2/10

Indep $\Rightarrow \mathrm{Cov}(X,Y)=0$

$\mathrm{Cov}(X,X) = \mathrm{Var}\,X$

$\mathrm{Cov}(X,Y) = \mathrm{Cov}(Y,X)$

$\mathrm{Cov}\left(\sum_i a_i X_i + b, \sum_j c_j Y_j + d\right) =$

$= \sum_{i,j} a_i c_j \cdot \mathrm{Cov}(X_i, Y_j)$

$\mathrm{Var} \sum_i X_i = \sum_i \mathrm{Var}\,X_i + 2 \sum_{i<j} \mathrm{Cov}(X_i, X_j)$

$\mathbb{E}|XY| \leq \cancel{SD\,X \cdot SD\,Y} \sqrt{\mathbb{E}X^2 \cdot \mathbb{E}Y^2}$

$-1 \leq \rho(X,Y) = \dfrac{\mathrm{Cov}(X,Y)}{SD\,X \cdot SD\,Y} \leq 1$

Correlation

$$X_i = \begin{cases} 1 & \text{if } A_i \\ 0 & \text{if } A_i^c \end{cases}$$ indicator of A_i

$X = \sum_{i=1}^{n} X_i$ = # of the A_i that occurring

$$\mathbb{E}X = \sum_{i=1}^{n} \underbrace{\mathbb{E}X_i}$$

$$\boxed{\mathbb{E}X_i X_j} - \mathbb{E}X_i \mathbb{E}X_j$$

$$\operatorname{Var}X = \sum_{i=1}^{n} \underbrace{\operatorname{Var}X_i} + 2 \sum_{1 \le i < j \le n} \operatorname{Cov}(X_i, X_j)$$

$$\mathbb{E}X_i = \mathbb{P}(A_i); \operatorname{Var}X_i = \mathbb{P}(A_i)(1 - \mathbb{P}(A_i))$$

Conditional expectations

$$\mathbb{E}(X \mid Y = y) = \sum_i x_i \cdot p_{X|Y}(x_i \mid y)$$

$\mathbb{E}(X \mid Y)$ is a fct of Y

is a r.v.

Tower rule: $\mathbb{E}\,\mathbb{E}(X \mid Y) = \mathbb{E}X$

$$\mathbb{E}\,\mathbb{E}(g(X)\,|\,Y) = \mathbb{E}\,g(X)$$

$$\operatorname{Var}\mathbb{E}(X\,|\,Y) + \mathbb{E}\operatorname{Var}(X\,|\,Y) = \operatorname{Var} X$$

$$\mathbb{E}\big(h(X)\cdot g(Y)\,\big|\,Y\big) = g(Y)\,\mathbb{E}(h(X)\,|\,Y)$$

$N \leftarrow$ Rnd.

$$\mathbb{E}\sum_{i=1}^{N} X_i = \mathbb{E}\,\underline{\underline{\mathbb{E}}}\Big(\sum_{i=1}^{N} X_i\,\Big|\,\underline{\underline{N}}\Big) = $$

$$= \mathbb{E}\Big[\sum_{i=1}^{N}\mathbb{E}(X_i\,|\,N)\Big]$$

$$M(t) = \mathbb{E}\,e^{tX}$$

hoping $M(t)<\infty$ has some interval of t's around 0.

$$M(0) = 1$$

$$M^{[n]}(0) = \mathbb{E}\,X^n$$

n^{th} derivative

Unique

If X, Y are indep., then

$$M_{X+Y}(t) = M_X(t)\, M_Y(t)$$

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⑦ Markov's ineq.

$X \geq 0$ r.v. $\quad \forall a > 0$

$$\mathbb{P}(X \geq a) \leq \frac{\mathbb{E}X}{a}$$

Chebyshev's: If $\operatorname{Var} X \leq \sigma^2 < \infty$

$$\mathbb{P}(|X - \underset{\displaystyle \mathbb{E}X}{\underset{\downarrow}{\mu}}| \geq b) \leq \frac{\sigma^2}{b^2}$$

$\mathbb{E}(g(Y)\cdot h(X)) = \mathbb{E}\,\mathbb{E}(g(Y)h(X)|Y) =$

$= \mathbb{E}[g(Y)\cdot\mathbb{E}(h(X)|Y)]$

11.4

Thm WLLN: If $X_1, X_2, \ldots$ iid $\forall \varepsilon > 0$.

$$\mathbb{P}\left(\left|\frac{X_1 + \ldots + X_n}{n} - \underbrace{\mu}_{\mathbb{E}X_i}\right| \geq \varepsilon\right) \xrightarrow[n\to\infty]{} 0$$

Thm: CLT : $X_1, \ldots,$ iid $\mathrm{Var}X_i = \sigma^2 < \infty$

$$\mathbb{P}\left(a < \frac{X_1 + \ldots + X_n - n\overset{\mathbb{E}X_i}{\mu}}{\sqrt{n}\,\sigma} \leq b\right) \xrightarrow{n\to\infty} \Phi(b) - \Phi(a)$$

iid = independent, identically distributed

DeMoivre-Laplace:

$X \sim \mathrm{Binom}(n, p)$ p fixed

$$\mathbb{P}\left(a < \frac{X - np}{\sqrt{np(1-p)}} \leq b\right) \xrightarrow{n\to\infty} \Phi(b) - \Phi(a)$$

Beispiel:

$$X \sim \text{Binom}(n,p) = \sum_{i=1}^{n} X_i$$

$$\begin{cases} 1 & \text{if trial } i \text{ succeeds} \\ 0 & \text{else} \end{cases}$$