School on interacting particle systems and random matrices

Simons Semester in Probability and Statistical Physics, Erdős Center Budapest, 16-20 June, 2025

Program

Monday, 16th June

10:00–11:00 *Registration and coffee*

11:00–11:50 Alice Guionnet
CLT and loop equations for Beta-ensembles (1)

11:50–14:00 *Lunch break*

14:00–15:00 Short talks:

14:00 Tian Yuan
The oriented swap process on half line

14:12 Leander Schnee
Equilibrium Fluctuations for a weakly asymmetric chain of anharmonic oscillators with a moving heat source

14:24 Carla Crucianelli
Interacting particle systems on sparse W-random graphs

14:36 Julian Zazueta Obeso
The Multivariate Markov-Krein Transform

14:48 Charlie Dworaczek Guera
Central Limit Theorem for β-Ensembles with Freud Weights: Application to the Variance Conjecture

15:15–16:05 Alice Guionnet/Thomas Buc-d'Alché
CLT and loop equations for Beta-ensembles (2)

16:05–16:35 *Coffee*

16:35–17:25 Bálint Virág
A brief user's manual to the directed landscape (1)

17:30–18:20 Charles Bordenave
Strong convergence of unitary representations: some applications (1)

18:30–20:00 *Wine reception*

Tuesday, 17th June

09:00–09:50 László Erdős
Multi-resolvent local laws and their applications (1)

09:50–10:20 *Coffee*

10:20–11:10 Alice Guionnet
CLT and loop equations for Beta-ensembles (3)

11:15–12:05 Bálint Virág
A brief user's manual to the directed landscape (2)

12:05–14:00 *Lunch break*

14:00–14:50 Charles Bordenave
Strong convergence of unitary representations: some applications (2)

14:50–15:20 *Coffee*

15:20–16:10 Ivan Corwin
The scaling limit of colored ASEP (1)

16:15–17:05 László Erdős
Multi-resolvent local laws and their applications (2)

17:20–18:20 Short talks:

17:20 Rémi Bonnin
A Wigner's theorem and a free probability theory for random tensors

17:32 Péter Juhász
Scaling limits in spatial birth-death systems with long-range interactions

17:44 Matteo Sfragara
Chaos and concentration in spatial growth models

17:56 Luke Peilen
Local Laws and Fluctuations for Riesz Gases

18:08 Ghurumuruhan Ganesan
Dissimilar Batch Decompositions of Random Datasets

Wednesday, 18th June

09:00–09:50 László Erdős
Multi-resolvent local laws and their applications (3)

09:55–10:45 Nina Gantert
Exclusion processes: classical results and new questions (1)

10:45–11:15 *Coffee*

11:15–12:05 Bálint Virág
A brief user's manual to the directed landscape (3)

12:10–13:00 Ivan Corwin
The scaling limit of colored ASEP (2)

Thursday, 19th June

09:00–09:50 Nina Gantert
Exclusion processes: classical results and new questions (2)

09:50–10:20 *Coffee*

10:20–11:10 Alice Guionnet
CLT and loop equations for Beta-ensembles (4)

11:15–12:05 Ivan Corwin
The scaling limit of colored ASEP (3)

12:05–14:00 *Lunch break*

14:00–14:50 Nina Gantert
Exclusion processes: classical results and new questions (3)

14:50–15:20 *Coffee*

15:20–16:10 Charles Bordenave
Strong convergence of unitary representations: some applications (3)

16:15–17:05 Ivan Corwin
The scaling limit of colored ASEP (4)

17:20–18:20 Short talks:

17:20 Amiya Das
Dynamics of chirped periodic and self-similar solitary waves in nonlocal nonlinear saturable media

17:32 Berend van Tol
Intertwining and propagation of mixture measures for IPS

17:44 Mixalis Louvaris
Density of growth-rates of subgroups of a free group and the non-backtracking spectrum of the configuration model

17:56 Sunita Rani
Spectrum of random centrosymmetric matrices; CLT and Circular law

Friday, 20th June

09:00–09:50 Charles Bordenave
Strong convergence of unitary representations: some applications (4)

