Blocking measures, hills, and hydrodynamics

Joint with
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Márton Balázs

University of Bristol

Probability and NonLocal PDEs
Swansea, 17 September, 2018.

Models

- Asymmetric simple exclusion
- Zero range

Classical knowledge

- Asymmetric hydrodynamics
- Symmetric hydrodynamics

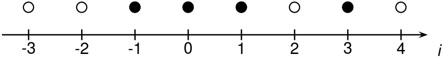
Blocking measures

- ASEP
- ZRP
- Further models

Hills

- Microscopic model
- Hydrodynamics

Asymmetric simple exclusion



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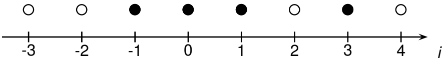

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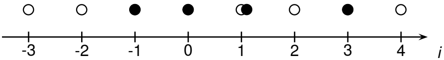

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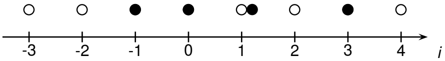

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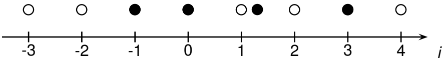

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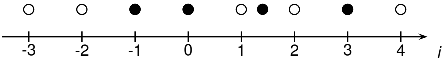

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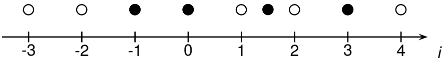

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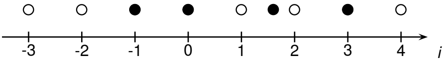

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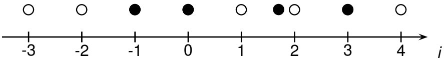

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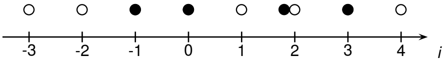

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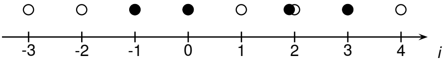

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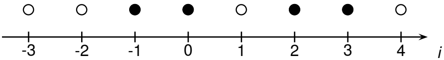

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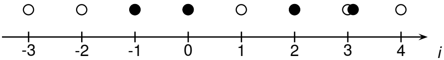

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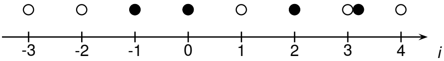

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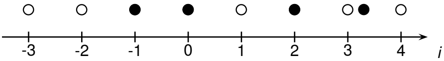

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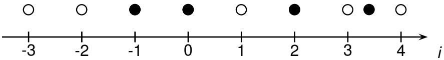

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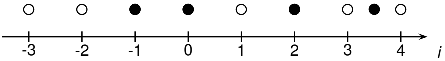

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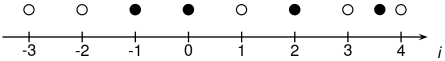

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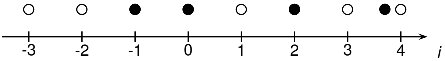

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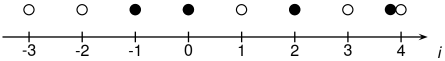

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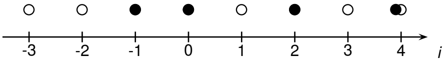

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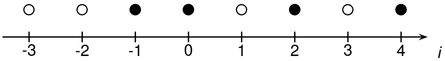

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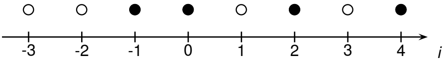

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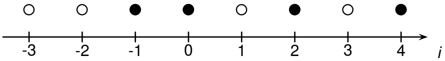

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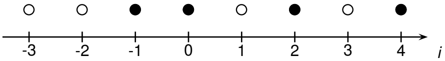

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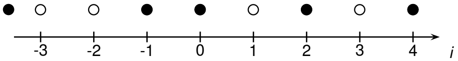

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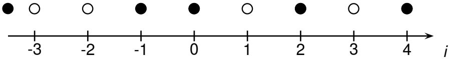

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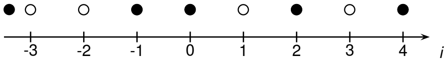

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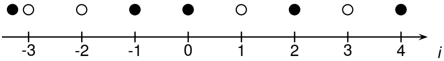

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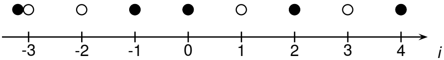

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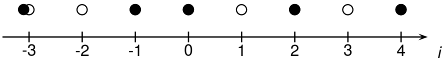

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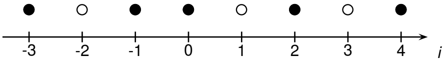

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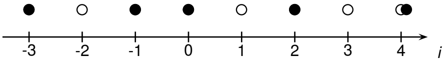

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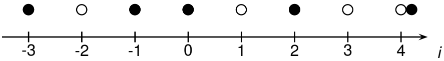

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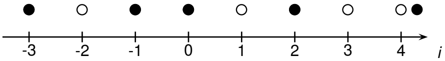

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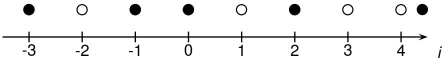

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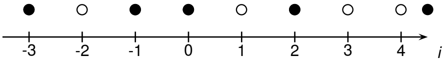

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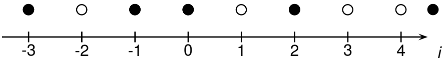

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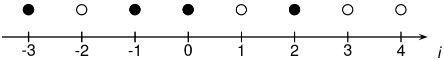

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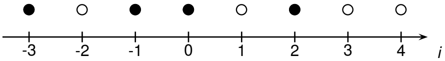

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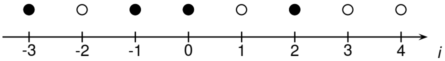

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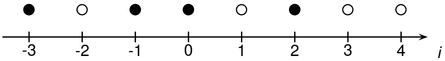

