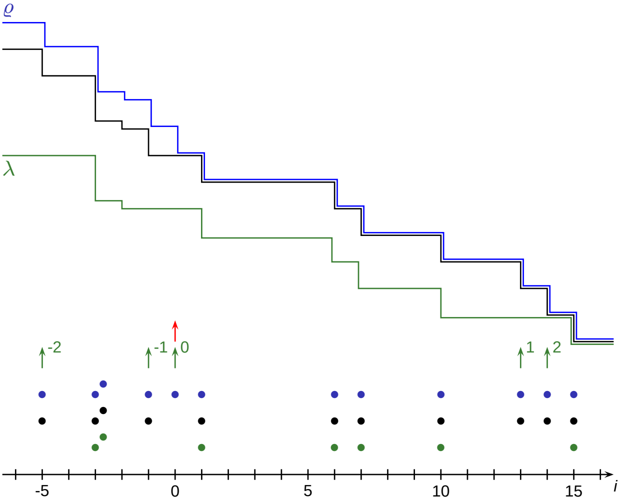

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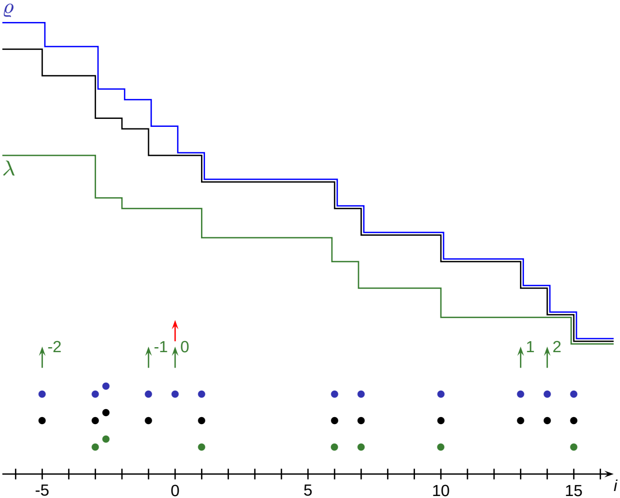

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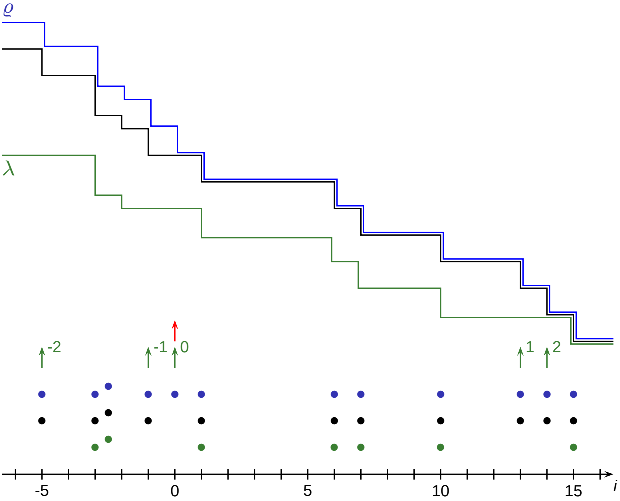

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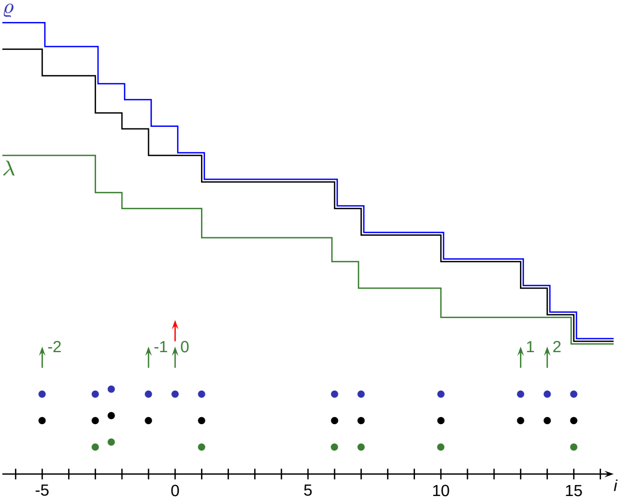

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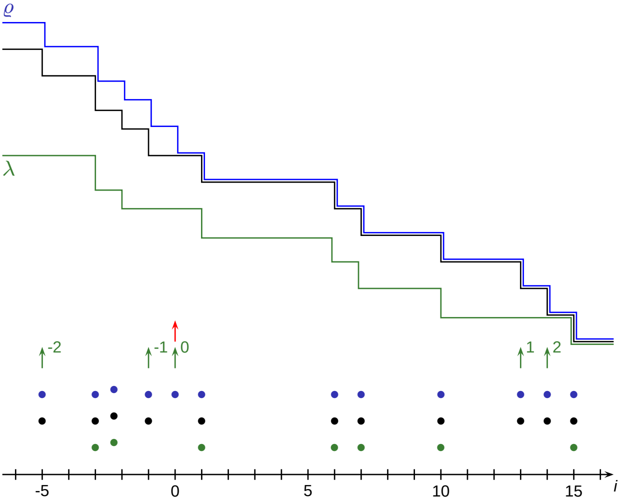

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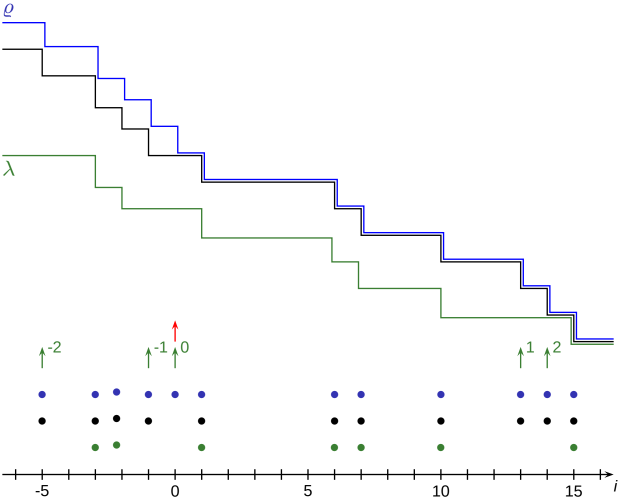

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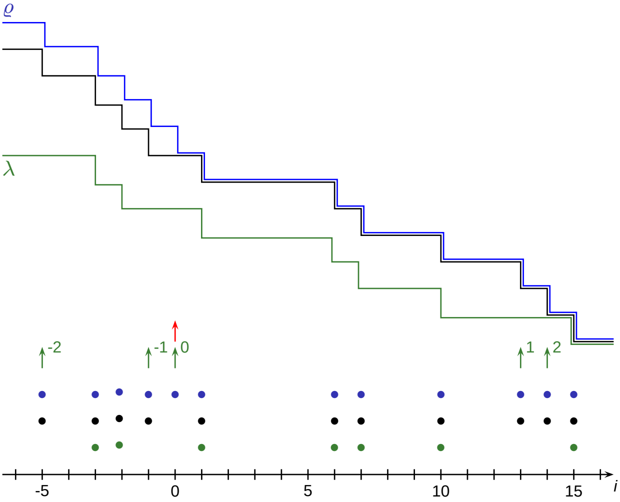

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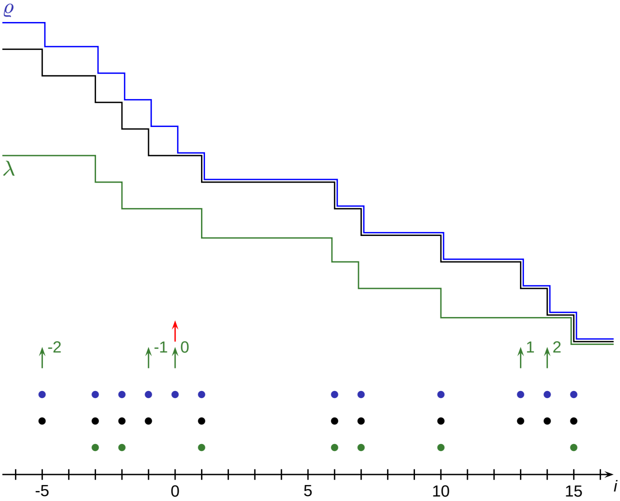

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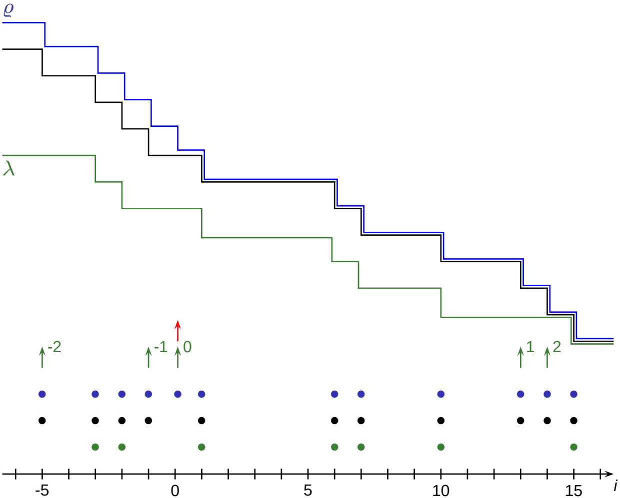

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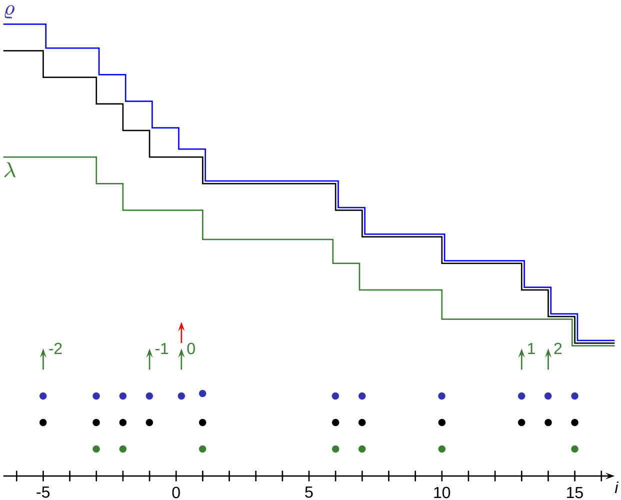

