A simple pension fund model (Simon Wood Feb 2018)

The purpose of this note is to produce a simple pension fund model to investigate the plausibility of the claim that the USS has become unaffordable. The model is aimed at establishing how a fund has to behave in order to be adequately funded over the long term. Note that over the last decade the USS fund has made an average annual return of 8.5% (net of management costs), while average CPI inflation has been 2.3%.

1 Basics

1. A pension scheme is a saving-for-retirement scheme in which participants club together to avoid the risk of running out of savings if they live a long time after retirement. The long lived are paid for by the short lived, all sides accepting the gamble.

2. In computing whether a scheme is likely to run out of money or not there are several considerations. 

(a) Inflation is irrelevant provided that salary/pension growth and investment returns are computed after subtracting inflation and the computation of pension entitlement discounts entitlement calculations appropriately by inflation, which is always the case. (Inflation is just a gradual change in the units of measurement - it doesn’t change the the underlying process.)

(b) Individuals vary in their salary trajectories up to retirement, and their longevity after retirement. The variability in level of salary at retirement is irrelevant provided it is independent of longevity (which is likely after controlling for gender).

(c) Individuals also vary in their length of service at retirement. This matters because shorter service lengths result in less time for the fund to grow.

(d) The rate of return on investment above inflation is the main unknown.

3. There are therefore three sources of uncertainty that matter, in decreasing order of importance.

(a) Future fund performance relative to inflation.

(b) Future changes in longevity/life-table structure.

(c) Future salary increases.

The last of these lies largely within the control of the scheme participants, and in any negotiation the effect on the pension fund is a powerful argument.

2 A simple model

We can think of the fund as being made up of a pension pot for each member. While they are working this grows, provided that the investment returns are not disastrous. After retirement it grows less quickly or shrinks. For the fund to be viable we require that the average balance at death is positive. If longevity is independent of salary, then this is equivalent to requiring that the expected balance at death is positive for all individuals. But actually the expected balance criterion is sufficient for viability in any case.

So all we need is a model for an individual pension pot, $P(t)$. Let $t_r$ denote retirement age, and $t_s$ the starting age. The key individual characteristic is their inflation adjusted salary profile between $t_s$ and $t_r$, $s(t)$, say. The key fund level characteristic is its growth rate in excess of inflation, $g(t)$, and the contribution rate as a proportion of salary, $\alpha$. Then for $t_s \leq t < t_r$ ($t$ has units of years)

$$\frac{dP}{dt} = g(t)P(t) + \alpha s(t) \text{ where } P(t_s) = 0.$$ 

After $t_r$, the inflation adjusted pension paid out is some functional of $s(t)$. In the USS case it is just

$$p = \int_{t_s}^{t_r} s(t)dt/75$$
At retirement a lump sum is paid out that is a multiple of $p$. In the USS case the multiple is 3. So for $t \geq t_r$ we have

$$\frac{dP}{dt} = g(t)P(t) - p \text{ where } P(t_r) = P(t_r^-) - 3p$$

($P(t_r^-)$ being the pension pot size before lump sum payout). Notice that the equation is correct when $P$ is negative, since a negative balance reduces the overall fund by this amount, with the growth of the reduction thereby being forgone.

Averaging $P$ over the distribution of age of death gives the expected balance at death. For simplicity dependants are not included in the above model (they shift the time of death upwards but may introduce a step reduction in $p$ at some random time).

### 2.1 Discrete version

For those who prefer a discrete model, let $s_i$, $g_i$ and $P_i$ denote the salary, fund growth and pension pot size, $i$ time units (months or years) after starting in the scheme. Then

$$P_i = P_{i-1} + g_i P_{i-1} + \alpha s_i \text{ where } P_0 = 0$$

up until retirement at $i = i_r$, say. The pension is then worked out as

$$p_y = \frac{1}{75} \sum_i s_i.$$  

$P_{i_r}$ is then set to $P_{i_r} - 3p_y$. Then the pension pot evolves as

$$P_i = P_{i-1} + g_i P_{i-1} - p,$$

where $p = p_y$ if the time unit is year, and $p = p_y/12$ if the time unit is month. Again we average $P_i$ over the distribution of death times to obtain the expected pension pot balance at death.

### 2.2 Remarks

1. The age at death distribution can be taken from ONS life tables, which are publicly available. Salary trajectories are harder to come by, but because benefits are career averaged, the sensitivity is anyway not very strong.

2. For a serious valuation this model would obviously be modified by adding in the details for death in service benefits and dependants. The latter is a substantial factor.

3. To value the whole scheme, the model can be run for each member, or a representative sample.

4. To value the whole scheme requires assumptions about the salary profiles for all members, the death age distribution and the fund performance relative to inflation. A decent assessment of scheme risk would explore the variability in these.