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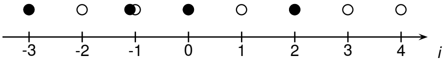

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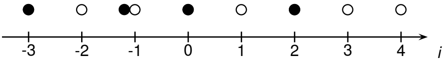

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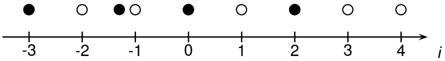

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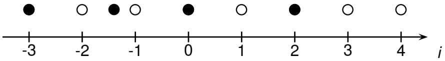

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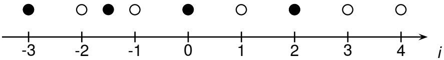

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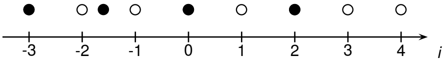

$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
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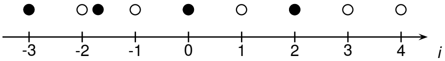

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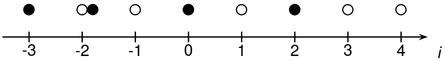

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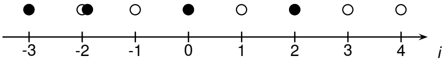

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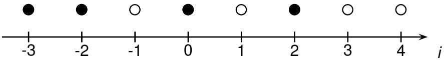

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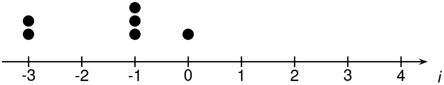

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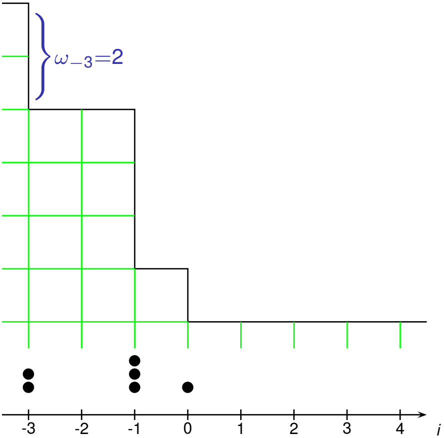

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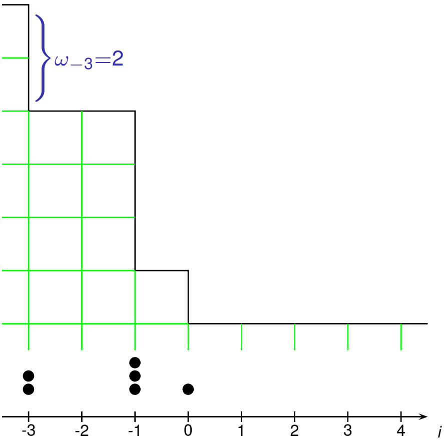

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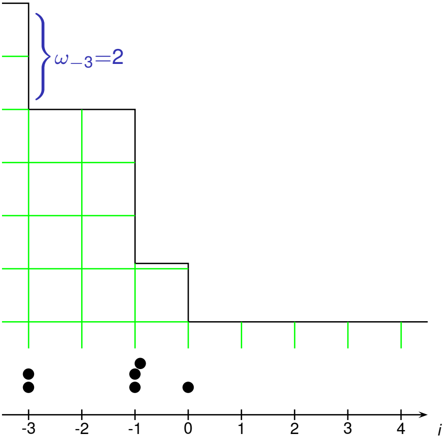

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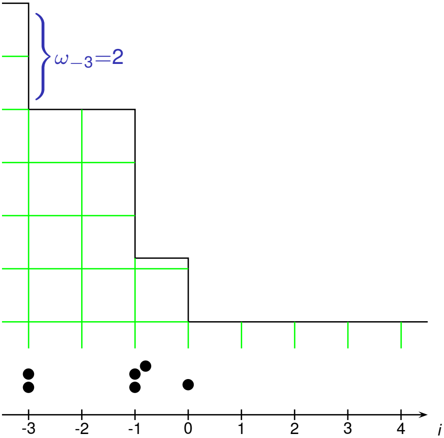

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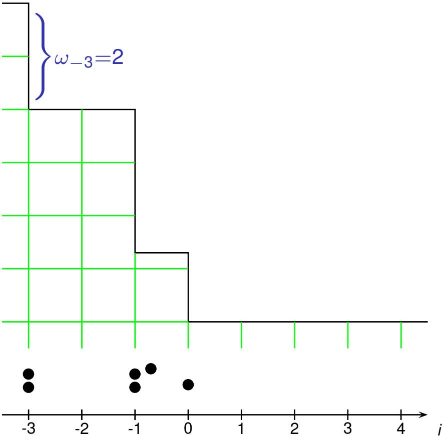

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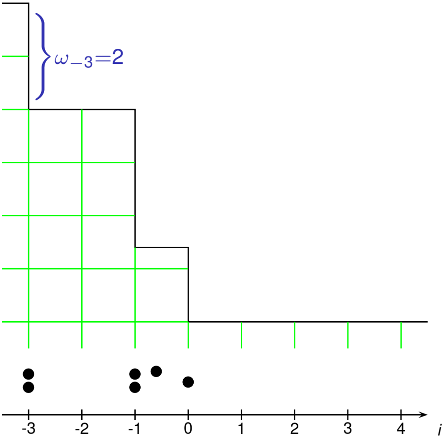

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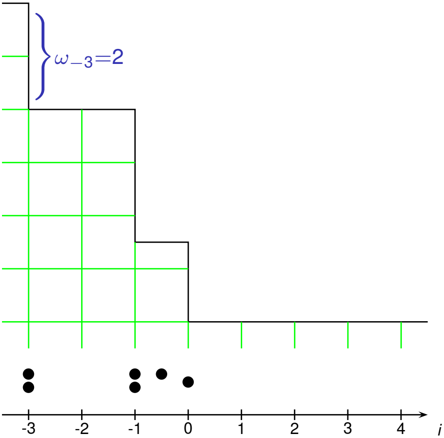

