Problems for the fourth week

1. Use the method of undetermined coefficients to find the general solution of the following five differential equations:

   (a) $y'' - 3y' - 4y = 3e^{2t}$

   (b) $y'' - 3y' - 4y = 2\sin t$

   (c) $y'' - 3y' - 4y = -8t\cos(2t)$

   (d) $y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8t\cos(2t)$

   (e) $y'' - 3y' - 4y = 2e^{-t}$

2. In each of the following four problems find the general solution of the differential equation

   (a) $y'' - 2y' - 3y = 3e^{2t}$

   (b) $y'' + 2y' + 5y = 3\sin(2t)$

   (c) $y'' - 2y' - 3y = -3te^{-t}$

   (d) $y'' + 2y' + y = 2e^{-t}$

3. Find the solution of the initial value problem:

$$y'' + 4y = t^2 + 3t, \qquad y(0) = 0, y'(0) = 2.$$

4. Find the general solution of

$$y'' + 4y = 3\csc t, \qquad (\csc t = 1/\sin t)$$

5. First use the method of variation of parameters to find the general solution of the following two differential equations. Then use the method of undetermined coefficients to check your answers.

   (a) $y'' - 5y' + 6y = 2e^{t}$.

   (b) $4y'' - 4y' + y = 16e^{t/2}$.

6. Find the general solution of the differential equation

$$y'' + y = \tan t, \qquad 0 < t < \frac{\pi}{2}.$$

7. Find the general solution of the differential equation

$$4y'' + y = 2\sec(t/2), \qquad -\pi < t < \pi \ (\sec t = 1/\cos t).$$

8. Consider the

$$t^2y'' - t(t+2)y' + (t+2)y = 2t^3, \quad t > 0.$$

First verify that the functions

$$Y_1 = t, \qquad Y_2 = te^t$$

form a fundamental solution of the corresponding homogenous equation $t^2y'' - t(t+2)y' + (t+2)y = 0$. Then find the general solution of the inhomogeneous equation.

9. A mass 4 lb stretches a spring 2 in. Supposed that the mass is displaced an additional 6 in (1/2 ft) in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 lb/sec. Formulate the initial value problem that governs the $y(t)$ motion of the mass.