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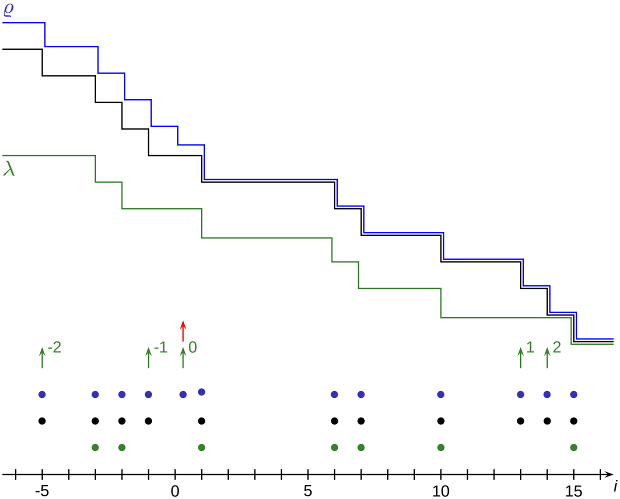

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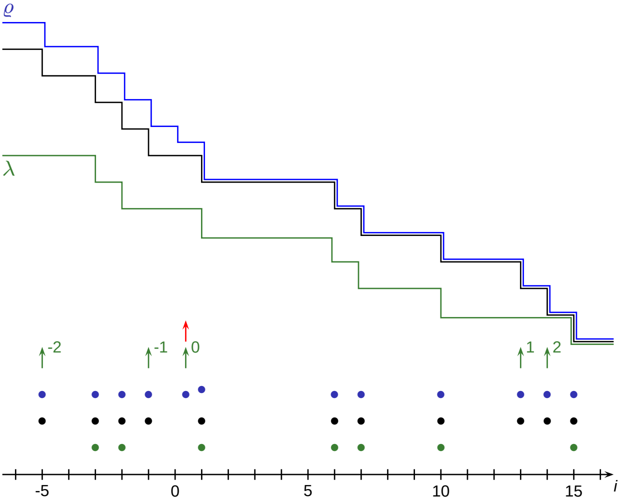

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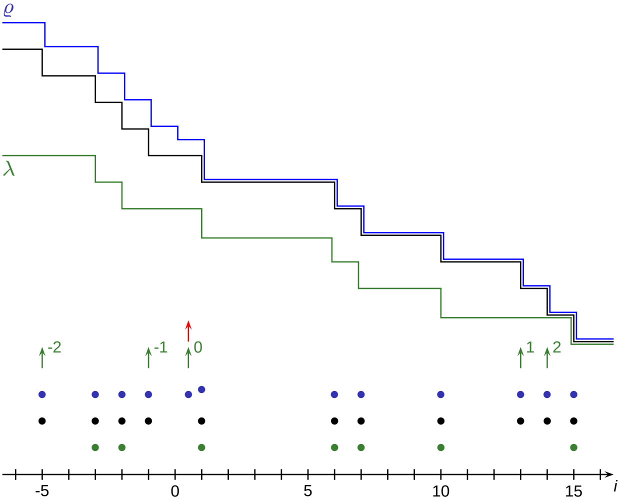

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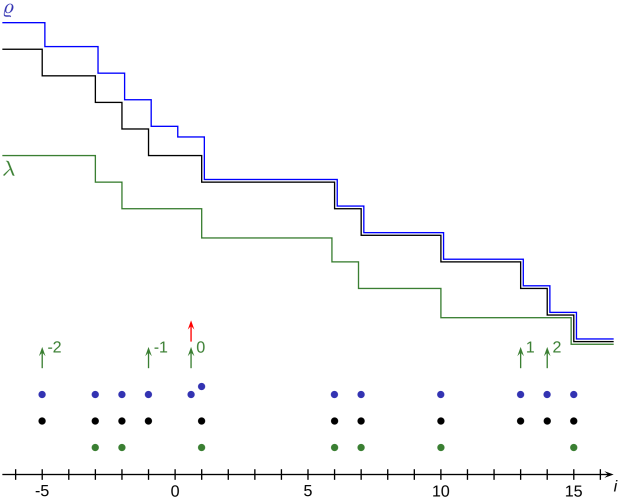

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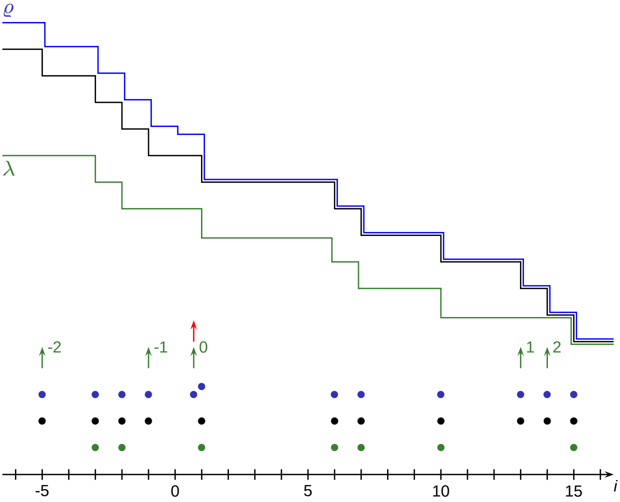

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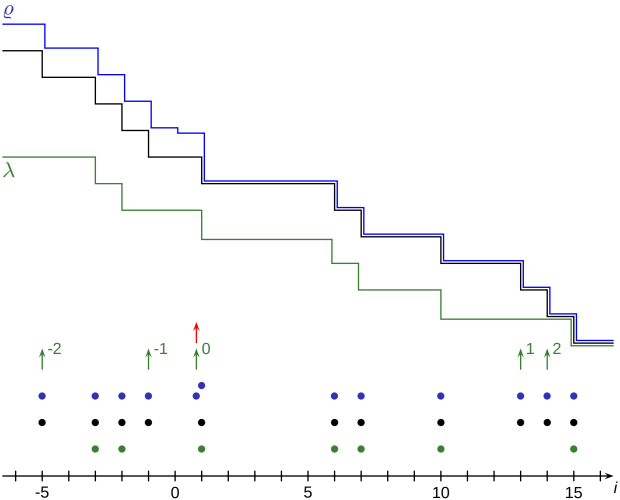

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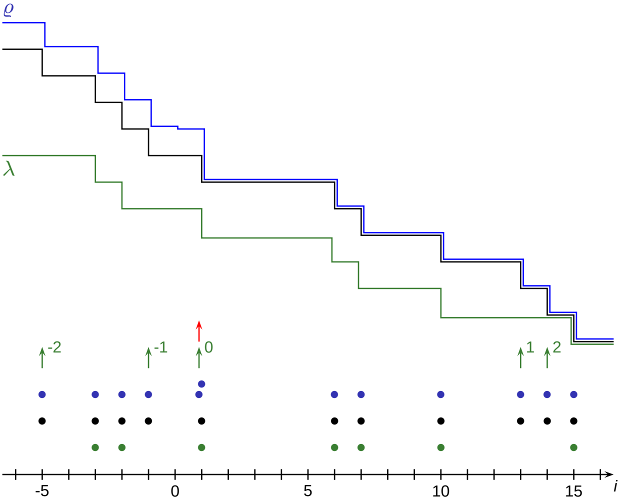

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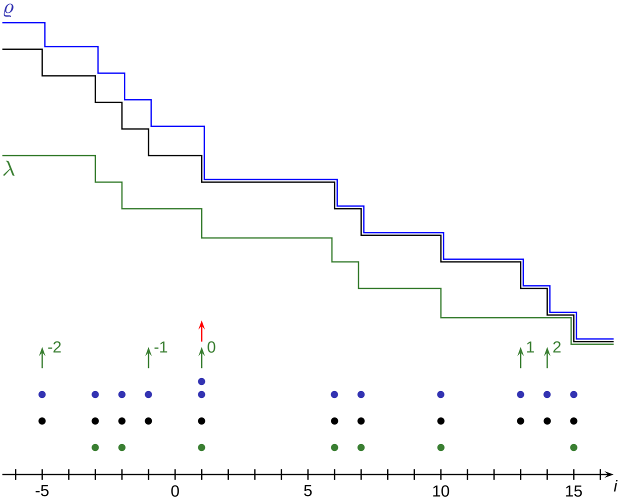

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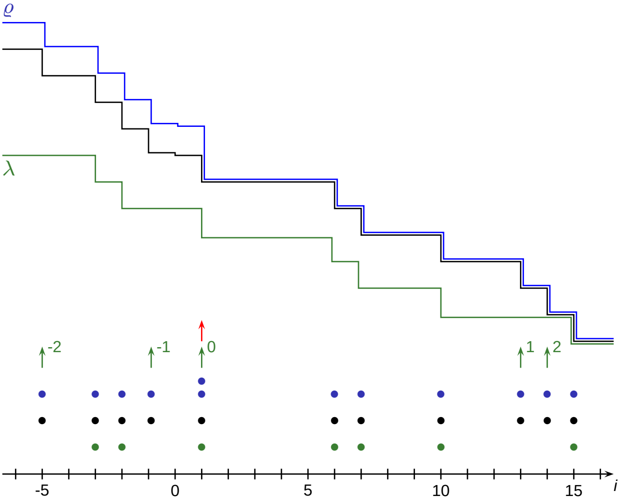

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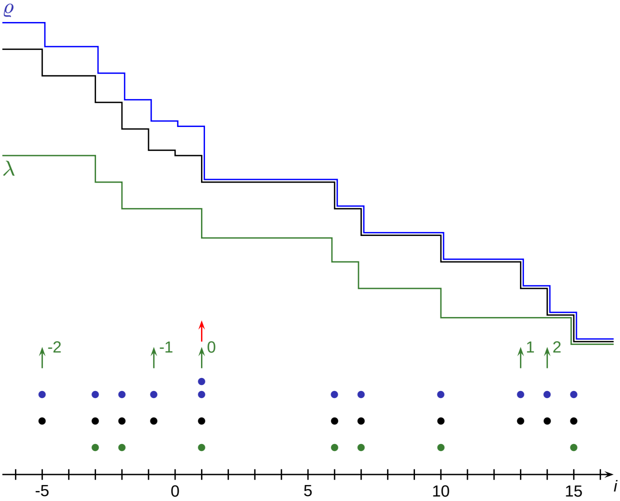

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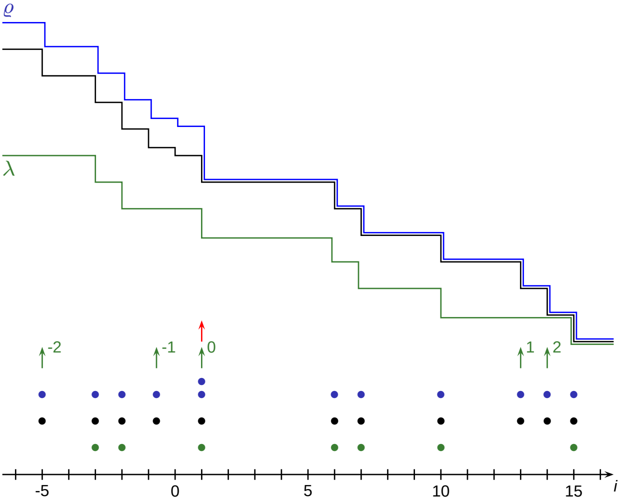

