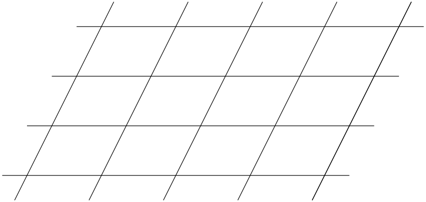
Planar lattices do not recover from forest fires

Ioan Manolescu
Joint work with Demeter Kiss and Vladas Sidoravicius

8 May 2015

What are forest fires?



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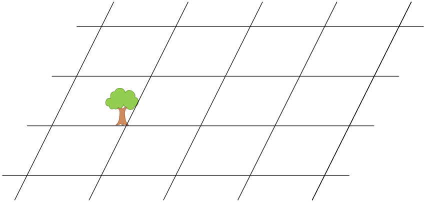

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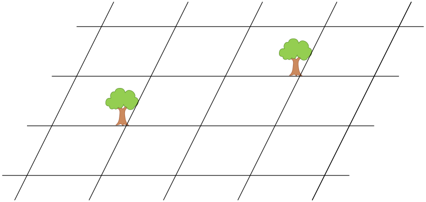

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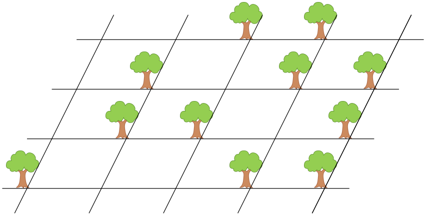

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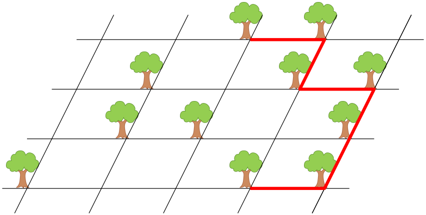

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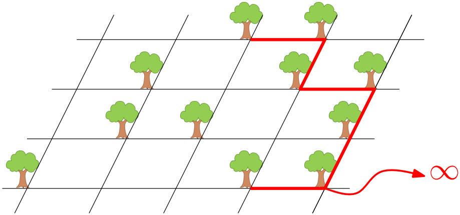

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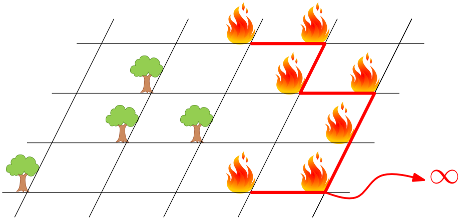

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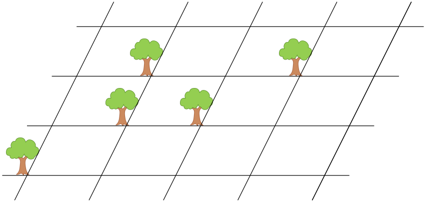

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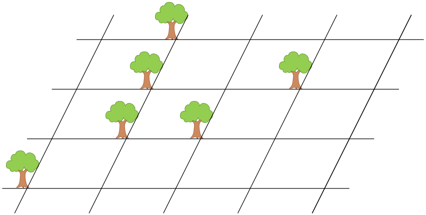

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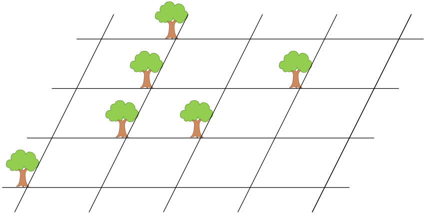

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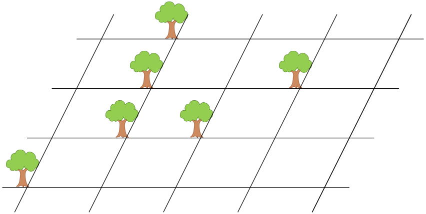

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Question: Does this make sense?

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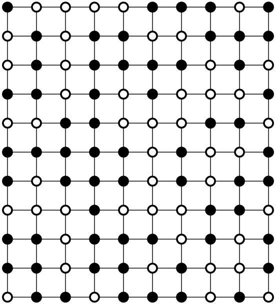
vertices are open with probability p, closed with probability $1 - p$, independently.



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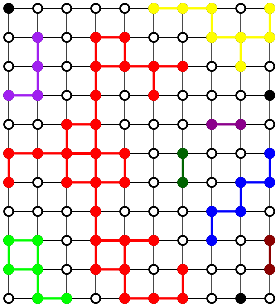

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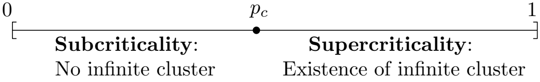

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0 p_c 1

Subcriticality:
No infinite cluster
Exponential tail for cluster size
Trivial large scale behaviour!