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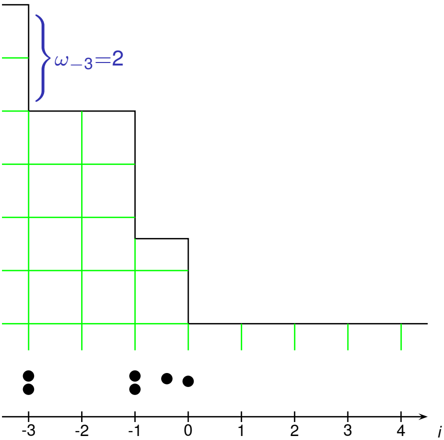

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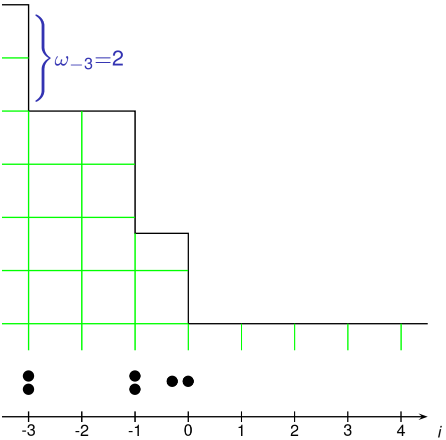

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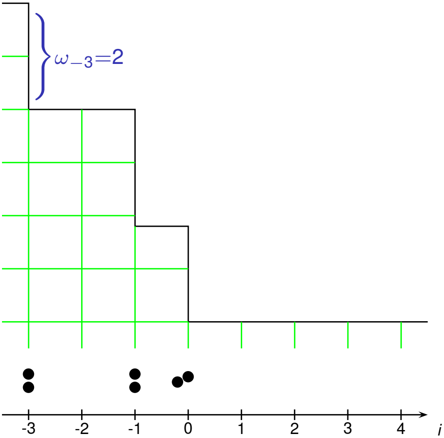

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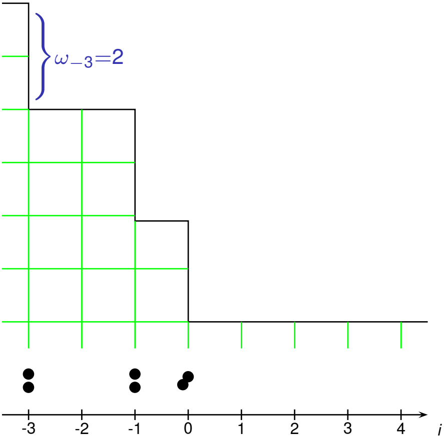

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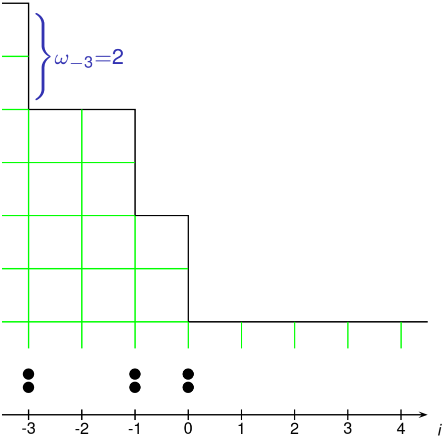

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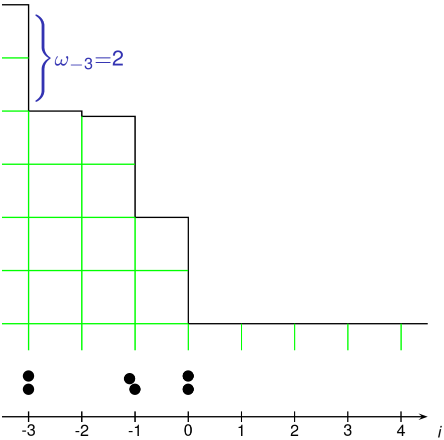

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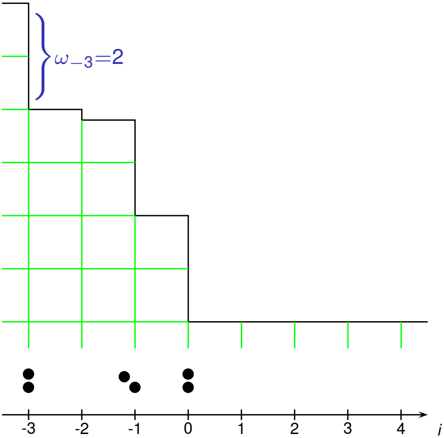

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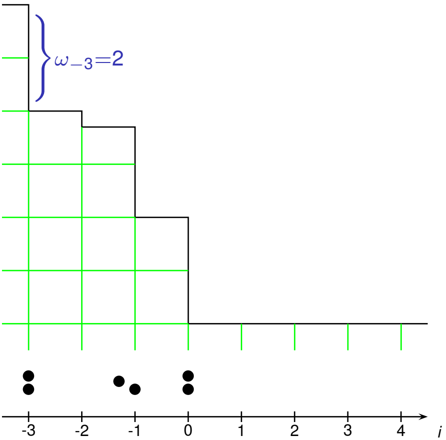

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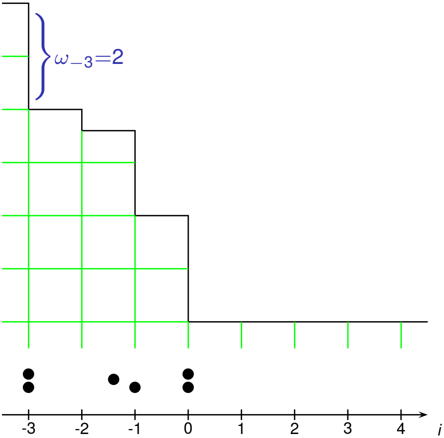