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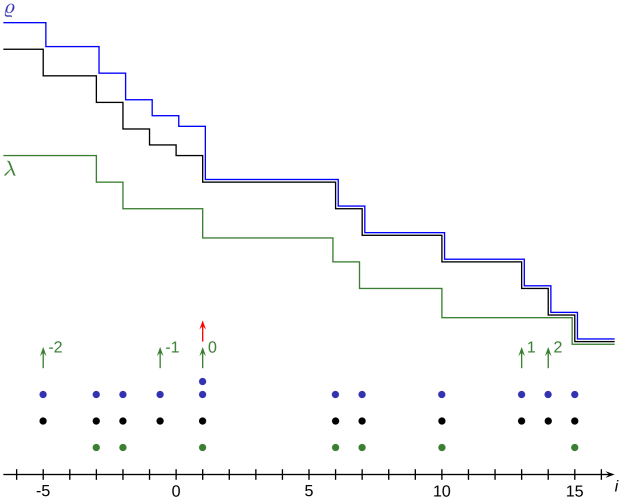

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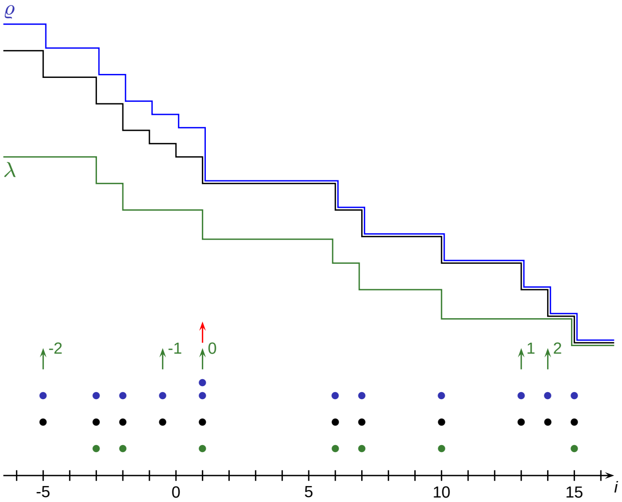

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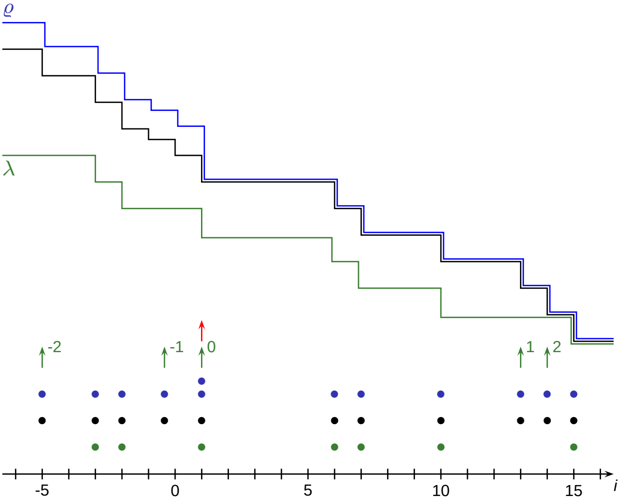

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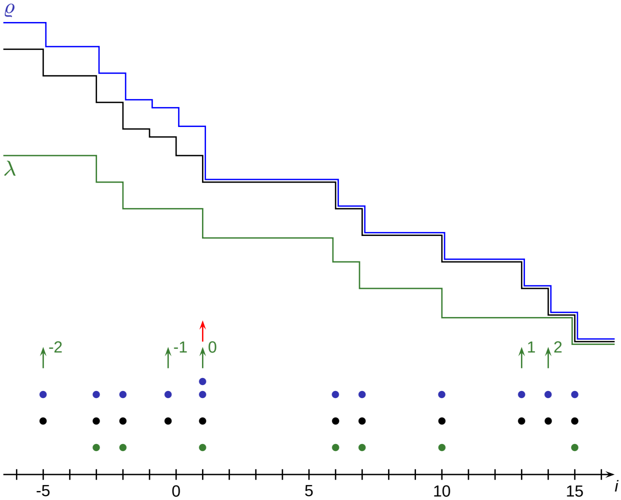

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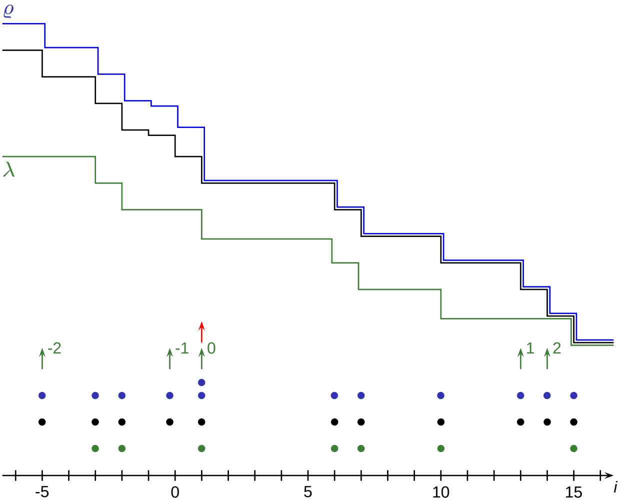

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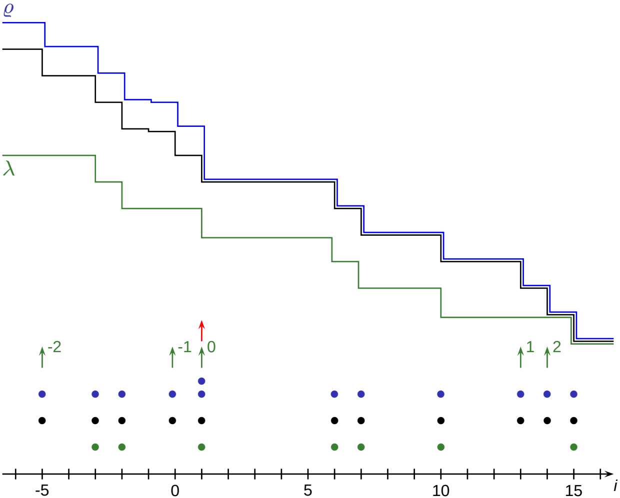

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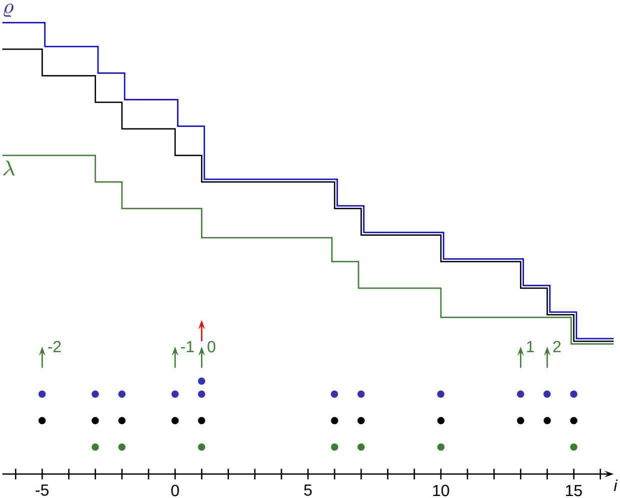

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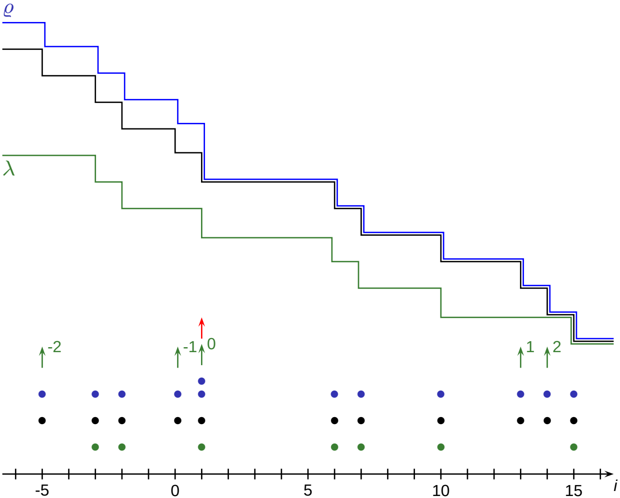

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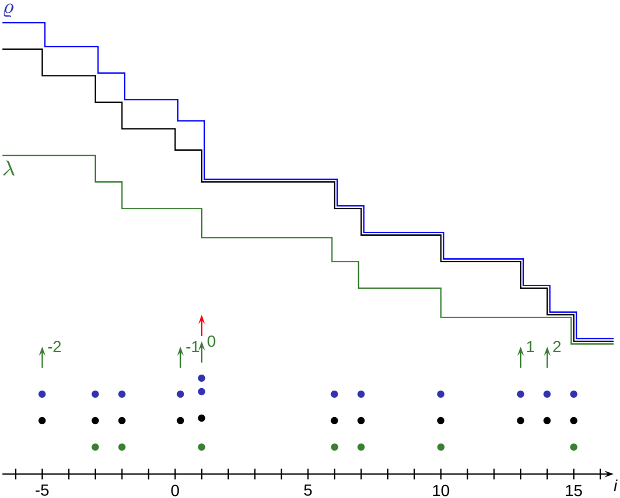

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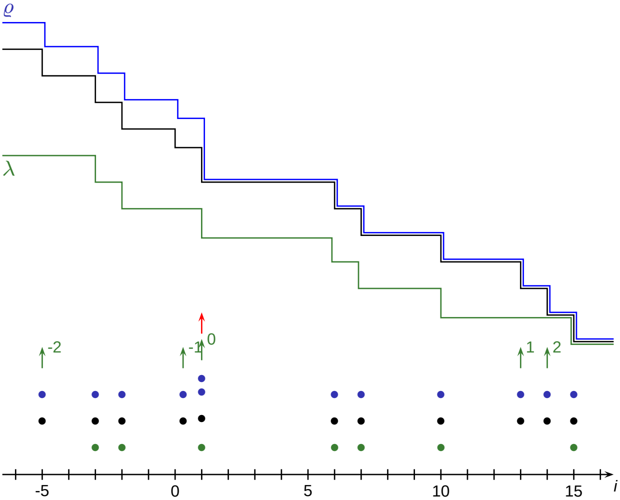

