Rényi's probability school and mathematical statistical physics in Hungary

Domokos Szász

Stochastics and Influences, 2025

I. Rényi and "modern" probability theory

1. **Diplom** Eötvös Univ. Supervisor: **L. Fejér** 1944
2. **Doctorate,** Szeged Univ. Supervisor: **F. Riesz** 1945
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II. My encounter with the mechanical theory of Brownian motion

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2. 1970: Dobrushin in Druskininkai: Markov Random Fields

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3. 1973: European Meeting of Statisticians, Budapest, 1972

Z. Cieselski on 1D ideal gas of identical particles:
- equilibrium: Harris (1965), Spitzer (1969). Wiener limit
- non-equilibrium: Szatzschneider (1975). Non-Wiener limit

4. 1975: Talk at Dobrushin-Sinai seminar

Major-Sz. AP. (1980): **general** *1D non-equilibrium model*

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III. Early MathStatPhys in Hungary

- 1972-: Learning seminar in Math. Institute on Preston: Gibbs States on Countable Sets (Holley Inequ.)
- **A. Krámli:** 1970-73, Grad studies with Sinai
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Some other classy directions of research in probability in Hungary

- Limit theorems
- Information theory
- Statistics
- Dynamical systems
- Applications in branches of mathematics
  - Discrete mathematics
  - Random fractals
  - Random matrices
  - Number theory
  - ...
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- Sz.-B. Tóth: Persistent RW in 1D random environment JSP (1984)

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V. Rayleigh gas

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Mechanical models of Brownian motion ($D = 1$) ★

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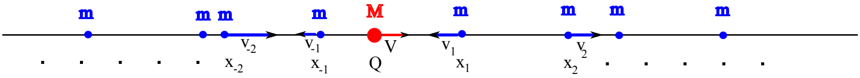

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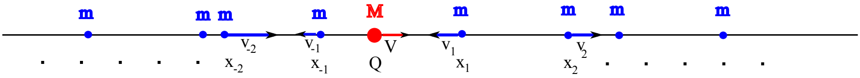

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V/2: Rayleigh gas. The model.

Münchhausen phase space:

$\mathfrak{X} = \mathbb{R} \times \Omega = \{x = (V, \omega) | V \in \mathbb{R}, \omega = (q_i, v_i)_{i \in I} \in \Omega\}$

Dynamics: uniform motion & elastic collision

$V^+ = \frac{M-1}{M+1} V^- + \frac{2}{M+1} v^-, \qquad v^+ = \frac{2M}{M+1} V^- - \frac{M-1}{M+1} v^-$

$S_t^M : \mathfrak{X} \to \mathfrak{X}$

Gibbs (invariant) measure: $\mu^M(d(V,\omega)) = dF^M(V)\nu(d\omega)$, where ν is Poisson measure on Ω of intensity $dxdF^1(v)$ and $dF^M(V) = \sqrt{\frac{M}{2\pi}} \exp\left(-\frac{MV^2}{2}\right) \qquad (M > 0)$

Notations: $V(x) = V, \quad \omega(x) = \omega$

$V_t^M(x) = V(S_t^M(x))$ and $Q_t^M(x) = \int_0^t V_s^M(x) ds$

TASK: understand $\lim_{A\to\infty} \frac{Q_{At}^M}{\sqrt{A}}$ (at least: $\lim_{t\to\infty} \mathbb{E}\frac{(Q_t^M)^2}{t} = \sigma_M^2$)

Harris, Spitzer ($M = m = 1$): $\frac{Q^1{}_{At}}{\sqrt{A}} \Rightarrow W^{\sigma_1^2}$ as $A \to \infty$

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V/3: Rayleigh gas: variance bounds

Theorem (Sinai-Soloveitchik (1986), Sz-Tóth (1986))

For $0 < M < \infty$

$$\underline{\sigma}^2 = (\sqrt{\frac{\pi}{8}}) = \liminf \mathbb{E}\frac{(Q_t^M)^2}{t} \leq \limsup \mathbb{E}\frac{(Q_t^M)^2}{t} = \overline{\sigma}^2\,(= \sigma_1{}^2) < \infty$$

Computer results:

- Omerti-Ronchetti-Dürr(1986): $\lim_{M\to\infty} \sigma_M = \underline{\sigma}$
  Contradicting Lebowitz-Goldstein-Dürr: $\sigma_M^2 \equiv \sigma_1^2$?
- Khasin(1987): $M \neq 1$ Non-Gaussian limit
- Boldrighini-Frigio-Tognetti(2002):
  $\underline{\sigma}^2 \approx .627 < \lim_{M\to 0} \sigma_M^2 = \sigma_0^2 \approx .74 < \overline{\sigma}^2 = .798$
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Péter Bálint-Bálint Tóth-I. Péter Tóth (2017): limiting model when $M \to 0$.