09:55–10:45 Bálint Virág
A brief user's manual to the directed landscape (4)

10:45–11:15 *Coffee*

11:15–12:05 Nina Gantert
Exclusion processes: classical results and new questions (4)

12:10–13:00 László Erdős
Multi-resolvent local laws and their applications (4)

Minicourse Abstracts

Charles Bordenave (Aix-Marseille Université):

Strong convergence of unitary representations: some applications

Mon 17:30, Tue 14:00, Thu 15:20, Fri 09:00

Strong convergence is a convergence for matrix algebras at the level of operator norms. Since the fundamental work of Haagerup-Thorbjørnsen in 2005 on the strong convergence of independent Gaussian unitary matrices, this notion has attracted a lot of attention. It plays a central role in recent advances in operator algebras, spectral graph theory or spectral geometry. In these lectures, we will explore the particularly intriguing case of strong convergence for unitary representations of finitely generated groups. Special emphasis will be placed on its applications, including the cutoff phenomenon for Markov chains and quantum ergodicity.

Ivan Corwin (Columbia University):

The scaling limit of colored ASEP

Tue 15:20, Wed 12:10, Thu 11:15, 16:15

I will describe how the Yang-Baxter equation and machinery of Gibbsian line ensembles provide a systematic route to extract the full space-time scaling limit of the colored asymmetric simple exclusion process and its close relative, the colored stochastic six vertex model. This is based on joint work with Amol Aggarwal and Milind Hegde.

László Erdős (IST Austria):

Multi-resolvent local laws and their applications

Tue 09:00, 16:15, Wed 09:00, Fri 12:10

Classical local laws in random matrix theory assert that the resolvents of large random matrices tend to be deterministic even for spectral parameters very close to the real axis. They are robust and provide essential a priori bounds for eigenvalue and eigenvector distributions that are routinely used in more sophisticated analysis. Products of resolvents also tend to be deterministic, but they are not simply given as a product of single resolvent approximations. In this series of lectures we present a theory of multi-resolvent local laws. The proofs are dynamical, they rely on the so-called zig-zag strategy; a successive alternate application of two different stochastic flows. In the second part of the lectures we focus on applications that include the proof of the Eigenstate Thermalisation Hypothesis and the Law of Fractional Logarithm for general Hermitian random matrices, as well as CLT for linear eigenvalue statistics and the Gumbel law for extremal eigenvalues for non-Hermitian random matrices.

Nina Gantert (TU Munich):

Exclusion processes: classical results and new questions

Wed 09:55, Thu 09:00, 14:00, Fri 11:15

We introduce exclusion processes on infinite and (sequences of) finite graphs, both in the symmetric and the asymmetric case. We address questions about the equilibrium distributions and their properties. We explain some results about the rate of convergence to equilibrium in the finite case. We also treat old and recent results about the speed of a tagged particle in exclusion processes on infinite graphs. No prerequisites are needed, and there will be a focus on open problems.

Alice Guionnet (ENS de Lyon UMPA):
CLT and loop equations for Beta-ensembles
Mon 11:00, 15:15, Tue 10:20, Thu 10:20

The study of global fluctuations in random matrices or Beta-ensembles requires the development of new tools due to the strong correlations of the eigenvalues. One of these is based on the analysis of so-called loop (or Dyson-Schwinger) equations. In this course, we will first recall how to use this approach to analyze (real) Beta-ensembles, following the work of Johansson and G-Borot. We will then discuss how this approach can be generalized to Beta-ensembles through the so-called Nekrasov equations, as introduced in joint work with Borodin and Gorin. Finally, we will discuss more recent developments such as tilings with not simply connected domains and complex potentials.

Bálint Virág (University of Toronto):
A brief user's manual to the directed landscape
Mon 16:35, Tue 11:15, Wed 11:15, Fri 09:55

The directed landscape is the conjectured scaling limit of first passage percolation in 2 dimensions. But how does one prove theorems about such an object? I will work through several examples from recent results.

