Probability 1, Autumn 2016, Problem sheet 6

To be discussed on the week 7 Nov. . . 11 Nov.
Problems marked with "PrCl" are discussed in the large problem class on Friday 4 Nov.
Mandatory HW's are marked with "**HW**", they are due on the week 14 Nov. . . 18 Nov, the latest.[1]
Solutions will be available on Blackboard on the 19th Nov.

HW 6.1 We draw randomly without replacement 3 balls from an urn that contains 3 red and 5 white balls. Denote by X the number of red balls drawn. Find the distribution of X, its expectation and standard deviation.

6.2 If $\mathbf{E}(X) = 1$ and $\mathbf{Var}(X) = 5$, find

(a) $\mathbf{E}[(2 + X)^2]$,

(b) $\mathbf{Var}(4 + 3X)$.

HW 6.3 From a random variable X, subtract its mean and then divide the result by the standard deviation. What is the expected value and variance of the new random variable obtained this way?

6.4 Steve has not prepared for his exam, where he has to answer 10 yes or no questions. A small part of the material dawns on him, and so he can give the correct answer to each question with probability 60%. What is the probability he will pass if that needs at least 8 correct answers?

6.5 Each student on a test has to answer 20 yes or no questions. Assume that independently for each question, a student knows the correct answer with probability 0.7, believes that he knows the correct answer, but he is wrong with probability 0.1, and doesn't know the answer with probability 0.2. In this latter case he answers yes or no with probability $\frac{1}{2} - \frac{1}{2}$. What is the probability that he will answer at least 19 questions correctly?

HW 6.6 On a multiple-choice exam with 4 possible answers for each of the 6 questions, what is the probability that a student would get 5 or more correct answers just by guessing?

6.7 On a quiz show a team of n people has to answer a yes-or-no question, where n is odd. Each of the team members can give the correct answer with probability p, independently of each other. They have to decide upon the strategy:

(a) either they pick one team member who will answer the question,

(b) or they do a majority voting to come up with the team's answer.

Show that (a) is preferable when $p < \frac{1}{2}$, it doesn't matter when $p = \frac{1}{2}$, and (b) is preferable when $p > \frac{1}{2}$. *HINT: Add two people to the team, thus increasing the number from n to $n + 2$. Does the probability of a correct answer increase, stay, or decrease?*

PrCl **6.8** A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be wrong when decoded?

HW 6.9 A walker starts from the origin of the integer line. At each step he moves one unit to the right with probability 1/2, and to the left with probability 1/2, independently of his previous steps. After 10 moves,

(a) What is the probability that he is at position 0?

(b) What is the probability that he is at position 1?

(c) What is the probability that he is at position (-2)?

(d) What is the probability that he is at position (-2), if he was at (-3) one step ago?

6.10 I have two coins, a fair one and a biased one, but I cannot distinguish them. The biased coin comes up heads with probability 3/4. I pick one of the two coins from my pocket, the fair one with probability 1/2 and the biased one with probability 1/2. Then I flip the chosen coin 30 times, and I find that it came up heads 25 times. What is the probability that I chose the biased coin?

6.11 A newsboy purchases papers at £1.00 and sells them at £1.50. However, he is not allowed to return unsold papers. If his daily demand is a binomial random variable with $n = 10$ and $p = 1/3$, approximately how many papers should he purchase so as to maximise his expected profit?

[1] Details of how to hand in are to be discussed with your tutor.

6.12 Approximately 80 000 marriages took place in a country last year. Estimate the probability that for at least one of these couples

(a) both partners were born on April 30;

(b) both partners celebrate their birthday on the same day of the year.

State your assumptions.

PrCl **6.13** There are 200 typos, randomly distributed, in a book of 400 pages. What is the probability that on page 13 there are more than one typos?

HW 6.14 How many raisins should there be in a muffin on average if we want at least one raisin in any given muffin with probability at least 0.95?

6.15 We find that 5% of a certain type of muffins have no raisins in it. What is the probability that a muffin from this series has more than two raisins in it?

6.16 Let $X \sim \mathrm{Poi}(\lambda)$. Show that

$$\mathbf{P}\{X \text{ is even}\} = \frac{1}{2}(1 + \mathrm{e}^{-2\lambda}).$$

HINT: What is $\mathbf{E}(-1)^X$*?*