Supercriticality:
Existence of infinite cluster
Unique infinite cluster
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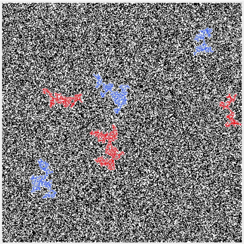

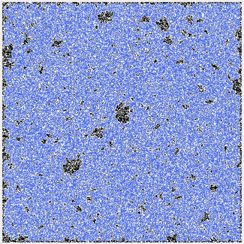

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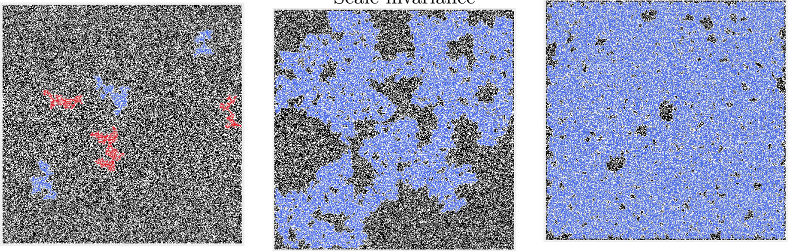
Criticality
No infinite cluster
Scale invariance



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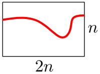
At $\mathbf{p_c}$... Crossing probabilities do not degenerate. (RSW)

$$\forall n, \quad \mathbf{P}_{p_c}\left[\quad \right] \geq \epsilon$$



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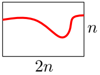
$\forall n,\ \mathbf{P}_{p_c}\Big[$ 

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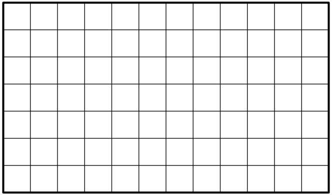

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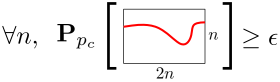

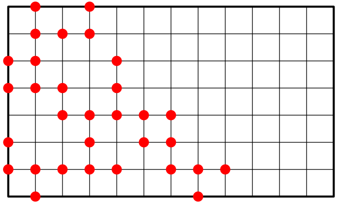

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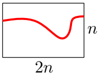
$\forall n, \ \mathbf{P}_{p_c}\Big[$ 

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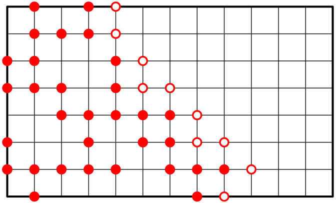

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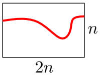
$\forall n, \ \mathbf{P}_{p_c}\Big[$ 

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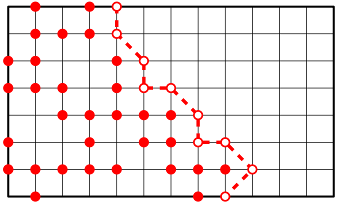

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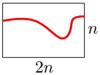
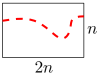
$\forall n,\ \mathbf{P}_{p_c}\Big[$ 

$\Big] \geq \epsilon$ $\qquad$ $\mathbf{P}_{p_c}\Big[$ 

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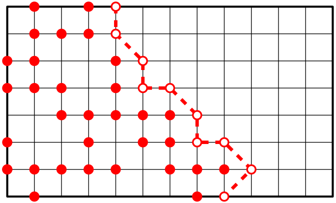

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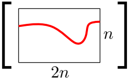
Existence of **critical exponents** (arm exponents)

$\forall n,\ \mathbf{P}_{p_c}$ 

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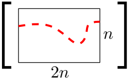
$\mathbf{P}_{p_c}$ 

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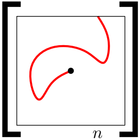
$\mathbf{P}_{p_c}$ 

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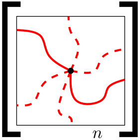
$\mathbf{P}_{p_c}$ 

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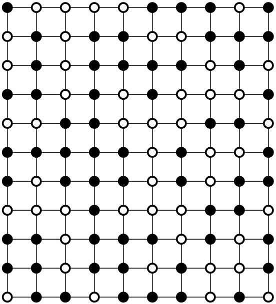
What is self-destructive percolation?

A planar lattice: here $\mathbb{Z}^2$.
Let $p, \delta \in [0, 1]$.
Two (site) percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).



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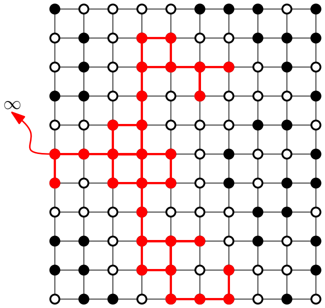

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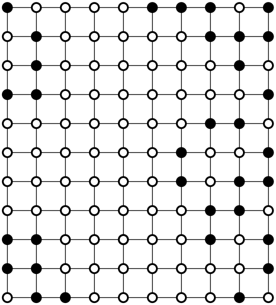

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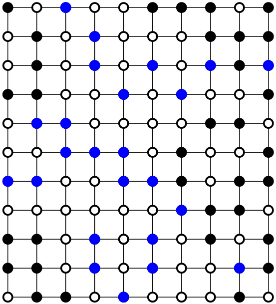

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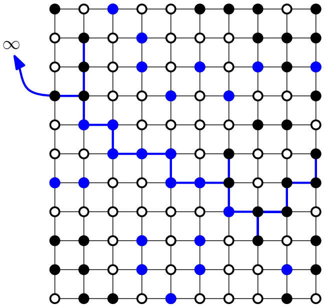

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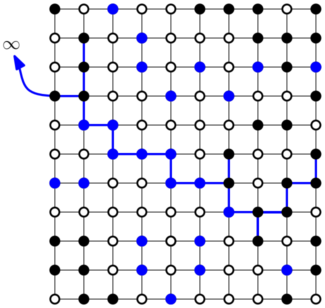

Question: $\delta_c(p) \to 0$ as $p \searrow p_c$?

