Problems for the first week

1. Find the general solution of the differential equation
$$y' = \frac{y}{x}.$$

2. Find the general solution of the differential equation
$$y' = \mathrm{e}^{x+y}.$$

3. Consider a sample of radioactive material which has mass $y(t)$ kg at time t. It has been observed that a constant factor of those radioactive atoms will spontaneously decay (into atoms of another element or into another isotope of the same element) during each time unit.

   (a) Find the differential equation which describes this process if the half-life of the material is $T = 100sec.$

   (b) Assume that at the beginning we had $1kg$ from this material. Find $y(t)$.

   The **half-life** of a radioactive material is the time for an amount of this material to decay to one-half of its original value.

4. Find the solution of the following initial value problem:
$$y' = \frac{\mathrm{e}^x}{y+1} \quad ; \quad y(0) = -4.$$

5. Assume that as a result of the drag force the decay of the speed of a moving object is proportional to the square of the speed of the object. Let $v(t)$ be the velocity as a function of the time. Write a differential equation for $v(t)$ and solve this differential equation.

6. Find the general solution of the differential equation
$$y' = \frac{1 + 2\mathrm{e}^y}{\mathrm{e}^y x \ln(x)}.$$

7. Find the general solution of the differential equation
$$(\mathrm{e}^{-2y} - \mathrm{e}^{-y})y' = \frac{\mathrm{e}^{x-y} + \mathrm{e}^{-x-y}}{\mathrm{e}^y + 1}.$$

8. Find the solution of the following initial value problem:
$$x + y - xy' = 0 \quad ; \quad y(1) = 1.$$

9. Find the general solution of the differential equation
$$x\mathrm{e}^{y/x} + y = xy'.$$

10. Find the general solution of the differential equation
$$xy' = y(\ln y - \ln x).$$