Stochastics and Influences

Simons Semester in Probability and Statistical Physics, Erdős Center Budapest,
30 June - 2 July, 2025
Happy birthday, Bálint!

Monday, 30$^{\text{th}}$ June

09:15–09:30 *Registration*

09:30–10:15 Domokos Szász
Rényi's probability school and mathematical statistical physics in Hungary

10:20–10:55 Károly Simon
Random Fractals

10:55–11:20 *Coffee*

11:20–11:55 Balázs Ráth
On the forest fires that Bálint ignited

12:00–12:35 Edward Crane
Markov chain couplings of Markov chains

12:35–14:30 *Lunch*

14:30–15:05 Jessica Jay
Connecting blocking measures of interacting particle systems with combinatorial objects

15:10–15:45 Tyler Helmuth
Statistical Mechanics and Hyperbolic Spin Systems

15:45–16:10 *Coffee*

16:10–16:45 Gábor Pete
Indistinguishability of infinite cycles in permutation and percolation processes

16:50–17:25 Benedek Valkó
Random walks with Bálint Tóth

17:30–19:00 *Wine reception*

Tuesday, 1$^{\text{st}}$ July

09:30–10:05 Christopher Lutsko
Diffusion in random Lorentz gas models

10:10–10:45 Péter Bálint
Generalized law of iterated logarithm in the infinite horizon Lorentz gas

10:45–11:10 *Coffee*

11:10–11:45 Imre Péter Tóth
Spectral gap in a stochastic energy exchange system motivated by a deterministic heat conduction model

11:50–12:25 Daniel Kious
A story of self-interacting random walks getting stuck in atypical behaviours

12:25–14:30 *Lunch*

14:30–15:05 Bálint Vető
The Brownian web and the Brownian web distance

15:10–15:45 Laure Dumaz
True self-repelling motion: recent applications to non-reversible Monte Carlo

15:45–16:10 *Coffee*

16:10–16:45 Ofer Busani
On the global solutions of the KPZ fixed point

16:50–17:25 Márton Balázs
Magic in particle systems

Wednesday, 2nd July

09:30–10:05 Miklós Zoltán Rácz
Phase transitions in the distinctness of neighborhoods in random graphs

10:10–10:45 Júlia Komjáthy
Phases of one-dependent first passage percolation

10:45–11:10 *Coffee*

11:10–11:45 Illés Horváth
Reconstruction versus prediction

<u>**Abstracts**</u>

Márton Balázs (University of Bristol):

Magic in particle systems

Tue 16:50

Explicit stationary distributions in particle systems give rise to some surprising identities – the first of which I was started on by Bálint 25 years ago. This one was in the language of martingales and second moments, and served very well for both classical and anomalous, KPZ-scaling results on current variances. The second round of identities came from Rains in last passage percolation, re-discovered some 20 years later by Emrah, Janjigian and Seppäläinen (EJS). Rather than second moments, this works on the exponential level via Radon-Nikodym type arguments, and yields much sharper results. There is a catch though: it has only been worked out for models that live in a quadrant while having nice, explicit stationary boundaries on two sides. I will explain how to port the EJS method to particle systems and what difficulties we are trying to sort to get rid of the quadrant-restriction.

Joint struggle with Elnur Emrah, Jess Jay, Ben Landon, Phil Sosoe, Erin Russell.

Péter Bálint (Budapest University of Technology and Economics):

Generalized law of iterated logarithm in the infinite horizon Lorentz gas

Tue 10:10

The dynamics of the infinite horizon Lorentz gas is characterized by the interplay of chaotic collisions and long flights. This is reflected in the superdiffusive scaling in the limit theorem for the position of the particle. In this talk, I would like to discuss results which naturally generalize the almost sure invariance principle and the law of iterated logarithm to this setting.

This is joint work with Dalia Terhesiu.

Ofer Busani (University of Edinburgh):

On the global solutions of the KPZ fixed point

Tue 16:10

The KPZ fixed-point is a random process appearing as the universal scaling limit of 1+1 random growth models. As the slope of an initial condition is a conserved quantity for this system, it is natural to ask, for a fixed realization of the system, how many eternal solutions (bi-infinite in time) of a fixed slope exist?

It is known that for a fixed realization, there is a 'good' set of slopes for which 'one force one solution' principle holds i.e. there exists a unique global solution to the KPZ fixed point with a prescribed slope. Outside the good set not much is known other than that for 'bad' slopes uniqueness fails. In particular, the number and nature of such 'bad slope' global solutions is unknown.

In this work we completely characterize all global solutions to the KPZ fixed point with a prescribed slope. In particular we show that for bad slopes there exists infinitely many global solutions.

Joint work with Sudeshna Bhattacharjee and Evan Sorensen.