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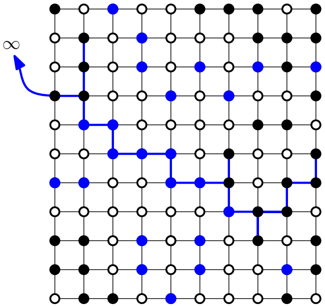

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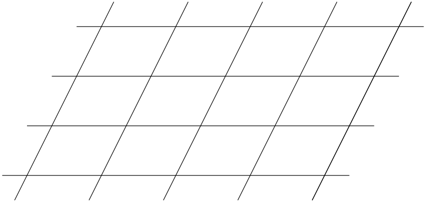
Theorem [Kiss, M., Sidoravicius] : There exists $\delta > 0$ such that, for all $p > p_c$,

$$\mathbb{P}_{p,\delta}(\text{infinite cluster in } \overline{\omega}^{\delta}) = 0.$$

In particular $\lim_{p \to p_c} \delta_c(p) > 0$

Back to forest fires: non-existence



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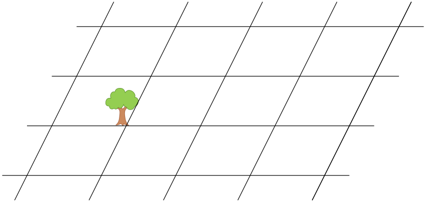

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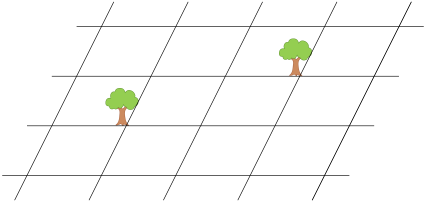

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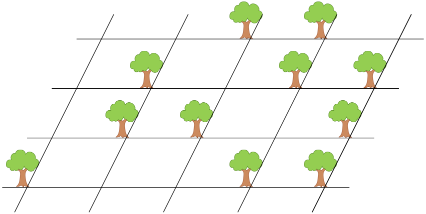

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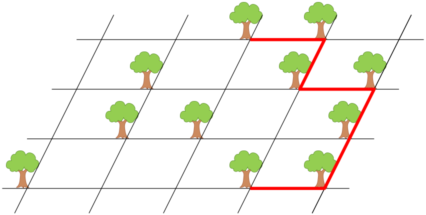

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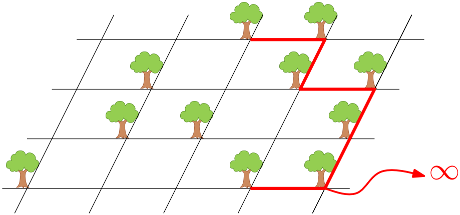

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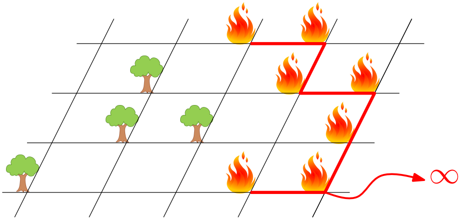

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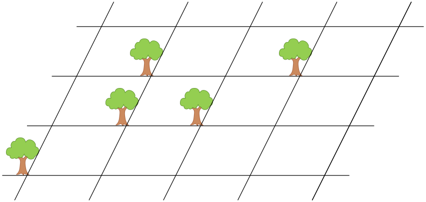

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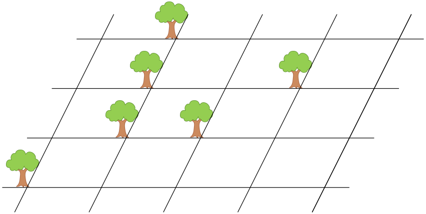

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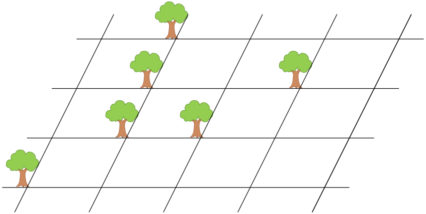
- Trees then continue to grow . . .

Back to forest fires: non-existence



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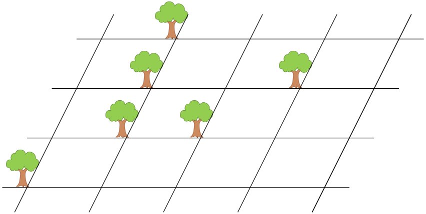

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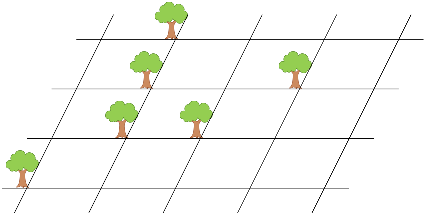

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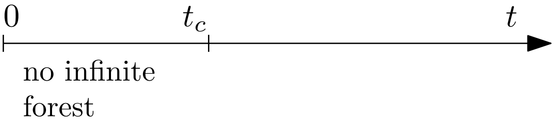

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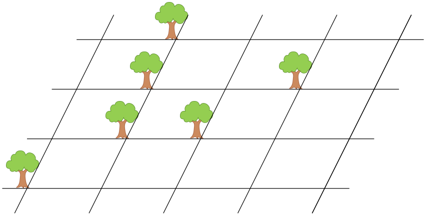

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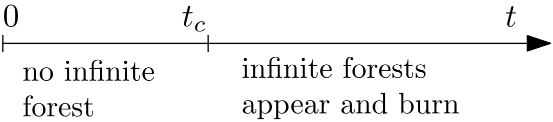

