Problems for the seventh week
Mathematics A3 for Civil Engineering students

1. Solve the following system of differential equations:

$$x' + 2y' - 3x + 4y = 2\sin t$$
$$2x' + y' + 2x - y = \cos t.$$

2. Solve the following system of differential equations:

$$x' - 2x + y = e^t$$
$$y' - 3x + 2y = t.$$

3. Solve the following system of differential equations:

$$x' - 2x + 5y = -\cos t$$
$$y' - x + 2y = \sin t.$$

4. Solve the following system of differential equations:

$$x' - x - y = e^{-2t}$$
$$y' - 4x + 2y = -2e^t.$$

5. Suppose that A and B are mutually exclusive events for which $\mathbb{P}\{A\} = 0.3$ and $\mathbb{P}\{B\} = 0.5$. What is the probability that

   (a) either A or B occurs;

   (b) A occurs but B does not;

   (c) both A and B occur?

6. A total of 28% of males smoke cigarettes, 7% smoke cigars, and 5% smoke both cigarettes and cigars.

   (a) What percentage of males smoke neither cigars nor cigarettes?

   (b) What percentage smoke cigars but not cigarettes?

7. A small community organization consists of 20 families, of which 4 have one child, 8 have two children, 5 have three children, 2 have four children, and 1 has five children.

   (a) If one of these families is chosen at random, what is the probability it has i children, $i = 1, 2, 3, 4, 5$?

   (b) If one of the children is randomly chosen, what is the probability this child comes from a family having i children, $i = 1, 2, 3, 4, 5$?

8. We flip a fair coin twice. What is the probability that at least one of our flips comes out Heads? What is the probability that *precisely* one of our flips comes out Heads?

9. What is the probability that, when rolling two fair dice, at least one of them shows a six? What is the probability that none of them show a six?

10. A pair of fair dice are rolled. What is the probability that the second die lands on a higher value than does the first?

11. Two symmetric dice have both had two of their sides painted red, two painted black, one painted yellow, and the other painted white. When this pair of dice are flipped, what is the probability that both land on the same color?

12. What is the probability that all three children of a family have the same sex? (We assume that each child is born girl or boy independently with probability 1/2.)

13. At least how many coin flips are needed in order to see at least one Heads with probability at least 90%?

14. We flip a fair die six times. What is the probability that each of the six numbers shows up?

15. Five people, designated as A, B, C, D, E, are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that

   (a) there is exactly one person between A and B;

   (b) there are exactly two people between A and B;

   (c) there are exactly three people between A and B?

16. What is the probability that in a classroom of 30 students there are no two birthdays that coincide? (We assume that each birthday is independently equally likely to be on any of the 365 days of the year. Do not bother with leap years.)

17. What is the probability that we will precisely have two hits on the lottery, where 5 numbers are drawn out of 90?

18. A forest contains 20 deers, of which 5 are captured, tagged, and then released. A certain time later 4 of the 20 deers are captured. What is the probability that 2 of these 4 have been tagged? What assumptions are you making?

19. There are 30 psychiatrists and 24 psychologists attending a certain conference. Three of these 54 people are randomly chosen to take part in a panel discussion. What is the probability that at least one psychologist is chosen?