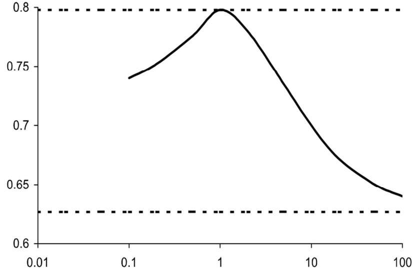
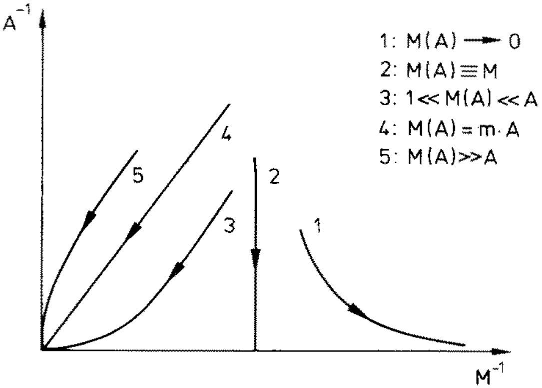


Fig. 2. Qualitative dependence $M \mapsto \sigma_M^2$ suggested by earlier numerical works.

Rayleigh: General picture, $D = 1$: Sz-T(1987)

Goal: understand $\lim_{A\to\infty} \frac{Q_{At}^{M(A)}}{\sqrt{A}}$



Some cases

- $M(A) \equiv M$: **Conjecture:** As $A \to \infty$, $\frac{Q^M}{\sqrt{A}} \Rightarrow T^M$ where T^M is a process w stationary increments and variance σ_M^2.
  NB: Harris, Spitzer: $T^1 = W^{\bar{\sigma}^2}$
- $1 \ll M(A) \ll A$:

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Theorem (Sz.-Tóth (1987))

If $A^{\frac{1}{2}+\varepsilon} \ll M(A) \ll A$ with $\varepsilon > 0$, then - as $A \to \infty$ -

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Method: Coupling

- $M(A) = cA$: Holley(1971): $\sqrt{A} V_{At}^{cA} \Rightarrow \eta^c$, $\frac{Q_{At}^{cA}}{\sqrt{A}} \sqrt{A} \Rightarrow \xi^c$
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Theorem (Sz.-Tóth (1987))

If $A^{\frac{1}{2}+\varepsilon} \ll M(A) \ll A$ with $\varepsilon > 0$, then - as $A \to \infty$ -

$$\frac{Q_A^{M(A)}}{\sqrt{A}} \Rightarrow W^{\underline{\sigma}^2}$$

Method: Coupling

- $M(A) = cA$: Holley(1971): $\sqrt{A} V_{At}^{cA} \Rightarrow \eta^c$, $\frac{Q_{At}^{cA}}{\sqrt{A}} \sqrt{A} \Rightarrow \xi^c$
  where η^c is an Ornstein-Uhlenbeck process

Rayleigh $D = 1$

(14, 17): S-S(1986), Sz-T(1986) Ref. 8: Holley (1971)

$d=1$	$A^{-1/2}Q^{M(A)}_{A\cdot} \Rightarrow$		$A^{-1/2}\tilde{Q}^{M(A)}_{A\cdot} \Rightarrow$
	Expected	Proved[a]	Proved[a]
$M(A) \ll 1$	$W^{\bar{\sigma}}$	??	Explodes
$M(A) \equiv M$	T^M	??	$W^{\tilde{\sigma}_M}$
	$\sigma_M \to \sigma$ if $M \to \infty$	Only known[14,17]: $\sigma \leqslant \sigma_M \leqslant \bar{\sigma}$	Also known: $\tilde{\sigma}^2_M \sim M^{-1/2}$ $(M \to 0)$ $\tilde{\sigma}^2_M \to \sigma^2$ $(M \to \infty)$ $\tilde{\sigma}^2_M \geqslant (1+1/M)^{1/2}\,\sigma^2_M$
$1 \ll M(A) \ll A$	W^{σ}	?? Known if $M(A) \gg A^{1/2+\varepsilon}$	W^{σ}
$M(A) = mA$	ξ^m	Ref. 8	ξ^m (Ref. 8)
$M(A) \gg A$	0	Trivial	0

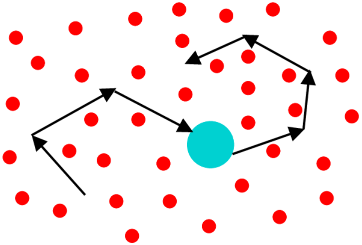
Rayleigh $D > 1$

Ref. 5: Dürr-Goldstein-Lebowitz(1981)

$d > 1$	$A^{-1/2} Q_{A\cdot}^{M(A)} \Rightarrow$		$A^{-1/2} \tilde{Q}_{A\cdot}^{M(A)} \Rightarrow$
	Expected	Proved[a]	Proved[a]
$M(A) \ll 1$	??	??	Explodes
$M(A) \equiv M$	$W^{\sigma_{d,R}^M}$	??	$W^{\tilde{\sigma}_{d,R}^M}$
	$\sigma_{d,R}^M \geqslant \underline{\sigma}_{d,R}$	??	—
$1 \ll M(A) \ll A$	$W^{\underline{\sigma}_{d,R}}$	?? Only if $M(A) \gg A^{1/2+\varepsilon}$	$W^{\underline{\sigma}_{d,R}}$
$M(A) = mA$	$\xi_{d,R}^m$	Ref. 5	$\xi_{d,R}^m$ (Ref. 5)
$M(A) \gg A$	0	Trivial	0 trivial

Random Lorentz gas

It is the Rayleigh gas when $m \equiv \infty$ (and $0 < M < \infty$).