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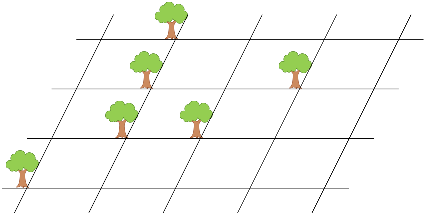

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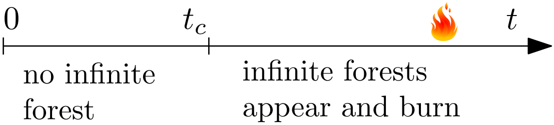

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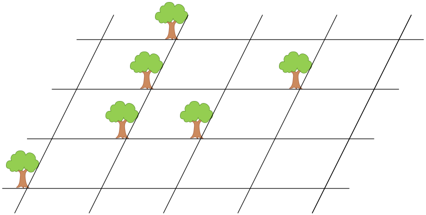

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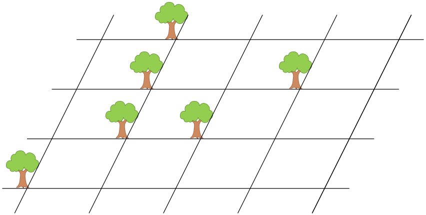

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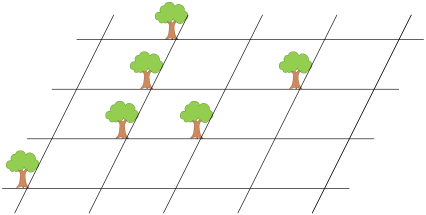

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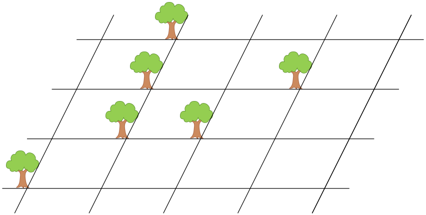

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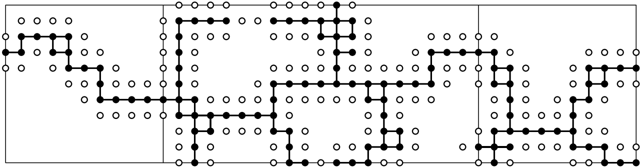
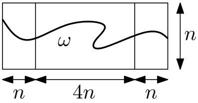
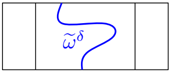


ω containing crossing

Proposition

For $\delta > 0$ small enough, as $n \to \infty$,

$\mathbb{P}_{p_c,\delta}\Bigg($ 

and 

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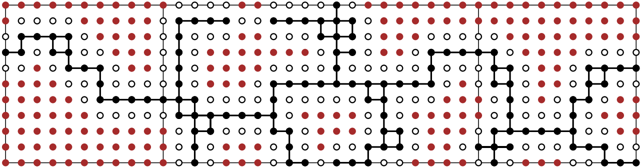

ω containing crossing $\xrightarrow{\text{delete crossing cluster}} \widetilde{\omega}$

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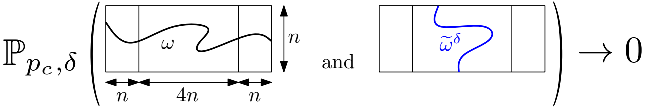

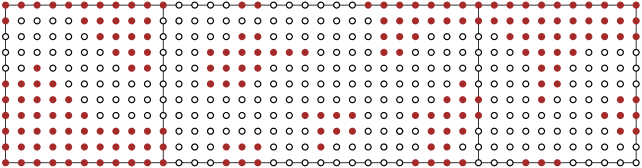

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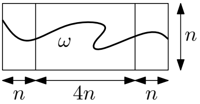
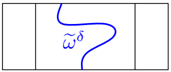
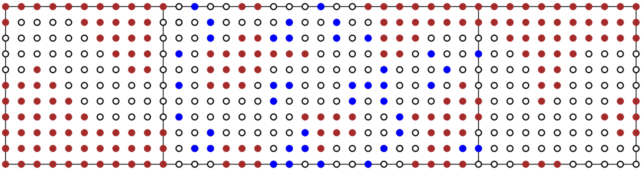
$\mathbb{P}_{p_c,\delta}\Bigg($  and  $\Bigg) \to 0$



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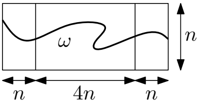

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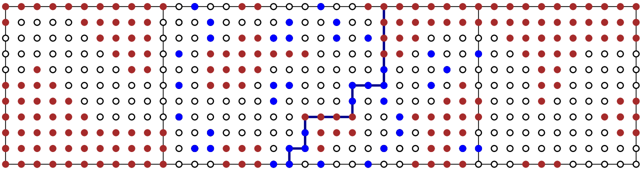

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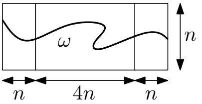