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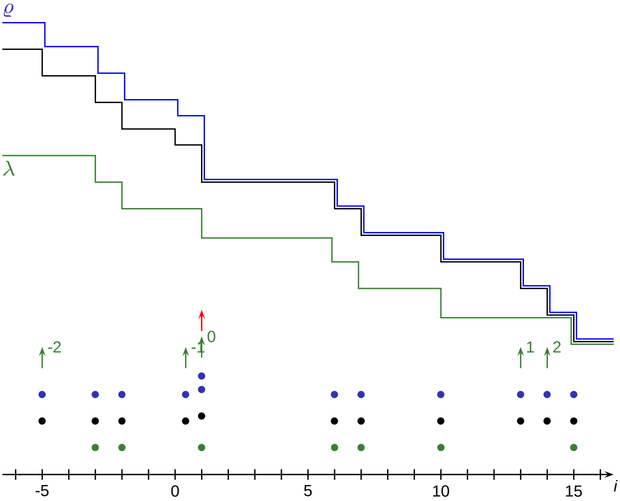

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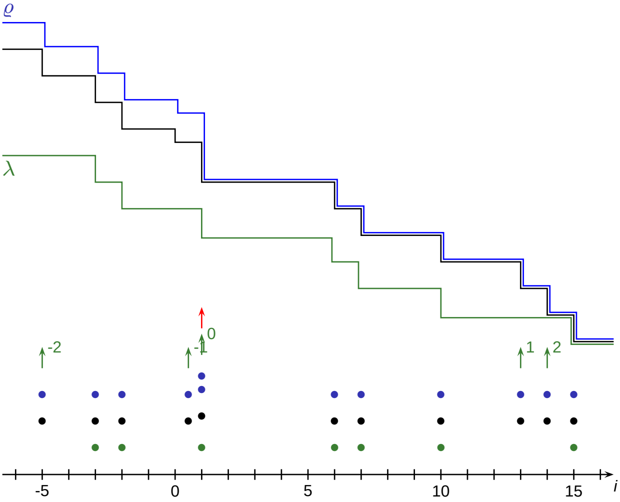

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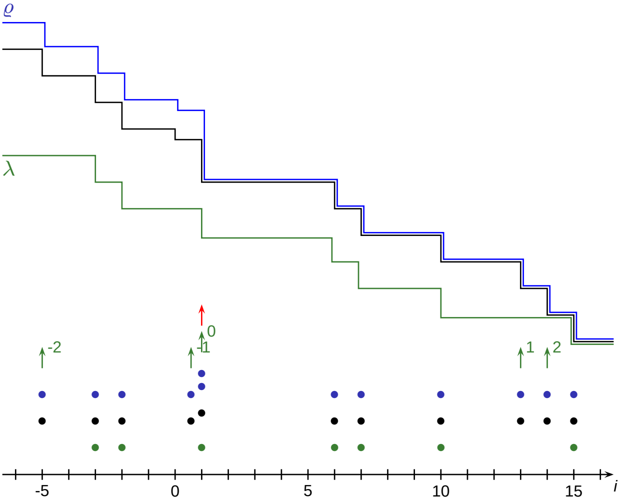

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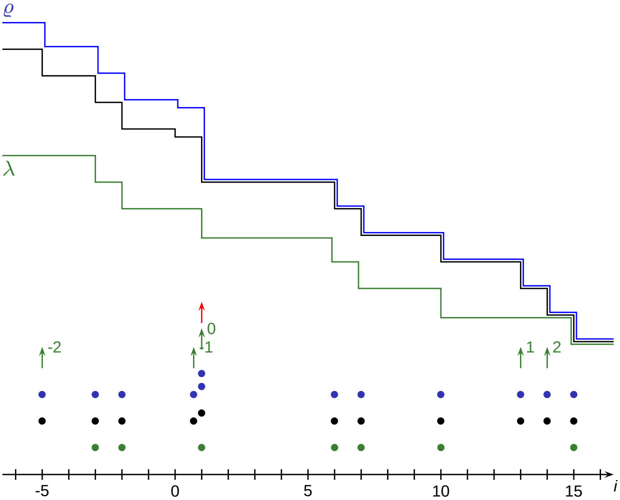

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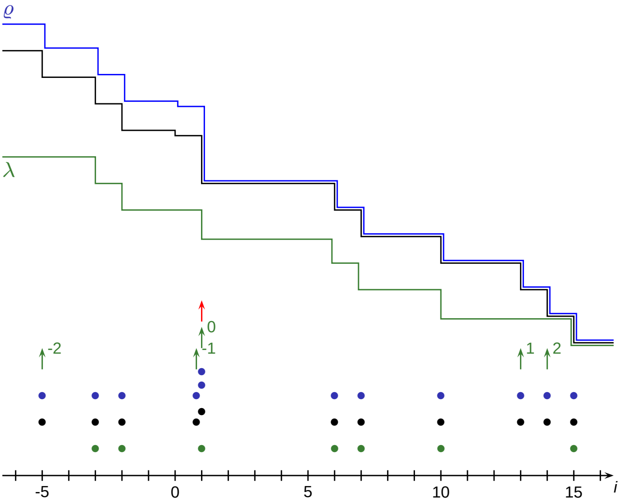

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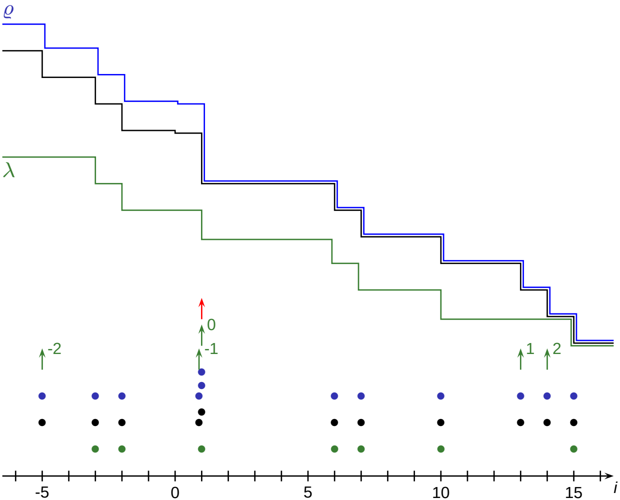

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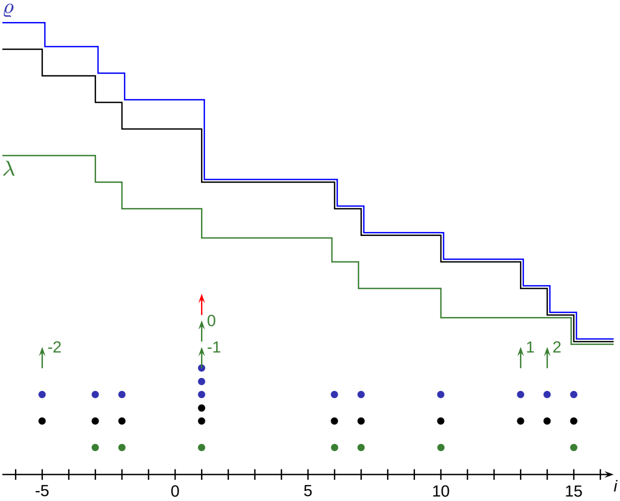

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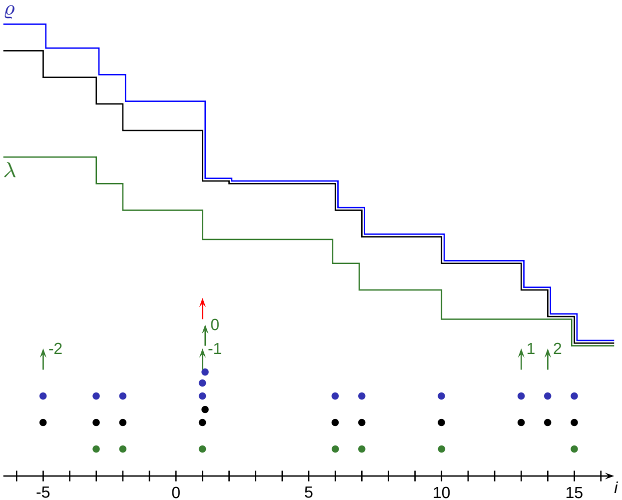

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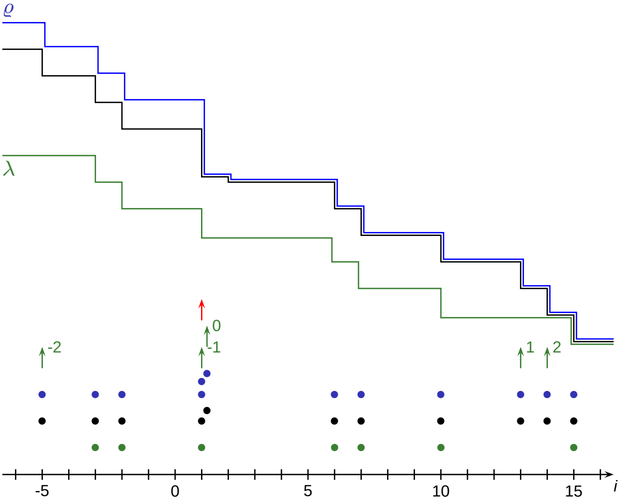

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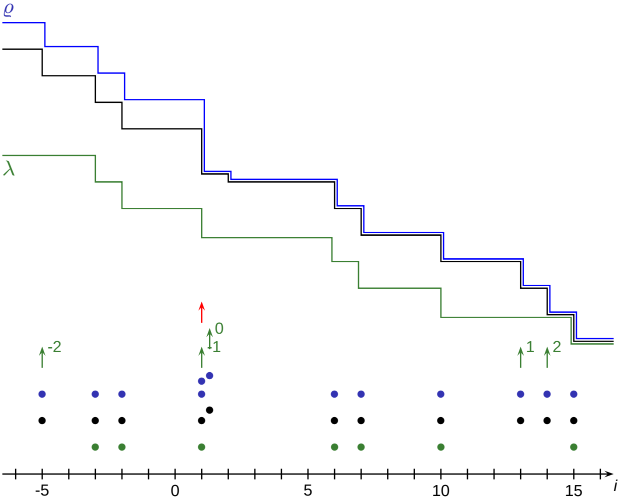