Edward Crane (University of Bristol):
Markov chain couplings of Markov chains
Mon 12:00

Given two Markov chains X and Y with state spaces A and B, and given a subset U of $A \times B$, how can you tell whether there exists a Markov chain with state space U that is a coupling of X and Y? I will present a few new results about this question. Strong lumpings (Dynkin's condition) and exact lumpings (the Pitman-Rogers condition) will also feature in the talk. Our motivation to study this problem came from studying a version of the Ráth-Tóth mean field forest fire model in which the edges are directed and fire only propagates in the direction of the arrows. For good initial conditions, the process of fire sizes and times has the same distribution in the directed model as its has in the original undirected model. We know this because we can couple the directed and undirected models to have identical fires. We wondered whether such a coupling could be chosen to be a Markov process with respect to its generated filtration.

Joint work with Erin Russell and Alexander Holroyd.

Laure Dumaz (DMA, ENS Paris):
True self-repelling motion: recent applications to non-reversible Monte Carlo
Tue 15:10

Tyler Helmuth (Durham University):
Statistical Mechanics and Hyperbolic Spin Systems
Mon 15:10

Spin systems with hyperbolic symmetries were initially considered as simplifications of / warmups for Efetov's method for random band matrices. Unexpectedly, it was later discovered that these spin systems compute certain observables for interesting statistical mechanics models, e.g., linearly reinforced walks and random forests. The basic behaviour of the statistical mechanics models has now been understood by making use of the spin system representations, but the most important phenomena remain conjectural. I'll explain these conjectures, their probabilistic translations, and (time permitting) comment on some ongoing related work.

Illés Horváth (Budapest University of Technology and Economics):
Reconstruction versus prediction
Wed 11:10

We consider observable processes in an environment that is not or just partially observable. Reconstruction is the question of identifying the environment based on the observation of the process, while prediction is the problem of predicting the future evolution of the process based on an observation over a finite time interval. We explore the relation of the two problems for an epidemic model on networks, but also provide a broader view.

Based on joint work with Dániel Keliger.

Jessica Jay (Lancaster University):
Connecting blocking measures of interacting particle systems with combinatorial objects
Mon 14:30

In 2018 Balázs and Bowen gave a purely probabilistic proof to a well-known combinatorial identity, the Jacobi triple product identity. This identity links an infinite sum to an infinite triple product and has interpretations across Mathematics and Physics. Probabilistically the identity is given as an equivalence of blocking measures via the Exclusion - Zero-range correspondence.

Recent research has found other examples of probabilistic proofs to identities of combinatorial significance by studying natural questions for certain interacting particle systems. This connection is not only interesting but can be very useful for proving new and sometimes surprising results both in combinatorics and probability.

In this talk we will see a selection of such results; new combinatorial identities and surprising probabilistic results which come from the connection with combinatorics.

Based on a selection of works joint with Daniel Adams, Márton Balázs, Dan Fretwell and Benjamin Lees.

Daniel Kious (University of Bath):
A story of self-interacting random walks getting stuck in atypical behaviours
Tue 11:50

Júlia Komjáthy (TU Delft):
Phases of one-dependent first passage percolation
Wed 10:10

In this talk we look at first passage percolation on spatial random graphs with power-law degree distributions where an underlying variable governs the expected degree of each node. To model a reasonably realistic scenario we assume that the transmission time through each edge depends on the expected degree-variables and also on the Euclidean length of the edge.

We show that in this setup there are 4 main universality classes of growth of the vertices reachable by time t, and a given graph may show all 4 phases by tuning only the transmission parameters but keeping the graph the same.

Finally for fun we show some pretty pictures where we apply the same model on a real dataset embedded in the Earth surface and find surprisingly good match with our theoretical results (which is a 'reverse' applied research since we searched for a dataset that fits our theory).

Joint with: John Lapinskas, Johannes Lengler, Ulysse Schaller. Simulations by Zylan Benjert.

Christopher Lutsko (University of Houston):
Diffusion in random Lorentz gas models
Tue 09:30

The Lorentz gas is an early model for the motion of electrons through metals and an early model of diffusion. We consider a cloud of non-interacting point particles moving through the compliment of an infinite array of obstacles in $\mathbb{R}^n$. While this model is simple in that the point particles do not interact with one another, in practice even this simple model is not fully understood depending on the obstacle configuration and interaction between point particles and obstacles. A holy grail in this field is to prove an invariance principle in the limit as the time goes to infinity. While this holy grail remains out of reach, I will present joint work with Bálint concerning a major step in that direction in a variety of contexts.