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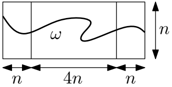
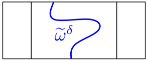
For $\delta > 0$ small, $\mathbb{P}_{p_c,\delta}$ (

and 

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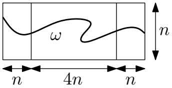
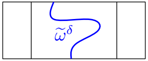
For $\delta > 0$ small, $\mathbb{P}_{p_c,\delta}\Big($ 

and 

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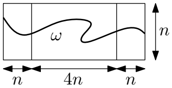
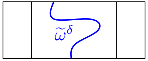
For $\delta' > 0$ small, $\mathbb{P}_{p,\delta'}\Big($ 

and 

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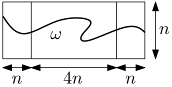
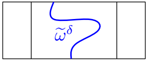
For $\delta' > 0$ small, $\mathbb{P}_{p,\delta'}\Big($ 

and 

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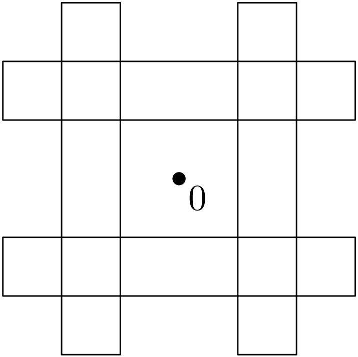
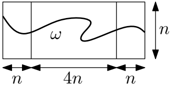
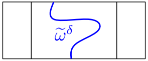


For $\delta' > 0$ small, $\mathbb{P}_{p,\delta'}\Bigg($ 

and 

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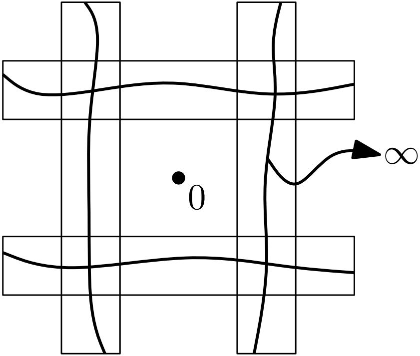
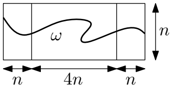
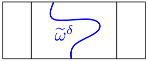


For $\delta' > 0$ small, $\mathbb{P}_{p,\delta'}\Big($ 

and 

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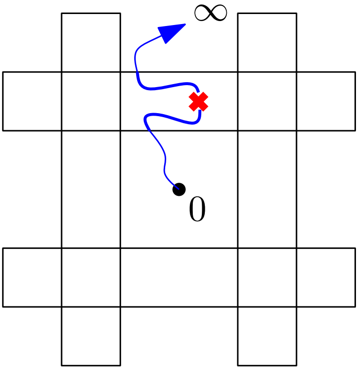
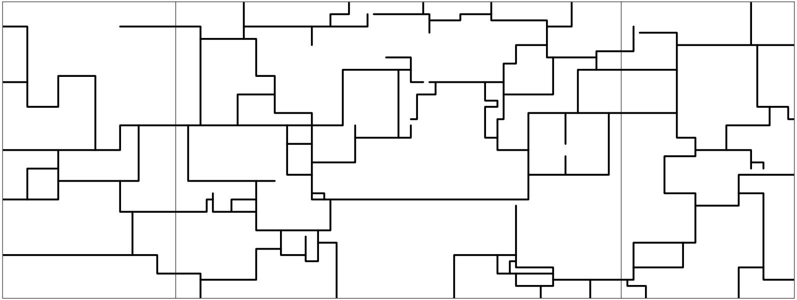
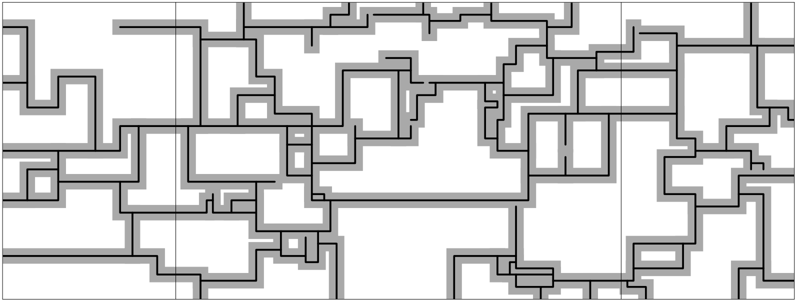
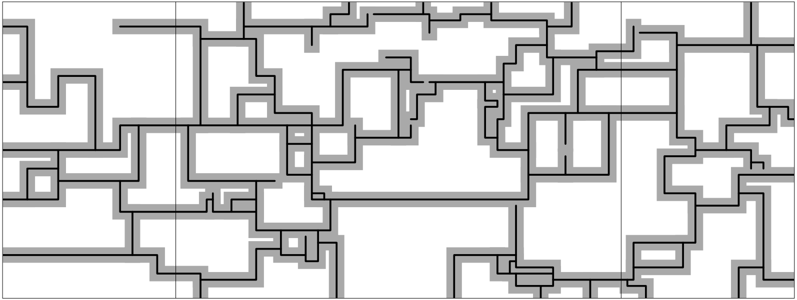
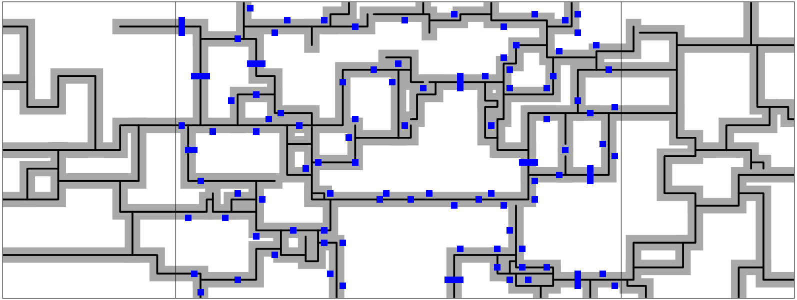