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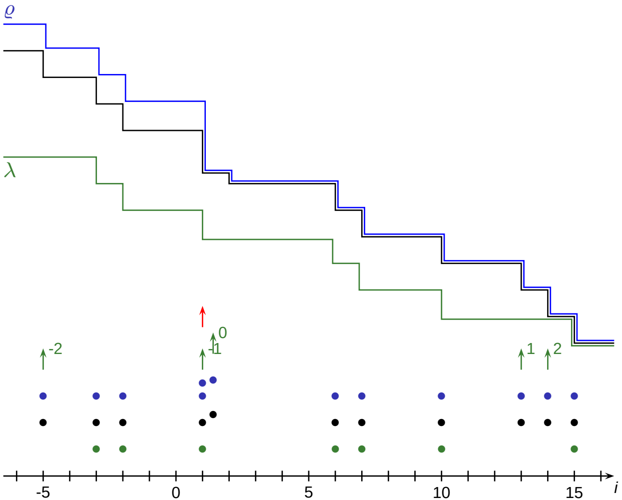

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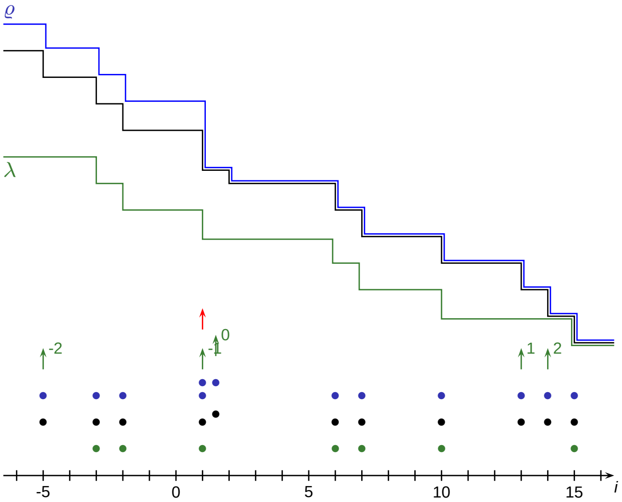

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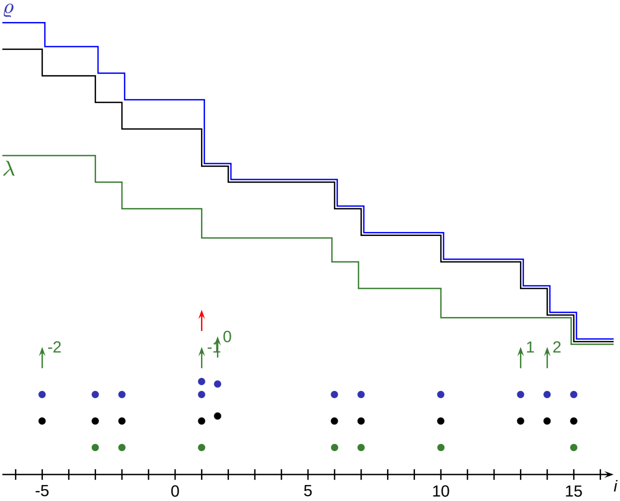

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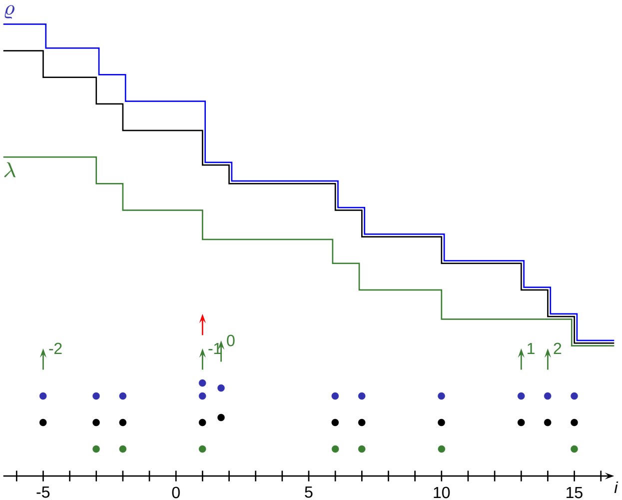

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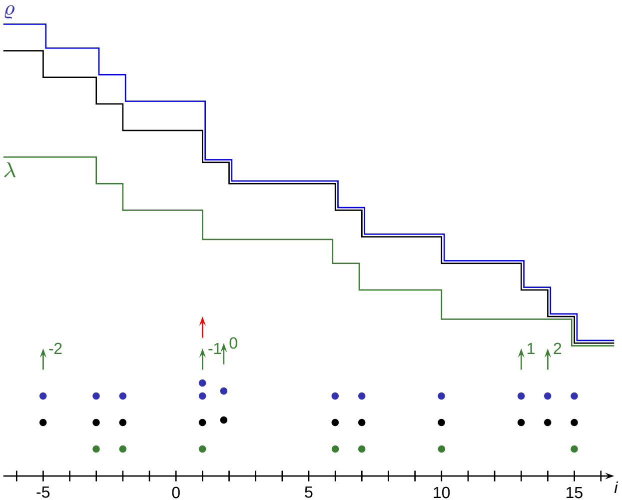

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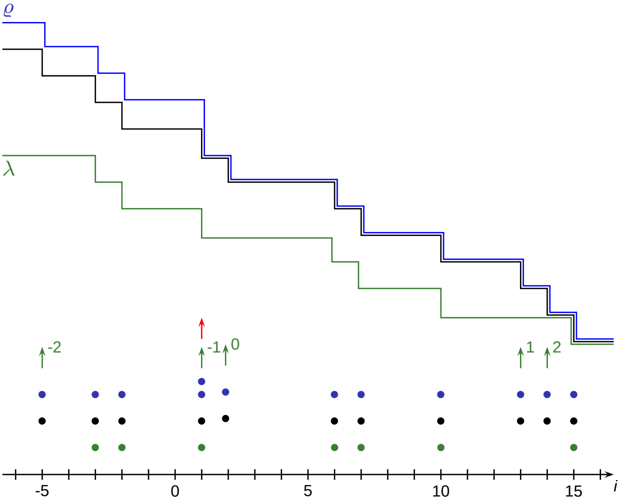

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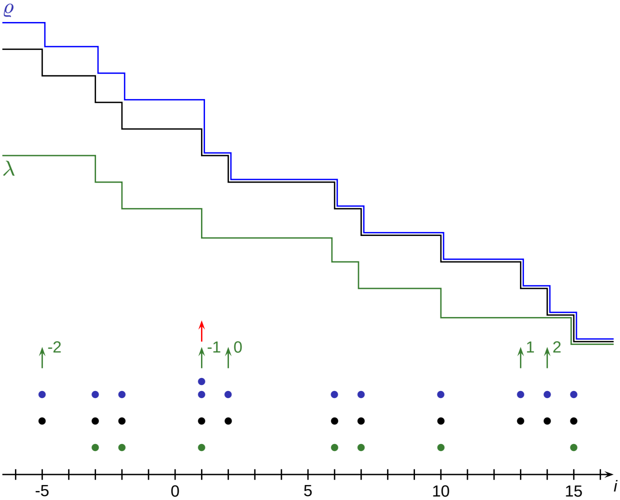

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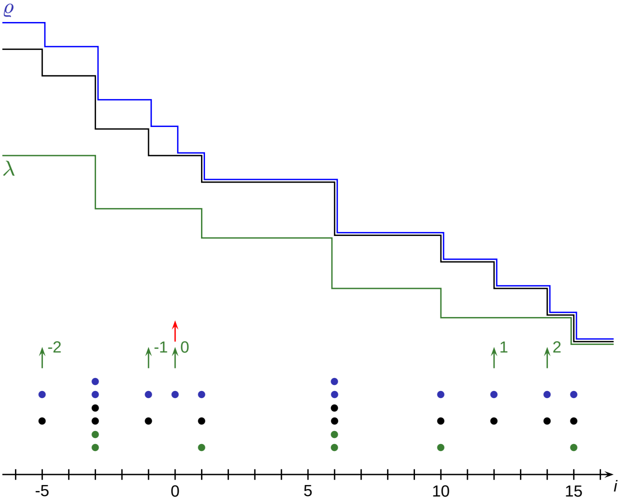

<u>Goal:</u> to understand $Q(t)$ on the background process of the ↑’s.

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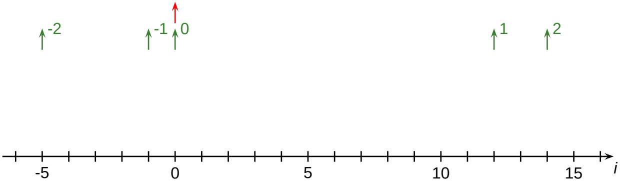

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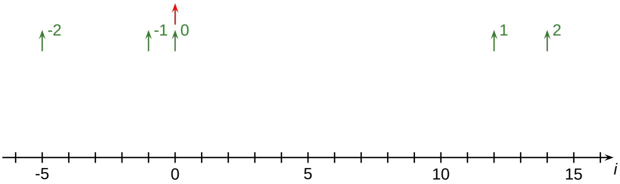

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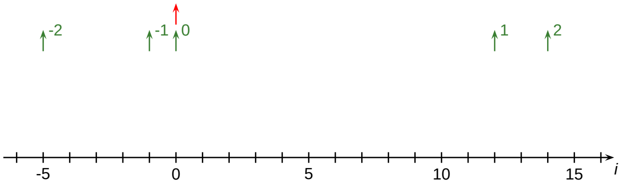

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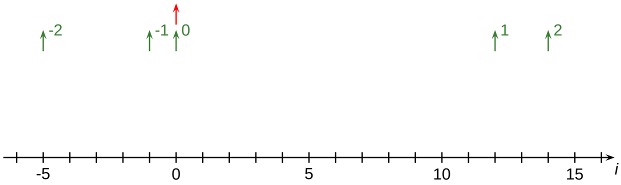

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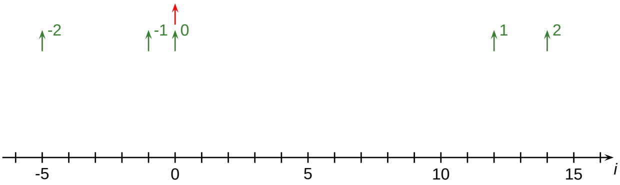

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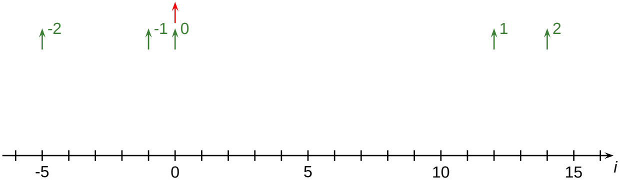

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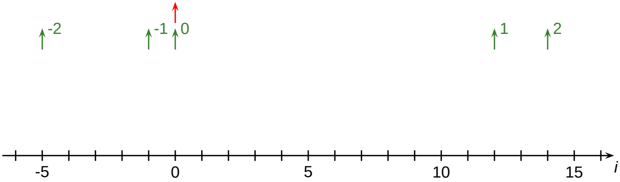

