Problems for the tenth week

Mathematics A3 for Civil Engineering students

1. Steve has not prepared for his exam, where he has to answer 10 yes or no questions. A small part of the material dawns on him, and so he can give the correct answer to each question with probability 60%. What is the probability he will pass if that needs at least 8 correct answers?

2. Each student on a test has to answer 20 yes or no questions. Assume that independently for each question, a student knows the correct answer with probability 0.7, believes that he knows the correct answer, but he is wrong with probability 0.1, and doesn't know the answer with probability 0.2. In this case he answers yes or no with probability $\frac{1}{2} - \frac{1}{2}$. What is the probability that he will answer at least 19 questions correctly?

3. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student would get 4 or more correct answers just by guessing?

4. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be wrong when decoded?

5. A wolverine starts from the origin of the integer line. At each step he moves one unit to the right with probability 1/2, and to the left with probability 1/2, independently of his previous steps. After 20 moves,

   (a) What is the probability that he is at position 0?

   (b) What is the probability that he is at position 1?

   (c) What is the probability that he is at position (-2)?

   (d) What is the probability that he is at position (-2), if he was at (-3) one step ago?

6. I have two coins, a fair one and a biased one, but I cannot distinguish them. The biased coin comes up heads with probability 3/4. I pick one of the two coins from my pocket, the fair one with probability 1/2 and the biased one with probability 1/2. Then I flip the chosen coin 30 times, and I find that it came up heads 25 times. What is the probability that I chose the biased coin?

7. When coin A is flipped, it lands heads with probability 0.4; when coins B is flipped, it land heads with probability 0.7. One of these coins is randomly chosen and flipped 10 times.

   (a) Are these coin flips independent of each other?

   (b) What is the probability that exactly 7 of the 10 flips land on heads?

   (c) Given that the first of these ten flips lands heads, what is the probability that we are using coin A?

   (d) Given that the first of these ten flips lands heads, what is the probability that exactly 7 of the 10 flips land on heads?

8. It is known that diskettes produced by a certain company will be defective with probability 0.01, independently of each other. The company sells the diskettes in packages of size 10 and offers a money-back guarantee that at most 1 of the 10 diskettes in the package will be defective.

(a) What is the probability that a box contain more than one defective diskettes?

(b) If someone buys 3 packages, what is the probability that he or she will return exactly 1 of them?

9. A newsboy purchases papers at HUF 100 and sells them at HUF 150. However, he is not allowed to return unsold papers. If his daily demand is a binomial random variable with $n = 10$ and $p = 1/3$, approximately how many papers should he purchase so as to maximize his expected profit?

10. Approximately 80 000 marriages took place in a country last year. Estimate the probability that for at least one of these couples

    (a) both partners were born on April 30;

    (b) both partners celebrate their birthday on the same day of the year.

    State your assumptions.

11. There are 200 typos, randomly distributed, in a book of 400 pages. What is the probability that on page 13 there are more than one typos?

12. How many chocolate chips should there be in a muffin on average if we want at least one chocolate chip in any given muffin with probability at least 0.99?

13. The Run With Us Movement organized a foot-race at the Danube Bend. Unfortunately, the track passed through an area infected with ticks. After the race, 300 contestants found one tick, 75 found two ticks on their bodies. Based on this information, approximate the number of contestants in this race.

14. Consider a roulette wheel consisting of 38 numbers – 1 through 36, 0, and double 0. If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that

    (a) Smith will loose his first 5 bets;

    (b) his first win will occur on his fourth bet?

15. An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If 2 of them are white and 2 are black, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 chosen are white. What is the probability that we shall make exactly n selections?

16. We repeatedly roll a die until we roll a 6. What is the expected number of times we roll that die? And if we roll two dice a time until we see a 6 on at least one of them?

17. We repeatedly roll a die until we see the same number twice in a row. What is the expected number of rolls we make?

18. Out of our 100 keys, only one opens the door in front of us. In the dark we don't see the keys we already tried, and so we might pick and try any given key more than once. What is the probability that we open the door by at most 50 trials? And what if we put away the ones we already tried?

19. If X is a geometric random variable, show analytically that

$$\mathbb{P}\{X = n + k \mid X > n\} = \mathbb{P}\{X = k\}. \tag{1}$$

Give a verbal argument using the interpretation of a geometric random variable as to why the preceding equation is true.

20. Mr. Bilk takes the tram to work each day, but he has no monthly pass nor ticket. Every day the ticket controller gets on the tram with probability 0.2, and then catches Mr. Bilk with probability 0.95. (Every day the ticket controller decides independently whether to get on Mr. Bilk's tram or not.)

    (a) What is the probability that Mr. Bilk has a "lucky week" that is, he won't get fined on the five working days of the week?

    (b) What is the probability that he will get fined exactly twice on the five working days of the week?

    (c) Given that Mr. Bilk had a lucky week, what is the probability that there was a ticket controller on his tram on each of the five working days?

    (d) What is the probability that he will get fined on Thursday the first time?

21. The suicide rate in a certain country is 1 suicide per 100 000 inhabitants per month.

    (a) Find the probability that in a city of 400 000 inhabitants within this country, there will be 8 or more suicides in a given month.

    (b) What is the probability that there will be at least 2 months during the year that will have 8 or more suicides?

    (c) Counting the present month as month number 1, what is the probability that the first month to have 8 or more suicides will be month number i, $i \geq 1$?