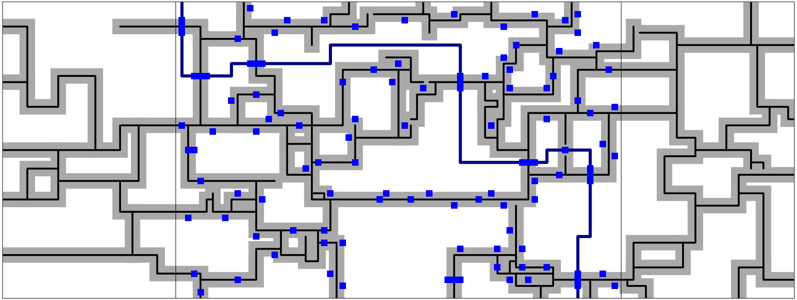
In both $\widetilde{\omega}^\delta$ and $\overline{\omega}^\delta$!



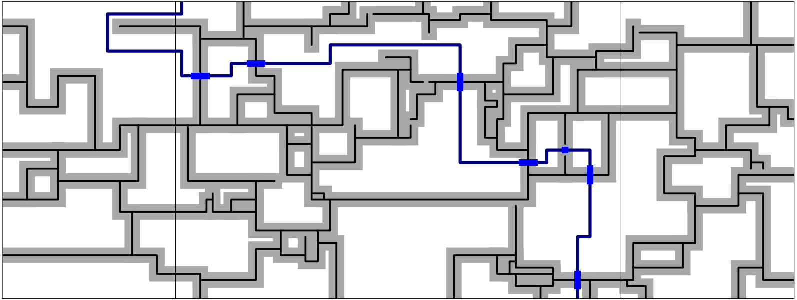








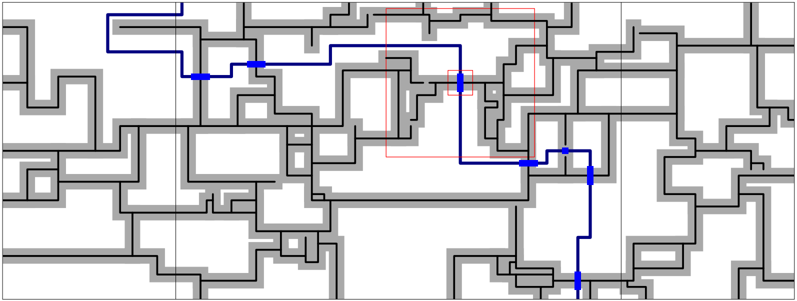
γ - vertical crossing with minimal number of enhanced points.



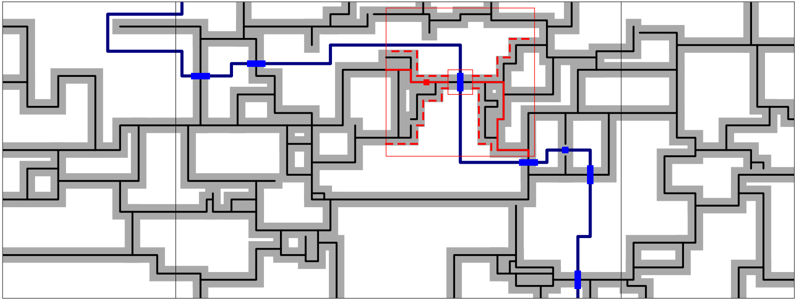
γ - vertical crossing with minimal number of enhanced points.

$\mathcal{X} = \{$enhanced points used by $\gamma\}$. If no crossing $\mathcal{X} = \emptyset$.

$$\mathbb{P}_{p_c,\delta}(\text{vertical crossing in } \widetilde{\omega}^{\delta}) = \sum_{X \neq \emptyset} \mathbb{P}_{p_c,\delta}(\mathcal{X} = X).$$



Annulus surrounding passage points but not containing passage points: 6 arms or 4 half-plane arms in ω (possibly with one defect).



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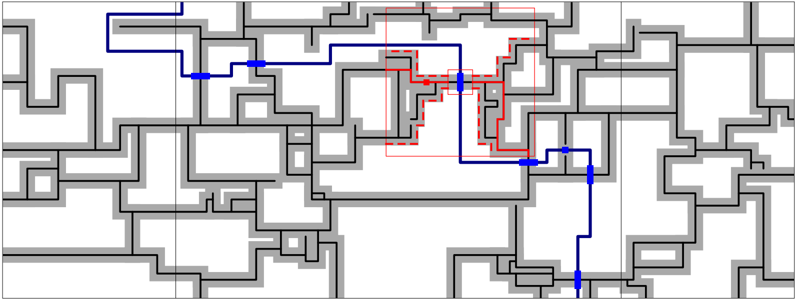

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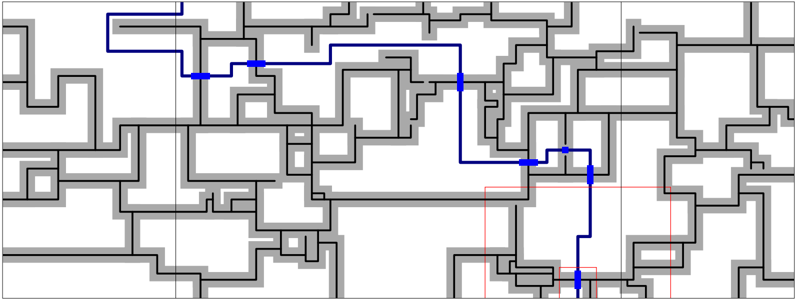