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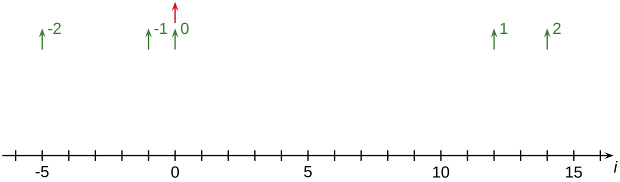

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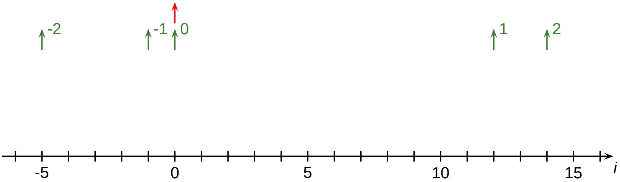

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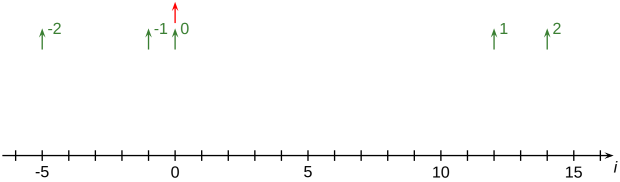

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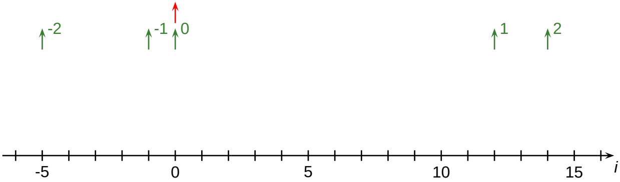

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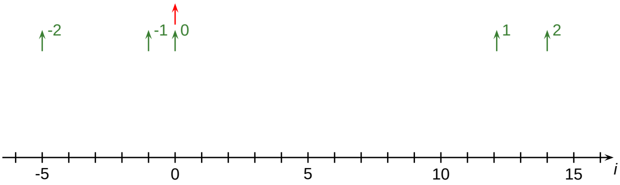

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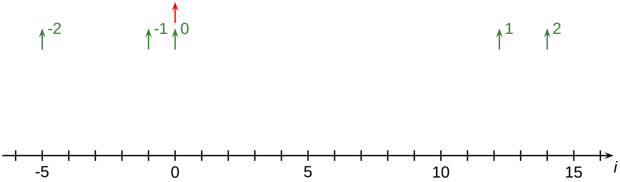

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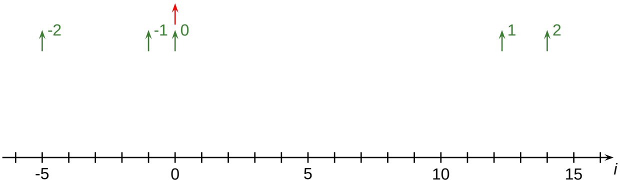

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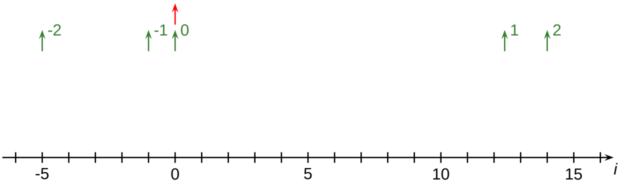

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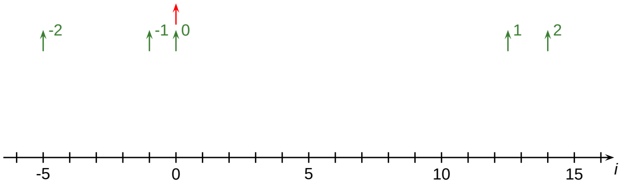

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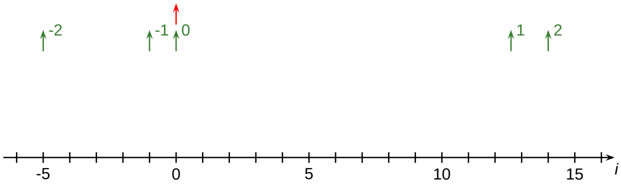

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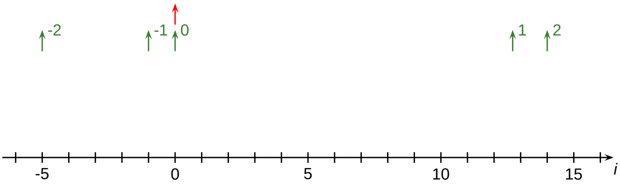

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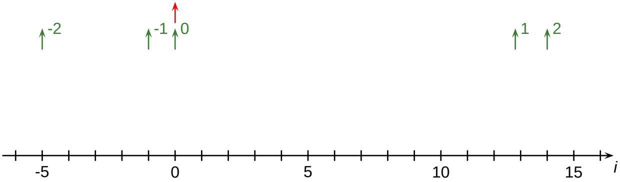

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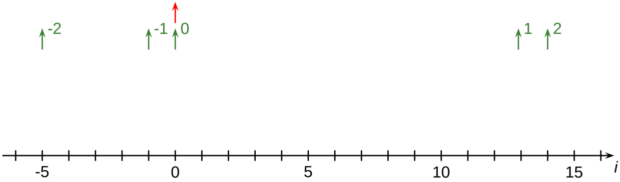

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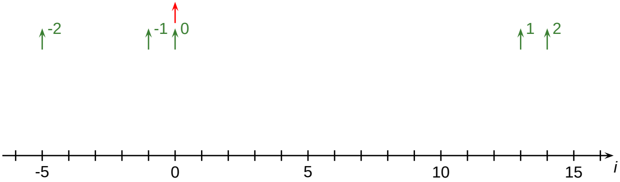

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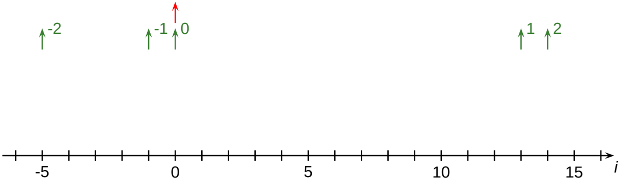

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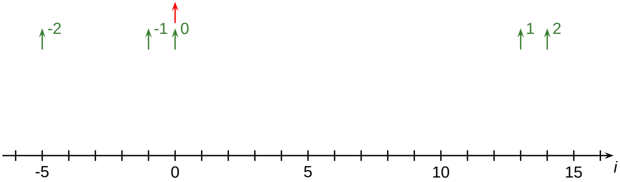

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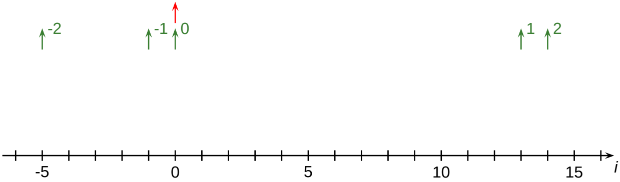

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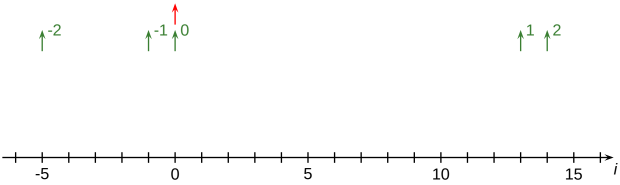

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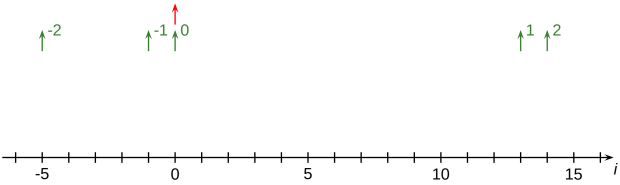

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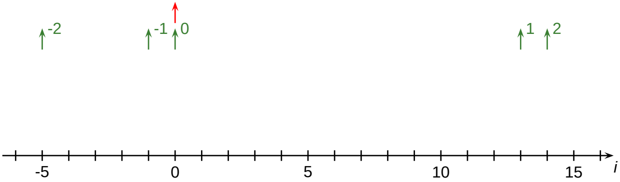

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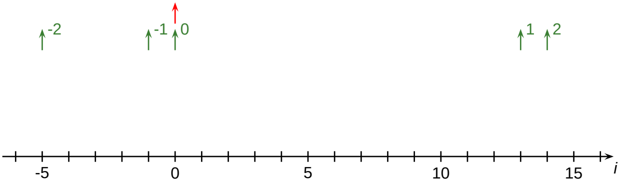

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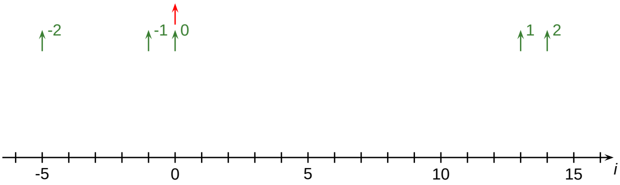

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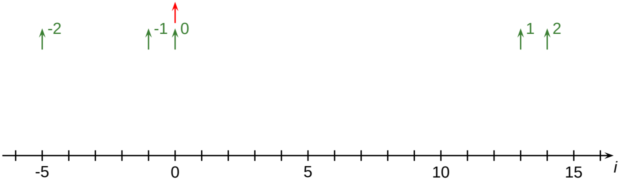

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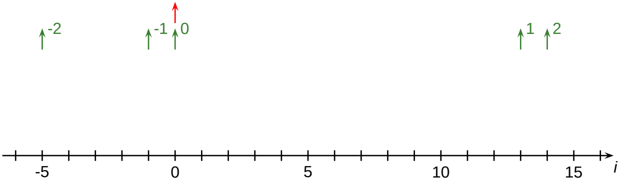

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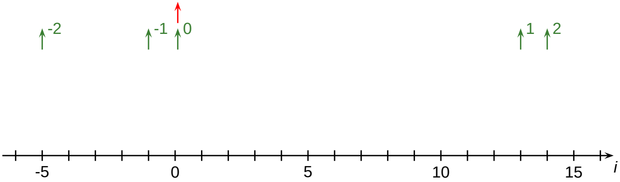

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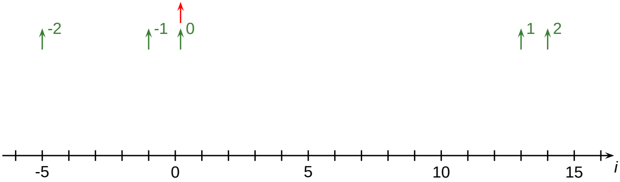

