Probability, Analysis and Dynamics '25

Bristol, 9-11 April, 2025

Abstracts of talks

Tim Austin (University of Warwick):
Notions of entropy in ergodic theory and representation theory
Wednesday 17:00 (G.10)

Entropy has its origins in thermodynamics and statistical mechanics. It was explored with mathematical rigour in Shannon's work on the foundations of information theory, and quickly found striking applications to ergodic theory in work of Kolmogorov and Sinai. Many variants and other applications have appeared in pure mathematics since, connecting probability, combinatorics, dynamics and other areas.

I will survey a few recent developments in this story, with an emphasis on some of the basic ideas that they have in common. I will focus mostly on (i) Lewis Bowen's "sofic entropy", which helps us to study the dynamics of "large" groups such as free groups, and (ii) a cousin of sofic entropy in the world of unitary representations, which leads to new connections with random matrices.

Sourav Chatterjee (Stanford University):
Clay Lecture: *Rigorous results for timelike Liouville field theory*
Friday 14:00 (G.10)

Liouville field theory has long been a cornerstone of two-dimensional quantum field theory and quantum gravity, which has attracted much recent attention in the mathematics literature. Timelike (or imaginary) Liouville field theory is a version of Liouville field theory where the coupling constant is made imaginary. I will talk about recent work that gives a rigorous construction of timelike Liouville theory in a restricted regime of parameters. The path integral for timelike Liouville field theory has a negative sign in front of the kinetic term, which makes it closer to a theory of quantum gravity than the usual (spacelike) Liouville field theory. Making sense of this "wrong sign" requires a theory of Gaussian random variables with negative variance. I will present such a theory, which will then be used to prove the timelike DOZZ formula for the 3-point correlation function in the restricted region. Expressions derived for the k-point correlation functions will also be presented, and I will show how these functions approach the correct semiclassical limits after insertion of heavy vertex operators.

Laure Dumaz (ENS Paris):
Some aspects of the Anderson Hamiltonian in 1D
Thursday 15:30 (G.10)

In this talk, we will present several results on the Anderson Hamiltonian with white noise potential in dimension 1. This operator formally writes «minus Laplacian plus white noise». It arises as the scaling limit of various discrete models and its explicit potential allows for a detailed description of its spectrum. We will discuss localization of its eigenfunctions as well as the behaviour of the local statistics of its eigenvalues. Around large energies, we will see that eigenfunctions are delocalized and the operator limit takes a simple form "$J\partial_t + 2$*2 noise matrix" that can be linked to the hyperbolic carousel operators introduced in the random matrix context by Valkó and Virág. Based on joint works with Cyril Labbé.

Manfred Einsiedler (ETH Zürich):
Effective equdistribution of closed orbits
Friday 10:00 (G.10)

We discuss equidistribution results of closed orbits on homogeneous spaces. The original results due to Mozes and Shah from 1995 relied heavily on the fundamental work of Ratner in unipotent dynamics. However, Ratner's theorems are ineffective – they do not provide any insights on the error rate. In various collaborations and increasing generality we have attacked that problem for closed orbits of noncompact semisimple subgroups. The latest results concerns "adelic quotients" and have interesting number theoretical applications. We outline the general method relying on (uniform) spectral gap, an effective ergodic theorem, and the shearing property of unipotent flows used by Ratner.

The talk is based on several joint works in various combinations with Lindenstrauss, Margulis, Mohammadi, Wieser, and Venkatesh.

Giovanni Forni (University of Maryland/Cergy Paris Université):
Finite codimension stability of invariant surfaces
Wednesday 12:00 (G.10)

Following a recent work of Alazard and Shao on the application of paradifferential calculus to KAM-type problems, we will present a finite codimension stability result for invariant surfaces of billiards in rational polygons (or geodesic flows on translation surfaces).

Nina Gantert (TU Munich):
Biased random walk on dynamical percolation
Wednesday 14:30 (G.10)

As an example for a random walk in random environment, we study biased random walk for dynamical percolation on the d-dimensional lattice. We establish a law of large numbers and an invariance principle for this random walk using regeneration times. Moreover, we verify that the Einstein relation holds, and we investigate the speed of the walk as a function of the bias. While for $d = 1$ the speed is increasing, we show that in general this fails in dimension $d \geq 2$. As our main result, we establish two regimes of parameters, separated by a critical curve, such that the speed is either eventually strictly increasing or eventually strictly decreasing. This is in sharp contrast to the biased random walk on a static supercritical percolation cluster, where the speed is known to be eventually zero.