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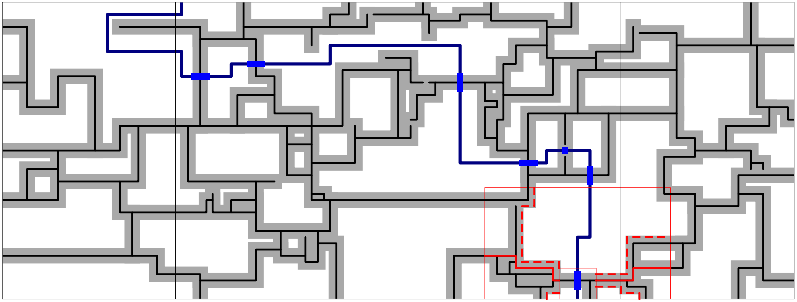

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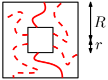
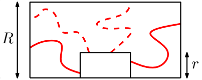
$\mathbb{P}_{p_c}\Big($ 

$\Big) \leq \left(\frac{r}{R}\right)^{2+\lambda}$ $\mathbb{P}_{p_c}\Big($ 

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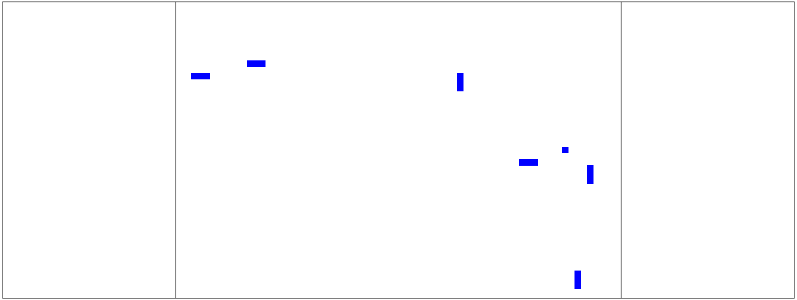
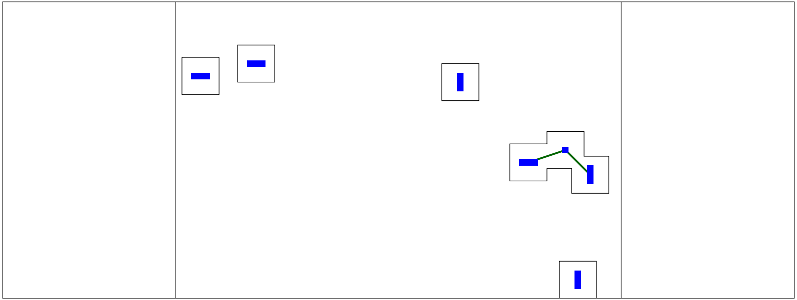


For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



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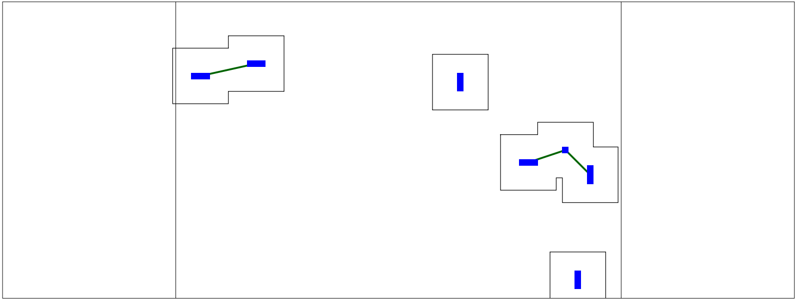

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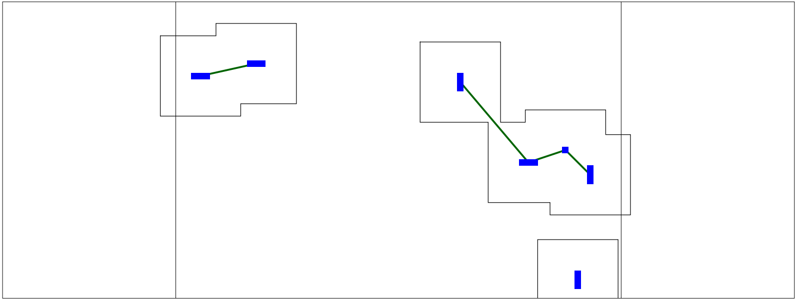

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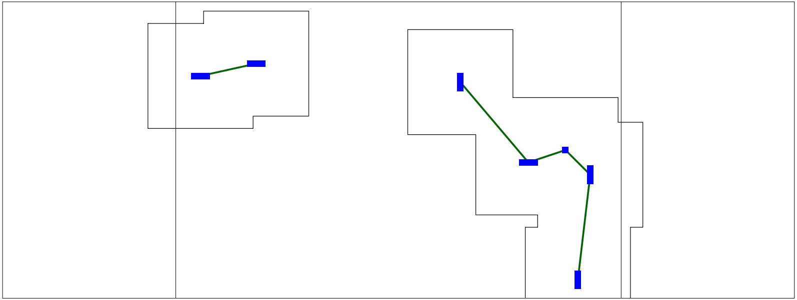