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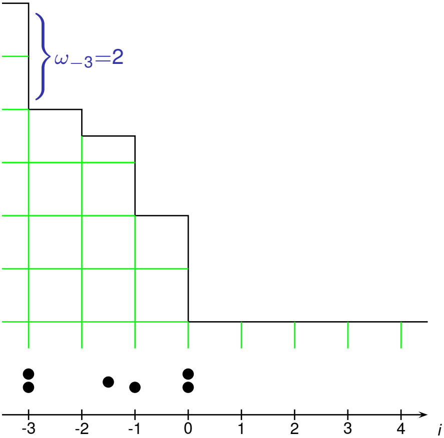

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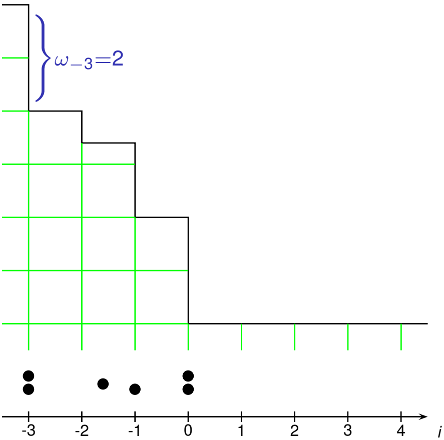

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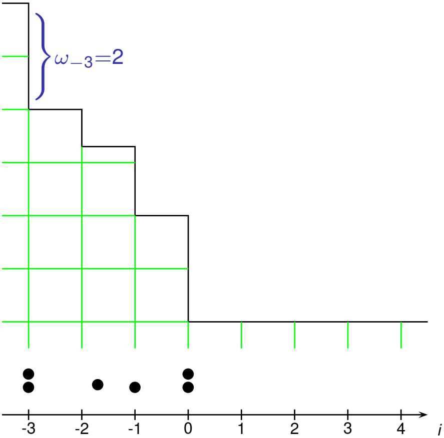

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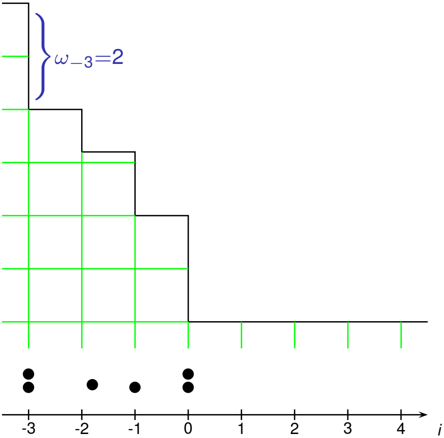

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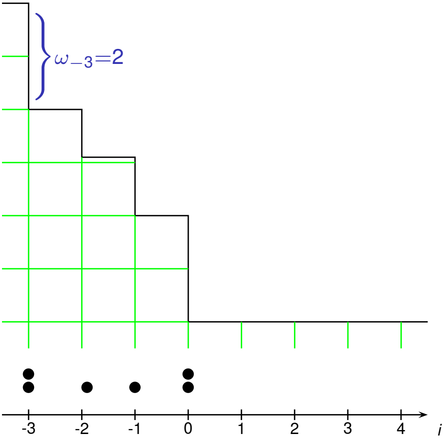

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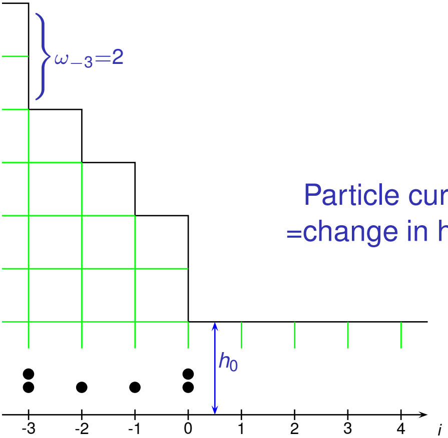

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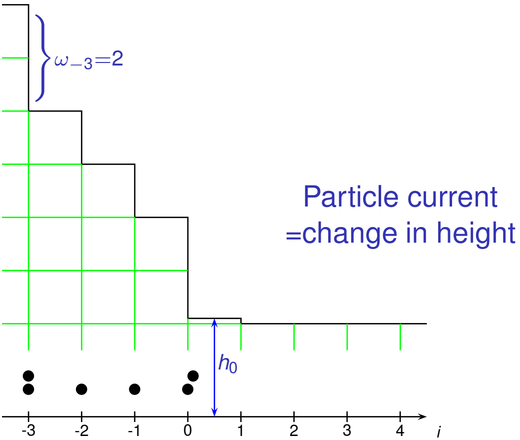

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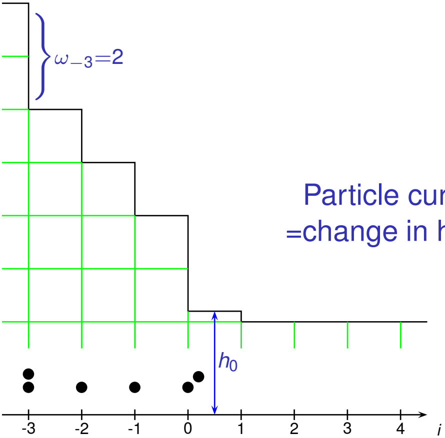

Particle current =change in height

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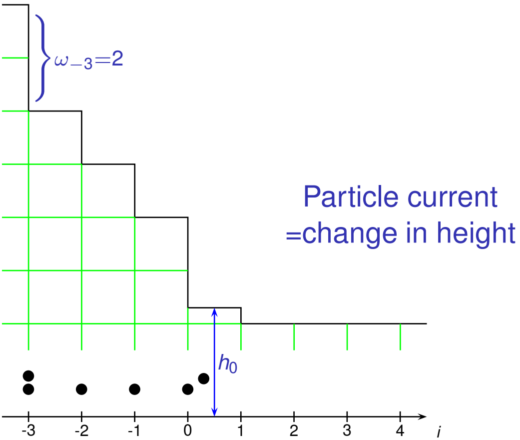