Based on joint work with Sebastian Andres, Dominik Schmid and Perla Sousi.

Alexander Gorodnik (Universität Zürich):
Optimal approximation exponents and density hypothesis
Thursday 14:00 (G.10)

We discuss the problem of quantitative distribution of orbits for group actions on homogeneous spaces, which involves analysis of the discrepancy functions and the approximation exponents. We develop analytic methods that allow estimating these quantities. It turns out that that density estimates on the automorphic spectrum, more specifically Sarnak's density hypothesis, can be used to establish the optimal approximation exponent generically. This is a joint work with Mikolaj Fraczyk and Amos Nevo.

Rachel Greenfeld (Northwestern University):
Integer distance sets
Wednesday 16:00 (G.10)

A set in the Euclidean plane is called an integer distance set if the distance between any pair of its points is an integer. All so-far-known integer distance sets have all but up to four of their points on a single line or circle; and it had long been suspected, going back to Erdős, that any integer distance set must be of this special form. In a recent work, joint with Marina Iliopoulou and Sarah Peluse, we developed a new approach to the problem, which enabled us to make the first progress towards confirming this suspicion. In the talk, I will discuss the study of integer distance sets, its connections with other problems, and our new developments.

Zemer Kosloff (University of Bristol/Hebrew University of Jerusalem):
Functional limit theorems in (truly) deterministic dynamical systems
Friday 11:30 (G.10)

This talk will be concerned with the following simulation question: Given a specific dynamical system, say an irrational rotation, and a self similar process can one find a function whose associated time series process converges in distribution to the process? Recent joint work with Dalibor Volny (Rouen) shows that the answer is yes for alpha stable Lévy motions. We will discuss this and (time permitting) an application of these results for other questions in dynamical systems.

Based on joint work with Dalibor Volny.

Eugenia Malinnikova (Stanford University):
Clay Lecture: *Uncertainty principles and spectral inequalities for Schrödinger operators*
Wednesday 10:30 (G.10)

A measurable subset of the real line is called "thick" if the measure of the intersection of this set with any interval of length one is bounded from below. The classical theorem of Logvinenko and Sereda states that if the Fourier transform of a function is supported in some interval, then the function itself can be sampled from a thick set. In this talk, we will consider Schrödinger operators with increasing potentials and functions with bounded spectra in corresponding space. The spectral inequalities provide estimates for sampling functions with bounded spectrum from (relatively) thick sets. We will give an overview of some recent results and describe an application of the spectral inequalities to control theory.

Jason Miller (University of Cambridge)
Title TBA
Thursday 11:30 (G.10)

Andrea Mondino (University of Oxford):
Smooth and non-smooth aspects of Ricci curvature lower bounds
Friday 9:00 (G.10)

After recalling the basic notions coming from differential geometry, the talk will be focused on spaces satisfying Ricci curvature lower bounds. The idea of compactifying the space of Riemannian manifolds satisfying Ricci curvature lower bounds goes back to Gromov in the 80s and was pushed by Cheeger and Colding in the 90s who investigated the fine structure of possibly non-smooth limit spaces. Around twenty years ago, a completely new approach via optimal mass transportation allowed Sturm and Lott-Villani to study non-smooth (metric-measure) spaces with Ricci curvature bounded below, in a synthetic sense. Such an approach has been refined in the last years giving new insights to the theory and yielding applications which seems to be new even for smooth Riemannian manifolds. The talk is meant to be an introduction to the topic, accessible to non-specialists, with outlooks on some recent exciting results.