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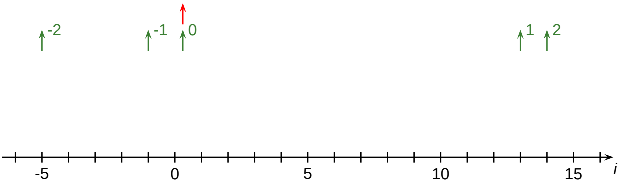

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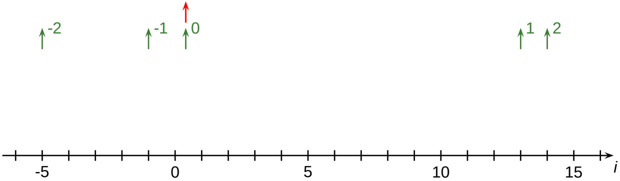

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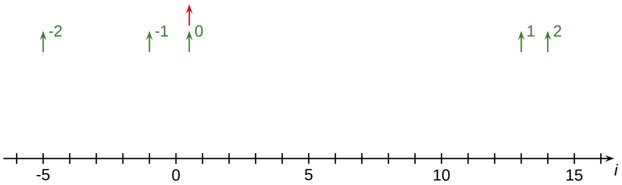

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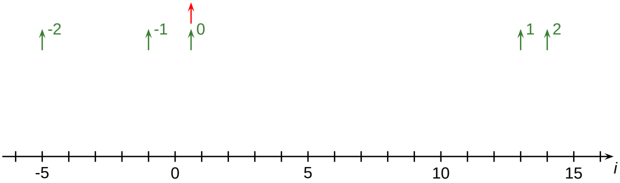

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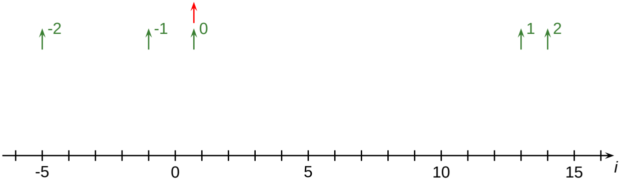

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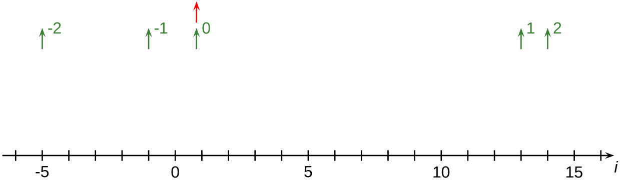

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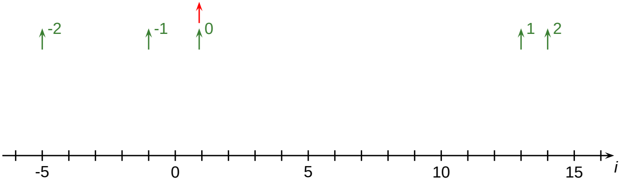

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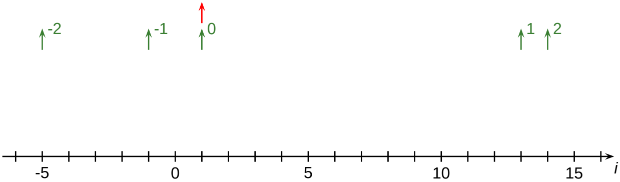

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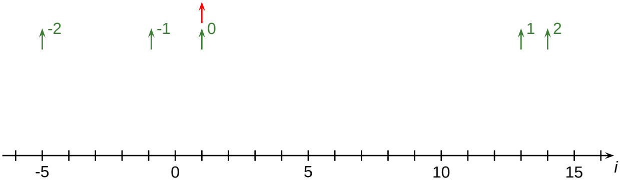

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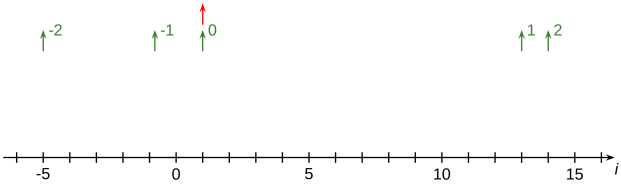

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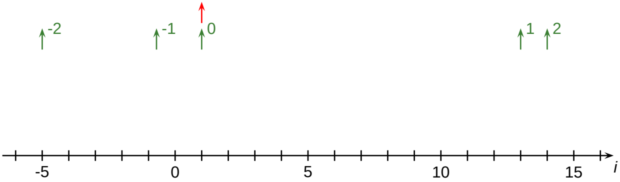

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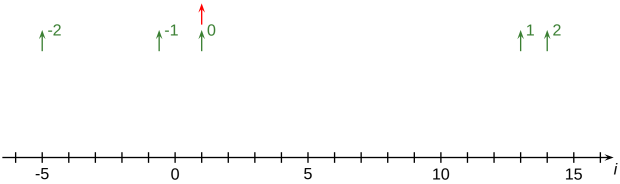

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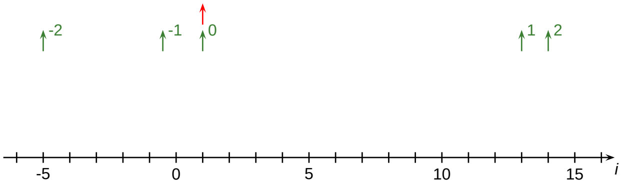

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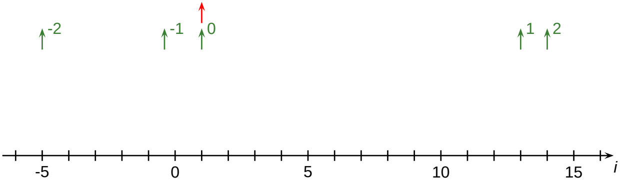

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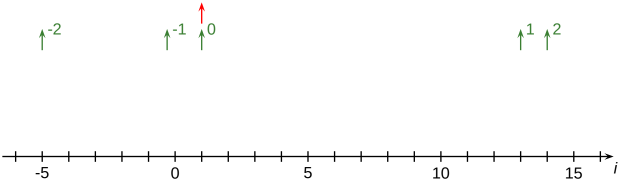

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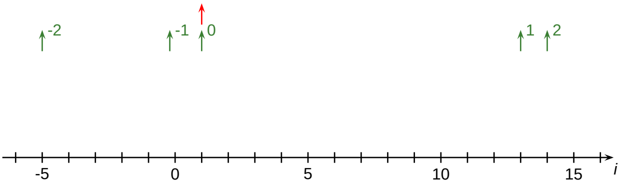

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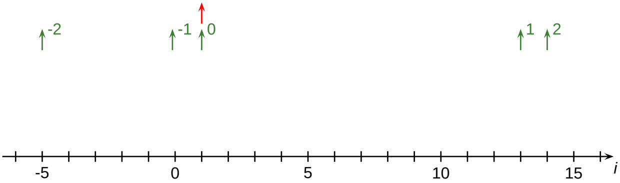

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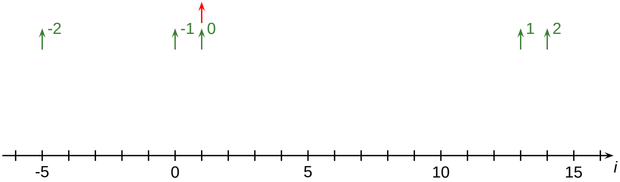

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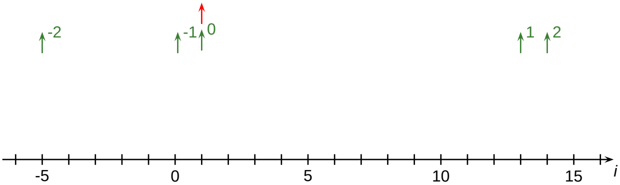

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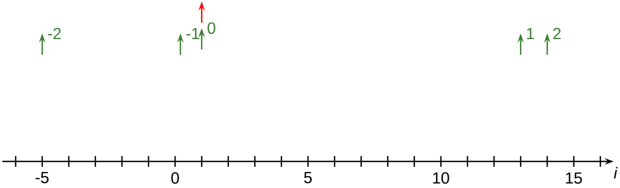

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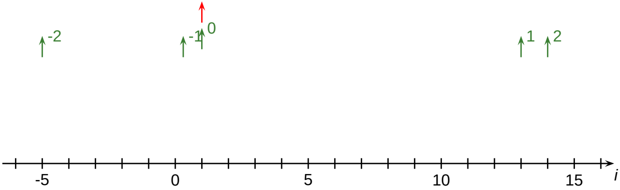

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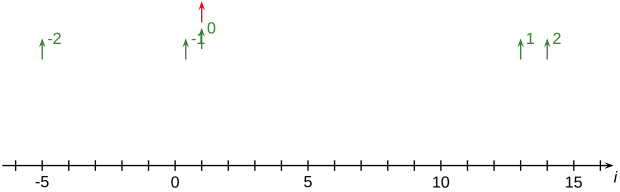

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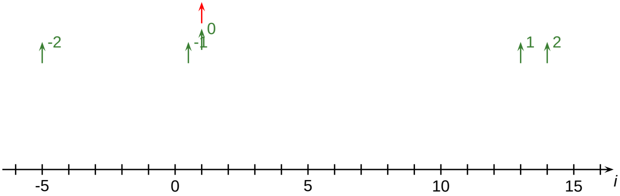

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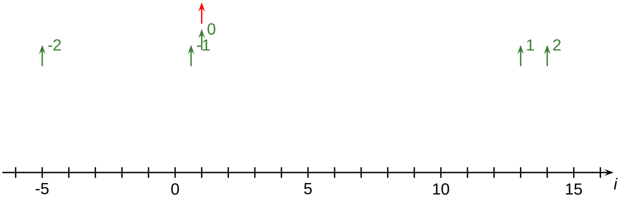

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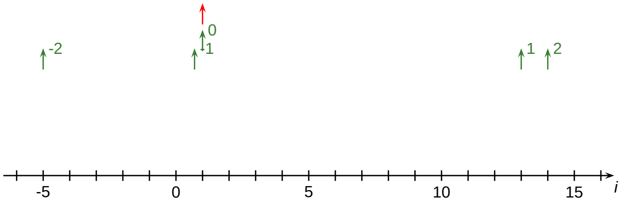

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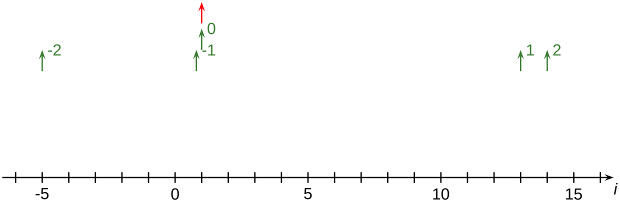