Gábor Pete (Budapest University of Technology and Economics / HUN-REN Alfréd Rényi Institute of Mathematics):
Indistinguishability of infinite cycles in permutation and percolation processes
Mon 16:10

A beautiful and very useful theorem of Lyons and Schramm (1999) is that infinite clusters in Bernoulli percolation on Cayley graphs (and more generally, in any insertion-tolerant invariant percolation) are indistinguishable by invariant events; e.g., either every infinite cluster is recurrent for simple random walk, or each is transient. In insertion-tolerant processes, one can have more than one infinite cluster only on non-amenable Cayley graphs, not Bálint's favourites. However, there are many natural invariant processes on $\mathbb{Z}^d$ where clusters can be finite or infinite cycles, and they have some sort of surgery-tolerance.

In joint work with Damis El A'lami and Ádám Timár, we prove indistinguishability of infinite cycles in many processes, including: the interchange process on any Cayley graph; biased corner percolation on $\mathbb{Z}^2$; the $O(n)$ loop model for amenable Cayley graphs. The first two processes were actually introduced by Bálint.

Miklós Zoltán Rácz (Northwestern University):
Phase transitions in the distinctness of neighborhoods in random graphs
Wed 09:30

When are the neighborhoods of vertices all distinct (i.e., nonisomorphic) in a random graph? I will discuss phase transitions for this question for Erdős–Rényi random graphs $G(n, p)$, some of which happen at curious values of p. I will also discuss motivation and implications of these results related to graph reconstruction problems and to average-case and smoothed analysis of graph isomorphism. This is based on joint works with Omer Angel, Nicolas Broutin, Julia Gaudio, Tom Hutchcroft, Nina Kamčev, Anirudh Sridhar, and Jifan Zhang.

Balázs Ráth (Budapest University of Technology and Economics / HUN-REN Alfréd Rényi Institute of Mathematics):
On the forest fires that Bálint ignited
Mon 11:20

I will review the origins of the forest fire model, the mean field version of which was introduced by Bálint. This model of self-organized criticality became a source of inspiration for multiple generations of researchers, including myself. The forest fire that Bálint ignited boils a melting pot in which inhomogeneous random graphs, explosive branching processes and controlled nonlinear PDE (as well as other juicy ingredients) form a delicious mix. This talk serves as an appetizer of the topic (spiced up with some anecdotes about the chef), with a dessert of open questions for those that are hungry for more.

Károly Simon (Budapest University of Technology and Economics):
Random Fractals
Mon 10:20

I will introduce a very popular family of random fractals called Fractal percolation or Mandelbrot percolation fractals. I will talk about classical results and some recent developments.

Domokos Szász (Budapest University of Technology and Economics):
Rényi's probability school and mathematical statistical physics in Hungary
Mon 09:30

In Hungary, modern probability theory – and, similarly, mathematical statistical physics – emerged under the influence of the leading Russian schools. This talk will outline how Bálint's interests developed from this foundation, which he notably enriched by exploring new, important, and elegant directions. I will also mention our joint results on mechanical models of Brownian motion.

Imre Péter Tóth (Budapest University of Technology and Economics):
Spectral gap in a stochastic energy exchange system motivated by a deterministic heat conduction model
Tue 11:10

Deriving Fourier's law of heat conduction rigorously for deterministic heat conduction models is one of the big challenges in mathematical statistical physics. A promising direction is the two-step approach of Gaspard and Gilbert. We start with a system of finitely many interacting billiard particles governed by classical (deterministic) Newtonian mechanics, and in the first part, by tuning the geometry, we obtain a (stochastic) Markov interacting particle system in the "rare interaction limit". In the second part, we study the hydrodynamic limit of this stochastic system as the number of particles goes to infinity.

In this talk I present a small but important step in the second part of this program: the spectral gap of the appearing Markov process is shown to decay as $1/N^2$ where N is the number of particles.

This is joint work with Eric Carlen and Gustavo Posta.

Benedek Valkó (University of Wisconsin Madison):
Random walks with Bálint Tóth
Mon 16:50

I will review some joint results (and joint experiences) with Bálint Tóth.

Bálint Vető (Budapest University of Technology and Economics):
The Brownian web and the Brownian web distance
Tue 14:30

The Brownian web consists of coalescing Brownian motions starting at every point of the plane, which was constructed by Tóth and Werner. In the talk, I plan to show the connection between the local time of the "true" self-avoiding walk and the Brownian web, and I mention some related Ray-Knight type results. Based on a recent joint work with Bálint Virág, I define the random walk web distance on the trajectories of coalescing random walks and its scale-invariant limit, the Brownian web distance, which can also be described in terms of the Brownian web. The Brownian web distance is integer-valued and has scaling exponents 0:1:2 as compared to 1:2:3 in the KPZ universality class, the Airy process still arises as its shear limit. A weighted version of the random walk web distance converges to a new explicit distribution that interpolates between the Gaussian and the GUE Tracy-Widom distribution.