

Problems for the eighth week
Mathematics A3 for Civil Engineering students

1. We roll a fair die. What is the probability that we roll a 6, if we know that:
 - the roll is an even number?
 - the roll is at least 3?
 - the roll is at most 5?
2. We roll two dice. What is the probability that at least one of the dice shows a 2, if we already know that the sum of the rolls is 6? And if we do not know anything?
3. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?
4. We repeatedly roll two dice at the same time, and only stop when at least one of them shows a six. What is the probability that the other also shows a six? (If you think it's $\frac{1}{6}$ then try again.)
5. There are 230 students and 30 teachers in a school. Once a flu epidemic occurs in the school, and the doctor finds the following data:

	Sick	Healthy
Boy	50	60
Girl	40	80
Teacher	10	20

A card is made for each of the 260 persons in the Doctor's office.

- (a) We randomly pick one of these cards. What is the probability that it belongs to:
 - i. a boy?
 - ii. a sick person?
 - iii. a sick boy?
- (b) If cards of boys, girls and teachers are stored in separate drawers, and I pick one card out of the girls' drawers, what is the probability I pick a card of a sick girl?
- (c) The Doctor's agile nurse collected all cards of sick persons into one large pile. Picking one of these randomly, what is the probability that the card belongs to a teacher?
- (d) If I draw two cards, one after the other, out of this pile, what is the probability that the first card belongs to a boy and the second one belongs to a girl? And that both belong to boys?

6. (a) I come from a family of two children. What is the probability that the other child is my sister?
- (b) The king comes from a family of two children. What is the probability that the other child is his sister?

7. Each of two balls is painted either black or gold and then placed in an urn. Suppose that each ball is colored black with probability $1/2$, and that these events are independent.

(a) Suppose that you obtain information that the gold paint has been used (and then at least one of the balls is painted gold). Compute the conditional probability that both balls are painted gold.

(b) Suppose, now, that the urn tips over and one ball falls out. It is painted gold. What is the probability that both balls are gold in this case?

8. In a certain community, 36 percent of the families own a dog, and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat.

(a) What percentage of families own both a dog and a cat?

(b) Out of those families who own a cat, what percentage owns also a dog?

9. An urn contains 3 red, 5 white and 6 green balls. We draw 3 balls one after the other. What is the probability that we draw a red first, a white second, and a green ball the third time, if drawings are made

(a) with replacement,

(b) without replacement?

10. Cockroach extinction is made in three steps in a building estate. The first step exterminates 60% of the cockroaches, but then they become more and more immune, hence the second step only exterminates 40%, and the third step 20% of them. What is the probability that a given cockroach

(a) survives the whole procedure?

(b) dies in the third step?

(c) survives the whole procedure, if he was seen alive after the first step?

11. Based on our information 60% of businesses with company A , and 70% of businesses with company B turn out to be favorable. We do two businesses with the company that contacts us sooner. Each A and B has probability $1/2$ to be the first to contact us. What is the probability that

(a) the first business we do will be favorable?

(b) both businesses we do will be favorable?

(c) we will do both favorable and unfavorable businesses?

12. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining the probability that each hand has an ace. Let E_i be the event that the i th hand has exactly one ace. Determine $\mathbb{P}\{E_1E_2E_3E_4\}$ by using the multiplication rule.

13. A recent college graduate is planning to take the first three examinations in the coming summer. She will take the first exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the conditional probability that she passes the second one is 0.8, and if she passes both the first and the second exams, then the conditional probability that she passes the third exam is 0.7.

(a) What is the probability that she passes all three exams?

(b) Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

14. Drunkard Druce spends $2/3$ of the day in pubs. The village has 5 pubs, and Druce is not choosy, he can be found in any of the 5 pubs with equal chance. Once we set out for finding him. We have looked for him in 4 pubs already, but we haven't found him. What is the probability that he will sit in the fifth pub?

15. 0.1% of drivers drive through the red lights of railway crossings. The crossing of that low-traffic road at the bottom of the hill is red in 5% of the time. I see from the hill that a car is just passing the crossing. What is the probability that the crossing is red at this moment?

16. At the factory, machines are produced on production lines A , B , and C . 25% of the machines are produced on production line A , 35% on line B , and 40% on line C . It is known that 5% of machines produced on production line A , 4% of machines produced on line B , and 2% of machines produced on line C , respectively, are defective. We select randomly one machine manufactured by the factory, and it turns out to be defective. What is the probability that it was produced on line A , B , or C ?

17. Once upon a time Odysseus met an intersection of three pathways. One of them lead to Athens, the other lead to Mycenae, and the third lead to Sparta, but he didn't know which route goes to which of these cities. He chose one of the routes by rolling a die, giving equal chance to each of these choices. He knew that, on average, Athenians only tell the truth in one case out of three, Mycenae citizens lie every second time, but people of Sparta are always honest. In the city he arrived, he asked the first man he met how much two times two was, and had four as answer. What is the probability that Odysseus finally reached Athens?

18. Suppose that an insurance company classifies people into one of three classes: good risks, average risks, bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15, and 0.3. If 20% of the population are good risks, 50% are average risks, and 30% are bad risks, what proportion of people have accidents next year? If a policyholder had no accidents last year, what is the probability that (s)he is a good, average, or bad risk?

19. "0" is sent with probability $1/3$, and "1" is sent with probability $2/3$ on a binary channel. Due to network noises, a sent "0" arrives as "1" with probability $1/4$, and a sent "1" arrives as "0" with probability $1/5$.

- (a) What is the probability that a "0" arrives?
- (b) We received a "0". What is the probability that it was also sent as a "0"?

20. A transport company wants to determine the average number of passengers on its buses. They have to decide between two methods:

- (a) They entrust n randomly selected passengers to count the total number of passengers on the buses they travel with. Then the company computes the average of the collected n answers.
- (b) The company asks n of its bus drivers to count the number of passengers on their buses, and computes the average of these n answers.

Which method would you recommend? Which method will give a larger result?