Abstracts of Short Talks

Rémi Bonnin (Aix-Marseille University):
A Wigner's theorem and a free probability theory for random tensors
Tue 17:20

We define higher-order analogues of the semicircular (or Wigner) law and the free Poisson (or Marcenko-Pastur) law. We prove the convergence of a Wigner-type (resp. Wishart) tensor to the semicircular law (resp. free Poisson law), and we present a free Central Limit Theorem. Next we build a notion of tensor freeness, with associated free cumulants. We develop a notion of free convolution for tensors and give the first examples of tensorial free convolution for some measures.

Joint work with Charles Bordenave.
Based on `arXiv:2412.02572`, `arXiv:2407.18881`, `arXiv:2404.14144`.

CANCELLED: Thomas Chouteau (Universidade de São Paulo):
Deformations of orthogonal polynomials with cubic potential and of discrete Painlevé I equation

Recent developments in integrable systems and random matrix theory have extended classical results and connected deformed models of determinantal point processes, random matrices, and orthogonal polynomials with integro-differential versions of Painlevé equations. In this talk, I will present some results from a work in progress in collaboration with G. Silva (ICMC-USP) and M. Yattselev (Purdue University). After recalling some results connecting random matrices and orthogonal polynomials(OPs), I will introduce a deformation of a family of OPs with respect to a cubic weight and describe how relevant quantities (coefficients of the three terms recursion, Christoffel-Darboux kernels) connect with a perturbation of the Painlevé I equation.

Carla Crucianelli (Princeton University):
Interacting particle systems on sparse W-random graphs
Mon 14:24

We consider a general interacting particle system with interactions on a random graph and study the large population limit of this system. When the sequence of underlying graphs converges to a graphon, we show convergence of the interacting particle system to a so-called graphon stochastic differential equation. This is a system of uncountable many SDEs of McKean-Vlasov type driven by a continuum of Brownian motions. We make sense of this equation in a way that retains joint measurability and essentially pairwise independence of the driving Brownian motions of the system by using the framework of Fubini extensions. The convergence result is general enough to cover nonlinear interaction, as well as various examples of sparse graphs. Moreover, we extend the results to the unbounded graphon case.

Amiya Das (University of Kalyani, India):
Dynamics of chirped periodic and self-similar solitary waves in nonlocal nonlinear saturable media
Thu 17:32

In this study, we consider a generalized nonlinear Schr odinger equation that incorporates both nonlocal response and saturable nonlinearity. We investigate traveling wave solutions arising from the interplay of competing nonlocal nonlinear effects within saturable media. Utilizing the similarity transformation method, we construct chirped self-similar solitary wave solutions under specific conditions. The dynamical structure of these self-similar waves is further explored within a switching control framework featuring periodically distributed gain. To gain deeper insight into the wave dynamics, we perform phase plane analysis and examine modulational instability characteristics.

Charlie Dworaczek Guera (KTH Stockholm):
Central Limit Theorem for β-Ensembles with Freud Weights: Application to the Variance Conjecture
Mon 14:48

The β-ensemble is a probabilistic model of N particles on the real line, confined by a potential V and interacting via a logarithmic repulsion. For certain choices of V, this probability measure coincides with the distribution of the spectrum of certain random matrix ensembles. In this talk, we focus on the potential $V(x) = |x|^p$ with $p \in (2, 3)$. Since V is not of class C^3, much of the existing literature – on CLTs, partition function expansions...– does not directly apply. We will demonstrate that a Central Limit Theorem still holds in this singular setting and establish the variance conjecture – a relaxed form of the KLS conjecture in geometric analysis – for a specific class of convex bodies. This is joint work with Ronan Memin and Michel Pain (both at IMToulouse).

Ghurumuruhan Ganesan (University of Bristol):
Dissimilar Batch Decompositions of Random Datasets
Tue 18:08

For better learning, large datasets are often split into small batches and fed sequentially to the predictive model. In this paper, we study such batch decompositions from a probabilistic perspective. We assume that data points (possibly corrupted) are drawn independently from a given space and define a concept of similarity between two data points. We then consider decompositions that restrict the amount of similarity within each batch and obtain high probability bounds for the minimum size. We demonstrate an inherent tradeoff between relaxing the similarity constraint and the overall size and also use martingale methods to obtain bounds for the maximum size of data subsets with a given similarity.