### 2.3 Dependents

The most serious deficiency in the above is the neglect of husbands, wives or other eligible dependants who substantially extend the period for which a pension is likely to be paid. This needs to be fixed.

The rules are that a surviving spouse continues to receive half the pension when the USS member dies. There seems to be no obvious simplification available here, and simulation is needed. I did the following, simulating with an annual time step:

1. A separate pension pot is simulated for each of $n$ USS members.

2. Each member has a sex, and is married to a partner of the opposite sex, with the age difference selected randomly from the age difference at marriage distribution for 1998, reported by the ONS.
3. I assumed that all members and their spouses survived to retirement at age 65.

4. After retirement each member has a yearly probability of death taken from the age and sex specific ONS life tables. Anyone over 100 has the annual mortality rate of a 100 year old (of their sex), with no-one surviving over 120.

5. When a member dies and has a surviving spouse, their pension is paid out at a 50% reduced rate until the spouse dies.

6. The dynamics of the pension pot after retirement are unchanged from the simple model, except for the step changes in pension payout rate.

An implementational sanity check is that the simulated average age at death corresponds to the life expectancy at 65 reported by the ONS for both sexes. It does.

Note that survival of everyone to retirement overstates liabilities, as in reality some members will be widowed by retirement. Similarly assuming 100% marriage overstates liabilities. Assuming only heterosexual partnership is likely to be a minor effect (some under estimation of female female liabilities offset by some overestimation of male-male).

3 Results

I set things up for someone joining at age 25 and retiring at 65, and on the basis of HESA data assumed a 40% female membership of the scheme (changing this to 50% makes very little difference). I based the marriage age gap on ONS Population Trends No 114 (left figure below) and mortality on the ONS National life tables, UK: 2014 to 2016 (below right, black male, red female).

In the absence of any real data I used the following inflation adjusted salary profile. Sensitivity analysis on the simple model without dependence suggests rather little sensitivity of the fund break even point to this assumption.
I then searched for the break even growth rates above inflation and the growth rate above inflation at which the fund never stops growing, even after retirement. I did this for two contribution rates: 26% of salary, as now, and 21% of salary, which is the figure without the ‘deficit reduction’ top up. The results are shown below, where the expected pension pot trajectory (over \( n = 50,000 \) replicates) is shown for various parameter combinations (the expectation is over the differing death profiles, of course). So at 21% contribution, break even requires the fund to outperform inflation by 1.85%, while at 26% contribution it needs to manage 1.1% above inflation. For unchecked growth we need 3.5% or 3% fund growth above inflation (these latter figures are rather robust to longevity assumptions — for example you get the same values without simulating to account for spouses).

4 Conclusions

The fund will be in surplus if it achieves growth of 2% over inflation, massively so if it achieves 3%. Hence the deficit appears artificial when considered against actual USS performance of 6% above inflation for the last decade. However the buffer is less extreme looking at the 20 year performance which was only 3.3% above inflation, as a result of low returns in the 10 years before the crash, followed by a 30% loss in one year as a result of the crash (giving inflation + .5% on average). The 2017 valuation assumes returns of 1% below CPI inflation over the coming decade.

It would be interesting to know whether this simple model really misses something that would substantially alter this conclusion.

Some notes:

1. The assumption of 100% marriage overstates liabilities.
2. Neglecting death before aged 65% probably overstates liabilities, since some members will be widowed by 65. But it is unclear how big an impact death in service would have.
3. The life tables used are not USS member specific and do not account for predicted future increases in longevity (all of which weirdly neglect the antibiotic resistance problem).
4. The age difference at marriage distribution is a snapshot at one point in time, and there is some evidence that the distribution is wider for unmarried couples.

5. Obviously the model is somewhat ‘steady state’ in nature, in that it is not considering elimination of any genuine deficits. However given the break even figures and historical USS performance it is hard to see how these could have arisen.

6. The major thing missing is the distribution of length of service at retirement. People who joined a long time ago and then left are good for the scheme, but late joiners cost the scheme, because their pension pot has not had long enough accumulating above inflation returns.

5 Sensitivity to longevity shifts

I multiplied the annual mortalities by 0.6 yielding a mean age at death of about 89.3 (about 5 years more than current rates yield). This is a much better life expectancy at 65 than any country in the world at present, and as if all USS members were as long lived as Japanese women (the greatest longevity in the world). It corresponds to women having a better life expectancy than Japanese women and men having a much better life expectancy than Icelandic men (the world record holders). For comparison the ONS analysis gives about 2 years extra life expectancy at 65 to higher managerial and professional classes (their highest category), relative to the average.

The break even rates are then 2.22% and 1.53% fund growth above inflation for contribution rates of 21% and 26% respectively.