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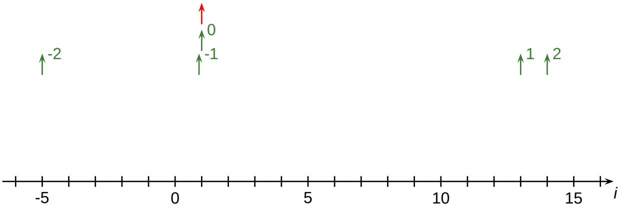

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Many second class particles plus one

$$m_Q(t) = [\text{the label of } \uparrow \text{ at } Q(t)] = \cancel{0}\,1$$

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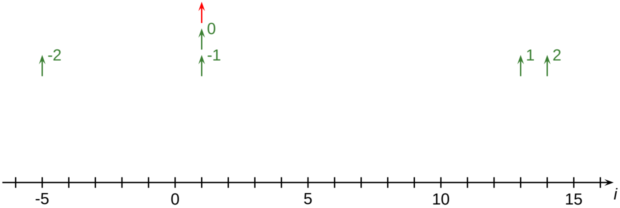

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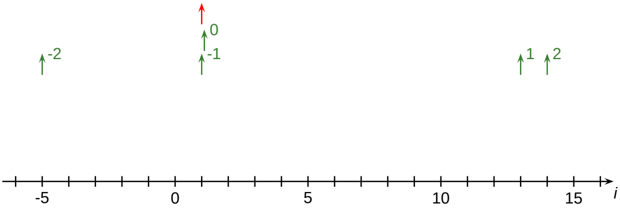

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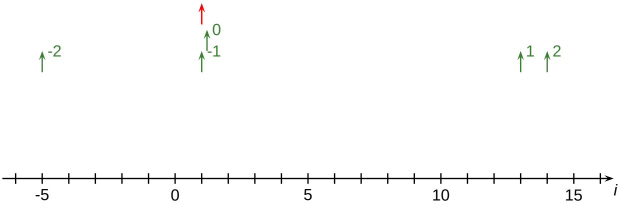

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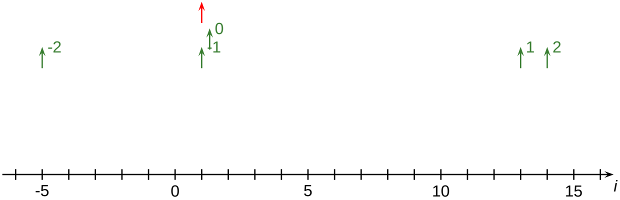

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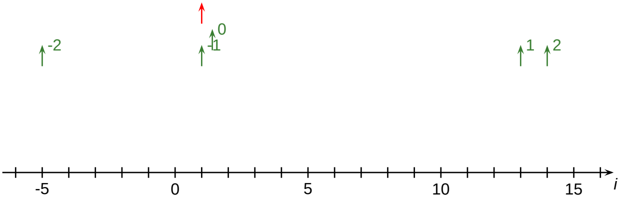

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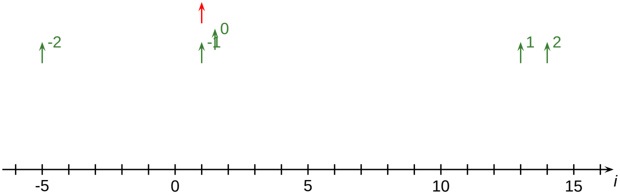

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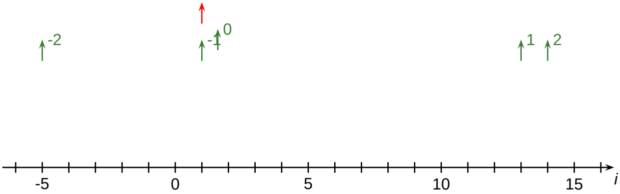

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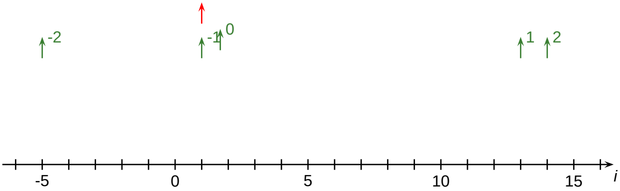

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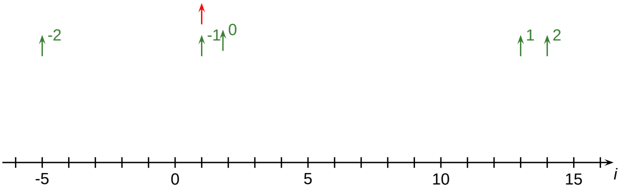

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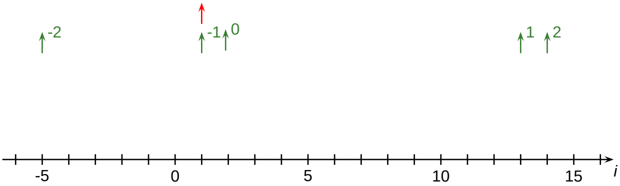

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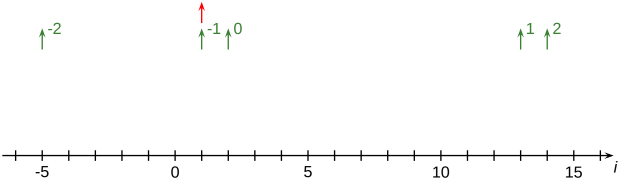

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Results

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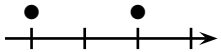
In their cases, we have

$$\lim_{t\to\infty} \frac{\mathbf{Var}(h_{Ct}(t))}{t^{1/2}} = \dots,$$

even convergence of the finite-dimensional distributions of the $h_{Ct}(t)$ process to Gaussian limits is known (Seppäläinen 2005, Ferrari and Fontes 1998, B., Rassoul-Agha and Seppäläinen 2006).

Nonconvex, nonconcave

And there are attractive asymmetric models with nonlinear, nonconvex and nonconcave hydrodynamics:

- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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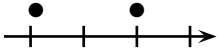
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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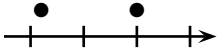
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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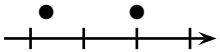
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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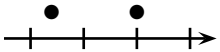
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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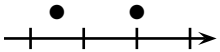
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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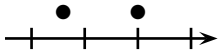
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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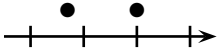
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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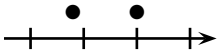
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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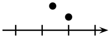
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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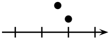
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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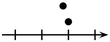
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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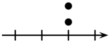
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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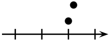
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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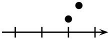
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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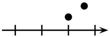
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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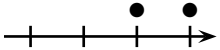
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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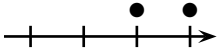
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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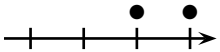
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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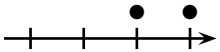
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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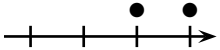
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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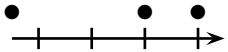
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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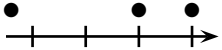
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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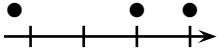
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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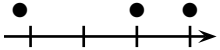
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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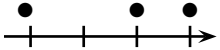
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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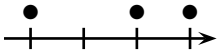
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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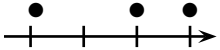
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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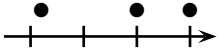
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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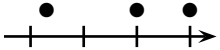
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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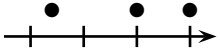
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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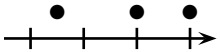
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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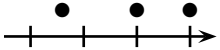
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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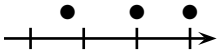
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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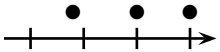
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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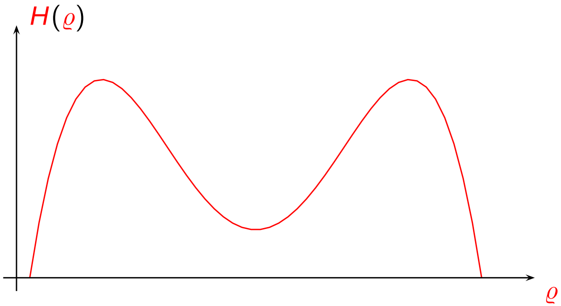

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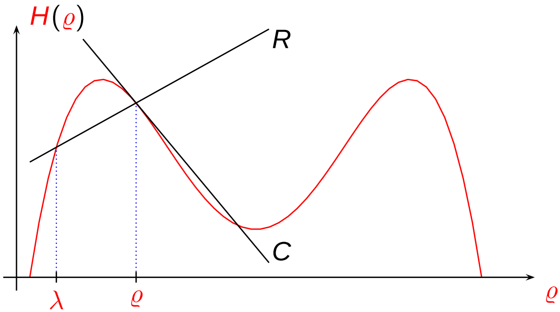

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$$Q(t) \overset{?}{\leq} X(t)$$

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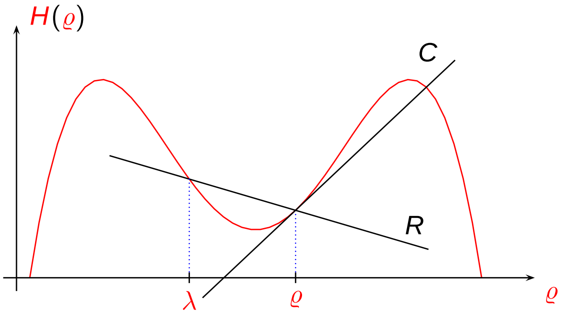

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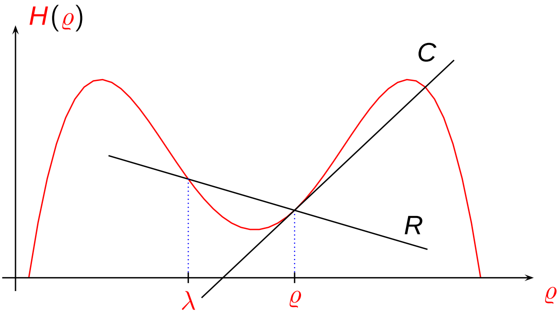

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Inequality changes with the density... ?

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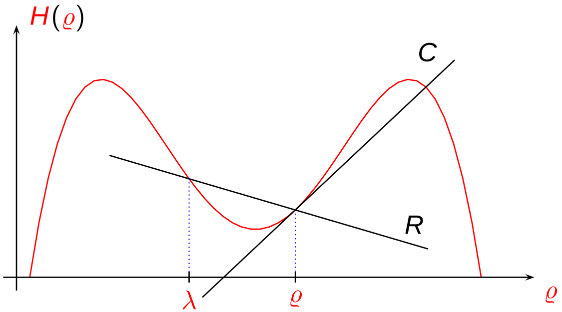

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Any coupling *must be very very tricky*.

Thank you.