Péter Juhász (Aarhus University, Denmark):
Scaling limits in spatial birth-death systems with long-range interactions
Tue 17:32

In this talk, we examine the interaction graph of a novel bipartite spatial birth-death particle system characterized by long-range interactions and heavy-tailed degree distributions. Studying such models is crucial for understanding complex systems, such as scientific collaboration networks. In our model, we represent authors as entities with exponentially distributed lifetimes and papers as timestamped events. Furthermore, regularly varying weights assigned to both authors and papers influence their spatial interaction range, resulting in a dynamic spatial interaction graph. In the large-system limit, we show that, depending on the tail index of the author–paper degree distribution, the scaling limit of the normalized author–paper edge count is a Gaussian autoregressive process in regimes with lighter tails, and a non-Markovian alpha-stable process in the heavy-tailed case.

Mixalis Louvaris (Gustave Eiffel University):
Density of growth-rates of subgroups of a free group and the non-backtracking spectrum of the configuration model
Thu 17:56

In this talk we will prove that the set of growth-rates of subgroups of a rank r free group is dense in $[1, 2r-1]$. Our main technical contribution is the probabilistic method. Specifically we prove a concentration result for the leading eigenvalue of the non-backtracking matrix in the configuration model, i.e. the uniform distribution amongst all graphs with a given degree sequence. Our main influence for the probabilistic claim are the papers [1] and [2]. This talk is based on the recent paper [3].

References

[1] C. Bordenave, M. Lelarge, and L. Massoulié, "Non-backtracking spectrum of random graphs: community detection and non-regular ramanujan graphs", in *2015 IEEE 56th Annual Symposium on Foundations of Computer Science.* IEEE, 2015, pp. 1347-1357.

[2] C. Bordenave, "A new proof of friedman's second eigenvalue theorem and its extension to random lifts", in *Annales Scientifiques de l'École Normale Supérieure*, vol. 4, no. 6, 2020, pp. 1393-1439.

[3] M. Louvaris, D. T. Wise, and G. Yehuda, "Density of growth-rates of subgroups of a free group and the non-backtracking spectrum of the configuration model", *arXiv preprint arXiv:2404.07321*, 2024.

Luke Peilen (Temple University):
Local Laws and Fluctuations for Riesz Gases
Tue 17:56

We study the statistical mechanics of the Riesz gas for general potential and inverse temperature. In particular, our analysis includes the one-dimensional log gas, which is of interest to the random matrix theory. By generalizing a bootstrap procedure used in the study of Coulomb gases, we prove local laws on a novel next order energy quantity that are valid down to microscopic length scales. Simultaneously, we exhibit a control on fluctuations of linear statistics that is also valid down to microscopic scales. Using these local laws, we recover and extend known CLT results for the log-gas and exhibit a CLT for Riesz gases in dimension 2 for kernels with a sufficiently small power. Our main technical novelty involves a generalization of the transport calculus introduced by Serfaty and her coauthors to general Riesz gases, where the nonlocality presents new difficulties. This is joint work with Sylvia Serfaty.

Sunita Rani (IIT Bhubaneswar):
Spectrum of random centrosymmetric matrices; CLT and Circular law
Thu 18:08

We analyze the asymptotic fluctuations of linear eigenvalue statistics of random centrosymmetric matrices with i.i.d. entries. We prove that for a complex analytic test function, the centered and normalized linear eigenvalue statistics of random centrosymmetric matrices converge to a normal distribution. We find the exact expression of the variance of the limiting normal distribution via combinatorial arguments. Moreover, we also argue that the limiting spectral distribution of properly scaled centrosymmetric matrices follows the circular law.