Difficulty: in RLG, memory effects are more substantial.

Lutsko, C., Tóth, B. Invariance Principle for the Random Lorentz Gas-Beyond the Boltzmann-Grad Limit. CMP (2020)
$D = 3$, Boltzmann-Grad scaling, diffusive limit,
$T = o((r|\log r|)^{-2})$.

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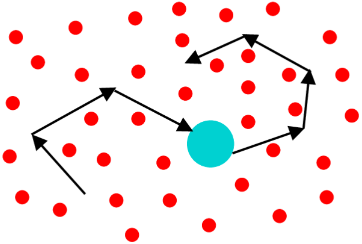

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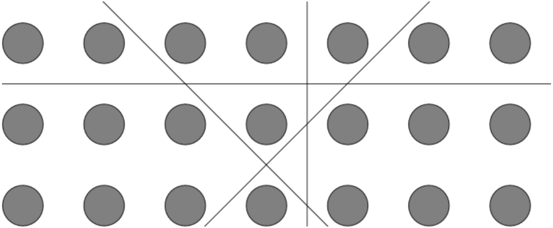
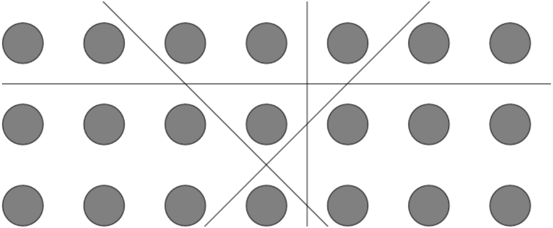
Periodic Lorentz Process, infinite horizon case



Marklof, J., Tóth, B. (2016), $D \geq 2$,
Boltzmann-Grad scaling, superdiffusive normalization $\sqrt{T \log T}$ rather than diffusive $\sqrt{T}$ (cf. Bleher(1992))

$D = 2$, superdiffusive normalization $\sqrt{T \log T}$, fixed scatterer size (!)
Sz-Varjú(2007): LT for discrete time process using Young towers
Dolgopyat-Chernov(2009): LT for flow & exp. correlation decay(!) using standard pairs

Periodic Lorentz Process, infinite horizon case



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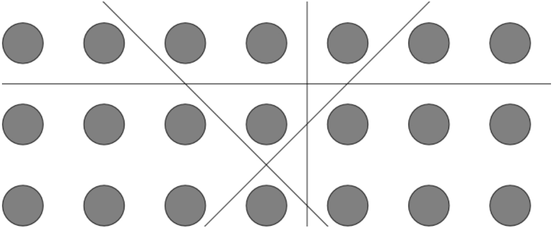
Periodic Lorentz Process, infinite horizon case



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Bálint's wide spectrum of interests

- Dynamical theory of Brownian Motion. Diffusion in deterministic dynamics
- Stochastic representations of quantum spin systems (in particular, stirring process, long cycle conjecture)
- Anomalous diffusion of random processes with long-range memory (in particular, Brownian web (Bálint and Wendelin Werner))
- Hydrodynamic limits of hyperbolic interacting particle systems
- ...

Students

Bálint Tóth

MathSciNet

Ph.D. Hungarian Academy of Sciences 1988

Dissertation: *On Dynamical Theories of Brownian Motion: Mechanical and Probabilistic Models*

Mathematics Subject Classification: 60—Probability theory and stochastic processes

Advisor: Domokos Szász

Students:
Click here to see the students ordered by family name.

Name	School	Year	Descendants
Balázs, Márton	Technical University of Budapest	2003	7
Valkó, Benedek	Technical University of Budapest	2004	5
Rath, Balazs	Budapest University of Technology and Economics	2010	
Vető, Bálint	Budapest University of Technology and Economics	2011	
Dumaz, Laure	Université Paris-Sud XI - Orsay	2012	1
Rudas, Anna	Budapest University of Technology and Economics	2013	
Horvath, Illes	Budapest University of Technology and Economics	2015	
Davidson, Angus	University of Bristol	2018	
Maxey-Hawkins, Felix	University of Bristol	2022	

According to our current on-line database, Bálint Tóth has 9
...ents and 21 descendants.
...me any additional information.


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HAPPY BIRTHDAY BÁLINT!