Problems for the third week

1. In each of the following problems (a) through (d) find the solution of the given initial value problem and compute $\lim_{t\to\infty} y(t)$.

   (a) $y'' + 5y' + 6y = 0, \qquad y(0) = 2, y'(0) = 3$

   (b) $y'' + y' - 2y = 0, \qquad y(0) = 1, y'(0) = 1,$

   (c) $y'' + 4y' + 3y = 0, \qquad y(0) = 2, y'(0) = -1,$

   (d) $y'' + 8y' - 9y = 0, \qquad y(1) = 1, y'(1) = 0.$

2. Consider the initial value problem

$$y'' + 5y' + 6y = 0, \qquad y(0) = 2, y'(0) = \beta,$$

   where $\beta > 0$.

   (a) Solve the initial value problem.

   (b) Determine the coordinates (t_m, y_m) of the maximum point of the solution as functions of β

   (c) Determine the smallest value of β for which $y_m \geq 4$.

   (d) Determine the behavior of t_m and y_m as $\beta \to \infty$.

3. In each problems (a) through (d) use Euler's formula to write the given expression in the form $a + ib$.

   (a) e^{-3+6i}

   (b) e^{1+2i}

   (c) $\mathrm{e}^{i\pi}$

   (d) 2^{1-i}

4. In each of the following problems (a) through (d) find the solution of the given initial value problem

   (a) $16y'' - 8y' + 145y = 0, \qquad y(0) = -2, y'(0) = 1$

   (b) $y'' + 4y = 0, \qquad y(0) = 1, y'(0) = 1,$

   (c) $y'' - 2y' + 5y = 0, \qquad y(\pi/2) = 0, y'(\pi/2) = 2,$

(d) $y'' + 2y' + 2y = 0, \qquad y(\pi/4) = 2, y'(\pi/4) = -2.$

5. In each of the following problems (a) through (d) find the solution of the given initial value problem

   (a) $y'' - y' + 0.25y = 0, \qquad y(0) = 2, y'(0) = \frac{1}{3}$

   (b) $9y'' - 12y' + 4y = 0, \qquad y(0) = 2, y'(0) = -1$

   (c) $9y'' + 6y' + 82y = 0, \qquad y(0) = -1, y'(0) = 2$

   (d) $y'' + 4y' + 4y = 0, \qquad y(-1) = 2, y'(-1) = 1$

6. Consider the initial value problem

$$4y'' + 12y' + 9y = 0, \qquad y(0) = 1, y'(0) = -4.$$

   (a) Solve the initial value problem and plot its solution for $0 \leq t \leq 5$.

   (b) Determine where the solution has the value zero.

   (c) Determine the coordinates (t_0, y_0) of the minimum points.

   (d) Change the second initial condition to $y'(0) = b$ and find the solution as a function of b. Then find the critical value of b that separates solutions that always remain positive from those that eventually become negative.