Leander Schnee (Freie Universit at Berlin):
Equilibrium Fluctuations for a weakly asymmetric chain of anharmonic oscillators with a moving heat source
Mon 14:12

We consider the scaling limit of the fluctuation field around the equilibrium of a particle system with a Hamiltonian of exponential type and added noise. Scaling the time by n^2 (diffusive) and the asymmetric Hamiltonian part by $n^{-1/2}$ (weak) it is known that the equilibrium fluctuations converge to the energy solutions of the stochastic Burgers equation (SBE). This result is directly related to the KPZ universality class. We investigate different ways to get a boundary condition for this limiting equation by looking at modified particle dynamics. In particular adding a heat source to the microscopic model that moves with the asymmetric drift of the system and scales like $n^{-\delta}$, we show that for $\delta \leq 1$ we get the SBE equation with one Dirichlet boundary which is exhibited in different ways depending on the different values of δ. For $\delta > 1$ we see the SBE on $\mathbb{R}$ without a boundary, which is known to be well-posed. This is a joint work in progress with Ana Djurdjevac, Cédric Bernardin and Patrícia Gonçalves.

Matteo Sfragara (University of Padova):
Chaos and concentration in spatial growth models
Tue 17:44

A decade and a half ago Chatterjee established the first rigorous connection between anomalous fluctuations (superconcentration) and a chaotic behaviour of the ground state in certain Gaussian disordered systems. We study the connection between chaos and concentration in spatial growth models, like first-passage percolation (FPP) and last-passage percolation (LPP), and we prove that they exhibit a chaotic behaviour. This extends previous work on the topic, and illustrates that this is a phenomenon that can be expected more widely. The notion of 'chaos' refers to the sensitivity of the optimal path (geodesic) when exposed to a slight perturbation. In FPP on $\mathbb{Z}^d$ the geodesic is the time-minimizing path from the origin to a vertex v, while in LPP on the square lattice $[0, n]^2$ the geodesic is the weight-maximizing up-right path from $(0, 0)$ to (n, n). This talk is based on two joint works with Daniel Ahlberg and Mia Deijfen (Stockholm University).

Berend van Tol (TU Delft):
Intertwining and propagation of mixture measures for IPS
Thu 17:44

We consider a class of stochastic models of discrete and continuous mass transport. For these models, one can prove that mixtures of equilibrium product measures are left invariant and the mixture measure evolves according to a Markov process which we call the "hidden parameter model". This generalizes earlier results to a larger class of models on general graphs. The models include discrete and continuous generalized KMP models, as well as discrete and continuous harmonic models. The results imply that in all these models, the non-equilibrium steady state of their boundary driven version is a mixture of product measures where the mixture measure is the stationary state of the corresponding hidden parameter model. For the harmonic models on the chain $\{1, \ldots, N\}$ with nearest neighbor edges, we obtain that the stationary measure of the hidden parameter model is the joint distribution of gapped order statistics of uniforms, with a purely probabilistic proof.

Tian Yuan (Max Planck/Leipzig University):
The oriented swap process on half line
Mon 14:00

The oriented swap process was first explored by Angel, Holroyd, and Romik. We extend the model to the half space case and prove several asymptotic results. Our approach relies on a relationship between multi-species particle systems and Hecke algebras, complemented by a detailed asymptotic analysis.

Julian Zazueta Obeso (CIMAT Guanajuato, Mexico):
The Multivariate Markov-Krein Transform
Mon 14:36

The Markov-Krein transform is a bijective transformation that links probability measures with certain signed measures on the real line. Its origins trace back to fundamental work in classical probability as Sergei Kerov explains in detail on 1997, this relation naturally appears in various contexts, including: Interlaced measures, The moment problem, Functions with positive imaginary part, Spectral shift function of self-adjoint operators and Young diagrams. Recently, this transform has emerged in areas related to non-commutative probability and random matrices, and these developments have led to an extension of the transform to a multivariate version. In the talk, I will provide an overview of the known results on the Markov-Krein transform, present examples and connections with random matrices, and discuss its recent extension to the multivariate setting. In particular, I will highlight its connections to non-commutative probability and random matrices, focusing on the relationships between multivariate moments and different types of cumulants: classical, Boolean, and free.