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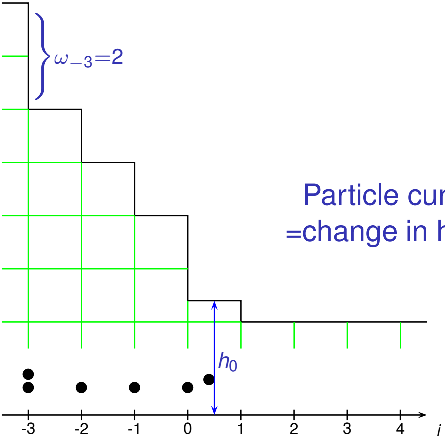

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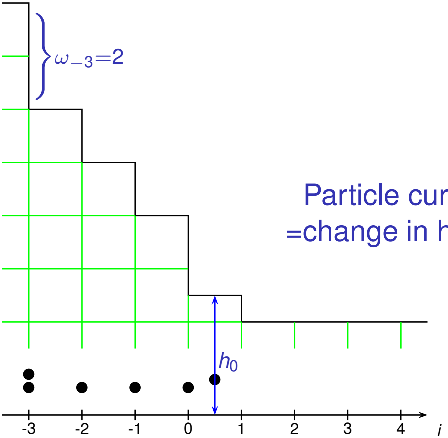

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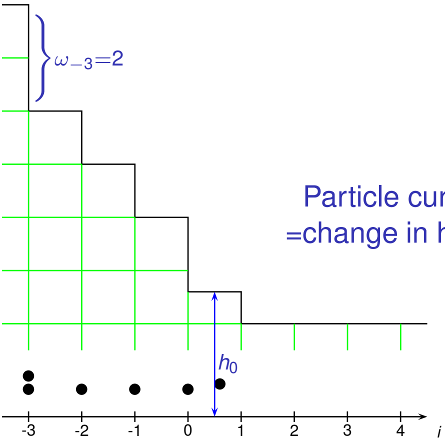

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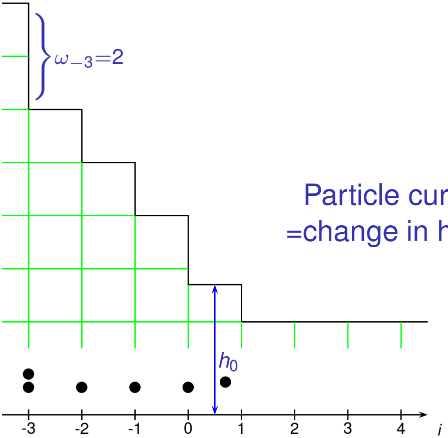

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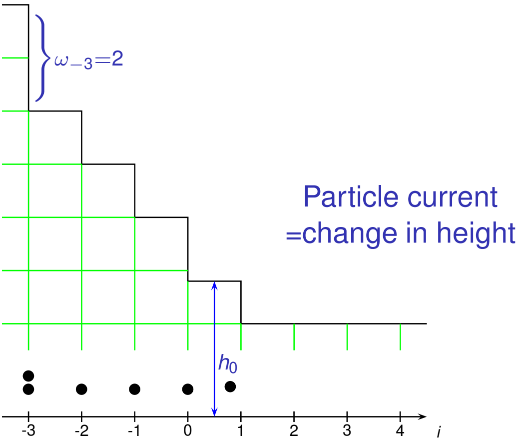

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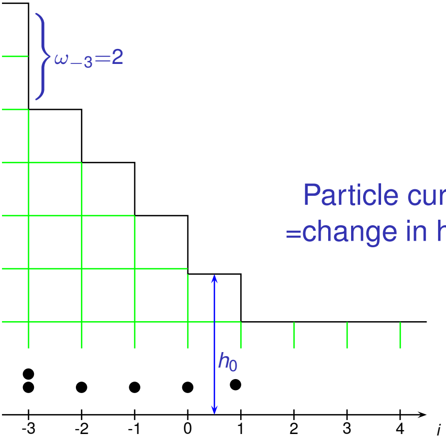

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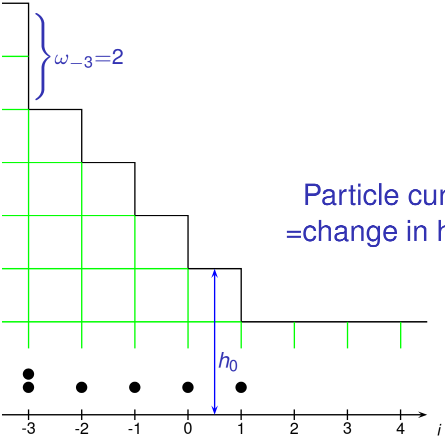

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The asymmetric zero range process

We need r non-decreasing.

Examples:

- 'Classical' ZRP: $r(\omega_i) = \mathbf{1}\{\omega_i > 0\}$.
- Independent walkers: $r(\omega_i) = \omega_i$.

Translation invariant measures

No surprise that constant density is stationary:

ASEP: $\eta_i \sim$ iid. Bernoulli(ϱ).

Classical ZRP: $\omega_i \sim$ iid. Geometric($\frac{1}{1+\varrho}$).

Independent walkers: $\omega_i \sim$ iid. Poisson(ϱ).

These are the only extremal translation-invariant distributions.