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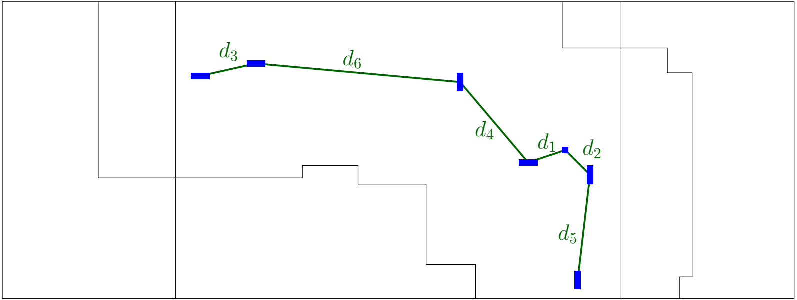

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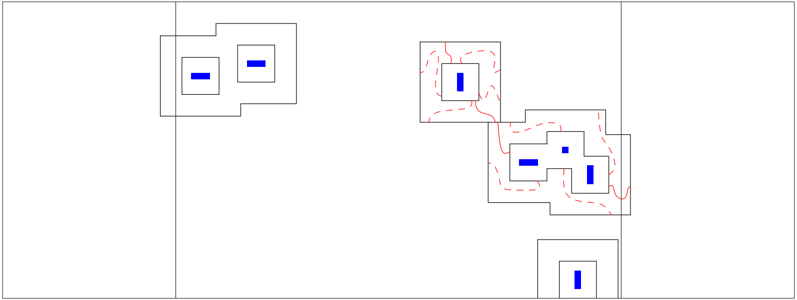
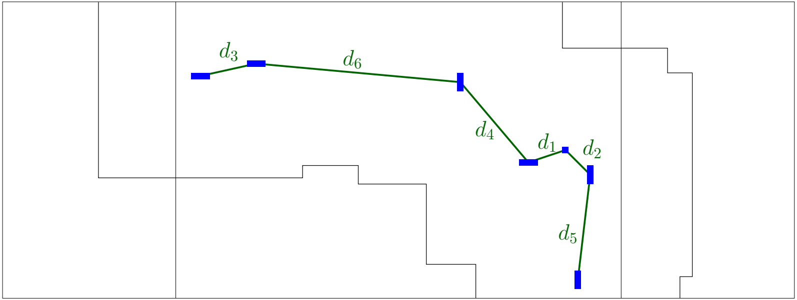


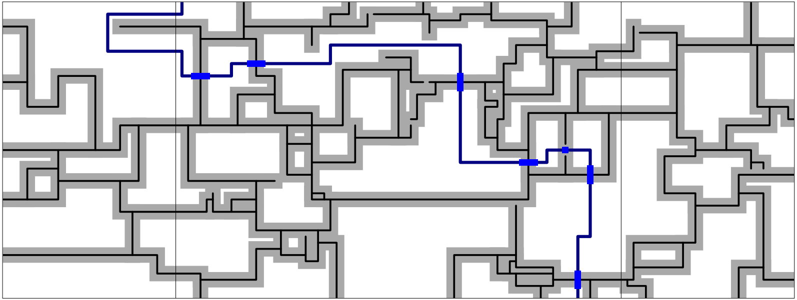
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



$$\mathbb{P}_{p,\delta}(\mathcal{X} = X) \leq c^k n^{-2-\lambda} \prod_j d_j^{-2-\lambda} \times \delta^k,$$

where $d_1, \dots, d_k$ are the merger times of X.

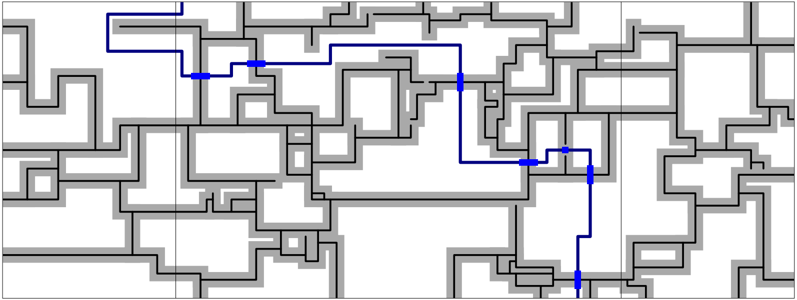
$$\#\{X \text{ with merger times } d_1, \dots, d_k\} \leq C^k n^2 \prod_j d_j.$$



$$\mathbb{P}(\text{vetical crossing in } \widetilde{\omega}^{\delta}) \leq n^{-\lambda} \sum_{X} \mathbb{P}_{p,\delta}(\mathcal{X} = X)$$

$$\leq n^{-\lambda} \sum_{\substack{k \geq 1 \\ d_1, \dots, d_k}} \left(\delta^k c^k \prod_k d_k^{-1-\lambda} \right) = n^{-\lambda} \sum_{k \geq 1} \left(\delta c \sum_{d \geq 1} d^{-1-\lambda} \right)^k \to 0,$$

for $\delta > 0$ small.



$$\mathbb{P}(\text{vetical crossing in } \widetilde{\omega}^{\delta}) \leq n^{-\lambda} \sum_{X} \mathbb{P}_{p,\delta}(\mathcal{X} = X)$$

$$\leq n^{-\lambda} \sum_{\substack{k \geq 1 \\ d_1,\dots,d_k}} \left(\delta^k c^k \prod_k d_k^{-1-\lambda} \right) = n^{-\lambda} \sum_{k \geq 1} \left(\delta c \sum_{d \geq 1} d^{-1-\lambda} \right)^k \to 0,$$

for $\delta > 0$ small.

Thank you!