Laura Monk (University of Bristol):
Typical hyperbolic surfaces have an optimal spectral gap
Thursday 16:30 (G.10)

The first non-zero Laplace eigenvalue of a hyperbolic surface, or its spectral gap, measures how well-connected the surface is: surfaces with a large spectral gap are hard to cut in pieces, have a small diameter and fast mixing times. For large hyperbolic surfaces (of large area or large genus g, equivalently), we know that the spectral gap is asymptotically bounded above by $\frac{1}{4}$. The aim of this talk is to present joint work with Nalini Anantharaman, where we prove that most hyperbolic surfaces have a near-optimal spectral gap. That is to say, we prove that, for any $\varepsilon > 0$, the Weil-Petersson probability for a hyperbolic surface of genus g to have a spectral gap greater than $\frac{1}{4} - \varepsilon$ goes to one as g goes to infinity. This statement is analogous to Alon's 1986 conjecture for regular graphs, proven by Friedman in 2003. I will present our approach, which shares many similarities with Friedman's work, and introduce new tools and ideas that we have developed in order to tackle this problem.

Joel Moreira (University of Warwick):
Infinite sumsets via ergodic theory
Thursday 10:00 (G.10)

We address the question of which infinite configurations are present in every set of natural numbers with positive upper density. It is possible to employ a variant of Furstenberg's correspondence principle to connect this question to a dynamical problem involving objects akin to arithmetic progressions. I will explore this connection and present recent joint work with Kra, Richter and Robertson where we prove that for any natural number k, any set A with positive upper density contains, up to a shift, all sums of at most k distinct elements of an infinite set B.

Florian Richter (EPFL Lausanne):
Ergodic methods in number theory and combinatorics
Thursday 9:00 (G.10)

Ergodic theory is the study of group actions on probability spaces, and it offers powerful tools for understanding the long-term behavior of complex, chaotic systems. In this talk, we offer a gentle introduction to the applications of ergodic theory in number theory and combinatorics, where it has solved problems that resisted all other approaches. This includes breakthroughs such as Furstenberg's proof of Szemerédi's theorem, the Green-Tao theorem on arithmetic progressions in primes, recent progress on Chowla's and Sarnak's conjectures regarding the randomness of prime factorizations, and solutions to Erdős's sumset conjectures. The latter will be explored in greater detail in the proceeding talk by Joel Moreira.

Bálint Tóth (University of Bristol/Rényi Institute Budapest):
Diffusion in the random Lorentz gas
Wednesday 9:30 (G.10)

Since the pioneering works of Hendrik Lorentz (1905) and Paul and Tatiana Ehrenfest (1912) the deterministic (Hamiltonian) motion of a point-like particle exposed to the action of a collection of fixed, randomly located short range scatterers has been a much studied model of physical diffusion under fully deterministic (Hamiltonian) dynamics, with random initial conditions. This model of physical diffusion is known under the name of "random Lorentz gas" or "random wind-tree model". Celebrated milestones on the route to better mathematical understanding of this model of true physical diffusion are the Kinetic Limits for the tagged particle trajectory under the so-called Boltzmann-Grad (a.k.a. low density), or weak coupling approximations [Gallavotti (1970), Spohn (1978), Boldrighini-Bunimovich-Sinai (1982), respectively, Kesten-Papanicolaou (1980)]. Under a second diffusive space-time scaling limit – done as a second step, after the kinetic approximations – the central limit theorem (CLT) and invariance principle (IP) for the tagged particle motion follow. However, the CLT/IP under bare diffusive space-time scaling (without first applying the kinetic approximations) remains a Holy Grail. In recent work we have obtained some intermediate results, partially interpolating between the two-steps-limit (first kinetic, then diffusive – as described above) and the bare-diffusive-limit (Holy Grail). We establish the Invariance Principle for the tagged particle trajectories under a joint kinetic+diffusive limiting procedure, performed simultaneously rather than successively, reaching significantly longer time scales than in earlier works. The Holy Grail (i.e., CLT under bare diffusive scaling) remains, however, beyond reach. I will present a survey of the main problems and (historic and more recent) results, accessible for a broad range of mathematicians.

Hong Wang (NYU Courant):
Kakeya sets in $\mathbb{R}^3$
Friday 15:00 (G.10)

A Kakeya set is a compact subset of $\mathbb{R}^n$ that contains a unit line segment pointing in every direction. Kakeya set conjecture asserts that every Kakeya set has Minkowski and Hausdorff dimension n. We prove this conjecture in $\mathbb{R}^3$ as a consequence of a more general statement about union of tubes. This is joint work with Josh Zahl.