Hydrodynamics: asymmetric ZRP

Take $p > q = 1 - p$, and AZRP with right rate $p \cdot r(\omega_i)$, left rate $q \cdot r(\omega_i)$.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = p\mathbf{E}r(\omega_{i-1}) + q\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

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Ballistic scaling (zoom out and speed up by factor L):

- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(Lt)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho}r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L)}\mathbf{E}\omega_i = qL\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\big) - pL\big(\mathbf{E}r(\omega_i) - \mathbf{E}r(\omega_{i-1})\big)$$

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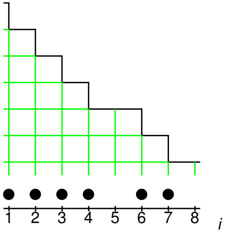
$$\frac{\partial}{\partial t}\varrho(t,\, x) = (q-p)\frac{\partial}{\partial x}G(\varrho(t,\, x)).$$

Hydrodynamics: asymmetric ZRP

$$\frac{\partial}{\partial t}\varrho(t,\,x)+(p-q)\frac{\partial}{\partial x}G(\varrho(t,\,x))=0$$

Classical ZRP: $G(\varrho)=\mathbf{E}^{\varrho}r(\omega)=\mathbf{E}^{\varrho}\mathbf{1}\{\omega>0\}=\frac{\varrho}{1+\varrho}$
concave, Burgers-type equation.

Independent walkers: $G(\varrho)=\mathbf{E}^{\varrho}r(\omega)=\mathbf{E}^{\varrho}\omega=\varrho$
linear, transport equation.



The stationary solution is constant density, linear slope.

Hydrodynamics: symmetric ZRP

Take $p = q = \frac{1}{2}$, and SZRP with right rate $\frac{1}{2}r(\omega_i)$, left rate $\frac{1}{2}r(\omega_i)$.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\mathbf{E}r(\omega_{i-1}) + \frac{1}{2}\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

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Diffusive scaling:

- $\varrho(t, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
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$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_i = \frac{L^2}{2}\big(\mathbf{E}r(\omega_{i+1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i-1})\big)$$

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$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\mathbf{E}r(\omega_{i-1}) + \frac{1}{2}\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

Diffusive scaling:

- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_i = \frac{L^2}{2}\big(\mathbf{E}r(\omega_{i+1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i-1})\big)$$

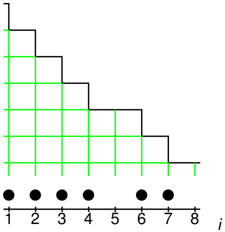
$$\frac{\partial}{\partial t}\varrho(t,\, x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\, x)).$$

Hydrodynamics: symmetric ZRP

$$\frac{\partial}{\partial t}\varrho(t,\,x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\,x))$$

Classical ZRP: $G(\varrho) = \mathbf{E}^{\varrho} r(\omega) = \mathbf{E}^{\varrho}\mathbf{1}\{\omega > 0\} = \frac{\varrho}{1+\varrho}$
concave, nonlinear heat equation.

Independent walkers: $G(\varrho) = \mathbf{E}^{\varrho} r(\omega) = \mathbf{E}^{\varrho}\omega = \varrho$
linear, heat equation.



The stationary solution is constant density, linear slope, or linearly changing G with current.

Hills

Can we model sedimentation and erosion processes with these surfaces?

Issues:

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- Hills are not always straight $\leftrightarrow$ translation invariance.

Hills

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Issues:

- Hills are not always straight $\leftrightarrow$ translation invariance.
- Most hillslopes are rather stationary $\leftrightarrow$ particle current.

Convex hills

Wikipedia

Concave hills

Stockphotos4free

Product blocking measures

Solution: block particles (no current) and make their rates asymmetric (non-constant density).

Can we have a reversible stationary distribution in product form:

$$\underline{\mu}(\underline{\omega}) = \bigotimes_i \mu_i(\omega_i);$$

$$\underline{\mu}(\underline{\omega}) \cdot \text{rate}\big(\underline{\omega} \to \underline{\omega}^{i \curvearrowright i+1}\big) = \underline{\mu}\big(\underline{\omega}^{i \curvearrowright i+1}\big) \cdot \text{rate}\big(\underline{\omega}^{i \curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

Here

$$\underline{\omega}^{i \curvearrowright i+1} = \underline{\omega} - \underline{\delta}_i + \underline{\delta}_{i+1}.$$

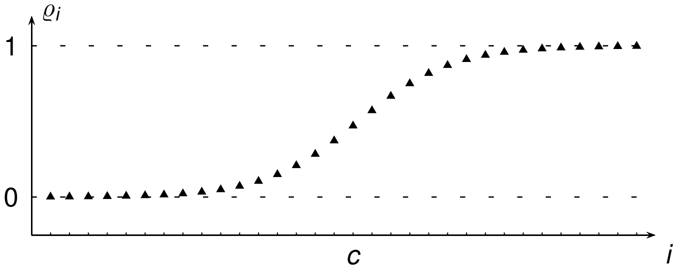
Asymmetric simple exclusion

$$\underline{\mu}(\underline{\eta}) \cdot \text{rate}\big(\underline{\eta} \to \underline{\eta}^{i\curvearrowright i+1}\big) = \underline{\mu}(\underline{\eta}^{i\curvearrowright i+1}) \cdot \text{rate}\big(\underline{\eta}^{i\curvearrowright i+1} \to \underline{\eta}\big)$$

ASEP: $\mu_i \sim \text{Bernoulli}(\varrho_i)$; $\underline{\eta}$

$$\varrho_i(1-\varrho_{i+1}) \cdot p = (1-\varrho_i)\varrho_{i+1} \cdot q$$

Solution: $$\varrho_i = \frac{(\frac{p}{q})^{i-c}}{1+(\frac{p}{q})^{i-c}} = \frac{1}{(\frac{q}{p})^{i-c}+1}$$



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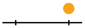

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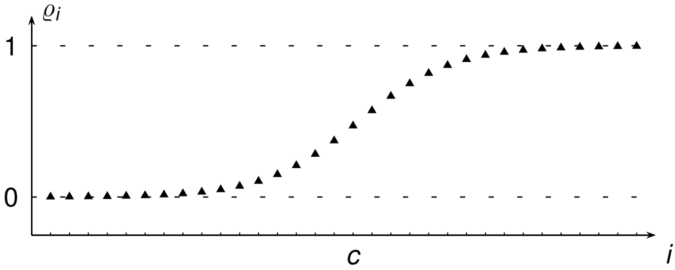

