Probability 1, Autumn 2014, Problem sheet 8

To be discussed on the week 24 Nov. . . 28 Nov.

Mandatory HW's are marked with "**HW**", they are due on the week 1 Dec. . . 5 Dec, the latest.[1]

Solutions will be available on Blackboard on the 6th Dec.

HW 8.1 The time, in years, a radio of a certain type functions is exponentially distributed with mean 6 years. Being a popular piece of engineering, this radio set has been manufactured for a long period. My friend and I both buy a used (and of course operational) radio of this type. It turns out that mine was made 9 years ago while my friend's set is only 2 years old.

(a) What is the probability that both my and my friend's radio will work 6 years from now?

(b) What is the chance that my radio stops working before my friend's does?

8.2 2% of electric components of a given type break down within 1000 hours of operation. Assume that time before breakdown has an exponential distribution. What is the probability that such a component will work for longer than the average?

8.3 For a memoryless light bulb, the probability that it operates for more than 2000 hours is 2/3. In a city 20 of these light bulbs are installed.

(a) What is the probability that after 1000 hours exactly 15 bulbs are operational?

(b) What is the probability that after 1000 hours exactly 15 bulbs are operational and after an additional 500 hours exactly 13 are operational?

8.4 The lifetime of the *probabilium* radioactive particle is exponentially distributed with mean value 3 years. What is its half-life (the amount of time required for half of the number of atoms of a sample to decay)?

8.5 If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute

(a) $\mathbf{P}\{X > 5\}$;

(b) $\mathbf{P}\{4 < X < 16\}$;

(c) $\mathbf{P}\{X < 8\}$;

(d) $\mathbf{P}\{X < 20\}$;

(e) $\mathbf{P}\{X > 16\}$.

8.6 Suppose that X is a normal random variable with mean (that is, expected value) 5. If $\mathbf{P}\{X > 9\} = 0.2$, approximately what is $\mathbf{Var}(X)$?

HW 8.7 The length of the Probability 1 lecture is well approximated by a normal random variable of mean $\mu = 52$ minutes and standard deviation $\sigma = 2$ minutes.

(a) What percentage of classes are over 55 minutes long?

(b) It's 55 past and the lecturer is still speaking. What is the probability that he will finish before 57 past?

8.8 What is the probability that the number of outcomes six is between 970 and 1050 if we roll a die 6 000 times?

8.9 How many times should a coin be tossed so as to having the number of heads between 47% and 53% of the number of all tosses with probability at least 0.95?

HW 8.10 Bob plays roulette in the casino. Every round he bets 10 tokens on 'red'. After 100 rounds he has lost 300 tokens. Is he reasonable when he thinks that the croupier is cheating? (On the game roulette one has 37 fields, numbered from 0 to 36. Out of these, '0' has color green, and 18 fields are red, and 18 are black. Betting on 'red' pays 10 extra tokens when the roll is red, and looses the 10 tokens bet when it's not red.)

8.11 Given are two very similar insurance companies each having 10 000 customers. At the beginning of the year each customer pays their insurance company a fee of £200, and during the year each customer independently puts in a claim for £800 of damages with probability $\frac{1}{4}$. Both companies have a capital of £40 000 from the previous year. An insurance company goes bankrupt if it cannot pay for these claims. Should these two companies unite? Let p_1 be the probability that at least one of the two companies goes bankrupt, and p_2 the probability that the united company goes bankrupt. Find the numerical values of p_1 and p_2 and conclude whether joining the two companies is a good idea.

[1] Details of how to hand in are to be discussed with your tutor.

8.12 A stick of length ℓ is broken randomly. What is the distribution function of the length of the shorter piece?

8.13 Let X be uniformly distributed on the interval $[-3, 4]$, and let $g(x) = |x-1| + |x+1|$. Determine the distribution function $F_Y(y)$ of the random variable $Y = g(X)$. Is this variable absolutely continuous? Is it discrete?

8.14 An explosion throws debris at a constant velocity v but in a uniform random direction α on $(0, \frac{\pi}{2})$. Such a piece of debris then lands at distance $R = \frac{v^2}{g} \cdot \sin(2\alpha)$ from the explosion site (g is the gravitational acceleration). Find the density of debris as a function of the distance from the explosion.

8.15 Let $X \sim \mathrm{Exp}(\lambda)$, and $c > 0$. Show that $cX \sim \mathrm{Exp}(\frac{\lambda}{c})$.

8.16 Variables Z and $2Z$ have the same distribution, What is it?

HW 8.17
(a) We randomly select a point on the $[0, 1]$ interval of the x axis. Let D denote the distance between this point and the point at coordinate $(0, 1)$ of the plane. Determine the density function of the distribution of the random variable D.

(b) We randomly select a point on the $[-1, 1]$ interval of the x axis. Let D denote the distance between this point and the point at coordinate $(0, 1)$ of the plane. Determine the density function of the distribution of the random variable D.