Problems for the fifth week

1. Solve the following initial value problem:

$$y'' = \frac{1}{\sqrt{1-x^2}}, \qquad y(0) = 3,\ y'(0) = 1$$

2. A rod is loaded by a bending moment that is proportinal to the value $f(x)$ at each coordinate x. It is known that the shape of this rod's median can be computed by solving the following differential equation:

$$\frac{y''}{(1+(y')^2)^{3/2}} = f(x).$$

Determine the shape of the rod if the bending moment follows

$$f(x) = 1 - x$$

and the initial conditions are given by

$$y(0) = y'(0) = 0.$$

3. (a) Consider the differential equation of free mechanical vibration without damping

$$my'' + ky = 0. \tag{1}$$

Solve it as an incomplete second order differential equation.

(b) Solve differential equation (1) as a second order linear equation.

(c) Prove that the solutions you obtained in 3a and 3b are the same.

4. Find the general solution of the following differential equations.

(a)

$$(y')^2 + 2yy'' = 0,$$

(b)

$$y'' = \frac{1}{4\sqrt{y}},$$

(c)

$$yy'' + (y')^2 = 1$$

5. Solve the following second order differential equation:

$$xy'' - y' = x^3.$$

6. Solve the following differential equations:

   (a) $2x\cos y + [2y\cos y - (x^2+y^2)\sin y]\,y' = 0$,

   (b) $xdy + ydx = 0$,

   (c) $\frac{x}{x^2+y^2}y' = \frac{y}{x^2+y^2}$,

   (d) $2x(\sin y + 1) + x^2\cos y \cdot y' = 0$.