Asymmetric zero range process

$$\underline{\mu}(\underline{\omega}) \cdot \text{rate}\big(\underline{\omega} \to \underline{\omega}^{i\curvearrowright i+1}\big) = \underline{\mu}\big(\underline{\omega}^{i\curvearrowright i+1}\big) \cdot \text{rate}\big(\underline{\omega}^{i\curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

AZRP, classical:

$$\mu_i(\omega_i)\mu_{i+1}(\omega_{i+1}) \cdot p\mathbf{1}\{\omega_i > 0\} = \mu_i(\omega_i - 1)\mu_{i+1}(\omega_{i+1} + 1) \cdot q$$

Solution: $\mu_i \sim \text{Geometric}\Big(1 - \Big(\dfrac{p}{q}\Big)^{i-\text{const}}\Big).$

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AZRP, independent walkers:

$$\mu_i(\omega_i)\mu_{i+1}(\omega_{i+1}) \cdot p\omega_i = \mu_i(\omega_i - 1)\mu_{i+1}(\omega_{i+1} + 1) \cdot q(\omega_{i+1} + 1)$$

Solution: $\mu_i \sim \text{Poisson}\Big(\Big(\frac{p}{q}\Big)^{i-\text{const}}\Big).$

Further models

In fact product blocking measures are very general.

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Other models can be *stood up*:

- ASEP
- q-exclusion
- Katz-Lebowitz-Spohn model

Product blocking measures

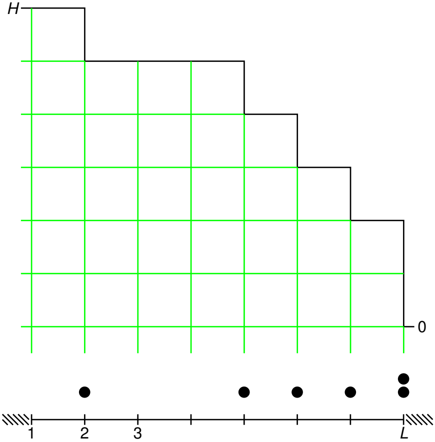
They are also very handy, due to reversibility.

Take a stationary, reversible Markov chain. Cut any of its edges. It stays reversible stationary w.r.t. the same distribution.

<u>In our case:</u> freeze the boundaries to obtain a stationary hill slope.

Microscopic model

Our choice: AZRP with frozen boundaries. $p > q$: convex



Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

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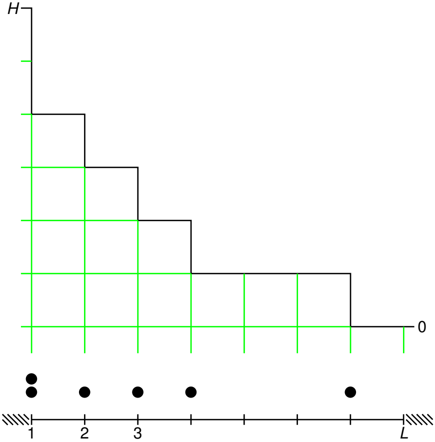

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Microscopic model

Notice:

- The height of the hill H is conserved, the product measure is *not* ergodic.
- One-site marginals, given H, are in general not explicit.
- Except for independent walkers, where ω_i are Binomial.

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- The height of the hill H is conserved, the product measure is *not* ergodic.
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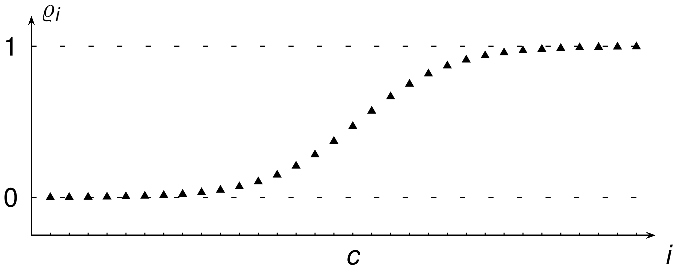
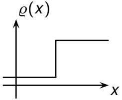
We won't be bothered by this.

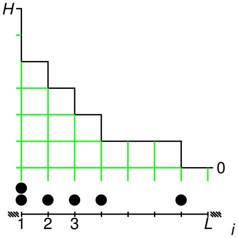
Hydrodynamics

Work in progress. . .



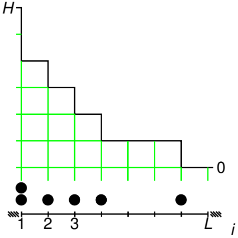
A blocking measure is a microscopic object. Here is its scaling limit:  , not very interesting.

Hydrodynamics



- Scaling parameter: L

Hydrodynamics



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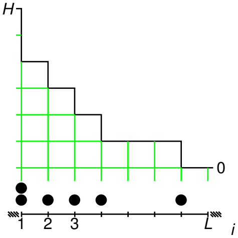
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^i = \left(\frac{p}{q}\right)^{Lx}$.

Hydrodynamics



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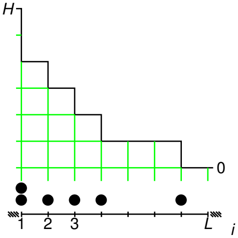

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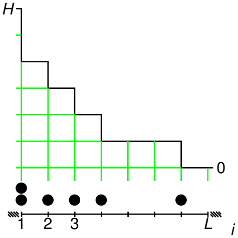

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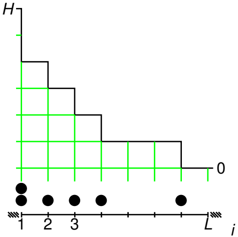

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For AZRP (rates $p \cdot r(\omega_i)$ right and $q \cdot r(\omega_i)$ left):

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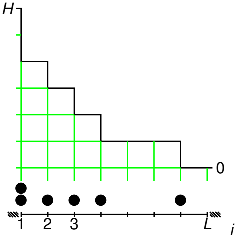

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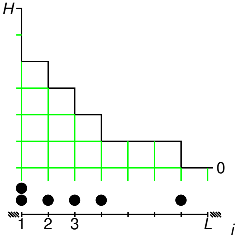

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which dictates diffusive scaling:

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Hydrodynamics

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Convection-diffusion type equation with Robin boundary.

Hydrodynamics

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\frac{\partial}{\partial t}\varrho(t,x) &= \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,x)) - 2\gamma\frac{\partial}{\partial x}G(\varrho(t,x)) \qquad (0 < x < 1)\\
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0 &= \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,0)) - 2\gamma G(\varrho(t,0))\\
0 &= \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,1)) - 2\gamma G(\varrho(t,1))
\end{aligned}$$

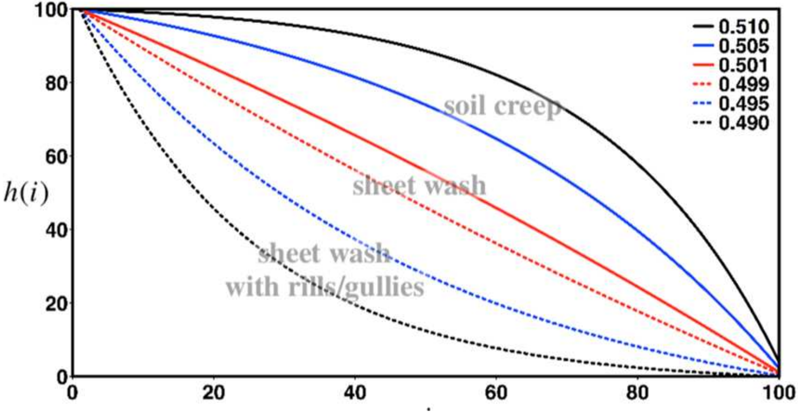
Convection-diffusion type equation with Robin boundary.

Doing the proper derivation is work in progress.

The time-stationary solution $G(\varrho(x)) = C\mathrm{e}^{4\gamma x}$ is consistent with the stationary blocking measure.

The stationary slope

$$G(\varrho(x)) = Ce^{4\gamma x}$$



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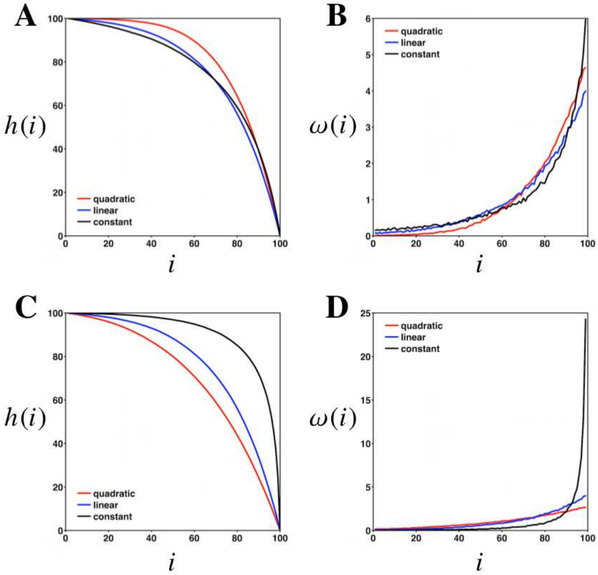


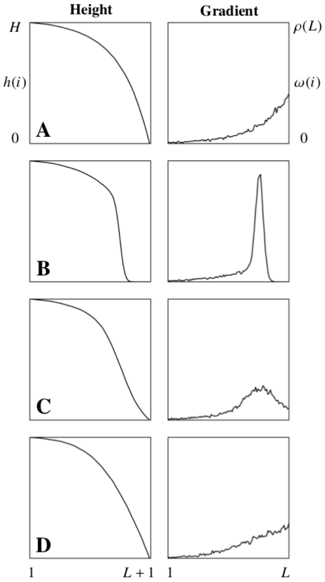
Dynamics

Space scale: $x \in [0, 1] \Leftrightarrow$ we $\in$ hill.

Problem 1: The stationary hillslope will not tell us the time scale.

$\rightsquigarrow$ Observe relaxation to stationarity in Nature and in the PDE.

Dynamics



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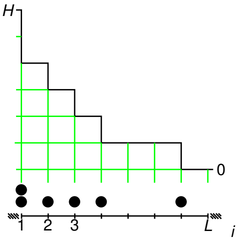

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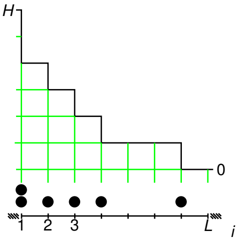
Notice: Hill particles $\neq$ our particles.

Dynamics



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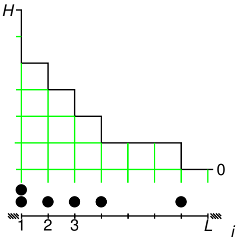

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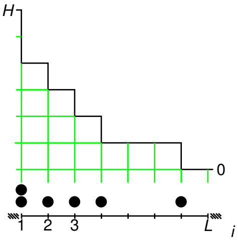

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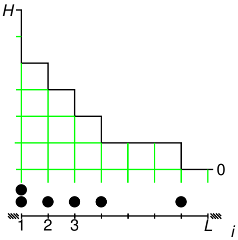

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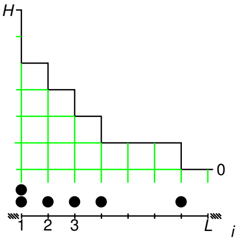

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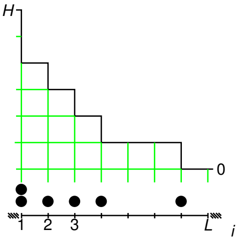

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One can then give an expected distance travelled by a hill particle. *Thank you.*